ELECTRICITY
AND
MAGNETISM

HIRST

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ELECTRICITY
AND
MAGNETISM

FOR ENGINEERING STUDENTS

by

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PREFACE TO FIRST EDITION

A sound knowledge of the elements of what is commonly called Electricity and Magnetism is essential to the electrical engineer; and a study of the subject should be begun before he enters the University or Technical College. For this purpose, however, most school textbooks are unsuitable; they are usually written from the physics rather than the engineering point of view, and in consequence include matter which is of little use to the electrical engineer, and omit much that is of importance to him.

As is implied in its title, the contents of the present volume are limited to those portions of the subject which form the basis of electrical engineering practice. Its primary object is to provide a textbook for students who are about to enter the Electrical Engineering Department of a University or Technical College; and it will suffice to carry them up to the standard of the Intermediate Examination in this subject. But it is hoped that it may be equally useful as a book of reference. It is the experience of every student that the memorization of formulæ does not help him much in solving any problem which differs at all from a standard type, and that the further he advances the more often he has to fall back on first principles. In the elementary stages, however, he rarely appreciates the importance of theoretical principles for which, at the time, he sees little practical application, so that he needs some book of reference, to which he may turn in later years.

Much thought has been given to the arrangement of the chapters and of the sections in each chapter so as to admit of an orderly development of the subject, although it has not been possible always to adhere to this. For instance, in order that the student may become familiar with the electric circuit, an early acquaintance with the practical units is necessary, and this prevents the development of these units from the absolute c.g.s. units in strictly logical sequence; but the cross references given should enable the student to follow this development at a later stage.

The nomenclature and symbols used throughout are those recommended by the British Standards Institution (B.S.S. 205 and 560). When there exist alternative names for a unit or property, no one term has been strictly adhered to since the student will meet with all of them in the course of his reading; but where there is a preference, this has been stated.

Apart from a brief reference in the opening chapters, the subject of electrostatics, which, although of great importance, is of little value to the student until he has reached a more advanced stage, has been relegated to the end of the book; and the last chapter has been designed to provide a foundation sufficient for the solution of many of the electrostatic problems which occur in electrical engineering.

Many of the examples are original, but a number have been taken from recent examination papers by kind permission of the Senate of the University of London and the Institution of Electrical Engineers, to whom the author wishes to express his thanks; the sources of all such questions are indicated in the text.

The author's warmest thanks are due to E. Tyler, Esq., D.Sc., F.Inst.P., for much helpful criticism in the manuscript stage, and for reading the proofs; and also to T. B. Worth, Esq., for checking the examples.

A. W. HIRST.

LEICESTER, *July, 1936.*

PREFACE TO THIRD EDITION

The general adoption of the rationalized M.K.S. system of units in the Engineering Departments of Universities and Colleges has made it desirable to bring the present volume into line with other textbooks.

The chapters dealing with circuits are little affected, but the aproach to electromagnetic and electrostatic theory differs considerably from that used with the classical C.G.S. system, so that considerable changes have been necessary in both matter and arrangement in these portions of the new edition.

The coming generations of students, brought up on the M.K.S. system, will be more fortunate than their elders, many of whom had to change systems in mid-course. Nevertheless, because of the wealth of technical literature expressed in C.G.S. units, the student will find that, as he advances, he must have some acquaintance with the C.G.S. system and the relation between corresponding expressions and units in the two systems. For this reason an appendix has been added, in which it is hoped that the C.G.S. units have been developed in as orderly and logical a manner as space permits.

The author's thanks are due to his colleague, D. Polak, Esq., B.Sc. (Tech)., A.M.I.E.E., for much helpful criticism.

Having regard to the extensive revision and re-arrangement which have taken place, it is almost too much to expect that every error has been detected, and the author will be grateful for information concerning any which may be discovered.

A. W. HIRST.

LEICESTER, *October*, 1958.

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LIST OF SYMBOLS

B, magnetic flux density (induction density).
C, capacitance.
D, electric flux density.
E, electromotive force
electric force.
F, magnetomotive force
force.
G, conductance.
H, magnetic field strength
magnetizing force.
I, current, moment of inertia.
J, intensity of magnetization.
L, self-inductance
length (in dimensions).
M, mutual inductance
magnetic moment
mass (in dimensions).
N, number of turns
speed (rev. per min.)
north.
P, power.
Q, quantity.
R, resistance
S, reluctance
south.
T, time (in dimensions)
period
torque.
V, potential difference
velocity.
W, work
energy
weight.

a, acceleration
cross-sectional area.
c, capacitance
number of circuits in parallel.
d, distance
diameter.
e, electromotive force.

f, force
frequency
linear acceleration.
g, acceleration due to gravity.
i, current.
k, radius of gyration.
l, length.
m, magnetic pole strength
mass.
n, number of turns
speed (revs. per sec.).
p, power
number of pairs of poles (in a machine).
q, quantity.
r, resistance
radius.
s, slip.
t, time
v, velocity
potential difference.
w, weight.
z, number of conductors in series.

α, resistance temperature coefficient
angular acceleration.
γ, conductivity.
δ, deflection.
ε (or κ), permittivity (specific inductive capacity or dielectric constant).
η, efficiency.
θ, angle.
μ, permeability.
ρ, resistivity.
σ, surface density.
Φ, magnetic flux.
φ, angle (particularly phase angle).
Ψ, electric flux.
ω, angular velocity.

CHAPTER I

Units and Mechanical Principles

This introductory chapter presents a summary of the common systems of units and of those mechanical principles which are employed in later chapters.

1. Fundamental and Derived Units

The magnitude of any physical quantity is measured in terms of a fixed amount of that particular quantity, which is called the *unit* of the quantity. For purely individual use the amount of this quantity which is taken as the unit might be quite arbitrary, but in order to facilitate the exchange of information, it is essential that the amount should be fixed at least nationally and, if possible, internationally.

The relationships between physical quantities are such that it is possible to select units for a few quantities known as *fundamental units* and then, by making use of these relationships, to derive from them the units of other quantities known as *derived units*. Such a system of units is called an *absolute system*, and the units are referred to as *absolute units*.

The units which are taken as fundamental are usually those of Length, Mass (or Force), and Time.

2. Dimensions of Units

The relation between the derived unit and the fundamental units is indicated by what is called the *dimensions* of the quantity. For example, the unit of area is derived from the unit of length and taken as the area of a square of which the sides are of unit length. Hence if length is denoted by $[L]$, the dimensions of area are $[L^2]$, and similarly the dimensions of volume are $[L^3]$. Velocity is measured by the number of units of length passed over in unit time, so that if $[T]$ denotes time, the dimensions of velocity are $\left[\frac{L}{T}\right]$ or $[LT^{-1}]$.

The equation showing the relationship between the derived unit and the fundamental units from which it is derived is called a dimensional equation, e.g.

$$[v] = [LT^{-1}].$$

The dimensions of each side of any equation expressing the relation between physical quantities must always be the same; this often serves as a valuable check on the correctness of an equation.

3. Systems of Units

The system of units which has been used almost universally for scientific purposes takes as the fundamental units of length, mass, and time, the centimetre, the gramme, and the second, and is known as the *centimetre-gramme-second* or, more shortly, the *C.G.S. system.*

An alternative system in which the units of length, mass, and time are the metre, the kilogramme, and the second, was put forward some fifty years ago as having certain advantages in the field of electrical engineering. This is known as the *metre-kilogramme-second system* or *M.K.S. system,* and in its *rationalized* form (see p. 419) was officially adopted by the International Electrotechnical Commission in 1950, and has superseded the C.G.S. systems for electrical engineering purposes. Most new textbooks are now written in M.K.S. units and they will be used in this book; but since almost all the technical literature of former years uses C.G.S. units, the serious student must be familiar with both systems.

The corresponding system of British units is based upon the foot, pound, and second as the fundamental units of length, mass, and time, and is known as the *foot-pound-second system* or *F.P.S. system.*

For general engineering purposes the *Gravitational system* is still commonly used. This takes as fundamental the units of length, *force,* and time. The units of length and time are the foot and the second, while the unit of force is the force with which the earth attracts a mass of one pound at one particular place—i.e. one *pound-weight*; hence, in this system the unit of mass is a derived unit.

The F.P.S. system has been little used by engineers but efforts are now being made to introduce it to replace the Gravitational System. It is consistent with the C.G.S. and M.K.S. systems and at the same time eliminates the confusion and possibility of serious error which arise from the loose use of the term pound for both mass and force.

The electrical engineer may have to make use of C.G.S., M.K.S., F.P.S. and Gravitational units and must be familiar with the relation between corresponding units in the four systems.

4. Fundamental Units

M.K.S. System.

Length $[L]$: The **metre,** originally intended to be one ten-millionth part of the quadrant of the earth measured from the pole to the equator.* The standard metre is the distance between two marks on a

* Later and more accurate methods have shown that the value of the units of length and mass does not conform exactly with these definitions, but this does not affect the validity or the utility of the system.

platinum-iridium bar deposited at the International Bureau of Weights and Measures at Sèvres near Paris.

Mass $[M]$: The **kilogramme.** The standard kilogramme is the mass of a platinum-iridium cylinder deposited at the International Bureau of Weights and Measures. (1 kilogramme contains 1000 grammes, and the gramme was intended to be the mass of 1 cubic centimetre of water at its maximum density.) (See footnote on p. 2.)

Time $[T]$: The **second.** This is the mean solar second, or 1/86,400 of the mean solar day, which is the *average* interval between successive transits of the sun across the meridian at any place during the course of a year.

C.G.S. System.

Length: the **centimetre.**
Mass: the **gramme.**
Time: the **second.**

F.P.S. System.

Length: the **foot,** which is $\frac{1}{3}$ of the distance between two marks on a platinum bar kept at the Standards Department of the Board of Trade.

Mass: the **pound,** the mass of a platinum cylinder kept at the Standards Department of the Board of Trade.

Time: the **second.**

British Gravitational System.

In this system, still commonly used by engineers, the units of length and time are the foot and the second respectively, but the third fundamental unit is taken as that of force.

Force: the **pound-weight,** the weight of a mass of one pound at sea-level and in latitude 45°.

1 inch = 0·0254 metre	1 ton = 1016 kilogrammes
1 mile = 1609·3 metres	1 metre = 39·37 inches
1 pound = 0·4536 kilogramme	1 kilogramme = 2·20 pounds

5. Derived Units

Derived units in common use are dealt with in the following sections and summarized in Table I, p. 13.

6. Velocity: $[v] = \left[\frac{L}{T}\right] = [LT^{-1}]$

Velocity is rate of motion and is measured by the distance travelled in unit time.

The M.K.S. unit is the **metre per second.**

The corresponding C.G.S. unit is the **centimetre per second** and the F.P.S. and Gravitational unit is the **foot per second.**

A common secondary unit is the **mile per hour.**

$$1 \text{ mile per hour} = 88 \text{ feet per minute} = 1{\cdot}47 \text{ feet per second} = 0{\cdot}45 \text{ metre per second}$$

The unit of angular velocity (ω) is the **radian per second.**

7. Acceleration: $[f] = \left[\frac{v}{T}\right] = [LT^{-2}]$

Acceleration is the rate of change of velocity and is measured by the change of velocity which takes place in unit time.

The M.K.S. unit is the **metre per second per second.**

The corresponding C.G.S. unit is the **centimetre per second per second** and the F.P.S. and Gravitational unit is the **foot per second per second.**

The acceleration due to gravity (g) varies slightly with latitude and height above sea-level. Its approximate value is

$$g = 9{\cdot}81 \text{ m. per sec. per sec.} = 32{\cdot}2 \text{ ft. per sec. per sec.}$$

A secondary unit in common use in traction is the *mile per hour per second,*

$$1 \text{ mile per hour per second} = 1{\cdot}47 \text{ ft. per sec. per sec.}$$

The unit of angular acceleration (α) is the **radian per second per second.**

8. Force: $[F] = [Mf] = [MLT^{-2}]$

Force is that which alters or tends to alter a body's state of rest or uniform motion in a straight line. (Newton's First Law of Motion.)

In the M.K.S., C.G.S., and F.P.S. systems it is measured in terms of the acceleration produced when acting on unit mass.

In the M.K.S. system unit force is that which gives unit acceleration (1 m. per sec. per sec.) to unit mass (1 kg.) and is called the **newton.**

The corresponding C.G.S. unit is that force which gives an acceleration of 1 cm. per sec. per sec. to a mass of 1 gm. and is called the **dyne.**

Since $1 \text{ kg.} = 10^3 \text{ gm. and } 1 \text{ m.} = 10^2 \text{ cm.}$

$$1 \text{ newton} = 10^3 \times 10^2 = 10^5 \text{ dynes.}$$

The F.P.S. unit is that force which gives an acceleration of 1 ft. per sec. per sec. to a mass of 1 lb. and is called the **poundal.**

In the Gravitational system force is taken as fundamental, the unit being a force equal to one pound-weight.

$$1 \text{ newton} = 10^5 \text{ dynes} = 7{\cdot}25 \text{ poundals} = 0{\cdot}225 \text{ pound-weight.}$$

9. Mass $[M]$

As stated in Sect. 4, in the M.K.S., C.G.S., and F.P.S. systems mass is taken as fundamental, the units being the kilogramme, the gramme, and the pound respectively.

In the Gravitational system mass is a derived quantity defined in terms of force and acceleration.

Unit mass is that mass to which unit force (1 pound-weight) gives unit acceleration (1 ft. per sec. per sec.).

Since a force of one pound-weight gives to a mass of one pound an acceleration of 32·2 feet per second per second, unit mass is 32·2 lb. Hence in the Gravitational system

$$\text{Mass} = \frac{\text{pounds weight}}{32{\cdot}2}.$$

10. Work $[W] = [FL] = [ML^2T^{-2}]$

The M.K.S. unit is the work done by a force of one newton acting through a distance of one metre, and is called the **joule.**

The C.G.S. unit is the work done by a force of one dyne acting through a distance of one centimetre, and is called the **erg.**

Since 1 newton = 10^5 dynes and 1 m. = 10^2 cm.,
1 joule = 1 newton-metre = 10^7 ergs.

The F.P.S. unit is the work done by a force of one poundal acting through a distance of one foot and is called the **foot-poundal.**

The Gravitational unit is the work done by a force of one pound-weight acting through a distance of one foot and is called the **foot-pound.**

1 joule = 10^7 ergs = 23·73 foot-poundals = 0·737 (0·74) ft.-lb.

11. Energy: $[W] = [ML^2T^{-2}]$

Energy is defined as capacity for doing work. The various forms of energy may be divided into two classes, *potential energy* and *kinetic energy.* For example, a body may possess:

Potential Energy.

1. Energy due to its position relative to other bodies or the earth's surface, e.g. a clock weight, or water in a mountain lake, etc.
2. Strain energy due to a distortion or displacement of its molecules, e.g. a coiled spring or a stretched wire.
3. Chemical energy, e.g. coal and other fuels, explosives, etc.

Kinetic Energy.

4. Energy due to its motion as a whole, e.g. a rifle bullet or a rotating flywheel.
5. Heat energy due to the motion of its molecules, e.g. steam.

Other forms of energy connected with electric and magnetic circuits will be considered later.

Since energy is capacity for doing work, its dimensions are those of work and it is measured in units of work, i.e. the foot-pound, the erg and the joule.

Heat Units of Energy.

Heat is a form of kinetic energy, and the relation between the common heat units and the mechanical units of energy was first determined experimentally by Joule, and is often known as *Joule's equivalent.*

The **gramme-calorie** is the amount of heat required to raise the temperature of one gramme of water by one degree Centigrade.

One gramme-calorie = 4·18 joules.

The **British Thermal Unit** (B.Th.U.) is the amount of heat required to raise the temperature of one pound of water by one degree Fahrenheit.

$$\text{One B.Th.U.} = 778 \text{ ft.-lb.} = 1054 \text{ joules.}$$

The **Centigrade Heat Unit** (C.H.U.) is the amount of heat required to raise the temperature of one pound of water by one degree Centigrade.

$$\text{One C.H.U.} = 1{\cdot}8 \text{ B.Th.U.} = 1400 \text{ ft.-lb.} = 1898 \text{ joules.}$$

A larger unit, the Therm, has recently come into use for certain purposes.

$$\text{One therm} = 100{,}000 \text{ B.Th.U.}$$

Electrical Units of Energy. (See p. 66.)

One joule = 1 watt-second = 10^7 ergs.

One Board of Trade unit (B.O.T. unit) = 1 kilowatt-hour

= $3{\cdot}6 \times 10^6$ joules = 1·34 horse-power-hour.

12. Kinetic Energy of a Moving Body

The relation between the mass of a body M, the force F acting on it, and the acceleration f produced is (Newton's Second Law of Motion)

$$F = Mf. \qquad \ldots \qquad (1)$$

Consider the simplest case of a body accelerating from rest under a constant force. The acceleration is

$$f = \frac{F}{M},$$

and at the end of a time T the velocity is

$$v = fT = \frac{F}{M}T,$$

so that

$$F = \frac{Mv}{T}.$$

Since the acceleration has been uniform the average velocity is half the maximum velocity, so that the distance travelled is

$$d = \frac{v}{2}T.$$

Therefore the work done by the force on the body during the time T is

$$W = Fd = \frac{Mv}{T} \cdot \frac{vT}{2} = \tfrac{1}{2}Mv^2. \qquad \ldots \qquad (2)$$

Neglecting frictional losses, all this work is stored in the body as kinetic energy; if the force is reversed, the retardation will have the same value as the acceleration, and the body will gradually give up its energy in overcoming the retarding force and finally come to rest in the same time and after travelling the same distance as in the acceleration period. Hence,

$$\text{Kinetic Energy of a moving body} = \tfrac{1}{2}Mv^2. \quad . \quad . \quad (3)$$

Example.—Calculate the kinetic energy of a car weighing 4000 lb. and travelling at 30 miles per hour.

If the brakes apply a retarding force of 1200 lb.-weight, how far will it travel before coming to rest?

Using F.P.S. units $M = 4000$ lb. $\left(\frac{4000}{32{\cdot}2}\text{ Gravitational units.}\right)$

$v = 44$ ft. per sec.

Kinetic energy $= \frac{1}{2}Mv^2 = \frac{1}{2} \times 4000 \times 44^2 = 3{\cdot}872 \times 10^6$ foot-poundals (120,000 ft.-lb.)

All this energy is dissipated in overcoming the retarding force, i.e.

$$1200 \times 32{\cdot}2 \times d = 3{\cdot}872 \times 10^6$$
$$d = 100 \text{ ft.}$$

13. Kinetic Energy of a Rotating Body. Moment of Inertia (I)

A rotating body possesses kinetic energy in virtue of the motion of its constituent particles, and the velocity of each of these depends upon its distance from the axis of rotation.

If the angular velocity of the body is ω, the velocity v of a particle of mass m at a distance r from the axis of rotation is equal to ωr, and its kinetic energy is

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}m\omega^2r^2,$$

and as the body is made up of a large number of particles, m_1, m_2, m_3, etc., situated at distances r_1, r_2, r_3, etc., from the axis of rotation (fig. 1), the total kinetic energy of the body is

$$\tfrac{1}{2}m_1\omega^2r_1^2 + \tfrac{1}{2}m_2\omega^2r_2^2 + \tfrac{1}{2}m_3\omega^2r_3^2 + \ldots,$$

or

$$W = \tfrac{1}{2}\omega^2(m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots). \quad . \quad . \quad . \quad (4)$$

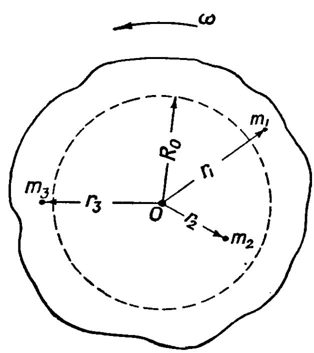


Fig. 1

The quantity in brackets is called the *moment of inertia* (I). In relation to rotational motion it is analogous to mass in relation to linear motion.

$$I = (m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots) = \Sigma(mr^2).$$

This may be written in the form

$$I = Mk^2$$

where M is the total mass; the length k is called the *radius of gyration* of the body about the axis of rotation, and is the radius at which the total mass might be concentrated without changing the moment of inertia.

Hence, from (4),

Kinetic energy of rotation
$= \frac{1}{2}$ (moment of inertia about axis of rotation) (angular velocity)^2,

or

$$W = \tfrac{1}{2}I\omega^2. \qquad (5)$$

The moments of inertia in a few common cases are given below:

1. *Solid cylinder, about axis of cylinder:*

$$I = M\frac{d^2}{8},$$

where d = diameter of cylinder.

2. *Hollow cylinder, about axis of cylinder:*

$$I = M\frac{(d_1^2 + d_2^2)}{8},$$

where d_1 = external diameter, d_2 = internal diameter.

3. *Cylindrical bar, about axis perpendicular to that of cylinder and passing through mid-point:*

$$I = M\left(\frac{l^2}{12} + \frac{d^2}{16}\right),$$

where l = length of cylinder, d = diameter of cylinder.

4. *Rectangular bar, about axis passing through centre of two opposite faces:*

$$I = M\left(\frac{l^2 + b^2}{12}\right),$$

where l and b are the lengths of adjacent edges of face through which axis passes.
M = mass in each case.

Example 1.—The moving system of an ammeter has a mass of 3 gm. and a radius of gyration of 1 cm. Determine the kinetic energy when the angular velocity is 6 radians per sec.

$I = Mk^2 = 3 \times (1)^2 = 3$ gm.-cm.^{2}
$\omega = 6$ radians per sec.
Kinetic energy $= \frac{1}{2}I\omega^2 = \frac{1}{2} \times 3 \times (6)^2 = 54$ ergs.

Example 2.—A flywheel consisting of a hollow cylinder having internal and external diameters of 4 ft. and 5 ft. weighs 4500 lb. Calculate the stored energy when it is rotating at 400 r.p.m.

$$I = \frac{M(d_1^2 + d_2^2)}{8} = 4500\left(\frac{25 + 16}{8}\right) = 23{,}050 \text{ lb.-ft.}^2$$
(716 gravitational units)

$$\omega = \frac{400 \times 2\pi}{60} = 41{\cdot}9 \text{ radians per sec.}$$

$$\text{Kinetic energy} = \tfrac{1}{2} \times 23{,}050 \times (41{\cdot}9)^2 = 20{\cdot}25 \times 10^6 \text{ foot-poundals}$$
$$= 629{,}000 \text{ ft.-lb.}$$

14. Power: $[P] = \left[\frac{W}{T}\right] = [ML^2T^{-3}]$

The power of an agent is measured by the rate at which it does work, i.e. by the amount of work done in unit time.

The M.K.S. unit of power is that of an agent which does work at the rate of **one joule per second** and is called the **watt.**

The C.G.S. unit of power is the **erg per second**

$$1 \text{ erg per sec.} = 10^{-7} \text{ watt.}$$

The F.P.S. unit is the **foot-poundal per second.**
The Gravitational unit is the **foot-pound per second.**

$$1 \text{ watt} = 23{\cdot}73 \text{ foot-poundals per second}$$
$$= 0{\cdot}737\ (0{\cdot}74) \text{ ft.-lb. per sec.}$$

$$\text{I horse-power} = 33{,}000 \text{ ft.-lb. per min.}$$
$$= 550 \text{ ft.-lb. per sec.} = 746 \text{ watts.}$$

15. Torque or Twisting Moment: $[T] = [FL] = [ML^2T^{-2}]$

A torque or twisting moment is the turning effect produced by a force about a given axis. When the line of action of the force is perpendicular to the axis, the torque is measured by the product of the force and the perpendicular distance between the axis and the line of action. For example, the torque produced by the force F about the axis through the point O perpendicular to the plane of the paper (fig. 2) is

$$T = Fr. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

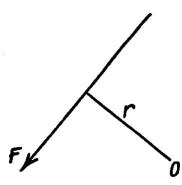


Fig. 2

Hence the dimensions of the unit of torque are the same as those of work or energy.

The M.K.S. unit of torque is the **newton-metre**: this distinguishes it from the unit of work which is always called the joule.

The C.G.S. unit is called the **dyne-centimetre**, which distinguishes it from the unit of work—the erg.

A secondary gravitational unit which is often more convenient is the **gramme-centimetre.**

$$1 \text{ gm.-cm.} = 981 \text{ dyne-cm.}$$

The F.P.S. unit is the **poundal-foot.**

The Gravitational unit is the *pound-foot,* which distinguishes it from the unit of work—the foot-pound.

$$1 \text{ newton-metre} = 0{\cdot}738\ (0{\cdot}74) \text{ lb.-ft.}$$

16. Angular Acceleration produced by a Torque.

The relationship between the moment of inertia (I) of a body, the torque (T) acting on it about a given axis, and the angular acceleration (α) produced about the axis is

$$T = I\alpha. \quad \ldots\ldots\ldots \quad (7)$$

It will be noticed that this is exactly similar to the relationship between the mass of a body, the force acting on it, and the linear acceleration produced.

Example.—The moving system of an electricity meter has a mass of 20 gm. and a radius of gyration of 2·5 cm. How long will it take to attain a speed of 100 r.p.m. under a torque of 0·5 gm.-cm.?

$$I = Mk^2 = 20 \times (2{\cdot}5)^2 = 125 \text{ gm.-cm}^2.$$

$$T = 0{\cdot}5 \text{ gm.-cm.} = 981 \times 0{\cdot}5 = 490{\cdot}5 \text{ dyne-cm.}$$

$$\therefore\ \alpha = \frac{T}{I} = \frac{490{\cdot}5}{125} = 3{\cdot}93 \text{ radians per sec. per sec.}$$

$$\omega = \frac{100}{60} \times 2\pi = 10{\cdot}47 \text{ radians per sec.}$$

$$\text{Time of acceleration} = \frac{10{\cdot}47}{3{\cdot}93} = 2{\cdot}66 \text{ sec.}$$

17. Relation between Torque, Work, Speed and Power

It is frequently necessary to calculate the work done per revolution and the power output from or input to a machine when the speed and the torque acting on the rotating member are known.

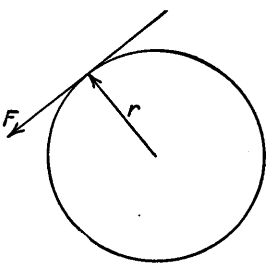


Fig. 3

Take the simple case of a pulley of radius r driven by a belt which exerts a tangential force F at the rim (fig. 3). The effect will be unaltered if the force is assumed to act at one point on the rim, which in one revolution travels a distance $2\pi r$. Hence

$$\text{Work done in one revolution} = 2\pi rF.$$

But $Fr = T$, the torque produced by F about the axis of rotation.

$$\therefore \text{ Work done per revolution} = 2\pi \text{ (torque)}. \quad \ldots \quad (8)$$

It follows that Torque $= \dfrac{\text{work done per revolution}}{2\pi}$, so that torque may also be defined as *Work done per radian.*

If T is the torque in newton-metres, and n is the speed in revolutions per second,

$$\text{Work done per revolution} = 2\pi T \text{ joules.}$$

$$\text{Power} = 2\pi nT \text{ joules per sec. or watts.} \quad . \quad . \quad . \quad (9)$$

If T is the torque in lb.-ft. and N is the speed in revolutions per minute,

$$\text{Work done per revolution} = 2\pi T \text{ ft.-lb.}$$

$$\text{Power} = \frac{2\pi NT}{33{,}000} \text{ horse-power.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Example 1.—Calculate (*a*) the power output, in watts and horse-power, of a motor which produces a torque of 26,300 lb.-ft. when running at 200 r.p.m., (*b*) the torque exerted by a motor running at 500 r.p.m. and producing a total power output of 100,000 watts.

$$(a) \text{ Power output} = \frac{2\pi \times 26{,}300 \times 200}{33{,}000} = 1000 \text{ h.p.}$$

$$26{,}300 \text{ lb.-ft.} = \frac{26{,}300}{0{\cdot}74} = 35{,}700 \text{ newton-metres.}$$

$$\text{Power output} = 2\pi \times 35{,}700 \times \frac{200}{60} = 746{,}000 \text{ watts.}$$

$$(b) \text{ Torque} = \frac{100{,}000}{2\pi \times 500/60} = 1910 \text{ newton-metres (1410 lb.-ft.).}$$

Example 2.—Calculate the energy stored in a flywheel in the form of a solid cylinder 9 ft. in diameter, weighing 12,000 lb., and rotating at 600 r.p.m.

How long will it take to come to rest under a constant retarding torque of 600 lb.-ft., and how many revolutions will it make during the retarding period?

$$\omega = \frac{600}{60} \times 2\pi = 62{\cdot}8 \text{ radians per sec.}$$

$$I = M\frac{d^2}{8} = 12{,}000 \times \frac{(9)^2}{8} = 121{,}500 \text{ lb.-ft.}^2$$

$$(= 3770, \text{ in gravitational units).}$$

$$\begin{aligned} \text{Kinetic energy} = \tfrac{1}{2}I\omega^2 &= \tfrac{1}{2} \times 121{,}500 \times (62{\cdot}8)^2 \\ &= 240 \times 10^6 \text{ foot-poundals} \\ &= 7{\cdot}45 \times 10^6 \text{ ft.-lb.} \\ &(= 13{,}540 \text{ horse-power-seconds).} \end{aligned}$$

Work done per revolution against retarding torque

$$= 2\pi \times 600 = 3770 \text{ ft.-lb.}$$

Number of revolutions in order to dissipate this energy

$$= \frac{7{\cdot}45 \times 10^6}{3770} = 1975.$$

$$\text{Average speed} = \frac{600}{2} = 300 \text{ r.p.m.} \quad \text{Time} = \frac{1975}{300} = 6{\cdot}58 \text{ min.}$$

18. Efficiency (η)

In all machines certain losses occur. In mechanical machines these are entirely frictional; in electrical machines there are, in addition, electrical and magnetic losses, but in both cases all the lost energy is converted into heat. Hence the output of a machine is always less than the input, and the ratio of the output to the input is called the *efficiency*.

$$\text{Efficiency} = \frac{\text{available power output}}{\text{power input}}$$

$$= \frac{\text{input} - \text{losses}}{\text{input}} \left(= 1 - \frac{\text{losses}}{\text{input}}\right)$$

or

$$= \frac{\text{output}}{\text{output} + \text{losses}}. \qquad \qquad (11)$$

Speaking more generally, losses occur whenever energy is converted from one form into another, whether through the agency of a machine or not.

For example, in the conversion of the chemical energy of coal into electrical energy, of the total energy available in one pound of coal about 10 per cent passes away in the hot flue gases and by conduction and convection, in spite of elaborate precautions which reduce these losses to a minimum; of the remainder which is passed on in the form of steam to the turbine, about 50 per cent is carried away by the condenser circulating water. Finally, of that which is transmitted mechanically through the shaft to the generator, some 5–8 per cent is absorbed in mechanical and electrical losses in the generator; so that the over-all thermal efficiency of even the most modern power stations seldom exceeds 30 per cent.

Example 1.—The torque on a steam turbine shaft is 40,000 lb.-ft., and the speed is 3000 r.p.m. If the mechanical efficiency is 90 per cent, determine the horse-power output.

$$\text{Horse-power input} = \frac{2\pi \times 40{,}000 \times 3000}{33{,}000} = 22{,}800.$$

$$\text{Output} = \text{input} \times \text{efficiency}.$$

$$\therefore \text{ Horse-power output} = 22{,}800 \times \frac{90}{100} = 20{,}500.$$

TABLE I

Mechanical Quantities and Units

Quantity	Symbol	Defining Equation in M.K.S. System	Dimensions	Units			
				M.K.S.	C.G.S.	F.P.S.	Gravitational
Length	L, l		L	metre	centimetre	foot	foot
Mass	M, m		M	kilogramme	gramme	pound	g pounds
Time	T, t		T	second	second	second	second
Velocity	v	$v = \frac{L}{T}$	LT^{-1}	m. per sec.	cm. per sec.	ft. per sec.	ft. per sec.
Acceleration	f	$f = \frac{v}{T}$	LT^{-2}	m. per sec. per sec.	cm. per sec. per sec.	ft. per sec. per sec.	ft. per sec. per sec.
Force	F	$F = Mf$	MLT^{-2}	newton	dyne	poundal	pound-weight
Work, Energy	W	$W = FL$	ML^2T^{-2}	joule	erg	foot-poundal	foot-pound
Power	P	$P = \frac{W}{T}$	ML^2T^{-3}	watt	erg per sec.	foot-poundal per sec.	foot-pound per sec.
Angular velocity	ω			radian per sec.	radian per sec.	radian per sec.	radian per sec.
Angular acceleration	α			radian per sec. per sec.	radian per sec. per sec.	radian per sec. per sec.	radian per sec. per sec.
Moment of inertia	I		ML^2	kilogramme-metre^2	gramme-cm.^{2}	pound-ft.^{2}	

Example 2.—The average output of a steam turbine is 20,000 h.p., and the coal consumption is 9 tons per hour. If the heating value of the coal is 11,000 B.Th.U. per lb., calculate the over-all thermal efficiency.

$$\text{Thermal efficiency} = \frac{\text{Equivalent heat output per hour}}{\text{Heat input per hour}}.$$

$$\begin{aligned}
\text{Energy output} &= 20{,}000 \times 33{,}000 \text{ ft.-lb. per min.} \\
&= 20{,}000 \times 33{,}000 \times 60 = 396 \times 10^8 \text{ ft.-lb. per hour.} \\
\text{Output in heat units} &= \frac{396 \times 10^8}{778} \\
&= 51 \times 10^6 \text{ B.Th.U. per hour.} \\
\text{Input in heat units} &= 9 \times 2240 \times 11{,}000 \\
&= 221{\cdot}8 \times 10^6 \text{ B.Th.U. per hour.} \\
\text{Thermal efficiency} &= \frac{51 \times 10^6}{221{\cdot}8 \times 10^6} = 23 \text{ per cent.}
\end{aligned}$$

19. Electrical and Magnetic Units

It has been shown in previous sections that the units of all mechanical quantities can be derived from the fundamental units of length, mass and time; and that the system most commonly used for scientific purposes has been the C.G.S. system in which the fundamental units are the centimetre, the gramme and the second.

In the case of electrical or magnetic phenomena a fourth concept is required. This is of a purely electrical nature, and usually involves a property—either the permeability or the permittivity—of the medium in which the phenomena occur. In the C.G.S. system these two properties, in the case of free space, were both arbitrarily assigned the value unity; but since the relation between them is such that they cannot both have the same value, this assumption gave rise to two systems of units—the electromagnetic C.G.S. system and the electrostatic C.G.S system. Of these the electromagnetic system was found to be the more generally useful, but since the size of the units was thought to be inconvenient, multiples or sub-multiples were used as *Practical Units*, e.g. 1 ampere $= 10^{-1}$ e.m. C.G.S. units, 1 volt $= 10^8$ e.m. C.G.S. units. (In contrast, 1 coulomb $= 3 \times 10^9$ e.s. C.G.S. units and 1 volt $= \frac{1}{300}$ e.s. C.G.S. units.)

In 1904 it was pointed out by Professor Giorgi in Italy, and almost simultaneously and quite independently by Professor David Robertson in England, that if the fundamental units were taken as the *metre*, the *kilogramme* and the *second*, and the permeability of free space was assigned a value of 10^{-7}, the system of units resulting—the Metre-Kilogramme-Second or M.K.S. system—would include all the Practical Units, i.e. these would then become absolute units in their own right instead of being multiples or sub-multiples of the C.G.S. units.

TABLE II

Electrical and Magnetic Quantities and Units

e.m. = electromagnetic C.G.S. units   e.s. = electrostatic C.G.S. units

Quantity	Symbol	Defining Equation	Dimensions	M.K.S. Unit and Relation to C.G.S. Unit
Current	I	$F = I_1 I_2/(\mu d^2)$	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}$	Ampere (A) = 0·1 e.m. unit
Quantity	Q	$Q = It$	$M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}$	Coulomb (C) = 0·1 e.m. unit
Electromotive Force	E	$E\,(\text{or } V) = W/Q$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}\mu^{\frac{1}{2}}$	Volt (V) = 10^8 e.m. units
Difference of Potential	V			= $\frac{1}{300}$ e.s. units
Resistance	R	$R = E/I$	$LT^{-1}\mu$	Ohm (Ω) = 10^9 e.m. units
Resistivity	ρ	$\rho = Ra/l$	$L^2T^{-1}\mu$	Ohm-metre
Magnetomotive Force	F	$F = NI$	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}$	Ampere-turn (AT) = $4\pi/10$ gilberts
Magnetizing Force	H	$H = F/l$	$M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}$	Ampere-turn per metre = $4\pi/10^3$ oersteds
Magnetic Flux	Φ	$e = -\,d\Phi/dt$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}\mu^{\frac{1}{2}}$	Weber (Wb) = 10^8 maxwells (e.m. lines)
Flux Density	B	$B = \Phi/a$	$M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}\mu^{\frac{1}{2}}$	Weber per metre² = 10^4 gauss
Reluctance	S	$S = F/\Phi$	$L^{-1}\mu^{-1}$	Ampere-turn per weber
Permeance	Λ	$\Lambda = \Phi/F$	$L\mu$	Weber per ampere-turn
Permeability	μ	$\mu = B/H$		Henry per metre = $10^7/4\pi$ e.m. units
Inductance	L	$e = -\,L\,di/dt$	$L\mu$	Henry (H) = 10^9 e.m. units
		$L = N\Phi/I$		
Electric Charge	Q		$M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}$	Coulomb = 3×10^9 e.s. units
Electric Flux	Ψ	$\Psi = Q$	$M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}$	Coulomb = $12\pi \times 10^9$ e.s. units
Electric Flux Density	D	$D = \Psi/a$	$M^{\frac{1}{2}}L^{-\frac{3}{2}}\mu^{-\frac{1}{2}}$	Coulomb per metre² = $12\pi \times 10^5$ e.s. units
(Displacement)				
Electric Force	E	$E = V/l$	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-2}\mu^{\frac{1}{2}}$	Volt per metre = $1/(3 \times 10^4)$ e.s. units
(Intensity)				
Permittivity	ε	$\varepsilon = D/E$		Farad per metre = $36\pi \times 10^9$ e.s. units
Capacitance	C	$C = Q/V$	$L^{-1}T^2\mu^{-1}$	Farad (F) = 9×10^{11} e.s. units

Permeability of free space (magnetic space constant) $\mu_0 = 4\pi \times 10^{-7}$ henry per metre.

Permittivity of free space (electric space constant) $\varepsilon_0 = \dfrac{1}{36\pi \times 10^9} = 8{\cdot}85 \times 10^{-12}$ farad per metre.

Further, if the permittivity of free space were then given its correct value relative to that of the permeability, the electromagnetic and electrostatic systems would be included in one comprehensive system.

This suggestion lay almost dormant for many years, but in 1935 the M.K.S. system was adopted unanimously by the International Electrotechnical Commission. This at first had little effect, one reason being that no decision had been reached on the important question of *rationalization* (see p. 419). Progress in general was retarded, although in particular fields it was accelerated, by the war. However, the Rationalized M.K.S. system was adopted by the International Electrotechnical Commission in 1950 and officially recognized by the Institution of Electrical Engineers in 1952; and from that date both the Institution and the University of London, together with other examining bodies, have gradually introduced M.K.S. units into their examination papers.

CHAPTER II

Electricity and Matter

1. Definition and Origin of the term Electricity

The study of a new subject is usually prefaced by a formal definition of that subject. Electricity is a term which cannot be defined in any precise and formal manner. The dictionary describes it as "*a powerful physical agent which makes its existence manifest by attractions and repulsions, by the production of light, heat, chemical decomposition and other phenomena*".

In other words, electricity can be defined only in terms of its effects. All knowledge of the laws by which it is governed is based on experimental evidence; and all theories as to its nature are attempts to explain observed facts. The many advances which have been made during the present century lead to the conclusion that all matter consists solely of electricity; so that the simplest definition in a very general form might be obtained by inverting the order of the terms, i.e. *everything is electricity.*

The modern view of the nature of electricity and its relation to matter is best illustrated by recalling some of the earliest-known experiments. Some 2500 years ago the Greek philosopher Thales discovered, possibly accidentally, that a piece of amber, when rubbed, acquired the property of attracting light bodies such as small pieces of cork or pith. About 2000 years later, in the sixteenth century, the English philosopher Gilbert found that other bodies, such as sulphur and glass, behaved in the same manner. He attributed their behaviour to what he called electric force (from *electron,* the Greek word for amber); and bodies possessing this property were spoken of as being electrified or charged with electricity.

2. Attraction and Repulsion of Charged Bodies

Later it was found that, under certain conditions, forces of repulsion as well as of attraction could be produced; and this led to the assumption that there were two kinds of electricity having opposite properties.

It is worth while discussing briefly some of these simple experiments, as they form an introduction to the modern electrical theory of the structure of matter.

A piece of ebonite, after being rubbed with fur, *attracts* small pieces of cork or pith, but *repels* a second piece of ebonite which has been electrified in a similar manner, as can be shown by suspending one piece by a silk thread, and bringing the other piece up to one end of it. Now whatever the nature of the process of electrification may be, it is clear that both pieces of ebonite must be charged with the *same* kind of electricity; so that experimental observation gives ground for the statement:

Bodies charged with electricity of the same kind exert forces of repulsion on each other.

A glass rod, when rubbed with silk, is also capable of attracting pieces of cork or pith; and if two glass rods electrified in the same manner are brought near each other, similar repulsive effects are observed, thus giving further support to the above statement. But if one of the pieces of electrified ebonite is brought near one of the electrified glass rods, *attraction* takes place. Now both these bodies are known to be electrically charged, and, as has already been observed, charges of *like* kind *repel* each other. Hence whatever the nature of the electric charge may be, it is clear that the charge on the ebonite is, in some way, different in character from that on the glass rod. These conclusions are summed up in the statement:

Bodies charged with electricity of different kinds exert forces of attraction on each other.

A brief expression of both these statements is contained in the fundamental law:

Like charges repel each other; unlike charges attract each other.

3. The Ether

The question at once arises: how does one charged body exert a force on another charged body at a distance from it? For many years this and other allied phenomena were explained in terms of the *ether*. This is an imaginary medium endowed with the properties required to fit in with observed facts; it is, for instance, assumed to be imponderable, perfectly elastic, and to fill all space.

Certain more recently observed phenomena, however, cannot be interpreted in terms of an ether, even when endowed with such convenient properties; and no attempt will be made here to explain the mechanism by which electric or magnetic forces are produced. The laws by which they are governed can be determined quite definitely without any such knowledge, so that their existence will be accepted simply as an experimental fact.

4. The Two Kinds of Electricity

The kind of electricity which appears on the glass rod (§ 2) is called *positive* electricity, and that on the ebonite *negative* electricity. These names, as originally given, were merely meant to express the opposite nature of the charges, and their allocation was quite arbitrary.

Further investigation shows that the charged ebonite *attracts* the fur with which it has been rubbed, and that the glass rod attracts the silk. On the other hand, the fur *repels* the glass rod, and the silk repels the ebonite. Now the glass rod is said to be positively charged so that the charge on the fur must also be positive, since the two repel each other. For the same reason, if the charge on the ebonite is called negative, that on the silk must also be negative. Therefore, when the ebonite is rubbed with fur a *negative* charge appears on the ebonite and a *positive* charge on the fur; this conclusion is further supported by the evidence of attraction between them. Similarly a positive charge appears on the glass and a negative charge on the silk. So, generally:

Whenever the rubbing together of two bodies produces electrification, charges of opposite sign are produced on each; and it can be shown that the magnitudes of the two charges are equal.

These observations gave rise to the theory that all uncharged bodies contain equal quantities of positive and negative electricity, the effects of which neutralize each other; and that the process of electrification is one of partial separation, rather than creation of electrical charges. During the process, some of the electricity of one kind is transferred from one body to the other so that the condition of equilibrium is upset. For reasons which will be explained later, it is assumed that it is the negative electricity which is thus transferred. The body to which it is transferred has an excess of negative electricity and therefore exhibits negative electrification; while the one from which it is transferred is left with an excess of positive electricity and so exhibits positive electrification.

5. Electrons and Protons

There is abundant experimental evidence that electricity is not continuous in structure, but is built up of separate entities in the form of almost inconceivably small particles. The elementary particle of negative electricity is called the *electron*, and the corresponding element of positive electricity the *proton*. The charge of the proton is of equal magnitude but opposite sign to that of the electron. All negative charges are built up of integral numbers of electrons, and all positive charges of integral numbers of protons. A body containing equal numbers of electrons and protons is electrically neutral.

The electron has the extremely small mass of 9×10^{-28} gm. The proton is a much more massive structure, its mass being 1840 times greater than that of the electron. The methods by which the charge and mass of electrons and protons have been measured are quite outside the scope of this work, but their determination is one of the most important achievements of modern physics.

It is clear from the figures given above that the electron is far more mobile than the proton, and for this reason it may be expected that transfer of electricity consists, in the main, of movements of electrons; the body to which they are transferred exhibits negative electrification due to an excess, and the body from which they are transferred possesses positive electrification due to a deficiency of electrons.

6. Structure of Matter

Until early in the present century all assumptions as to the structure of matter were based on the molecular and atomic theories propounded by Dalton in 1801. Matter is not continuous in structure but is built up of small entities called molecules, a molecule being defined as the smallest portion of a substance which can exist in the free state. Further, each molecule is composed of one or more atoms. An atom is the smallest portion of a substance which can take part in a chemical reaction, and may or may not be able to exist in the free state. All molecules of the same substance and all atoms of the same element were assumed to be equal in weight and similar in structure.

Chemical theory is based on these hypotheses, but they do not explain how the atoms are held together in the molecule nor why the different elements possess different properties. The discovery of the electron and proton, however, has greatly extended our knowledge of the structure of the atom. Experimental and theoretical evidence largely due to Rutherford and Bohr, led to the view that all atoms were composed wholly of electrons and protons. In other words the basic constituents of all matter are the same, and the distinctive properties of the various elements are due to differences in the number and arrangement of the electrons and protons of which the molecule is made up.

According to the Rutherford-Bohr theory (1914) the neutral atom contains equal numbers of electrons and protons and therefore possesses no resultant charge. The protons are concentrated at the centre along with a certain number of electrons, and form what is called the *nucleus*, while the remaining electrons revolve in one or more orbits or *shells* round the nucleus, the whole forming a miniature planetary system.

If, by some means, a neutral atom gains one or more electrons, the resultant charge on the atom is negative. The loss of electrons on the other hand leaves an excess of protons and results in a positive charge.

The discovery, by Chadwick in 1932, of the *neutron,* which has the same mass as the proton but possesses no charge, led to some modification of the Rutherford-Bohr atom. It is now held that the nucleus consists entirely of protons and neutrons. The number of units of charge is equal to the number of protons, and (since the mass of the electrons is so small) the atomic weight is almost equal to the sum of the protons and neutrons.

The lightest of all the atoms, and the simplest in structure, is that of hydrogen, which consists of one proton with one electron revolving round it. The oxygen atom (atomic weight 16) contains a nucleus of 8 protons and 8 neutrons surrounded by 8 electrons, while the nucleus of uranium (atomic weight 238) contains 92 protons and 146 neutrons, surrounded by 92 electrons.

The number of protons in the nucleus determines the characteristics of the element, but the number of neutrons may vary somewhat. This accounts for the fact that the atomic weights of certain elements are not whole numbers. Chlorine, for example, has an atomic weight of 35·5. This is because there are two slightly different types of chlorine nucleus, one containing 17 protons and 18 neutrons and the other 17 protons and 20 neutrons, the two occurring in such a proportion as to give a resultant atomic weight of 35·5. Such variations are called *isotopes*, and it has been found that most elements have several isotopes. At the present time the best known example is uranium. The commonest form, as described above, has an atomic weight of 238, but there are other forms, one of which has a nucleus containing 92 protons and 143 neutrons (atomic weight 235). This isotope is particularly suitable for nuclear fission and has attained great importance in connection with the atomic bomb and as an atomic fuel.

Although the atom is extremely small, the size of its constituent particles is so much less that they are separated by relatively large distances. A solid must therefore be conceived as a sponge-like structure, in which the electrons and nucleus occupy but little of the total space taken up.

7. Conductors and Insulators

All the materials so far considered have been non-metals. A metal rod, held in the hand and rubbed with silk, showns no sign of electrification, although if held by a handle of glass, ebonite or similar material and submitted to the same process, it is found to possess the properties associated with charged bodies.

The ease with which an electron can be removed from or added to an atom differs in different substances. In such materials as glass and ebonite, the process takes place only with difficulty. Friction between the glass and the silk tears away an electron from some of the surface atoms of the glass which are therefore left positively charged, but this charge is confined to the spot where it was produced and has little effect on the surrounding atoms. Such materials are called *insulators* or *dielectrics*.

On the other hand, in certain substances the electrons in the outer shell are able to break away very easily, so that there are always present a certain number of *free* electrons, which for simplicity can be imagined as moving about inside the solid somewhat like the molecules of a gas. These free electrons are therefore at liberty to move under the influence of electric forces, which, under suitable conditions, may superimpose on their random motion a directive effect. Such materials are called *conductors*, and a progressive motion of the electrons through the solid constitutes an *electric current*. Only electrons possess this freedom: experimental evidence shows that the denser protons are always firmly anchored to the atoms of which they form the nucleus.

Although in a good insulator there are no (or very few) free electrons so that no appreciable electric current can flow through it, the application of electric forces causes a displacement of the electrons relative to the protons in the atom, thus giving rise to effects which will be discussed later.

The terms conductor and insulator are to some extent relative, and there is no sharp line of distinction between a poor conductor and a poor insulator. Further, there are no perfect conductors nor perfect insulators, although perfection is more nearly reached in the latter class. All metals are good conductors, while non-metals are either very poor conductors or insulators; although they may become conductors at high temperatures, glass for example. In general, good conductors of electricity are good conductors of heat and vice versa.

When a metal rod held in the hand is rubbed, electrons are transferred to it from the rubber, just as in the case of ebonite and fur, but since the metal is a very good conductor and the human body a fair conductor, the excess of electrons immediately passes away through the body to the earth. The addition of an *insulating* handle confines the electrons to the rod, which remains negatively charged; but as they are free to move inside the metal, they distribute themselves over the whole surface and the charge is not confined to the spot where it was produced, as in an insulator.

In addition, certain liquids, chiefly solutions of acids, salts and bases, act as moderately good conductors (although the process of conduction is different); while other liquids, notably oils, are extremely good insulators. Gases are normally classified as insulators, but under certain conditions act as conductors (see Chap. XV).

8. The Electric Current

Although the modern conception of an electric current is that of a stream of electrons, great advances had been made in the study of electricity and in its applications before the discovery of the electron; and so far as electrical engineering, in a general sense, is concerned, a knowledge of the mechanism of conduction is interesting rather than essential. Early scientists explained their observations by supposing that electricity was an immaterial fluid, which flowed through a conductor as through a tube. The nature of the flow was unknown and it was necessary to explain it in terms of something which was familiar. Use was therefore made of what is often called the *hydraulic analogy*, i.e. that of water flowing through a pipe. This analogy, if not pressed too far, is of great assistance in visualizing the electric circuit and can be used to give at least a superficial explanation of some of the phenomena associated with it. In spite of more detailed and accurate knowledge, it is probable that most electrical engineers still think, even if subconsciously, of the electric current in a conductor as flowing like water through a pipe.

On the other hand, in certain branches, such as the study of the thermionic valve, X-rays and other vacuum phenomena, it is essential to consider the motion of the electrons.

9. Direction of Flow

Just as water flows from a higher level to a lower level, the electric current was imagined to flow from a point of higher electric level, or *potential*, to a point of lower potential, the body with the positive charge being assumed to be at the higher potential. Hence when oppositely charged bodies were brought into contact, it was considered (and still is) that an electric current flowed from the positively charged body to the negatively charged body.

An electron, however, being negatively charged, is repelled by a negative charge and attracted by a positive charge; it therefore moves from *negative to positive*; so that what is commonly termed an electric current in one direction is composed of a stream of electrons moving in the *opposite* direction.

For all general purposes, the direction of flow is immaterial, as long as everyone is agreed upon the same direction; hence it is still customary to assume that the direction of flow of the current in a circuit is from the positive terminal to the negative terminal.

CHAPTER III

Electricity and Energy

1. Association of Electricity with Energy

The statement sometimes made that "*electricity is a form of energy*" is incorrect, at least if the word energy is used in its normal sense. Water, for example, is not, in the generally accepted sense, a form of energy; but it may possess energy. Its position may give it potential energy; its temperature or velocity may endow it with kinetic energy, due in the former case to the motion of the molecules within the liquid and in the latter case to the motion of the liquid as a whole. So also electricity can be associated with energy, and, from an engineering point of view, electricity is well defined as "*a means of converting energy from one form to another and of conveying energy from one point to another*".

In the previous chapter it was shown that experimental evidence leads to the conclusion that all matter is composed solely of electricity. A metal rod therefore consists of electricity, but in its neutral state, when the quantities of opposite kinds of electricity are equal, it possesses no available energy in the electrical sense, just as when lying on the floor it possesses no available energy in the mechanical sense.

By lifting the rod on to the table it can be endowed with potential energy of position, all of which can be transformed into kinetic energy by allowing it to fall to the floor. By rubbing the same rod, fitted with an insulating handle, partial separation of the two kinds of electricity takes place, causing the rod to become electrically charged; and in virtue of this charge it possesses potential energy.

If two such rods charged with opposite kinds of electricity are connected by a conductor such as a piece of wire, the excess of electrons on the negatively charged rod will travel along the wire to supply, either partially or completely, the deficiency of electrons in the positively charged rod. Thus, by means of the electric current, the potential energy of the charge has been transformed into the kinetic energy of the heat produced in the wire.

Electrical engineering is mainly concerned with electricity in motion, although the study of phenomena associated with electricity at rest (charged bodies) is becoming of increasing importance. (See Chapter XIV.)

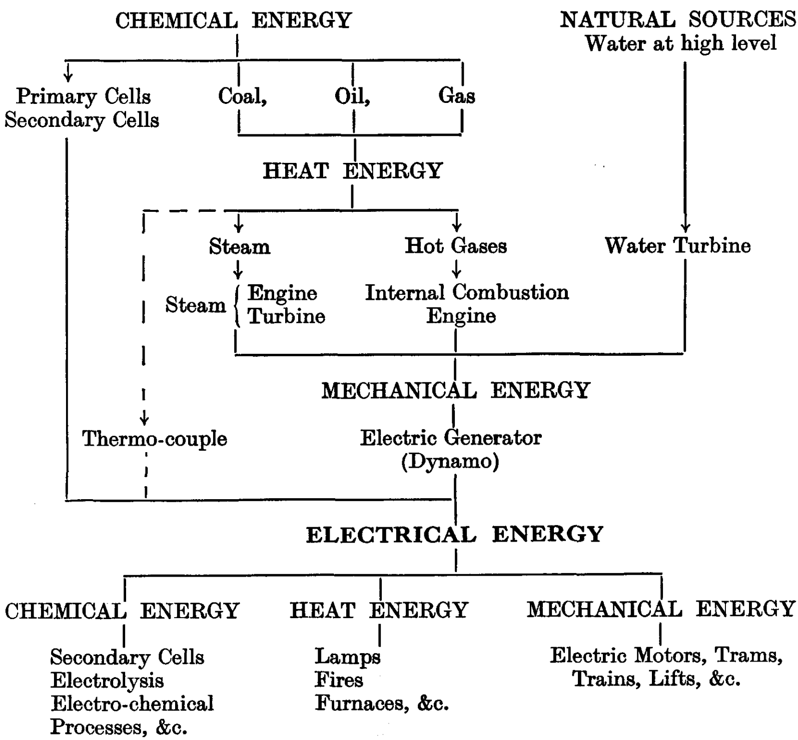
2. Energy Transformations

Transformation of energy from one form to another is ceaselessly going on in the world around, and is so much a part of everyday life that the process is often not recognized. Man himself is continuously converting the potential energy stored chemically in food into various forms of kinetic energy, such as the heat energy which maintains his body at a constant temperature, or the mechanical energy evidenced by his various movements.

The potential energy of coal is converted into kinetic heat energy in the domestic fire or in the boiler furnace. In the latter case the heat energy is transferred to the water and, in the form of steam, is conveyed to the steam engine or steam turbine, in which it is converted into mechanical energy. In an internal-combustion engine the conversion of the chemical energy of the fuel into heat energy and thence into mechanical energy takes place inside the engine cylinder.

Mechanical energy, however produced, can then, by means of a

TABLE I



dynamo, be converted into electrical energy; and electrical energy can be reconverted very easily into any other form. Some of the common sequences of energy transformation are shown in Table I.

3. Advantages of Energy in the Electrical Form

From the table shown above it will be seen that, in order to obtain heat energy from an electric fire instead of directly from a lump of coal, four transformations of energy are required. Each such transformation entails certain losses, particularly in the conversion of the heat energy of steam into mechanical energy, where even under the best conditions some 70 per cent of the energy is lost. Yet the tendency in modern practice is to convert greater and greater quantities of the chemical energy of coal into electrical energy for subsequent reconversion into other forms, rather than to use it more directly.

The reason for this is that electrical energy is the most convenient of all forms of energy from the point of view both of transmission and utilization. Only in this form can energy be conveyed conveniently and efficiently over great distances. This property alone has made possible the utilization of immense quantities of energy, chiefly in the form of water power, obtained from natural sources which would otherwise have continued to run to waste simply because they are situated at great distances from the nearest point at which the energy can be used.

There are several ways in which energy can be transmitted over short distances with considerable efficiency, e.g. by belts, shafting, and air or water under pressure; each of these methods has its own particular advantages, but the range of all of them is limited. It is only necessary to imagine the difficulties in the erection and maintenance of a shaft 100 miles long or even of a compressed-air main of the same length to realize the impossibility of using such methods on any but a small scale.

On the other hand, by means of three wires suitably supported on poles or towers, energy sufficient for the needs of a large town can be transmitted with high efficiency over a distance of many miles. Further, the ease with which electrical energy can be converted into other forms just when and where required, and the convenience and delicacy with which the conversion can be controlled, make certain processes feasible which would otherwise be out of the question, and often enable economies to be effected which far outweigh the cost of the losses involved in the additional energy transformations.

For general industrial purposes, the use of the electric motor, either for the individual drive of large machines or the group drive of a number of small machines, increases efficiency and decreases maintenance costs by the elimination of much of the shafting and belting; the ease and convenience of control increase output; while the control gear can be fitted with safety and other automatic devices impossible with any other form of drive.

For traction, the electric motor with its large starting torque and consequent high acceleration, coupled with the absence of fumes, enables heavy and frequent suburban services to be maintained, either as surface tramway or underground railway systems, which would be impossible with any other form of motive power. It has also solved the problem of main line service in countries where there are long tunnels or gradients (as in Switzerland, Norway, U.S.A., etc.) and where water power is plentiful and coal expensive.

Electric light, owing to its flexibility, absence of fumes, and the ease with which it can be applied and controlled under varied conditions, has no rival for domestic, industrial and artistic purposes.

Electrical energy is very largely used in metallurgical processes, both for providing intense heat in electric furnaces and in electrochemical processes such as electroplating and refining.

Medical uses of electricity are continually increasing, not only in the form of X-rays in diagnosis and therapy but also as a means of nerve stimulation and heat treatment for curative purposes.

Finally, as far as present knowledge goes, electricity offers the only means of communication over long distances, by the transmission of sound or vision, either with or without the aid of wires.

4. Electromotive Force

It is impossible to consider the motion of a body without assuming the existence, as a cause of the motion, of some kind of *urge* which, in mechanics, is called a *force*. Similarly some kind of urge is necessary in order to cause a progressive motion of the electrons, and to this urge was given the name *electromotive force* (E.M.F.). Hence an E.M.F. exists in every circuit in which a current is flowing. It is important to note, however, that an E.M.F. is not a *force* in the ordinary mechanical sense. A more exact definition will be given later.

In mechanics there are certain forces which cause motion and others which oppose it. In order to cause motion to take place, the sum of the applied forces must exceed that of the opposing forces by the amount necessary to overcome friction; and the total expenditure of energy or work done is the sum of the energy expended in overcoming the opposing forces and that converted into heat by friction.

Similarly in an electric circuit there are two kinds of E.M.F., those which cause the motion of the electrons constituting the current and those which oppose it; the latter are usually called *back-E.M.F.s* or *counter-E.M.F.s*. In order to cause a current to flow, the sum of the *forward E.M.F.s* must exceed the sum of the counter-E.M.F.s by the amount required to overcome the resistance of the circuit.

The existence of an E.M.F. always indicates that an energy transformation will take place if the circuit is completed so that a current flows. A forward E.M.F. indicates a transformation from some other form *to* the electrical form, while a counter-E.M.F. indicates a transformation *from* the electrical form to some other form. The total expenditure of energy is the sum of the energy expended in overcoming the counter-E.M.F.s (which is converted into other forms depending on the nature of the counter-E.M.F.) and that converted into heat due to the resistance of the circuit. The magnitude of the E.M.F. is measured in terms of the quantity of energy received or given out by each unit of electricity. This subject is discussed more fully in Chapter VI.

CHAPTER IV

Effects of an Electric Current and their Applications

In this chapter some of the effects produced by an electric current, and their practical applications, are described. The current is assumed to be flowing along a conductor, but no precise inquiry is made as to its origin. The conductor forms part of a complete circuit in which an E.M.F. of some kind exists, but the methods by which such E.M.F.s are produced are discussed in later chapters.

1. The Effects of an Electric Current

In fig. 1 there are shown:

(*a*) A glass jar containing water to which a little acid has been added, into which dip two carbon or platinum plates. Such an arrangement is known as an *electrolytic cell*, or *voltameter*, and the plates are termed *electrodes*.

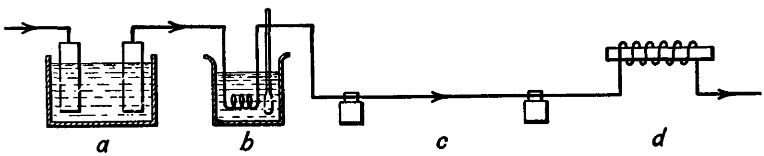


Fig. 1.—The effects of an electric current

(*b*) A beaker containing water in which a coil of wire and a thermometer are immersed.

(*c*) A straight length of wire stretched with its length pointing approximately north and south, underneath which is placed a small compass needle (see fig. 2).

(*d*) A long coil of insulated wire.

These four pieces of apparatus are connected together by suitable conductors so that an electric current passes through each in turn in the direction shown by the arrows.

If an examination is made while the current is flowing, the following facts may be observed:

(*a*) In the electrolytic cell, bubbles of gas rise from each plate, but much more rapidly from one than from the other.

(*b*) The thermometer indicates an increase in the temperature of the water; the temperature continues to rise and, if the current is large enough, the water will eventually boil.

(*c*) The straight wire is not visibly affected, but the small compass needle, which was previously pointing in the direction of the length of the wire, is deflected so that its axis is inclined to that of the wire by an angle which depends upon the strength of the current (fig. 2).

(*d*) One end of the compass needle, brought near to each end of the coil in turn, is attracted in one case and repelled in the other; the coil, in fact, behaves like a weak bar magnet. The effect is greatly increased if an iron rod is placed inside the coil, and the rod becomes magnetized sufficiently strongly to pick up small pieces of iron and steel.

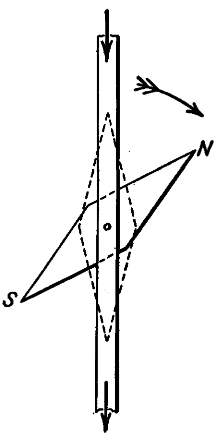


Fig. 2.—Magnetic effect of an electric current

Thus the same electric current is capable of producing effects which may be divided into three classes:

(1) Chemical effects, as in (*a*).

(2) Heating effects, as in (*b*).

(3) Magnetic or mechanical effects, as in (*c*) and (*d*).

Most uses of electricity, in spite of their apparent variety, are applications of one or more of these effects. The heating and magnetic effects are universal, i.e. no current can flow without evidence of them, although by suitable arrangement the effects may be increased, reduced, or even in some cases neutralized. The chemical effect takes place only in liquid conductors, and can be eliminated when not required.

2. Chemical Effects (see also Chapter VIII)

In the foregoing experiment (*a*), if the bubbles of gas rising from each electrode are collected and examined, the gas which is evolved the more rapidly can be shown to be hydrogen; the other is oxygen. These gases must have come from the water, so that the passage of the current has apparently caused some kind of chemical action to take place.

In contrast with solid conductors, a non-metallic liquid conductor or *electrolyte* does not allow a current to pass without undergoing chemical change; it is split up into two portions, which appear respectively at the two electrodes by which the current enters or leaves the liquid. This phenomenon is known as *electrolysis*. In some cases the constituent portions are liberated in the free state, but very often secondary reactions occur immediately. When water (H_2O) is electrolysed, no such reactions appear to occur,* and both its constituents, hydrogen and oxygen, are liberated in the free state.

Electrolytes consisting of solutions of metallic salts are split up into a metallic and a non-metallic portion, which are liberated at the electrodes. If copper sulphate ($CuSO_4$) is substituted for water, a film of pure copper is deposited on the plate from which the hydrogen was previously evolved, while at the other a secondary reaction takes place and oxygen is liberated. Similarly, if a solution of a silver salt is used, silver is deposited and oxygen evolved.

Further, if, in the case of a salt of a metal, an electrode of the *same* metal is substituted for the carbon plate at which oxygen is evolved, this gas is no longer liberated. Instead the metal gradually goes into solution, one gramme of the metal being dissolved for every gramme which is deposited on the other plate. Under these conditions electrolysis is a process by which the metal is transferred from one plate to the other.

3. International Units of Quantity and Current

Faraday showed, experimentally, that the weight of any particular metal so deposited by a given quantity of electricity (i.e. a given number of electrons) is always the same, being quite independent of such conditions as the temperature or the strength of the solution. Thus, quantity of electricity can be measured in terms of the weight of metal deposited. The accurate measurement of quantity in this way can be carried out comparatively easily, and although, officially, the units of quantity and current are defined electromagnetically, it is often convenient to define them electrolytically in terms of the weight of silver deposited from a solution of silver nitrate (§§ 8, 9, pp. 59, 60).

4. Practical Applications of Chemical Effects

Electroplating.

Electrolysis has been used for many years for the purpose of depositing a layer of one metal upon another, either for decorative or preservative purposes, the metals most commonly used being gold, silver, chromium, nickel, zinc, and copper. Gold and silver plate consist of articles of baser metals covered with a thin film of the noble metal. Many parts of motor-cars and cycles are plated with nickel (or more recently chromium), not only to enhance their appearance, but also to preserve them from corrosion.

The process is carried out in a large tank containing a solution of a salt of the metal to be deposited, in which are hung plates of the

* This is not strictly true. See § 7, p. 122.

same metal which forms the electrodes through which a current is passed into the solution. The articles to be plated are also immersed in the bath and so connected that they serve as the electrodes through which the current leaves the solution, as shown diagrammatically in fig. 3. The metal is thus transferred from the plates to the articles, and the process is continued until a layer of sufficient thickness has been deposited.

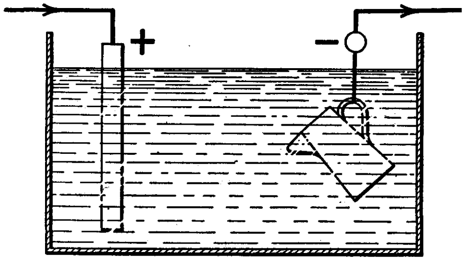


Fig. 3.—Electroplating bath

Electrotyping.

This is a similar process extensively used for obtaining accurate metallic copies (usually in copper) of objects in either relief or intaglio. A cast of the object is taken in some plastic material such as paraffin-wax or papier mâché, and the surface is rendered conducting by covering it with a thin film of graphite. The cast is then hung in a plating bath and metallic copper deposited over the whole surface. When the required thickness has been obtained, the cast is broken away, leaving an exact metallic copy of the original.

The process is largely used in the printing industry. Type, as produced in linotype or monotype machines, soon becomes worn. When a large edition is required, electrotypes are made from the original in copper, and can be used for printing a very large number of copies without noticeable deterioration.

In the manufacture of gramophone records, an electrotype master record is made from the original, and from this all subsequent copies are obtained.

Electrolytic Extraction and Refining.

This process is used in the extraction of metals such as copper, zinc and aluminium, from their ores. Almost all aluminium is obtained by the electrolysis of the ore in the molten state between carbon electrodes.

A similar process is used in the refining of gold, silver and zinc, and more particularly copper, in which a very small percentage of impurity greatly reduces its conductivity, so that further refining is necessary if it is to be used for electrical purposes. In addition, the impurities may contain traces of silver and gold which are worth recovering.

In the electrolytic process, bars of crude copper and thin plates of pure copper are hung in a bath of copper sulphate, and pure copper is transferred from the former to the latter by passing an electric current through the solution. The impurities are left behind and either dissolve in the solution or fall to the bottom of the tank. By this process copper may be obtained with a purity as high as 99·8 per cent.

Other Electrolytic Processes.

Hydrogen and oxygen are produced commercially in large quantities by the electrolysis of water.

Similar processes are used for the production of sodium carbonate, sodium bicarbonate, caustic soda, bleaching powder, etc.

5. The Heating Effect

Whenever an electric current passes through a conductor, heat is produced at a rate which depends upon the magnitude of the current and the dimensions and material of the conductor (see Chap. VII). For certain purposes the conversion of electrical energy into heat is definitely required, but in all other cases such conversion represents a loss of energy, and must be made as small as is economically possible, If the section and material of the conductor are suitable for the current which it is to carry, the rate of heat production is small and the heat is carried away by the surrounding air with little or no appreciable rise in temperature. On the other hand, if the section is much reduced or a material of inferior conducting properties is used, heat is produced more rapidly, the temperature of the conductor rises, and it may become red-hot or even melt.

6. Practical Applications of the Heating Effect

Fuses.

The possibility of raising the temperature of a conductor to fusing point, by the passage of an electric current, is used for the protection of both domestic and industrial electrical installations from the effects of abnormally large currents, which might occur as the result of a fault. A short portion of each circuit consists of a wire which is thinner than that in the rest of the circuit, and often of a material of inferior conducting properties and of lower melting-point. This wire, which is suitably enclosed, is called a *fuse*; it is capable of carrying the normal current continuously, but any abnormal increase raises its temperature and, if the overload is maintained, causes it to melt and break the circuit.

The Electric Lamp.

The most common source of artificial light is still the incandescent lamp. This consists essentially of a glass bulb containing a filament of tungsten wire, which is heated to incandescence by the passage of a current. The proportion of the energy radiated as light depends upon the working temperature. This has been gradually increased during recent years from the comparatively low temperature possible with carbon filament lamps, through higher values in the metal filament vacuum lamp, to the enormous figure of about 2600° C. in the gas-filled lamp.

The Electric Furnace.

The efficiency of the heat engine cycle is so low that not more than about 20 per cent of the heat energy of the equivalent coal is available in the electric furnace, even under the best conditions. Nevertheless, the ease with which the heat can be localized and the temperature controlled, the high temperatures attainable (up to 3000° C.), and the absence of products of combustion, make the electric furnace more economical and more efficient than any other type of furnace for many metallurgical and chemical processes, such as the smelting and refining of iron, steel, aluminium and zinc, and the manufacture of calcium carbide, carborundum, graphite, phosphorus, etc.

There are several different types of furnace, and the heat is produced in various ways—by heating elements in the furnace walls; by an arc between carbon electrodes; or by the circulation of eddy currents induced electromagnetically in the body of the metal to be heated, when it is placed near a coil carrying an alternating current (see § 10, p. 151).

Electric Welding.

Welding is the process of uniting two pieces of metal by fusion. The requisite high temperature at the junction is produced either electrically or by the oxyacetylene flame.

In resistance welding, the metallic surfaces to be joined are pressed together and a large current is passed across the joint. In the more widely used process of arc welding, the two pieces are held in position, and a metal filler in the molten state is deposited between them by the electric arc.

The development of welding has revolutionized many constructional methods. In particular, it has led to the replacement of complicated castings, e.g. the frames of motors and generators, by structures built up of mild steel plates with welded joints. Such structures are lighter and stronger than castings, and are cheaper to produce, owing to the saving of the time and expense involved in pattern-making, moulding and casting.

Domestic Appliances.

Most electrical domestic appliances are applications of the heating effect produced by the passage of a current through a spiral or coil of wire. In the electric fire, the spirals of wire are raised to a bright red heat, which is distributed by direct radiation, and by conduction and convection to the surrounding air. The electric cooker is in effect a low-temperature electric furnace, the heating elements being built into the walls. In the electric iron and electric kettle the heating coils are solidly enclosed in the base or, in the latter case, immersed in the water. The immersion heater consists of a heating element, usually thermostatically controlled, immersed in the hot-water cylinder in such a position that circulation is aided by convection currents in the water.

7. The Magnetic Effect

A compass needle in the neighbourhood of a conductor experiences a force which causes it to be deflected when a current flows (fig. 2, p. 30). Similar effects are observed when the needle is placed near a bar magnet, and are attributed to the *magnetic field* surrounding it (see § 14, p. 47). Hence a conductor carrying a current appears to be surrounded by a magnetic field, which must be due to the current, since there is evidence of it only when a current is flowing. This magnetic effect, like the heating effect, can be termed universal, for every electron, when in motion, sets up a magnetic field; although, by suitable arrangement, the magnetic forces produced may be, to some extent, controlled or even neutralized.

If the straight wire in § 1, p. 29, is bent into the form of a circle with its plane in the magnetic meridian, and the compass needle placed at its centre (fig. 22, p. 161), the magnetic forces causing deflection are much increased. Such a piece of apparatus is termed a *galvanometer*, and may be used in order to compare the magnitude of different currents; indeed, the C.G.S. unit of current is defined in terms of the magnetic force produced at the centre of such a loop (see p. 403).

An iron core placed inside the coil in fig. 1 (p. 29) behaves as a magnet when a current is passed through the coil, but loses most of its magnetism when the current ceases. Such a magnet is called an *electromagnet*. It can be made much more powerful than a permanent magnet, and the temporary nature of its properties is, in practice, one of its most valuable characteristics.

8. Practical Applications of the Magnetic Effect

The operation of all electrical machinery is due to the application of the magnetic effect in one form or another, but only a few simple examples are given here.

The Electric Bell.

The electric bell (fig. 4 *) is a familiar example in which the temporary character of an electromagnet is an essential feature.

* This type is illustrated rather than the more modern and compact under-dome type, as being simpler and more familiar.

When the circuit is completed by pressing the button, the soft iron cores *A* become magnetized by the current passing through the coils which surround them, and attract the soft iron armature *B* which causes the hammer to strike the bell; at the same time this movement causes the interruption of the circuit at *C*. When the current ceases, the cores lose almost all their magnetism, and the armature, returning to its initial position under the influence of the spring *D*, completes the circuit; the cycle is then repeated.

The Lifting Magnet.

Large electromagnets are used extensively for lifting iron and steel in the form of plates, girders, pig iron, scrap iron, etc., which in many cases would be difficult to handle in any other way.

At the other end of the scale large electromagnets are also used in hospitals and factories for removing pieces of iron or steel from the eye.

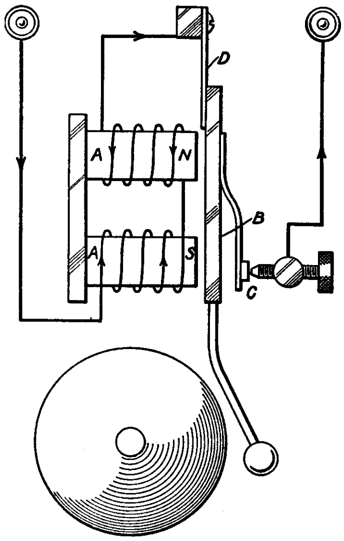


Fig. 4.—Electric Bell

The Magnetic Brake.

In hilly districts electric trams are fitted with magnetic track brakes. These are electromagnets of the shape shown in fig. 5, suspended on springs between the wheels and normally hanging just clear of the track. When a current passes through the coil, the magnet is pulled down on to the rail and acts as a powerful brake, directly through friction between the pole pieces and the rail and indirectly by applying the brake shoes to the wheels.

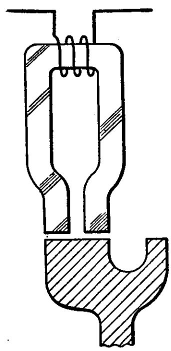


Fig. 5.—Magnetic track brake

Motors and Generators.

These constitute by far the most important application of the magnetic effect. A detailed description of their operation is given in Chapter XIII; only a few remarks will be made here.

In a motor, the magnetic forces set up by the passage of a current through the coils of the winding produce a torque which causes the movable portion of the machine (called the armature or the rotor) to rotate; electrical energy is thus converted into mechanical energy.

In a generator (or dynamo, which is the older term), the armature is caused to rotate *against* this torque by some external means; and in this way mechanical energy is converted into electrical energy. The design of a motor and that of a generator are essentially the same, and in many cases the same machine may be used in either capacity as required.

Miscellaneous Examples.

The following are additional examples of the use of the magnetic effect of an electric current:

(i) Magnetic chucks and couplings; solenoids for operation of points and signals; contactors (magnetic switches) in control gear.

(ii) Motor-car magnetos and ignition coils; induction coils (§ 25, p. 175) for generation of X-rays; transformers (§ 23, p. 337).

(iii) Telephone receivers; loudspeakers; gramophone pick-ups.

CHAPTER V

Magnetism

1. Magnetism and Electromagnetism

In the older textbooks, magnetism, which is concerned chiefly with the properties of permanent magnets, was treated as a separate subject quite distinct from electromagnetism, which deals with the magnetic effects of electric currents. The discovery of the electron, and advances which have since been made in our knowledge of the structure of the atom, show that *all* magnetic effects are due to the *motion of electrons*, whether this takes place as a progressive movement along a conductor or as a rotation or spin inside the atom. Thus magnetism and electromagnetism are of common origin, and should be looked upon as merely different sections of the same subject.

The electrical engineer is mainly concerned with electromagnetism. Nevertheless a brief treatment of the properties of permanent magnets is essential and also serves as an introduction to some of the terms and concepts used in connection with the study of electromagnetism.

2. Natural and Artificial Magnets

It was known in very early times that certain iron ores (magnetite, Fe_3O_4) possessed the property of attracting small pieces of iron; and pieces of this ore were called *magnets*, from Magnesia, a district in Asia Minor, in which it was first discovered. Later it was found that such a *natural* magnet, when freely suspended, came to rest with its longer axis pointing approximately north and south. The importance of this property as an aid to navigation was soon recognized, and gave rise to the alternative name of *loadstone* (from an Anglo-Saxon word meaning way or course).

Natural magnets are now of little practical importance; for most purposes *artificial* magnets are more convenient. Magnetic properties may be developed in pieces of iron or steel either by suitable treatment with natural or artificial magnets, or by means of an electric current (§ 12, p. 45). A straight bar or rod of magnetized steel is called a *bar magnet*; another common form, in which the bar is bent so that the ends approach each other, is known as a *horseshoe magnet*. The magnets used in electrical instruments (see Chapter X) are of the latter type.

3. Properties of Magnets

The common properties of magnets are so familiar that they will only be briefly enumerated.

(1) A magnet attracts pieces of iron or steel, but has no apparent effect on other materials such as brass, wood, glass, etc. The force of attraction increases rapidly as the piece of iron approaches the magnet. If the magnet is dipped into iron filings, it can be clearly seen that this force is greatest in the neighbourhood of the ends known as the *poles* and negligible near the middle of its length.

(2) Pieces of unmagnetized iron or steel, when brought near a magnet, exhibit magnetic properties, e.g. the power of attracting iron filings, most or all of which disappear when the magnet is removed; they are said to be *magnetized by induction*. No such effect is apparent in other materials.

(3) A bar magnet, pivoted or suspended so that it is free to turn in a horizontal plane, at a distance from all other magnets or pieces of iron, comes to rest with its length lying along a line directed approximately north and south. If the ends of the magnet are marked, it is found that, whatever the initial position, the magnet finally comes to rest with the same end pointing towards the north.

(4) If one end of a second bar magnet is brought near each end of the suspended magnet in turn, a force of attraction is observed in one case and a force of repulsion in the other.

4. Magnetic and Non-magnetic Materials

The observations (1) and (2) in the preceding paragraph show that materials may be divided roughly into two classes. Those which can be magnetized or exhibit magnetic properties when in the neighbourhood of magnets are called *magnetic materials*; and those which exhibit no such properties are called *non-magnetic materials*. Among magnetic materials only iron and steel are of importance for engineering purposes; a few other substances, notably nickel and cobalt, show similar effects, but to a much smaller degree.

It will be seen later that this rough classification can be considerably extended, but it is sufficient for present purposes (see § 1, p. 210).

5. Poles of a Magnet

The localities, near the ends of a magnet, where the magnetic properties appear to be concentrated are known as the *poles* of the magnet. The pole which, when the magnet is freely suspended, as in § 3, points towards the north is called the *north-seeking*, or more shortly the *north* pole; the other is the *south* pole.

The pole of a magnet is a region, not a definite point, but for purposes of calculation it is generally assumed to be located at a point near the end of the magnet; and the straight line joining the poles is known as the *axis* of the magnet. If the length of the magnet is great compared with its diameter, e.g. length/diameter $= 100$, it can be assumed with little error that the poles are situated at the ends.

It will be shown later that the two poles of any magnet are always of *equal* strength.

6. Attraction and Repulsion

The observations made in §3, (3) and (4), may now be considered in more detail.

Suppose that each of the two bar magnets is suspended, in turn, away from other magnets and masses of iron, and that, after the magnet has come to rest, the pole which points to the north is marked. On repeating the experiment described in § 3, (4), it is found that repulsion takes place between the two north poles and between the two south poles, but that there is attraction between a north and a south pole.

Now, whatever the nature of magnetism may be, it can reasonably be assumed that the kind of magnetism possessed by each of the north poles is the same; and similarly that both south poles possess magnetism of the same kind. Experiment shows that two poles *similarly* magnetized set up mutually *repulsive* forces; hence the *attractive* forces existing between a north and a south pole must be due to the fact that their magnetism is of *different* kinds. This conclusion is embodied in the fundamental statement:

Like poles repel each other; unlike poles attract each other.

It will be noticed that this law is identical in form with that governing the forces between charged bodies obtained in § 2, p. 18.

7. The Earth as a Magnet (see also § 19, p. 51)

The directional properties of a suspended magnet may be explained by assuming that the earth acts as a magnet having poles which are situated near the geographical poles. These poles exert forces of attraction or repulsion on the poles of every magnet on the earth's surface, and those which are free to move swing round under the influence of these forces and finally settle in a definite position of equilibrium. There is no tendency for the magnet to *move as a whole* in any direction, which shows that the poles of each magnet, though of opposite kind, are of equal strength. This can be demonstrated by placing a bar magnet on a piece of cork floating in a bowl of water; the magnet *turns* until it points north and south, but does not move as a whole in any direction.

It may be noted here that, since unlike poles attract each other, the pole of a magnet which points to the north must possess magnetism of *unlike* kind and should properly be called a south pole. The term north pole therefore must be regarded as an abbreviation of north-seeking pole, and not as a description of the kind of magnetism which it possesses relative to that possessed by the earth's pole of the same name. In any case the choice of terms is arbitrary, so that as long as one convention is universally accepted, the nature of the convention is unimportant.

8. The Compass Needle

The observations of § 3, (3) and (4), p. 39, and many other experiments can be carried out much more conveniently if the suspended bar magnet is replaced by a compass needle. This consists of a thin piece of hard steel, strongly magnetized and with the north- and south-seeking poles plainly distinguished; the needle is carefully balanced

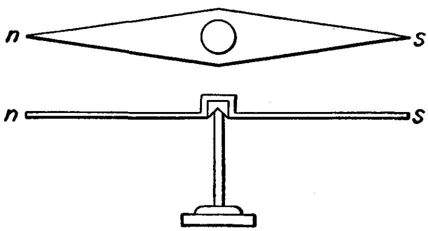


Fig. 1.—Compass needle

and fitted at the centre with a glass or jewelled bearing which rests on a hard steel pivot, as shown in fig. 1. Such needles are made in various lengths, depending upon the purpose for which they are to be used, and form a convenient means of ascertaining polarity, of making rough comparisons of pole strength, etc.

9. The Molecular Theory of Magnetism

The magnetic properties of a bar magnet appear to be concentrated chiefly in the neighbourhood of its ends; but if such a magnet is broken in halves,* that portion of the bar near the centre, which previously showed little or no evidence of magnetic properties, immediately develops a polarity, which can be shown, by means of a compass needle, to be of equal strength but opposite kind to the existing pole (fig. 2). If each of these halves is broken, the same effect is observed, and however far the subdivision is carried, each portion becomes a magnet having north and south poles of equal strength. *It is therefore impossible to isolate a single pole.*

In view of these facts, it is not unnatural to imagine that if the

* It is obviously impossible to break an ordinary bar magnet, but the experiment may be carried out with a hardened knitting needle or a piece of clock spring.

process of subdivision were carried on indefinitely, the final breakage would result in the separation of the molecules, each of which would act as a small magnet.

On this assumption, Weber and Ewing built up the *molecular theory of magnetism*, which in its present form accounts for most of the pro-

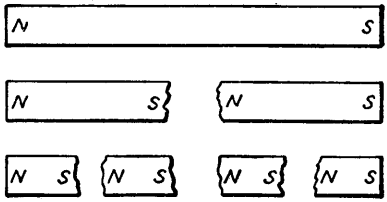


Fig. 2.—Effect of breaking a magnet

perties of magnetic materials. Each molecule of such a material is imagined to contain one or more small magnets which are capable of movement within the molecule. In the unmagnetized state the molecular magnets are arranged in groups, held together by the magnetic forces between them, which neutralize each other so far as external effects are concerned: such a grouping is indicated in fig. 3*a*, in which

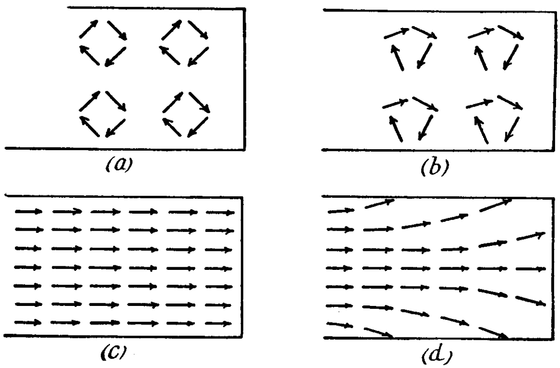


Fig. 3.—Molecular theory of magnetism

the north poles are distinguished by arrowheads. The possibility of such an arrangement is shown in fig. 4. Imagine a steel ring divided into six segments, all of which are equally magnetized. Each segment by itself possesses the usual properties of a bar magnet; but if the six segments are fitted together so that everywhere north and south poles are in contact, they will be held firmly in position by the magnetic forces between them but will exhibit no polarity or external magnetic effects.

The process of magnetization consists in " lining up " or orienting these molecular magnets so that the general direction of their axes is more nearly parallel to that of the bar (fig. 3*b*). There is then a preponderance of north poles pointing towards the right-hand end, and of south poles pointing towards the left-hand end. At the centre of the bar, the effect of each individual pole is neutralized by that of neighbouring poles of opposite polarity, as was the case with the poles at the ends of each segment of the ring in fig. 4; but, as the ends of the bar are approached, the extent of the neutralization decreases, and the cumulative effects of all the north poles increase until the right-hand end is reached; similarly, the cumulative effects of the molecular south poles produce south polarity at the left-hand end.

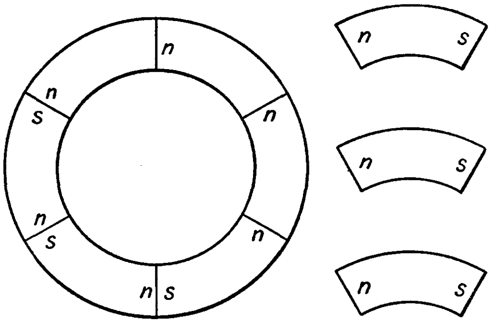


Fig. 4.—Molecular theory of magnetism

Evidently, when all the molecular magnets have been turned so that their axes are parallel to that of the bar, the limit of magnetization has been reached, and the pole strength cannot be increased further. This is found to be the case in practice, and the iron is then said to be *saturated* (fig. 3*c*). Such a state of saturation can be produced by surrounding the iron by a coil of wire (similar to that shown in fig. 1*d*, p. 29) through which an electric current is passing, but it cannot be maintained permanently. As soon as the magnetizing force is removed, the mutually repulsive forces between similar poles cause a partial rotation of the molecular magnets near the ends of the bar, so that the arrangement becomes more like that shown in fig. 3*d*. This also accounts for the observed fact that the poles are not located definitely at the ends, and that evidences of polarity can be observed for a considerable distance from each end towards the centre of the bar.

Weber, to whom the molecular theory was originally due, assumed, without attempting to explain, that the molecular magnets were actually permanent magnets. Ampère, with greater insight, assumed that they were due to currents flowing continuously inside the atoms. Modern ideas of the structure of the atom are in agreement with Ampère's hypothesis, and suggest that these molecular currents are due to the motion of electrons about the nucleus and about their own axes, and that the magnetic effects are due chiefly to the latter, i.e. the electron spin.

In the most recent theories of magnetism these elementary magnets are regarded as being of a size which is much greater than that of a molecule, although still small compared with that of a crystal of the material. These aggregates of molecules are known as *domains*, and the process of magnetization consists in the orientation of these domains rather than of the individual molecules.

The term *molecular magnet* has, however, been retained as being a simple concept and sufficient for the present purpose.

10. Magnetic Induction

Any account of the molecular theory of magnetism gives, almost automatically, an explanation of the process of magnetization by induction. If one pole of a permanent magnet (say the south pole) is brought near to the end of an unmagnetized bar, the repulsive force acting on the molecular south poles and the attractive force acting on the molecular north poles produce torques which cause partial orientation of the axes of the molecular magnets (fig. 5). Hence the bar exhibits north polarity at the end next to the south pole of the permanent magnet, and south polarity at the farther end. When the magnet is removed, the molecular magnets swing back to some extent under the influence of internal forces but do not resume completely their original grouping; the extent of their return depends upon the completeness of the previous orientation and upon the material.

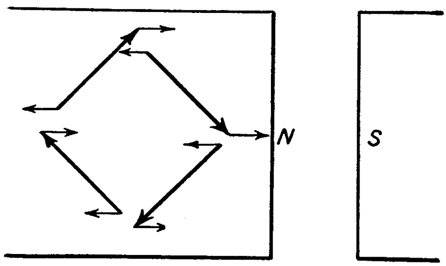


Fig. 5.—Magnetic induction

In soft iron, the molecular magnets move comparatively freely under the influence of an external magnet, but return almost completely to their original groupings when this is removed, i.e. soft iron is easily magnetized by induction but loses almost all its magnetism when the inducing magnet is removed. As will be seen later, this property makes soft iron particularly suitable for certain parts of electrical machines, and its use in the electric bell (p. 35) has already been mentioned. In hard steel, on the other hand, more powerful external magnetic forces are required to produce the same degree of orientation; but once deflected, the molecular magnets swing back to a much smaller extent when the external force is removed, i.e. hard steel is more difficult to magnetize but retains a large portion of its magnetism, and is therefore a suitable material for permanent magnets.

This explanation, though admittedly somewhat superficial, is sufficient for present purposes; the subject is considered in more detail in Chapter XI.

The process by which a magnet attracts an unmagnetized piece of iron can now be followed. The approach of one pole of a permanent magnet magnetizes the piece of iron by induction, so that a pole of opposite kind is produced in the portion nearest the inducing pole and one of the same kind in the farther portion (fig. 6). The iron is now subjected to two forces, one of attraction due to its induced south pole and the other of repulsion due to the induced north pole; but as magnetic forces decrease very rapidly with increase of distance, the repulsive force due to the farther pole is much smaller than the attractive force of the nearer pole, so that the resultant force is one of attraction. The process may be extended from one piece of iron to another, i.e. one which is itself magnetized by induction can induce magnetism in another. In fig. 7 a nail is shown hanging from the north pole of a bar magnet. At its lower end a north pole is induced which will induce a south pole at the upper end of a second nail suspended from it, and a north pole at the lower end. If the bar magnet is strongly magnetized,

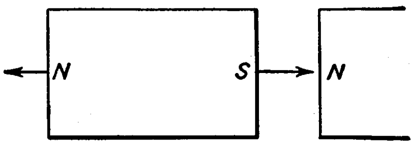


Fig. 6.—Attraction of unmagnetized piece of iron by induction

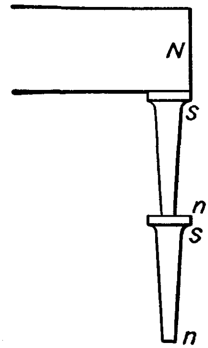


Fig. 7.—Experiment to illustrate magnetic induction

the process may be repeated several times, although the strength of the induced poles becomes progressively less until the attractive force is insufficient to support the weight of the nail.

11. Repulsion as Evidence of Permanent Magnetism

From the preceding section, it follows that the existence of a force of attraction between two pieces of iron away from other magnets shows that one of them is magnetized, but not necessarily both; the other may be magnetized by induction. But if, on presenting in turn each end of one of the pieces to one end of the other, *repulsion* occurs in one case, both pieces must be *permanently* magnetized, since under no conditions can *similar* polarity be induced at the adjacent end.

12. Methods of Magnetizing a Piece of Steel

The facts mentioned in § 10 explain the success of a common laboratory method of magnetizing a piece of steel (sometimes called the method of double-touch). The bar to be magnetized is laid with its ends resting on the opposite poles of two bar magnets (fig. 8), which induce in the bar poles of opposite polarity. Two other magnets, with opposite poles adjacent, are then held almost vertically and drawn in opposite directions along the bar from the centre to the end which rests on the similar pole. By repeating this stroking process many times, a proportion of the molecular magnets are " combed out " so that their axes lie more or less along that of the bar; and, if the steel is hard, a considerable amount of magnetism is retained when the inducing magnets are removed. The magnetization produced by this method does not penetrate far into the steel, and is chiefly confined to a thin layer next the surface; hence in order to produce a powerful magnet in this way, it would be necessary to build it up from a number of thin magnetized strips.

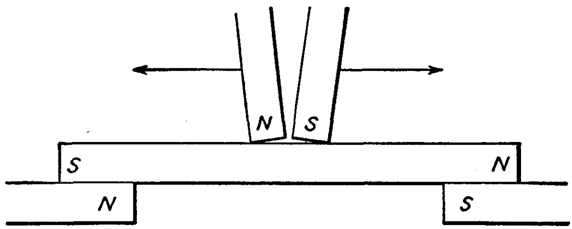


Fig. 8.—Method of magnetizing a piece of steel

A better method, by which the steel may be magnetized much more strongly, is to place the bar inside a long coil similar to that shown in fig. 1 (p. 29), through which a current is passed. Magnets are usually made, commercially, either in this way or by placing the steel across the poles of a very powerful electromagnet of horseshoe form.

13. Coulomb's Law. Unit Pole Strength

As a result of Coulomb's classical series of researches, carried out with a torsion balance, it was shown that the force between magnet poles is proportional to the product of their pole strengths, inversely proportional to the square of the distance between them, and also depends upon the nature of the medium between them, being inversely proportional to a property of the medium called the *permeability*.

$$F \propto \frac{m_1 m_2}{\mu d^2}$$

This led to the definition of unit pole strength, as being that of a pole which repels a similar pole placed unit distance (1 cm.) away in free space (or air, very nearly) with unit force (1 dyne): and this formed the basis of the Electromagnetic C.G.S. System of units. Many of these were found to be of inconvenient size, and multiples or sub-multiples of them were used as Practical Units, e.g. the volt, the ampere, the ohm, which are now absolute units in the M.K.S. System.

14. The Magnetic Field—Direction of Field

A *magnetic field* is a portion of space in which magnetic forces exist. The space surrounding a magnet (or current-carrying conductor) is therefore a magnetic field. The field actually extends indefinitely in all directions, but in practice the term is applied only to the comparatively restricted region, in the neighbourhood of the magnet, where these forces can be detected.

Each pole of a small compass needle placed in the magnetic field in the neighbourhood of a bar magnet is acted upon by two forces* (fig. 9). The pole n is attracted by the pole S of the magnet and repelled by the pole N, while the pole s is repelled by S and attracted

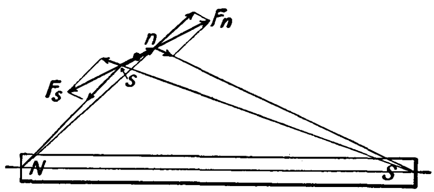


Fig. 9.—Torque on small compass needle in magnetic field

by N. The resultants of these pairs of forces form a couple which causes the needle to turn until its axis is in line with them. Since the resultant forces produced by the magnetic field on the two poles of the compass needle are in opposite directions, some convention must be adopted in order that the " direction of the field " may have a precise meaning, and, therefore,

The direction of the magnetic field at any point is taken as the direction of the force which would act upon a north pole placed at that point.

Hence the north pole n of the compass needle indicates the direction of the field; and the general direction of the lines of magnetic flux in air is from north to south.

15. Magnetic Flux

Since the magnetic field is invisible and intangible, it is necessary to employ some means by which it may be expressed and visualized. The method, used originally by Faraday, and universally adopted, is to ascribe the properties of the field to the existence of a *magnetic*

* The effect of the earth's field is here neglected.

flux set up by the magnet (or current); and this is visualized in terms of *lines of magnetic flux*.

Lines of magnetic flux are imaginary lines drawn in the field so that their direction at any point is that of the magnetic field at that point.

In the C.G.S. system magnetic flux is measured in terms of these lines. The M.K.S. unit of flux (see p. 146) does not involve the idea of lines of flux; nevertheless the concept is still of great assistance in visualizing the magnetic field.

16. Plotting of Magnetic Fields

With the aid of a small compass needle it is possible to plot some of the lines of flux due to a magnet. A bar magnet is laid on a sheet of paper and a small compass needle placed near one pole (fig. 10), say the north pole. When it has come to rest, a mark is made on the paper as near as possible to the north end of the needle, which is of course pointing *away from* the magnet pole. The needle is then advanced and its position adjusted until its *south* pole is immediately over the mark, a second mark being made at the new position of the north pole.

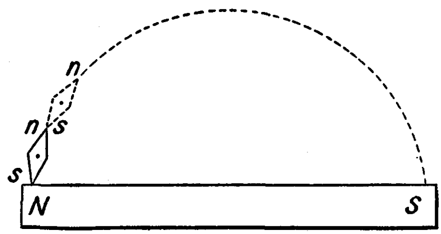


Fig. 10.—Use of compass needle for plotting magnetic field

If the process is repeated again and again, the compass needle eventually arrives at the south pole of the magnet, and the succession of dots lie on a curve which represents one of the lines of flux. By choosing different starting points, a number of such lines may be traced.

In all probability it will be noticed that the curves so plotted are not symmetrical; this is due to the effect of the earth's magnetic field (see § 19, p. 51). Each pole of the compass needle is in reality acted upon by a third force due to the earth's field, in addition to the two due to the bar magnet. The lines of flux just plotted represent the resultant of the field due to the magnet and that due to the earth; and the amount of distortion depends upon the direction of the axis of the magnet relative to that of the earth's field.

This method of plotting lines of flux is obviously a tedious one. A far more complete picture of the field may be obtained very quickly by means of iron filings sprinkled on a piece of paper laid over the magnet. Each filing becomes magnetized by induction and, when friction is reduced by tapping the paper, sets itself with its longer axis in the direction of the magnetic field at that point. The filings therefore arrange themselves in curved chains stretching between the poles, and giving a very complete picture of the field distribution in the plane of the paper.

17. Typical Field Distributions

The field distribution round a bar magnet is shown in fig. 11*a*. A horseshoe magnet is formed by bending a bar magnet until the poles approach each other; the field distribution is shown in fig. 11*b*, from which it will be seen that between the poles there is a strong field, and the lines of flux are almost straight. It must be remembered that these figures show the distribution in the plane of the paper only; in reality, the field extends in all directions.

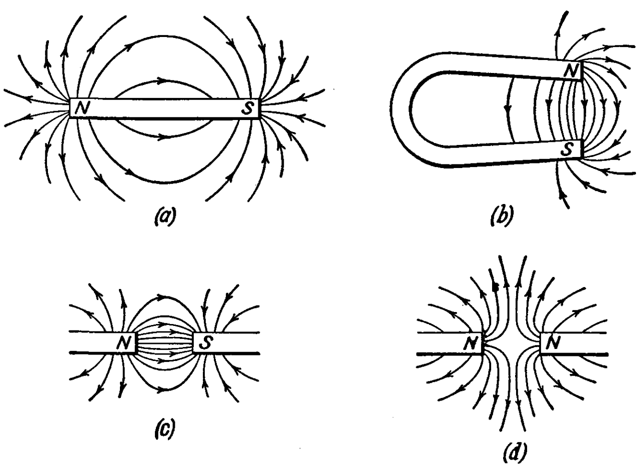


Fig. 11.—Examples of magnetic fields

Fig. 11*c* shows the field surrounding the north pole of one magnet and the south pole of another, from which it can be seen that many of the lines issuing from the north pole of the one enter the south pole of the other.

Since the lines of flux are only convenient concepts they may be endowed with any properties which are necessary in order to explain the known facts; and the force of attraction which is known to exist between unlike poles can be explained by assuming that there is a tension along each line, so that it behaves like a stretched elastic thread. This assumption entails another, for such a tension acting alone would cause all the lines to pass from one pole to the other by the shortest possible path; in order to explain the characteristic curvature of the lines, it must also be assumed that they exert lateral repulsive forces on each other.

The latter assumption also serves as an explanation of the repulsive force between two similar poles. The field distribution round two such poles is shown in fig. 11*d*. Instead of crossing the space between the two poles, the fluxes due to the two poles remain quite distinct, and, owing to the repulsion between them, are bent round towards their opposite poles much more rapidly than in the case of an isolated bar magnet; the resultant repulsion between the two sets of lines may be looked upon as causing the repulsive force actually observed between like poles.

18. Effect of placing a Piece of Unmagnetized Iron in a Magnetic Field

In § 10, p. 44, it was shown that a piece of unmagnetized iron, when in the neighbourhood of a magnet, becomes magnetized by induction. To place the piece of iron near the magnet is to place it in a magnetic field; so that the more general statement may be made:

Unmagnetized iron or steel when placed in a magnetic field becomes magnetized by induction.

As has already been seen, the magnetic field deflects the molecular magnets and brings their axes more into line with that of the field. The resultant directed effect of these molecular magnets is to produce

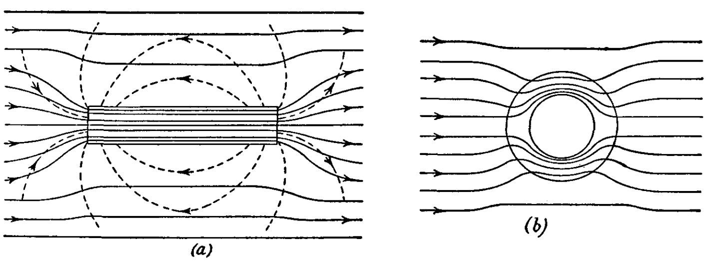


Fig. 12.—Effect of placing an unmagnetized piece of iron or steel in a magnetic field

poles at opposite faces and thus make available, outside the iron, a proportion of the magnetic forces which are inherent in the molecules of the iron, but which in the unmagnetized bar are entirely neutralized internally.

The magnetic effect of the induced poles is stronger than that of the original field which, by its directive effect on the molecular magnets, produced them. The field is now the resultant of the original field and that due to the induced poles, which is indicated by the broken lines in fig. 12*a*. Near the poles the latter is in the same direction as the original field, and therefore strengthens it so that the flux density is increased; whereas in the neighbourhood of the middle of the piece of iron it is in the opposite direction, so that the field is weakened and the flux density decreased. Thus the lines of magnetic flux appear to crowd into the iron as though it offered an easier path than does the air.

The total flux therefore consists of two portions, one due to the original field, and the other much greater portion due to the poles induced in the iron by the field. While the physicist would consider the two portions separately, the electrical engineer usually deals with the flux as a whole, and explains the increase produced by the presence of the iron by assigning to the medium in which the flux is produced a property called the *permeability* (μ), somewhat analogous to conductivity, and assuming that the permeability of iron is greater than that of air. Permeability is defined as the ratio of the flux density (B) to the magnetic field strength or magnetizing force (H):

$$\text{Permeability} = \frac{\text{Flux Density}}{\text{Magnetizing Force}} \quad \text{or} \quad \mu = \frac{B}{H}.$$

The permeability of free space, air, and other non-magnetic materials, has the same constant value; that of magnetic materials is much greater but is not constant and depends on a number of factors (see Chap. XI).

If a narrow slot is imagined to be made in a permanent magnet (or piece of iron lying in a magnetic field) in a direction perpendicular to the direction of the field, it is an experimental fact that poles of opposite polarity are developed on the walls of the slot. Hence whereever such a slot is cut a magnetic field is set up, represented by lines of magnetic flux crossing the slot. It is therefore assumed that these lines form closed loops, passing from the north pole to the south pole in the air and returning from the south pole to the north pole inside the iron.

19. The Earth's Magnetic Field

In § 7, p. 40, the fact that a freely suspended magnet turns so that its axis points roughly north and south was explained by assuming that the earth acted as a magnet having poles situated near the geographical poles. Actually the earth's magnetic field is better represented by supposing it to be due to a short magnet placed at the centre, with its axis inclined at about 17° to the geographical axis, as shown in fig. 13*a*. The ends of the diameter formed by producing the axis are called the *magnetic poles*, N_M, S_M. Some of the lines of flux due to such a magnet are shown in the figure, from which it can be seen that the direction of the field is horizontal (i.e. tangential) near the equator and vertical (i.e. radial) at the magnetic poles. At other points, the direction can be resolved into two components, one horizontal and one vertical, as shown in fig. 13*b*.

The horizontal component (H_E), which exercises the directive effect on the compass needle, is in general the more important; and the term "the earth's magnetic field" without further qualification usually refers to this component. It has a maximum value near the equator which diminishes to zero at the magnetic poles.

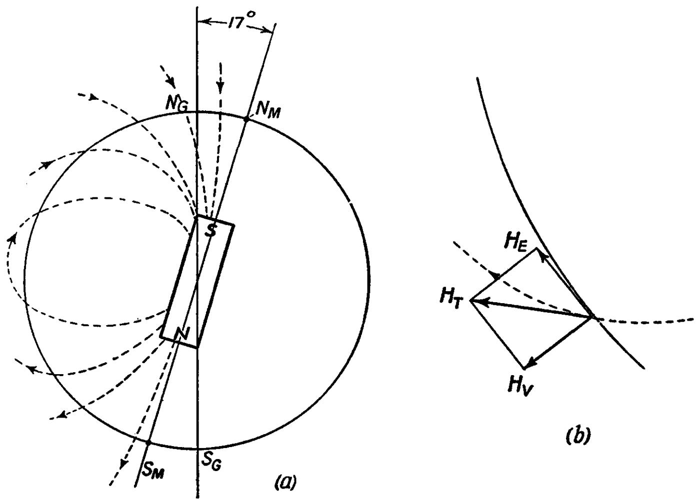


Fig. 13.—The earth's magnetic field

The vertical component of the earth's field (H_V) is zero in the neighbourhood of the equator and increases to a maximum at the magnetic poles. A compass needle taken from the equator to the poles would experience a decreasing torque about the vertical axis, and an increasing torque about a horizontal axis, which would cause it to dip until at the north magnetic pole it would, if free, stand vertically with the north pole pointing downwards.

It is probable that the earth's magnetic field will continue to be measured in C.G.S. units. At Greenwich the value of the horizontal component is about 0·19 oersted and that of the vertical component about 0·43 oersted.

20. Declination and Dip

The following terms are used in connection with the earth's magnetic field:

The magnetic meridian at any point is the vertical plane passing through that point and containing the direction of the earth's field, or in other words passing through the magnetic poles.

The geographical meridian at any point is the plane containing that point and the geographical poles.

The declination is the angle between the magnetic and geographical meridians, measured east or west from the geographical meridian.

The inclination or dip is the angle between the earth's field and its horizontal component.

In the plan view, looking directly down on the geographical north pole N_G (fig. 14), it can be seen that at all points lying on the circle XX, containing both magnetic and geographical poles, the magnetic and geographical meridians coincide and the compass needle points true north and south. At all other points, such as P, the north pole of the compass needle points either east or west of the geographical north. The declination at a given place varies gradually from year to year. In the British Isles it is about 9° to 14° W.

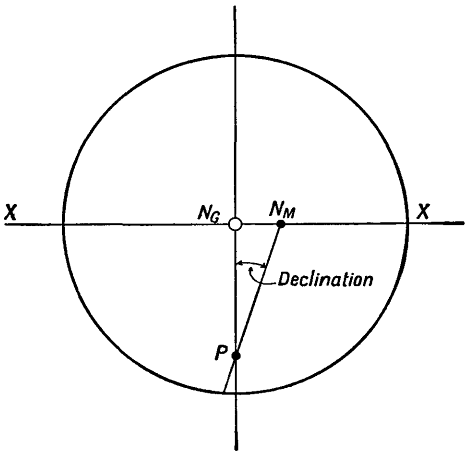


Fig. 14.—To illustrate declination

The *dip* or *inclination* is measured with a *dip-circle*, which consists of a compass needle, carefully balanced before magnetization, mounted on a horizontal axis and moving in front of a circular graduated scale. In the British Isles the dip is about 67°.

In actual practice it is found that the values of field strength, declination and dip do not vary uniformly with position as has been assumed; they are affected by the irregularity with which deposits of magnetic materal occur in the earth's crust. There are, in addition, small periodic variations which occur daily and annually; and superimposed on these, a third very slow change, of which records have not been kept for a sufficient time to determine whether it is periodic or not.

21. The Ship's Compass

In the ship's compass the needle is attached to and rotates with the card, so that the point on the card marked north always indicates magnetic north. The single needle is replaced by several needles, and the arrangement is such that the magnetic moment is large to give a strong directive effect, and the period of swing is long so that oscillation due to movements of the ship is reduced to a minimum. The whole moving system is immersed in a mixture of water and alcohol (to damp out vibration) contained in a brass bowl mounted on two pairs of gimbals placed at right angles, so that the plane of the bowl is unaffected by movements of the ship.

The steel hull of the ship, during construction, acquires magnetism induced by the earth's field, some trace of which it afterwards retains; the direction of the magnetic axis depends upon the direction of the slipways on which the ship was built. In addition, when in service, the ship possesses some temporary magnetism due to the earth's field which varies in magnitude and direction as it changes its position. This permanent and temporary magnetization causes errors in the compass reading, which are corrected by placing masses of iron and permanent magnets in the neighbourhood of the compass.

CHAPTER VI

The Electric Circuit

1. The Electric Circuit

Electrical engineering is usually concerned with the motion of electricity along certain definite paths. Each such path forms a closed loop and is referred to as the *electric circuit.*

2. Electromotive Force, E.M.F. (E)

An E.M.F. has already been described (p. 27) as an urge tending to cause or to oppose the motion of electricity and, if such motion takes place, indicating a conversion of energy. If the motion is in the direction of the E.M.F., energy is converted into the electrical form from some other form and the circuit receives energy at the point where the E.M.F. exists. If the direction of motion is opposite to that of the E.M.F., energy is converted from the electrical form into some other form and the circuit gives out energy at this point; the E.M.F. is then referred to as a *back-* or *counter-E.M.F.* In either case the E.M.F. is measured in terms of the energy received or given out by each unit quantity of electricity.

The energy conversion may take place in various ways. Two conducting plates of different materials, placed in a liquid which reacts chemically with one of them, form the seat of a *chemical E.M.F.* The most familiar example of such a *cell,* in which chemical energy is converted into electrical energy, is that used in flash-lamp or wireless high-tension batteries, which consists of a zinc and a carbon plate in contact with a solution of ammonium chloride (sal ammoniac). A conductor, when moved through a magnetic field so that it cuts the lines of force, is the seat of a *generated E.M.F.*, through which mechanical energy is converted into electrical energy as in the dynamo.

On the other hand, if the conductor carries a current from an external source, and moves under the influence of the force set up, the E.M.F. generated opposes the current, and acts as a back- or counter-E.M.F., in overcoming which electrical energy is converted into mechanical energy, as in the motor.

These and other forms of E.M.F. will be discussed more fully later.

3. Mechanism of Flow of Electricity in a Simple Circuit

The property of conduction in a solid is attributed to the existence of free electrons in its interior (§ 7, p. 21). In a cell of the type mentioned in the previous section and represented diagrammatically in fig. 1*a*, the E.M.F. causes a displacement of these free electrons, so that there is a deficiency in the carbon plate, which in consequence is positively charged, and an excess in the zinc plate, which becomes negatively charged. This displacement represents the absorption of a small quantity of energy which is stored as potential energy in the charged plates.

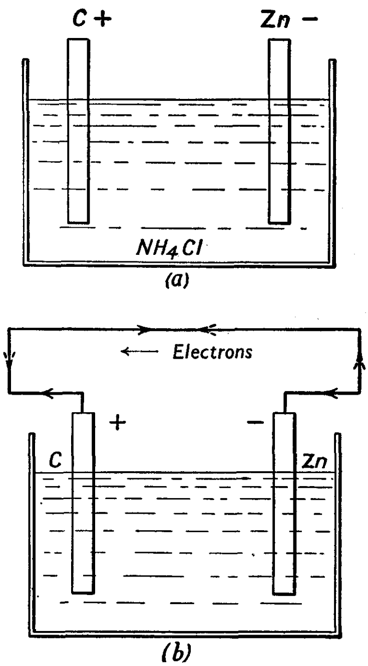


Fig. 1.—Mechanism of flow in simple circuit

If now the plates are joined by a conductor, the excess of electrons in the zinc plate pass along the conductor to supply the deficiency in the carbon plate. But the E.M.F. of the cell causes a continuous withdrawal of electrons from the carbon plate so that it is maintained positively charged. Hence a stream of electrons passes round the circuit from the zinc to the carbon plate outside the cell, and from the carbon plate to the zinc plate inside the cell; this stream constitutes an *electric current*, which, as the result of early conventions, is assumed to flow in the *opposite* direction (§ 9, p. 23). The direction of the electron stream is shown in fig. 1*b* by the broken arrows and that of the current by the full-line arrows.

In this case the conversion of energy is continuous. Each electron receives a small quantity of energy, supplied by the chemical decomposition of the zinc, most of which it gives up in the external portion of the circuit, where, in this simple case, it is converted directly into heat.

4. The Electric Current

In this simple circuit the energy received by each electron is gradually dissipated in repeated collisions with the atoms. It might naturally be expected that its velocity would continually decrease during its progress through the circuit, but because of the forces exerted on each electron by the electrons behind it, the average progressive velocity round the circuit is everywhere the same. This can best be realized by imagining the circuit to be an endless trough completely filled with marbles which represent the electrons (fig. 2); if a force of constant value is applied to each marble as it passes one particular point in the trough, all the marbles will travel round with uniform velocity. Hence under constant conditions the same number of electrons, i.e. the same quantity of electricity, passes any given point in the circuit in the same time, so that the *rate of flow* is the same at all points. This rate of flow is what has already been referred to as the electric current, so that *in any simple circuit the current has the* *same value at every point.* Although, even when the current is small, the number of electrons passing any point per second is very large ($6 \cdot 3 \times 10^{18}$ for a current of 1 ampere), the number involved is so great that the average velocity along the conductor is very small compared with the random thermal velocities—only a few centimetres per minute.

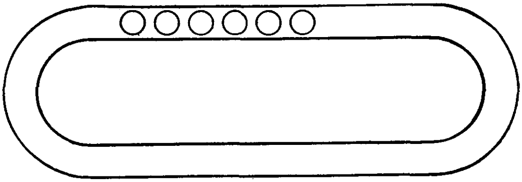


Fig. 2.—Mechanical analogy to electric current

5. The Hydraulic Analogy

It is natural to imagine that the electric current flows along a conductor, very much as water flows through a pipe; and such an analogy has been employed very frequently in order to represent the electric circuit in a familiar way. Provided that it is not pressed too far, the analogy yields much useful information, and will be used from time to time in succeeding sections.

6. Difference of Potential, P.D. (V)

If it is assumed that the positively charged carbon plate of the cell shown in fig. 1 is at a higher level or *potential* than the negatively charged zinc plate, the current may be imagined to flow in the external circuit from the point of higher potential to the point of lower potential,

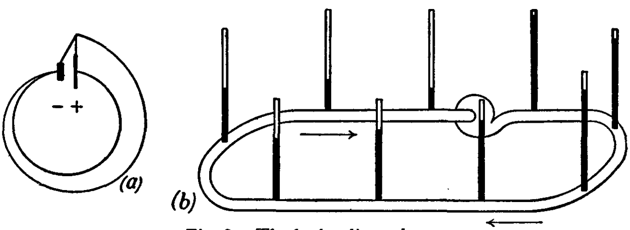


Fig. 3.—The hydraulic analogy

somewhat as water flows from a higher level to a lower level; and to be forced from the point of lower potential to that of higher potential inside the cell, which can be assumed to act like a pump in a hydraulic circuit.

This is illustrated graphically in fig. 3*a*, and the equivalent hydraulic circuit is shown in fig. 3*b*. Water (electricity) flows along a pipe (conductor), and manometer tubes at intervals indicate the gradual fall of pressure (potential) round the circuit. From the point of lowest pressure (potential) the water (electricity) is raised to the point of highest pressure (potential) by the pump (source of E.M.F.), which must be driven by some external means and therefore represents a conversion of energy. The whole of the energy received by the circuit at the pump is, in this simple case, converted directly into heat.

There is therefore a continuous fall in pressure along a pipe in which water is flowing; the difference in pressure between the ends of any particular section is that required to overcome the opposition offered to the flow by friction, and indicates that a portion of the energy is being turned into heat in that section. Similarly, when a current is flowing along a conductor, there is a continuous fall of potential along

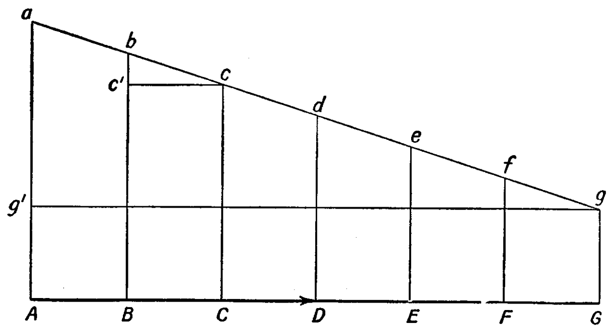


Fig. 3c.—Fall of potential along conductor

its length. In order to maintain the current there must be a *difference of potential* (P.D.) between the ends of any section of the conductor, in order to overcome the resistance of that section, and this difference of potential indicates that some of the energy is being turned into heat.

Resistance is that property of a conductor which opposes the passage of a current and by which some of the energy is turned into heat; but it must not be assumed that resistance and friction are similar in nature.

In fig. 3c a current is flowing from A to G along a uniform conductor AG. The potential at A is represented by Aa and that at G by Gg, and there is a continuous fall from A to G. The P.D. between A and G, given by $Aa - Gg = g'a$, is required to overcome the resistance of the length of conductor AG. Similarly, considering any section such as BC, the P.D. between B and C, given by $Bb - Cc = c'b$, is necessary to overcome the resistance of the section BC.

Both E.M.F. and P.D. are concerned with the movement of electricity and with the energy associated with it; and both are measured in the same unit, although they are by no means synonymous terms (§ 19, p. 67).

7. Units

As mentioned on p. 14 many of the C.G.S. units were considered unsuitable in size for general use, and multiples or sub-multiples were taken as *Practical Units*, which were adopted internationally, and are now all included in the M.K.S. System as absolute units in their own right.

The solution of simple problems is of great value in assisting the student to appreciate the properties and characteristics of the electric circuit. This requires an acquaintance with the more common units when it is too early for them to be developed in logical sequence. For example, the unit of current is defined electromagnetically in terms which at present would be unintelligible. Hence in the following sections *quantity* and *current* are defined electrolytically. Nevertheless, for the sake of completeness and for future reference, the electromagnetic definition of unit current is added.

8. The Unit of Quantity of Electricity (Q)—the Coulomb

It is a matter of experimental fact that when an electric current passes through an electrolyte such as a solution of a metallic salt, the amount of a particular metal deposited on one of the electrodes by a given quantity of electricity is always the same (Faraday's laws, § 1, p. 118). This enables quantity of electricity to be determined by weighing, and since such a determination can be carried out with great accuracy, the unit of quantity may be defined in this way.

The unit of quantity is that quantity of electricity which deposits 0·001118 gm. of silver from a solution of silver nitrate in a standard silver voltameter, and is called the coulomb (cmb.).

Although the coulomb is defined in terms of the weight of silver deposited from a solution of silver nitrate, it is usual in all but the most accurate determinations to use a solution of copper sulphate. The equivalent weight of copper is 0·000328 gm.; hence the coulomb can be defined as that quantity of electricity which deposits 0·000328 gm. of copper in a copper voltameter.

9. The Unit of Current (I)—the Ampere

The definition of the unit of current follows at once from that of the unit of quantity.

$$\text{Current} = \text{rate of flow} = \frac{\text{quantity}}{\text{time}}.$$

Unit current is that rate of flow in which unit quantity (one coulomb) passes any given point in the circuit in unit time (one second) and is called the ampere.*

* This corresponds with the compound unit, gallons per minute or c. ft. per second; there is no single unit expressing the rate of flow of liquids.

From the definition of the coulomb given in the previous section, it is clear that the unit of current may be defined alternatively thus:

The ampere is that steady current which deposits 0·001118 gm. of silver per second in a standard silver voltameter.

1 ampere = 1 coulomb per second.
1 coulomb = 1 ampere-second
(*not* 1 ampere per second).
1 ampere-hour = 3600 coulombs.
1 milliampere = 10^{-3} ampere.
1 microampere = 10^{-6} ampere.

The formal definition of unit current in the M.K.S. system is as follows:

Unit current—the ampere—is that current which, flowing in two infinitely long parallel conductors, with their centres unit distance (1 metre) apart, in free space, produces a force between them of 2×10^{-7} newtons per metre length.

It may be noted that the ampere is one-tenth of the C.G.S. unit of current.

10. Measurement of Quantity and Current by Voltameter (see also § 9, p. 123)

If a steady current is passed through a solution of silver nitrate for a time t seconds, and the weight of silver deposited is w gm., then:

Total quantity of electricity (Q) passed through the solution

$$= \frac{w}{0{\cdot}001118} \text{ coulombs;}$$

$$\text{and the current } (I) = \frac{\text{quantity}}{\text{time}}$$

$$= \frac{w}{0{\cdot}001118 \times t} \text{ amperes.}$$

It should be noted that current measured in this way must have a *constant* value throughout the determination; but the same weight of silver will be deposited, however much the value of the current varies, provided that the total number of coulombs is the same. In this case I is the *average* value of the current.

If a solution of copper sulphate is used instead of silver nitrate, then

$$Q = \frac{w}{0{\cdot}000328} \text{ coulombs,}$$

$$I = \frac{w}{0{\cdot}000328\, t} \text{ amperes.}$$

Example.—A steady current passed through a copper voltameter for 30 min. produces a deposit of 1 gm. of copper. Determine the quantity of electricity passed through the solution, and the value of the current.

$$Q = \frac{1}{0{\cdot}000328} = 3050 \text{ coulombs}, \qquad I = \frac{3050}{30 \times 60} = 1{\cdot}69 \text{ ampere}.$$

11. Unit of E.M.F.—the Volt

In the hydraulic circuit illustrated in fig. 3*b*, p. 57, each pound, or cubic foot, of water in passing through the pump (which is analogous to a source of E.M.F.) receives a certain quantity of energy, in virtue of the increase in pressure produced. It would therefore be quite possible to measure the pressure, not in lb. per sq. in. but in ft.-lb. per lb., or, alternatively, ft.-lb. per c. ft.; and E.M.F., as has already been stated, is measured in this way.

Unit E.M.F. exists at a point where each unit of electricity (1 coulomb) receives or gives out unit quantity of energy (1 joule), and is called the volt.

Hence 1 volt = 1 joule per coulomb

or 1 joule = 1 volt-coulomb.

1 millivolt = 10^{-3} volt. 1 microvolt = 10^{-6} volt.

It may be noted here that since the coulomb is one-tenth of the C.G.S. unit and the joule is 10^7 C.G.S. units (ergs),

$$1 \text{ volt} = \frac{10^7}{10^{-1}} = 10^8 \text{ C.G.S. units}$$

The E.M.F. of certain types of cell under constant conditions has such a constant value that they may be used as concrete standards: and the volt may be defined as $\frac{1}{1{\cdot}0186}$ of the E.M.F. of a Weston standard cell at a certain temperature (see § 18, p. 131).

12. Relation between Current and E.M.F.

It can be shown experimentally that **in any given circuit, under constant conditions, the current is directly proportional to the E.M.F.; i.e.**

$$I \propto E, \quad \text{or} \quad \frac{E}{I} = \text{a constant}.$$

Similarly, **in any given portion of a circuit, under constant conditions, the current is proportional to the P.D. between the ends of that portion; i.e.**

$$I \propto V, \quad \text{or} \quad \frac{V}{I} = \text{a constant}.$$

This relationship, which is of fundamental importance, was first pointed out by Ohm and is known as *Ohm's law*. The constant, i.e. the ratio of the E.M.F. or P.D. to the current, is known as the *resistance* (R) of the circuit, or portion of the circuit.

13. Resistance (R). Unit of Resistance—the Ohm

In practice, relative motion of any kind is always attended by some form of opposition and the production of heat which, in the case of water flowing through a pipe, is due largely to the friction between the water and the inner surface of the pipe. Since an electric current consists of a stream of electrons moving inside the conductor, it is natural to suppose that there will exist some kind of opposition to their motion, and this has been found to be the case, although it must not be inferred that the opposition is of a nature similar to that set up by pipe friction. It is in fact due to innumerable collisions between the electrons and the atoms of the conductor, by which part of the energy associated with each electron is turned into heat.

Resistance can be defined as that property of a conductor which opposes the flow of a current through it and by which part of the energy associated with the current is converted into heat. Its value depends upon the material, dimensions and temperature of the conductor. These factors are discussed in detail in § 7, p. 98 *et seq*.

Two of the quantities connected by Ohm's law having been defined, the third may be defined in terms of these.

Unit resistance is that of a circuit in which unit E.M.F. produces unit current, or of a conductor in which unit current flows when there is unit P.D. between its ends.

The unit of resistance is that of a circuit in which an E.M.F. of one volt causes a current of one ampere to flow, or of a conductor in which a current of one ampere flows when the P.D. between its ends is one volt.

This unit is called the ohm.

It may be noted that since the ampere is 10^{-1} C.G.S. units and the volt is 10^8 C.G.S. units, the ohm is $\frac{10^8}{10^{-1}} = 10^9$ C.G.S. units.

The values of very high resistances and of *insulation resistances* (see § 6, p. 97) are usually expressed in *megohms*:

$$1 \text{ megohm} = 10^6 \text{ ohms}.$$

Also

$$1 \text{ microhm} = 10^{-6} \text{ ohms}.$$

The *conductance* (G) of a circuit is the reciprocal of its resistance:

$$G = \frac{1}{R}.$$

The unit of conductance is the *mho*; a circuit which has a resistance of n ohms, has a conductance of $\frac{1}{n}$ mho.

The unit is little used in simple circuits.

14. Ohm's Law

It follows, from the preceding sections, that when an E.M.F. E volts exists in a circuit of resistance R ohms, the current produced is I amperes where $I = E/R$; and that when a P.D. V volts exists between the ends of a portion of a circuit of resistance R ohms, the current in that portion is I amperes where $I = V/R$.

Hence the general form of *Ohm's law* is:

$$I = \frac{V \text{ or } E}{R}, \qquad \ldots\ldots\ldots \quad (1)$$

where

$$\begin{aligned} I &= \text{current in amperes,} \\ V \text{ or } E &= \text{P.D. or E.M.F. in volts,} \\ R &= \text{resistance in ohms.} \end{aligned}$$

It is important for the student to realize that the relationship holds for any portion of a circuit as well as for the whole, provided that appropriate values are used.

If the conductance of the circuit is substituted for the resistance, equation (1) becomes

$$I = (V \text{ or } E)G. \qquad \ldots\ldots\ldots \quad (2)$$

Example 1.—A circuit of which the total resistance is 5 ohms contains a cell having an E.M.F. of 2 volts. Determine the current in the circuit (fig. 4*a*).

In this case the law is applied to the whole circuit.

$$E = 2 \text{ volts}, \; R = 5 \text{ ohms}.$$

$$\therefore\; I = \frac{E}{R} = \frac{2}{5} = 0{\cdot}4 \text{ ampere}.$$

Example 2.—A coil is connected to two terminals between which is a P.D. of 100 volts. If the current is 20 amperes, what is the resistance of the coil? (Fig. 4*b*.)

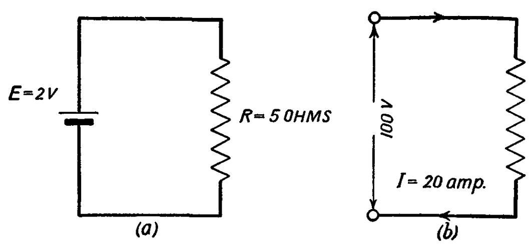


Fig. 4

Here the law is applied merely to that portion of the circuit containing the terminals and the coil.

$$V = 100 \text{ volts}, \ I = 20 \text{ amperes}.$$

$$\therefore R = \frac{V}{I} = \frac{100}{20} = 5 \text{ ohms}.$$

Example 3.—A coil of 100 ohms resistance is connected to a supply, and the current is found to be 2·3 amperes. What is the voltage of the supply?

$$R = 100 \text{ ohms}, I = 2{\cdot}3 \text{ amperes}.$$

$$\therefore \ V = IR = 2{\cdot}3 \times 100 = 230 \text{ volts}.$$

15. Experimental Demonstration of Ohm's Law

At this stage it is desirable that the student should carry out an experimental demonstration of Ohm's law, although it is necessary to anticipate somewhat, and to make use of information which is given in later sections.

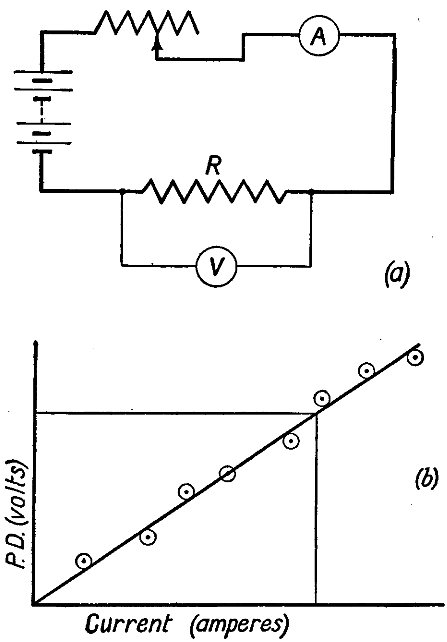


Fig. 5

In fig. 5*a* is shown a simple circuit consisting of a coil of constant resistance R connected in series with a variable resistance to some source of supply such as a battery. The current in the circuit is measured in amperes by means of the ammeter A, and the P.D. between the ends of the coil is measured in volts by means of the voltmeter V. (These instruments are described in Chapter X.)

It will be noticed that if the value of the variable resistance is *decreased* the current *increases* and vice versa; and also that the ammeter and voltmeter readings increase or decrease *simultaneously*.

The current is now increased in a series of steps, and a set of readings is taken of corresponding values of the current and the P.D. between the ends of the coil. If these readings are plotted as shown in fig. 5*b*, it will be found that (apart from small experimental errors) the points all lie on *a straight line passing through the origin*, thus showing clearly that *the current in any portion of a circuit of constant resistance is proportional to the P.D. between its ends.*

Since $R = V/I$, the resistance (in ohms) of the coil may be found from any pair of readings by dividing the P.D. by the current. These, however, are subject to experimental error, and a more accurate result is obtained by taking values of V and I corresponding with a point on that straight line which passes through the origin and lies most evenly *among* the experimental points.

The value of the current should not be sufficient to cause an appreciable rise in the temperature of the coil during the experiment.

16. Energy (W) and Power (P)

In the hydraulic circuit of fig. 3*b*, p. 57, it is clear that the energy received by each pound or cubic foot of water in passing through the pump is proportional to the increase in pressure. Hence the total energy received during any given time is proportional to the product:

(Quantity passing through pump in given time) × (rise in pressure).

Similarly, the total energy given out in any section of the pipe is proportional to:

(Quantity passing through section in given time) × (fall in pressure).

In the electric circuit exactly similar relationships hold. From the definition of E.M.F. in § 11, p. 61, it follows that:

1 joule of energy is received by each coulomb at a point where the E.M.F. is 1 volt; or

1 joule of energy is given out by each coulomb in passing along a section of a circuit between the ends of which there is a P.D. of 1 volt.

Hence

$$\textbf{Energy} = \textbf{(Quantity)} \times \textbf{(E.M.F. or P.D.)};$$

or

$$W = QE \text{ (or } QV\text{) joules, where } Q = \text{quantity in coulombs,}$$

and

$$E \text{ or } V = \text{E.M.F. or P.D. in volts.}$$

Power is *rate at which energy is expended* or rate of doing work.

$$\therefore \text{ Power} = \frac{\text{energy}}{\text{time}} = \frac{(\text{quantity}) \times (\text{E.M.F.})}{\text{time}}.$$

But

$$\frac{\text{quantity}}{\text{time}} = \text{rate of flow or current.}$$

Hence

$$\textbf{Power} = \textbf{(Current)} \times \textbf{(E.M.F. or P.D.)};$$

i.e. $$P = \frac{W}{t} = \frac{QE}{t} = E\frac{Q}{t} = EI \text{ or } VI \text{ joules per sec.,}$$

where

$$I = \text{current in amperes,}$$
$$E \text{ or } V = \text{E.M.F. or P.D. in volts.}$$

17. Unit of Power—the Watt

The unit of power is that of a circuit in which energy is expended at the rate of one joule per second, e.g. in which a current of one ampere is produced by an E.M.F. of one volt; or in a section of a circuit in which the current is one ampere and between the ends of which the P.D. is one volt.

This unit is called the Watt.

Hence

$$1 \text{ watt} = 1 \text{ joule per sec.}$$

and

$$P = EI \text{ or } VI \text{ watts.}$$

Some idea of the magnitude of this unit may be gained by noting that in an electric lamp such as is used in a small living-room, the power input is usually 100 watts.

Also

$$1 \text{ horse-power} = 746 \text{ watts,}$$

and therefore

$$1 \text{ watt} = \frac{33{,}000}{746} = 44{\cdot}2 \text{ ft.-lb. per min.} = 0{\cdot}737 \text{ ft.-lb. per sec.}$$

For many purposes the watt is too small a unit, and the **kilowatt** (kW.), which is 1000 times as large, is used.

$$1 \text{ kilowatt} = 1000 \text{ watts} = 1{\cdot}34 \text{ horse-power.}$$

Even this unit is inconveniently small for some purposes and the output of modern generating stations is expressed in **megawatts.**

$$1 \text{ megawatt (MW.)} = 1000 \text{ kilowatts.}$$

18. Units of Energy

Since

$$1 \text{ watt} = 1 \text{ joule per sec.,}$$
$$1 \text{ joule} = 1 \text{ watt-second,}$$

i.e. (energy in joules) = (power in watts) (time in seconds);

$$W = Pt = VIt \text{ joules.}$$

This unit, the joule or watt-second, is too small for many purposes. A larger unit is the *watt-hour*, which is the energy expended in a circuit by a power of 1 watt acting for 1 hour.

$$1 \text{ watt-hour} = 3600 \text{ watt-seconds} = 3600 \text{ joules.}$$

A still larger unit, the *kilowatt-hour* (kWh.) is used for commercial and industrial purposes. It is the energy expended in a circuit by a power of one kilowatt acting for one hour.

$$\begin{aligned} 1 \text{ kilowatt-hour} = 1000 \text{ watt-hours} &= 3{\cdot}6 \times 10^6 \text{ joules} \\ &= 2{\cdot}65 \times 10^6 \text{ ft.-lb.} \end{aligned}$$

This unit is also known as the *Board of Trade Unit* (B.O.T. unit), or simply as the *Unit*, and on it all electricity charges are based.

Example 1.—A cell having an E.M.F. of 1·5 volts supplies a current of 2 amperes. Find (*a*) the power (in watts) acting in the circuit; (*b*) the total energy (in joules) supplied by the cell in one hour.

(*a*) Power (watts) = E.M.F. (volts) × current (amperes)
= 1·5 × 2 = 3 watts.

(*b*) Energy (joules) = power (watts) × time (sec.)
= 3 × 60 × 60 = 10,800 joules.

Example 2.—What is the weekly cost of a 100-watt lamp which is used for 3 hours a day when electrical energy costs 5*d*. per unit?

Total energy consumption = 100 × 3 × 7 = 2100 watt-hours
= 2·1 kW.-hours or B.O.T. units.

Cost = 5 × 2·1 = 10·5*d*.

19. Difference between E.M.F. and P.D.

The function of an electric circuit is to take in energy at one or more points and, by means of the electric current, to transfer it in the electrical form to other points where it is converted into other forms of energy. Energy is not stored *in* the circuit: all the energy received by each coulomb in one portion is given out in other portions of the circuit.

In a circuit containing resistance only all the energy is converted directly into heat. In other cases only a small portion of the energy is turned into heat, and the remainder is converted into other forms at points in the circuit where back-E.M.F.s exist, e.g. into mechanical energy (back-E.M.F. of motor, p. 324), or stored in the potential form, as chemical energy (back-E.M.F. of secondary cell, p. 135), or in the magnetic field (E.M.F. of self-inductance, p. 166), or in the electrostatic field (E.M.F. of a condenser, p. 362).

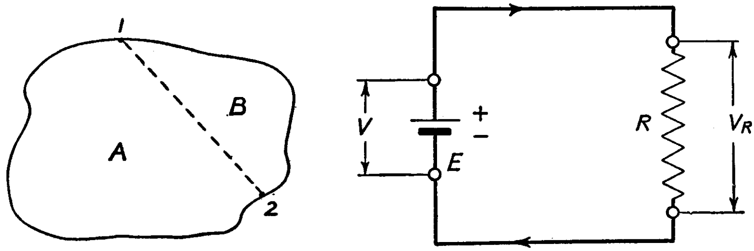
In all cases except that of the conversion into heat, the transformation is reversible, and if the *forward* E.M.F. is removed, the counter-E.M.F. either sets up a current in the reverse direction or maintains the existing current until all the stored energy is dissipated. The conversion of electrical energy, by resistance, into heat, due to collision between the electrons and the atoms of the conductor, is, however, an irreversible process: there is no counter-E.M.F., and if the forward E.M.F. in a circuit containing resistance only is removed, the current immediately ceases, since there is no stored energy by which it may be maintained.

An E.M.F.:

(1) can exist at a point;

(2) is associated with a *conversion* of energy into or from the electrical form;

(3) is measured by the energy received or given out by each coulomb of electricity on passing the point at which the E.M.F. exists.



(a) Fig. 6.—Difference between E.M.F. and P.D. (b)

On the other hand, a P.D.:

(1) is concerned with two points in the circuit;

(2) is associated with a *transfer* of energy in the *electrical form* from one part to the other of the two parts into which the points divide the circuit. For example, if there is a P.D. between the points 1 and 2 in the circuit shown in fig. 6*a*, a current flowing round the circuit will transfer energy from part A to part B or vice versa;

(3) is measured by the energy so transferred by each coulomb passing between the points.

These differences will be made clearer by means of an example. In fig. 6*b* is shown a simple circuit consisting of a cell connected to a resistance coil.

Inside the cell chemical energy is converted into electrical energy and, if the E.M.F. is E volts, each coulomb in passing through the cell receives E joules of energy. But since the cell, like every other portion of the circuit, possesses resistance (the internal resistance), a small portion of the total energy is turned into heat *inside* the cell. The quantity of energy available for transfer by each coulomb from the cell to the external circuit, which is represented by the P.D. between the cell terminals, is therefore less than the total energy represented by the E.M.F., i.e. V is less than E.

Similarly, the connections between the cell and the coil possess resistance, by which a further quantity of the energy is converted into heat. Hence the P.D. between the ends of the coil, which represents the energy transferred by each coulomb to the coil, is less than the P.D. of the cell, i.e. V_R is less than V.

This can be shown very easily by experiment. If a high resistance voltmeter (see Chapter X) is connected to the cell before the main circuit is closed, the current taken is so small that the reading may be taken as indicating the E.M.F. Directly the main circuit is closed and an appreciable current is taken from the cell the voltmeter reading falls, since the P.D. is now less than the E.M.F. Further, if the voltmeter is connected to the terminals of the coil, the reading is still lower, showing that the P.D. across the coil is less than that between the terminals of the cell.

This difference between the P.D. and the E.M.F. of a cell which is supplying a current can also be regarded, in the light of Ohm's law, as due to a portion of the E.M.F. being used in overcoming the internal resistance of the cell, so that there is a "resistance drop" inside the cell. Similarly, there is a further "resistance drop" in the connections, and the P.D. between the ends of the coil is equal to the E.M.F. of the cell minus the sum of the "resistance drops" in the cell and the the connections.

Example.—A battery has an E.M.F. of 50 volts and an internal resistance(r_b) of 0·15 ohms, and supplies a number of lamps having an effective resistance (R) of 10 ohms, through leads having a resistance (r_l) of 0·25 ohms.

From these data, we can make the following deductions.

Total resistance of circuit, $R_t = r_b + r_l + R = 0{\cdot}15 + 0{\cdot}25 + 10 = 10{\cdot}4$ ohms.

Current $I = \dfrac{E}{R_t} = \dfrac{50}{10{\cdot}4} = 4{\cdot}8$ amperes.

P.D. at battery terminals $= V_b = E - Ir_b = 50 - 0{\cdot}72 = 49{\cdot}28$ volts.

P.D. at lamp terminals $= V_b - Ir_l = E - Ir_b - Ir_l$

$= 49{\cdot}28 - 1{\cdot}2 = 48{\cdot}08$ volts.

Total energy received by each coulomb		= 50	joules.
Energy per coulomb turned into heat in battery		0·72	,,
,, ,, ,,	transferred to external circuit	= 49·28	,,
,, ,, ,,	turned into heat in leads	= 1·2	,,
,, ,, ,,	transferrred to lamps	= 48·08	,,
Power output from battery	= 49·28 × 4·8	= 236·54	joules per sec.
		= 236·54	watts
Power loss in leads	= 1·2 × 4·8	= 5·76	,,
Power input to lamps	= 48·08 × 4·8	= 230·78	..

20. Series and Parallel Connections

Electric circuits in general consist of a number of sections which may be connected either in *series* or in *parallel*, or in a combination of the two.

In a series connection the end of one section is connected to the beginning of the next (fig. 7*a*). The characteristic of such a connection is that the current is the same throughout and that the P.D. between the ends of the circuit is the sum of the P.D.s between the ends of each section.

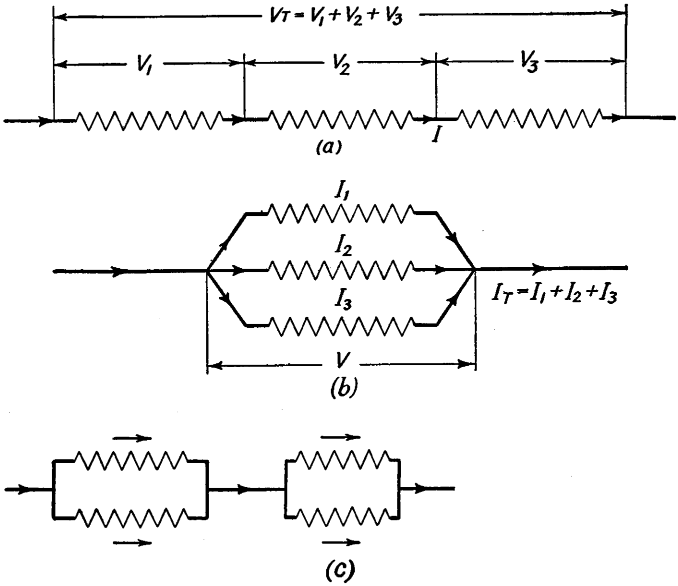


Fig. 7.—Series and parallel connection of resistances

In a parallel connection, corresponding ends of each section are joined to common points (fig. 7*b*), so that the total current is distributed among the various branches. The characteristic of a parallel connection is that the total current is the sum of the currents in the various branches and that the P.D. across each branch is the same.

A *series-parallel* connection, in which a number of sections, each consisting of two or more branches in parallel, are joined in series, is shown in fig. 7*c*.

21. Resistances in Series

If a number of sections are connected in series, the statement that the total resistance is the sum of the resistances of the several sections is almost self-evident.

Three such sections, having resistances R_1, R_2, and R_3, are shown in fig. 8. When a current is flowing, there is a continuous fall of potential from A to B, and the P.D.s across the sections are V_1, V_2, V_3; so that the P.D. between A and B is

$$V_T = V_1 + V_2 + V_3. \quad \ldots \ldots \quad (a)$$

(The student should verify this by experiment.)

Since the current (I) is the same in each section, by Ohm's law

$$V_1 = IR_1,\ V_2 = IR_2,\ V_3 = IR_3,$$

and if R_T = total or resultant resistance of the three sections, i.e. that single resistance which would have the same effect,

$$V_T = IR_T.$$

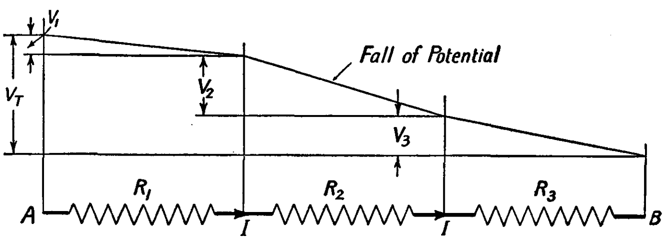


Fig. 8.—Resistances in series

Substituting these values in (a) above, we find

$$IR_T = IR_1 + IR_2 + IR_3;$$
$$\therefore\ R_T = R_1 + R_2 + R_3. \quad \ldots \ldots \quad (3)$$

The total resistance of any number of resistances in series is the sum of the separate resistances.

It will be realized that every conductor, however short, consists of a large number of elements connected in series, so that its resistance is the sum of the resistances of all the elements.

22. Resistances in Parallel

Three sections of resistance R_1, R_2, and R_3 are shown in fig. 9, connected in parallel. The total current I_T splits up at A, a portion flowing through each branch, and reunites at B.

Now

$$I_T = I_1 + I_2 + I_3. \quad \ldots \ldots \quad (b)$$

(This is one of Kirchhoff's laws, p. 83, and can be verified experimentally by the student.)

Again, since the P.D. across each branch is the same,

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad I_3 = \frac{V}{R_3}.$$

Further, if R_J is the joint or effective resistance of the three sections,

$$I_T = \frac{V}{R_J}.$$

Substituting these values in (*b*) above,

$$\frac{V}{R_J} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3};$$

$$\therefore \frac{1}{R_J} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \quad \ldots \ldots \quad (4)$$

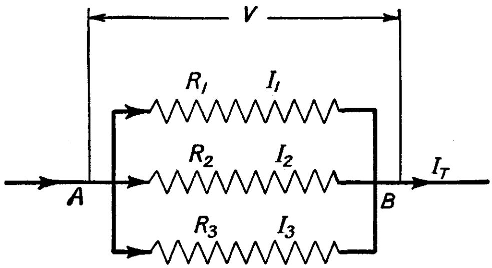


Fig. 9.—Resistances in parallel

The reciprocal of the joint resistance is equal to the sum of the reciprocals of the resistances of the parallel branches.

An equivalent form of the relation is

$$R_J = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}. \quad \ldots \ldots \quad (5)$$

The reciprocal of the resistance is the *conductance* (p. 62). Hence the above relationship may also be expressed in the form:

The total conductance is equal to the sum of the conductances of the parallel branches:

$$G_T = G_1 + G_2 + G_3. \quad \ldots \ldots \quad (6)$$

Similar results to those of (4), (5), (6) clearly hold for any number of branches.

Example 1.—Three coils of resistance 10 ohms, 5 ohms and 3 ohms are connected in series to a 230-volt supply. Determine the current and the P.D. across each coil.

$$R_T = 10 + 5 + 3 = 18 \text{ ohms.}$$

$$I = \frac{V}{R_T} = \frac{230}{18} = 12{\cdot}78 \text{ amperes.}$$

$$\begin{aligned} V_1 &= IR_1 = 12{\cdot}78 \times 10 = 127{\cdot}8 \text{ volts.} \\ V_2 &= IR_2 = 12{\cdot}78 \times 5 \;\; = \;\; 63{\cdot}9 \;\; ,, \\ V_3 &= IR_3 = 12{\cdot}78 \times 3 \;\; = \;\; \underline{38{\cdot}3} \;\; ,, \\ & \qquad\qquad\qquad\qquad\quad 230{\cdot}0 \;\; ,, \end{aligned}$$

Example 2.—The same three coils are connected in parallel. If the total current is 20 amperes, determine the P.D. across the coils and the current in each.

$$\frac{1}{R_J} = \frac{1}{10} + \frac{1}{5} + \frac{1}{3} = 0{\cdot}63.$$

$$R_J = \frac{1}{0{\cdot}63} = 1{\cdot}58 \text{ ohm.}$$

(It may be noted here that the joint resistance is always *less* than that of the branch of *lowest* resistance.)

$$V = I_T R_J = 20 \times 1{\cdot}58 = 31{\cdot}6 \text{ volts.}$$

$$I_1 = \frac{V}{R_1} = \frac{31{\cdot}6}{10} = 3{\cdot}16 \text{ amperes.}$$

$$I_2 = \frac{V}{R_2} = \frac{31{\cdot}6}{5} = 6{\cdot}32 \;\; ,,$$

$$I_3 = \frac{V}{R_3} = \frac{31{\cdot}6}{3} = \underline{10{\cdot}53} \;\; ,,$$

$$I_T = 20{\cdot}00 \;\; ,,$$

Example 3.—In the circuit shown in fig. 10, determine (*a*) the total current, (*b*) the current in each of the parallel branches, (*c*) the P.D. across the coil *A* (the figures denote the resistances in ohms).

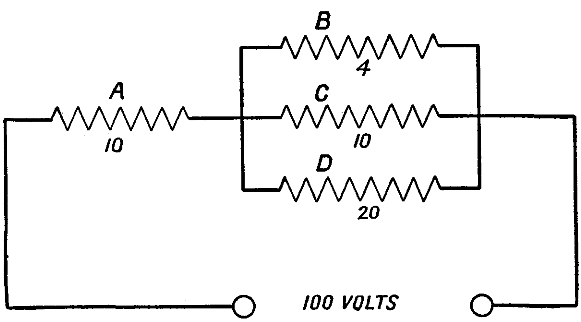


Fig. 10

If R_J = joint resistance of B, C and D,

$$\frac{1}{R_J} = \frac{1}{4} + \frac{1}{10} + \frac{1}{20} = 0{\cdot}4, \quad R_J = 2{\cdot}5 \text{ ohms},$$

i.e. the joint effect of B, C and D as far as the total current is concerned is that of a single coil of resistance 2·5 ohms, which is connected in series with A.

$$\text{Hence the total resistance } R_T = 2{\cdot}5 + 10 = 12{\cdot}5 \text{ ohms.}$$

$$\text{Total current } I_T = \frac{V}{R_T} = \frac{100}{12{\cdot}5} = 8 \text{ amperes.}$$

P.D. across the parallel branches B, C, $D = I_T R_J = 8 \times 2{\cdot}5 = 20$ volts.

$$\therefore\ I_B = \frac{20}{4} = 5 \text{ amperes.}$$

$$I_C = \frac{20}{10} = 2 \quad ,,$$

$$I_D = \frac{20}{20} = \underline{1} \quad ,,$$

$$8 \quad ,,$$

P.D. across $A = I_T R_A = 8 \times 10 = 80$ volts, or alternatively, since A is the only other coil in the circuit,

$$V_A = 100 - 20 = 80 \text{ volts.}$$

Example 4.—In the circuit shown in fig. 11, the P.D. across coil A is 100 volts. Find the current in each coil, the P.D. across E, and the power in F (the figures denote the resistances in ohms).

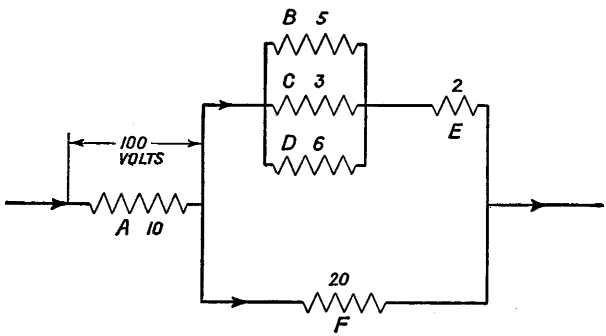


Fig. 11

The total current passes through A,

$$\therefore\ I_T = I_A = \frac{100}{10} = 10 \text{ amperes.}$$

The joint resistance of B, C and D is given by

$$\frac{1}{\frac{1}{5} + \frac{1}{3} + \frac{1}{6}} = 1{\cdot}43 \text{ ohm.}$$

This is in series with E, so that the total resistance of upper branch $=$ $1{\cdot}43 + 2 = 3{\cdot}43$ ohms. Therefore joint resistance of the two main branches is

$$R_J = \frac{1}{\frac{1}{3{\cdot}43} + \frac{1}{20}} = 2{\cdot}93 \text{ ohms.}$$

Next, since the total current is 10 amperes, the P.D. across the parallel branches is

$$I_T R_J = 10 \times 2{\cdot}93 = 29{\cdot}3 \text{ volts.}$$

Hence
$$I_E = \frac{29{\cdot}3}{3{\cdot}43} = 8{\cdot}54 \text{ amperes.}$$

$$I_F = \frac{29{\cdot}3}{20} = 1{\cdot}46 \text{ ampere.}$$

P.D. across $F = 29{\cdot}3$ volts; therefore power $= V_F I_F = 29{\cdot}3 \times 1{\cdot}46 = 42{\cdot}8$ watts.
P.D. across $E = I_E R_E = 8{\cdot}54 \times 2 = 17{\cdot}08$ volts.
Therefore P.D. across B, C and $D = 29{\cdot}3 - 17{\cdot}08 = 12{\cdot}22$ volts.

Hence
$$I_B = \frac{12{\cdot}22}{5} = 2{\cdot}44 \text{ amperes.}$$

$$I_C = \frac{12{\cdot}22}{3} = 4{\cdot}07 \quad \text{,,}$$

$$I_D = \frac{12{\cdot}22}{6} = 2{\cdot}04 \quad \text{,,}$$

23. Grouping of Cells

This subject was of more importance when primary cells of comparatively small size formed the chief source of electrical energy, particularly for laboratory purposes; but it is dealt with here as forming a useful illustration of the application of Ohm's law in simple circuit calculations.

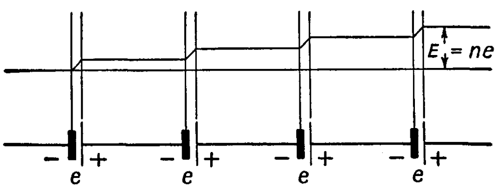


Fig. 12.—Grouping of cells: series

Cells may be connected in series or in parallel or in series-parallel; and when so grouped are spoken of, collectively, as a *battery*. As mentioned in § 19, p. 68, each cell possesses an internal resistance, the value of which depends upon the size and type of cell and, in some cases, is very small indeed.

Series Grouping.

If n cells each having an E.M.F. e and internal resistance r are connected in series (fig. 12), i.e. the $-$ve terminal of one cell connected to the $+$ve terminal of the next, each coulomb of electricity receives e joules of energy in passing through each cell. The total energy received is ne joules, so that the total E.M.F. of the battery is

$$E = ne \text{ volts.}$$

Also, since the cells are in series, the total internal resistance is

$$R = nr \text{ ohms.}$$

The source of E.M.F. has been likened to a pump raising electricity from a lower to a higher potential; a number of cells in series is equivalent to a pump with several stages in each of which the electricity receives a definite increase in energy.

The total E.M.F. of a number of cells in series is the sum of the individual E.M.F.s, and the total internal resistance the sum of the individual resistances.

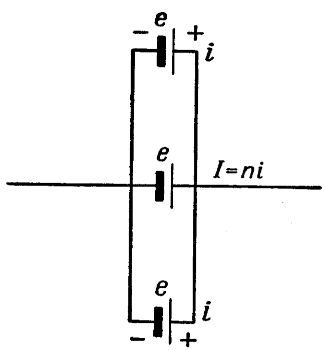


Fig. 13.—Grouping of cells: parallel

Increasing the number of cells in a circuit does not produce a proportionate increase in current; the total E.M.F. is increased but so also is the total internal resistance, and hence the total resistance of the circuit.

If the external resistance is R, the current is

$$I = \frac{ne}{R + nr}.$$

If R is small, an increase in the number of cells produces little increase in current, since the total resistance is greatly increased by the increase in internal resistance.

On the other hand, if R is large, so that nr is almost negligible and $I = \frac{ne}{R}$ (very nearly), the value of the current is almost proportional to the number of cells.

Hence a series grouping is most suitable when the external resistance is large.

Parallel Grouping.

In this method of connection, which can be used only with cells having E.M.F.s of equal value, all the terminals of similar polarity are joined together (fig. 13).

If n cells of E.M.F. e and internal resistance r are connected in parallel, the effect is equivalent to increasing the size of the cell n times. The E.M.F. of the battery is e (that of one cell), while the internal resistance is

$$r_J = \frac{1}{n} r.$$

If one cell is capable of supplying a current of i amperes, then the battery can supply a current of ni amperes.

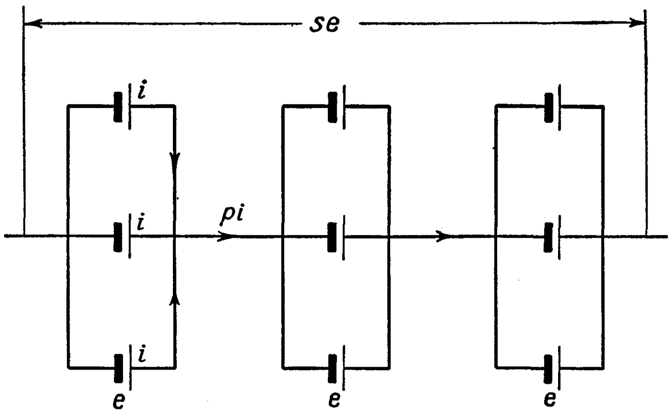


Fig. 14.—Grouping of cells: series-parallel

If the external resistance is R, the current is

$$I = \frac{e}{R + \frac{1}{n} r}.$$

If R is large, the reduction of the internal resistance by the addition of further cells (which do not increase the battery E.M.F.) has little effect on the total resistance and therefore on the current.

On the other hand, if R is small, so that the internal resistance forms a considerable portion of the total, an increase in the number of cells produces a considerable increase in current.

Hence the parallel connection is most suitable when the external resistance is small.

Series-parallel Grouping.

In cases where the E.M.F. of one cell is too low and its internal resistance too high, the cells may be connected in a number of parallel groups and these groups then connected in series (fig. 14).

If n cells are arranged in s groups in series, each group consisting of p cells in parallel, then

$$sp = n.$$

E.M.F. of each group $= e$. Internal resistance of each group $= \frac{1}{p} r$.

Total E.M.F. $= se$. Total internal resistance $= \frac{s}{p} r$,

and, if the resistance of the external circuit is R,

$$I = \frac{se}{R + \frac{s}{p} r}.$$

Example 1.—In the circuit shown in fig. 15, the battery consists of 12 cells arranged in series-parallel, each group containing 3 cells in parallel. The E.M.F. of each cell is 1·5 volt and the internal resistance of each is 0·1 ohm.

Determine (*a*) the battery E.M.F., (*b*) the current, (*c*) the battery P.D.

(*a*) The E.M.F. of each group is that of one cell, i.e. 1·5 volt; the internal resistance of each group is $\frac{0{\cdot}1}{3} = 0{\cdot}033$ ohm; and since there are four groups in series,

The E.M.F. of the battery $E = 4 \times 1{\cdot}5 = 6$ volts.

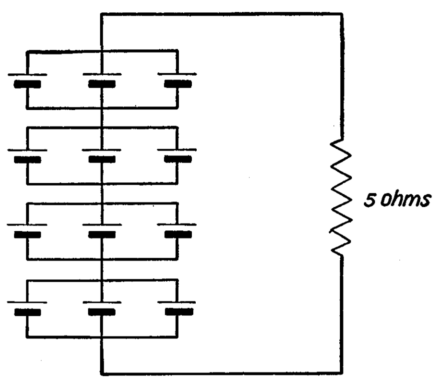


Fig. 15

(*b*) The internal resistance $R_B = 4 \times 0{\cdot}033 = 0{\cdot}13$ ohm.

$$\text{The total resistance} = 0{\cdot}13 + 5 = 5{\cdot}13 \text{ ohms}$$

$$\therefore \text{ The current} = \frac{E}{R_T} = \frac{6}{5{\cdot}13} = 1{\cdot}17 \text{ ampere.}$$

(*c*) The P.D. of the battery is the E.M.F. less the internal "resistance drop":

$$V_B = E - IR_B$$
$$= 6 - (1{\cdot}17 \times 0{\cdot}13) = 5{\cdot}85 \text{ volts.}$$

Example 2.—Twelve cells, each of E.M.F. 1·5 volts and internal resistance 0·1 ohm, are arranged in series-parallel so as to give a total E.M.F. of (*a*) 4·5 volts, (*b*) 6 volts. Calculate the current when connected to an external resistance of 2 ohms.

(*a*) Three groups, each of four cells in parallel.

$$\text{E.M.F.} = 4{\cdot}5 \text{ volts; internal resistance } \frac{0{\cdot}1}{4} \times 3 = 0{\cdot}075 \text{ ohm.}$$

$$\text{Total resistance} = 2 + 0{\cdot}075$$
$$= 2{\cdot}075 \text{ ohms.}$$

$$\text{Current} = \frac{4{\cdot}5}{2{\cdot}075} = 2{\cdot}17 \text{ amperes.}$$

(*b*) Four groups, each of three cells in parallel.

$$\text{E.M.F.} = 6 \text{ volts; internal resistance} = \frac{0{\cdot}1}{3} \times 4 = 0{\cdot}133 \text{ ohm.}$$

$$\text{Total resistance} = 2 + 0{\cdot}133$$
$$= 2{\cdot}133 \text{ ohms.}$$

$$\text{Current} = \frac{6}{2{\cdot}133} = 2{\cdot}81 \text{ amperes.}$$

24. Principle of Wheatstone Bridge

The principle of the Wheatstone bridge, which is a direct deduction from Ohm's law, is of great importance as forming the basis of important methods of measuring resistance.

The bridge consists of four resistances (usually referred to as the arms of the bridge) arranged as shown in fig. 16. A galvanometer* is connected across the points B and D, and a cell across the points A and C.

Assume, in the first place, that the galvanometer is disconnected. At A the circuit splits into two parallel branches which reunite at C; a current entering at A divides, and part passes through the upper branch, consisting of R_a and R_b in series, while the remainder flows through the lower branch, composed of R_s and R_x in series. There is therefore a continuous fall of potential along each branch, and since the points A and C are common to both, the total fall is the same in each case.

* A galvanometer is an instrument used for indicating small currents (see Chap. X).

If this portion of the circuit is imagined as a river, dividing at A and reuniting at C, it is clear that since the fall in level is the same in each branch, wherever a point is chosen on one branch, a corresponding point at the *same level* can be found on the other; and *no water will flow through a pipe line laid between them.*

Similarly, whatever point is considered in the branch ADC, a corresponding point at the same potential can be found in the other branch ABC; if these are connected, no current will flow and there will be no deflection of the needle of a galvanometer included in the connection. When this is the case, the bridge is said to be *balanced.*

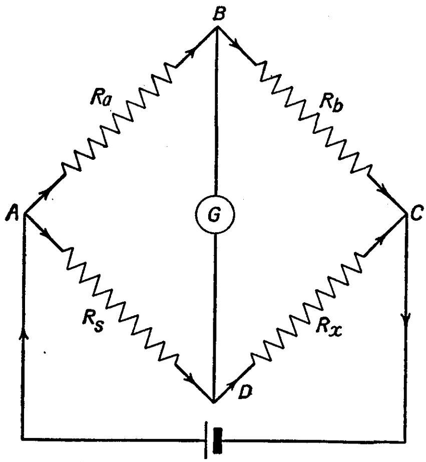


Fig. 16.—Principle of Wheatstone bridge

Suppose B and D are such corresponding points. Since no current passes along BD, the current in R_a must be the same as that in R_b, and the current in R_s the same as that in R_x, i.e.

and
$$\left.\begin{aligned} I_a &= I_b \\ I_s &= I_x \end{aligned}\right\} \quad \ldots \ldots \quad (7)$$

(It should be noted that I_a may have a value very different from I_s.)

Also, if B and D are at the same potential, the fall in potential or P.D. between A and B, V_a, must be the same as that between A and D, V_s; and similarly, the P.D. between B and C, V_b, is the same as that between D and C, V_x; i.e.

$$V_a = V_s, \quad V_b = V_x.$$

But

$$V_a = I_aR_a, \quad V_b = I_bR_b, \text{ etc. (Ohm's law).}$$

Hence

$$I_a R_a = I_s R_s,$$

and

$$I_b R_b = I_x R_x.$$

By division,

$$\frac{I_a R_a}{I_b R_b} = \frac{I_s R_s}{I_x R_x}.$$

But, by equation (7),

$$I_a = I_b, \quad \text{and} \quad I_s = I_x.$$

Therefore,

$$\frac{R_a}{R_b} = \frac{R_s}{R_x}, \qquad \text{. . . . . (8)}$$

or

$$R_x = R_s \cdot \frac{R_b}{R_a}. \qquad \text{. . . . . (9)}$$

This relationship between the resistances of the arms of a balanced bridge enables the value of one of them (R_x) to be determined, provided we know either:

(i) the value of all the others, or

(ii) the value of *one* (e.g. R_s) and the *ratio* of the other two (R_b/R_a).

The apparatus used and the adjustments by which a balance is obtained are described in detail in Chapter VII.

25. Principle of Potentiometer

This principle forms the basis of methods used for the accurate measurement of E.M.F. and hence of current and resistance.

If a steady current flows along a wire of uniform section and composition, there is a uniform fall of potential along its length, and the P.D. between any two points is proportional to the distance between them (since $R \propto l$, § 7, p. 98), i.e.

$$\frac{V_{AB}}{V_{AC}} = \frac{ab}{ac} = \frac{AB}{AC} \text{ (fig. 17}a\text{).}$$

(The student should verify this by experiment.)

In fig. 17*b*, a steady current flows along such a wire from the cell *C*. A galvanometer, in series with a second cell *D*, is connected between *A* and a sliding contact *S*. The arrangement is such that the terminals of both cells which are connected to *A* are of the same polarity, which in the figure is assumed to be positive.

Assuming, in the first place, that the cell *D* is omitted, *A* is at a higher potential than *S* (since the current *AB* is flowing from *A* to *B*) and therefore a current will flow through the galvanometer from left to right, the galvanometer connection, in fact, forming a branch circuit in parallel with the portion of the wire AS. On the other hand, if the cell C were disconnected, so that this P.D. disappeared, the E.M.F. of D would cause a current to pass through the galvanometer from right to left. When both cells are in circuit, the direction of the current in the galvanometer depends upon whether the P.D. between A and S is greater or less than the E.M.F. of D.

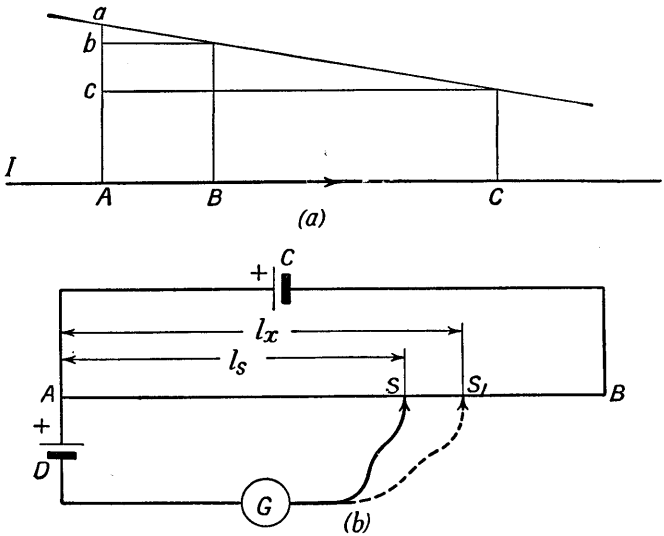


Fig. 17.—Principle of potentiometer

The value of the P.D. can be varied by moving the slider S, so that a point may be found at which the two are equal and no current flows in the galvanometer, as is indicated by the absence of any deflection. Suppose this point is at S. If another cell D_1, having a somewhat greater E.M.F., is now substituted for D, the point of balance is found to be at S_1, a point on the wire somewhat farther from A.

Hence,

$$\frac{\text{E.M.F. of } D}{\text{E.M.F. of } D_1} = \frac{AS}{AS_1},$$

so that if one E.M.F. (E_s) is known and the lengths AS and AS_1 are carefully measured, the value of the other E.M.F. (E_x) can be found. Thus

$$\frac{E_s}{E_x} = \frac{AS}{AS_1} = \frac{l_s}{l_x};$$

$$\therefore\ E_x = E_s \frac{l_x}{l_s}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

In practice the cell D is a Weston standard cell (§ 18, p. 131), the E.M.F. of which is known with great accuracy.

It should be noticed that this is a method of measuring the actual value of the E.M.F. At the point of balance no current is taken from the cell, so that there is no internal resistance drop; whereas when a voltmeter is used, the reading given is that of the P.D., which, since a small current is taken, is less than the true E.M.F. by the amount of the internal resistance drop.

Potentiometer methods are capable of very great accuracy and may also be used for the measurement of current and resistance. The current to be measured is passed through a standard resistance and the P.D. between its terminals measured by potentiometer; then $I = v/R$. Resistance may be measured by comparison with a known resistance. The two are connected in series and the P.D. across each measured by potentiometer. Then $R_x/R_s = v_x/v_s$.

Alternating current potentiometers are also used for the accurate measurement of inductance, capacitance, and frequency.

26. Kirchhoff's Laws

The determination of the current distribution in more complicated circuits or networks is facilitated by the use of two generalized relationships known as Kirchhoff's laws. *Kirchhoff's First Law* follows from the fact that electricity cannot accumulate at any point in a circuit,

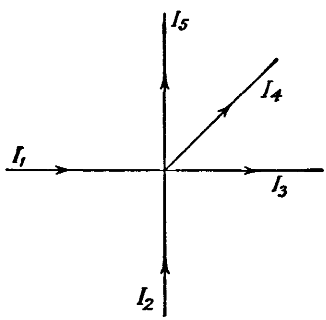


Fig. 18.—To illustrate Kirchhoff's first law

and is almost self-evident. If several conductors meet at a point, the total current flowing *away* from the point must be equal to the total current flowing *towards* the point. If the current flowing towards the point is assumed positive and that flowing away negative, the law can be expressed in the form:

The algebraic sum of the currents, in conductors meeting at a point, is zero.

Thus, referring to fig. 18,

$$I_1 + I_2 - I_3 - I_4 - I_5 = 0,$$

or

$$I_1 + I_2 = I_3 + I_4 + I_5.$$

In a simple circuit containing resistance only, Ohm's law states that

$$E = IR,$$

i.e. the E.M.F. is equal to the product of the current and the resistance of the circuit. In many circuits, however, where some of the energy is converted into forms other than heat, the current is opposed by one or more back- or counter-E.M.F.s in addition to the resistance; and in such circuits the *forward* or driving E.M.F. must exceed the sum of the counter-E.M.F.s by an amount sufficient to maintain the current, i.e.

$$\Sigma E_f - \Sigma E_b = \Sigma IR,$$

or

$$\Sigma E = \Sigma IR,$$

where ΣE is the *algebraic* sum of all the E.M.F.s and ΣIR that of the products of current and resistance. This is *Kirchhoff's Second Law*:

In any closed circuit the algebraic sum of all the E.M.F.s is equal to the algebraic sum of the products of current and resistance in the various portions of the circuit.

Looked at from the energy point of view, since energy cannot be stored *in* the circuit, all the energy taken in by each coulomb at points where there is a forward E.M.F. is given out in other portions of the circuit. The total energy received by each coulomb is represented by ΣE_f: of this, the portion represented by ΣE_b is converted into various other forms of energy depending on the nature of the back-E.M.F., while the remainder is turned into heat in overcoming the resistance of the circuit.

Kirchhoff's Second Law can be applied equally well in the case of a closed mesh (such as often occurs in complicated networks) made up of conductors carrying currents in which there are no E.M.F.s. In such cases

$$\Sigma IR = 0,$$

i.e. the algebraic sum of all the resistance drops, taken in order round the mesh, is zero.

It has been seen that there must be a P.D. between the ends of any portion of a circuit in order to maintain the current in that portion; and referring to the simple circuit of fig. 6*b*, the P.D. between the ends of the resistance R is given by

$$V_R = IR.$$

But if this portion of the circuit contains, in addition, an opposing E.M.F. E_b as shown in fig. 19, the P.D. must exceed this by an amount sufficient to maintain the current, i.e.

$$V_R = E_b + IR.$$

For this circuit as a whole, Kirchhoff's Second Law states that

$$E - E_b = Ir + Ir_l + IR,$$

where r is the internal resistance of the battery, and r_l that of the leads (R includes the internal resistance of the cell producing E_b).

This equation may be written in the form

$$E_b + IR = V_R = E - Ir - Ir_l,$$

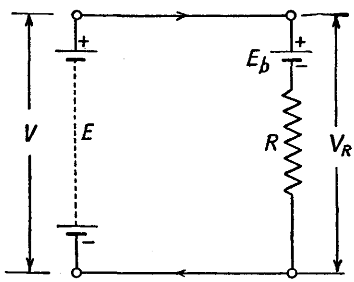


Fig. 19

where V_R is the P.D. between the load terminals, or in the form

$$E_b + IR + Ir_l = V = E - Ir,$$

where V is the P.D. between the battery terminals.

From this it follows that,

The P.D. between two points in a closed circuit is given by the algebraic sum of the E.M.F.s and the resistance drops in either of the two portions into which the points divide the circuit.

Looked at from the energy point of view, the terminal P.D. of the battery, V, represents the energy transferred by each coulomb from the battery to the external circuit, i.e. the total energy received less that turned into heat inside the battery; while the P.D. between the load terminals, V_R, represents the energy given up by each coulomb in the load portion of the circuit, which is less than that represented by V by the amount turned into heat in the leads.

Example.—To illustrate the use of Kirchhoff's laws.

Find the current in each branch of the bridge network shown in fig. 20. Internal resistances of battery and cell may be neglected.

Mark the probable directions of the currents in each branch: a wrong direction will be indicated by a negative value in the final result.

Use the minimum number of symbols to represent the current in each branch, e.g. if I_1, I_2 and i represent the currents in AB, AD and DB respectively, the currents in BC and DC can be expressed in terms of these, using Kirchhoff's First Law.

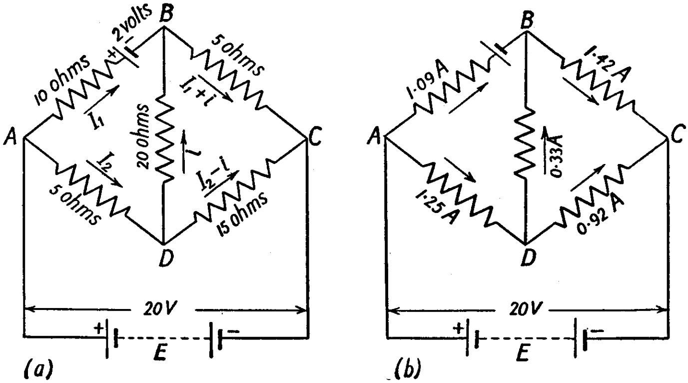


Fig. 20

In the circuit $EABC$, from Kirchhoff's Second Law,

$$20 - 2 = 10I_1 + 5(I_1 + i),$$

or
$$15I_1 + 5i = 18. \quad \ldots \ldots \quad (a)$$

In the circuit $EADC$,

$$20 = 5I_2 + 15(I_2 - i),$$

or
$$20I_2 - 15i = 20. \quad \ldots \ldots \quad (b)$$

In the mesh BCD, in which there is no E.M.F.,

$$20i + 5(I_1 + i) - 15(I_2 - i) = 0,$$

or
$$5I_1 - 15I_2 + 40i = 0. \quad \ldots \ldots \quad (c)$$

From (a), $i = 3{\cdot}6 - 3I_1$, and substituting this in (c),

$$115I_1 + 15I_2 - 144. \quad \ldots \ldots \quad (d)$$

From (a) and (b), $$45I_1 + 20I_2 = 74. \quad \ldots \ldots \quad (e)$$

From (d) and (e), $I_1 = 1{\cdot}09$ amperes,

$I_2 = 1{\cdot}25$ amperes, and therefore

$i = 0{\cdot}33$ ampere.

This gives the current distribution shown in fig. 20*b*.

EXAMPLES

1. The P.D. between the ends of a coil in which a current of 15 amperes is flowing is 100 volts. Calculate (*a*) the resistance of the coil, (*b*) the power input (in kW.), (*c*) the energy input in 12 hours (in watt-hours).

2. An electric motor takes a current of 15 amperes at 500 volts. If the efficiency of the machine is 90 per cent, what is the output in horse-power?

3. An electric motor has an efficiency of 92 per cent. When the output is 100 h.p., calculate:

(*a*) the power input in kW.;
(*b*) the hourly cost of electrical energy at $\frac{1}{2}d.$ per unit.

4. In a certain building there are:

20–100 watt lamps,
40–60 „ „
30–40 „ „

Calculate the total energy consumption per week if all the lights are in use for 3 hours per day.

5. A battery has an E.M.F. of 6 volts, but when connected to a coil having a resistance of 20 ohms, the P.D. between its terminals is 5·5 volts. What is the internal resistance of the battery?

6. A dry battery having an E.M.F. of 3 volts is connected directly to a lamp which takes 2 watts. If the current is 0·75 ampere, what is the internal resistance of the dry battery, and what fraction of the total energy output of the battery is used in the lamp?

7. A battery having an E.M.F. of 100 volts and internal resistance of 0·1 ohm supplies a motor through leads having a resistance of 0·05 ohm. If the motor efficiency is 85 per cent, calculate the output in horse-power when the current taken is 50 amperes.

8. Three coils have resistances of 10, 15 and 20 ohms. Find (*a*) total resistance when connected in series; (*b*) joint resistance when connected in parallel.

9. Explain what is meant by the electromotive force of a voltaic cell.

A voltmeter of 100 ohms resistance reads 1·35 volt when connected to the terminals of a certain cell. This reading falls to 0·93 volt when a 10-ohm coil is also connected across the cell terminals. Find the E.M.F. and internal resistance of the cell. [Lond. Inter. B.Sc.(Eng.).]

10. A battery of 10 cells, each of E.M.F. 2 volts and internal resistance 0·1 ohm, is connected to a circuit consisting of a variable resistance R in series with a group of three resistances of 1, 2, and 3 ohms connected in parallel.

What must be the value of R in order that the current in the 2-ohm resistance may be 1 ampere?

When R has this value, what is the P.D. across the battery, the parallel group and the variable resistance? [Grad. I.E.E.]

11. State Ohm's law.

When a cell is connected to a 5-ohm resistance, a high resistance voltmeter connected across its terminals reads 0·94 volt. When a second 5-ohm resistance is placed in parallel with the first, the voltmeter reading is 0·83 volt.

What would be the voltmeter reading if (*a*) a third 5-ohm coil was connected in parallel, (*b*) all the coils were disconnected? [Lond. Genl. Sch.]

12. A certain relay has 2 coils each of 100 ohms resistance, and is placed in a circuit of which the resistance is already 150 ohms and in which the E.M.F. is 30 volts.

Calculate the current in the circuit when the relay coils are connected (*a*) in series, (*b*) in parallel.

13. Define the units in which electrical power and energy are measured.

An electric heating coil having, when in use, a resistance of 75 ohms costs $\frac{3}{4}d.$ per hr. for electricity. If two wires touch so that the resistance is reduced to 65 ohms, what will be the cost of running if the applied voltage remains the same? Give reasons. [Lond. Genl. Sch.]

14. A battery consists of 12 cells each of E.M.F. 1·5 volt and resistance 0·1 ohm. When arranged so that 4 groups (each containing 3 cells in parallel) are in series, and connected to a certain coil, the total current is 5·3 amperes.

What will be the total current if the battery is rearranged so that 3 groups (each containing 4 cells in parallel) are in series and it is then connected to the same coil?

15. In a Wheatstone bridge network $ABCD$, the resistances are,

$$AB = 10,\ BC = 15,\ AD = 20,\ DC = 25 \text{ and } BD = 10 \text{ ohms.}$$

If the P.D. between A and C is 20 volts, find the current distribution in the network, and the resistance between A and C.

16. In a Wheatstone bridge $ABCD$ the resistances of the arms are,

$$AB = 750,\ BC = 750,\ AD = 500,\ DC = 490 \text{ ohms.}$$

If the galvanometer connected between B and D has a resistance of 500 ohms and the P.D. between A and C is 2 volts, find the current in the galvanometer.

17. Two 250-ohm coils are connected in series across constant voltage mains. What is the resistance of a voltmeter which when connected across one of the coils, reduces the voltage across it by 1 per cent? Find also the percentage change which would occur in this P.D. if the voltmeter had a resistance equal to one-tenth that of the coil. [Grad. I.E.E.]

18. Two batteries connected in parallel have electromotive forces of 80 and 90 volts and internal resistances of 0·2 and 0·22 ohm respectively. The combination is connected through a 5-ohm resistor to a 200-volt d.c. supply, the positive poles of the batteries being connected to the positive poles of the supply. Find the magnitude and direction of the current in each battery and of the current from the supply.

Calculate also the power dissipated in the 5-ohm resistor. [J.S.A.,* 1947.]

19. A torch battery consists of two dry cells in series, each with an outer casing made of 11 grammes of sheet zinc. The lamp in the torch is labelled 3V, 0·3A. Estimate the total number of hours of use which may be obtained from the battery, if half the zinc finally remains unused, and if a possible fall in battery voltage may be ignored.

At a battery price of 8*d.*, calculate the ratio between the cost per kilowatt-hour of this energy, and the cost of a public electric supply at 4*d.*/kWh.

(Electrochemical equivalent of zinc $= 3{\cdot}37 \times 10^{-4}$ gramme per coulomb.)

[J.S.A., 1948.]

* J.S.A. stands for Joint Section A Examination of the Institutions of Civil Engineers and Electrical Engineers.

CHAPTER VII

Resistance and the Heating Effect—Thermo-Electricity

1. The Measurement of Resistance

The determination of the resistance of a conductor or circuit is the most common of all measurements which have to be made either in the laboratory or in industry. The method employed depends upon the value of the resistance and the accuracy required; those in most frequent use are

(*a*) The ammeter and voltmeter method, which is a direct application of Ohm's law.

(*b*) The Wheatstone bridge method, carried out with a slide-wire bridge, a post-office box, or elaborations of these instruments.

(*c*) Comparison with a known resistance by means of a potentiometer.

(*d*) Method of substitution.

2. The Ammeter and Voltmeter Method

This is the general industrial method when very great accuracy is not required. By choosing suitable instruments, it can be used over a wide range; and, particularly in the case of machine windings, the necessary readings may be obtained while the machine is running and during the progress of other tests.

The method consists in measuring simultaneously the current (I) passing through the resistance and the P.D. (V) between its ends. Then, by Ohm's law,

$$R = \frac{V}{I}.$$

In order to reduce experimental errors, several sets of readings are obtained, using different values of current, from which the mean value of the resistance is calculated.

The accuracy of the results obviously depends upon that of the instruments used (see Chapter X); but in addition several other precautions are necessary.

(1) Resistance depends upon temperature (§ 12, p. 104), and the passing of a current raises the temperature of the conductor; so that when the value of the resistance at the room temperature is required, the current used should be small and the readings taken as quickly as possible.

(2) The best arrangement of the instrument connections depends upon the value of the resistance. The natural arrangement is as shown in fig. 1*a*, in which the voltmeter is connected directly across the resistance R. Most voltmeters require a small current to operate them, so that this is equivalent to placing a high resistance in parallel with R. The ammeter indicates the total current, which is greater than the current through R, so that the value of R calculated from the readings is lower than the true value (it is, of course, the joint resistance of R and the voltmeter in parallel). The importance of the error clearly depends upon the relative values of the voltmeter resistance and the resistance to be measured. If the latter is small so that the voltmeter current is a very small fraction of the total, the error is negligible unless great accuracy is required. Hence the connections in fig. 1*a* are suitable if the value of R is small compared with the resistance of the voltmeter.

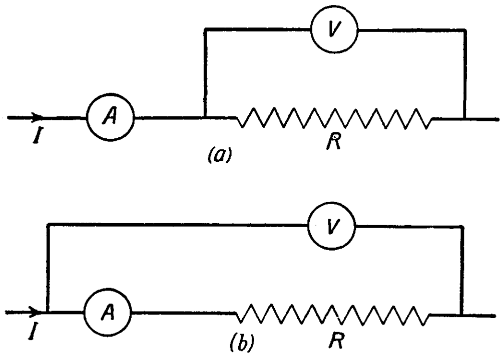


Fig. 1.—Measurement of resistance—ammeter and voltmeter method

When R is large, the better arrangement is to connect the voltmeter across both ammeter and resistance (fig. 1*b*). In this case the voltmeter current is not included in the ammeter readings. On the other hand, the voltmeter reading includes the P.D. across the ammeter, but since this has essentially a low resistance (§ 2, p. 189) the error is usually negligible.

When the resistance of the instruments is known, corrections can be made for the voltmeter current in the first case and for the P.D. across the ammeter in the second case, so that the resistance can then be determined with an accuracy limited only by that of the instruments and of the observations, say within 2 per cent, using industrial-grade instruments.

3. Wheatstone Bridge Methods

When a higher degree of accuracy is required than is readily obtainable by the previous method, the measurement may be made either with the slide-wire bridge or the post-office box. Both these instruments make use of the Wheatstone bridge principle, but the arrangement and method of operation are somewhat different in the two cases. The only instrument employed is a galvanometer, and the accuracy of the results depends upon that of the resistances forming the other arms of the bridge and upon the sensitivity of the galvanometer.

Slide-wire Bridge (Metre Bridge).

This consists of a straight wire of some resistance alloy, of uniform section and usually either 1 metre or $\frac{1}{2}$ metre long, stretched between two terminal blocks A and C (fig. 2). A sliding contact B with a spring key is arranged so that contact can be made at any point on the wire, and the position of this point read off on a scale placed alongside the wire. Between A and a third terminal block D is connected a resistance

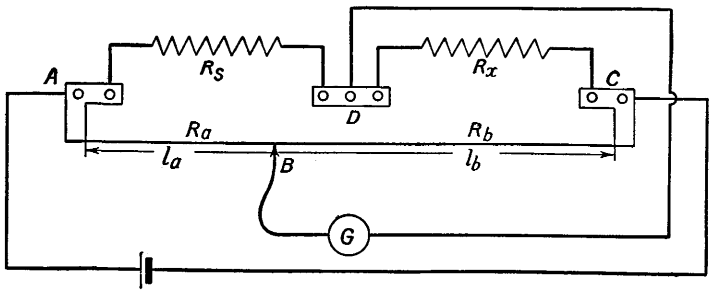


Fig. 2.—Slide-wire bridge

of known value, and between D and C the resistance to be measured. The galvanometer is connected between D and B and a cell between A and C. A reference to fig. 16, p. 80, will show that the arrangement forms a Wheatstone bridge, corresponding points being similarly lettered.

In order to balance the bridge the resistances of the arms R_a and R_b are varied simultaneously by moving the slider along the wire. As the balance is approached the galvanometer deflection decreases until at the balance point it becomes zero; beyond this it reverses.

When balanced it follows (§ 24, p. 79) that

$$\frac{R_a}{R_b} = \frac{R_s}{R_x} \quad \text{or} \quad R_x = R_s \frac{R_b}{R_a}. \quad . \quad . \quad . \quad . \quad (1)$$

But since the wire is uniform, its resistance is proportional to its length, so that

$$R_x = R_s \frac{l_b}{l_a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence a knowledge of the actual resistance of the slide wire is not required.

In order to secure accurate results several precautions are necessary:

(1) To avoid undue heating of the slide-wire and resistance coils, a key is often placed in the battery circuit. This should be pressed *before* the galvanometer key, so that if R_x is inductive (§ 19, p. 166) the current may attain a steady value before the galvanometer is connected.

(2) If the galvanometer is not very sensitive, the balance point may be indefinite. In such cases it is advisable to note the points which give the least observable deflection in either direction and to take the balance point as being the mean of the two.

(3) To facilitate adjustment the galvanometer should be so connected that the direction of the deflection indicates that in which the slider should be moved to reduce the deflection.

(4) To avoid damage to the galvanometer, the key should be *tapped* very lightly until near the balance point: the galvanometer is often protected by a *shunt* (§ 14, p. 200), which is removed before the final balance is made.

(5) Since the ratio $\frac{R_b}{R_a}$ can be varied between 0 and ∞, a balance point can always be obtained whatever the values of R_s and R_x, but errors of observation have least effect when it is at the middle of the scale. This occurs when $R_s = R_x$, so that the value of R_s should be as near that of R_x as possible. If R_x is quite unknown, a preliminary experiment with any value of R_s will enable a suitable value of R_s to be chosen for a more accurate determination.

(6) It is advisable to repeat the measurement with R_s and R_x interchanged and to take the mean of the results. It should be noted that the effective value of R_s and the measured value of R_x include the resistance of the connections to the bridge terminals, so that the latter should either be measured separately or be negligible.

When used carefully, a good slide-wire bridge gives rapid and accurate measurements of resistance over a wide range, and in a somewhat more elaborate form, such as the Carey-Foster bridge, is capable of great accuracy.

Post-office Box.

This piece of apparatus, while more portable than the slide-wire bridge, cannot be balanced so rapidly, and is not capable of such high accuracy. The two arms R_a and R_b are not continuously variable, but can be set to give a number of definite ratios, and the bridge is balanced by adjusting the value of the R_s arm.

The three arms—R_a, R_b, known as the ratio arms, and R_s, the variable arm—are made up of a number of coils of known resistance, connected to massive brass blocks mounted on the top of the box which contains them (fig. 3*a*). Between each pair of blocks is a tapering hole provided with a tapered plug ground to fit (fig. 3*b*). When the plug is in position, the resistance between the adjacent blocks is negligibly small, but when it is removed the resistance is that of the coil connected between them. Hence by removing or replacing a plug the resistance of the arm is increased or decreased by a known amount. The resistances of the three coils in each ratio arm are 10, 100, and 1000 ohms, so that R_b/R_a may have values of 100, 10, 1, $\frac{1}{10}$, $\frac{1}{100}$, while those in the variable arm are so arranged (like a set of weights) that R_s may be adjusted in 1-ohm steps from 1 to 11,110 ohms. Two keys are provided, in the battery and galvanometer circuits respectively, the connections, indicated by the broken lines in fig. 3*a*, being made inside the box.

A reference to fig. 16 (p. 79) will show that here again the arrangement forms a Wheatstone bridge; corresponding points in the two figures are similarly lettered. (In some P.O. boxes the terminals are

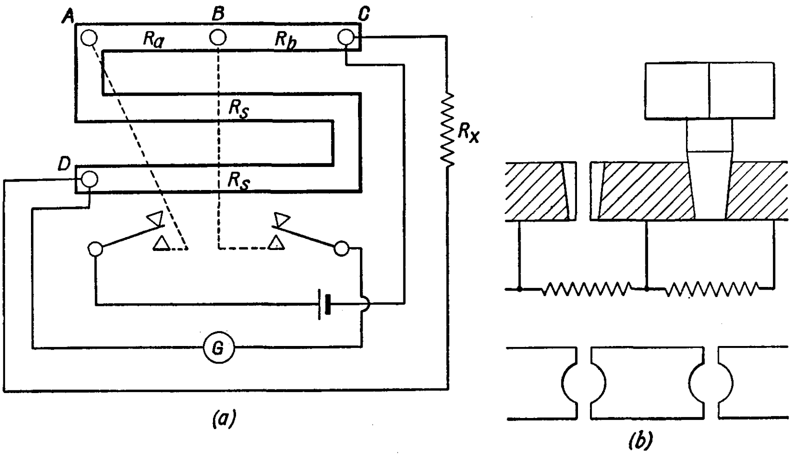


Fig. 3.—Post-office box

unmarked, while in others the old Post Office nomenclature is retained, but the student should learn to connect up the instrument from the schematic diagram of fig. 16, p. 79.) Hence, when the bridge is balanced,

$$R_x = R_s \cdot \frac{R_b}{R_a}. \quad \ldots\ldots\ldots \quad (3)$$

If R_b and R_a are equal, $R_b/R_a = 1$, so that $R_x = R_s$; since the smallest unit in the variable arm is 1 ohm, R_x can be measured only to the nearest ohm and it is usually impossible to obtain an exact balance. When R_x is less than 1111 ohms, greater accuracy is obtained by making $R_b/R_a = \frac{1}{10}$ ($R_b = 10$, $R_a = 100$ or $R_b = 100$, $R_a = 1000$). Then $R_x = \frac{1}{10} R_s$ and can be obtained to the nearest tenth of an ohm;

e.g. if $\qquad R_s = 9722$ ohms, $R_x = 972{\cdot}2$ ohms.

If R_x is less than 111 ohms and $R_b/R_a = \frac{1}{100}$ ($R_b = 10$, $R_a = 1000$), then $R_x = \frac{1}{100} R_s$ and can be measured to the nearest hundredth of an ohm;

e.g. $\qquad R_s = 3425$ ohms, $R_x = 34{\cdot}25$ ohms.

Conversely, if R_x is greater than 11,110 ohms, R_b/R_a can be made either 10 or 100, in which case $R_x = 10R_s$ or 100 R_s.

The theoretical range of measurement extends therefore from $1 \times \frac{1}{100} = 0{\cdot}01$ ohm to $11{,}110 \times 100 = 1{,}111{,}000$ ohms, but the accuracy obtainable is not the same throughout.

The following are some points to be noted in using the apparatus:

(1) The battery key should always be pressed before the galvanometer key to allow the current to become steady. The coils in the instrument are wound non-inductively (§ 21, p. 169) but the resistance under test may be inductive.

(2) To facilitate adjustment a mark should be made on each side of the galvanometer indicating whether the direction of deflection corresponds with the value of the R_s arm being too large or too small. This can be determined by withdrawing the plug marked " Infinity " which breaks the R_s arm; the direction of deflection then corresponds with R_s being too large.

(3) If the value of R_x is entirely unknown, a preliminary test should be made with $R_b/R_a = 1$. This will give some idea of its value, and the determination can then be repeated with a suitable ratio.

(4) If no point of balance can be obtained, and a change of 1 ohm in the value of R_s causes the deflection to reverse, the correct value of R_s may be estimated from the magnitude of the two deflections. For example, if $R_s = 41$ ohms produces a deflection of 10 divisions to the left, and $R_s = 42$ ohms produces a deflection of 15 divisions to the right, the deflection would be reduced to zero when

$$R_s = 41 + \frac{10}{10 + 15} = 41{\cdot}4 \text{ ohms.}$$

(5) The terminal marking on some boxes is such that the positions of the battery and galvanometer are interchanged, but this does not affect the working of the bridge or the relationship of the resistances.

4. Comparison of Resistances by Potentiometer

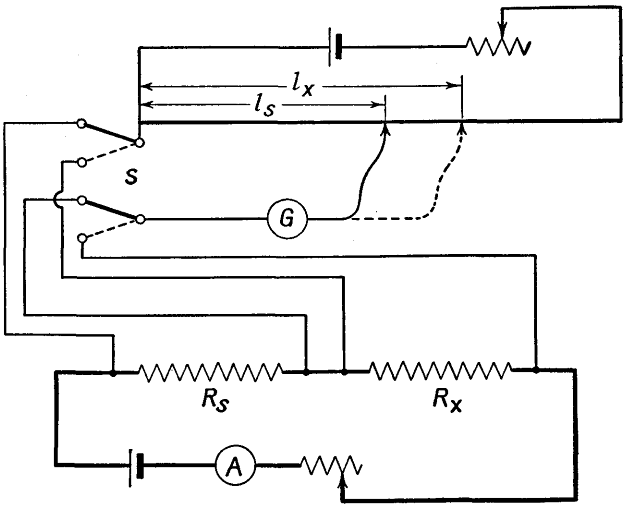
When great accuracy is required in the measurement of a low resistance, the unknown resistance may be compared with a standard resistance by measuring with a potentiometer the P.D. across each when the same current is flowing. The two resistances R_s and R_x are connected in series, and a steady current is passed through them (fig. 4). By means of a multiple contact switch S, the P.D. across each in turn is measured quickly. Then

$$\frac{R_s}{R_x} = \frac{V_s}{V_x}$$

or

$$R_x = R_s \frac{V_x}{V_s} = R_s \frac{l_x}{l_s}. \qquad (4)$$

The current is adjusted to a suitable value by means of a variable resistance and ammeter. It is of course essential that the value of this current and that in the potentiometer wire should remain constant during the readings of V_s and V_x. To check this, the first P.D. is measured again after the second has been measured; if there is no change, it can be assumed that the currents have remained constant.



·Fig. 4.—Comparison of resistances by potentiometer

5. Method of Substitution

This method, although simple in principle, is particularly suitable for the measurement of high resistances, and is used in a modified form in the determination of the insulation resistance of cables.

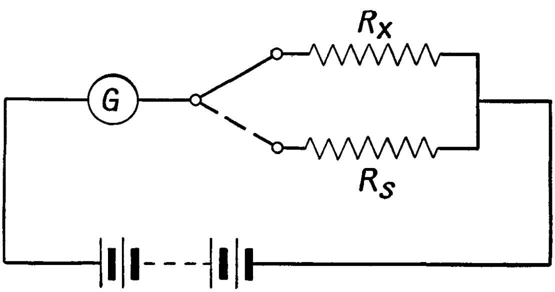


Fig. 5.—Measurement of resistance—method of substitution

The unknown resistance, in series with an ammeter or galvanometer, is connected to a battery of suitable E.M.F. and the deflection of the galvanometer noted (fig. 5). By means of a two-way switch, a variable known resistance is substituted for the unknown resistance and adjusted until the same deflection is produced, i.e. the current in the circuit is the same. If the battery P.D. is unchanged, it is clear that the value of the known resistance must be the same as that of the unknown resistance.

Alternatively, if a suitable variable resistance is not available, a known *fixed* resistance may be substituted for the unknown resistance and the deflection noted. (In this case the galvanometer, if used, must be one in which the deflection is proportional to the current.) Then, if

δ_x = deflection with unknown resistance (R_x),
δ_s = deflection with known resistance (R_s),

and if the battery P.D. has remained constant, we have

$$\frac{\delta_x}{\delta_s} = \frac{I_x}{I_s} = \frac{V}{R_x} \cdot \frac{R_s}{V} = \frac{R_s}{R_x}.$$

$$\therefore\ R_x = R_s \frac{\delta_s}{\delta_x}. \qquad \qquad (5)$$

Example 1.—In a slide-wire bridge of which the wire is 100 cm. long and the standard resistance R_s is 100 ohms, a balance is obtained at a point 47·15 cm. from the end to which the unknown resistance R_x is connected. On interchanging R_s and R_x, the balance point occurs 52·75 cm. from the same end.

What is the value of R_x?

1st determination: $R_s = 100$ ohms,

$$l_b = 47{\cdot}15 \text{ cm.},\ l_a = 100 - 47{\cdot}15 = 52{\cdot}85 \text{ cm.};$$

$$\therefore\ R_x = 100 \times \frac{47{\cdot}15}{52{\cdot}85} = 89{\cdot}21 \text{ ohms}.$$

2nd determination: $R_s = 100$ ohms,

$$l_b = 100 - 52{\cdot}75 = 47{\cdot}25 \text{ cm.},\ l_a = 52{\cdot}75 \text{ cm.};$$

$$\therefore\ R_x = 100 \times \frac{47{\cdot}25}{52{\cdot}75} = 89{\cdot}57 \text{ ohms}.$$

Mean value of $R_x = 89{\cdot}39$ ohms.

Example 2.—In measuring an entirely unknown resistance with a post-office box, it is found that with a ratio $R_b/R_a = 1$, $R_s = 420$ was too small and $R_s = 421$ too large.

Describe how the value of R_x can be obtained more accurately.

The preliminary experiment shows that R_x lies between 420 and 421 ohms. If $R_b = 100$, $R_a = 1000$, $R_x = \frac{1}{10}R_s$; and if a balance is obtained when $R_s = 4203$ (say),

$$R_x = \frac{1}{10} \times 4203 = 420{\cdot}3 \text{ ohms}.$$

Alternatively, suppose that, when $R_b/R_a = 1$, $R_s = 420$ produces a deflection of 3 divisions to the left and $R_s = 421$ produces a deflection of 7 divisions to the right, then

$$R_s = 420 + \frac{3}{10} = 420{\cdot}3 \text{ ohms}.$$

Example 3.—An unknown resistance in series with a proportional galvanometer was connected to a battery and the deflection produced was 12 divisions. On substituting a known resistance of 1000 ohms, the deflection was increased to 17 divisions.

What is the value of the unknown resistance?

Assuming the battery P.D. was constant,

$$\frac{\delta_x}{\delta_s} = \frac{R_s}{R_x}, \quad R_x = R_s \frac{\delta_s}{\delta_x} = 1000 \times \frac{17}{12} = 1417 \text{ ohms.}$$

6. Insulation Resistance

In an electric circuit, the flow of the current is confined to the prescribed path by covering the conductor with, or supporting it on, an insulating material.

For reasons of safety most electrical supply systems are connected to the earth at *one* point. If any other point in the circuit is connected to earth through faulty insulation, a low-resistance path is provided through which a large leakage current may pass (fig. 6); and since the resistance of the earth path is usually small, the current will

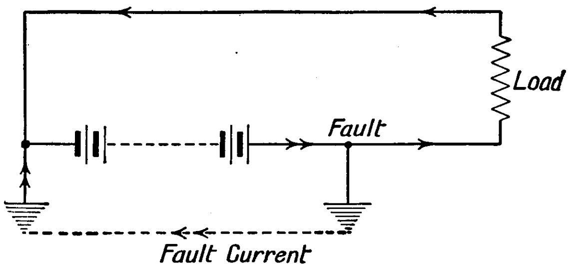


Fig. 6.—Effect of an earth

be large and may cause damage. Hence, before being put into service every electrical circuit, whether it is a wiring installation or a machine winding, is tested in order to discover any fault in the insulation due either to material or workmanship. The test consists in measuring what is called the *insulation resistance* (expressed in megohms) between the conductors of the circuit and the earth, the value of which must not fall below a figure which depends upon the size of the installation and the working P.D.

The current actually passing *through* the insulating covering of the conductor is very small unless there is a definite fault, but leakage takes place through the films of dirt and moisture deposited on exposed surfaces at switches, lampholders, etc., so that the more extensive the installation the larger the number of leakage points and the lower the insulation resistance. The value of the insulation resistance is a somewhat variable quantity and depends upon several factors, one of which is the state of the weather, e.g. an increase in the humidity causes the leakage to increase and the insulation resistance to fall.

The measurement of insulation resistance is an operation of such frequent occurrence that special instruments are employed for the purpose (such as the megger, the principle of which is described in § 17, p. 208).

On the other hand, the metallic enclosures of all circuits and apparatus are connected solidly to earth, so that if the insulation becomes faulty, the large leakage current will melt the fuses and thereby isolate the faulty circuit from the supply. If not earthed, on the occurrence of a fault the enclosure would attain the potential of the conductor in contact with it, and any person touching it would be liable to receive a shock.

7. Relation between Resistance and the Dimensions of a Conductor

It has already been mentioned (§ 6, p. 57) that a pipe offers resistance to the flow of water through it, so that there must be a difference in pressure between its ends to maintain the flow. Further, it is almost self-evident, and can be verified experimentally, that in order to maintain the same *rate of flow*, i.e. the same current, the difference in pressure between the ends of the pipe:

(1) must be increased if the length of pipe is increased;

(2) must be increased if the bore of the pipe is decreased, or decreased if the bore is increased;

(3) depends upon the state of the inner surface of the pipe, i.e. must be greater if it is rough, and less if it is smooth.

Similar but even simpler relationships exist between the electrical resistance and the dimensions and nature of a conductor. Experiment shows that at constant temperature the resistance:

(1) is proportional to the length ($R \propto l$);

(2) is inversely proportional to the sectional area $\left(R \propto \frac{1}{a}\right)$;

(3) depends upon the material.

8. Resistivity or Specific Resistance (ρ)

Some description of the mechanism of conduction has been given on p. 22. In certain elements one or two of the electrons in the outer shell, which is shielded to some extent from the attractive effect of the nucleus by the intervening shells, are relatively loosely attached, so that, in any mass of the element many of these outer or *conduction electrons* become detached and wander about with random motion between the atoms. The size of the electron is so small compared with that of the atom that the solid may be conceived as a very open sponge-like structure, in which the movement of the conduction electrons is similar to that of the molecules in a mass of gas. When placed in an electric field, e.g. by connecting a wire of the material to the terminals of a cell, the electrons experience an attractive force due to the positive electrode and a repulsive force due to the negative electrode, so that a general drift towards the positive is superimposed on the random motion. Such materials are called *conductors* and the drift of the electrons constitutes an electric current (assumed by convention to flow in the *opposite* direction). The nuclei of the atoms which, having lost an electron, exhibit a positive charge experience forces opposite to those acting on the electrons, but, in a solid, are unable to move.

While the random velocities of the electrons are relatively high the drift velocity is very small. The charge of an electron is about $1{\cdot}6 \times 10^{-19}$ cmb., so that in a circuit in which a current of 1 ampere is flowing, electrons are passing any given point at the rate of about $6{\cdot}24 \times 10^{18}$ per second. But the number of conduction electrons is so vast—about 10^{23} per cu.cm. in the case of copper—that if this current is being carried by a copper wire of 0·029 in. (0·074 cm.) diameter, the drift velocity is only about 0·015 cm. per sec., i.e. less than 1 cm. per *minute*.

In any one element the atoms are arranged in a particular pattern called the *crystal lattice*. The drift of the electrons is impeded by repeated collisions with the atoms in the lattice, and this constitutes what is called the resistance of the conductor. It is clear therefore that the conducting properties of the material depend upon the available conduction electrons per unit volume and upon the pattern of the crystal lattice, and will therefore differ in different materials.

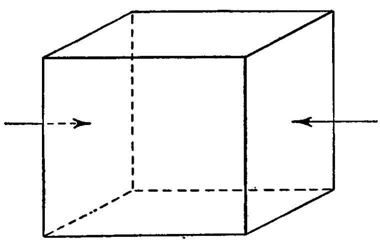


Fig. 7.—To illustrate resistivity

It would be absurd to infer, from the fact that a certain iron wire had a lower resistance than a copper wire of different dimensions, that iron is a better conductor than copper. In order to compare the conducting properties of materials, it is necessary to determine the resistance of conductors of *equal dimensions*. It should be noticed that the condition of equal volumes is insufficient since resistance depends upon the *ratio* of length to section; for example, a cubic inch of copper may be an inch cube or may be drawn into a mile of fine wire, and the value of the resistance will be very different in the two cases.

The resistivity (or specific resistance) (ρ) of a material is the resistance of a conductor of that material having unit length and unit cross-section, i.e. the resistance between the opposite faces of a unit cube of the material (fig. 7).

It is possible to calculate the resistance of any conductor when its dimensions and the resistivity of the material are known. Thus:

The resistance of a conductor of unit length and unit section is ρ.

$\therefore$ The resistance of a conductor of length l and unit section is ρl.

$\therefore$ The resistance of a conductor of length l and section a is $\frac{\rho l}{a}$.

Hence
$$R = \frac{\rho l}{a}.$$

9. The Unit of Resistivity—the Ohm-inch or Ohm-centimetre

From the above equation,

$$\rho = \frac{a}{l} R. \qquad \ldots \ldots \ldots \quad (6)$$

Therefore the dimensions of ρ are

$$\frac{(\text{length})^2}{\text{length}} \times \text{resistance, i.e. length} \times \text{resistance.}$$

The length of the side of the unit cube is usually taken as 1 in. or 1 cm., according to the system of units which is being used; so that the unit of resistivity is either the *ohm-inch* or the *ohm-centimetre.* The logical result of the adoption of the M.K.S. system is that the unit of resistivity should be the ohm-metre, but since the resistivity of good conducting materials is already expressed in microhm-cm. (or microhm-inches) and if expressed in ohm-metres would be only one-hundredth of these values, it would be less convenient and is unlikely to be adopted at present, if ever.

It may be noted that resistivity is sometimes expressed in *ohms per centimetre (or inch) cube.* This expression is cumbersome and liable to misinterpretation as *ohms per cubic centimetre,* which is entirely wrong since resistance depends on the *ratio,* and not on the product, of the length and the cross-section.

Hence, if

ρ = resistivity in ohm-cm. (ohm-in.),

l = length in cm. (in.),

a = cross-section in sq. cm. (sq. in.),

$$R = \frac{\rho l}{a} \text{ ohms.} \qquad \ldots \ldots \ldots \quad (7)$$

The resistivity of a material depends upon its temperature (§ 11, p. 102) and the value usually stated is that at 0° C. The resistivity of a metal also depends to some extent upon the treatment to which it has been subjected; e.g. the hard drawing of copper, which improves its mechanical strength, increases its resistivity, while annealing decreases the resistivity. Values of ρ for some common metals and alloys are given on p. 103. It will be noticed that the metal with the lowest resistivity, i.e. the best conductor, is silver; fortunately, copper is almost as good, and the former is employed only in very small quantities for special purposes. Until a few years ago, copper was almost the only material used for conductors, and it is still used almost universally for machine windings and cables; but of late years aluminium, on account of its lightness, has been employed to a very large extent in overhead lines, and is now being used in certain types of cable.

The resistivity of alloys is always higher than that of their constituents and, in general, much less affected by temperature. Alloys such as German silver (copper, nickel, and zinc) and particularly manganin (copper, nickel, and manganese) are much used in making resistance coils, such as are used in the post-office box and other apparatus.

The *conductivity* (γ) of a material is the reciprocal of its resistivity:

$$\gamma = \frac{1}{\rho}, \qquad (8)$$

and is expressed in mhos per cm. or per in.

The term is little used in connection with simple d.c. circuits, but is of importance in a.c. circuits; it is also the property which is somewhat analogous to permeability in the magnetic circuit (§ 11, p. 155).

Since the *conductance* (G) of a conductor is the reciprocal of its resistance (R),

$$G = \gamma \frac{a}{l}. \qquad (9)$$

Example 1.—Find the resistance of a coil of mean diameter 2·5 cm., containing 100 turns of manganin wire 0·056 cm. in diameter. The resistivity of manganin is 42×10^{-6} ohm-cm.

$$l = 100 \times 2{\cdot}5\pi = 785{\cdot}4 \text{ cm.}, \quad a = 0{\cdot}7854 \times (0{\cdot}056)^2 \text{ sq. cm.}$$

$$R = \rho\frac{l}{a} = \frac{42 \times 10^{-6} \times 785{\cdot}4}{0{\cdot}7854 \times 0{\cdot}003136} = 13{\cdot}4 \text{ ohms.}$$

Example 2.—A cable 1 mile in length and having a conductor section of 0·5 sq. in. carries a current of 200 amperes. Find the P.D. between the ends and the power loss in the cable. [$\rho = 0{\cdot}7 \times 10^{-6}$ ohm-in.]

$$l = 5280 \times 12 \text{ in,}, \quad a = 0{\cdot}5 \text{ sq. in.}$$

$$R = \rho\frac{l}{a} = \frac{0{\cdot}7 \times 10^{-6} \times 5280 \times 12}{0{\cdot}5} = 0{\cdot}0887 \text{ ohm.}$$

$$V = IR = 0{\cdot}0887 \times 200 = 17{\cdot}74 \text{ volts.}$$

$$\text{Power loss} = VI = 17{\cdot}74 \times 200 = 3548 \text{ watts.}$$

10. The Measurement of Resistivity

The values of resistivity in the table on p. 103 have been obtained entirely experimentally, by determining the resistance of a conductor of the particular material, of which the dimensions and temperature are known accurately, and using the relation (6), viz. $\rho = (a/l)R$.

The resistivity of copper at room temperature may be measured by determining the resistance of a piece of copper wire of which the length and diameter have been carefully measured. In order to reduce errors in measuring the length, and to ensure that the resistance is not too small, the wire should be not less than 2 yards long and the diameter not greater than 0·022 in. (No. 24). Such a a piece has a resistance of about one-tenth ohm. Two diameters at right angles should be measured at several points along the wire and the mean value taken, from which the cross-section (a) is obtained. The resistance is best determined by means of a slide-wire bridge, a suitably low value of R_s being used. Before the wire is removed from the bridge, two fine scratches should be made close to the terminals, and the length (l) between these scratches carefully measured. Then

$$R_x = R_s \frac{l_b}{l_a}, \quad \text{and} \quad \rho = \frac{a}{l} R_x.$$

If the experiment is repeated, the end portions containing the scratches should be cut off before replacing the wire.

The resistivity of any other material can be obtained in a similar manner.

Example.—In determining the resistivity of copper, the resistance of a copper wire 72·1 in. long and 0·022 in. diameter was found to be 0·13 ohm. Calculate the resistivity.

$$l = 72{\cdot}1 \text{ in.}, \quad a = 0{\cdot}7854 \times (0{\cdot}022)^2 = 0{\cdot}00038 \text{ sq. in.}$$

$$\rho = \frac{a}{l} R = \frac{0{\cdot}00038 \times 0{\cdot}13}{72{\cdot}1} = 0{\cdot}685 \times 10^{-6} \text{ ohm-inches.}$$

11. Relation between Resistivity and Temperature—Resistance Temperature Coefficient (α)

It is an experimental fact that the resistivity of all pure metals increases with rise in temperature. This is often explained as due, in some way, to the increased thermal agitation of the atoms, which makes collisions between electrons and atoms more frequent. But there is no simple explanation of the further experimental facts (a) that the effect of temperature on the resistivity of alloys is very much smaller and in some cases, e.g. manganin, extremely small; and (b) that the resistivity of non-metallic conductors, e.g. carbon, and of insulators, decreases with rise in temperature.

Within the normal temperature range for machine windings and cables, it may be taken that in metals the relationship is a linear one, i.e. that the resistivity increases or decreases uniformly as the temperature is raised or lowered.

The resistivity of metals continues to decrease as the temperature falls, and in some cases, notably lead, a critical temperature (only a few degrees above the absolute zero) is reached, below which the resistivity suddenly becomes exceedingly small. Such materials at temperatures below this critical value possess what is termed *super-conductivity*. An exceedingly small E.M.F. is sufficient to cause a large current to flow, and a current once excited in the circuit may take many hours to die away.

Within the linear range, if the resistivity at a given temperature (usually 0° C.) is known, the resistivity at any other temperature may be calculated by using the **resistance-temperature coefficient.**

The resistance-temperature coefficient (α) is that fraction of the resistivity at 0° C. by which the resistivity increases for each °C. rise in temperature. (Cf. coefficient of linear expansion.)

It follows that if at 0° C. the resistivity is ρ_0, then

at 1° C. the resistivity is $\rho_0 + \alpha\rho_0 \quad = \rho_0(1 + \alpha)$,
at 2° C. the resistivity is $\rho_0 + 2\alpha\rho_0 = \rho_0(1 + 2\alpha)$,
at t° C. the resisitivity is $\rho_0 + t\alpha\rho_0 \quad = \rho_0(1 + \alpha t)$.

Hence

$$\rho_t = \rho_0(1 + \alpha t). \qquad \text{(10)}$$

The relationship is more exactly expressed in the form

$$\rho_t = \rho_0(1 + \alpha t - \beta t^2).$$

At normal machine temperatures the third term is negligibly small, but it becomes of importance at the temperature of, for example, the element of an electric fire or the filament of an electric lamp.

For copper $\alpha = 0{\cdot}0043$ per °C., i.e. the resistivity increases by about $\frac{1}{230}$ for each °C. rise in temperature. The value for some other common metals and alloys is given in the table below: *

	Resistivity (ρ) at 0° C. Ohm-inches	Ohm-centimetres	Temp. Coeff. (α) per °C.
Silver	$0{\cdot}61 \times 10^{-6}$	$1{\cdot}54 \times 10^{-6}$	0·0040
Copper	0·65	1·65	0·0043
Aluminium	1·05	2·66	0·0038
Iron (soft)	4·56	11·6	0·0063
Mercury	37·07	94·07	0·00088
German silver (Cu, Ni, Zn)	10·46	26·6	0·00044
Manganin (Cu, Mn, Ni)	16·58	42·11	0·00001
Constantan (Cu, Ni)	19·35	49·1	—0·00002

The very low value of α in the case of manganin, previously referred to, makes the alloy suitable for use in standard resistance coils, and for most purposes the resistance of such coils can be considered constant.

The temperature coefficient of non-metals and electrolytes is negative, i.e. the resistivity falls as the temperature rises, but the only conductor of importance in this group is carbon. Most insulating materials also have a negative temperature coefficient.

The difference in the effect of temperature on the resistivity of a metal and of carbon can be seen clearly by comparing the behaviour of a carbon and a metal filament lamp. In a carbon lamp the resistance at room temperature is very much higher than at the working temperature, so that the initial value of the current is less than its normal value, and the filament takes a short but

* Since the resistivity of metals depends upon the previous mechanical treatment, and in the case of alloys upon the exact composition, these figures may differ in some cases from those obtained experimentally or given elsewhere.

quite appreciable time to attain its normal brightness. On the other hand, the cold resistance of a metal filament lamp is only about one-tenth of that at the working temperature; consequently the initial current is many times the normal and the filament attains its normal brightness almost instantaneously.

12. Relation between Resistance and Temperature

In practice, it is more important to know the effect of temperature on the resistance of a given conductor or coil, rather than its effect on the resistivity of the material; but since the resistance of a conductor of given dimensions is proportional to the resistivity, it follows that if

$$\rho_t = \rho_0(1 + \alpha t),$$

then

$$R_t = R_0(1 + \alpha t). \qquad . \quad . \quad . \quad . \quad . \quad (11)$$

In most cases it is a question of calculating the resistance at some higher temperature when it has been measured, not at 0° C. but at normal air temperature.

From equation (11) above,

at the lower temperature t_1 $R_1 = R_0(1 + \alpha t_1)$, . . . ($a$)

at the higher temperature t_2, $R_2 = R_0(1 + \alpha t_2)$. . . . ($b$)

Dividing (b) by (a)

$$\frac{R_2}{R_1} = \frac{R_0(1 + \alpha t_2)}{R_0(1 + \alpha t_1)} = \frac{1 + \alpha t_2}{1 + \alpha t_1}. \qquad . \quad . \quad . \quad . \quad (12)$$

Hence

$$R_2 = R_1\left(\frac{1 + \alpha t_2}{1 + \alpha t_1}\right). \qquad . \quad . \quad . \quad . \quad . \quad (13)$$

This is an important relationship which is of frequent occurrence, but it need not be memorized as, provided equation (11) is remembered, it can always be written down.

13. Measurement of Rise in Temperature by Increase in Resistance—Resistance Thermometry

The definite relationship between resistance and temperature may be used for the measurement of temperature, for, since

$$\frac{R_2}{R_1} = \frac{1 + \alpha t_2}{1 + \alpha t_1},$$

by subtracting 1 from each side we get

$$\frac{R_2 - R_1}{R_1} = \frac{1 + \alpha t_2 - (1 + \alpha t_1)}{1 + \alpha t_1} = \frac{\alpha}{1 + \alpha t_1}(t_2 - t_1).$$

Hence

$$t_2 - t_1 = \frac{1 + \alpha t_1}{\alpha R_1}(R_2 - R_1), \quad . \quad . \quad . \quad . \quad (14)$$

i.e.

$$(\text{increase in temperature}) = \frac{1 + \alpha t_1}{\alpha R_1}(\text{increase in resistance}).$$

(This equation need not be memorized; it can always be developed from equation (12).)

This is a method in general use for the measurement of the temperature of the windings of electrical machines. The resistance is measured at air temperature when the machine is cold, and again after running for the specified time at full load; the temperature rise, and hence the actual temperature, is then calculated from the increase in resistance, by using the equation given above. The temperature of a coil is highest at the centre and lowest at the surface, the difference between the two values depending upon the thickness and construction of the coil. A thermometer measures the *surface* temperature, but the value calculated from the increase of resistance is the *average* temperature, and gives a closer indication of the temperature inside the coil.

The high temperatures of furnaces can be measured in a similar manner. A coil of wire of high melting-point, usually of platinum, and enclosed in a tube of refractory material, forms the fourth arm of a Wheatstone bridge. The resistance of the coil is measured at air temperature and again when inserted in the furnace; and the temperature rise and hence the actual temperature can be obtained from this increase in resistance. Such an instrument is called a *resistance* or *platinum thermometer*, and the bridge is often arranged to be direct reading.

14. Experimental Determination of Temperature Coefficient

The temperature coefficient of metals can be obtained experimentally by measuring, at various temperatures, the resistance of a coil of wire of the particular material, immersed in an oil bath fitted with a thermometer and stirred and heated over a burner (fig. 8). The measurement is made most easily with a slide-wire bridge, which can be balanced more rapidly than a post-office box.

Heating must be carried out slowly and the oil well stirred, so that the thermometer reading may indicate the actual coil temperature. The resistance should be measured at every 10° or 15° C., up to about 120° C., the burner being removed and the oil well stirred before each reading. Intermediate readings may be obtained as the oil cools; such readings, due to the more gradual and uniform change in temperature, are usually more reliable than those obtained during the heating period, but the time taken is longer, owing to the slow fall in temperature in the later stages.

While the resistance of the coil increases with rise in temperature that of the connections between the coil and bridge is unchanged. The resistance measured is that of the coil and the connections, so that the resistance of the latter must be obtained separately and subtracted in each case.

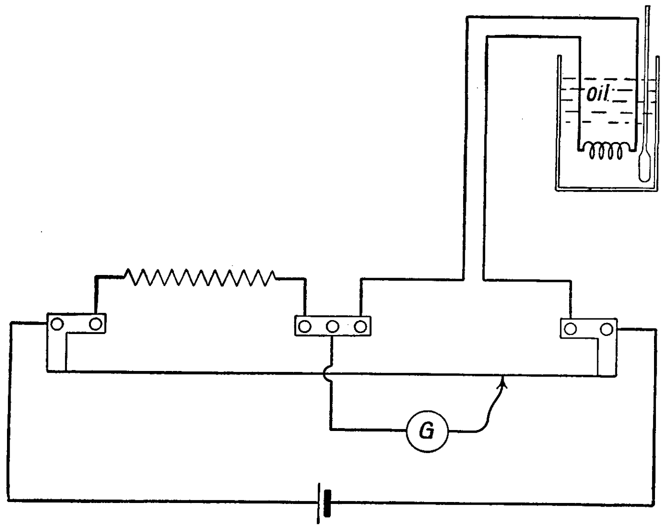


Fig. 8.—Determination of resistance-temperature coefficient

The coil resistance is then plotted against temperature. The relationship is found to be a linear one (fig. 9). If the straight line is drawn which lies most evenly among the points, the intercept on the resistance axis is the resistance at 0° C. (R_0); and by taking another point on the straight line the resistance (R_t) at another temperature (t) is found. Then from equation (11),

$$R_t = R_0(1 + \alpha t)$$

or

$$\alpha = \frac{R_t - R_0}{R_0 t}. \qquad (15)$$

Example 1.—The resistance of a coil of copper wire at 10° C. is 20 ohms; determine the resistance at 40° C. (α = 0·0043 per °C.).

Here

$$R_1 = 20,\ t_1 = 10,\ t_2 = 40.$$

$$\frac{R_2}{R_1} = \frac{1 + \alpha t_2}{1 + \alpha t_1}.$$

$$\therefore R_2 = R_1 \cdot \frac{1 + \alpha t_2}{1 + \alpha t_1} = 20 \times \frac{1 + (40 \times 0{\cdot}0043)}{1 + (10 \times 0{\cdot}0043)}$$

$$= 20 \times \frac{1{\cdot}172}{1{\cdot}043} = 22{\cdot}5 \text{ ohms.}$$

Example 2.—Find the resistance of a mile of aluminium wire 0·1 in. diameter at 30° C. (At 0° C. $\rho = 1{\cdot}05 \times 10^{-6}$ ohm-in., $\alpha = 0{\cdot}0038$ per °C.)

$$\rho_{30} = \rho_0(1 + 30\alpha) = 1{\cdot}05 \times 10^{-6}(1 + \overline{30 \times 0{\cdot}0038}) = 1{\cdot}17 \times 10^{-6} \text{ ohm-in.}$$

$$R_{30} = \rho_{30}\frac{l}{a} = 1{\cdot}17 \times 10^{-6} \times \frac{5280 \times 12}{0{\cdot}7854 \times (0{\cdot}1)^2} = 9{\cdot}44 \text{ ohms.}$$

Alternatively, R_0 might be found from ρ_0 and then

$$R_{30} = R_0(1 + 30\alpha).$$

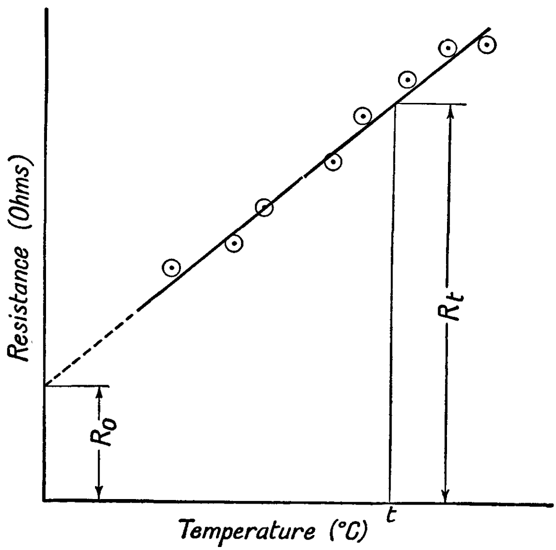


Fig. 9.—Relation between resistance and temperature

Example 3.—A coil at 10° C. when connected to a 50-volt supply takes a current of 1 ampere. At the end of an hour the current has fallen to 0·8 ampere. If the P.D. has remained constant, what is the average temperature of the coil? ($\alpha = 0{\cdot}0043$ per °C.)

From Ohm's law,
$$R_{10} = \frac{50}{1} = 50 \text{ ohms,}$$

$$R_t = \frac{50}{0{\cdot}8} = 62{\cdot}5 \text{ ohms.}$$

$$\frac{R_t}{R_{10}} = \frac{1 + \alpha t}{1 + 10\alpha}$$

from equation (12), which is easier to remember than equation (14).

Hence
$$\frac{R_t - R_{10}}{R_{10}} = \frac{\alpha(t - 10)}{1 + 10\alpha},$$

$$\frac{62{\cdot}5 - 50}{50} = \frac{0{\cdot}0043t - 0{\cdot}043}{1{\cdot}043},$$

from which
$$t = 70{\cdot}7° \text{ C.}$$

15. The Production of Heat in a Conductor—Joule's Law

Heat is produced in a conductor carrying a current by the partial dissipation of the energy associated with the current, by repeated collisions between the electrons and the atoms. The portion of the total energy of each coulomb so converted into heat in any section of a conductor is proportional to the P.D. between its ends; the heat produced is

$$W = VQ \text{ joules } (\S\ 16, \text{ p. } 65),$$

and the rate of heat production is

$$P = \frac{W}{t} = \frac{VQ}{t} = V\frac{Q}{t} = VI \text{ watts},$$

where V = P.D. (volts) between the ends of the section considered, Q = quantity (coulombs), I = current (amperes), t = time (sec.).

If the resistance of the section is R ohms, then by Ohm's law

$$V = IR.$$

Hence

$$\boldsymbol{P = (IR)I = I^2R} \textbf{ watts.} \quad . \quad . \quad . \quad . \quad (16)$$

The rate at which heat is produced in a conductor is proportional to its resistance and to the square of the current

This statement is known as *Joule's law*.

The energy which is thus converted into heat when a current flows through a conductor is referred to as the *resistance loss*, the *I^2R loss*, or the *copper loss*.

16. Relation between Electrical and Heat Units of Energy—Joule's Equivalent

If a coil of wire is completely immersed in a calorimeter containing water, all the heat produced in the coil by the passage of a current is imparted to the water. By measuring the current and the P.D. between the ends of the coil, the electrical energy input in a given time is obtained; and a knowledge of the effective weight of water, and the rise in temperature produced, enables the heat input to the water to be calculated. Assuming there has been no loss of energy, the relation between the electrical and the heat units of energy may be obtained by equating these two quantities.

The apparatus is arranged as shown in fig. 10. The calorimeter is fitted with a lid through which passes a thermometer and stirrer, and is enclosed in an outer jacket packed with cotton-wool; the coil is made of a resistance alloy. The ammeter indicates the current, and the voltmeter connected directly to the terminals measures the P.D. across the coil. The product of these two gives the power input, which is measured directly in this way rather than calculated from the resistance of the coil, as it automatically takes account of any change in resistance of the coil and also of the fact that a small current passes through the water, so that the effective resistance is less than that of the coil measured out of the water.

During the experiment the current is kept constant by means of a variable resistance, and the P.D. and temperature are read at frequent intervals, say about every half minute, the water being well stirred.

Immediately the temperature of the water rises above that of the surrounding air, some heat is lost. This is kept as small as possible by surrounding the calorimeter by poor conductors, but it cannot be entirely eliminated. Hence more accurate results will be obtained if the power input and the weight of water are such that the rise in temperature is fairly rapid, so that the duration of the test is reduced.

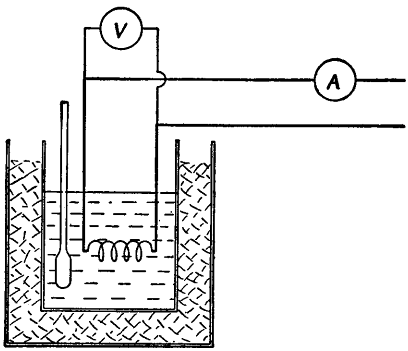


Fig. 10.—Determination of Joule's equivalent

If W =	weight of water,	gm.
w =	water equivalent of calorimeter, stirrer and coil,	gm.
t_1 =	initial temperature,	°C.
t_2 =	final temperature,	°C.
I =	constant value of current,	amperes
V =	average value of the P.D.,	volts
t =	heating period,	sec.

then

Average value of power input $= VI$ watts;
Total energy input $= VIt$ watt-seconds or joules; and
Heat energy absorbed by water $= (W + w)(t_2 - t_1)$ calories.

Hence, assuming that there is no loss of heat,

$$VIt = (W + w)(t_2 - t_1),$$

which gives

$$1 \text{ calorie} = \frac{VIt}{(W + w)(t_2 - t_1)} \text{ joules.}$$

A correction may be made for the heat losses which occur during the experiment, as follows. At the conclusion of the heating period, the fall in temperature which occurs during a cooling period equal to half the heating period is observed.

Now the rate of heat loss is proportional to the temperature rise, so that, since the rise in temperature during the heating period is uniform, the average rate of heat loss is half the maximum rate of loss.

$$\begin{aligned}\therefore \text{ Total heat loss} &= \text{(average rate of loss) (time)}\\ &= \tfrac{1}{2}\text{ (maximum rate of loss) (time)}\\ &= \text{(maximum rate of loss) }(\tfrac{1}{2}\text{ time}).\end{aligned}$$

The maximum rate of loss occurs at the end of the heating period, and since the fall in temperature is slow, it may be assumed with little error that during the cooling period (equal to half the heating period) the rate of loss remains constant at its maximum value.

Hence total heat lost during heating period is equal to the heat lost during the cooling period, i.e.

$$\text{Total heat lost} = (W + w)(t_2 - t_3),$$

where t_3 is the temperature at the end of the cooling period.

Thus the total heat absorbed by the water must be increased by this quantity, and is $(W + w)(t_2 - t_1 + t_2 - t_3)$.

$$\therefore 1 \text{ calorie} = \frac{VIt}{(W + w)(t_2 - t_1 + t_2 - t_3)} \text{ joules.}$$

In another method, cooling losses are eliminated by starting with the water a few degrees below room temperature and allowing heating to proceed until the temperature exceeds that of the room by the same amount; the average temperature during the experiment is therefore equal to the room temperature, and the water gains as much heat from the room during the first half of the heating period as it loses during the second half. This method, however, usually requires some artificial means of cooling.

As the result of careful experiment it has been found that

$$1 \text{ calorie} = 4{\cdot}18 \text{ joules}$$

and hence

$$1 \text{ B.Th.U.} = 1054 \text{ joules.}$$

The number of joules equivalent to 1 calorie (or, more generally, the number of units of mechanical or electrical energy equivalent to 1 unit of heat energy) is known as *Joule's equivalent*. The relationship forms the foundation of all calculations connected with the heating effects of electric currents, and the use of electrical energy for heating purposes.

Example 1.—In a determination of Joule's equivalent, the following data were obtained:

Weight of water	90 gm.
Water equivalent of calorimeter, coil and stirrer ..	2 gm.
Initial temperature	10° C.
Final temperature	20° C.
Heating time	6 min.
Temperature after cooling for 3 min.	18·5° C.
Current	3 amperes.
Average P.D.	4·14 volts.

Find the value of the equivalent in joules per calorie.

Heat gained by water $= 92 \times (20 - 10) = 920$ calories.

Heat lost by cooling $= 92 \times (20 - 18{\cdot}5) = 138$ calories.

Total heat output of coil $= 1058$ calories.

Energy input $= 4{\cdot}13 \times 3 \times 360 = 4460$ joules.

$$\therefore 1 \text{ calorie} = \frac{4460}{1058} = 4{\cdot}22 \text{ joules.}$$

(The correct value is 4·18 joules.)

Example 2.—The heating element of an electric kettle has a resistance of 53 ohms. If the efficiency is 90 per cent, how long will it take to boil 3 pints of water at 60° F. when connected to a 230-volt supply, and what will be the cost at $\frac{3}{4}d.$ per unit? (1 gal. of water = 10 lb.)

3 pints of water weigh 3·75 lb

Heat absorbed by water $= 3{\cdot}75 \times (212 - 60) = 570$ B.Th.U.

$= 570 \times 1054 = 601{,}000$ joules.

$$\text{Total heat output of element} = \frac{601{,}000 \times 100}{90} = 667{,}800 \text{ joules.}$$

$$\text{Current} = \frac{230}{53} = 4{\cdot}34 \text{ amperes.}$$

Power input $= 4{\cdot}34 \times 230 = 1000$ watts.

Energy input $= 1000t$ watt-seconds or joules, where $t =$ time in seconds.

$\therefore 1000t = 667{,}800.$

$t = 668$ sec. $= 11$ min. 8 sec.

Energy input $= 667{,}800$ watt-seconds

$$= \frac{667{,}800}{3600} = 185{\cdot}6 \text{ watt-hours}$$

$= 0{\cdot}186$ kW.-hr. or unit.

Cost $= 0{\cdot}186 \times 0{\cdot}75 = 0{\cdot}14$ pence.

17. The Heating of Conductors

The heating effect produced by an electric current is a most valuable property and makes possible the many and varied uses of electrical energy for heating purposes; but, as has already been pointed out, it is a universal effect and cannot be entirely eliminated when not wanted. In conductors used to convey energy from one point to another, or in the windings of electrical machines, the conversion of part of the energy into heat which cannot be utilized is not only a dead loss but may raise the temperature of the conductor to a dangerous value.

From equation (16) p. 108, it is clear that the rate of heat production may be reduced by decreasing either the resistance of the conductor or the current. The resistance of a given length of conductor is proportional to the resistivity, hence the almost universal use of copper, which has the lowest resistivity of all metals except silver.

It is also inversely proportional to the cross-section, so that the losses may be reduced by increasing the section. But this increases the weight and cost of the conductor, and there is an economic limit beyond which the saving in losses does not justify the increase in cost. The section of long cables and overhead lines carrying large currents is decided partly on such economic principles, but in the case of the smaller currents used in internal power and lighting circuits, the choice is generally based on considerations of mechanical strength, resistance drop, or rise in temperature.

The heating loss is reduced rapidly by decreasing the current, but if the power transmitted is to be the same, a corresponding increase in P.D. is necessary. Hence by raising the P.D. at which a given power is transmitted, the losses in a given conductor are reduced; or, for the same losses, the conductor section may be decreased. In the normal domestic distribution system the voltage is limited for safety to a low value (240 volts), but one of the chief reasons for the use of very high P.D.s in long transmission lines is to enable the section and therefore the weight and cost of the conductor to be reduced without increasing the losses.

18. Temperature Rise of Conductors

The rate at which heat is produced by a given current in a conductor of given resistance is always the same, but the rise in temperature produced depends upon a number of factors. At the start all the heat is absorbed in raising the temperature, but directly this rises above the surroundings, some of the heat is dissipated by conduction and convection. The rate of loss increases as the temperature increases, and therefore the rate of rise of temperature decreases until a steady temperature is reached when the rate of loss is equal to that of production. The value of this steady temperature depends largely upon the area of cooling surface and the surrounding conditions.

The laws of heat flow are similar to those of current flow. To cause heat to pass from one point to another a difference in temperature (P.D.) is necessary, and the rate of heat flow (current) is proportional to this difference in temperature and inversely proportional to the thermal resistance (electrical resistance) of the path; while the thermal resistance, as in the case of electrical resistance, is proportional to the length and inversely proportional to the section of the path, and also depends on the nature of the material.

Consider three conductors (*a*), (*b*) and (*c*) of the same material, of equal length, section and resistance, and carrying equal currents so that heat is produced at the same rate in each. (*a*) is bare and forms part of an overhead line; (*b*) is a portion of a house wiring installation, is insulated, and lies alongside another conductor in a length of conduit; while (*c*) is in the middle of a coil forming part of a machine winding.

It is obvious that in (*a*), which is in intimate contact with the air, by which the heat is carried away rapidly by convection, the temperature rise will be least. In (*b*) the heat is first passed through the insulation (all insulators are bad conductors of heat) and is then conveyed, partly by direct conduction and partly through the air inside, to the conduit and hence to the outer air; so that the temperature rise will be much higher than in (*a*). Finally, in (*c*) the heat has to pass through several alternate layers of copper and insulation before reaching the air; so that the temperature rise will be higher even than in the case of (*b*). It is therefore clear that from the point of view of temperature rise no general law can be given for the current-carrying capacity of a conductor of given size. It depends greatly on surrounding conditions, and the limit is often set by other factors. In case (*a*) a comparatively high temperature would be harmless and the limit would be set by the losses. In (*b*) the limiting factor would be either mechanical strength or resistance drop; when carrying the maximum current allowed by these considerations, the temperature rise would probably be small. In (*c*) the injurious effect on insulation caused by prolonged exposure to temperatures of 100° C. and over would cause temperature rise to be the limiting factor.

19. Applications of the Heating Effect

(See Chapter IV, § 6, p. 33.)

THERMO-ELECTRICITY

20. The Peltier Effect

When a current passes through a circuit consisting of different metals, experiment shows that, superimposed upon the general heating due to the resistance of the conductors, there is an additional heating or cooling effect at the junction of the metals. For instance, if the circuit consists of iron and copper, as shown in fig. 11, and a current is flowing in the direction shown, the heating at the junction (*b*) is greater, and that at junction (*a*) less than the average heating

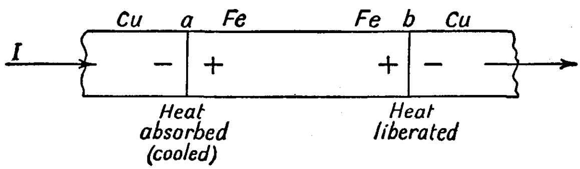


Fig. 11.—The Peltier effect

due to the resistance, i.e. junction (*b*) is heated and junction (*a*) is *cooled.* If the direction of the current is reversed, it is found that junction (*a*) is heated and junction (*b*) is cooled; which shows clearly that the effect is quite different from the normal resistance heating which is independent of the direction of the current.

This phenomenon is known as the *Peltier effect.* It is assumed to be due to abrupt changes of potential at the junctions. A P.D. always exists between two dissimilar metals in contact (§ 11, p. 125), and in the case of copper and iron, the potential of iron is positive relative to copper. Hence each coulomb of electricity in crossing the junction *a* from copper to iron is raised in potential, and therefore receives an increase in energy which is derived from the heat absorbed at this junction; while in crossing junction *b* from iron to copper, the potential of each coulomb is lowered and the energy so lost appears as heat at the junction. If in the portion *ab* platinum, which is negative relative to copper, is substituted for iron, the P.D. across each junction is reversed, so that for the direction of current shown in fig. 11 *a* is heated and *b* is cooled.

This effect is entirely distinct from the normal heating due to resistance in that:

(1) it depends upon the *direction* of the current;

(2) it is proportional to the *current* instead of to the (current)^2.

The effect is comparatively small and needs sensitive apparatus for its detection and measurement. In the copper-iron circuit considered above, the heat liberated or absorbed is about $1{\cdot}7 \times 10^{-4}$ calories per coulomb.

21. Thermo-E.M.F.s

If the E.M.F. producing the current in the above circuit is removed and the whole circuit allowed to attain a uniform temperature, no current flows, since the P.D.s at the junctions are in opposite directions and neutralize one another. But if heat is supplied to junction *a*, thus raising its temperature, a current flows round the circuit, the direction at this junction being from copper to iron (fig. 12), i.e. the direction associated with absorption of heat at this junction, in the Peltier experiment. This effect was discovered by Seebeck some years before the discovery of the Peltier effect.

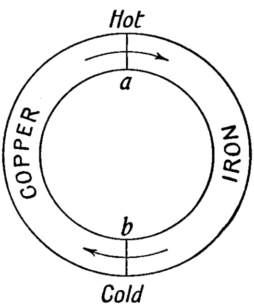


Fig. 12.—T urrents

If junction *b* is heated instead of *a*, the direction of the current is reversed; so that the direction of current flow is such as will, through the Peltier effect, absorb heat from the hot junction and liberate it at the cold junction. Hence heat is transferred from the hot junction to the cold junction by direct conversion into electrical energy. In fact the circuit may be looked upon as a heat engine in which heat is taken in at the hot junction (engine stop valve) at high temperature and converted directly into electrical energy. A portion of this is reconverted into heat by the resistance of the circuit, some can be utilized in other ways, e.g. for operating a galvanometer, and the remainder is rejected at low temperature at the cold junction (steam condenser).

The conversion of heat energy into electrical energy requires the existence of an E.M.F., hence:

If a closed circuit consists of two different metals and one junction is at a higher temperature than the other, a thermo-E.M.F. is produced, which causes a thermo-electric current to flow in the circuit, and represents the direct conversion of heat energy into electrical energy.

Such a junction is called a *thermo-couple*. The E.M.F. produced depends upon the particular metals in contact, the difference in temperature of the junctions, and the mean temperature of the junctions; but the relationship is not a simple one and will not be discussed fully here. The E.M.F. is comparatively small and is usually measured in micro-volts or milli-volts; for example, in an iron-copper circuit in which the temperature of the hot junction is 150° C. and that of the cold junction is 50° C., the E.M.F. is 660 micro-volts, i.e. 0·00066 volt.

22. Use of Thermo-couples for Temperature Measurement

From the engineering point of view, the chief use of thermo-E.M.F.s is in the measurement of temperature. For this purpose, one couple is raised to the temperature to be measured while the other is kept at a known low temperature, and the E.M.F. is read by means of either a high-resistance galvanometer or a potentiometer. The couple employed depends upon the temperature to be measured. Up to about 500° C. copper-nickel, copper-iron, or iron-eureka couples are used; above this the couple often consists of platinum and a platinum alloy.

It can be shown that the E.M.F. of a couple consisting of two metals A and B is not affected by the insertion of any number of other metals between A and B, provided that all the junctions so formed are at the same temperature. This result is of importance in practice, since it means that at the hot end the two metals forming the couple may be soldered or brazed together, and at the cold end the necessary instruments may be inserted without affecting the value of the E.M.F.

As previously stated, the relationship between the E.M.F. and the difference in temperature is not a simple one, and in general the couple is calibrated by measuring the actual E.M.F. produced with the hot and cold couples at known temperatures. In some cases, however, the E.M.F. is almost proportional to the temperature difference over a considerable range. In a copper-nickel couple, for instance, it has a value of about 27 micro-volts per °C., so that the E.M.F. is given approximately by

$$E = (t_2 - t_1) \times 27 \times 10^{-6} \text{ volts},$$

from which

$$t_2 - t_1 = \frac{E}{27 \times 10^{-6}} \text{ °C.},$$

or

$$t_2 = \frac{E}{27 \times 10^{-6}} + t_1 \text{ °C.}$$

The time lag between change of temperature and change of E.M.F. is extremely small, so that rapid variations in temperature may be recorded by means of thermo-couples. They are also often used for measuring temperatures at points which would otherwise be inaccessible. Examples of such uses are the measurement of internal coil and winding temperatures, the couple being embedded in the coil during manufacture; and the measurement of the temperature at various points in a gas-engine piston, in which case the couple is attached inside the piston and connected to the instruments through flexible leads.

EXAMPLES

1. Describe the Wheatstone bridge method of measuring resistance.

Draw a diagram of a metre-bridge balanced at 75 cm. from one end. (*a*) If the resistance at the same end is 100 ohms, mark the value of the other resistance on the diagram. (*b*) If a second 100 ohms resistance is placed in parallel with the first, where will the balance point occur? [Lond. Matric.]

2. In a post-office box the ratio arms are adjusted so that $R_b = 10$, $R_a = 100$, but an exact balance cannot be obtained. When $R_s = 501$ ohms the galvanometer deflection is 6 divisions to the left, and when $R_s = 502$ ohms the deflection is 4 divisions to the right. What is the value if R_x?

3. In an experiment with a half-metre bridge, a balance is obtained 38·22 cm. from the end to which a known resistance of 50 ohms is connected. On interchanging the known and unknown resistances the balance occurs at a point 11·68 cm. from the same end.

What is the value of the unknown resistance?

4. Find the resistance of a circular coil of copper wire containing 1500 turns, the mean diameter being 6 in. and the diameter of the wire 0·028 in. [$\rho = 0{\cdot}68 \times 10^{-6}$ ohm-in.]

5. It is required to supply a 2-volt wireless valve taking a current 0·1 ampere from a 4-volt accumulator by placing a resistance of German-silver wire in series with it.

Calculate (*a*) the value of the resistance,
(*b*) the length of German-silver wire 0·031 cm. in diameter.
[$\rho = 30{\cdot}48 \times 10^{-6}$ ohm-cm.]

6. A piece of iron wire 20 metres long has a resistance of 3 ohms. What is its diameter? [$\rho = 10{\cdot}2 \times 10^{-6}$ ohm-cm.]

7. The resistance of an inch cube of copper between opposite faces at 20° C. is $0{\cdot}67 \times 10^{-6}$ ohms. Find its resistance when drawn into a fine wire a mile long.

8. Find the resistance of 1 mile of aluminium wire 0·104 in. diameter, at 40° C. [At 0° C., $\rho = 1{\cdot}13 \times 10^{-6}$ ohm-in., $\alpha = 0{\cdot}004$ per °C.]

9. A coil of copper wire at 20° C. when connected to a 500-volt supply takes a current of 1 ampere. After some hours the current has fallen to 0·85 ampere the supply pressure remaining constant. What is the average temperature of the coil? [$\alpha = 0{\cdot}0043$ per °C.]

10. Two conductors, one of copper and the other of iron, are connected in parallel, and at 20° C. carry equal currents. What proportion of the total current will pass through each if the temperature is raised to 100° C.? [Copper $\alpha = 0{\cdot}0043$ per °C., iron $\alpha = 0{\cdot}0063$ per °C.]

11. The steel-cored aluminium conductor used on the overhead transmission lines of the National Grid consists of a central core of 7 strands of steel wire 0·11 in. diameter, surrounded by 30 strands of aluminium wire of the same diameter.

If a mile of this conductor is connected to a 20-volt d.c. supply, calculate the total current. [Steel $\rho = 5{\cdot}3 \times 10^{-6}$ ohm-in., aluminium $\rho = 1{\cdot}13 \times 10^{-6}$ ohm-in.]

12. A calorimeter of water containing a heating coil has a total water equivalent of 100 gm. A current of 5 amperes is passed through the coil for 10 minutes at an average P.D. of 2 volts, and the rise in temperature is 14·2° C.

Neglecting losses, calculate Joule's equivalent in joules per calorie.

13. What are the laws governing transformation of electrical energy into heat?

Calculate the current required to raise the temperature of 1 litre of water from 20° C. to boiling-point in 10 min., the resistance of the heater being 50 ohms and the efficiency 80 per cent. [Grad. I.E.E.]

14. A steady current of 5 amperes is passed through a platinum wire situated along the axis of a long glass tube, through which water is flowing at the rate of 300 c. cm. per min. The temperature of the water as it enters is 15° C. and as it leaves, 19° C.

Calculate the resistance of the wire. [Grad. I.E.E.]

15. Explain the terms joule, watt, and kilowatt-hour.

An electric kettle is marked 500 watts–230 volts and is found to take 15 min. to raise 1 kg. of water from 15° C. to boiling-point. Calculate the percentage of the energy which is employed in heating the water. [Lond. Matric.]

16. An electric radiator is required to dissipate 1 kW. when connected to a 230-volt supply. If the coils of the radiator are of wire 0·5 mm. in diameter, having a specific resistance of 60 microhms per cm. cube, estimate the necessary length of wire. [Grad. I.E.E.]

17. An electric motor taking a current of 50 amperes is connected with the 230-volt supply mains 500 yd. away by cable having a copper section of 0·15 sq. in.

Calculate (*a*) the P.D. at the motor terminals,

(*b*) the power loss in the connecting cable as a percentage of the motor input. [$\rho = 0{\cdot}67 \times 10^{-6}$ ohm-in.]

18. A transmission line 1 mile long has conductors of aluminium and supplies at 500 volts a 50-h.p. motor having an efficiency of 90 per cent. If the losses in the line are not to exceed 10 per cent of the input to the motor at full load, calculate the necessary sectional area of the conductors. [$\rho = 1{\cdot}13 \times 10^{-6}$ ohm-in.]

19. A coil is wound with insulated copper wire of diameter 0·032 in.; the copper weighs 10 lb. If the coil carries a current of 1 ampere and has reached a steady temperature, the average throughout the coil being 60° C., calculate the voltage between the terminals of the coil. (Temperature coefficient of copper = 1/234·5 from and at 0° C. Density of copper = 0·32 lb./in.^{3} Resistivity of copper = $0{\cdot}7 \times 10^{-6}$ ohms, in inch units, at 20° C.) [J.S.A., 1947.]

20. The field coil of a salient-pole alternator has the following dimensions:

Mean length of turn = 95 cm.; number of turns = 70; copper tape cross-section, 15 mm. × 3 mm. Determine the resistance of the coil at 70° C., and the heat generated in calories per second when carrying a current of 100 amperes. (Resistivity of copper at 20° C. is 1/58 ohm per metre length of 1 mm.^{2} cross-section. Temperature coefficient of copper is 1/234·5 per degree C. 1 calorie is equivalent to 4·18 joules.) [J.S.A., 1945.]

CHAPTER VIII

Electrolysis—Primary and Secondary Cells

1. Electrolysis

A superficial description of the phenomenon of electrolysis has been given in § 2, p. 30; it may be summarized briefly as follows:

(1) Certain liquids, chiefly solutions of acids, salts, and bases, act as conductors and are known as *electrolytes*. In contrast with a solid conductor, which is unchanged except for possible indirect effects due to heating, the passage of a current through an electrolyte is accompanied by chemical changes.

(2) The electrolyte is split up into two portions, which are liberated at the electrodes by which the current enters or leaves the liquid. Here they may either appear in the free state or take part in secondary reactions.

(3) In the case of metallic salts the metal is liberated at one of the electrodes, and the mass of any particular metal so liberated by each unit of electricity is always the same.

Further investigation shows that:

(4) The metal in the case of a metallic salt or the hydrogen in the case of acids is liberated at the electrode by which the current *leaves* the liquid. This electrode is called the *cathode*, while that by which the current enters is called the *anode*.

(5) If cells containing solutions of various metallic salts are connected in series, so that the same current traverses each, it is found by experiment that the amounts of metal deposited on each cathode in the same time, i.e. by the same quantity of electricity, are proportional to their chemical equivalents.* A similar relationship holds with regard to the amounts of the substances liberated at the anodes, but the determination is often complicated by secondary reactions.

These experimental results were embodied by Faraday in two laws:

(1) **The mass of a substance liberated from an electrolyte is proportional to the quantity of electricity which has passed through the electrolyte.**

* The chemical equivalent is the number of units, by weight, of the element, which can combine with or replace unit weight of hydrogen (or more correctly 8 units by weight of oxygen).

(2) **The mass of a substance liberated from an electrolyte by a given quantity of electricity is proportional to its chemical equivalent.**

2. Electrochemical Equivalent

The mass of a substance liberated from an electrolyte by unit quantity of electricity is called its *electrochemical equivalent*; it is usually expressed in grammes per coulomb. From Faraday's second law it follows that the electrochemical equivalents of substances are proportional to their chemical equivalents.

The electrochemical equivalents of some common elements are given below:

Hydrogen	0·0000104	gm. per coulomb
Oxygen	0·0000829	,, ,,
Silver	0·001118	,, ,,
Copper	0·000329	,, ,,
Sodium	0·000239	,, ,,

3. Electrolytic Dissociation

The preceding sections contain an account of the experimental facts of electrolysis; a brief description of the mechanism by which it is assumed to take place will now be given.

The structure of the molecule is held together by electric forces existing between the electrons and the protons of which it is composed (§ 6, p. 20). The magnitude of the forces between oppositely charged bodies depends upon the nature of the medium by which they are separated, and in water they have a value which is much lower than in air. Consequently when a substance is dissolved in water the structure of each molecule is greatly weakened; and it is assumed that molecules, in consequence of repeated collisions with other molecules, are continually splitting up and reforming. The portions into which a molecule is split may each consist of a single atom or a group of atoms, and are called *ions*. Each ion possesses either a positive or a negative charge; in the ions produced from one molecule the charges are equal and opposite and, in the normal state, neutralize each other. Hydrogen and the metals form positive ions; the non-metals form negative ions.

The process by which the molecules are split up is called *electrolytic dissociation* and the solution is said to be *ionized*. There are no free conduction electrons in an electrolyte, as in a solid conductor; but the charges, the motion of which constitutes the current, are carried by the ions.

4. Dissociation of Metallic Salts and Acids

Most of the common electrolytes are solutions of metallic salts or acids. In the former the metal forms the positive ion and the acid portion the negative ion; in the latter the hydrogen (which, in the formation of a salt, is replaced by a metal) forms the positive ion. The charge of each ion consists of a number of electrons or protons equal to the valency * of the element or group.

For example, one molecule of sodium chloride produces one sodium ion positively charged with one proton, and one chlorine ion negatively charged with one electron:

$$NaCl = \underset{+}{Na} + \underset{-}{Cl}.$$

One molecule of copper sulphate produces one copper ion charged with two protons, and one sulphion charged with two electrons:

$$CuSO_4 = \underset{+\,+}{Cu} + \underset{-\,-}{SO_4}.$$

One molecule of sulphuric acid produces two hydrogen ions, each with one proton, and one sulphion with two electrons:

$$H_2SO_4 = \underset{+}{H} + \underset{+}{H} + \underset{-\,-}{SO_4}.$$

5. Mechanism of Electrolysis

During the constant interchange of partners of which the process of dissociation consists, there are, at any instant, a number of ions (the number depending upon the strength of the solution) which are

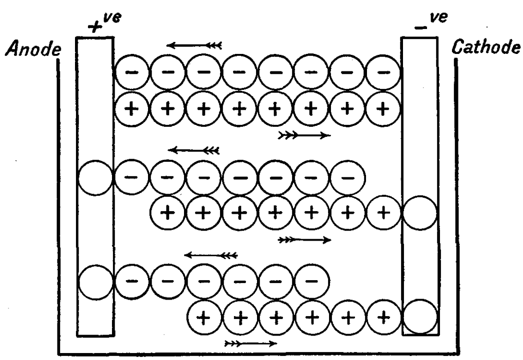


Fig. 1.—Mechanism of electrolysis

without partners. If two electrodes between which there is a P.D. are immersed in the solution, these free ions will experience forces tending to move the positive ions towards the negative electrode (cathode) and the negative ions towards the positive electrode (anode). There is thus set up a progressive movement of oppositely charged

* The valency of an element (or group) is the number of atoms of hydrogen which one atom of it can combine with or replace.

ions in opposite directions towards the electrodes, where they are discharged and set free. The process is to be regarded not so much as a free movement of individual ions but rather as a directed interchange of partners all along the line, in which each ion is passed on from molecule to molecule and which results in the freeing of positive ions at the cathode, and of negative ions at the anode. This is illustrated diagrammatically in fig. 1.

6. Electrolysis of Copper Sulphate

Fig. 2 shows a cell containing copper sulphate into which dip two carbon electrodes connected to a battery.

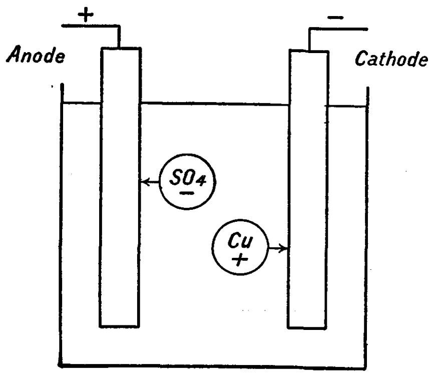


Fig. 2.—Electrolysis of copper sulphate

As has been seen, each molecule of copper sulphate forms one copper ion with two protons, and one sulphion with two electrons. The copper ion moves towards the cathode (—), where it is discharged by the removal of two electrons and deposited in the metallic form, producing the layer of copper referred to on pp. 31–3 and 59–61. The sulphion moves to the anode (+), where it gives up its surplus electrons and is liberated. Then being unable to exist in the free state it reacts with a molecule of water to form sulphuric acid (which remains in solution) and oxygen (which is evolved as a gas):

$$2SO_4 + 2H_2O = 2H_2SO_4 + O_2.$$

It will be noticed that each copper ion removes two electrons from the cathode and each sulphion gives up two to the anode; this results in a transfer of electrons from cathode to anode, which constitutes a current passing through the liquid from anode to cathode.

If the anode is of copper instead of carbon, each sulphion on liberation unites with an atom of copper to form copper sulphate:

$$Cu + SO_4 = CuSO_4.$$

In the first case the copper deposited is obtained from the solution, which gradually grows weaker, whereas in the second case, for every atom of copper deposited on the cathode an atom is dissolved from the anode so that the strength of solution remains unchanged. In the commercial applications of electrolysis, such as electroplating and refining, the anode always consists of the metal to be deposited.

7. Electrolysis of Water

Pure water is a poor conductor, and a little sulphuric acid is usually added to increase its conductivity. In the electrolysis it is probable that it is the acid which is electrolysed, the water being decomposed by the subsequent secondary reactions at the anode.

The dissociation of a molecule of sulphuric acid produces two hydrogen ions and one sulphion (§ 4). Each hydrogen ion is discharged at the cathode (fig. 3); the atoms then combine in pairs and are liberated in the gaseous state. Each sulphion on liberation at the anode attaches a molecule of water to reform sulphuric acid and liberate oxygen:

$$2SO_4 + 2H_2O = 2H_2SO_4 + O_2.$$

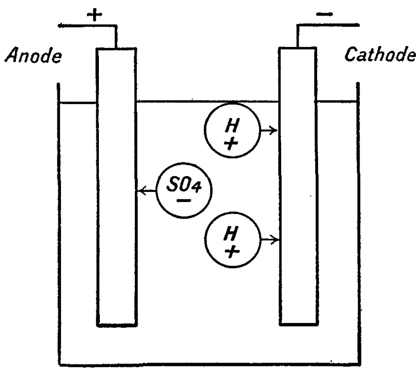


Fig. 3.—Electrolysis of water

Hence hydrogen is evolved at the cathode and oxygen at the anode; and since each sulphion produces a molecule of sulphuric acid, the total quantity of acid remains unchanged and the effect is exactly the same as if the water was electrolysed directly.

For each sulphion discharged at the anode, two hydrogen ions are discharged at the cathode. The liberation of a molecule of oxygen involves the liberation of two sulphions and therefore of four hydrogen ions or two hydrogen molecules. Hence the volume of hydrogen produced is twice that of the oxygen, a fact which can be verified experimentally by collecting the gases:

$$2H_2SO_4 = \overbrace{4H}^{2H_2 \text{ liberated at cathode}} + 2SO_4$$

$$2SO_4 + 2H_2O = \underset{\text{reformed}}{2H_2SO_4} + \underset{\substack{\text{liberated}\\\text{at anode}}}{O_2}$$

8. Resistance of Electrolytes—E.M.F. of Polarization

As with solid conductors, the resistance of a liquid conductor is proportional to the length of path and inversely proportional to the cross-section, and depends also upon the nature of the solution and its concentration; the temperature coefficient is negative. The direct measurement of resistance is made difficult by *polarization* (§ 14, p. 127) at the electrodes, which produces a counter- or back-E.M.F.; and the resistance between electrodes, unless measured by special methods, is a variable quantity which is greater than the true resistance.

When the products of electrolysis possess potential chemical energy, the process represents an absorption of energy. For instance, the hydrogen and oxygen evolved in the electrolysis of water, if collected together, form an explosive mixture which, when fired, results in the reformation of the water accompanied by the evolution of light and heat. This energy can have been derived only from the energy absorbed during electrolysis, which is represented by the back-E.M.F. of polarization. Hence, of the total energy input, part is turned directly into heat through the resistance of the electrolyte, as in a solid conductor, and the remainder is used in overcoming the back-E.M.F. of polarization, and is stored as potential chemical energy in the products of electrolysis. In the case of water the polarization E.M.F. is about 1·7 volt, so that unless the supply pressure exceeds this, electrolysis will not proceed.

If the products of the electrolysis of water are allowed to collect in contact with their respective electrodes and the original source of E.M.F. is then removed, on completing the circuit through an ammeter or galvanometer a current will be observed flowing in the *opposite* direction to that of the original current, and will continue to flow until the whole of the hydrogen and oxygen evolved has been reabsorbed. This is a simple example of a secondary cell or *accumulator* (§ 19, p. 131), in which the chemical reactions are reversible and the chemical energy stored in the products of electrolysis is converted back into electrical energy.

9. The Measurement of Quantity and Current—Voltameters

An electrolytic cell can be used for determining the electrochemical equivalent of a metal by carefully weighing the amount deposited on the cathode by a known quantity of electricity. Conversely, if the electrochemical equivalent is known, the quantity of electricity or the value of the steady current may be obtained from the weight of metal deposited. When used for this purpose the apparatus is known as a *voltameter*.

The units of quantity and current have already been defined electrolytically (pp. 59–61). The most accurate determinations have been made by means of a silver voltameter. This consists of a platinum bowl, which itself constitutes the cathode on which the silver is deposited, containing a solution of silver nitrate, in which is immersed a plate of pure silver forming the anode. For ordinary laboratory purposes the copper voltameter is quite satisfactory. A common form is shown in fig. 4; the electrodes are of copper sheet and there is an anode plate on both sides of the cathode, so that each face of the latter is equally effective. The electrolyte is copper sulphate.

10. Calibration of an Ammeter by Copper Voltameter

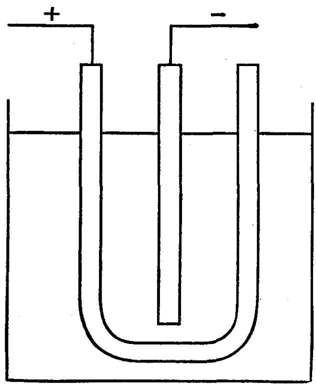


Fig. 4.—Copper voltameter

The copper voltameter may be used conveniently for calibrating a low-reading ammeter. A steady current, as indicated by the ammeter, is passed through the cell for a known time and the true value of the current determined from the weight of the copper deposited. This process is repeated at various points on the scale.

Although the weight of copper deposited per coulomb is independent of the current, an excessive current affects the character of the deposit, which becomes powdery and falls to the bottom of the cell. If the film of copper is to be firmly adherent, the current density should not exceed about 0·02 ampere per sq. cm. of cathode area, which, with the arrangement shown in fig. 4, includes both sides of the plate. A preliminary adjustment of the current is made, using a trial cathode of the same size. The test cathode after careful cleaning and weighing is then inserted and the steady current allowed to pass for a given time. (In order to minimize errors in weighing, the weight of deposit should not be less than 0·5 gm., which requires about 1500 coulombs.) At the conclusion the cathode is removed and carefully dried, first in filter paper and then in warm air; the warm air above a bunsen flame may be used if the plate is not held near enough to cause oxidation. The plate is then weighed (when cold, to prevent errors due to convection currents), dried again, and reweighed, the process being repeated until there is no further loss in weight.

Then, if the weight of copper deposited in t sec. is w gm.,

$$Q = \frac{w}{0{\cdot}000329} \text{ coulombs,}$$

$$I = \frac{Q}{t} = \frac{w}{t \times 0{\cdot}000329} \text{ amperes,}$$

from which the ammeter error at that point on the scale may be determined.

Example.—In calibrating a 0–2-ampere ammeter the following data were obtained:

Initial weight of cathode	50·120 gm.
Final weight of cathode	50·701 gm.
Ammeter reading	1·0 ampere.
Time	30 min.

$$w = 50{\cdot}701 - 50{\cdot}120 = 0{\cdot}581 \text{ gm.},$$

$$Q = \frac{0{\cdot}581}{0{\cdot}000329} = 1766 \text{ coulombs},$$

$$I = \frac{1766}{30 \times 60} = 0{\cdot}981 \text{ ampere.}$$ Hence ammeter reading is 1·9 per cent high.

11. Contact Difference of Potential

Between dissimilar metals in contact a P.D. exists, the magnitude and direction of which depend upon the nature of the metals.

The reason for this is not fully understood, but it may be assumed to be due to the diffusion of some of the free conduction electrons from one metal into the other. The metal from which the electrons pass is left positively charged, while that to which they pass becomes negatively charged. The process thus establishes a P.D. opposing further diffusion which ceases when the P.D. attains a certain value. The direction in which diffusion takes place and the limiting value of the P.D. depend upon the particular metals in contact.

In the case of zinc and copper, for instance, the P.D. is about 0·7 volt, the zinc being at the higher potential, i.e. positively charged; on the other hand, zinc when in contact with aluminium is at the lower potential.

A somewhat similar effect takes place when a metal is in contact with an electrolyte. There is a tendency either for some of the metal (as in the case of zinc) to go into solution in the form of positively charged ions, thus leaving the electrode negatively charged; or for some of the positive ions in the solution to be deposited on the electrode (as in the case of copper), which becomes positively charged. Whether it is the first or the second process which occurs depends upon the particular metal and electrolyte in contact. In either case equilibrium occurs when the P.D. established attains a value sufficient to cause the process to cease; and this value again depends upon the metal and the nature and concentration of the solution.

12. The Simple Voltaic Cell

When two electrodes of the same material are immersed in an electrolyte of uniform concentration, the P.D. between each metal and the electrolyte is the same, so that the P.D. between the electrodes is zero. But if the electrodes are dissimilar, the P.D.s between each and the solution are different in value and there is therefore a resultant P.D. between the electrodes (fig. 5).

If a zinc plate is placed in dilute sulphuric acid, some of the zinc enters the solution in the form of positive zinc ions, which combine with the negative sulphions in the solution to form zinc sulphate ($\underset{++}{\mathrm{Zn}} + \underset{--}{\mathrm{SO_4}} = \mathrm{ZnSO_4}$); but this action ceases when the potential of the zinc has fallen to about 0·62 volt below that of the solution. Similarly, if a copper plate is placed in the same solution, positive hydrogen ions from the solution are deposited on it until the potential has risen to about 0·46 volt above that of the solution. Hence, as shown in fig. 5*b*, there is a P.D. between the plates of 0·62+0·46 = 1·08 volt.

When this P.D. is established, no further action takes place (if the zinc is pure) as long as there is no external connection between the plates; but if these are joined by a conductor, electrons immediately move from the zinc to the copper

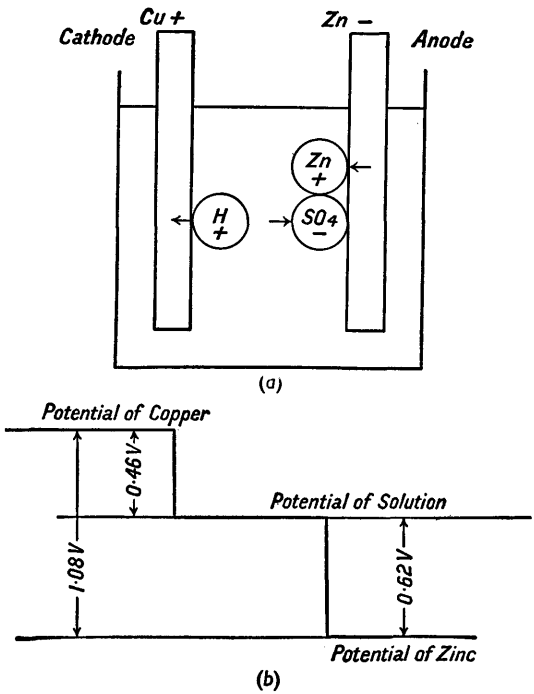


Fig. 5.—Simple voltaic cell

plate. This loss of electrons from the zinc plate tends to raise its potential and releases more positive zinc ions, which go into solution in order to maintain the potential at its equilibrium value. At the same time, the arrival of electrons at the copper plate discharges some of the hydrogen ions, which are evolved as a gas, and enables more to pass out of the solution. Thus electrons are continually entering the solution at the copper plate (cathode), leaving it at the zinc plate (anode) and returning through the external connection.

If the plates are connected by a conductor, a current flows from the copper (positive) to the zinc (negative) outside the cell and from the zinc to the copper inside, the zinc being gradually dissolved with the formation of zinc sulphate, and hydrogen being evolved at the copper plate. The current, which will continue until either the circuit is broken or all the zinc has entered the solution, is generally looked upon as being produced by the E.M.F. of the cell. This chemical E.M.F., which is the sum of the contact P.D.s between the electrodes and the solution, represents the conversion of the chemical energy derived from the solution of the zinc in the acid into electrical energy.

Such a cell—first used by Volta—is called a *primary cell*, since it can be recharged only by supplying fresh active material. The mechanism of current flow through the electrolyte is exactly the same as in the examples of electrolysis previously described. In fact the primary cell is a particular example of an electrolytic cell in which an E.M.F. is set up due to the reaction of one of the electrodes with the electrolyte. Any two dissimilar metals immersed in a solution which reacts with one of them constitute a primary cell, but certain combinations are more suitable than others.

13. Polarity of Anode and Cathode

In both the primary cell and the electrolytic cell the electrode by which the current *enters* the cell is called the *anode*, and that by which it *leaves*, the *cathode*; but they are often also referred to by their polarity in relation to the *external* circuit, and this sometimes causes confusion, particularly in the case of secondary cells.

In the electrolytic cell the *anode*, since it is connected to the positive terminal of the supply, is often called the *positive* electrode, and therefore the cathode is called negative. But in a primary cell the *anode* through which the current enters the cell is the electrode through which it *leaves* the external circuit and is therefore referred to as the *negative* electrode, while the cathode is called positive. In the electrolysis of water, for instance, the hydrogen ions are shown in fig. 3, p. 122, as moving to the negative plate; while in the simple cell the same ions are represented as being deposited on the copper plate, which is described as positive (fig. 5). In both instances, however, the plate in question is acting as the *cathode*; and the statement that *the hydrogen ions move towards the cathode* is true in both cases.

14. Polarization

An ammeter, included in the external circuit of the simple cell just described, indicates a gradual decrease in the current flowing. If the copper plate is removed and wiped, the current rises immediately after it is replaced, but again decreases after a short period. This decrease is due to some of the hydrogen bubbles which fail to escape and collect on the positive plate (cathode); the effect of this layer of gas is twofold:

(1) It acts as a shield, reducing the active area of plate and thus increasing the internal resistance.

(2) Positively charged hydrogen ions collect in the layer and exert

repulsive forces on other hydrogen ions which are approaching, thus producing the effect of a counter-E.M.F.

This phenomenon is called *polarization*, and a cell in this condition is said to be *polarized*. It renders the simple voltaic cell useless for practical purposes; and the differences in the various types of primary cell consist largely in the different methods used to overcome this defect. The most general method is to surround the cathode with either a solid or liquid *depolarizer* which oxidizes the hydrogen as it is liberated, and is prevented from mixing with the electrolyte by enclosing it in a porous pot.

15. Local Action

The statement in § 12, p. 125, that after the potential of each electrode relative to the solution has been established no further action takes place until the external circuit is completed, is true only if the zinc is pure. Commercial zinc contains traces of iron and lead, and these form small local cells short-circuited by the main body of the electrode. The *local action* of these parasitic cells cannot be controlled, so that the zinc is gradually dissolved even though there is no external connection between the plates.

Such local action is prevented by rubbing the surface of the zinc with mercury. The action of the mercury is not fully understood, but it may be assumed that it covers up the impurities and maintains a film of pure zinc amalgam at the surface.

16. Primary Cells

At the time when primary cells formed the chief source of small currents for laboratory and other purposes, there were several types in common use, e.g. the Daniell, Grove, Bunsen, and Leclanché cell; but of these only the Leclanché cell has survived.

The *Daniell* cell, which is the simple voltaic cell with the addition of a depolarizer, consists of a copper (+) and zinc (—) plate in sulphuric acid. The copper plate is surrounded by a solution of copper sulphate (the depolarizer) contained in a porous pot, which allows the passage of the ions but prevents the two solutions from mixing. The hydrogen liberated at the copper plate (cathode) reacts with the copper sulphate to form sulphuric acid and copper, which is deposited on the cathode:

$$Zn + H_2SO_4 = ZnSO_4 + H_2$$

$$H_2 + CuSO_4 = H_2SO_4 + Cu.$$

The cell has an E.M.F. of 1·1 volt and was at one time much used for telegraphic purposes.

In the *Grove* cell the copper plate is replaced by one of platinum and the copper sulphate by strong nitric acid, which acts as a depolarizer:

$$Zn + H_2SO_4 = ZnSO_4 + H_2$$

$$H_2 + 2HNO_3 = 2H_2O + 2NO_2.$$

In the *Bunsen* cell the expensive platinum plate is replaced by a carbon plate. In both cases the anode is zinc and the electrolyte sulphuric acid. The E.M.F. of both cells is about 1·9 volt and the internal resistance is low, so that they are capable of supplying com-

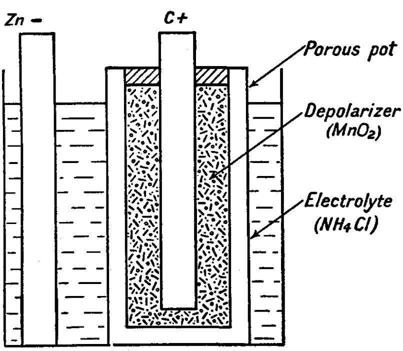


Fig. 6*a*.—Leclanché cell

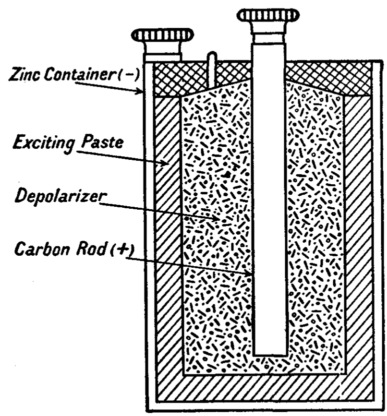


Fig. 6*b*.—Dry cell

paratively large currents; but the fumes of nitrogen peroxide produced by the reduction of the nitric acid by the hydrogen are objectionable.

The only primary cell of importance today, apart from standard cells (§ 18, p. 131), is the Leclanché cell, which is used in large numbers for domestic bell and telephone circuits, and in far greater quantities in the " dry " form for flash lamp, cycle lamp and wireless batteries. The cell, of which a section is shown in fig. 6*a*, consists of a zinc (—) and a carbon (+) rod in a solution of sal ammoniac or ammonium chloride (NH_4Cl). The carbon is surrounded by a solid depolarizer, manganese dioxide (MnO_2), mixed with crushed carbon to reduce the resistance and contained in a porous pot.

In consequence of dissociation, there are present in the solution negative chlorine (Cl) ions and positive ammonium (NH_4) ions:

$$NH_4Cl = \underset{+}{NH_4} + \underset{-}{Cl}.$$

The positive zinc ions react with the negative chlorine ions from the electrolyte to form zinc chloride,

$$\underset{++}{Zn} + \underset{--}{2Cl} = ZnCl_2,$$

while the positive ammonium ions move to the carbon plate, where they are discharged by electrons arriving through the external circuit from the zinc plate, and immediately decompose to form ammonia and hydrogen:

$$2NH_4 = 2NH_3 + H_2.$$

The hydrogen, which would otherwise collect and cause polarization, is then oxidized by the manganese dioxide with the formation of water:

$$2MnO_2 + H_2 = Mn_2O_3 + H_2O.$$

The solid depolarizer is less rapid in action than a liquid, and hydrogen gradually accumulates; but, if allowed to rest, the excess is gradually oxidized and the cell recovers. Hence the Leclanché cell is suitable in cases where the current is small and the duty intermittent; it has the further advantages that it is inexpensive, emits no objectionable fumes, and requires little attention.

The E.M.F. of the cell is 1·5 volt and its internal resistance, the value of which depends upon its size and construction, is moderately high. In the usual domestic form the zinc is in the form of a rod; by using a zinc plate bent in circular form to surround the porous pot, the internal resistance is reduced; the substitution of a canvas bag for the porous pot has a similar effect.

17. Dry Cells

The term dry cell is a misnomer, since moisture is essential to the working of the cell; but in this type the electrolyte is in paste form and unspillable, so that the cell is portable. Most of the dry cells which are used in cycle lamp, flash lamp and wireless batteries are of the Leclanché type.

A section of a dry cell is shown in fig. 6*b*. The zinc plate is in the form of a hollow cylinder and serves as the container. The carbon rod or plate in the centre is surrounded by a mixture of manganese dioxide and crushed carbon contained in a sack, and the annular space between the sack and the container is filled with a paste made of plaster of Paris, zinc chloride, sal ammoniac, flour and water. To prevent drying, the top is sealed with a layer of pitch containing a small gas vent.

The E.M.F. of the dry cell is that of the Leclanché, i.e. 1·5 volt, but the resistance in the larger sizes is considerably lower than in the liquid porous pot type and is of the order of one-tenth of an ohm.

In certain types of battery, such as those used in radio sets and other electronic apparatus, the conventional cylindrical shape (fig. 6*b*) has been replaced by a flattened disc form in order to economize in space.

18. Standard Cells

Certain cells, of which the E.M.F., in the absence of polarization, is extremely constant and affected only slightly by changes of temperature, are used as standards of E.M.F. and are known as *standard cells.*

In the *Latimer Clark* cell the electrodes are mercury (+) and zinc (—), the electrolyte being zinc sulphate and the depolarizer mercurous sulphate. The E.M.F. of the cell is 1·434 volt at 15° C., and falls by about one part in 1400 per °C. This cell has been replaced almost

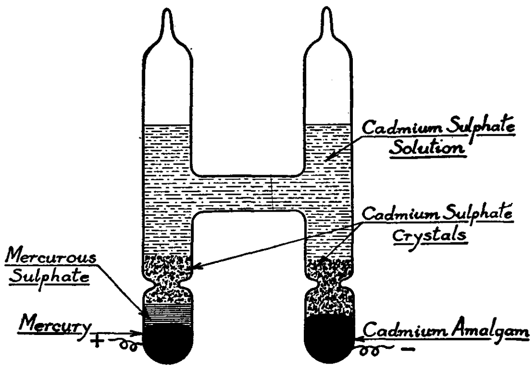


Fig. 7.—Weston cadmium cell

entirely by the *Weston normal cadmium* cell (fig. 7), which is less affected by variations in temperature. The electrodes are mercury (+) and mercury-cadmium amalgam (—), the electrolyte is a saturated solution of cadmium sulphate, and the depolarizer, which forms a layer over the mercury, is mercurous sulphate. The E.M.F. of the cell is 1·0186 volt at 20° C., and falls by only one part in 25,000 per °C. rise in temperature.

A standard cell is never used as a source of current. It serves as a standard of E.M.F., generally in conjunction with a potentiometer, so that at the time of measurement no current passes through the cell; and during the period of adjustment it is protected by a large series resistance.

19. Secondary Cells—the Lead-acid Cell

In the primary cell, electrical energy is obtained from chemical energy by an irreversible reaction, so that the cell can be recharged only by adding a fresh quantity of active materials. In the *secondary cell* or *accumulator*, the combination of electrodes and electrolyte is such that the reaction is reversible; and the cell may be recharged by passing a current through it in the opposite direction, which causes the reformation of the active materials.

The lead cell has innumerable applications and is used in all sizes, from the small units of which certain wireless " high tension batteries " are made up, to the very large cells employed in the battery of a central telephone exchange or in submarines for propulsion when submerged. Probably the most familiar examples of the lead cell are the " low tension battery " of the wireless set and the car lighting and starting battery.

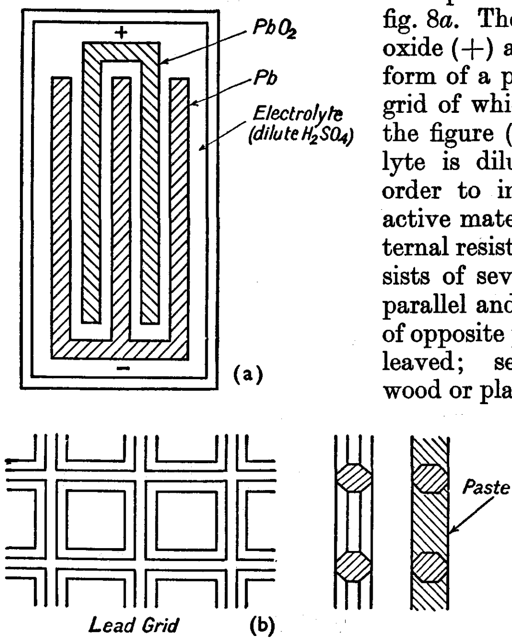


Fig. 8.—The lead cell

A plan of a lead cell is shown in fig. 8*a*. The electrodes are lead peroxide (+) and lead (—), each in the form of a paste supported in a lead grid of which one form is shown in the figure (fig. 8*b*), and the electrolyte is dilute sulphuric acid. In order to increase the quantity of active material and decrease the internal resistance, each electrode consists of several plates connected in parallel and so arranged that plates of opposite polarity are closely interleaved; separators of perforated wood or plastic sheets, or glass tubes are placed between the plates to prevent accidental contact.

The reactions which take place are complex but may be represented approximately as follows:

Discharge.

During discharge the behaviour is that of a primary cell. At the negative plate, which acts as the anode, lead ions react with sulphions from the electrolyte to form lead sulphate:

$$Pb + SO_4 = PbSO_4.$$

At the same time hydrogen ions from the electrolyte are discharged at the positive plate (cathode), and the liberated hydrogen reduces the lead peroxide to lead oxide, which is attacked by the acid to form lead sulphate:

$$PbO_2 + 2H = PbO + H_2O,$$

$$PbO + H_2SO_4 = PbSO_4 + H_2O.$$

Thus each plate on discharge is converted into lead sulphate, and since the hydrogen is used up in the process there is no polarization. The water formed at the cathode causes the specific gravity of the acid to fall during discharge.

Charge.

During charge the behaviour is that of an electrolytic cell, and it must be noted that the positive plate now acts as the anode and the negative as the cathode. The first stage of the reactions is similar to that in the electrolysis of water (§ 7, p. 122). The sulphions move to the positive plate (anode) and react with the water to produce sulphuric acid and oxygen; and the latter attacks the lead sulphate and forms lead peroxide and more sulphuric acid:

$$SO_4 + H_2O = H_2SO_4 + O,$$

$$O + H_2O + PbSO_4 = PbO_2 + H_2SO_4.$$

At the same time the hydrogen ions liberated at the negative plate (cathode) reduce the lead sulphate to metallic lead:

$$PbSO_4 + 2H = H_2SO_4 + Pb.$$

Each of the above reactions produces sulphuric acid, so that the specific gravity of the electrolyte rises during charge. The specific gravity in the fully charged cell is 1200–1220 (water 1000) and falls to about 1180 at the end of discharge. These figures vary slightly with different makes of cell, but in any particular case the specific gravity gives a good indication of the state of charge; the charge indicators on the cells used in battery operated radio receivers are simply indicating hydrometers.

The state and appearance of the cell when charged and discharged may be described briefly as below:

Charged

Positive plate.	Dark brown, lead peroxide (PbO_2)	Sp. gr. 1200–1220.
Negative plate.	Grey, metallic lead (Pb)	

Discharged.

Positive plate.	Light brown, lead sulphate ($PbSO_4$)	Sp. gr. 1170–1180.
Negative plate.	Greyish white, lead sulphate ($PbSO_4$)	

If the charging current is continued after the plates have been fully converted, electrolysis of the water continues, the hydrogen and oxygen are evolved freely, and the cell is said to " gas ". The accumulation of gases round the plates causes the E.M.F. of the cell to rise to about 2·5 volts, but as these disperse at the end of charge, the E.M.F. falls to 2 volts. During the greater portion of the discharge the E.M.F. remains almost constant at this value, but drops towards the end to about 1·8 volt (fig. 9): when the E.M.F. has fallen to this value, the cell is considered to be discharged.

The internal resistance is very low, of the order of 0·005 to 0·0005 ohm, depending on the size, and as there is no polarization the cell can supply steady currents for long periods. The capacity is stated

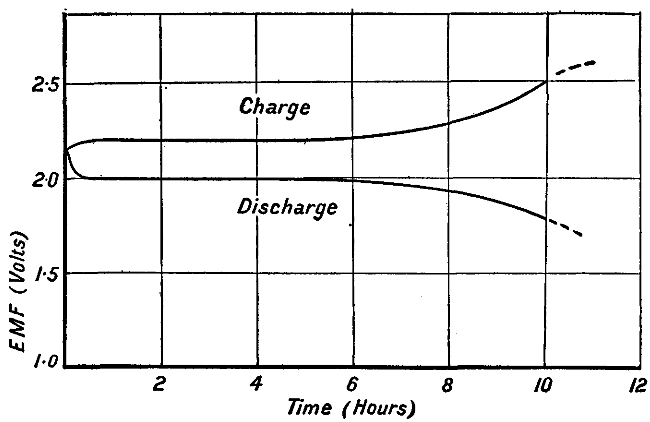


Fig. 9.—Characteristic curves of lead cell

in terms of quantity, i.e. ampere-hours, and is usually based on a 10-hour rate, e.g. a 100-ampere-hour cell will give a current of 10 amperes for 10 hours before the E.M.F. falls below 1·8 volt. If the discharge is continued beyond this point or the cell is left discharged for long periods, the normal lead sulphate changes into an insoluble variety which decreases the capacity and increases the resistance of the cell. Values of charge and discharge current given by the makers should not be exceeded, as excessive currents cause the plates to buckle and the paste to fall out; hence an accumulator should never be short-circuited.

Excessive discharge rates also reduce the ampere-hour capacity of the cell. The acid in the outer layers of material is diluted more rapidly than it can be removed by diffusion, and its higher resistance causes the P.D. to fall rapidly, even though much of the material in the inner layers is still unchanged.

During charge the charging P.D. must exceed the E.M.F. of the cell, which acts as a counter-E.M.F., by the amount required to overcome the internal resistance. Of the total energy input per coulomb, the greater part, proportional to that portion of the P.D. used in overcoming the counter-E.M.F., is stored chemically in the plates or used in electrolysing the water, while the small remainder is converted into heat.

The ampere-hour efficiency is of the order of 85–90 per cent; but the watt-hour efficiency is considerably lower because of the rise in E.M.F. during charge, particularly in the final stages.

Example.—A battery consists of 120 cells each having an internal resistance of 0·005 ohm. At the end of the charge, the E.M.F. of each cell has risen to 2·5 volts. What is the P.D. necessary to maintain a charging current of 10 amperes?

Total internal resistance	$= 120 \times 0{\cdot}005 = 0{\cdot}6$ ohm.
Resistance drop	$= 0{\cdot}6 \times 10 = 6$ volts.
Battery E.M.F.	$= 120 \times 2{\cdot}5 = 300$ volts.
Charging P.D.	$= 300 + 6 = 306$ volts.

20. Secondary Cells—Alkaline Cells

In the nickel-iron cell, sometimes called the Edison cell, the active materials are nickel hydroxide ($Ni(OH)_3$) (positive) and iron (Fe) (negative). In both cases the materials are enclosed in perforated steel tubes or pockets which are assembled in nickel-steel frames to form the plates. The electrolyte is potassium hydroxide (caustic potash —KOH) contained in a nickel-steel box.

The reactions which occur are very complex. Briefly, on discharge, the nickel hydroxide is reduced to a lower hydroxide ($Ni(OH)_2$) and the iron oxidized to iron hydroxide ($Fe(OH)_2$); on charge the reaction is reversed.

The specific gravity of the electrolyte remains almost constant, as neither water nor potassium hydroxide is produced in the reactions; and for this reason, also, the ampere-hour capacity is less reduced by high rates of discharge than in the case of the lead-acid cell.

Owing to the higher internal resistance the efficiencies are lower than in the lead-acid cell.

In the nickel-cadmium cell the iron is replaced by cadmium, but the construction, reactions, and characteristics are similar to those of the nickel-iron cell.

The cell is inherently more robust than the lead cell and will withstand much rougher usage. Excessive charge and discharge rates do little harm, and the cell can be left discharged for long periods without injury. The chief disadvantages compared with the lead cell, which have prevented its more extensive use, are the low E.M.F., 1·2 volt, the higher internal resistance, and higher cost.

EXAMPLES

1. State Faraday's laws of electrolysis. What is meant by the electrochemical equivalent of an element, and how is it related to the chemical equivalent?

Given that the electrochemical equivalent of copper is 0·000329 gm. per cmb., calculate the current required to deposit 5 gm. of copper in 1 hr. on an article being plated. [Grad. I.E.E.]

2. A current of 5 amperes passed through a vat of silver nitrate deposits 6·71 gm. of silver in 20 min. Hence calculate the electrochemical equivalent of silver.

3. Two voltameters containing copper sulphate and silver nitrate respectively are connected in series. If the weight of copper deposited is 15 gm., what weight of silver is deposited in the same time?

4. Calculate the value of the steady current necessary to decompose 100 gm. of water in 20 hr.

5. In calibrating a low-reading ammeter by means of a copper voltameter, the following readings were obtained:

Ammeter reading (amperes)	0·25	0·5	0·75	1·0
Time (min.)	120	60	40	30
Weight of copper deposited (gm.)	0·65	0·60	0·58	0·59

Determine the true value of the current in each case and hence the percentage error (electrochemical equivalent of copper = 0·000329 gm. per coulomb).

6. An ammeter is calibrated by means of a copper voltameter, and the value of the steady current as indicated on the instrument is 2 amperes. If 5·5 gm. of copper are deposited in $2\frac{1}{2}$ hr., determine the error in the reading.

7. A battery consists of 50 cells each having a resistance of 0·003 ohm. At the end of the charging period the E.M.F. of each cell is 2·5 volts and the charging P.D. is 128 volts.

Calculate (*a*) the charging current,
(*b*) the resistance loss as a percentage of the total power input.

8. Determine the variation in the charging P.D. required to charge a battery of 200 lead cells at a constant current of 12 amperes. The resistance of each cell is 0·01 ohm and the E.M.F. rises from 1·8 volt at the beginning to 2·4 volts at the end of the charge.

9. A high resistance voltmeter connected to an accumulator indicates an E.M.F. of 2 volts; but when supplying a current of 100 amperes the reading falls to 1·9 volt. What is the internal resistance of the accumulator?

10. A battery consisting of 80 cells, each having an internal resistance of 0·0025 ohm, is to be charged from a 240-volt supply through leads having a resistance of 0·1 ohm. Find the value of the external series resistance required to limit the initial charging current to 20 amperes, assuming the E.M.F. per cell at the beginning of charge is 1·9 volts.

If the external resistance is later reduced to 2 ohms, what is the final value of the charging current when the E.M.F. per cell has risen to 2·5 volts?

CHAPTER IX

Electromagnetism

1. Magnetic Effects of a Current

The observations made in § 1, p. 30, and § 7, p. 35, are summarized below.

Assuming a current flowing from south to north in a conductor lying in the magnetic meridian:

(1) A small compass needle lying below it is deflected in a counter-clockwise direction (fig. 1*a*).

(2) The same needle placed above it is deflected in a clockwise direction (fig. 1*b*).

(3) Whatever the position of the needle, reversal of the current reverses the deflection.

(4) The angle of deflection can be increased by (*a*) bringing the needle closer to the conductor, (*b*) increasing the current.

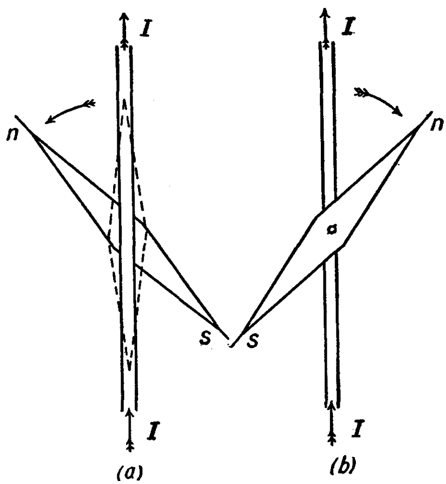


Fig. 1.—Magnetic effect of a current in a straight conductor

From these observations it is evident that:

(*a*) the magnetic field is opposite in direction above and below the conductor: (1) and (2);

(*b*) the field is reversed in direction when the current is reversed: (3);

(*c*) the strength of the field (or **magnetizing force**) at a given point is dependent on the strength of the current, and for a given current, increases in strength as the conductor is approached: (4).

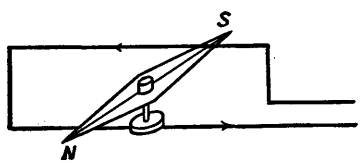


Fig. 2.—Principle of simple galvanometer

Further, if the circuit is arranged as in fig. 2 the magnetic effect is increased, since the conductors lying on opposite sides of the needle, and carrying current in *opposite* directions, exert deflecting forces on the needle in the *same* direction. Similarly, if a coil of several turns is substituted for the single turn, each turn contributes an equal effect and the deflecting forces are proportionately increased, i.e. the magnetic effect of a current I flowing through a coil of N turns is the same as that of a current NI flowing through a single turn of the same dimensions, and therefore:

(*d*) the magnetizing force is proportional to the product (current × turns), usually called the *ampere-turns*.

Hence, by using a large number of turns a large deflection may be obtained with a very small current; this principle is used in the construction of sensitive galvanometers.

2. Examples of Field Distribution

Field surrounding a Long Straight Conductor.

A more complete investigation of the field surrounding a long straight conductor can be made by means of the arrangement shown in fig. 3. The conductor is held in a vertical position and some distance from the return portion of the circuit; and a number of small exploring compasses are placed on a horizontal piece of cardboard surrounding the conductor. When no current is flowing, all the needles lie approximately parallel to each other and in the magnetic meridian; but immediately the circuit is completed each needle is deflected until its axis appears to be tangential to an imaginary circle having the conductor as its centre.

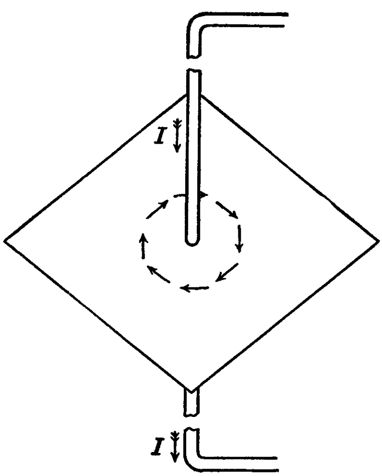


Fig. 3.—Magnetic field due to current in a long straight conductor

If the direction of the current is downwards, the north poles of the needles all point in a clockwise direction round the circle as shown in fig. 3; when the current is reversed the deflection is reversed, and the north poles point in a counter-clockwise direction round the circle. Since the direction of the longitudinal axis of the needle represents that of the field at its pivot (§ 15, p. 47), it appears that the lines of flux are circular in shape. The truth of this inference may be shown by replacing the compass needles by iron filings; these form closed chains surrounding the wire, which, if the return portion of the circuit is sufficiently far away, are seen to be concentric circles having the wire as centre * (fig. 4*a*, *c*).

The direction of the field for a given current can be deduced from observations (1) and (2) (p. 137), or from the direction of the north poles of the needles in the experiment described above. It is shown in fig. 4*b*, from which it can be seen that the relative directions of the field and the current are the same as the relative rotational and axial motions of a right-handed screw (fig. 4*d*), e.g. an ordinary wood-screw.†

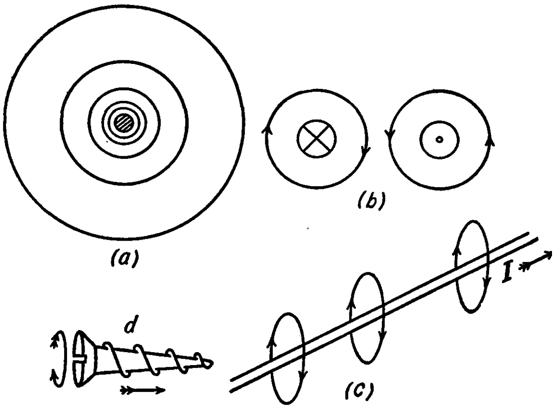


Fig. 4.—Relative directions of current and lines of force

The field surrounding a long straight conductor carrying a current, and far removed from other conductors, is circular and concentric with the conductor. Its direction is that in which a right-hand screw must be turned in order to move it in the direction of the current.

It will be shown later than the magnetizing force is proportional to the current and inversely proportional to the distance from the centre.

Field surrounding a Loop or Short Coil.

If the outward and return conductors of the loop are close together, the field distribution, as obtained by iron filings, is as shown in fig. 5*a*; the lines of flux, although still closed loops, are distorted owing to the presence of the neighbouring conductor. By applying the "screw rule" given in the previous section, it can be seen that the lines of

* The earth's horizontal field produces some distortion, but if the current is sufficiently large the effect is negligible in the neighbourhood of the wire.

† In fig. 4*b* the cross-section of the conductor is represented by a circle. A cross in the circle indicates a current flowing *away from*, and a dot in the circle a current flowing *towards* the observer.

flux contributed by each conductor have the same general direction inside the loop, and that at the centre of the loop this direction is axial (fig. 5*b*).

This figure also represents the field surrounding any short multi-turn coil carrying a current. The similarity between it and the field due to a short bar magnet (fig. 11*a*, p. 49) is obvious; and, in fact, any such coil behaves like a bar magnet. In fig. 5*b* the lower side of the coil, from which the lines issue, exhibits north polarity and the upper side south polarity. The polarity under given conditions can be obtained by means of a modification of the screw rule.

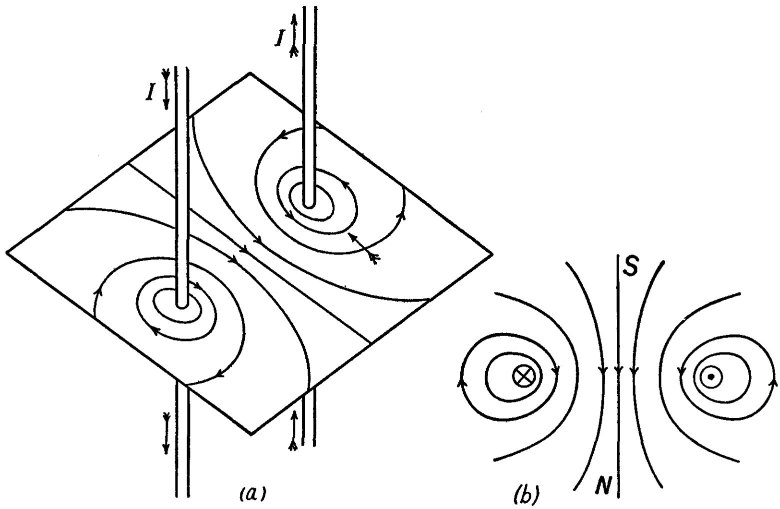


Fig. 5.—Magnetic field due to current in a loop

If it is imagined that the right hand is laid on the outside of the coil so that the fingers point in the direction in which the current is flowing, the thumb points in the direction of the north pole.

It will be shown later that the magnetizing force at the centre is proportional to the ampere-turns and inversely proportional to the radius of the coil.

Field due to a Long Coil or Solenoid.

A long coil or solenoid may be looked upon as made up of a number of short coils, all wound in the same direction, connected in series and placed axially end to end. Since a short coil carrying a current acts like a short bar magnet the long coil may be expected to behave like a number of short bar magnets placed end to end with unlike poles adjacent (fig. 6). Since the pairs of adjacent poles very largely neutralize each other as far as external effects are concerned, the arrangement is equivalent to a long bar magnet. That a long solenoid has magnetic properties similar to those of a long bar magnet is borne out by the experimental observations summarized below, some of which have been mentioned in § 1, p. 29.

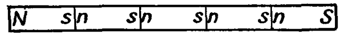


Fig. 6.—Magnetic effect of long coil

(1) Each end of a compass needle is attracted by one end of the solenoid and repelled by the other; the position of the poles is indefinite, and evidences of polarity can be obtained well towards the middle of the solenoid.

(2) The polarity as ascertained in (1) is found to agree with that determined by the right-hand rule given above.

(3) The field distribution in one plane, as determined by means of iron filings,* is shown in fig. 7, and the similarity between it and the field due to a bar magnet is evident. But it will be noticed that a large number of the lines of flux emerge through the sides of the coil before reaching the end; this causes the indefinite position of the poles referred to in (1).

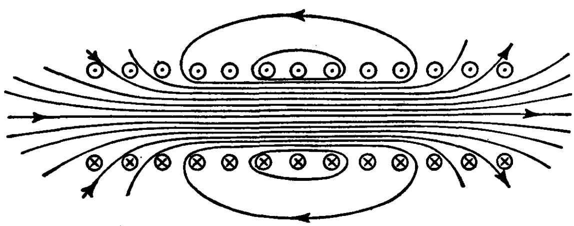


Fig. 7.—Magnetic field due to current in long coil

The increasing leakage through the walls of the solenoid is accounted for by the considerations given below. The field set up by a single turn is shown in fig. 8*a*. In fig. 8*b* three such turns are shown placed side by side, from which it will be seen that the general direction of the field due to each is from right to left inside and from left to right outside the coil. But *between* any two turns the fields set up by adjacent portions of these turns are in *opposite* directions, and to a great extent neutralize each other, so that the general form of the resultant field is as shown by the full line.

Actually the field strength between any two turns is due not to those two turns alone, but to all the turns lying on either side of the point considered; so that complete neutralization occurs only at the centre

* The field distribution *inside* the coil may be obtained by winding a widely spaced coil through holes in a card and using a large current.

of the coil. A section of a short solenoid of 18 turns is shown in fig. 8*c*. The 9 turns on the right of the mid-point *A* produce a magnetizing force directed outwards between turns 9 and 10, while the 9 turns on the left produce at the same point a force of equal strength directed inwards. These two neutralize each other completely, and no evidence of polarity is shown at this point.

But the numbers of turns on each side of any point not at the centre are unequal. At *B*, for instance, the outwardly directed force between turns 7 and 8 produced by the 11 turns on the right is greater than the inwardly directed force due to the 7 turns on the left. Hence there is a resultant force directed outwards, and some lines of flux emerge at *B* and enter the coil at *C*, where the conditions are exactly reversed.

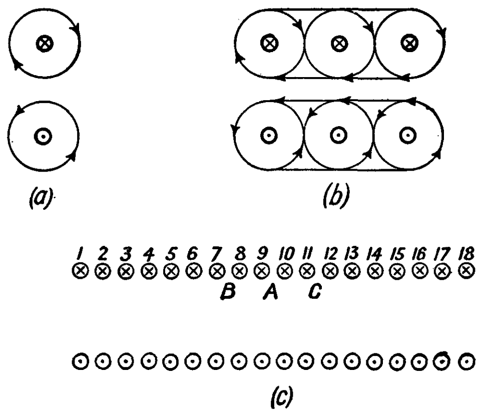


Fig. 8.—Magnetic field due to current in long coil

As the end of the coil is approached, the want of balance and the corresponding leakage increase rapidly, until at either end the magnetizing force on the axis of the coil has fallen to about half the value at the centre (see § 17, p. 164).

3. Principles of Electromagnetic Induction

In the year 1831, Michael Faraday, knowing that a magnetic field is produced by an electric current, was investigating the possibility of a converse effect, i.e. the production of a current by a magnetic field, when he made a number of observations which were to form the foundation of modern electrical engineering.

His observations may be summarized as follows:

(1) If one pole of a bar magnet is inserted into a coil connected to a galvanometer (fig. 9*a*), a deflection is produced while the magnet is approaching and entering the coil, but ceases as soon as it becomes

stationary. When the magnet is withdrawn (fig. 9*b*), a deflection is again produced, but in the opposite direction. Both deflections are reversed in direction if the opposite pole of the magnet is used.

The same effect is produced if the magnet is held stationary and the coil is moved.

It can be shown that these deflections are not due to any direct action of the magnet upon the galvanometer, by arranging that the latter is at some distance from the coil.

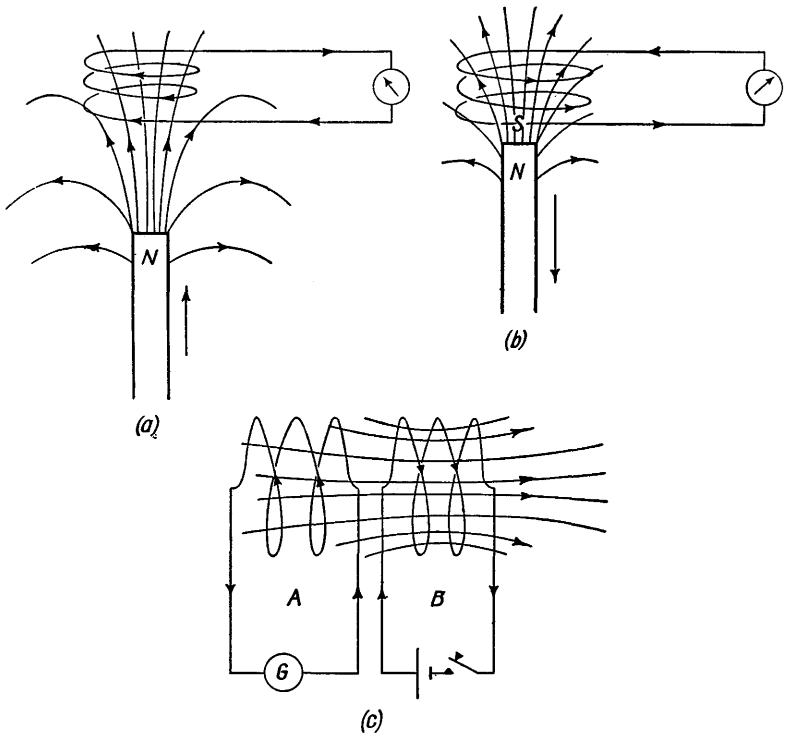


Fig. 9.—Principle of electromagnetic induction

(2) If instead of a magnet another coil, in which a current is flowing, is used, the effects produced are similar. In this case the directions of both deflections are reversed if the current is reversed in the second coil.

In both (1) and (2) the magnitude of the deflection is increased if the velocity of the relative motion is increased; but the duration of the deflection is shorter.

(3) If the two coils are placed side by side with their axes in line (fig. 9*c*), a momentary deflection of the galvanometer occurs in one

direction when the switch is closed and the current *begins to flow* in coil B, and in the opposite direction when the switch is opened and it *ceases*; but there is no deflection as long as the current is steady.

The effect is greatly increased if an iron core is inserted in the coils.

In all three cases the deflection is reduced if the resistance of the galvanometer circuit is increased, and vice versa.

In all these experiments there is one common feature. The deflection occurs only when the magnetic conditions in the neighbourhood of the coil are changing. In (1), when the magnet is brought near, more of the lines of flux issuing from it pass through the coil, i.e. there is an increase in the flux linked with it (fig. 9*a*); similarly, when the magnet is withdrawn, there is a decrease in the flux linked with it (fig. 9*b*). In (2), exactly the same processes occur except that the flux is produced not by a bar magnet, but by a coil carrying a current, which behaves like a bar magnet. It is obvious that it does not matter which of the two moves as long as there is suitable relative motion, and that the more rapid the motion the greater is the *rate of change* of the flux linked with the coil. In (3), when the current starts in coil B, a magnetic field is set up and some of the flux enters coil A, while when the current ceases, the field collapses and the flux is withdrawn (fig. 9*c*).

It appears, therefore, that the deflection is caused by a change in the flux passing through or linked with coil A, and that its direction depends upon whether the flux is increasing or decreasing. The deflection indicates the existence of a current in the coil and therefore of an E.M.F. by which it is produced. And since an increase in the resistance of the circuit results in a decrease in deflection, i.e. a decrease of current, it is reasonable to assume that the primary result of the change in flux is the production of an E.M.F.

These deductions may be summarized for the present as follows:

Whenever the value of the flux linked with a circuit changes, an E.M.F. is induced in the circuit. The direction of the E.M.F. depends upon the nature of the change and its magnitude upon the rate at which it takes place.

This E.M.F. is called the *E.M.F. of electromagnetic induction*, or more shortly, the *induced E.M.F.*

4. The Laws of Faraday and Lenz

Further investigation by Faraday showed that the magnitude of the induced E.M.F. is directly proportional to the rate at which the change in the flux linked with the coil occurs; and, later, Lenz stated the factors upon which the direction of the E.M.F. depends.

The deflection of the galvanometer involves a certain expenditure of energy, and, since the movement is caused by a current, a small amount of heat is produced in the circuit, which represents a further expenditure of energy. Since there is no battery in the circuit, the only source of this energy lies in the movement of the magnet. Hence in some way the work done in moving the magnet into the coil is greater when the circuit is closed than when it is open.

Experiment shows that if (say) the north pole of the magnet is approaching the coil (fig. 9*a*) the direction of the induced E.M.F. is such that the current produced makes the near end of the coil have north polarity, i.e. the ampere-turns of the coil *oppose* the increase of flux through the coil; and at the same time the work done in overcoming the repulsive force between them reappears as electrical energy in the circuit. Similarly, when the magnet is withdrawn, the E.M.F. and current reverse, and the ampere-turns of the coil now attempt to maintain the flux, i.e. to *oppose* the decrease in flux; while at the same time work is done in overcoming the attractive force set up between the north pole of the magnet and the south polarity developed on the nearer end of the coil (fig. 9*b*), which appears as electrical energy in the circuit.

Again, when the current is started in coil B (fig. 9*c*), the E.M.F. induced in coil A produces a current in the *opposite* direction which *opposes* the increase in the flux; while when the current in B ceases, the induced E.M.F. and current in A reverse and tend to maintain the flux, i.e. to *oppose* the decrease in flux.

Hence the direction of the induced E.M.F. is always such that the magnetic effect of the current produced by it *opposes* the change in flux to which it is due: the current flows only if the circuit is complete but the E.M.F. exists even when it is open.

These experimental facts are summarized in the **Laws of Lenz and Faraday,** which are combined as follows:

Whenever there is a change in the flux linked with a circuit, an E.M.F. is induced in the circuit. The magnitude of the E.M.F. is proportional to the rate of change, and the direction is such that the E.M.F. (or the current produced by it) opposes the change to which it is due.

5. Equivalence of Rate of Change and Rate of Cutting Flux

Since every line of flux forms a closed loop, it may be imagined that when the flux is increasing the additional lines become linked with the circuit by cutting through it. An increase of Φ lines in the linking flux means that Φ lines, in entering, have cut the circuit at some point or other; similarly for a decrease of Φ lines. Any explanation of the process of cutting is unnecessary since the lines are mental concepts and have no physical existence. (Cf. § 17, p. 49.)

It is possible to look upon the E.M.F. as being induced either (*a*) by the change in the value of the total flux linked with the circuit, or (*b*) by the flux cutting the circuit, or being cut by the circuit. The same result is obtained whichever point of view is adopted, but one method may be more convenient than the other according to circumstances. For instance, in a dynamo or electric generator in which the change of flux through the coils of the winding is produced by the motion of the coils through the magnetic field, method (*b*) is more convenient; whereas in the case of a transformer where there is no relative motion, method (*a*) is generally used (see Chapter XIII).

6. Unit of Magnetic Flux (Φ)

The E.M.F. induced in a coil is proportional to the rate of change of the flux and the number of turns with which it is linked, i.e. to the total change in flux-linkage (Faraday's law).

Hence unit rate of change of flux is that which induces unit E.M.F. in a circuit of one turn.

The unit of magnetic flux is called the *Weber* and can be defined as that flux which when withdrawn uniformly from a circuit of one turn in one second, induces an E.M.F. of one volt.

Hence $\quad 1 \text{ volt} = 1 \text{ weber per second}$

or $\quad 1 \text{ weber} = 1 \text{ volt-second } (= 10^8 \text{ C.G.S. lines}).$

If the rate of change is not uniform the E.M.F. induced at any instant is

$$e = \frac{d\Phi}{dt} \text{ volts,}$$

and if the flux is fully linked with a coil of N turns, since an equal E.M.F. is induced in each turn,

$$e = N\frac{d\Phi}{dt} \text{ volts.} \qquad (1)$$

7. Unit of Flux Density (B)

In the C.G.S. system in which the flux is measured in lines which can be visualized, the idea of flux density, measured in terms of the number of lines crossing unit area in a plane perpendicular to the direction of the lines, i.e. $B = \Phi/a$, is easily grasped, and unit flux density is clearly a density of one line per sq. cm. (1 gauss).

In the M.K.S. system the flux is not expressed in lines, although it is usually visualized in this way, but, as before, $B = \Phi/a$, so that unit flux density is a density of *one weber per sq. metre*.

$$\left(1 \text{ weber per m.}^2 = \frac{10^8 \text{ lines}}{10^4 \text{ cm.}^2} = 10^4 \text{ lines per cm.}^2 = 10^4 \text{ gauss.}\right)$$

8. E.M.F. generated in a Conductor moving through a Magnetic Field.

Consider a conductor, of effective length l metres, lying in and perpendicular to a field of uniform density B webers per square metre and forming the lower side of a single turn (fig. 10) which is moving downwards in a direction perpendicular to the field with a constant velocity of V metres per second.

In a time t the conductor has moved downwards through a distance Vt metres and the flux passing through the turn has increased by an amount $\Phi = BlVt$ webers. Hence the generated E.M.F. is

$$E = \frac{\Phi}{t} = \frac{BlVt}{t} = BlV \text{ volts.}$$

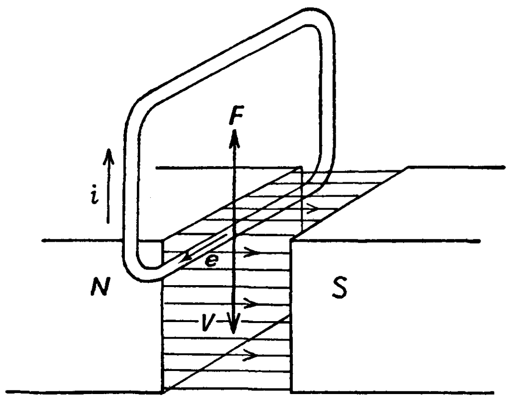


Fig. 10.—E.M.F. generated in moving conductor

It must be noted that only the lower side of the turn is assumed to be moving through the field. If both sides were in the field, equal E.M.F.s would be generated in both, which would oppose each other round the turn, so that the resultant E.M.F. would be zero. Looked at from the other point of view, under these conditions the total flux passing through the turn would be unchanged, so that the generated E.M.F. would be zero.

If either the flux density or the velocity (or both) is not uniform,

$$e = Blv \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where B and v are the instantaneous values of flux density and velocity.

The relative direction of E.M.F., flux, and motion can be found by the method given on p. 150.

9. Force acting on a Current-carrying Conductor lying in a Magnetic Field

Experiment shows that a conductor carrying a current and lying in a magnetic field experiences a force in a direction perpendicular to that of both current and field; the force is proportional to the current, the effective length of conductor, and the strength of the field.

Referring again to fig. 10 suppose that the generated E.M.F., e volts, causes a current, i amperes, to flow round the turn. In accordance with Lenz's law the direction of the generated E.M.F. and current is such that the conductor experiences a force *opposing* the motion.

The electrical power is ei watts, and in a time dt the electrical energy output (which in this case is all turned into heat) is

$$ei\,dt \text{ joules.}$$

The only source of this energy is the work done in overcoming the opposing force, and during this time the conductor has moved through a distance dx so that

$$\text{Work done} = F\,dx \text{ joules,}$$

where F is the force acting on the conductor in newtons.

Hence
$$F\,dx = ei\,dt = Bl\,vi\,dt$$
$$= Bl\frac{dx}{dt}i\,dt = Bl\,dx\,i. \quad \dots \quad (3)$$

Therefore
$$F = Bli \text{ newtons.}$$

A conductor l metres long, lying in and perpendicular to a field of density B webers per square metre and carrying a current I amperes, experiences a force

$$\mathbf{F = BlI \text{ newtons}} \quad \dots\dots \quad (4)$$

in a direction perpendicular to both current and flux.

In electrical machinery it is usually arranged that the length of the conductor is perpendicular (or very nearly so) to the field; but if this is not so, the effective length is its perpendicular component, $l \sin\theta$, where θ is the angle of inclination to the field.

Referring again to equation (3),

$$F\,dx = Bli\,dx;$$

$l\,dx$ is this area swept by the conductor, so that $Bldx$ represents the total flux cut by the conductor in this time dt.

Hence, when a conductor carrying a current moves through a magnetic field,

Total work done = Current × Total flux cut.

When, as is usual, the current is measured in amperes and a flux in webers,

$$\textbf{Total work done} = \mathbf{I\Phi} \textbf{ Joules.} \quad . \quad . \quad . \quad . \quad (5)$$

This is a general expression of fundamental importance.

The important results of the last two sections may be summarized as follows:

A conductor lying in a magnetic field and carrying a current experiences a force. If the conductor is moved by external means against the force, an E.M.F. is generated in the conductor. At the same time work is done *on* the conductor which is converted into electrical energy at the rate of EI joules per second or watts, where E is the E.M.F. generated, in volts, and I is the current in amperes, and the apparatus acts as a *generator*. On the other hand, if the conductor moves under the influence of the force, and therefore in the *opposite* direction, work is done *by* the conductor. At the same time an E.M.F. is generated, which, since it is now in the opposite direction, opposes the current (and is often called a back-E.M.F.), and electrical energy is converted into mechanical energy at the rate of EI joules per second or watts, and the apparatus acts as a *motor*.

This is a particular illustration of the general principle that if the current is in the same direction as the E.M.F. mechanical energy (or some other form of energy) is being converted into electrical energy; while if the current is flowing *against* the E.M.F., electrical energy is being converted into some other form of energy.

In both cases,

$$\text{Total work done} = \text{Current} \times \text{Total flux cut.}$$

Various rules have been devised in order to determine rapidly the direction of the force when that of current and flux is given; the best known is Fleming's Left-hand Rule.*

The author's experience has shown that such rules are difficult to remember; that referred to above is, in addition, easily confused with Fleming's Right-hand Rule. The method given below, although it may take a little longer and at first necessitate a rough sketch, can be used in place of either of the " hand " rules and is much less liable to lead to error.

In fig. 11 is shown at (*a*) a uniform field and at (*b*) the section of a conductor with its associated field. If the conductor is placed in the field, it will be seen that on the left-hand side the field produced by the current is in the same direction as and therefore strengthens the main field, while on the other side it opposes and weakens it. The

* The rule is: place the thumb, forefinger, and middle finger of the left hand mutually at right angles. If the forefinger points in the direction of the flux, and the middle finger in that of the current, the thumb indicates the direction of the force.

resultant field produced is shown in (*c*). Now, remembering that lines of flux are assumed to behave like stretched elastic threads which tend to shorten (§ 17, p. 49), it is clear that there is a force on the conductor urging it to the right. If the direction of the current is reversed, the main field is strengthened on the right and weakened on the left, so that the direction of the force is reversed (*e*); the same effect is produced by reversing the field instead of the current (*d*); but if both are reversed, the direction of the force is unchanged (*f*).

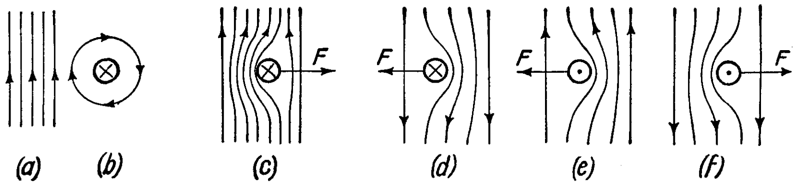


Fig. 11.—Relative directions of flux, current and force

After a little practice the student will find it unnecessary to sketch more than the direction of the main field, and later even this may be omitted.

The direction of E.M.F. for given directions of flux and motion may be ascertained from Fleming's Right-hand Rule,* which is the complement of and possesses the same disadvantages as the Left-hand Rule.

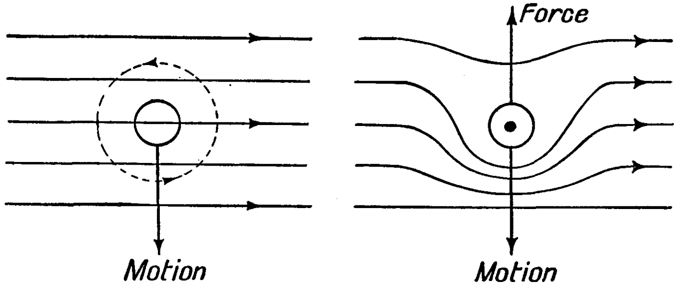


Fig. 12.—Relative direction of flux, motion, and E.M.F.

Mistakes are much less likely to occur if a method is adopted similar to that described above. It has been shown that the direction of the induced E.M.F. is such that the current due to it produces a force opposing the motion. Hence if a sketch is made of a portion of the field and the conductor placed in it with the direction of motion indicated (fig. 12), it is easy to see which direction of current is neces-

* The rule is: place the thumb, forefinger and middle finger of the right hand mutually at right angles. If the thumb points in the direction of motion, and the forefinger in that of the flux, then the middle finger indicates the direction of the induced E.M.F.

sary to produce a force opposing the motion; and this must be the direction of the E.M.F. In the figure, for example, the motion is *downwards*, so that the direction of the E.M.F. must be such that the current produced by it causes an *upward* force, i.e. out of the paper.

10. Eddy Currents

Previous sections have dealt with the E.M.F. induced in a conductor or a coil, in cases where a *current* flows only if the circuit is completed, and then only in a definite path. An E.M.F. is induced, however, in any mass of metal when it moves through a magnetic field or when the flux passing through it changes; and in many cases the flow of the current in local circuits inside the metal cannot be controlled. The cross-section of a piece of metal lying in a magnetic field, as shown in fig. 13, can be looked upon as made up of thin shells, each

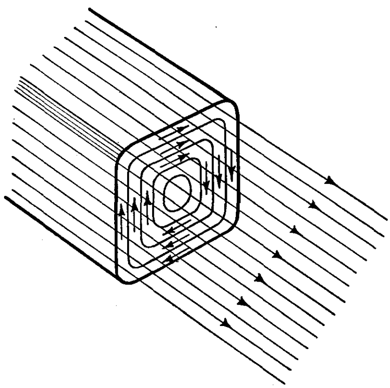


Fig. 13.—Production of eddy currents

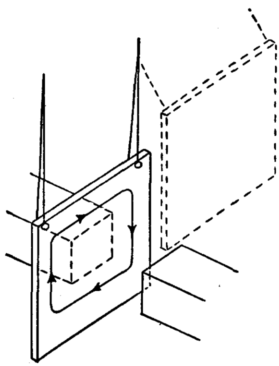


Fig. 14.—Effect of eddy current

shell being linked with a certain amount of flux. If there is any change in the flux linked with the shells, an E.M.F. is induced which causes a current to flow round the shells while the change is taking place. The arrows show the direction of the current when this flux is increasing. Such currents are called *eddy currents*, and represent the conversion of energy into heat which, in most cases, cannot be utilized and therefore constitutes a loss.

The production of eddy currents and their effects can be illustrated in a striking manner by allowing a heavy copper plate to swing edge-ways between the poles of an electromagnet (fig. 14). As long as the magnet is unexcited, the plate swings freely, and the amplitude diminishes only slowly as the original energy is gradually dissipated by the friction. But when the magnet is excited so that the plate moves through the strong field between the poles, E.M.F.s are induced which, since the resistance of the copper plate is low, cause large eddy currents; the forces produced by these *oppose* the motion, and the plate comes to rest almost immediately. In this case the original energy is turned into electrical energy and then into heat.

In electrical apparatus, some or all of the iron which forms the magnetic circuit carries an alternating magnetic flux: and if the metal were solid the eddy currents and consequent heating losses would be large. Eddy currents cannot be eliminated, but are much reduced by *laminating*, i.e. building up the parts forming the magnetic circuit of thin sheets insulated electrically from each other. The eddy currents in each lamination are thus isolated; the eddy E.M.F. is small and the resistance of the long narrow path is large, so that the eddy current is very small. The current and consequently the power loss are still further reduced by using special alloyed iron (§ 5, p. 221), which has a high resistivity.

Eddy currents also occur in conductors; they are negligibly small except where the cross-section is considerable, in which case the conductors are usually laminated.

In some cases eddy currents may be usefully employed, as in the induction furnace (§ 6, p. 34) and in the damping of instruments (§ 6, p. 191).

11. The Magnetic and Electric Circuits

Whenever a current flows in an electric circuit, a magnetic flux is set up in a magnetic circuit which is linked with it; and every magnetic flux requires for its production and maintenance a current flowing in an electric circuit linked with it. This statement includes the fluxes associated with permanent magnets where the electric circuits are within the atoms (see § 9, p. 41). The interlinkage of the magnetic and electric circuits is as shown in fig. 15, the relationship being similar to that between two adjacent links in a chain.

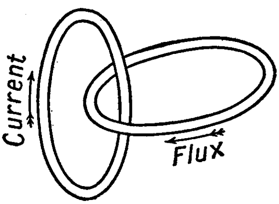


Fig. 15.—Interlinkage of magnetic and electric circuits

The term *electric circuit* has been used frequently in preceding sections, and refers to the particular path in which the electric current is constrained to flow; similarly, the *magnetic circuit* refers to the path taken by the magnetic flux considered, although this is not so definite as the electric circuit. An electrical machine consists essentially of a magnetic circuit and an electric circuit, interlinked and suitably arranged in relation to each other: so that magnetic circuit calculations are as necessary and as important as those connected with the electric circuit.

Since there are certain similarities between the two circuits, and since the relationships between the fundamental quantities are similar in form, it is helpful to consider the magnetic circuit by analogy with the electric circuit.

Current and flux may be considered as corresponding quantities in that their existence gives the circuits their characteristic properties.

The current in an electric circuit is due to an electromotive force. Similarly, the flux in a magnetic circuit may be considered as due to a *magnetomotive force* (F)* which is proportional to the current and to the number of turns in the interlinking electric circuit, and, in the

* F is the standard symbol for both Magnetomotive Force and Force in the mechanical sense, but the context will usually prevent any confusion between them.

M.K.S. system, is measured in ampere-turns.

The M.K.S. unit of magnetomotive force is the ampere-turn,

so that

$$F = NI \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Further, just as resistance is a property of the electric circuit which opposes the current, so *reluctance* is a property of the magnetic circuit which can be considered as opposing the establishment of a flux; and both resistance and reluctance depend in a similar way on the dimensions and material of the circuit.

In the electric circuit the fundamental quantities are related by Ohm's law,

$$\text{Current} = \frac{\text{electromotive force}}{\text{resistance}}$$

and the relation between the corresponding quantities in the magnetic circuit is similar in form,

$$\text{Flux} = \frac{\text{magnetomotive force}}{\text{reluctance}}.$$

Although this analogy is helpful it must be realized that the two circuits are *essentially different,* as can be seen from the following considerations.

(1) There is no *flow* in a magnetic circuit, the term magnetic *flux* being a misnomer; the statement that a magnetic flux has been set up really means that a state of magnetic strain has been produced in the material of the circuit.

(2) In the electric circuit the maintenance of a current requires a continuous expenditure of energy; but the maintenance of a magnetic flux, once it has been established, requires no energy at all, e.g. the flux of a permanent magnet. The energy expenditure in the case of an electromagnet takes place, not in the magnetic circuit but in the electric circuit associated with it, and is due to the fact that there is no material which is a perfect conductor.

A further difference lies in the fact that whereas the conductivity of a good conductor such as copper is about 10^{20} times that of a good insulator, so that it is possible to confine the electric current almost entirely to the desired path, the maximum permeability of a ferromagnetic material is only a few thousand times that of air or other non-magnetic material. There is therefore no material which can be described as a *magnetic insulator*, so that in most magnetic circuits the flux cannot be confined wholly to a definite path. An exception to this statement is the case of the ring solenoid (§ 18, p. 165), in which the magnetizing winding is distributed uniformly over the whole length of a magnetic circuit which is homogeneous and uniform in cross-section; no external poles are produced and the flux lies entirely inside the solenoid. On the other hand, in the long straight solenoid (§ 2, p. 142) the path of the flux returning outside the solenoid extends indefinitely in all directions and the term magnetic circuit has no precise meaning.

In practice, the whole or as much as possible of the desired path is made of ferromagnetic materials, so that most of the flux set up lies within it; but since the circuit is usually neither homogeneous nor uniform in section, and since the magnetizing winding is concentrated on only a portion of it, the magnetic flux cannot be wholly confined, and a part of it, known as *leakage flux*, follows other paths in the surrounding air.

Similar conditions would obtain in an electric circuit if it is imagined that all the conductors are uninsulated and immersed in a poor conductor such as a dilute acid. Indeed, if the disposition of the electric circuit is made similar to that of the magnetic circuit considered, the distribution of the leakage currents is similar to that of the leakage flux.

Because of this, the simple Ohm's law relationship, although it is true for every magnetic circuit, can seldom be applied directly since, whenever magnetic leakage occurs, the precise calculation of the effective reluctance and the total flux is very difficult.

Returning to the electric circuit,

$$\text{Resistance} = \text{resistivity} \times \frac{\text{length}}{\text{cross-sectional area}}$$

and, replacing the resistivity (ρ) by the reciprocal property of conductivity (γ) where $\rho = \dfrac{1}{\gamma}$,

$$\text{Resistance} = \frac{1}{\text{conductivity}} \times \frac{\text{length}}{\text{area}}$$

so that

$$\text{Current} = \frac{\text{electromotive force} \times \text{conductivity} \times \text{area}}{\text{length}}$$

from which, dividing by the area,

$$\text{Current density} = \text{E.M.F. per unit length} \times \text{conductivity}.$$

In a similar way, in the magnetic circuit,

$$\text{Reluctance} = \text{reluctivity} \times \frac{\text{length}}{\text{cross-sectional area}}$$

$$= \frac{1}{\text{permeability}} \times \frac{\text{length}}{\text{area}}$$

where permeability $= \dfrac{1}{\text{reluctivity}}$ and is a property of the material analogous to conductivity.

Hence,

$$\text{Flux} = \frac{\text{magnetomotive force} \times \text{permeability} \times \text{area}}{\text{length}}$$

from which, dividing by the area,

$$\text{Flux density} = \text{M.M.F. per unit length} \times \text{permeability}.$$

Now the magnetomotive force per unit length is called the **magnetizing force** (H), so that

The M.K.S. unit of magnetizing force is the ampere-turn per metre.

Hence,

$$\underset{(webers\ per\ metre^2)}{\text{Flux density}} = \text{permeability} \times \underset{(ampere\text{-}turns\ per\ metre)}{\text{magnetizing force}}$$

i.e.

$$B = \mu H \quad \ldots \ldots \ldots \quad (7)$$

The permeability of materials is expressed in terms of the permeability of free space (μ_0) (see § 13) in the form

$$\mu = \mu_r \mu_0$$

where μ_r is termed the *relative permeability,* and is the ratio of the flux density set up in the material to that set up in free space by the same value of magnetizing force, e.g. if the flux density (B) set up in a certain sample of iron is 500 times that set up in free space by the same value of magnetizing force (H),

$$\mu_r = 500.$$

The permeability of air and most other non-magnetic materials is so nearly that of free space that for most purposes it can be taken as being the same, i.e. for these materials $\mu_r = 1$ and $\mu = \mu_0$.

The properties of magnetic materials are considered in Chapter XI. It is sufficient for the present to say that the value of μ_r (which may be as high as 3000 in soft iron under certain conditions) depends not only on the material but also on the existing flux density, and that in general it decreases as the flux density increases, i.e. as the iron approaches saturation.

12. Magnetizing Force due to a Long Straight Conductor

Experiment shows that the lines of magnetic flux surrounding a long straight conductor, far removed from other conductors, are circles concentric with the conductor (see fig. 16), and considerations of symmetry show that the magnetizing force must have the same value at every point on one of these circles. The magnetomotive force due to a current I flowing in the one turn made up of this conductor and the necessary return conductor (however far removed) is I ampere-turns. Hence at a distance r from the centre (see fig. 16) the length of magnetic path is $2\pi r$ metres and the magnetizing force is

$$H = \frac{F}{l} = \frac{I}{2\pi r} \text{ ampere-turns per metre} \quad . \quad (8)$$

and the flux density due to this magnetizing force is, in free space (or air),

$$B = \mu_0 H = \mu_0 \frac{I}{2\pi r} \text{ webers per square metre.} \quad (9)$$

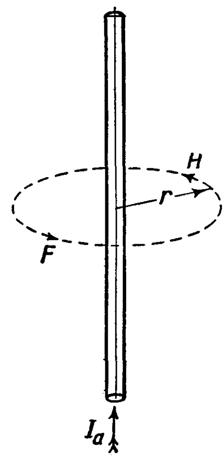


Fig. 16.—Field strength in neighbourhood of long straight conductor

13. Force between Parallel Conductors—Permeability of Free Space

In general, mechanical forces exist between two conductors, each of which is carrying a current. In most cases the forces are small, but where the conductors are close together and the currents are large, they can attain high values, which may cause damage unless due allowance has been made in the design and arrangement of the apparatus.

The determination of the magnitude and direction of the forces between parallel conductors is simply a case of determining the force acting on a current-carrying conductor (either one of the two conductors) lying in a magnetic field (produced by the other conductor).

It follows, from the reasoning of § 9 that,

Repulsive forces exist between conductors carrying currents in *opposite* directions.

Attractive forces exist between conductors carrying currents in the *same* direction.

Hence, in any coil carrying a current there is,

(*a*) an attractive force between turns causing a tendency for the coil to shorten axially,

(*b*) a repulsive force between diametrically opposite elements of each turn, which is equivalent to a radial bursting force.

Under normal conditions these forces are small but they may reach destructive values under fault conditions (see p. 158).

Consider two long parallel conductors A and B with their centres d metres apart, and each carrying a current I amperes (fig. 17).

The magnetizing force set up by A at the point B is (from equation 8)

$$H = \frac{I}{2\pi d} \text{ ampere-turns per metre}$$

which produces a flux density

$$B = \mu_0 H = \mu_0 \frac{I}{2\pi d} \text{ webers per square metre}$$

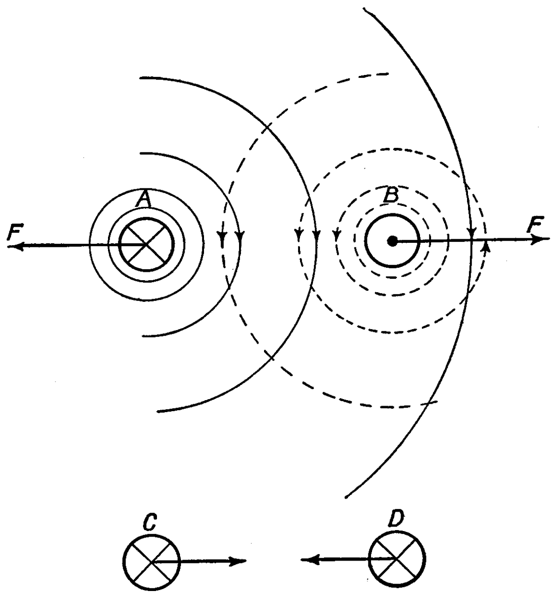


Fig. 17.—Force between conductors

where μ_0 is the permeability of free space (or air very nearly).

Hence the force acting on conductor B (either of attraction or repulsion depending on the relative directions of the currents) is (from equation 4)

$$F = BlI = \mu_0 \frac{I}{2\pi d} lI = \frac{\mu_0 l I^2}{2\pi d} \text{ newtons,}$$

and, if the conductors each carry unit current (1 ampere) and are unit distance (1 metre) apart, the force experienced by unit length (1 metre) is

$$F = \frac{\mu_0}{2\pi} \text{ newtons.}$$

But, by definition (p. 60), when unit current flows in two conductors unit distance apart, the force experienced per unit length is 2×10^{-7} newtons.

Hence $$\frac{\mu_0}{2\pi} = 2 \times 10^{-7}$$

or $$\mu_0 = 4\pi \times 10^{-7}.$$

The unit in which permeability should be expressed has been a matter of considerable controversy.

The somewhat analogous property—conductivity—can be expressed in *mhos per metre* where the mho is the unit of conductance; but the unit of permeability cannot be expressed in this way because there is no name for the unit of permeance.

Since, however, $\mu = B/H$ it is possible to express the unit of permeability as the (*weber per square metre*) *per* (*ampere-turn per metre*). This although self-explanatory is very cumbersome, but can be shortened by suitable substitutions into *henry per metre*, which is the unit most frequently employed. This again involves certain assumptions and is not entirely satisfactory. In any case it conveys little meaning to the student, and at this stage it is probably best simply to say that the unit of permeability is $10^7/4\pi$ times the permeability of free space, i.e. that $\mu_0 = 4\pi \times 10^{-7}$ M.K.S. units.

It has also been suggested that since free space—a vacuum—contains nothing magnetizable, a better term for μ_0 is *magnetic space constant*. This term is used by some writers, but here the term *permeability of free space* will be retained.

It will be noticed that the force set up between the conductors is proportional to (current)^2. The inherent rigidity of machine windings is well able to withstand the forces set up by currents of normal values: but if a fault occurs, the current may reach momentarily 10 to 20 times the normal value and produce 100 to 400 times the normal forces. Hence windings liable to large fault currents must be very strongly braced.

14. Ampère's Law

Ampère's experiments showed that the magnetic effect at any point P (fig. 18) due to a current in an element of conductor varies as the current, the length of the element, and the sine of the angle between the direction of the current and the line joining the element to this point, and inversely as (the distance of the element from the point)^2

i.e. $$\delta H \propto \frac{I \, . \, \delta l}{x^2} \sin \theta.$$

Applying this to the case of a straight conductor (fig. 19),

$$\delta H \propto \frac{I \,.\, \delta l}{x^2} \sin \theta = K \frac{I \,\delta l}{x^2} \cos \alpha$$

Fig. 19*c* shows part of fig. 19*a* in greater detail. The element δl subtends at P a small angle $\delta\alpha$, so that the chord ab and the arc are very nearly equal, and in the limit,

$$ab = x \,.\, d\alpha$$

and $dl = \dfrac{ab}{\cos \alpha} = \dfrac{x \,.\, d\alpha}{\cos \alpha}$

Hence

$$dH = K \frac{I}{x^2} \frac{x \,.\, d\alpha}{\cos \alpha} \cos \alpha = K \frac{I}{x} d\alpha$$

$$= K \frac{I}{r} \cos \alpha \, d\alpha.$$

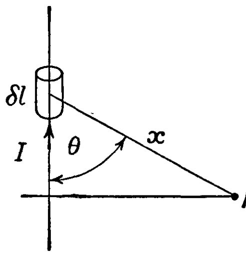


Fig. 18.—Magnetic field due to current in long coil

For an element immediately opposite P, angle $\alpha = 0$, and if the conductor is assumed to extend indefinitely in both directions, the

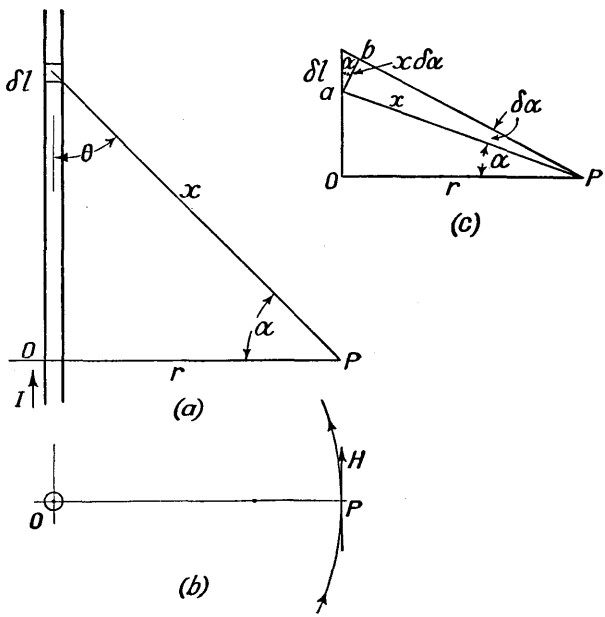


Fig. 19.—Field strength at a point near long straight conductor

limiting value of α is $\pi/2$. Hence, since corresponding current elements in the conductor on both sides of O produce equal effects, the resultant magnetizing force at P is

$$H = \frac{2K}{r} I \int_0^{\pi/2} \cos\alpha\, d\alpha = 2K \frac{I}{r}\Big[\sin\alpha\Big]_0^{\pi/2}$$

$$= K\frac{2I}{r} \text{ ampere-turns per metre.}$$

But it has been shown in the previous section that

$$H = \frac{I}{2\pi r} \text{ ampere-turns per metre.}$$

Hence $$K\frac{2I}{r} = \frac{I}{2\pi r} \quad \text{or} \quad K = \frac{1}{4\pi}$$

so that in the M.K.S. system, Ampère's law becomes

$$dH = \frac{1}{4\pi}\cdot\frac{I\,dl}{x^2}\sin\theta \text{ ampere-turns per metre.} \qquad . \quad . \quad . \quad (10)$$

15. Magnetizing Force at Centre of Circular Loop or Short Coil

When the conductor takes the form of a circular loop (fig. 20) all the elements are equidistant from the centre and perpendicular to the radius, so that they all produce equal magnetic effects at the centre. Hence since $x = r$, and $\sin\alpha = 1$ for every element,

$$H = \frac{1}{4\pi}\cdot\frac{I}{r^2}\int dl = \frac{1}{4\pi}\cdot\frac{I}{r^2}2\pi r = \frac{I}{2r} \text{ ampere-turns per metre.}$$

If the loop is replaced by a coil of N turns, the axial length of which is so short that it can be assumed that

$$OB = OA = r \text{ and angle } BOX = AOX = 90° \quad \text{(fig. 21),}$$

then each turn will produce an equal effect at the centre and

$$H = \frac{NI}{2r} \text{ ampere-turns per metre.} \qquad . \quad . \quad . \quad (11)$$

One of the earliest instruments used for the accurate measurement of current was the *tangent galvanometer.*

In its simplest form it consists of a circular loop or short coil at the centre of which is pivoted a short compass needle (fig. 22). If the instrument is placed so that with no current flowing the plane of the coil coincides with the axis of the compass needle (i.e. with the coil in the magnetic meridian), the field H_C at the centre of the coil is along the axis and therefore perpendicular to the plane of the coil and to the earth's horizontal field H_E. Further, if the needle is short compared with the radius of the coil, it may be assumed that the field H_C has a uniform value (equal to its value at the centre) throughout the range of movement of the needle.

Hence the needle sets itself along the resultant field, and

$$\tan \theta = \frac{H_C}{H_E}.$$

Since H_C is a function of the current,

$$I = kH_E \tan \theta$$

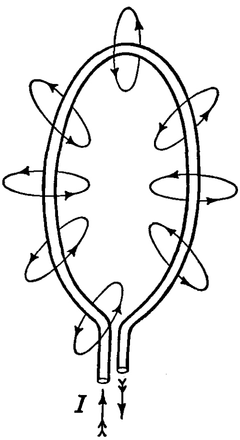


Fig. 20.—Field due to circular loop

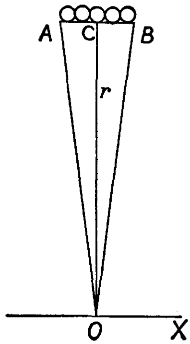


Fig. 21

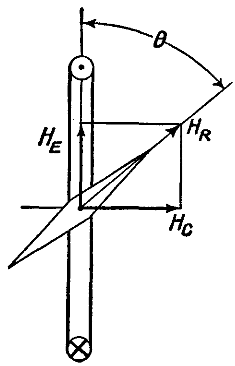


Fig. 22.—Principle of tangent galvanometer

For general purposes the tangent galvanometer has now been entirely superseded by various types of precision *indicating* instruments. It is interesting, however, as an example of an *absolute* instrument, since the constant k involves only the number of turns and the diameter of the coil, and, provided the strength of the earth's field is known, the instrument needs no calibration.

16. Magnetizing Force at any other Point on the Axis of Short Coil

An element of the loop at A (fig. 23), perpendicular to the paper and of length dl, produces at P a magnetizing force

$$dH_A = \frac{1}{4\pi} I \frac{dl}{x^2}$$

in the direction PC.

A diametrically opposite element at B produces at P an equal effect

$$dH_B = \frac{1}{4\pi} I \frac{dl}{x^2}$$

in the direction PD.

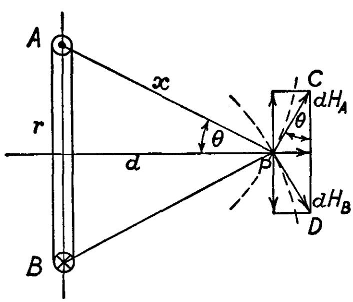


Fig. 23.—Relative directions of flux, current and force

The components of dH_A and dH_B perpendicular to the axis are equal and opposite and neutralize each other, and the axial components are

$$dH_A \sin\theta = dH_B \sin\theta.*$$

This is true for every pair of diametrically opposite elements, so that the direction of the resultant magnetizing force lies along the axis at all points and is given by

$$H_P = \int dH \sin\theta = \frac{1}{4\pi}\frac{I}{x^2}\int dl \sin\theta$$

$$= \frac{1}{4\pi} I \frac{2\pi r}{x^2} \sin\theta$$

$$= \tfrac{1}{2} I \frac{r}{x^2} \sin\theta \text{ ampere-turns per metre,}$$

and for a short coil of N turns,

$$H_P = \tfrac{1}{2} NI \frac{r}{x^2} \sin\theta \qquad (12)$$

$$= \tfrac{1}{2} NI \frac{r^2}{(r^2 + d^2)^{3/2}} \text{ ampere-turns per metre.}$$

17. Magnetizing Force on the Axis of a Long Solenoid

The section of a long solenoid, containing n turns per metre, is shown in fig. 24.

The magnetizing force at any point P on the axis is the resultant of the magnetic effects of all the elementary sections, each containing $n \,.\, dl$ turns, of which the solenoid can be considered to consist.

The magnetizing force at P due to the section AB is (from equation 12)

$$dH = \tfrac{1}{2} n \, dlI \frac{r}{x^2} \sin\theta$$

$$= \tfrac{1}{2} nI \frac{r}{x^2} x \, d\theta$$

$$= \tfrac{1}{2} nI \sin\theta \, d\theta$$

since $dl \sin\theta = x \, d\theta$ (fig. 24b)

If the solenoid is assumed to be very long compared with its diameter and P_1 is considered to be the mid-point, the limiting values of θ

* It should be noticed that this angle θ is not the angle θ of Section 14 and fig. 18. In this case the latter angle has a value of $\pi/2$ for every element. Hence equation (10) becomes

$$dH = \frac{1}{4\pi} I \frac{dl}{x^2}$$

as stated above.

will be $\pi/2$ for the elementary section at the mid-point and a very small value, approaching zero, for the end sections.

Hence, at the mid-point of an infinitely long solenoid, the resultant magnetizing force is

$$H_1 = 2 \cdot \tfrac{1}{2} nI \int_{\theta=0}^{\theta=\pi/2} \sin\theta\, d\theta$$

$$= nI\Big[-\cos\theta\Big]_0^{\pi/2} = nI[-0+1]$$

$$= nI \text{ ampere-turns per metre,} \quad . \quad . \quad . \quad . \quad (12a)$$

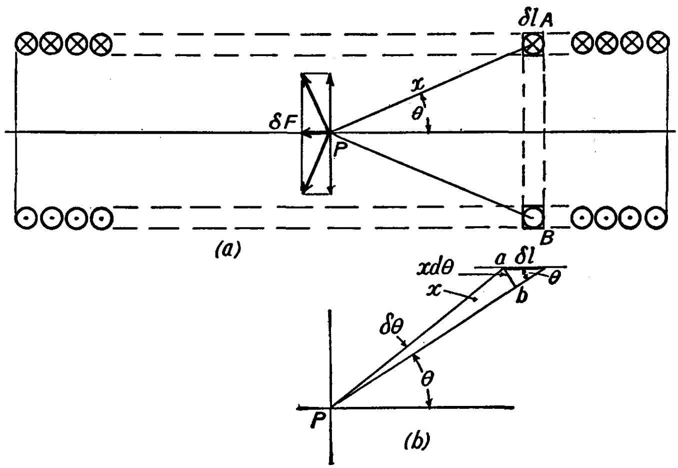


Fig. 24.—Field strength on axis of solenoid

and this is almost true for any solenoid of which the length is great compared with its diameter. For instance, if $l = 10d$ the value of θ for the end sections is 5·7°, so that

$$H_1 = 2 \cdot \tfrac{1}{2} nI \int_{\theta=5\cdot7^\circ}^{\theta=90^\circ} \sin\theta\, d\theta = nI[-0+0\cdot955]$$

$$= 0\cdot995 nI \text{ ampere-turns per metre,}$$

i.e. 99·5 per cent of that for an infinitely long solenoid. Hence there is little error in assuming that at the midpoint of a solenoid of length l and continuing N turns,

$$H = nI = \frac{NI}{l} \text{ ampere-turns per metre.} \quad . \quad . \quad (13)$$

At any point other than the midpoint, e.g. at P_2 (fig. 25a),

$$H = \tfrac{1}{2}nI\left[\int_{\theta_2}^{\pi/2} \sin\theta\, d\theta + \int_{\theta_2'}^{\pi/2} \sin\theta\, d\theta\right] = \tfrac{1}{2}nI\,[\cos\theta_2 + \cos\theta_2'].$$

Again considering a coil in which $l = 10d$, at a distance from the midpoint equal to the radius,

$$\cos\theta_2 = \cos\tan^{-1}\tfrac{1}{9} = 0{\cdot}994 \quad \text{and} \quad \cos\theta_2' = \cos\tan^{-1}\tfrac{1}{11} = 0{\cdot}996.$$

Hence $$H_2 = \tfrac{1}{2}nI\,[0{\cdot}994 + 0{\cdot}996] = 0{\cdot}995nI.$$

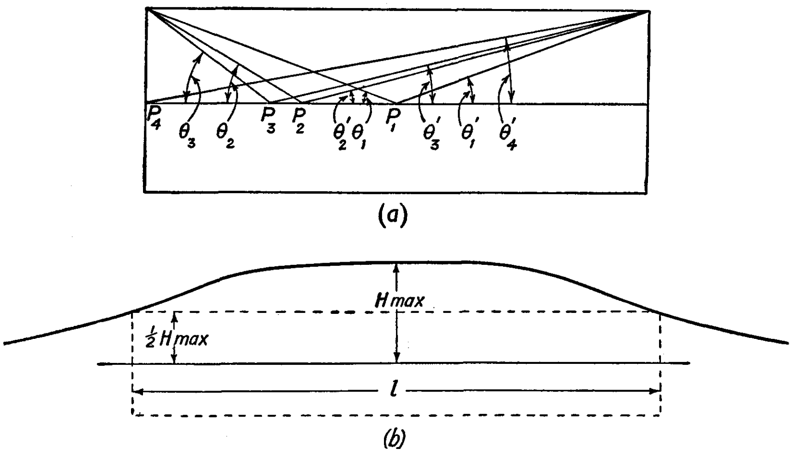


Fig. 25.—Variation of field strength along axis of solenoid

Halfway between the midpoint and the ends,

$$\cos\theta_3 = \cos\tan^{-1}\tfrac{1}{5} = 0{\cdot}981 \quad \text{and} \quad \cos\theta_3' = \cos\tan^{-1}\tfrac{1}{15} = 0{\cdot}998.$$

Hence $$H_3 = \tfrac{1}{2}nI\,[0{\cdot}981 + 0{\cdot}998] = 0{\cdot}989nI$$

while at the ends

$$\cos\theta_4 = 0, \quad \cos\theta_4' = \cos\tan^{-1}\tfrac{1}{20} = 0{\cdot}999$$

$$H_4 = \tfrac{1}{2}nI\,[0 + 0{\cdot}999] = 0{\cdot}499nI.$$

If the value of H is obtained in this way for a number of points along the axis, it will be found that for some distance on either side of the mid-point, the value is almost constant, but diminishes, at a gradually increasing rate, to about half the maximum value at the ends, as shown in fig. 25b.

18. Magnetizing Force due to a Ring or Toroidal Coil

A uniformly wound coil in the form of a ring is shown in fig. 26; it can be assumed to have been formed by bending a long solenoid into a circle so that the two ends meet.

The field distribution due to a straight solenoid was considered on p. 142 from which it was seen that at any point on the surface of the solenoid, with the exception of the mid-point, the M.M.F.s due to the portions of the coil on either side of the point considered are unequal, so that there is a resultant M.M.F. which causes flux to enter or leave the solenoid, between the turns, at a rate which increases as the ends are approached. In a ring-shaped coil, however, whatever point such as A (fig. 26) is considered, equal numbers of turns are distributed symmetrically on both sides of it, so that conditions at every point are similar to those at the mid-point of a straight solenoid, and no flux either enters or emerges. Hence the flux lies wholly inside the coil which produces no poles or external magnetic effects.

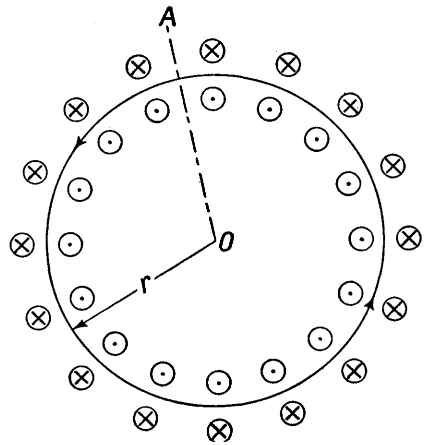


Fig. 26—Field inside a ring solenoid

It also follows, from considerations of symmetry, that at all points on the axis of the coil (or on any circle concentric with it), the magnetizing force has a constant value

$$H = \frac{\text{M.M.F.}}{\text{length of path}} = \frac{NI}{l} = \frac{NI}{2\pi r} \text{ ampere-turns per metre.}$$

The value of H naturally decreases from the inner radius to the outer radius, and has a mean value

$$H = \frac{NI}{2\pi R} \text{ ampere-turns per metre,} \quad . \quad . \quad . \quad (14)$$

where R is the mean radius.

If the coil is wound on a core of non-magnetic material, the mean flux density is

$$B = \mu_0 H = \mu_0 \frac{NI}{2\pi R} \text{ webers per square metre,} \quad . \quad (15)$$

and if the cross-section of the core is a the total flux set up is

$$\Phi = Ba = \mu_0 \frac{NI}{2\pi R} a \text{ webers.} \quad . \quad . \quad . \quad . \quad (16)$$

If a ring of magnetic material is substituted, the flux will be greatly increased, but μ $(=\mu_r\mu_0)$ is no longer constant over the cross-section, since it decreases as H increases, from the outer to the inner radius. However, if the radial thickness of the ring is small compared with its mean diameter, so that there is little difference between the maximum and minimum values of H, there is very little error in assuming that

$$\Phi = \mu_r\mu_0 \frac{NI}{2\pi R} a \text{ webers,} \quad . \quad . \quad . \quad . \quad (17)$$

where μ_r is the relative permeability at the mean diameter.

The absence of poles and the ease with which the magnetizing force can be calculated makes the ring-shaped iron specimen particularly suitable for use in certain magnetic measurements (see p. 226).

19. E.M.F. of Self Inductance

It has been seen that an E.M.F. may be induced in a circuit by a change in the flux linked with the circuit, the change being produced either by a change in the current flowing in a neighbouring circuit or by relative motion between the circuit and a magnetic field. There is yet another way in which an E.M.F. may be induced—by a change in the flux set up *by the current in the circuit itself.* Such an E.M.F. is called an *E.M.F. of self inductance.*

Consider a coil about to be connected to some source of supply. Before connection there is no current in the coil and no flux associated with it. Immediately connection is completed, a current starts to flow and the magnetic field begins to grow. The flux linked with the coil increases and an E.M.F. is induced which opposes the current (§ 4, p. 145) and delays its growth, so that it takes an appreciable time to reach its steady value. As the current rises, this E.M.F. gradually decreases and finally disappears when the steady value is attained.

During the period of growth, while the current is flowing *against* the E.M.F. of self inductance, *energy is absorbed and stored in the magnetic field.* When the circuit is broken the field collapses and the decreasing flux induces an E.M.F. in the opposite direction which tends to maintain the current. In this way *the stored energy is given back* and dissipated in the arc which appears as the switch contacts separate.

The property of an electric circuit in virtue of which any change in the flux linked with it induces in it an E.M.F. is called *inductance.*

When the flux is set up by the current in the circuit itself, the latter is said to possess *self inductance* (L); when the flux is due to the current in a neighbouring circuit, the two circuits are said to possess *mutual inductance* (M). The term *inductance* without further qualification is commonly used to express the property of *self inductance.*

Self inductance can be looked upon as similar, in many ways, to the mechanical property of *mass* or *inertia.* Inertia is that property of a body which *opposes* change of motion and by which energy is stored up while the body is in motion. It is a matter of common knowledge that a heavy object, such as a garden roller, can be put into motion or accelerated only by the application of a considerable force. During acceleration this force is greater than that required to overcome frictional resistance, and the excess energy is stored kinetically and remains so stored until retardation takes place. In order to retard and bring the body to rest, an opposing force must be applied in overcoming which the stored energy is dissipated. This process may take place gradually, as when the roller is allowed to run freely and the energy is given up slowly in overcoming frictional forces; or it may take place rapidly, as when the roller is allowed to run into a wooden fence, in which case either the whole or a part of the energy is given up rapidly and the forces and destructive effects are correspondingly large.

In the case of an inductive circuit, while the flux is increasing (corresponding with the acceleration period in the above illustration) energy is *stored up magnetically*, and remains so stored as long as the flux is constant; but when the flux decreases (corresponding with the retardation period) this *energy is given back.* Further, if the decrease is rapid, the energy may be released sufficiently rapidly to produce destructive effects.

The self inductance of a circuit is that property whereby any change in the current flowing induces an E.M.F. opposing the change, and in virtue of which energy is stored in the magnetic field established by the current.

20. Unit of Inductance—the Henry

The value of an induced E.M.F. depends not only upon the rate of change of the flux but also upon the number of turns with which it is linked. Hence the inductance of a circuit, the property to which the E.M.F. of self inductance is attributed, depends upon the flux linkage ($N\Phi$) of the circuit.

Inductance may be measured in terms of:

(1) The flux linkage produced when unit current flows in the circuit;

(2) the E.M.F. induced when the current is changing at unit rate;

(3) the energy stored when unit current flows in the circuit.

For the present the first two only will be considered.

The unit of inductance is that of a circuit in which unit current produces unit flux linkage.

Hence if a current I in a coil of N turns sets up a flux Φ, all of which is linked with all the turns, the self inductance is

$$L = \frac{N\Phi}{I}.$$

But in a circuit possessing unit inductance, a current changing at unit rate will cause the flux linkage to change at unit rate, and therefore an E.M.F. of unit value will be induced. Hence, alternatively,

A circuit possesses unit inductance when unit E.M.F. is induced by a current changing at unit rate.

If a current changing at a rate di/dt causes an E.M.F. e to be induced, the inductance of the circuit is

$$L = \frac{e}{di/dt}$$

and therefore

$$e = L\frac{di}{dt}.$$

Strictly $e = -L\,di/dt$. The negative sign indicates that the induced E.M.F. is in such a direction as to oppose the change, e.g. if the current is increasing (di/dt positive), the E.M.F. is in the opposite direction; if decreasing (di/dt negative), the E.M.F. is in the same direction as the current.

The unit of inductance is that of a circuit in which a current of 1 ampere produces a flux-linkage of 1 weber-turn; or in which a current changing at the rate of 1 ampere per second induces an E.M.F. of 1 volt.

This unit is called the henry.

$$L = \frac{N\Phi}{I} \text{ henrys, } \qquad (18)$$

$$e = -L\frac{di}{dt} \text{ volts. } \qquad (19)$$

It is often convenient to use submultiples of the henry.

One millihenry $= 10^{-3}$ henrys.

One microhenry $= 10^{-6}$ henrys.

Example 1.—Determine the inductance of a coil of 500 turns in which a current of 5 amperes produces a flux of 5 micro-webers.

$$L = \frac{N\Phi}{I} = \frac{500 \times 5 \times 10^{-6}}{5}$$
$$= 0{\cdot}0005 \text{ henry.}$$

Example 2.—A coil has an inductance of 0·01 henry. Determine the E.M.F. of self inductance when the current is changing at the rate of 500 amperes per second.

Since the inductance is 0·01 henry, a current of 500 amperes produces a total flux linkage of

$$N\Phi = 500 \times 0{\cdot}01 = 5 \text{ weber-turns.}$$

Hence a current changing at the rate of 500 amperes per sec. causes the flux linkage to change at the rate of 5 weber-turns per second.

$$\therefore\ e = 5 \text{ volts.}$$

Alternatively:

$$e = L\frac{di}{dt} = 0{\cdot}01 \times 500 = 5 \text{ volts.}$$

21. Non-inductive Coils

All circuits possess some inductance, but by suitable arrangement this may be made very small, and the circuit is then termed *non-inductive.*

Resistance coils as used in the post-office box (§ 3, p. 92) were described as being wound non-inductively. In such cases the ends of the necessary length of wire are first of all brought together, and the coil is then wound with the double strand (fig. 27), so that each turn is

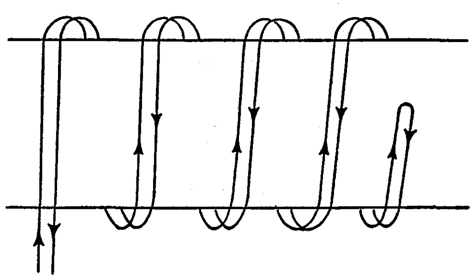


Fig. 27.—Principle of non-inductive winding

in close contact with a similar turn carrying a current in the opposite direction. The magnetic effect of one turn then almost entirely neutralizes that of the other, and the resultant flux and therefore the inductance is very small. Actually to eliminate inductance completely the outward and return conductors would have to be coincident.

22. Calculation of Self Inductance of a Coil

The exact calculation of the self inductance of a coil is not easy except in special cases.

There is usually some leakage (§ 11, p. 154), particularly when the magnetic circuit lies in air or some other non-magnetic material, so that all the flux is not linked with all the turns, and allowance must be made for this by means of correction factors. It is clear, however, that if without changing the dimensions of the coil the number of turns is doubled, then for the same current the magnetizing force is doubled and therefore the flux is doubled. At the same time, each line of flux is linked with twice as many turns, so that $N\Phi$ (and hence the inductance of the coil) becomes *four* times as great. Hence:

The inductance of a coil of given dimensions is proportional to (number of turns)^2, assuming that the permeability of the magnetic circuit remains constant.

When the magnetic circuit lies wholly or largely in iron, the flux produced by a given current, and therefore the inductance, is greatly increased, while the effect of leakage is of less importance. On the other hand, since the permeability of the iron decreases as the flux density increases, the flux set up is not proportional to the current, and the inductance decreases as the iron becomes saturated.

One of the simplest cases is that of a ring solenoid as shown in fig. 26 (p. 165):

Let

R = mean radius of ring (m.),
a = cross-section of core (sq. m.),
N = number of turns,
μ = permeability of core ($=\mu_r\mu_0$)
l = length of magnetic path ($= 2\pi R$ for a ring) (m.).

When a current I flows in the coil, the magnetizing force is

$$H = \frac{NI}{l} \text{ ampere-turns per metre (eqn. 14, p. 165)}$$

and the flux density produced

$$B = \mu H \text{ webers per sq. m.}$$

(which can be assumed constant over the cross-section if the radius of the turn is small compared with R).

Therefore the total flux set up is

$$\Phi = Ba = \frac{NI}{l}\mu a \text{ webers.}$$

In this case all the flux is linked with all the turns, hence

$$L = \frac{N\Phi}{I} = \frac{N}{I}\frac{NI}{l}\mu a$$

$$= N^2 \frac{\mu a}{l} \text{ henrys.} \quad \ldots \ldots \quad (20)$$

This expression is approximately true for any coil in which the magnetic circuit lies wholly in the same material, and in which the winding is fairly uniformly distributed over the whole circuit. It is also true for a solenoid, if corrected for leakage effects by multiplying by a factor which depends upon the ratio diameter/length.

When the magnetic circuit is of iron, the value of μ is that corresponding with the actual value of the current at which the inductance is to be calculated.

Example 1.—Calculate the inductance of a ring-shaped coil having a mean diameter of 20 cm., wound on a wooden core 2 cm. in diameter. The winding is evenly distributed and contains 500 turns.

$\mu = \mu_0 = 4\pi \times 10^{-7}, l = \pi \times 0{\cdot}2 = 0{\cdot}628\text{m}, a = \frac{\pi}{4} \times 2^2 \times 10^{-4} = 3{\cdot}14 \times 10^{-4}\text{m}^2.$

$$H = \frac{500I}{0{\cdot}628} = 797I \text{ AT/m.}$$

$$B = \mu_0 H = 4\pi \times 10^{-7} \times 797I = 1000I \times 10^{-6} \text{ Wb/m}^2.$$

$$\Phi = Ba = 1000I \times 10^{-6} \times 3{\cdot}14 \times 10^{-4} = 0{\cdot}314I \times 10^{-6} \text{ Wb.}$$

$$L = \frac{N\Phi}{I} = \frac{500 \times 0{\cdot}314I \times 10^{-6}}{I} = 0{\cdot}000157 \text{ henry}$$

(0·157 millihenry).

Example 2.—If the wooden core is replaced by an iron core which has a permeability $\mu_r = 600$ when the current is 5 amperes, find L.

$$H = \frac{500 \times 5}{0{\cdot}628} = 3980 \text{ AT/m.}$$

$$B = \mu_r \mu_0 H = 600 \times 4\pi \times 10^{-7} \times 3980 = 3 \text{ Wb/m}^2.$$

$$\Phi = Ba = 3 \times 3{\cdot}14 \times 10^{-4} = 0{\cdot}942 \times 10^{-3} \text{ Wb.}$$

$$L = \frac{N\Phi}{I} = \frac{500 \times 0{\cdot}942 \times 10^{-3}}{5} = 0{\cdot}0942 \text{ henry}$$

(when $I = 5$ amperes).

23. Mutual Inductance (M)

When two circuits are so situated that some of the flux due to one is linked with the other, and a change of the current in the one induces an E.M.F. in the other (§ 3, p. 143), they are said to possess *mutual inductance.*

Two circuits possess a mutual inductance of 1 henry when a current of 1 ampere in one circuit produces a flux linkage of 1 weber-turn in the other; or when a current changing at the rate of 1 ampere per second in the one induces an E.M.F. of 1 volt in the other.

Assuming that a fraction k of the flux due to the first circuit is linked with the second, and that a current I_1 in the first produces a flux Φ_1, then the flux linked with the second circuit (of N_2 turns) is $k\Phi_1$, and

$$M = \frac{kN_2\Phi_1}{I_1} \text{ henrys,} \quad . \quad . \quad . \quad . \quad . \quad (21)$$

and the E.M.F. induced in the second by a varying current in the first is

$$e_2 = -M \frac{di_1}{dt} \text{ volts, } \quad \ldots\ldots \quad (22)$$

the negative sign having the same significance as in equation (19).

Further, since the inductance is *mutual*, the E.M.F. induced in the first circuit by a varying current in the second circuit is

$$e_1 = -M \frac{di_2}{dt} \text{ volts.}$$

Example. The self inductance of a coil of 500 turns is 0·2 henry. If 60 per cent of the flux is linked with a second coil of 10,000 turns, calculate (*a*) the mutual inductance, (*b*) the E.M.F. induced in the second coil by a current in the first coil changing at the rate of 1000 amperes per sec.

$$L_1 = \frac{N_1\Phi_1}{I_1}. \quad \therefore \frac{\Phi_1}{I_1} = \frac{L_1}{N_1} = \frac{0{\cdot}2}{500}.$$

(*a*) Flux linked with second coil $= 0{\cdot}6\Phi_1$, i.e. $k = 0{\cdot}6$.

$$M = \frac{kN_2\Phi_1}{I_1} = \frac{0{\cdot}6 \times 0{\cdot}2 \times 10{,}000}{500}$$
$$= 2{\cdot}4 \text{ henrys.}$$

(*b*)
$$e_2 = M\frac{di_1}{dt}$$
$$= 2{\cdot}4 \times 1000$$
$$= 2400 \text{ volts.}$$

24. Relation between Self and Mutual Inductance

The self inductance of two coils A and B in which it is assumed that all the flux is linked with all the turns, can be expressed as

$$L_A = \frac{N_A\Phi_A}{I_A} \quad \text{and} \quad L_B = \frac{N_B\Phi_B}{I_B}$$

from which

$$\frac{\Phi_A}{I_A} = \frac{L_A}{N_A} \quad \text{and} \quad \frac{\Phi_B}{I_B} = \frac{L_B}{N_B}. \quad \ldots\ldots \quad (23)$$

The mutual inductance between them is

$$M = \frac{k_BN_A\Phi_B}{I_B} = \frac{k_AN_B\Phi_A}{I_A}$$
$$= \frac{k_BN_AL_B}{N_B} = \frac{k_AN_BL_A}{N_A} \text{ from (23),}$$

where k is that fraction of the flux set up by either coil which can be considered as wholly linked with the other. Therefore,

$$M^2 = k_Ak_BL_AL_B$$

or

$$M = \sqrt{k_Ak_BL_AL_B}. \quad \ldots\ldots \quad (24)$$

The maximum value of M, if all the flux set up by either coil was wholly linked with the other, i.e. when $k_A = k_B = 1$, would be

$$M_0 = \sqrt{(L_A L_B)}.$$

The ratio $\dfrac{M}{M_0} = \dfrac{M}{\sqrt{(L_A L_B)}}$ is called the *coefficient of coupling.*

The effective self-inductance of a circuit consisting of two portions depends not only on the self inductance of each portion but also on the mutual inductance between them.

Consider two coils A and B, connected in series so that the current flows round both in the same direction as indicated by the solid arrows (fig. 28a). Coil A produces a flux Φ_A, part of which is linked with B;

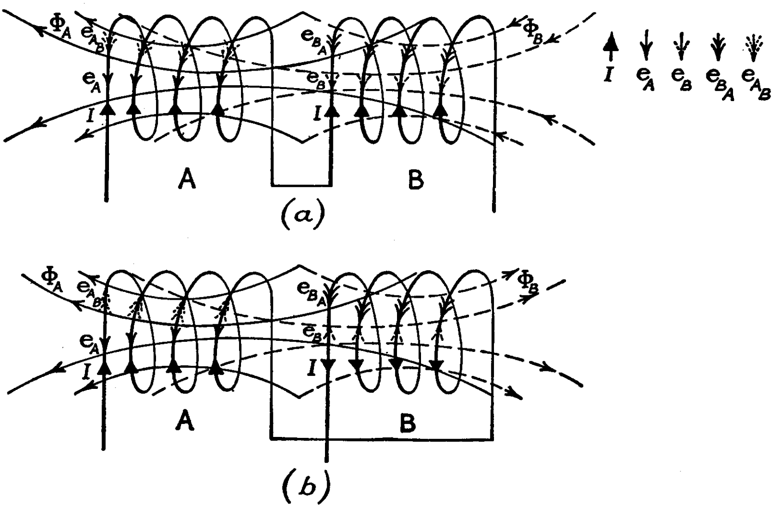


Fig. 28.—To illustrate the relation between self and mutual inductance

and similarly coil B produces a flux Φ_B part of which is linked with A. Assuming that at the instant considered the current and flux are increasing, the direction of the E.M.F.s of self inductance e_A and e_B in each coil is (in accordance with Lenz's law) such as to oppose the current. Further, since the direction of the flux set up is the same in both coils, the E.M.F.s of mutual inductance, e_{AB} induced by Φ_B in A and e_{BA} induced by Φ_A in B (indicated by the broken arrows), are in the same direction as the E.M.F.s of self inductance.

Hence the resultant E.M.F. is

$$e_r = e_A + e_B + e_{AB} + e_{BA}$$

or $$L_r \frac{di}{dt} = L_A \frac{di}{dt} + L_B \frac{di}{dt} + M \frac{di}{dt} + M \frac{di}{dt}$$

from which $$L_r = L_A + L_B + 2M.$$

Now suppose that the connections to coil B are reversed (or that the coil is turned through 180°). The directions of both current and flux are reversed (fig. 28*b*) and consequently the E.M.F. of self inductance e_B is reversed, but, since it still *opposes* the current, it acts, relative to the circuit as a whole, in the *same* direction as e_A. On the other hand, since Φ_B is reversed relative to Φ_A, the E.M.F.s of mutal inductance e_{AB} and e_{BA} are now in the opposite direction to the E.M.F.s of self inductance.

Hence the resultant E.M.F. is

$$e_r = e_A + e_B - e_{AB} - e_{BA}$$

or $$L_r \frac{di}{dt} = L_A \frac{di}{dt} + L_B \frac{di}{dt} - M \frac{di}{dt} - M \frac{di}{dt}$$

from which $$L_r = L_A + L_B - 2M.$$

Hence the general expression for the resultant inductance is

$$L_r = L_A + L_B \pm 2M. \quad . \quad . \quad . \quad . \quad . \quad (25)$$

It is thus possible to vary the effective inductance of the circuit by an amount $4M$, continuously and without change of resistance, by turning one coil relative to the other, through an angle of 180°. This is the principle of the *variometer*, used in radio circuits, which consists of two short concentric coils connected in series, the inner of which can be turned through 180°. If the self inductances of the coils are equal, and since the coefficient of coupling is almost unity,

$$M = \sqrt{(L \, . \, L)} = L$$

and the effective inductance can be varied between

$$L_{\text{min}} = L + L - 2M = 0,$$

and $$L_{\text{max}} = L + L + 2M = 4L.$$

A convenient method of measuring the mutual inductance between two coils consists in measuring the total inductance (L_1) when they are connected in series, and repeating the measurement (L_2) with the connections to one coil reversed.

Then $L_1 = L_A + L_B + 2M$ and $L_2 = L_A + L_B - 2M$,

from which $$M = \tfrac{1}{4}(L_1 - L_2).$$

25. The Induction Coil

Two of the most important examples of the application of mutual inductance are the *induction coil* and the *transformer*; the latter is described in Chapter XIII.

Before the growth of a.c. supply systems and the development of the high-voltage transformer, induction coils were extensively employed for the purpose of obtaining high voltages for experimental purposes; and they are used today in large numbers for producing the high-tension ignition spark in internal-combustion engines, and for medical purposes.

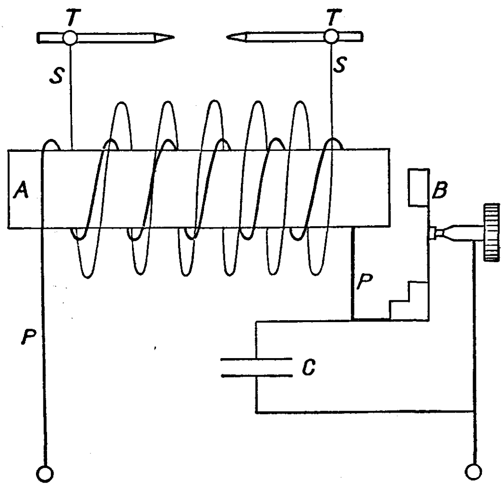


Fig. 29.—The induction coil

The induction coil consists essentially of two coils wound closely one on the top of the other, so that most of the flux produced by the inner coil is linked with the outer, i.e. so that their mutual inductance is very large. The inner coil *PP* (fig. 29), called the *primary* winding, consists of comparatively few turns wound on an iron core *A*, and is provided with an interrupter *B*, which in its simplest form is similar to that of an electric bell. The outer coil *SS*, called the *secondary* winding, consists of a very large number of turns of fine wire, and is connected to two terminals *TT* separated by an air gap of variable length (or in the case of an ignition coil, to the points of the sparking plug).

When the primary is connected to the supply, a large flux is set up in the iron core and induces an E.M.F. of mutual inductance in each turn of the secondary. Owing to the large number of turns in series, the resultant E.M.F. is very high and, if the air gap is not too long, causes a spark to pass through the air between the terminals. Meanwhile the interrupter B has been attracted by the magnetized core and the primary circuit is broken; the flux collapses and induces a secondary E.M.F. in the opposite direction. The hammer is then returned to its initial position by the spring, the primary circuit is again completed and the process repeated. The rate at which the flux collapses can be made much greater than that at which it grows, so that the E.M.F. at the "*break*" is much higher than that at the "*make*", and unless the spark gap is very short, a spark occurs only when the primary circuit is *broken.*

Although the secondary E.M.F. is one of mutual inductance, an E.M.F. of self inductance is induced in the primary winding when the circuit is broken, which causes a spark as the contacts of the interrupter separate. In order to reduce the destructive effect of the spark, a capacitor C (§ 23, p. 374) is connected across the contacts which absorbs the energy released as the magnetic field collapses. The addition of the capacitor also accelerates the collapse of the flux and thus increases the value of the secondary E.M.F. obtained.

Example.—The flux set up by a current of 5 amperes in the primary of an induction coil is 0·002 weber. Assuming that all the flux is linked with the secondary, which contains 20,000 turns, determine (*a*) the mutual inductance, (*b*) the average E.M.F. if the primary circuit is broken in $\frac{1}{500}$ sec.

(*a*)
$$M = \frac{N_2 \Phi_1}{I_1} = \frac{20{,}000 \times 0{\cdot}002}{5}$$
$$= 8 \text{ henrys.}$$

(*b*) Average rate of change of current $= \dfrac{5}{0{\cdot}002} = 2500$ amperes per sec.
$$e_2 = M\frac{di}{dt} = 8 \times 2500 = 20{,}000 \text{ volts.}$$

Alternatively:

Average rate of change of flux $= \dfrac{0{\cdot}002}{0{\cdot}002} = 1$ weber per sec.

Average E.M.F. per turn $= 1{\cdot}0$ volt.

Average secondary E.M.F. $= 1{\cdot}0 \times 20{,}000 = 20{,}000$ volts.

26. Growth of a Current in an Inductive Circuit—Helmholtz Equation

Consider the case of an inductive coil connected to supply terminals between which is maintained a constant P.D. In the circuits considered hitherto there have been no counter-E.M.F.s so that the whole of the P.D. has been used in overcoming the resistance,

$$V = IR.$$

This is true also in the case of the inductive circuit provided that the current has attained its steady value. But when the coil is first connected to the supply, the current must grow from zero to its steady value, and in so doing it sets up an increasing magnetic flux, which induces an E.M.F. of self inductance which opposes the current (§ 19, p. 166). During the growth period, therefore, there is, in addition to the resistance of the circuit, an opposing E.M.F. of self inductance proportional to the *rate of growth* and at any instant

$$V - L\frac{di}{dt} = iR.$$

This may be written

$$V = L\frac{di}{dt} + iR, \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

so that the supply P.D. (V) may be considered as made up of two portions, one of which is used in overcoming the resistance and maintaining the current at the value which it has attained, while the other, by overcoming the E.M.F. of self inductance, causes it to grow still further.

As the current increases the portion of the supply P.D. which overcomes the resistance (iR) must increase in order to maintain the increased current. But since the total value of the supply P.D. is constant, that portion overcoming the E.M.F. of self inductance (Ldi/dt) must decrease; and since the inductance (L) is constant,* the rate of growth (di/dt) must decrease. Hence as the current increases its *rate of increase* becomes less; and the E.M.F. of self inductance decreases continuously until it finally disappears when the current reaches its steady value, i.e.

$$L\frac{di}{dt} = 0. \quad \therefore \; V = IR.$$

A curve showing the growth of the current is given in fig. 30. Theoretically its rate of growth is always proportional to the amount by which it *falls short* of its steady value (i.e. the curve is exponential), which it reaches only after an infinite time. In practice, however, the departure from steady conditions is usually negligible after a few seconds even in highly inductive circuits.

Equation (26) may be integrated as follows. Dividing by R,

$$\frac{V}{R} = i + \frac{L}{R}\frac{di}{dt}.$$

But $\frac{V}{R} = I$, the steady value of the current.

$$\therefore I = i + \frac{L}{R}\frac{di}{dt}$$

* Actually in an iron-cored coil L decreases somewhat as the current increases.

or

$$I - i = \frac{L}{R}\frac{di}{dt},$$

which can be written

$$\frac{di}{I - i} = \frac{R}{L}dt.$$

On integrating both sides, [since $\int \frac{di}{I - i} = -\log_e(I - i)$]

$$-\log_e(I - i) = \frac{R}{L}t + K,$$

where K is the constant of integration.

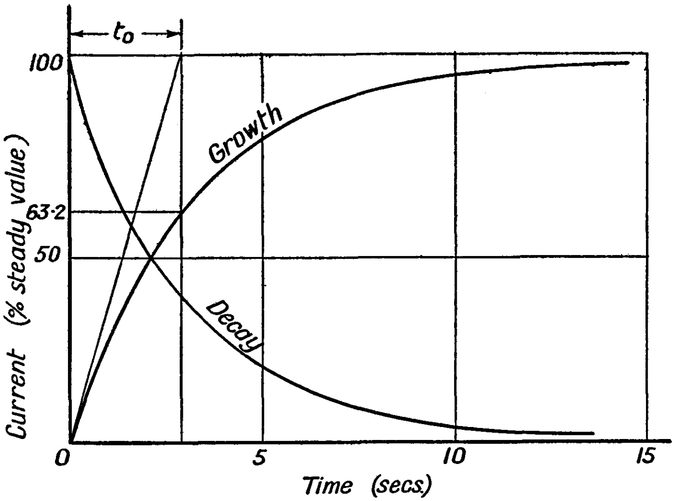


Fig. 30.—Growth and decay of current in inductive circuit

But when $t = 0$, $i = 0$. $\therefore K = -\log_e I$.

$$\therefore -\log_e(I - i) = \frac{R}{L}t - \log_e I,$$

$$\therefore \quad \log_e(I - i) - \log_e I = -\frac{R}{L}t,$$

$$\therefore \quad \log_e\left(\frac{I - i}{I}\right) = -\frac{R}{L}t,$$

$$\therefore \quad \frac{I - i}{I} = e^{-\frac{R}{L}t}$$

and finally,

$$i = I\left(1 - e^{-\frac{R}{L}t}\right), \qquad \qquad (27)$$

This is often referred to as the Helmholtz equation.

27. The Time Constant

At the instant when the coil is connected to the supply, $t = 0$ and $i = 0$. Hence $iR = 0$ and

$$V = L\frac{di}{dt},$$

so that the initial rate of growth is

$$\frac{di}{dt} = \frac{V}{L}.$$

If this rate of growth were maintained, the current would attain its steady value in a time t_0 where

$$t_0 = \frac{I}{di/dt} = \frac{V/R}{V/L} = \frac{L}{R}. \qquad \ldots\ldots \quad (28)$$

This quantity t_0 is called the *time constant* of the circuit.

Actually, since the rate of growth decreases, the current, instead of attaining its steady value in a time t_0, reaches a value obtained by substituting $t = t_0 = L/R$ in equation (27). This value is

$$\begin{aligned} i &= I(1 - e^{-\frac{RL}{LR}}) \\ &= I(1 - e^{-1}) \\ &= I\left(1 - \frac{1}{2{\cdot}718}\right) \\ &= 0{\cdot}632I. \end{aligned}$$

It also follows that since

$$I - i = \frac{L}{R}\frac{di}{dt}, \quad \frac{I - i}{\frac{di}{dt}} = \frac{L}{R} = t_0,$$

i.e. at *any point* on the growth curve, if the rate of growth at that point were maintained, the current would reach its steady value in a further time equal to the time constant.

Thus the time constant may be defined as:

(*a*) the time in which the current would attain its steady value if the initial rate of growth were maintained;

(*b*) the time in which the current actually attains 63·2 per cent of its steady value;

(*c*) the time, starting from any point on the growth curve, in which the current would attain its steady value, supposing that the rate of growth at the point considered were maintained.

It is clear that the time constant can be reduced by increasing the resistance of the circuit. In cases where it is imperative that the current in an inductive coil shall attain its steady value rapidly, a non-inductive resistance is sometimes placed in series with it; but this necessitates an increased P.D. to maintain the same current.

28. Decay of Current in an Inductive Circuit

If the circuit is now broken at the terminals, the E.M.F. of self inductance reverses and, in an endeavour to maintain the current, causes an arc between the separating contacts. As the separation of the contacts and the lengthening of the arc produce a rapid increase in the resistance of the circuit, the current diminishes and finally ceases when the arc breaks. But the rate at which the resistance increases is quite indefinite, so that in general the conditions under which a current decays are very complex.

The simpler conditions necessary for mathematical treatment can be obtained if it is imagined that the terminals are short-circuited and the source of supply disconnected, simultaneously, so that a closed circuit is left, having the same resistance and carrying a current which at this instant is still at its steady value. The current immediately starts to decrease, but the decreasing flux induces an E.M.F. of self inductance which tends to maintain the current and retards its decrease. Since $V = 0$, equation (26) becomes

$$-\frac{Ldi}{dt} = iR.^* \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

At any instant the rate of decrease will be such that the induced E.M.F. ($-Ldi/dt$) is just sufficient to maintain the current at the value to which it has decayed at that instant.

If the resistance is unaltered, the decay curve is a reflection of the growth curve shown in fig. 30. Theoretically the current never reaches its zero value, but in practice it becomes negligibly small in the course of some seconds.

From equation (29)

$$\frac{di}{i} = -\frac{R}{L}dt.$$

By integration,

$$\log_e i = -\frac{R}{L}t + K.$$

When $t = 0$, $i = I$, so that $K = \log_e I$.

$$\therefore \log_e i - \log_e I = -\frac{R}{L}t,$$

$$\log_e \frac{i}{I} = -\frac{R}{L}t,$$

and

$$i = Ie^{-\frac{R}{L}t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

In a time equal to the time constant ($t_0 = L/R$) the current decays to a value

$$i = Ie^{-\frac{R}{L}\frac{L}{R}}$$

$$= I\frac{1}{e} = 0{\cdot}368I.$$

* The E.M.F. of self inductance is still denoted by $-Ldi/dt$, but its direction is reversed because di/dt is now a rate of *decrease*.

As in the case of the growth, the *time constant* can be defined as,

(*a*) the time in which the current would fall to zero if the initial rate of decay were maintained,

(*b*) the time in which the current actually falls to 36·8 per cent of its initial value,

(*c*) the time, starting from any point on the decay curve, in which the current would fall to zero, supposing that the rate of decay at the point considered were maintained.

Example.—A coil having a resistance of 15 ohms and an inductance of 10 henrys is connected to a 75-volt supply. Determine (*a*) the value of the current 0·67 sec., and 2 sec., after connection: (*b*) the time taken for the current to reach a value of 4 amperes.

Here the steady value of the current is

$$I = \frac{75}{15} = 5 \text{ amperes.}$$

(*a*) The time constant is $t_0 = \dfrac{L}{R} = \dfrac{10}{15} = 0{\cdot}67$ sec.

When $t = 0{\cdot}67$, $i = 0{\cdot}632 \times 5 = 3{\cdot}16$ amperes.

When $t = 2$, $i = 5(1 - e^{-15\times 2/10})$

$$= 5\left(1 - \frac{1}{e^3}\right) = 4{\cdot}75 \text{ amperes.}$$

(*b*) When $i = 4$ amperes,

$$4 = 5(1 - e^{-3t/2}).$$

$$\therefore e^{-3t/2} = \frac{5-4}{5} = 0{\cdot}2.$$

Taking logarithms of both sides,

$$-\frac{3}{2}\, t \log_{10} e = \log_{10} 0{\cdot}2,$$

$$t = -\frac{2}{3}\frac{\log_{10} 0{\cdot}2}{\log_{10} 2{\cdot}718} = -\frac{2}{3}\cdot\frac{\bar{1}{\cdot}301}{0{\cdot}434}$$

$$= 1{\cdot}075 \text{ sec.}$$

29. Energy stored in a Magnetic Field

It has been seen that whenever a current flows against an opposing E.M.F. electrical energy is converted into some other form.

In an inductive circuit, during the growth period, while the current is flowing against the opposing E.M.F. of self inductance, electrical energy is taken from the supply and stored up in the magnetic field as what may be termed magnetic strain energy (p. 166).

That portion of the P.D. which overcomes the resistance (§ 26, p. 177) represents the energy per coulomb which is converted into heat by the resistance of the circuit; while the remainder, which overcomes the inductance E.M.F., represents the energy per coulomb stored in the magnetic circuit. Immediately after connection, during the early stages of the growth period when the current is small, most of the energy is stored, but as the current increases the proportion converted into heat increases, until finally, under steady conditions, the whole of the energy input is converted into heat and the total quantity stored remains constant.

An increase in the P.D. causes a further rise in current and a consequent increase in stored energy; while a fall in the P.D. results in a liberation of some of the stored energy.

Consider a coil of inductance L, such as the ring solenoid shown in fig. 26, p. 165. Let the coil be connected to a supply, and at some instant during the growth period let the current be represented by i and the E.M.F. of self inductance by e.

Then the instantaneous value of the power (p) absorbed is

$$p = ei$$
$$= \left(L\frac{di}{dt}\right)i \text{ watts,}$$

and the energy stored in a time dt is

$$p\,dt = \left(L\frac{di}{dt}\right)i\,dt = Li\,di \text{ joules.}$$

Hence the total energy stored during the growth of the current from zero to its steady value I is

$$\int p\,dt = L\int_0^I i\,di = L\left[\frac{i^2}{2}\right]_0^I$$
$$= \tfrac{1}{2}LI^2 \text{ joules;}$$

When a current of I amperes is flowing in a circuit of inductance L henrys, the energy stored in the magnetic circuit is

$$\tfrac{1}{2}LI^2 \textbf{ joules.} \quad \ldots \ldots \quad (31)$$

Further, let

$N =$ number of turns in coil,
$l =$ mean length of magnetic path (m.),
$a =$ cross-section of magnetic circuit (m.^{2}).

During the growth of the current, the instantaneous value of the magnetizing force is

$$H = \frac{Ni}{l} \text{ ampere-turns per metre}$$

so that

$$i = \frac{lH}{N} \text{ amperes,}$$

and if Φ and B are the corresponding values of flux and flux density produced the E.M.F. of self inductance is

$$e = N\frac{d\Phi}{dt} = Na\frac{dB}{dt} \text{ volts [since } \Phi = Ba].$$

$$\therefore p = ei = Na\frac{dB}{dt}\cdot\frac{lH}{N} = al\,H\frac{dB}{dt} \text{ watts,}$$

and the energy stored in a time dt is

$$p\,dt = ei\,dt = alH\,dB \text{ joules.}$$

Hence the total energy stored while the flux is growing to its maximum value Φ_m is

$$al\int_0^{B_m} H\,dB \text{ joules}$$

where B_m is the flux density corresponding to Φ_m.

Now al is the volume of the magnetic circuit in cubic metres, so that

$$\text{Energy stored per cubic metre} = \int_0^{B_m} H\,dB \text{ joules.} \quad . \quad (32)$$

The integral $\int H\,dB$ represents the area between the curve relating B and H (the B-H curve of the material, § 2, p. 211) and the B-axis (fig. 31).

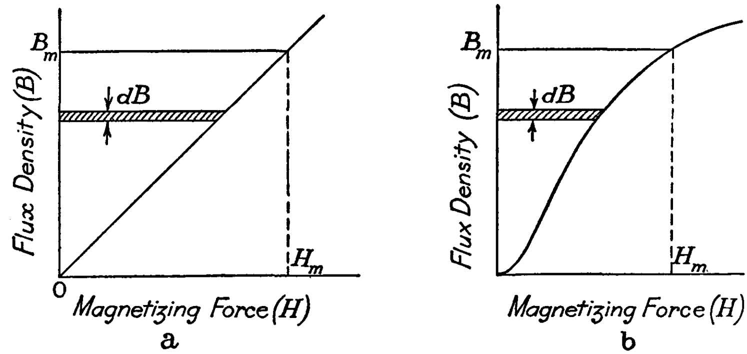


Fig. 31.—Energy stored in an air gap

Hence

The energy stored in a magnetic circuit in establishing a flux density B_m is $\int_0^{B_m} H\,dB$ joules per cubic metre, and is represented by the area between the B-H curve and the B-axis.

Energy stored in an Air Gap (or Other Non-magnetic Portion of Magnetic Circuit).

Since the permeability is constant, the B-H curve is a straight line (fig. 31*a*) and $H = B/\mu_0$, so that

$$\int_0^{B_m} H\,dB = \frac{1}{\mu_0}\int_0^{B_m} B\,dB = \tfrac{1}{2}\frac{B_m^{\,2}}{\mu_0}.$$

Hence

Energy stored in an air gap in establishing a flux density B_m is

$$\frac{1}{2}\frac{B_m^{\,2}}{\mu_0} \text{ joules per cu. m.} \quad . \quad . \quad . \quad . \quad . \quad (33)$$

Energy stored in a Magnetic Material.

When the magnetic circuit lies in a magnetic material the B–H curve is not a straight line, and the area under the curve (fig. 31b) can be found only by direct measurement after drawing the curve.

Energy stored per cubic metre in establishing a flux density B_m

$$\propto \text{(area under } B\text{–}H \text{ curve between } B = 0 \text{ and } B = B_m).$$

The measured value of the area, however, clearly depends on the scales employed in plotting B and H, so that in general terms the expression becomes

Energy stored in joules per cubic metre

= area under curve in terms of B and H.

For example, if the scales are such that

on the H-axis 1 cm. represents p ampere-turns per metre,
on the B-axis 1 cm. represents q webers per sq. metre

Energy stored in joules per cubic metre

$$= \text{(area under curve in square centimetres)} \times p \times q. \quad (33a)$$

30. Force of Attraction between Two Magnetized Surfaces

The expression obtained in the previous section for the energy stored in the air portion of a magnetic circuit can be used in order to determine the force of attraction (or magnetic pull) between two magnetized surfaces.

Fig. 32 shows two poles of rectangular section separated by an air gap, between which is a uniform field of density B webers per sq. m.

From equation (33) above, the energy stored in the air gap is $\frac{1}{2}B^2/\mu_0$ joules per cu. m., and since the volume of the gap is abx cu. m., it follows that

$$\text{Total energy stored} = \frac{1}{2}\frac{B^2}{\mu_0}abx \text{ joules.}$$

Now imagine that the lower pole is moved downwards through a distance dx, the flux density remaining constant. The motion is opposed by the force of attraction, so that the work done in moving the pole is

$$F\,dx \text{ joules,}$$

where F is the force in newtons.

At the same time the volume of the air gap has been increased by $ab\,dx$ cu. m., so that the stored energy is increased by

$$\frac{1}{2}\frac{B^2}{\mu_0}ab\,dx \text{ joules.}$$

Now the only source of this increase in energy is the work done in moving the lower pole. Hence

$$F\,dx = \tfrac{1}{2}\frac{B^2}{\mu_0}ab\,dx.$$

$$\therefore\ F = \tfrac{1}{2}\frac{B^2}{\mu_0}ab \text{ newtons,}$$

$$f = \frac{F}{ab} = \tfrac{1}{2}\frac{B^2}{\mu_0} \text{ newtons per sq. m.}$$

When a flux density of B webers per sq. m. exists between two magnetized surfaces, the attractive force between the two surfaces is

$$\tfrac{1}{2}\frac{B^2}{\mu_0} \textbf{ newtons per square metre.} \quad . \quad . \quad . \quad (34)$$

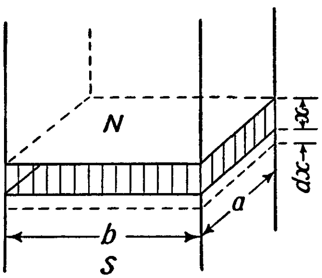


Fig. 32.—Energy stored in an air gap

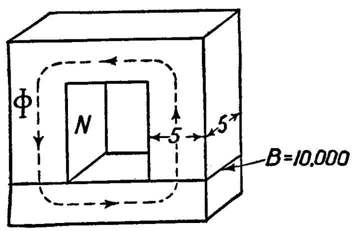


Fig. 33

Example 1.—One of the field coils of a large dynamo contains 1500 turns, and a current of 5 amperes produces a flux of 0·06 weber. Calculate (*a*) the inductance, (*b*) the stored energy, (*c*) the average value of the E.M.F. of self inductance if the circuit is broken in 0·02 sec.

(*a*) $$L = \frac{N\Phi}{I} = \frac{1500 \times 0{\cdot}06}{5}.$$
$$= 18 \text{ henrys.}$$

(*b*) Stored energy $= \tfrac{1}{2}LI^2 = \tfrac{1}{2} \times 18 \times (5)^2 = 225$ joules (166 ft.-lb.).

(*c*) Average induced E.M.F. $= \dfrac{N\Phi}{t} = \dfrac{1500 \times 0{\cdot}06}{0{\cdot}02}$
$$= 4500 \text{ volts.}$$

Alternatively,

$$\text{E.M.F.} = L\frac{di}{dt} = \frac{18 \times 5}{0{\cdot}02} = 4500 \text{ volts.}$$

Example 2.—A horseshoe magnet has poles 5 cm. square. Find the pull (in lb.) between the poles and the keeper when the flux density at the contact surface is 1 weber per sq. m. (fig. 33).

$$f = \tfrac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2 \times 4\pi \times 10^{-7}}$$

$$= 398 \times 10^3 \text{ newtons per sq. m.}$$

Total contact area (2 poles) $= 50 \times 10^{-4}$ sq. m.

Total pull $= 398 \times 10^3 \times 50 \times 10^{-4}$

$= 1990$ newtons $= 448$ lb. $\left[\begin{array}{l}1 \text{ newton} = \\ 0{\cdot}225 \text{ lb.}\end{array}\right]$

EXAMPLES

1. The flux linked with a coil of 20 turns is changing at the rate of 0·01 weber per sec. What is the value of the E.M.F. induced in (*a*) each turn, (*b*) the coil?

2. The flux linked with a coil of 100 turns is reduced from 0·04 weber to 0·02 weber in 0·1 sec. What is the average value of the induced E.M.F.?

3. The two ends of a coil of 1500 turns are connected together. If the resistance of the coil is 20 ohms, at what rate must the flux linked with it change, in order to produce a current of 5 amperes?

4. A conductor 12 in. long on the armature of a dynamo cuts through a magnetic field of 60,000 lines per sq. in. at a speed of 300 ft. per min. What is the value of the induced E.M.F.? (1 weber $= 10^8$ C.G.S. lines.)

5. Calculate the strength of the magnetic field at the centre of a short coil, 20 cm. in diameter, consisting of 5 turns and carrying 5 amperes.

6. Calculate the magnetic field strength (*a*) at the centre of a circular coil of 4 turns 10 cm. in diameter, (*b*) in the interior of a solenoid of length 50 cm. uniformly wound with 500 turns. The current in each case is 2·5 amperes.
Make sketches of the lines of force which would be produced. [Grad. I.E.E.]

7. Determine the magnetizing force and the total flux at the centre of a solenoid 100 cm. long and 8 cm. in diameter, uniformly wound with 1000 turns, carrying 10 amperes.

8. A straight helix 1 metre in length and 2 cm. in diameter is uniformly wound with 2000 turns of insulated wire. Calculate the magnetic field at the centre of the helix when a current of 1·5 ampere is passed through it.
How does the field strength at any point depend upon the position of that point with respect to the helix? [Grad. I.E.E.]

9. A conductor 12 in. long lies in and perpendicular to a field of 6 webers per square metre. If the total force acting on the conductor is 12 lb., find the current flowing in it.

10. Calculate the work done by a conductor carrying a current of 100 amperes in cutting through a total flux of 0·05 weber.

11. Calculate the force in lb. per ft. between two conductors 1 in. apart each carrying a current of 5000 amperes in opposite directions.

12. If a coil of large diameter having 2 turns carries a momentary current of 10,000 amperes, and the turns of the coil are 2 cm. apart, estimate the force of attraction between them in kg. per cm. length. [Grad. I.E.E.]

13. Calculate the magnetizing force along the axis of a ring solenoid, having a mean diameter of 20 cm. and uniformly wound with 500 turns, carrying a current of 5 amperes.

14. An iron ring, having a mean diameter of 25 cm. and a cross-section of 2 sq. cm., is uniformly wound with 400 turns. Calculate the flux density and the total flux produced by a current of 5 amperes if the relative permeability of the iron is 450.

15. Determine the inductance of a coil of 750 turns in which a current of 4 amperes produces a flux of 800 lines.

16. A ring solenoid is 30 cm. in mean diameter and uniformly wound on a wooden core 4 cm. in diameter, with 900 turns. Calculate its inductance.

17. What is the value of the E.M.F. induced in a circuit of which the inductance is 0·5 henry, when the current is changing at the rate of 250 amperes per sec.?

18. (*a*) If, in the solenoid in question 16, the wooden core is replaced by an iron core of the same size and having a relative permeability of 500, calculate the inductance.

(*b*) If the current in the solenoid is 5 amperes and the circuit is broken in 0·01 sec., determine the average value of the induced E.M.F.

19. A long solenoid is wound with 10 turns per cm. of length. A secondary winding situated inside and at the centre of the solenoid is 5 cm. in diameter and contains 200 turns.

Calculate the mutual inductance between the two coils.

20. The field windings of an electrical machine consist of 8 coils connected in series, each containing 1200 turns. When the current is 5 amperes, the flux linked with each coil is 0·05 weber. Calculate (*a*) the inductance of the circuit, (*b*) the energy stored (in joules), (*c*) the average value of the induced E.M.F. if the circuit is broken in 0·1 sec.

21. A circuit, of which the inductance is 3 henrys and the resistance 10 ohms, is connected to a 100-volt supply. Calculate (*a*) the time-constant of the circuit, (*b*) the value of the current 0·5 sec. after connection to the supply.

22. If the resistance of the field circuit of the machine in question 20 is 100 ohms:

(*a*) Calculate the current 0·3 sec. after connection to a 500-volt supply.

(*b*) Calculate the time taken for the current to grow to 4·9 amperes.

(*c*) If a non-inductive resistance of 100 ohms is connected across the terminals of the field circuit, calculate the current which flows 0·5 sec. after the 500-volt supply has been disconnected.

23. In a certain circuit connected to a 500-volt supply, the current has a steady value of 10 amperes and reaches a value of 8 amperes in 1 sec. after switching on.

Calculate (*a*) the inductance of the circuit; (*b*) the increased P.D. necessary to cause the current to reach a value of 9·6 amperes in the same time, and the steady value under these conditions.

24. Find an expression for the growth of a current in an inductive circuit.

If the final current in an inductive circuit is 2·0 amperes, and the time taken to reach 1·0 ampere is 0·05 sec., find the initial rate of rise of the current.

[Lond. B.Sc.(Eng.)].

25. A horseshoe magnet has two poles, each 10 cm. square. Calculate the magnetic pull (in kg.) between the poles and the armature when the flux density at the contact surface is 0·8 weber per sq. m.

26. The area of each pole face of the magnetic track brake of an electric tram is 12 sq. in. Calculate the flux density at the contact surface, in order that the adhesive force between magnet and rail may be 2000 lb.

27. An iron ring, having a mean diameter of 20 cm. and cross-section of 5 sq. cm., is divided diametrically by two fine saw cuts. The two halves are then put together and the ring is wound uniformly with 500 turns. If the relative permeability of the iron is 300, calculate the force (in lb.) necessary to pull the two halves apart when the current is 5 amperes.

28. A ring of non-magnetic material has a cross-sectional area of 5 sq. cm. and a mean diameter of 20 cm. It is wound uniformly with 450 turns of copper wire of 1 mm. diameter. Calculate the approximate resistance and inductance of the coil. Neglect the thickness of the insulation. Resistivity of copper is $\frac{1}{58}$ ohm for 1 m. and 1 mm.^{2}. [Grad. I.E.E.]

CHAPTER X

Measuring Instruments

1. Classification of Instruments

Mention of certain instruments has already been made in previous sections, but a description of their operation would have been premature.

There are many kinds of instruments for the measurement of various electrical quantities such as current, potential difference, quantity, power, energy, frequency, etc., but they may be divided broadly into:

(1) *Indicating instruments*, which indicate the actual value of the quantity at the instant considered, e.g. ammeters, voltmeters and wattmeters.

(2) *Integrating instruments*, which measure the quantity of electricity or the electrical energy supplied in a given time, e.g. ampere-hour meters (which are essentially coulomb-meters) and watt-hour meters.

(3) *Recording instruments*, which are indicating instruments in which a pen is substituted for a pointer, and which give a continuous record, on a moving roll of paper, of the variations of the particular quantity considered.

The instruments most commonly employed both industrially and in the laboratory are indicating instruments; and of these the only ones which will be described here are:

Ammeters, for the measurement of current.

Voltmeters, for the measurement of E.M.F. and P.D.

Galvanometers, used for the detection and comparison of very small currents but not, in general, for direct measurement.

2. Ammeters and Voltmeters

With the exception of electrostatic instruments (§ 12, p. 199), all measuring instruments depend for their operation upon an electric *current*; but since the current in any portion of a circuit of *constant* resistance is proportional to the P.D. between the ends of that portion, P.D. can be measured by means of the effects of the current which it produces. There is no *essential* difference between ammeters, voltmeters and galvanometers of the same type; and differences in detail are due chiefly to the difference in the magnitudes of the currents involved.

An *ammeter* is connected in *series* with the circuit and carries the whole, or a known portion, of the current to be measured (fig. 1*a*); and in order to reduce the power loss (I^2R) its resistance must be *low*.

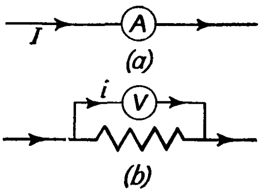


Fig. 1.—Connection of ammeters and voltmeters

A *voltmeter* is connected across the two points between which the P.D. is to be measured, i.e. in *parallel* with the portion of the circuit between them (fig. 1*b*). It carries a current proportional to the P.D. ($i = V/R$); and in order to reduce the power loss ($i^2R = V^2/R$) the resistance must be high.

A *galvanometer* may have either a low or a high resistance, according to the purpose for which it is to be used.

3. Essentials of an Indicating Instrument

An indicating instrument consists essentially of a pointer or mirror attached to a moving system, either suspended or pivoted in jewelled bearings, upon which act two torques:

(1) *The deflecting torque*, produced by the operating current, which causes a partial rotation of the spindle and consequent deflection of the pointer from its zero position. This deflection would, in general, be somewhat indefinite were it not for:

(2) *The controlling torque*, which is a torque, increasing as the deflection increases, which opposes the deflecting torque and tends to restore the system to its zero position. The deflection therefore increases until the controlling torque is equal and opposite to the deflecting torque, so that there is a definite angle of deflection corresponding with each value of the deflecting torque, i.e. with each value of the current.

For satisfactory operation a third torque is necessary, the *damping torque*, which is operative only when the system is moving, and the object of which is to bring it to rest quickly without prolonged oscillation about its steady position.

4. The Deflecting Torque

Any one of the three effects of a current, i.e. chemical, heating or magnetic, may be used to measure a current.

Although the unit of current was at one time defined in terms of the chemical effect, this method is clearly more suitable for the measurement of *quantity* and cannot be used in an *indicating* instrument.

The heating effect is by its nature more suitable for the measurement of power or energy, but is used indirectly in one form of indicating instrument known as the *hot-wire* type (§ 11, p. 199).

With a few exceptions, the deflecting torque in all measuring instruments depends upon the *magnetic effect* of a current.

5. The Controlling Torque

In sensitive galvanometers, the controlling torque is usually due to the twisting of the suspending fibre in conjunction with the earth's horizontal field or a magnetic field produced by an external control magnet.

In ammeters and voltmeters, however, the torque is produced either by gravity or by the twisting of a spiral hair-spring.

Gravity Control.

A small weight W is attached to the spindle so that in the zero position it hangs vertically (fig. 2*a*). When the spindle turns, the weight is lifted into the position W' and produces an opposing torque which is proportional to the *sine of the angle* of deflection. This type of control is simple and unvarying, but necessitates a vertical position and therefore cannot be used in portable instruments.

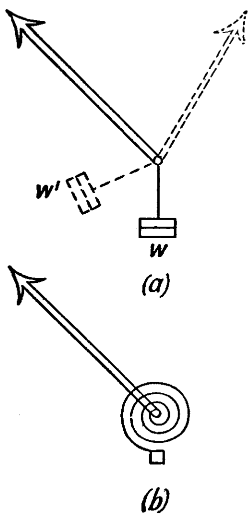


Fig. 2.—Gravity and spring control

Spring Control.

The spindle is connected to one end of a fine phosphor-bronze hair-spring, the other end of which is fixed. When the spindle turns, the spring is tightened and produces an opposing torque proportional to the *angle* of deflection. Instruments with spring control can be used in any position, and most modern instruments have this type of control.

6. The Damping Torque

The moving system, if acted upon by the deflecting and controlling torques only, would, owing to its inertia, swing beyond the steady or equilibrium position and execute a number of oscillations before coming to rest; and if the current were fluctuating might not come to rest at all. To prevent this a *damping* torque is required, which must oppose the motion and must be dependent on it so that it disappears when the pointer comes to rest and does not affect the value of the steady deflection.

This torque is usually supplied either by air friction (solid friction is obviously inadmissible) or by eddy currents (§ 10, p. 151) induced in certain parts of the moving system as the result of its motion through a magnetic field (electromagnetic damping).

When the damping is such that the pointer moves up to its steady position and comes to rest without oscillation, it is said to be *dead-beat*. In most instruments the damping is made somewhat less than this critical value, so that the pointer makes one or two small oscillations before coming to rest.

7. Types of Instrument

Instruments may be classified according to the way in which the deflecting torque is produced:

(1) *Moving magnet type,* used in galvanometers only.

(2) *Moving iron type*, (3) *Moving coil type*: in general use in the laboratory and in industry.

(4) *Dynamometer type,* used chiefly in the form of wattmeters both in the laboratory and in industry, and for precision ammeters and voltmeters.

(5) *Induction type,* used industrially on a.c. circuits only.

(6) *Hot-wire type,* used only for special purposes.

(7) *Electrostatic type,* used for voltmeters only, particularly for measurement of very high P.D.s.

Only types (1), (2) and (3) will be described in any detail, and type (5) will be omitted.

8. Moving Iron Instruments

There are two types of moving iron instruments—the *attraction* type, in which the torque is produced by a piece of soft iron, mounted eccentrically on the spindle, which is drawn into the coil when a current passes through it (fig. 3); and the *repulsion* type, in which the torque is due to the repulsive force between two pieces of iron similarly magnetized. Modern instruments are usually of the latter type, which is shown in fig. 4. It consists of a coil carrying the current to be measured, inside which are two pieces of soft iron, one fixed and the other attached to the spindle. When a current passes through the coil these are similarly magnetized, and the repulsive force set up between adjacent poles of the same polarity produces the deflecting torque on the

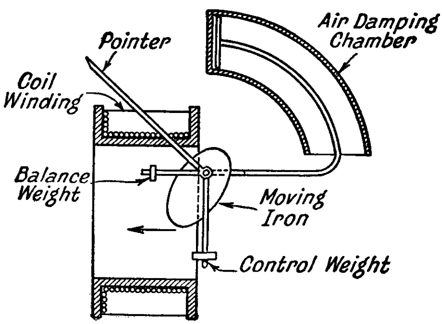


Fig. 3.—Moving iron instrument—attraction type

[From *Electrical Measuring Instruments*—Golding (Sir Isaac Pitman & Sons, Ltd.)]

spindle. The force is roughly proportional to the (current)^2 and inversely proportional to the (distance apart)^2, so that the deflection is not proportional to the current and the scale is unevenly divided and crowded at the lower end; although it can be improved by substituting for the iron rods specially shaped pieces of sheet iron. A reversal of the current reverses the polarity of *both* irons, so that the direction of the force between them is unchanged; hence the instrument is " unpolarized " and can be used on either direct or alternating current circuits.

The damping torque is produced by air friction; electromagnetic damping requires a permanent magnet, the presence of which would affect the accuracy of the instrument. An aluminium piston (or vane) is attached to the spindle and moves with very little clearance in a curved cylinder (or circular box), so that the air must pass from one side to the other through the narrow annular space between them. Hence motion in either direction is opposed by the difference in pressure on the two sides of the piston (or vane), which disappears when the motion ceases. The piston type is shown in fig. 3 and the vane type in fig. 4.

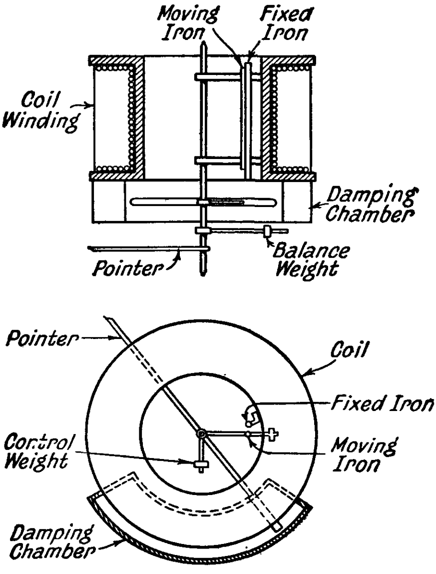


Fig. 4.—Moving iron instrument—repulsion type
[From *Electrical Measuring Instruments*—Golding (Sir Isaac Pitman & Sons, Ltd.)]

In any given moving system the torque depends upon the *ampere-turns*, so that the current range can be varied by altering the number of turns on the operating coil. For example, if the number of turns is *halved*, *twice* the current will be required to produce the same deflection, and the range is therefore *doubled*. Hence, in an ammeter, as the current range increases the number of turns decreases, but the section of each turn increases.

The same movement may be used for a voltmeter, but, since the operating current is small, the coil contains many turns of fine wire. Further, in order to ensure that the resistance shall be little affected by temperature, a condition which is essential for accuracy, the operating coil, wound with copper wire, forms only a small portion of the total resistance, the remainder being supplied by a separate coil of manganin wire with a negligible temperature coefficient, often termed a *swamping resistance.* The range of the instrument is varied by altering the value of the swamping resistance; e.g. if the total resistance is *doubled, twice* the P.D. is necessary to produce the same deflection, and the range is therefore *doubled.*

The moving iron instrument, since it is suitable for a.c. circuits, is the most widely used of all types, and can be made of high accuracy. It can also be used on d.c. circuits, but usually with somewhat lower accuracy. However, improvements in design and materials have allowed the development of instruments which can be used with equal accuracy for a.c. or d.c. measurements and which have scales which are almost uniform over the working range.

Since the operating field is comparatively weak, instruments of this type are affected by stray magnetic fields unless surrounded by a soft-iron shield or iron case.

9. Moving Coil Instruments

In a moving coil instrument the deflecting torque is produced by the force acting on a conductor carrying the current to be measured, and lying in a uniform magnetic field of constant strength.

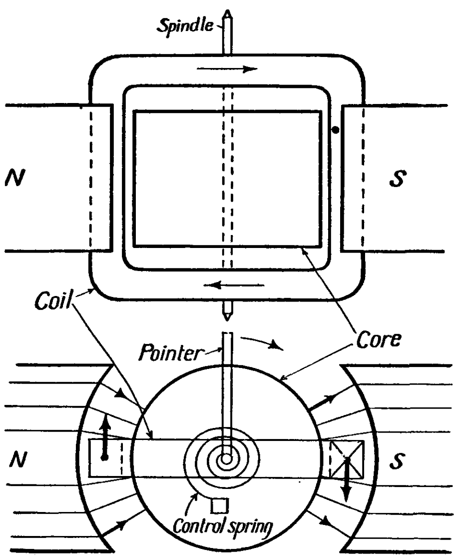


Fig. 5.—Principle of moving coil instrument

A light rectangular coil of fine wire is mounted on the spindle so that its sides lie on the annular air gaps between the two poles of a permanent magnet and a soft iron cylinder (fig. 5). The current is led into and out of the coil by means of the two control hair-springs, one above and one below the coil. If the relative directions of the flux due to the permanent magnet and of the current are as shown, it can be seen by applying the method of § 9, p. 150, that the sides of the coil will experience forces which produce a clockwise deflecting torque.

The insertion of the circular iron core reduces the air path to two short annular air gaps of equal length, across which the flux passes radially. The sides of the coil, therefore, lie in a strong field of *uniform* density, so that the force and therefore the deflecting torque (t_d) is proportional to the current, whatever the position of the coil within the range of movement, i.e.

$$t_d \propto I \text{ or } t_d = k_d I,$$

and since the control is by spring, the controlling torque (t_c) is proportional to the angle of deflection, i.e.

$$t_c \propto \theta \text{ or } t_c = k_c \theta.$$

The steady value of the deflection is that which makes the two torques equal, i.e.

$$t_d = t_c, \text{ or } k_d I = k_c \theta.$$

$$\therefore \theta = \frac{k_d}{k_c} I = k_0 I,$$

that is, the angle of deflection is proportional to the current so that the scale is evenly divided.

Reversal of the current will obviously reverse the torque, since the direction of the field of the permanent magnet is constant. Moving coil instruments are therefore *polarized* instruments and can be used only on d.c. circuits.

The damping is electromagnetic. The coil is wound on a copper or aluminium frame which forms one complete turn. When it moves through the field under the deflecting torque, an E.M.F. is induced in the sides which causes a current to flow round the frame. The sides of the frame are therefore subjected to forces which set up a torque opposing the motion (§ 10, p. 151); and this torque quickly damps out the oscillation of the coil, but disappears as it comes to rest. In fact the frame acts as a generator by which the kinetic energy of the coil is converted into electrical energy and then into heat.

Spring control is always used, not only because the springs serve as electrical connections to the coil, but because gravity control would not give the evenly divided scale which is one of the advantages of the type. The use of the springs to carry the current limits the latter to a small value. In an ammeter the current in the coil is only a small fraction of the total current, the greater portion of which passes through a low-resistance *shunt* (§ 14, p. 200) connected in parallel with it. By employing external shunts, one instrument can be used over a number of different ranges.

The moving coil instrument is probably the most accurate of all types of instrument, and is in general use for accurate measurements in d.c. circuits. Its chief advantages lie in the even scale, the almost ideal method of damping, and the fact that one instrument can be used over a very wide range. Since the working field is very strong, stray magnetic fields have little effect. The uniform scale, the small operating current and other advantages of the moving coil instrument have led to its use, in conjunction with a rectifier or thermo-couple, for the measurement of small alternating voltages and currents, particularly at high frequencies, e.g. in communication circuits.

10. Dynamometer Instruments

In instruments of this type, the permanent magnet of the moving coil type is replaced by a fixed coil, usually in two halves, which provides an almost uniform field in which the moving coil is placed (fig. 6*a*). The coils are air-cored, so that errors due to the use of iron are avoided. In voltmeters, the two coils are connected in series, while in

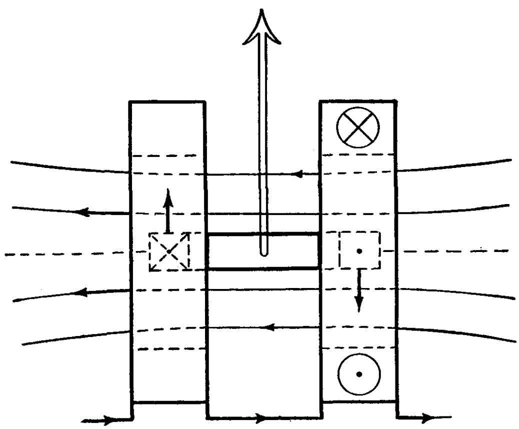


Fig. 6*a*.—Principle of dynamometer type instrument

ammeters (seldom used) the moving coil is connected across a shunt in series with the fixed coils (fig. 6*c*). In both cases the torque is very nearly proportional to the (current)^2, so that the scale is uneven. Reversing the current reverses the field due to the fixed coil as well as the current in the moving coil, so that the direction of the torque is unchanged; and instruments of this type can be used with equal accuracy on both a.c. and d.c.

If the fixed coil carries the whole or a known portion of the current in a circuit, while the moving coil is connected across the ends of the circuit and carries a current proportional to the P.D., then the deflecting torque is proportional to the *power* in the circuit, and the instrument acts as a *wattmeter* (see below).

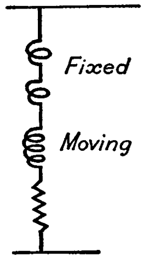


Fig. 6*b*.—Voltmeter

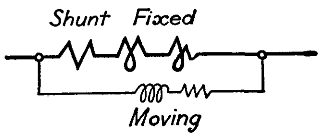


Fig. 6*c*.—Ammeter

Instruments of this type are used chiefly for a.c. measurements, particularly in the form of wattmeters; on d.c. circuits they are inferior to good moving coil instruments, and more expensive.

Air damping is used, as the presence of the magnet necessary for electromagnetic damping would affect the accuracy; and since the operating field is comparatively weak, the instrument is affected by stray fields unless shielded.

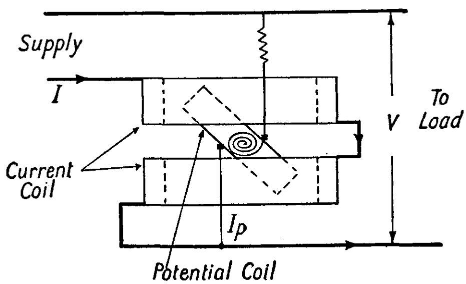


Fig. 7.—Dynamometer wattmeter

The dynamometer instrument used as a wattmeter is shown in fig. 7. The fixed or current coil carries the main circuit current or a known fraction of it, while the moving or potential coil carries a current proportional to the P.D. Considering the case of the d.c. circuit first, since there is no iron present, the flux density set up by the fixed coils is proportional to the current, i.e.

$$B_f = k_f I,$$

while the current in the potential coil is proportional to the P.D.,

$$I_p = k_p V.$$

Hence the deflecting torque is

$$T_d = kB_f I_p = k_0 VI,$$

i.e. the torque is proportional to the *power*.

In the a.c. case all the above relations hold for instantaneous values, so that at every instant,

$$t_d = k_0 vi,$$

i.e. the torque is proportional to the power at that instant. But the moving system is prevented by its inertia from following the variations in deflecting torque which take place during the cycle, and takes up a position corresponding with the *average value* of the torque, i.e. it indicates the *average value* of the power, which is what is required. Since the field set up by the fixed coils in not *radial* (as in moving-coil instruments), the *torque per watt* varies with the position of the moving coil, so that the scale is not entirely uniform.

Laboratory wattmeters are usually provided with series resistances of different values by which the voltage range of the potential coil circuit can be varied. Two current ranges, one of which is double the other, are obtained by connecting the two halves of the current coil in parallel or series respectively.

11. Hot-wire Instruments

In this type the current to be measured, or a known portion of it, is passed through a fine platinum-iridium wire, and the expansion due to the heating effect produced is used to cause a movement of the pointer. Since the expansion is small, it is usually magnified by means of the "double-sag" arrangement shown in fig. 8. At a point near the centre of the heated wire is connected another wire W, the other end of which is fixed, and from a point near the centre of W a silk thread T passes round the spindle and is held taut by a spring. When the hot wire expands it sags; this produces a larger sag in W, and the spring draws the silk thread T to the left and rotates the spindle and pointer.

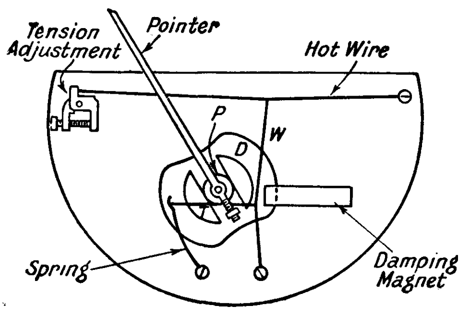


Fig. 8.—Hot-wire instrument

[From *Electrical Measuring Instruments*—Golding (Sir Issac Pitman & Sons, Ltd.)]

The heating effect and therefore the expansion and deflection are proportional to the (current)^2, so that the scale is uneven and crowded at the lower end. The power loss is greater than in other types, and the instrument is sluggish in action, as the wire takes an appreciable time to attain a steady temperature; slip between the thread and the spindle also often causes a zero error. Its great advantage is that the reading is independent of frequency and wave-form (§ 3, p. 244), and it has been largely used for measurements at radio-frequencies.

12. Electrostatic Instruments

Instruments of this type are essentially voltmeters. The deflecting torque is produced by the attractive force between oppositely charged plates and is small unless the P.D. is large, so that the instrument is used chiefly for the measurement of high voltages.

It takes the form of a quadrant electrometer, and consists of four metal quadrants separated by air gaps, forming a box inside which moves a metal vane (fig. 9). Opposite quadrants are connected together and each pair is connected to one terminal of the supply, while one pair is also connected to the moving vane. The figure shows that under these conditions the vane is repelled by the quadrants *aa* and attracted by the quadrants *bb*.

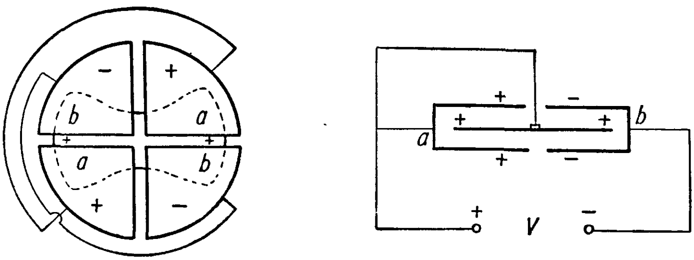


Fig. 9.—Principle of electrostatic voltmeter

In the high-voltage type the plane of the box is vertical, and the vane is mounted on a horizontal spindle supported on knife-edges. In order to make the instrument suitable for lower voltage ranges, a number of the quadrants are placed one above the other, and a similar number of vanes are mounted on a common vertical spindle which is suspended, or (in the portable type) supported in jewelled bearings. By this means the range may be brought down as low as 150 volts.

Electrostatic instruments are laboratory rather than industrial instruments. They have the advantage that the reading is independent of wave-form and frequency, but their accuracy is not very high and their use is generally restricted to the measurement of high voltages or to cases where it is necessary that the operating current should be negligibly small.

13. Accuracy of Instruments

Ammeters and voltmeters are divided by the British Standards Institution (B.S. 89) into two grades:

Substandard.—These are precision instruments, used only when very great accuracy is required and for checking other instruments. The permissible error depends upon the type and varies from 0·2 to 0·5 per cent of the maximum reading, the lowest figure being for moving coil instruments.

Industrial Grade.—These are in general use for industrial and laboratory purposes where reasonable accuracy is required. The permissible error is 1 to 1·5 per cent, depending upon the type.

14. Extension of Range

Every instrument reading is subject to certain errors of observation, the importance of which naturally increases as the angle of deflection decreases. Hence when accurate readings are required, it is advisable to use an instrument with a range not much exceeding the value of the quantity to be measured, so that the deflection is large; and it is often convenient to be able to vary the range to suit conditions.

Ammeters.

In § 8, p. 193, it was pointed out that in moving iron instruments if the number of turns in the operating coil is decreased, the current to produce the same deflection is proportionately increased. It is therefore possible to make such an instrument with several ranges by bringing out tappings from the operating coil so that the number of turns in circuit can be varied, but it is difficult to obtain equal accuracy on all ranges. Most moving iron instruments have one range only, although two ranges can be obtained satisfactorily by dividing the operating coil and connecting the sections either in series or in parallel, as in the wattmeter.

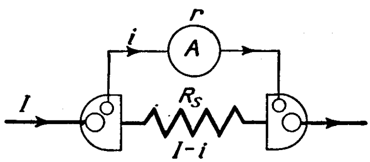


Fig. 10.—Ammeter and shunt

Moving coil instruments, on the other hand, must be used in conjunction with a shunt unless the current does not exceed a few milliamperes, and so are inherently suitable for use as multi-range instruments. When the shunts are incorporated in the instrument, the number of ranges does not often exceed three, but with separate shunts the number is unlimited.

Suppose that the resistance of the instrument is r, and that the current to produce full deflection is i, and let it be required to calculate the resistance of a shunt R_s which, when connected across the instrument terminals, will cause full deflection to represent a total current I.

The shunt and the instrument form two resistances in parallel (fig. 10), and of the total current I, a current i is to pass through the instrument and $I - i$ through the shunt.

Now the P.D. across each resistance is the same.

$$\therefore\ ir = (I - i)R_s,$$

or

$$R_s = r \cdot \frac{i}{I - i}, \qquad \qquad (1)$$

and, by solving for i,

$$\frac{i}{I} = \frac{R_s}{r + R_s}. \qquad \qquad (2)$$

If the range is to be increased n times, i.e. if $I = ni$, then, by substituting for I in (1),

$$R_s = \frac{r}{n-1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Thus, for example, if the range is to be increased 10 times, the shunt must have a resistance $\frac{1}{9}$ of that of the instrument, so that $\frac{1}{10}$ of the total current passes through the instrument and $\frac{9}{10}$ through the shunt.

It is essential that the resistance shall be little affected by temperature, so that the shunt usually consists of thin strips (or a wire for small currents) of manganin, soldered to two heavy copper blocks each having a large terminal for the main connection to the circuit and a small terminal for the instrument, as indicated in fig. 10.

The resistance r includes that of the instrument leads, so that when using external shunts the same pair or a pair of equal resistance should always be used.

Example.—The resistance of a moving coil instrument is 5 ohms, and full-scale deflection is caused by a current of 15 milliamperes.

(*a*) Calculate the resistance of a shunt which will enable it to be used as an ammeter with a range of 30 amperes.

(*b*) If the scale is marked up to 15, what current will be indicated by readings of 9·72 and 14·73 when the instrument is connected to the shunt?

(*a*) At full deflection $i = 0{\cdot}015$ ampere, $I = 30$ amperes.

$\therefore$ Current in shunt $= 30 - 0{\cdot}015 = 29{\cdot}985$ amperes.

$\therefore 0{\cdot}015 \times 5 = 29{\cdot}985 \times R_s$,

$$R_s = \frac{0{\cdot}075}{29{\cdot}985} = 0{\cdot}002501 \text{ ohm.}$$

Alternatively,

$$n = \frac{30}{0{\cdot}015} = 2000,$$

$$R_s = \frac{5}{2000-1} = \frac{5}{1999} = 0{\cdot}002501 \text{ ohm.}$$

(*b*) Since $n = 2000$,

A reading of 9·72 represents $9{\cdot}72 \times 2000$ milliamperes $= 19{\cdot}44$ amperes.

A reading of 14·73 represents $14{\cdot}73 \times 2000$ milliamperes $= 29{\cdot}46$ amperes.

Or, more simply, since a full-scale deflection of 15 divisions represents 30 amperes, the multiplying factor is 2 so that,

A reading of 9·72 represents $9{\cdot}72 \times 2 = 19{\cdot}44$ amperes.

" " " 14·73 " $14{\cdot}73 \times 2 = 29{\cdot}46$ "

Voltmeters.

The range of a voltmeter, whether of moving iron or moving coil type, can be extended by connecting in series with it an additional resistance coil (often called a multiplier).

Suppose that the resistance of the instrument is r, that full deflection is produced by a P.D. v, and that it is required to determine the resistance R_m to be added so that full-scale deflection will represent a P.D. V.

The deflection depends upon the current in the operating coil, and the added resistance must be such that when the P.D. across the instrument and multiplier is V the current is the same as when the P.D. across the instrument alone is v (fig. 11), i.e.

$$\frac{V}{R_m + r} = \frac{v}{r}.$$

$$\therefore R_m = \frac{V}{v} r - r. \qquad \ldots\ldots \quad (4)$$

If the range is to be increased n times, i.e. $V = nv$, then, by substituting for V in (4),

$$R_m = r(n - 1). \qquad \ldots\ldots \quad (5)$$

For example, if the range is to be increased 10 times, the total resistance must be increased 10 times, and the resistance to be added is 9 times that of the instrument.

Such resistances are wound non-inductively of manganin wire.

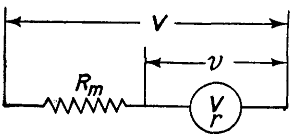


Fig. 11.—Voltmeter and multiplier

Example 1.—The instrument in the previous example is to be used as a voltmeter reading up to 150 volts. Calculate the necessary series resistance.

$$i = 0{\cdot}015 \text{ ampere}, \quad r = 5 \text{ ohms}.$$

$$\therefore v = 0{\cdot}075 \text{ volt}.$$

$$\therefore R_m = \left(5 \times \frac{150}{0{\cdot}075}\right) - 5$$

$$= 9995 \text{ ohms}.$$

Example 2.—The resistance of a 75-volt voltmeter is 5000 ohms. What resistance must be added to increase its range to 300 volts?

The total resistance must be such that the current with a P.D. of 300 volts is the same as that now produced by a P.D. of 75 volts, i.e.

$$\frac{300}{5000 + R_m} = \frac{75}{5000},$$

$$R_m = \left(5000 \times \frac{300}{75}\right) - 5000$$

$$= 15{,}000 \text{ ohms}.$$

15. Galvanometers

The compass needle placed below the straight wire shown in fig. 2 (p. 30) constitutes a galvanometer in its simplest form; a more elaborate and sensitive arrangement is the tangent galvanometer described in § 15, p. 160. For the detection of very small currents, the sensitivity is increased by eliminating the friction of the pivots, by decreasing the controlling torque, and by increasing the effective length of pointer.

Pivot friction is eliminated by suspending the moving portion by a single silk or quartz fibre or very thin phosphor-bronze strip. The effective length of the pointer is increased by using a beam of light. The movement carries a small mirror, by which a beam of light from a lamp is reflected on to a scale (fig. 12) so that any rotation of the mirror causes a movement of the spot of light on the scale. By this

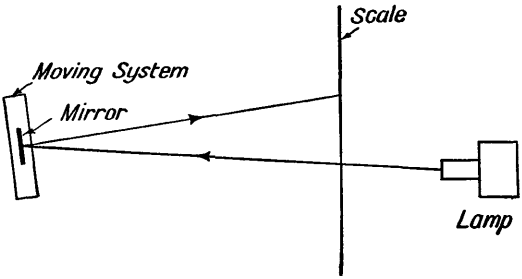


Fig. 12.—Principle of reflecting instrument

means, the equivalent of a *weightless* pointer is obtained, the effective length of which (since the angle turned through by the reflected beam is *twice* the angle of rotation of the mirror) is equal to *twice* the distance of the scale from the mirror. Galvanometers in which this arrangement is used are called *reflecting galvanometers.*

Galvanometers maybe either of the *moving magnet* or *moving coil* type, but the former is now rarely used.

Moving Magnet Galvanometer (Kelvin or Thomson type)

The moving magnet type is essentially a sensitive form of tangent galvanometer. It consists of a small mirror to the back of which are attached several very small steel magnets with similar poles pointing in the same direction, suspended by a fine quartz fibre in the centre of a coil through which passes the current to be measured. It is shown diagrammatically in fig. 13*a*. The forces exerted on the poles of the small magnets by the magnetic field set up by the coil produces a deflecting torque exactly as in the tangent galvanometer (fig. 22, p. 161). The controlling torque is supplied by the torsion of the fibre and also by a magnetic field. The latter may be that due to the earth alone as in the tangent galvanometer, or it may be the resultant of the earth's field and that of an external permanent magnet mounted on the galvanometer case, known as a control magnet.

In a still more sensitive form of the instrument there are two coils, at the centre of each of which is a group of small magnets, similar poles pointing in opposite directions in the two groups (fig. 13*b*). The current traverses the coils, which are connected in series, in *opposite* directions, so that the deflecting torque is in the *same* direction

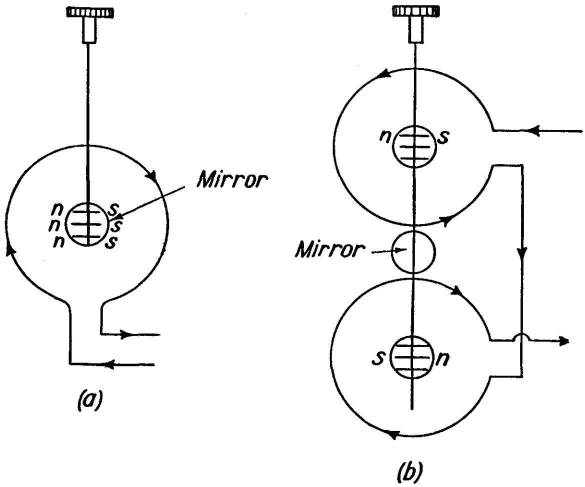


Fig. 13.—Principle of moving magnet galvanometer

in both cases. But the controlling torques exerted by the earth's field are in *opposite* directions and, if the moments of the magnets are almost equal, very nearly neutralize each other; so that the controlling torque is reduced to that of the torsion of the fibre, while the deflecting torque due to the two groups may be doubled.

Such an arrangement is known as an *astatic* system.

For general laboratory purposes moving magnet galvanometers have been superseded by instruments of the moving coil type.

Moving Coil Galvanometers (D'Arsonval type).

The principle of this instrument is the same as that of the moving coil ammeter described in § 9, p. 194, and in the less sensitive but more robust pointer types, such as are often used with the slide wire bridge or post-office box, the coil is pivoted and the general mechanical construction is similar.

In the more sensitive reflecting type, the coil is suspended by a very fine phosphor-bronze strip attached to a torsion head by which the zero may be adjusted. This strip serves as one connection to the coil, the other being supplied by a loosely coiled spiral of very fine wire which exerts no appreciable torque (fig. 14). As in the moving coil ammeter, the electromagnetic damping is provided by eddy currents flowing in the metal frame on which the coil is wound.

16. Ballistic Use of Galvanometers

When a steady current flows in a circuit, the quantity of electricity passing in a given time is easily determined by measuring the current. If the current is a varying one, the quantity may still be measured by means of a voltameter (§ 9, p. 123), provided it is sufficient to produce a weighable deposit. It is often necessary, however, to measure the quantity of electricity circulated by a current of such short duration (a *transient* current) that the use of a voltameter is quite impossible; in such cases the measurement may be made by a *ballistic* galvanometer.

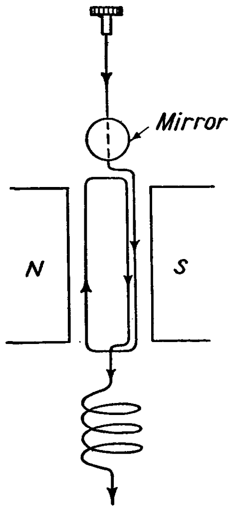


Fig. 14.—Principle of moving coil galvanometer

Any galvanometer can be used ballistically provided certain conditions are fulfilled, but the moving coil type is, in general, the most suitable, and for satisfactory operation certain modifications in the design are desirable.

It is clear that when a transient current passes through the instrument, no steady deflection can be produced, but, *provided the current has ceased before there has been any appreciable rotation of the moving system*, it can be shown that the *amplitude of the first swing* (θ) is proportional to the *quantity* of electricity circulated by the transient current, i.e.

$$Q \propto \theta \quad \text{or} \quad Q = k_g\theta, \qquad \ldots \ldots \quad (6)$$

where k_g is a constant usually expressed in microcoulombs per scale division.

In order to fulfil the condition stated, the moving system must be arranged so as to have a large moment of inertia; and in order that the first swing shall be large, the damping torque is generally made small * by winding the coil on a non-metallic frame. Thus, compared with a " dead beat " instrument, the ballistic galvanometer has a heavy moving system and a long period of swing, and, unless artificially damped, when set swinging takes a long time to come to rest.

* A small damping torque is not essential, and it is often more convenient to use an instrument in which there is considerable damping.

Let
t_d = deflecting torque,
t_c = controlling torque,
i = current in coil,
θ = angle of deflection,
θ_1 = angle of deflection at end of first swing,
T = time during which current flows,
α = angular acceleration of moving system,
ω = angular velocity of moving system,
I = moment of inertia of moving system.

It is assumed that the galvanometer is of the moving coil type, so that (1) the deflecting torque is proportional to the current,

$$t_d \propto i, \text{ or } t_d = k_d i, \quad \ldots \ldots \quad (7)$$

where k_d is a constant; (2) the controlling torque, supplied by the torsion of the suspending strip, is proportional to the angle of deflection,

$$t_c \propto \theta \text{ or } t_c = k_c \theta, \quad \ldots \ldots \quad (8)$$

where k_c is a constant.

The effect of the transient current is to give a sudden twist to the moving system, which, owing to its large inertia, does not move appreciably from its zero position until the current has ceased.

The angular acceleration produced by this twist is

$$\alpha = \frac{t_d}{I} \text{ (§ 16, p. 10),}$$

so that the angular velocity which has been produced when the current ceases is, from (7),

$$\omega = \int_0^T \alpha\, dT = \frac{1}{I}\int_0^T t_d\, dT, = \frac{k_d}{I}\int_0^T i\, dT. \quad \ldots \quad (9)$$

But $\int_0^T i\, dT = Q$, the quantity of electricity circulated by the transient current,

$$\therefore \omega = \frac{k_d}{I} Q,$$

or

$$I\omega = k_d Q. \quad \ldots \ldots \quad (10)$$

Hence the kinetic energy with which the system begins its swing is

$$\tfrac{1}{2} I\omega^2 = \tfrac{1}{2}\frac{I^2\omega^2}{I} = \frac{1}{2I} k_d^2 Q^2, \text{ from (10).}$$

Assuming that there are no friction or damping losses, when the end of the swing is reached all this energy has been used in overcoming the controlling torque, i.e.

$$\frac{1}{2I} k_d^2 Q^2 = \int_0^{\theta_1} t_c\, d\theta,$$

from (8),

$$= k_c \int_0^{\theta_1} \theta\, d\theta = \tfrac{1}{2} k_c \theta_1^2.$$

$$\therefore \frac{1}{2I} k_d^2 Q^2 = \tfrac{1}{2} k_c \theta_1^2,$$

from which

$$Q^2 = I\frac{k_c}{k_d^2}\theta_1^2,$$

or

$$Q = \frac{\sqrt{I \cdot k_c}}{k_d}\,\theta_1 = k_0\theta_1.* \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

In practice, since there is always some damping due to currents induced in the coil itself by its motion through the field, and to air friction, the amplitude of the first swing is never as great as the calculated value, and

$$Q = mk_0\theta_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where m is a factor greater than unity, called the *damping factor*.† More generally, $Q = mk_0'\delta_1$, where δ_1 is the deflection in scale divisions.

Since the damping torque and hence the damping factor depend upon the total resistance of the galvanometer circuit, it is usual to obtain the galvanometer constant k_g $(= mk_0')$ experimentally (§ 8, p. 224), using a circuit of the same resistance as that in which it is to be used.

In electrical engineering, one of the most important uses of the ballistic galvanometer is in connection with the magnetic testing of iron, which is described in § 6, p. 222.

17. Instruments for the Measurement of Insulation Resistance

It was stated in § 6, p. 97, that the measurement of insulation resistance is an operation of such frequent occurrence in connection with electrical machinery and wiring installations that special instruments have been developed for the purpose, such as the megger and the metrohm. The underlying principle of these is shown diagrammatically in fig. 15.

Such instruments are essentially of the moving coil type, but the moving system consists of two coils inclined at an angle and rigidly connected to the spindle. One of these, P, is connected directly to a d.c. supply (which is generally provided by a hand-driven generator) and carries a current proportional to the P.D. The other, C, is connected in series with the resistance to be measured, X, and carries a current which depends upon the magnitude of this resistance. The direction of the current in each coil is such that the torques produced are in opposite directions, and the system takes up a position at which the two torques are equal; so that no control springs are necessary.

If the unknown resistance is very high, so that the current in coil C is negligibly small, the system rotates under the torque exerted by coil P until the pointer takes up the position P_1 marked infinity on

* It can be shown that $k_0 = \frac{T_0}{2\pi}\frac{k_c}{k_d}$, where T_0 is the natural period of oscillation of the moving system.

† The damping factor may be obtained by setting the system swinging and observing the amplitude of successive swings. Then

$$m = \sqrt{\frac{\theta_1}{\theta_2}} = \left(\frac{\theta_1}{\theta_n}\right)^{\frac{1}{2n-2}},$$

where θ_1 and θ_n are the amplitudes of the 1st and nth swings.

the scale. But if the value of X is such that the current in C is appreciable, the position at which the two torques are equal occurs farther down the scale at a point which becomes progressively lower as the resistance of X decreases and the current through C increases. Varia-

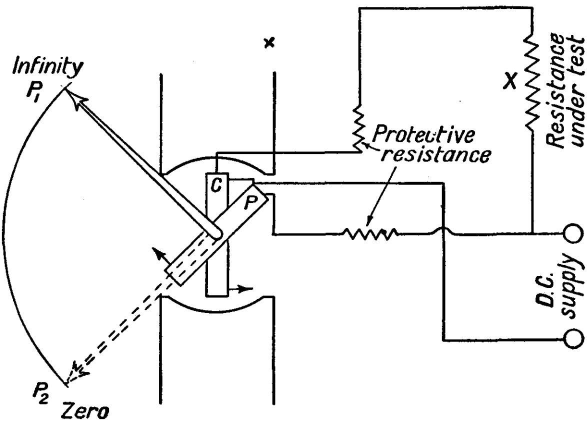


Fig. 15.—Principle of ohm-meter

tions in the supply pressure have no effect on the readings since each coil is affected equally. Each position of the pointer therefore corresponds with a definite value of the unknown resistance, and the scale is calibrated directly in megohms.

EXAMPLES

1. A moving iron ammeter with a range of 10 amperes has an operating coil containing 30 turns. How many turns will there be on the operating coil of an instrument of similar type having a range of (*a*) 20 amperes, (*b*) 1 ampere?

2. The movement of the instrument in question (1) is to be used in a voltmeter having a range of 200 volts. The total resistance of the instrument is 4000 ohms. How many turns must the operating coil contain?

3. A moving iron instrument with a range of 100 volts has a total resistance of 2000 ohms, of which the resistance of the operating coil, which is wound with copper, is 200 ohms at 15° C. The series resistance is wound with wire having a negligible temperature coefficient. At a temperature of 15° C. the instrument reads correctly at 100 volts. What is the error at 25° C., supposing that the deflection at this portion of the scale is proportional to the current? ($\alpha = 0{\cdot}0043$.)

4. A moving coil instrument has a resistance of 5 ohms, and a current of 15 milliamperes causes full deflection. Calculate the resistance of (*a*) a shunt to extend the range to 50 amperes; (*b*) a series resistance to extend the range to 100 volts.

5. What is the multiplying factor of (*a*) a shunt having a resistance of 0·005005 ohm; (*b*) a series resistance having a resistance of 9995 ohms, when used with the instrument of question (4)?

6. An instrument of resistance 15 ohms gives a reading of 25 when a current of 32 milliamperes passes through it. Find the value of the series resistance which will convert it into a direct reading voltmeter.

7. Give a sketch showing the construction of a moving coil voltmeter.

If the moving coil consists of 100 turns wound on a square former which has a length of 3 cm. and the flux density in the air gap is 600 lines per sq. cm. (0·06 weber per sq. m.), calculate the turning moment acting on the coil when it is carrying a current of 12 milliamperes. [Lond. B.Sc.(Eng.).]

8. Distinguish between Potential Difference and E.M.F. in a simple circuit.

A voltmeter has two ranges with the scales marked 0–2 volts, 0–10 volts, the resistance of the instrument on the respective scales being 50 ohms and 250 ohms. When the instrument is connected across the terminals of a battery the reading is 1·8 volts on the lower range and 2·2 volts on the higher range. Calculate (*a*) the internal resistance, (*b*) the E.M.F. of the battery. [Grad. I.E.E.]

9. A galvanometer gives a full-scale deflection when the current in the coil is 0·2 milliampere. Explain how to convert the galvanometer into an ammeter to measure currents up to 2 amperes, the resistance of the coil being 1000 ohms.

CHAPTER XI

The Magnetization of Iron and Steel—Magnetic Circuit Calculations

1. Magnetic Classification of Materials

In § 4, p. 39, materials, in respect of their magnetic properties, were divided into two classes, magnetic and non-magnetic; and for most practical purposes such a distinction is sufficient. Investigations show, however, that most materials are affected to some extent when placed in a strong magnetic field. It is probable that *all* materials would be affected provided the field were sufficiently powerful, and that each could then be placed in one of the following divisions:

(*a*) *Ferromagnetic Materials.*—This group includes what are generally termed the magnetic materials, i.e. iron, steel, nickel, and cobalt, to which must be added certain alloys of copper, aluminium and manganese such as the *Heusler alloys* and the permanent-magnet alloys of iron, aluminium, nickel, and cobalt. Of these, iron and steel are the materials of most importance from an engineering point of view, although nickel in its magnetic properties is not greatly inferior to cast iron. When a piece of a ferromagnetic material is placed in a magnetic field, *opposite* polarity is induced in the end nearest the inducing pole (fig. 1*a*), with the result that it sets itself with its longer axis *in line with* the field. The permeability is greater than that of free space but is not constant and depends, among other things, on the strength of the magnetizing force; and the phenomenon of *hysteresis* is exhibited (§ 3, p. 215).

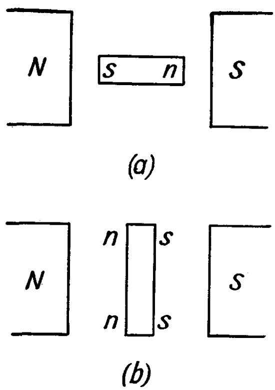


Fig. 1.—To illustrate (*a*) paramagnetism and (*b*) diamagnetism

(*b*) *Paramagnetic Materials.*—In these materials, of which manganese, platinum and oxygen are examples, the induced polarity is the same as in ferromagnetic materials, but the effects are much more feeble. The value of the permeability seems to be independent of that of the magnetizing force, and no hysteresis is observed.

(*c*) *Diamagnetic Materials.*—This group includes all the other so-called non-magnetic materials. The effects, when they can be detected, are in most cases very feeble, even in the most powerful fields available.

In these materials, however, the polarity induced is opposite to that in a ferromagnetic or paramagnetic material, i.e. the *same* polarity in the portion nearest the inducing pole (fig. 1*b*), with the result that the piece sets itself with its longer axis *perpendicular* to the direction of the field. The value of the permeability, which is less than that of free space, is independent of that of the magnetizing force, and no hysteresis effects are exhibited.

Diamagnetic effects, however, are exceedingly small, even in the case of bismuth, by which they are exhibited most strongly; and for all practical purposes the assumption that the permeability of all non-magnetic materials is constant, and equal to that of free space, is sufficiently correct.

In § 9, p. 43, the phenomenon of magnetic induction was explained in terms of Ampère's intramolecular current rings, which, in the light of modern knowledge, are provided by the motion of the electrons in the atom. In addition to their orbital rotation round the nucleus, electrons are assumed to have a spinning motion about their own axes, which is thought to be the chief source of magnetic effects.

In an isolated atom the axes of spin are all parallel, but the spin may be in either direction. If in any given orbit the number of electrons spinning in the two directions is unequal, a resultant magnetic effect is produced. In iron there are 26 electrons moving in four orbits. In three of these the number spinning in each direction is the same, so that there is no resultant magnetic effect; but in the fourth the numbers are different, and it is this inequality which gives iron (and to a lesser extent, nickel and cobalt) its magnetic properties.

Conditions are somewhat modified in a mass of magnetic material. Metals are crystalline in structure, and each crystal contains a certain number of atoms arranged in a definite pattern in what is known as the *crystal lattice*. The spinning electrons in neighbouring atoms in the crystal react on each other, and this reaction may either increase or decrease the resultant magnetic moment to an extent which depends on the relative size and spacing of the atoms. In ferromagnetic materials these dimensions are such that the reaction effects greatly increase the resultant magnetic moment. In paramagnetic materials the dimensions have values such that the reaction effect is very much less.

As stated on p. 44 modern magnetic theory assumes that each crystal is divided into a number of regions, called *domains*, in each of which the magnetic axes of all the atoms are parallel, although the direction is different in different domains. Hence the elementary magnetic entity is the domain rather than the atom.

2. The Magnetization Curve, or B-H Curve, of a Magnetic Material

As stated in the previous section, the permeability of a ferromagnetic material is not constant but depends upon various factors; hence B is not a simple function of H and the relation between them can be determined only by experiment.

In electrical engineering, magnetic problems generally consist in obtaining the value of the magnetizing force H to produce a given flux density B; and since the determination of μ involves the measurement of corresponding values of B and H, a curve relating these two quantities directly is of much greater use than one relating either of them with μ. Hence the magnetic characteristics of iron and steel are usually expressed in the form of *B-H curves*, often called *magnetization curves* and sometimes *saturation curves*.

The magnetization curve of cast steel is shown in fig. 2, and the shape is typical of all such curves; further examples are shown in

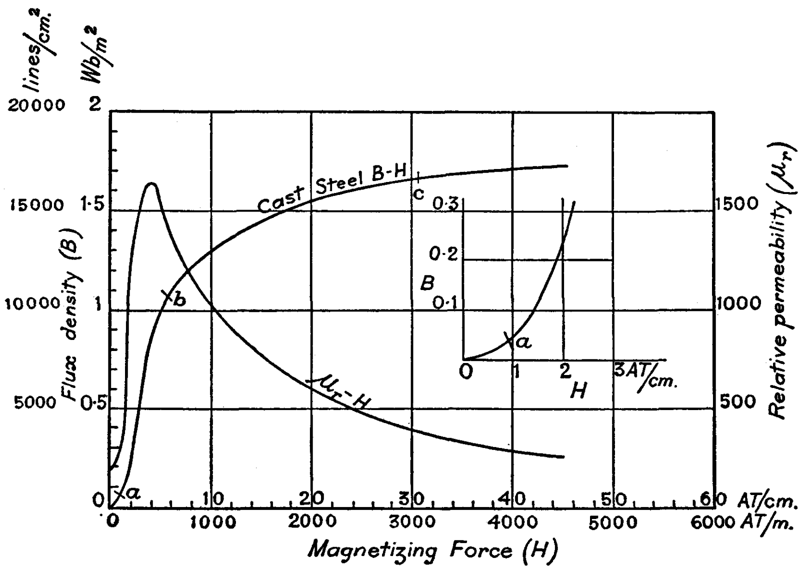


Fig. 2.—Magnetization curve for cast steel

fig. 9 (p. 220). The curve is obtained by subjecting a specimen of the material, placed inside a magnetizing coil, to a gradually increasing magnetizing force and measuring the flux density produced as described in § 6, p. 222.

For any pair of corresponding values of B and H,

$$\text{Permeability } (\mu) = \frac{B}{H}$$

and

$$\text{Relative permeability } (\mu_r) = \frac{\mu}{\mu_0} = \frac{1}{\mu_0} \cdot \frac{B}{H}.$$

The second curve shows the relation between μ_r and H. A μ_r-B curve can be obtained in a similar way.

The B-H curve shows that the process of magnetization can be divided roughly into three stages:

(*a*) At very low values of H, μ is small and increases slowly at first, so that the initial slope of the B-H curve (Oa, fig. 2) is small, and B is almost proportional to H. This can be seen clearly in the inset in fig. 2, in which the initial portions of the curves are plotted to a larger scale.

(*b*) As H continues to increase, the value of μ grows very rapidly, so that a small change in H produces a large change in B and the slope of the B-H curve becomes very steep (ab, fig. 2).

(*c*) μ_r quickly reaches a maximum value (about 1650 in this case) and begins to decrease slowly, so that the slope of the B-H curve decreases and the iron is said to be reaching *saturation* (bc, fig. 2). If H continues to increase, μ_r continues to decrease and would ultimately reach a value of unity. At this stage the iron is fully saturated, the intensity of magnetization can be increased no further, and the slope of the curve becomes that of the B-H curve of a non-magnetic material.

Except for experimental purposes, the magnetization of iron seldom approaches the saturation value, as the attendant power losses both in the magnetizing coils and in the iron itself would be uneconomically large. The flux densities used in the magnetic circuits of machines are generally such that the working point lies somewhere on the portion of the curve (fig. 2) between b and c.

The B-H curve for air and other non-magnetic materials is, of course, a straight line through the origin, and if drawn to the scales used for ferromagnetic materials, the slope would be so small that it could hardly be distinguished from the H axis. Since B is proportional to H such a curve is unnecessary,

$$B \text{ (in Wb. per sq. m.)} = 4\pi \times 10^{-7}\, H \text{ (in A.T. per m.)}.$$

It is sometimes necessary to convert this relation into centimetres or inch units.

Since $\quad 1 \text{ weber per sq. m.} = 10^4 \text{ lines per sq. cm.}$

and $\quad 1 \text{ ampere-turn per m.} = 10^{-2} \text{ ampere-turns per cm.},$

$$\begin{aligned} B &\text{ (in lines per sq. cm.)} \\ &= 10^4 \times (4\pi \,.\, 10^{-7}) \times 10^2\, H \text{ (in ampere-turns per cm.)}, \\ &= 1{\cdot}26\, H \text{ (in ampere-turns per cm.)}; \end{aligned}$$

and since $\quad B \text{ (in lines per sq. in.)} = 2{\cdot}54^2\, B \text{ (in lines per sq. cm.)}$

and $\quad H \text{ (ampere-turns per in.)} = 2{\cdot}54 \times 10^{-2}\, H \text{ (ampere-turns per m.)},$

$$\begin{aligned} B \text{ (in lines per sq. in.)} &= 2{\cdot}54^2 \times 10^4 \times 4\pi \times 10^{-7} \times \frac{10^2}{2{\cdot}54}\, H \text{ (in ampere-turns per in.)} \\ &= 3{\cdot}2\, H \text{ (in ampere-turns per in.)}. \end{aligned}$$

The molecular theory of magnetism previously discussed in § 9, p. 41, offers a satisfactory explanation of the shape of the *B-H* curve and of the related phenomenon of *hysteresis*. A reference to that section will remind the student that when a piece of unmagnetized iron or steel is subjected to a gradually increasing magnetic force, each molecular magnet * experiences a torque tending to turn its axis into line with the field, this torque being resisted by the forces between the molecules forming a group such as is shown in fig. 3*a*. The three stages in the process of magnetization described in the previous section can now be explained as follows:

(*a*) In the initial stage, when the magnetizing force is very small, only a slight deflection of the molecules takes place (fig. 3*b*). This is sufficient to disturb the internal equilibrium of the groups so that poles are produced at the ends; but is not sufficient to break up the grouping so that the rate of increase of magnetization and therefore of flux density is small and almost proportional to the strength of the field. Further, when the magnetization force is removed, the molecules resume their original positions and the magnetism disappears. This stage corresponds with the portion *Oa* of the curve (fig. 2).

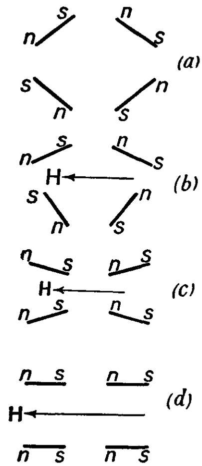


Fig. 3.—To illustrate molecular theory of magnetism

(*b*) If the magnetizing force continues to increase, the deflection of the molecular magnets also increases until a stage is reached when the internal resisting forces are insufficient to preserve the original grouping, which breaks up as shown in fig. 3*c*. The molecules then combine into fresh groups, in which they are held in equilibrium by their mutual forces and the forces due to the external field, and in which the general direction of the magnetic axes more closely approaches that of the field. The break-up and the reformation of the grouping produces a rapid increase in the intensity of magnetization represented by the steep portion of the curve *ab*, fig. 2.

(*c*) After the reformation of the groups, further increase in the magnetizing force serves only to turn the molecules still more nearly into line with the field (*bc*, fig. 2); until finally, when the " lining-up " process is complete (fig. 3*d*), the iron is entirely saturated and the intensity of magnetization can increase no further.

In the actual process these three stages are not sharply defined, and the smoothness of the *B-H* curve shows that in the earlier portions, stages (*a*) and (*b*), and in the later portions, stages (*b*) and (*c*), are occurring simultaneously.

* Although in modern theory the magnetic entity is the *domain*, the term molecular magnet has been retained as being a simpler concept, sufficient for present purposes.

3. Hysteresis

If, after a state of saturation is approached, the magnetizing force is gradually reduced, the deflection of the molecular magnets and therefore the intensity of magnetization decrease. As the deflecting torques decrease still further, some of the groups begin to break up under the influence of internal forces; but this does not occur until the magnetizing force has decreased to a value considerably less than that at which their formation took place during the process of magnetization. Hence *the change in the magnetic state of the iron lags behind the change in the magnetizing force*, so that the descending portion of the curve lies above the ascending portion (fig. 4); this is known as the *hysteresis* effect.

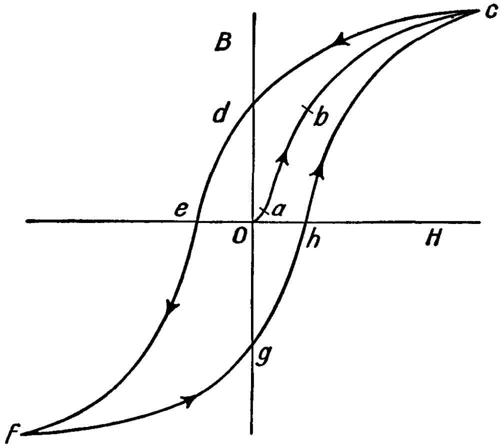


Fig. 4.—Hysteresis loop

Even when the magnetizing field is reduced to zero, not all the molecular magnets have resumed their original grouping, so that the iron retains some of its magnetism. This is illustrated by the fact that the descending curve cuts the B-axis at some point d, the intercept Od representing what is called the *residual flux density* (residual magnetism).* In order to reduce the flux density to zero, it is necessary to apply a magnetizing force in the *reverse* direction, which completes the rupture of the remaining molecular groups. This value of the reversed force Oe is known as the *coercive field*, and is a measure of the " tenacity " with which the iron retains its magnetism; both residual flux density and coercive field are important quantities in connection with the design of permanent magnets.

* The value of the residual flux density after the material has been magnetized to saturation is called the *remanence*.

When this reversed magnetizing force is further increased, the iron becomes magnetized in the opposite direction and the curve *ef* is traced out. If on reaching a value equal and opposite to the previous maximum the reversed field is reduced to zero, the iron retains a residual flux density *Og*, in the opposite direction to *Od*, to destroy which a coercive field *Oh* (in the original or positive direction) is required. Finally, if the magnetizing force is increased once more to the original positive maximum, the curve *hc* completes what is known as the *hysteresis loop* of the material.

The iron has now been taken through a complete magnetic cycle and the same loop is traversed once for each successive cycle of magnetization, provided that the maximum values of *B* and *H* are the same; the original *B-H* curve *Oc* is not retraced, and can be obtained only by starting with the iron in a completely demagnetized state. If the maximum value of *H* is decreased, the loop traced is similar in shape but smaller in area, as shown in fig. 5; conversely, the area is increased by increasing the maximum value of *H*.

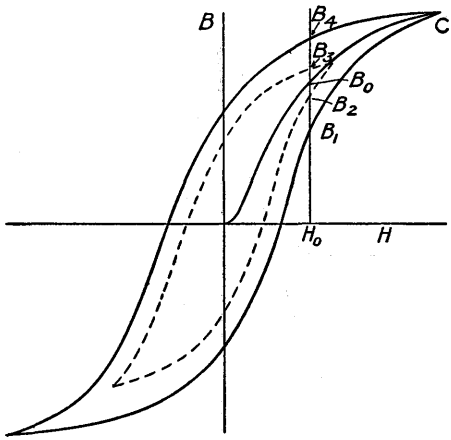


Fig. 5.—Hysteresis loops for different maximum values of *B* and *H*

Energy is stored potentially during the deflection of the molecular magnets, and some of it is converted into heat by the rapid vibrations which are set up on the breaking up of a group. Hence when iron is taken through a magnetic cycle, energy is absorbed and converted into heat. This is known as the *hysteresis loss*; it occurs in all the iron portions of electrical machinery which pass through recurrent cycles of magnetization, and considerably influences the design of and choice of materials for these parts.

It has been stated already that in ferromagnetic materials the curve showing the relation of *B* to *H* cannot be defined mathematically;

actually, except under specified conditions, the relationship between the two quantities is entirely indefinite. In fig. 5 three such sets of conditions are represented by the original *B-H* curve and the two hysteresis loops. By considering these, it can be seen that for a given value of the magnetizing field H_0, the flux density may have *five* different values, one, B_0, on the true *B-H* curve when the iron has been previously demagnetized, and the others, B_1, B_2, B_3, B_4, depending upon whether the magnetizing field is increasing or decreasing and upon the maximum value reached during the cycle.

Hence in any ferromagnetic material the value of B produced by a given value of H depends upon the magnetic treatment to which the material has been subjected beforehand; in other words, upon its previous *magnetic history.*

4. The Hysteresis Loss

In § 29, p. 181, it was shown that the energy absorbed per cu. cm., during the magnetization of a material, is proportional to the area under the *B-H* curve. In the case of air and other non-magnetic materials, there is no hysteresis effect and *all* the energy absorbed is restored when the magnetizing force is removed: since the *B-H* curve is a straight line, the area can be calculated, and it has been shown that the energy stored in establishing a flux density B is $\frac{1}{2}B^2/\mu_0$ joules per cu. m. (equation (33), p. 183).

For ferromagnetic materials, however, the area must be *measured*; and only a *portion* of the energy is returned on the removal of the magnetizing force. If a piece of unmagnetized iron or steel is subjected to a magnetizing force H_m (fig. 6) which establishes a flux density B_m, the energy absorbed by each cu. m. is represented by the area *Oab* under the *B-H* curve. When H is reduced to zero, the energy restored to the circuit is represented by the smaller area *abc*, under the descending curve; the remainder, proportional to the area *Oac*, is converted into heat by the vibration of the molecular magnets, as explained in the previous section.

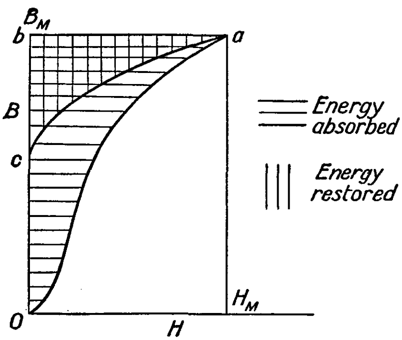


Fig. 6.—Energy stored in a magnetic material

Applying this principle to the complete loop in fig. 7, and starting from the instant when the magnetizing force is zero and about to grow positively, the energy absorbed in destroying the residual flux density *Og* and establishing a flux density B_m in the positive direction is represented by the area *ghab*, and that restored as the force is removed by the area *abc*. Similarly, during the other half of the cycle, the energy absorbed in destroying the residual flux density *Oc*, and in establishing the flux density $-B_m$, is proportional to the area *cdef*, and that restored when the field is reduced to zero, by the area *efg*.

The difference between the quantity of energy absorbed and that restored is the quantity of energy which is turned into heat and constitutes the *hysteresis loss*. It is obviously proportional to the area (*ghab* — *abc*) + (*cdef* — *efg*), i.e. to the *area of the loop*. Hence from § 29 p. 184, it follows that:

Energy loss per cycle in joules per cu. m.

= area of the loop expressed in terms of B and H.

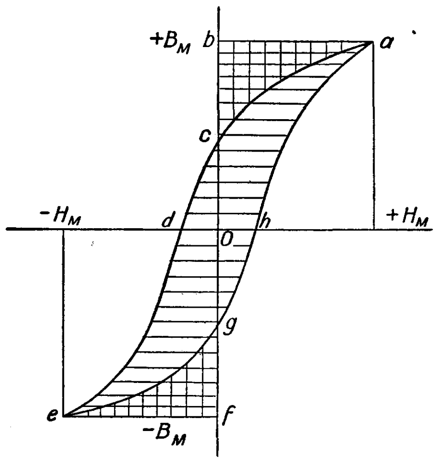


Fig. 7.—The hysteresis loss

For example, if the scales employed in drawing the loop are such that:

On the horizontal axis, 1 cm. = p ampere-turns per metre,

On the vertical axis, 1 cm. = q webers per square metre,

then,

Energy loss per cycle in joules per cu. m.

$$= (\text{area of loop in sq. cm.}) \times p \times q. \quad . \quad . \quad . \quad (1)$$

It is clearly more convenient for purposes of calculation if the hysteresis loss can be represented (if only approximately) by a mathematical expression. The area of the loop depends upon the nature of the material and upon the maximum value of the flux density reached during the cycle (fig. 5). The investigations of Steinmetz have shown that for a given material, and over a considerable range of densities, the area is nearly proportional to the n^{th} power of the maximum flux density, so that

$$\textbf{Energy loss in joules per cu. m. per cycle} = \eta B_m^n, \quad . \quad (2)$$

where, for normal densities n has a value which lies between 1·5 and 1·8, but approaches 2 as the iron becomes saturated.

If the flux is alternating at a frequency of f cycles per second,

$$\textbf{Hysteresis loss in watts per cu. m.} = \eta B_m^{1\cdot 6} f, \quad . \quad (2a)$$

where η is a constant depending on the material, which, when B is measured in webers per sq. m. has values ranging from 250 for silicon steel to 12,500 for a magnet steel.

It will often be more convenient to express the hysteresis loss in watts per cu. cm., in which case the above values will become 0·00025 and 0·0125 respectively.

A piece of iron may be taken through a magnetic cycle by being placed in an alternating field, one cycle of which has just been considered, or it may be rotated in a constant field, when it passes through one cycle per revolution (fig. 8); but the loss per cycle is not the same in the two cases, even when the material and the maximum flux density are the same. Either of these conditions, or a combination of both, may occur in various parts of electrical machines, so that the expression above gives only approximate results.

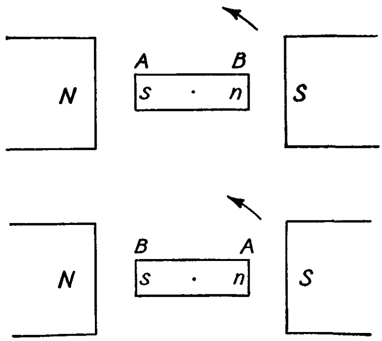


Fig. 8.—Rotational hysteresis

5. Properties of Ferromagnetic Materials

The magnetic circuit (§ 12, p. 231) of electrical machines and apparatus consists almost entirely of iron and steel; and the hysteresis loop and B-H curve serve as the criteria of the suitability of the material for use under given conditions.

Typical B-H curves are shown in figs. 2 and 9. In these curves the magnetizing force is expressed in *ampere-turns per cm.*; the ampere-turn per cm. or per in. is usually more convenient than the M.K.S. unit of H. Comparative hysteresis loops, the maximum flux density being the same in each case, are given in fig. 10.

The relative permeability of pure iron is high, reaching a value of about 4000 webers at a flux density of about 0·7 weber per sq. m. This decreases as the proportion of carbon increases, but the low carbon steels are not greatly inferior magnetically to pure iron. The *B-H* curve for mild steel is shown in fig. 9, and that for cast steel (fig. 2) is generally similar.

Cast iron is, magnetically, a poor material, as is shown by the *B-H* curve. In electrical machine construction it has been replaced to a large extent by steel; and its use is restricted to portions which either do not form part of the magnetic circuit, or in which the cross-section necessary for mechanical strength is ample magnetically.

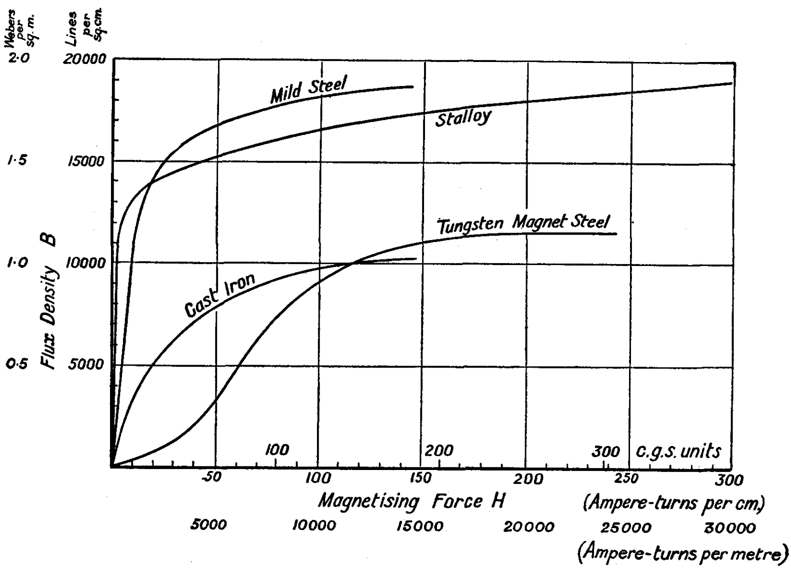


Fig. 9.—Typical magnetization curves

Mild steel has a high permeability (fig. 9) which, together with its general mechanical properties, leads to its extensive use for parts of machines in which the flux is constant. The area of the loop (fig. 10*b*) shows that large hysteresis losses would occur if subjected to alternating magnetization.

The addition of a small percentage of tungsten or cobalt greatly reduces the permeability (fig. 9) and increases the area of the loop (fig. 10*a*), but at the same time increases the value of the residual flux density and the coercive field; thus these materials, although quite unsuitable for use in electrical machines, have been widely used for permanent magnets. They have now been superseded, in many cases, by new materials.

The development, in 1930, of iron alloys containing aluminium and nickel, and aluminium, nickel, copper, and cobalt (known by such trade names as *Alni*, *Alnico*, *Alcomax*) which have exceedingly high values of coercive force (about eight times that of tungsten steel) has revolutionized the design of permanent magnets by enabling great reductions to be made in size and weight.

The addition of a small quantity of silicon (1–5 per cent) to the iron increases the permeability at low densities and somewhat decreases it at high densities; a more important effect, however, is that it produces a decrease in the hysteresis loss, and an increase in the electrical resistance of the iron, which reduces eddy-current losses (§ 10, p. 151). Such alloyed irons, of which *stalloy* is a well-known example, are used for all parts of electrical machines, such as armatures and transformer cores (see Chapter XIII), which are continuously subject to cycles of

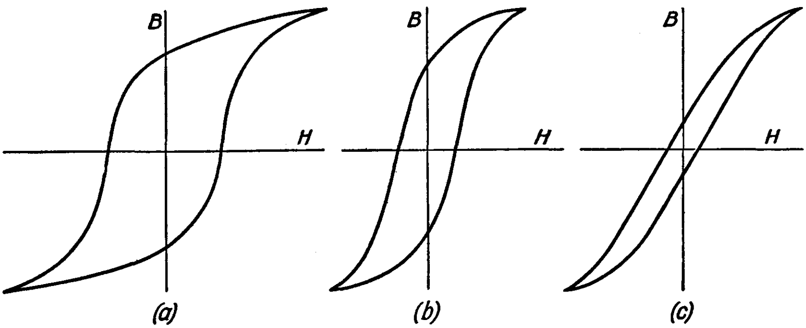


Fig. 10.—Typical hysteresis loops

magnetization. A typical loop is shown in fig. 10*c*, the narrowness of which indicates the small hysteresis loss; while the *B-H* curve is shown in fig. 9.

If nickel is added to the iron, the effect is first of all to decrease its magnetic properties until, with about 30 per cent of nickel, the resulting material is almost non-magnetic and also possesses a higher electrical resistance. Such materials, of which *no-mag* is an example, are often used for structural parts of electrical machines which are exposed to the influence of fluctuating or alternating magnetic fields and in which, if they were made of cast iron or steel, the hysteresis and eddy-current losses would be excessive.

The addition of further quantities of nickel causes a rapid increase in the magnetic properties and gives materials such as *permalloy* and *mumetal*. The latter has a low hysteresis loss and a very high permeability at low flux densities (to which it owes its name). The maximum value of the relative permeability reaches as high as 30,000 but occurs at a flux density of about 0·25 weber per sq. m. (as compared with about 4500 at 0·5 weber per sq. m. in the case of stalloy). This disadvantage, and the high cost, restricts its use except in special cases, as in the cores of current transformers, where it is essential that the magnetizing ampere-turns shall be as small as possible.

The addition of manganese to the steel renders it intensely hard, and so greatly reduces the permeability that it can be made almost non-magnetic.

The permeability of all ferromagnetic materials decreases with increase in temperature, and iron when heated to a high temperature ceases to be magnetic. If allowed to cool, it suddenly regains its magnetic properties at a temperature of about 750° C., known as the critical temperature, at which the occurrence of some profound molecular change is also indicated by a sudden rise in temperature called *recalescence*.

6. The Magnetic Testing of Materials

A knowledge of the magnetic characteristics of materials is essential to the designer of electrical apparatus; and since these characteristics can be obtained only by experiment, magnetic testing is an important operation in the manufacture both of materials and apparatus.

Such tests consist, essentially, in placing a sample of the material, of suitable shape and known dimensions, inside a magnetizing coil, and of measuring the flux density (B) produced by various values of the magnetizing force (H). The specimen is preferably in the form of a ring so that the effect of the poles is eliminated, and the flux density is determined by measuring, with a ballistic galvanometer or fluxmeter (§ 11, p. 231), the *change in flux* produced by a given change in the magnetizing force.

The specimen is wound with two coils, a magnetizing coil and a secondary or search coil which is connected to a ballistic galvanometer. Any change in the flux set up in the specimen, which is all linked with the search coil, induces in it an E.M.F., which produces a current in the galvanometer circuit and, consequently, a deflection. In the next section it is shown that the initial swing of the galvanometer, which under certain conditions is proportional to the quantity of electricity passed through it (§ 16, p. 205), is, under the same conditions, proportional also to the *total change in flux* linked with the search coil.

From measurements of the change in flux produced by a known change in the magnetizing force, either the B-H curve or the hysteresis loop may be drawn.

7. Measurement of Flux Change by Ballistic Galvanometer

Suppose that a coil of n turns is connected to a ballistic galvanometer, the total resistance of the circuit being r, and that the flux linked with the coil changes by an amount $\Delta\Phi$ in a time t.

The change of flux induces an E.M.F. in each turn of the coil, and the *average* value of the resultant E.M.F. is

$$E = n\frac{\Delta\Phi}{t} \quad \text{(equation (1), p. 146).}$$

The *average* value of the current produced by the E.M.F. is

$$I = \frac{E}{r} = n\frac{\Delta\Phi}{rt},$$

and the quantity of electricity circulated by this current is

$$Q = It = \frac{n\,\Delta\Phi\,t}{rt} = \frac{n\,\Delta\Phi}{r}, \quad \ldots\ldots \quad (3)$$

i.e. the quantity of electricity circulated is proportional to the *change* in flux, but *independent* of the time during which the change takes place. If, however, the time is so short that the flux change, and therefore the current, has ceased before appreciable movement of the galvanometer has taken place, then, by equation (6), p. 205,

$$Q = k_g\theta,$$

where θ is the amplitude of the first swing, and k_g is the galvanometer constant. Thus, from (3),

$$\frac{n\Delta\Phi}{r} = Q = k_g\theta,$$

$$\therefore\ \Delta\Phi = \frac{r}{n}k_g\theta. \quad \ldots\ldots\ldots \quad (4)$$

The change in the flux linked with a circuit connected to a ballistic galvanometer is proportional to the amplitude of the first swing, provided that the change is complete before the system has moved appreciably from the zero position.

Example.—A ballistic galvanometer is connected to a coil of 40 turns. The total resistance of the circuit is 900 ohms, and the galvanometer constant corresponding with this resistance is 0·26 micro-coulomb per division. Calculate the deflection produced by a change of 100 micro-webers (10,000 C.G.S. lines) in the flux linked with the coil.

If the change of flux takes place in a time t sec.,

$$\text{Average E.M.F. } (E) \text{ induced in coil} = \frac{100 \times 10^{-6} \times 40}{t} = \frac{4 \times 10^{-3}}{t} \text{ volts.}$$

$$\text{Average current } (I) \text{ during the change} = \frac{E}{r} = \frac{4 \times 10^{-3}}{900t} = \frac{40 \times 10^{-6}}{9t} \text{ amperes.}$$

Quantity of electricity circulated $= It = \dfrac{40 \times 10^{-6}}{9}$ coulombs

$= 4{\cdot}44$ micro-coulombs.

Deflection of galvanometer $= \dfrac{4{\cdot}45}{0{\cdot}26} = 17{\cdot}1$ divisions.

It is often more convenient to write equation (4) in the form,

$$n \, . \, \varDelta\Phi = k_g'\theta,$$

where k_g' is the galvanometer constant expressed directly in terms of *flux linkage* per division.

The value of k_g' depends upon the sensitivity of the galvanometer, the number of turns in the search coil, and the resistance of the galvanometer circuit. It is determined experimentally as described in the next section.

8. Calibration of Ballistic Galvanometer for Flux Measurements

The galvanometer is calibrated by measuring the deflection caused by a known change of flux produced by either a standard magnet or a long solenoid.

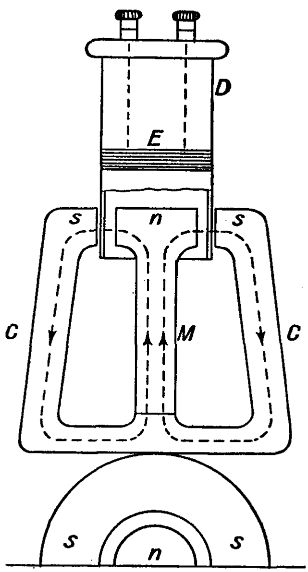


Fig. 11.—Hibbert standard magnet

As shown in § 16, p. 205, the amplitude of the first swing of the galvanometer falls short of its theoretical undamped value, owing to the damping torque which is always present to a greater or less extent. The amount of damping depends considerably upon the resistance of the galvanometer circuit; the quantity of electricity circulated by a given change of flux depends also upon this resistance (equation (3), p. 223). For both these reasons it is necessary that the value of the galvanometer constant should be that corresponding with a resistance equal to that of the galvanometer circuit during the actual magnetic test; so that the circuit should contain the search coils of the specimen and also of the standard magnet (or long solenoid) during both the calibration and the magnetic test.

Calibration by Standard Magnet.

A standard magnet of the Hibbert type is shown in section and plan in fig. 11. It consists of a cylindrical bar magnet M bolted to a hollow pot-shaped cast-iron yoke C. The upper end of the bar magnet forms one pole, while the upper edge of the yoke forms the other, the two being separated by a narrow annular air-gap.

A thin brass cylinder D, in the middle of which is wound a narrow search coil E, slides through the gap but is normally held in the position shown by means of a trigger. When released it falls, so that the coil cuts the whole of the flux crossing the gap in a time which, since the motion is due to gravity, is constant. The flux crossing the gap, which by suitable treatment of the magnet during manufacture may be made a very constant quantity, is carefully measured and stamped on the yoke.

If the value of this flux is Φ, and the search coil has n_s turns, the total change in flux linkage is $n_s\Phi$; and, if this produces a galvanometer swing of δ divisions, the constant for the particular value of circuit resistance is

$$k_g' = \frac{n_s\Phi}{\delta} \text{ weber-turns per division.} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

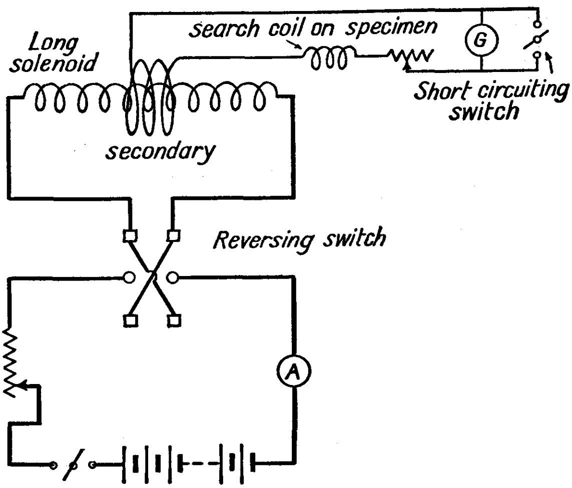


Fig. 12.—Calibration of ballistic galvanometer by long solenoid

Calibration by Long Solenoid.

The magnetizing force in an infinitely long solenoid is uniform across the section and given by

$$H = \frac{NI}{l} \text{ ampere-turns per metre (equation (13), p. 163),}$$

and this expression is very nearly true *at the centre of* any solenoid in which the length is great compared with the diameter, i.e. length not less than ten times diameter.

For calibration purposes, such a solenoid is uniformly wound in either one or two layers on a wooden or other non-magnetic core, a search coil being wound under the magnetizing winding, or placed inside the solenoid at the middle of its length. The number of turns on both coils, the length of the magnetizing coil, and diameter of the search coil must be known accurately; and the axial length of the latter must be small, so that it can be assumed that the field inside the solenoid is constant over this short distance.

The apparatus is connected as shown in fig. 12. The search coil is connected in series with the search coil on the specimen, and with a variable resistance, to the ballistic galvanometer; while the solenoid is connected in series with a reversing switch, ammeter and variable resistance to a suitable d.c. supply.

Let the

length of the solenoid (m.)	$= l_s$,
number of turns on the solenoid	$= N_s$,
number of turns on the search coil	$= n_s$,
area of cross-section of the search coil (sq. m.)	$= a_s$.

When a current I is flowing in the solenoid, the magnetizing force at the centre is

$$H = \frac{N_s I}{l_s} \text{ ampere-turns per metre,}$$

and the flux linked with the search coil is

$$\Phi = Ba_s = \mu_0 \frac{N_s I}{l_s} a_s \text{ webers.}$$

Since there is no ferromagnetic material present, there is no hysteresis, and on switching off the magnetizing current the whole of the flux disappears, and the change in flux is Φ. In order to obtain a larger change of flux for the same magnetizing current, the latter is usually reversed, so that the original flux disappears and an equal flux is set up in the opposite direction; this gives a *total change* of 2Φ.

In this case, if I is reversed,

$$\text{Total change in flux linkage} = 2n_s\Phi,$$

and if this produces a galvanometer swing of amplitude δ divisions,

$$\begin{aligned} k_g' &= \frac{2n_s\Phi}{\delta} \\ &= \frac{2n_s}{\delta} \mu_0 \frac{N_s I}{l_s} a_s \text{ weber-turns per division.} \qquad (6) \end{aligned}$$

In carrying out the calibration several readings * are taken of the swing produced on reversing various values of magnetizing current. Since μ is constant, the flux produced, and therefore the deflection, is proportional to the current, i.e. the curve relating δ and I is a straight line through the origin. The values of δ and I are plotted, the best straight line (which must pass through the origin) is drawn through them, and k_g' is calculated from the corresponding values of δ and I at some point on this line.

9. Determination of *B-H* curve of Iron Specimen (Method of Reversals)

In a ring solenoid the lines of flux form circles inside the ring and concentric with it (§ 18, p. 165), the magnetizing force at every point on the circle being constant. If the specimen is in the form of a ring, no external poles are produced, so that no correction for demagnetizing

* The galvanometer is usually provided with a short-circuiting switch, which is closed between readings; this increases the damping and brings the system quickly to rest, and also prevents disturbance of the galvanometer during adjustments to the magnetizing current.

effects is necessary. Further, if the magnetizing winding is distributed uniformly and the radial thickness of the section is small compared with the mean radius of the ring, it can be assumed that the magnetizing force and the permeability have constant values over the whole cross-section equal to their values at the mean radius.

On account of these advantages, the ring form of specimen is in general use in laboratory work, and also in industrial testing where great accuracy is required; it may be either solid or, in the case of sheet material, made up of ring-shaped laminations. On the other hand, such specimens are not easy to produce and must be wound individually by hand; and for industrial purposes the apparatus is often modified so that the specimen may be in the form of a straight rod which is slipped inside a wound magnetizing coil, the magnetic circuit being completed by a massive iron yoke of negligible reluctance (fig. 12*a*). For accurate results, however, suitable allowances must be made for the reluctance of the joints between the specimen and the yoke. In what follows, however, a ring specimen is assumed.

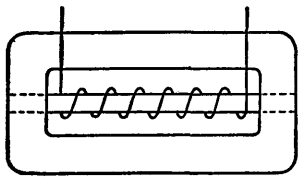


Fig. 12*a*

The search coil is placed as close to the iron as possible, but it need not be distributed uniformly over the whole ring. Above this, and well insulated from it, the magnetizing coil is wound in several layers as uniformly as possible.

The flux density produced by a given magnetizing force is obtained from the deflection of the galvanometer produced by the reversal of the magnetizing current. In ferromagnetic materials the relation between B and H is definite only if the previous magnetic history is known (§ 3, p. 217). A repeated alternation of H between equal positive and negative values, produced by reversing the magnetizing current, provides such a history. After a number of reversals the specimen attains what is known as the " cyclic state ", in which the same hysteresis loop is always traversed, and the total flux change produced by a reversal of H between the same limits is always the same. The " cyclic state " must be produced, by repeated reversals, every time the value of H is changed, i.e. between each set of readings.

The apparatus is arranged as shown in fig. 13, which is generally similar to fig. 12, except that the long solenoid is replaced by the specimen, the magnetizing coil of which is connected through the ammeter, reversing switch and variable resistance of the d.c. supply. The procedure is, briefly, as follows:

(1) The variable resistance in the galvanometer circuit is adjusted until the deflection produced by the reversal of the maximum value of the magnetizing current to be used is just within the limits of the scale.

(2) The galvanometer is then calibrated for this circuit resistance either by a standard magnet or by the long solenoid; in the latter case the d.c. supply is temporarily transferred to the solenoid.

(3) The specimen must now be demagnetized by gradually reducing the magnetizing current and at the same time throwing the reversing switch backwards and forwards so that the iron is carried through a number of cycles diminishing in value, until the magnetizing force is reduced either to zero or to a value well below the lowest point at which a reading is to be made. During this process the galvanometer is short-circuited.

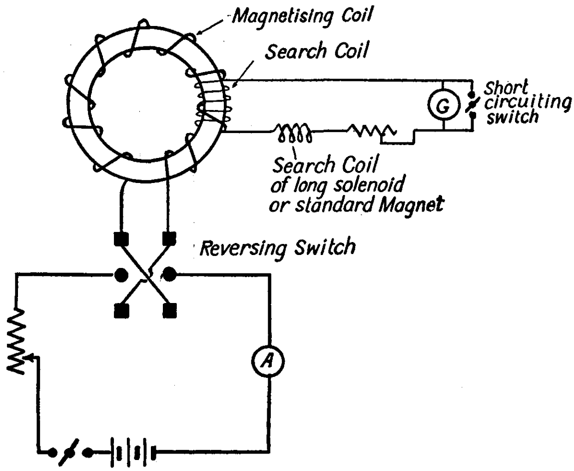


Fig. 13.—Magnetic testing—ballistic method

(4) The magnetizing current is now set at the lowest value required and reversed an even number of times (say 10).* The short-circuit is removed from the galvanometer, the reversing switch thrown over, and the deflection noted.

(5) The galvanometer is short-circuited and the reversing switch thrown back. The current is now adjusted to the next value, and (4) is repeated.

(6) Steps (4) and (5) are repeated for each value of the magnetizing current until the maximum is reached.

By reversing an *even* number of times and always throwing the switch back at the end of a reading as in (5), the deflection is always in the same direction. The readings of current and deflection should be plotted as the test proceeds, so that the current steps may be varied to suit the curvature of the curve.

* The cyclic condition of the specimen can be checked by observing the deflection produced by a second reversal of the same current in the *same direction*; if the value is not the same, the reversals must be continued until successive deflections are equal.

Let

mean length of path in specimen (m.)	$= l_I$,
($= \pi \times$ mean diameter)	
net iron cross-section (sq. m.)	$= a_I$,
number of turns on magnetizing coil	$= N_I$,
number of turns on search coil	$= n_I$,
galvanometer constant (flux linkages per division)	$= k_g'$.

If a current I in the magnetizing coils sets up a flux Φ in the specimen, and a reversal of I produces a deflection δ,

$$\text{Total change of flux linkage} = k_g'\delta = 2n_I\Phi.$$

$$\therefore \Phi = \frac{k_g'\delta}{2n_I} \text{ webers, } \qquad (7)$$

and

$$B = \frac{\Phi}{a_I} = \frac{k_g'}{2n_I a_I}\delta \text{ webers per sq. m. } \qquad (8)$$

The corresponding value of H is

$$H = \frac{N_I I}{l_I} \text{ ampere-turns per metre.}$$

The values of B and H for each set of readings are calculated in the same way, and from these the B-H curve of the material is drawn; in this method the curve is obtained as the locus of the peaks of a number of hysteresis loops corresponding with the various values of H.

Example.—In the determination of the B-H curve of a sample of iron by the method of reversals, the following data were obtained:

Length of standard solenoid	50 cm. (0·5 m.)
Number of turns on solenoid	350.
Cross-section of secondary coil	12 sq. cm. (12×10^{-4} sq. m.)
Number of turns on secondary coil ..	200.
Mean diameter of ring specimen	15 cm. (0·15 m.)
Net iron section of specimen	2 sq. cm. (2×10^{-4} sq. m.)
Number of turns on magnetizing coil ..	500.
Number of turns on search coil	10.
Total resistance of galvanometer circuit ..	900 ohms.

A reversal of a current of 5 amperes in the standard solenoid produces a deflection of 12 divisions on the galvanometer.

Calculate the flux density produced in the specimen when the reversal of a current of 4 amperes in the magnetizing coil produces a deflection of 35 divisions; and determine the permeability of the specimen at this flux density.

Calibration of Galvanometer.

At the centre of the solenoid

$$H = \frac{N_s I}{l_s} = \frac{350 \times 5}{0{\cdot}5} = 3500 \text{ A.T. per m.}$$

Flux linked with search coil

$$= \Phi = Ba_s = 4\pi \times 10^{-7} \times 3500 \times 12 \times 10^{-4}$$
$$= 5{\cdot}28\ \mu\text{Wb. (528 C.G.S. lines).}$$

Total change of linkage on reversing

$$= 2\Phi n_s = 2 \times 5{\cdot}28 \times 200$$
$$= 2112\ \mu\text{Wb.-turns},$$
$$k_g' = \frac{2112}{12}$$
$$= 176\ \mu\text{Wb.-turns per division (17,600 C.G.S. flux-linkages per division).}$$

Test on Specimen.

Total change of flux linkage on reversal

$$= k_g'\delta = 176 \times 35$$
$$= 6160\ \mu\text{Wb.-turns.}$$

Hence

$$2\Phi n_I = 6160.$$
$$\therefore\ \Phi = \frac{6160}{2 \times 10}$$
$$= 308\ \mu\text{Wb. (30,800 C.G.S. lines),}$$

and

$$B = \frac{308 \times 10^{-6}}{2 \times 10^{-4}} = 1{\cdot}54\ \text{Wb. per sq. m. (15,400 lines per sq. cm.).}$$
$$H = \frac{500 \times 4}{0{\cdot}15\pi} = 4250\ \text{A.T. per m.}$$
$$\mu = \frac{B}{H} = \frac{1{\cdot}54}{4250}.$$
$$\mu_r = \frac{\mu}{\mu_0} = \frac{1{\cdot}54}{4250} \times \frac{1}{4\pi \times 10^{-7}} = 288.$$

10. Determination of Hysteresis Loop (Method of Stepped Reversals)

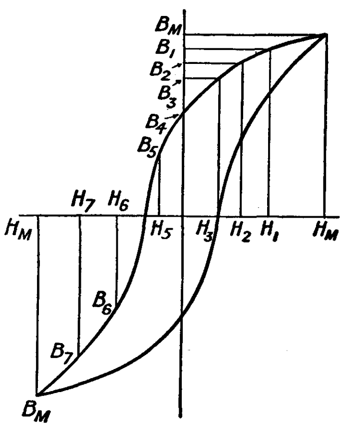


Fig. 14.—Determination of hysteresis loop

If, after the iron has attained the cyclic state, corresponding with the maximum value of the magnetizing field which is to be applied, H_m, instead of being reversed, is reduced to some value H_1 (fig. 14), the flux density is reduced from a value B_m to some value B_1. The change of flux so produced can be measured by a ballistic galvanometer, and if B_m is known B_1 can then be calculated. By increasing the magnetizing field again to H_m and reducing it progressively to H_2, H_3, etc., the corresponding values B_2, B_3, etc., can be obtained in a similar manner; finally, the deflection produced by switching off the magnetizing current altogether gives B_4, the residual flux density.

If now the magnetizing current is again increased to the original maximum and then reversed to a small value, so as to produce a field H_5 in the opposite direction, the corresponding flux density B_5 can be calculated, and, by gradually increasing the reversed value, B_6, B_7, and finally B_m, corresponding to a full reversal of the current, are obtained.

These readings enable half the hysteresis loop to be drawn, and the other half can be completed from symmetry. The method will not be described in detail, but the connections are exactly as in fig. 13, except that a variable rheostat is included in one position of the reversing switch.

11. The Fluxmeter

In some cases it is difficult to make the flux change sufficiently rapid to fulfil the condition that the change should have ceased before appreciable movement of the galvanometer takes place; and in such cases it is preferable to use a *fluxmeter*.

This is a ballistic galvanometer in which the restoring torque is almost negligibly small and the electromagnetic damping large. Under these conditions the amplitude of the first swing is proportional to the flux change and *independent of the time of change*. The instrument is usually calibrated directly in flux-linkages and is therefore more convenient, but gives less accurate results than the ballistic method.

MAGNETIC CIRCUIT CALCULATIONS

12. Typical Magnetic Circuits

The magnetic circuits employed in some common types of electrical apparatus and machines are shown in fig. 15. Corresponding directions of current and flux are indicated, and the interlinkage of the two circuits should be noted in each case.

The magnetic circuit of a simple lifting magnet such as has been mentioned in previous sections is shown in (*a*). When the lower limb (often called the armature) is in close contact with the poles, the circuit is almost entirely in iron and the reluctance is small; but it is greatly increased if the armature is separated from the poles, even by a short *air-gap*.

The magnetic circuit, or cores, of two common types of transformer are shown in (*b*) and (*c*). They are built up of thin sheets or laminations in order to reduce eddy currents (§ 10, p. 151) and, to economize in material, each square is formed of rectangular strips interleaved at the corners; the small air-gaps which occur at these joints somewhat increase the reluctance of the circuit, although this effect is reduced by interleaving. In (*c*) it should be noticed that there are two magnetic circuits in parallel; the flux passing up the centre core divides, half returning along each of the side limbs.

Fig. (*d*) shows the magnetic circuit of a 4-pole d.c. machine, in which there are four magnetic circuits. In order to allow the central circular portion to rotate (see Chapter XIII), an air gap is necessary which is responsible for a large portion of the reluctance of the circuit. The *magnetic* difference of potential between the tips of adjacent poles

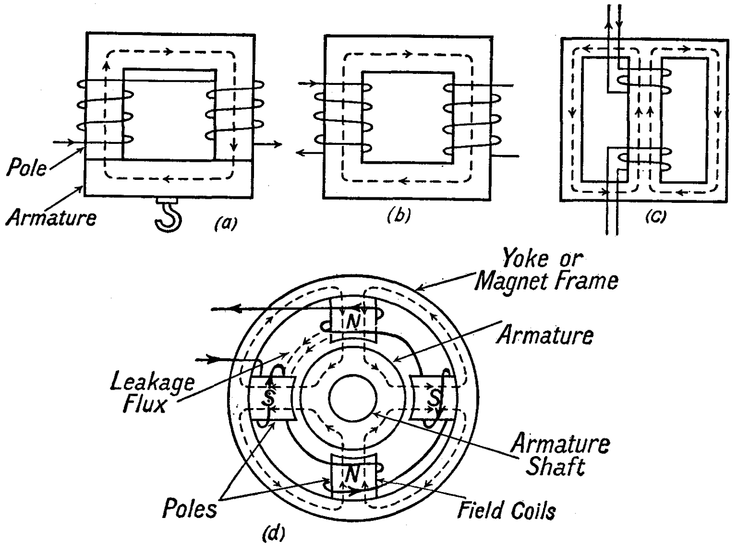


Fig. 15.—Typical magnetic circuits

is therefore large, and a considerable amount of leakage flux passes directly across from pole to pole without passing through the armature (see § 14, p. 234).

13. Reluctances in Series

When different portions of an electric circuit consist of different materials or are of different dimensions, the total resistance is obtained by adding together the resistances of the various portions, calculated separately.

$$\text{Total resistance} = \rho_1 \frac{l_1}{a_1} + \rho_2 \frac{l_2}{a_2} + \rho_3 \frac{l_3}{a_3} + \ldots$$

$$= \frac{1}{\gamma_1} \frac{l_1}{a_1} + \frac{1}{\gamma_2} \frac{l_2}{a_2} + \frac{1}{\gamma_3} \frac{l_3}{a_3} + \ldots .$$

where γ $(=1/\rho)$ is the conductivity.

Similarly, when the different portions of a magnetic circuit consist of different materials or are of different dimensions,

$$\text{Total reluctance} = \frac{1}{\mu_1} \frac{l_1}{a_1} + \frac{1}{\mu_2} \frac{l_2}{a_2} + \frac{1}{\mu_3} \frac{l_3}{a_3} + \ldots, \quad . \quad (9)$$

where $\frac{1}{\mu_1} \frac{l_1}{a_1}$, etc., are the reluctances of the various portions.

It follows therefore that, since M.M.F. = flux × reluctance,

$$\text{M.M.F.} = \Phi\left(\frac{1}{\mu_1}\frac{l_1}{a_1} + \frac{1}{\mu_2}\frac{l_2}{a_2} + \frac{1}{\mu_3}\frac{l_3}{a_3} + \ldots\right). \quad (10)$$

In electrical engineering, one of the most common problems is to determine the number of ampere-turns which must be provided by the magnetizing coil in order to set up a given flux in a magnetic circuit. Provided that the dimensions and the permeability of each portion of the circuit are known and that leakage can be neglected, it is possible to obtain the total M.M.F. required from equation (10) above, as illustrated in the example below.

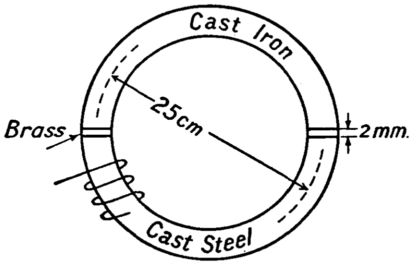


Fig. 16

In practice, however, it is usually more convenient to work directly from the B–H curves of the materials: the method of procedure is therefore somewhat different and is described in § 15, p. 234.

Example.—A metal ring, 25 cm. in mean diameter, 5 sq. cm. in cross-section and uniformly wound with 500 turns, consists of two semicircular pieces, of cast iron and cast steel respectively, separated at the junctions by two pieces of brass each 2 mm. thick (see fig. 16).

Calculate the current required to produce in the ring a flux of 300×10^{-6} weber (30,000 lines).

(For cast iron, $\mu_r = 175$; for cast steel, $\mu_r = 1500$.)

Reluctance of cast-iron portion

$$= \frac{1}{\mu}\frac{l}{a} = \frac{1}{175 \times 4\pi \times 10^{-7}} \times \frac{0{\cdot}25\pi}{2 \times 5 \times 10^{-4}} = 3{\cdot}575 \times 10^6.$$

Reluctance of cast-steel portion

$$= \frac{1}{1500 \times 4\pi \times 10^{-7}} \times \frac{0{\cdot}25\pi}{2 \times 5 \times 10^{-4}} = 0{\cdot}416 \times 10^6.$$

Reluctance of brass portion (μ_r for brass = 1)

$$= \frac{1}{4\pi \times 10^{-7}} \times \frac{0{\cdot}4 \times 10^{-2}}{5 \times 10^{-4}} = 6{\cdot}37 \times 10^6.$$

$$\text{Total reluctance} = 10{\cdot}361 \times 10^6.$$

$$\begin{aligned} \text{M.M.F.} &= \text{flux} \times \text{reluctance} \\ &= 300 \times 10^{-6} \times 10{\cdot}361 \times 10^6 \\ &= 3108 \text{ ampere-turns.} \end{aligned}$$

$$\text{Current in magnetizing coil} = \frac{3108}{500} = 6{\cdot}2 \text{ amperes.}$$

14. Magnetic Leakage

Just as the current in any portion of an electrical circuit is proportional to the P.D. between its ends, so the flux in any portion of a magnetic circuit can be looked upon as due to the difference of *magnetic* potential between the ends of the section considered. When the reluctance of the particular section is large, as in the case of an air gap, the magnetic P.D. between its ends is high, and considerable leakage occurs. A simple magnetic circuit with an air gap is shown in fig. 17*a*, and its electrical counterpart in fig. 17*b*. The M.M.F. produced by the concentrated magnetizing coil at A corresponds with the E.M.F. of the cell at A'. In the magnetic circuit, most of the reluctance is

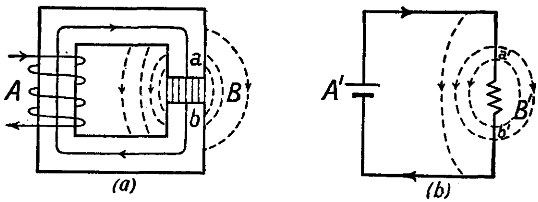


Fig. 17.—Corresponding magnetic and electric circuits

provided by the *air gap* * at B. In the electric circuit most of the resistance is concentrated at B'. Consequently the P.D. across $a'b'$ is little less than that of the cell, and if it is imagined that the whole electric circuit is immersed in a conducting liquid, leakage currents will flow along the paths indicated. Similarly, the magnetic P.D. between a and b is large, and in this case the permeability of the surrounding medium is the same as that of the section ab of the circuit; hence some flux is set up in the leakage paths indicated on fig. 17*a* by the broken lines.

Leakage flux is difficult to calculate without simplifying assumptions, and in the magnetic circuits used in electrical machinery, the effect of leakage, when it cannot be neglected, is usually taken into account by the use of empirical *leakage factors.*

15. Use of *B-H* Curves in Magnetic Calculations

In equation (10), p. 233, it was found that

$$\text{M.M.F.} = \Phi\left(\frac{1}{\mu_1}\frac{l_1}{a_1} + \frac{1}{\mu_2}\frac{l_2}{a_2} + \frac{1}{\mu_3}\frac{l_3}{a_3} + \dots\right);$$

* Short portions of a magnetic circuit, which do not consist of ferromagnetic materials, are often referred to as *air gaps*, even though they may consist of some other non-magnetic material.

but $\frac{\Phi}{a} = B$, the flux density in the portion considered, so that

$$\text{M.M.F.} = \frac{B_1}{\mu_1} l_1 + \frac{B_2}{\mu_2} l_2 + \frac{B_3}{\mu_3} l_3 + \ldots,$$

and since $\frac{B}{\mu} = H$,

$$\textbf{M.M.F.} = \boldsymbol{NI = H_1 l_1 + H_2 l_2 + H_3 l_3 + \ldots}, \quad (11)$$

where H_1, H_2, etc., are the values of the magnetizing force corresponding with the flux densities B_1, B_2, etc., in each portion of the circuit obtained from the *B-H* curve appropriate to the material, and l_1, l_2, etc., are the mean lengths of magnetic path in each portion.

In such curves H is often expressed in ampere-turns per cm. or ampere-turns per inch and it is clear that equation (11) still holds true provided l is measured in cm. or inches.

The procedure in a magnetic calculation, which is best illustrated by an actual example, is briefly as follows:—

The magnetic circuit is divided into a number of portions, wherever there is a change either of flux density or of material; and in each of these the flux density (B) and the mean length of magnetic path (l) is determined. From the *B-H* curve appropriate to the material, the value of the magnetizing force (H) corresponding with that of B is read off. The product Hl then gives the M.M.F. (ampere-turns) for that particular portion; and the total M.M.F. is the sum of the M.M.F.s for the various portions. When there are several portions, it is convenient to arrange the calculation in tabular form.

As a simple illustration the example of § 13, p. 233, is worked out in this way below:

Example 1 (as on p. 233).—A metal ring, 25 cm. in mean diameter, 5 sq. cm. in cross-section, and uniformly wound with 500 turns, consists of two semicircular pieces, of cast iron and cast steel respectively, separated at the junctions by two pieces of brass each 2 mm. thick. Calculate the current required to produce in the ring a flux of 300 micro-webers (30,000 lines).

Cast-iron portion.

$$\text{Flux density} = \frac{300 \times 10^{-6}}{5 \times 10^{-4}} = 0{\cdot}6 \text{ weber per sq. m. (6000 lines per sq. cm.).}$$

$$\text{Length of magnetic path} = \frac{25\pi \times 10^{-2}}{2} = 0{\cdot}393 \text{ m.}$$

From *B-H* curve for cast iron (fig. 9, p. 220), when

$$B = 0{\cdot}6 \text{ weber per sq. m.}$$
$$H = 2700 \text{ ampere-turns per m.}$$

M.M.F. for cast-iron portion $= Hl = 2700 \times 0{\cdot}393 = 1060$ ampere-turns.

Cast-steel portion.

Flux density = 0·6 weber per sq. m.

Length of magnetic path = 0·393 m.

From *B-H* curve for cast steel (fig. 2, p. 212), when

$B = 0{\cdot}6$ weber per sq. m.

$H = 320$ ampere-turns per m.,

M.M.F. for cast-steel portion $= Hl = 320 \times 0{\cdot}393 = 126$ ampere-turns.

Air Gaps.

Flux density = 0·6 weber per sq. m.

Length of path $= 2 \times 0{\cdot}2 = 0{\cdot}4$ cm. (two gaps in series) $= 0{\cdot}4 \times 10^{-2}$ m.

M.M.F. for air gaps $= \dfrac{0{\cdot}6}{4\pi \times 10^{-7}} \times 0{\cdot}4 \times 10^{-2} = 1915$ ampere-turns.

Total M.M.F. $= 1060 + 126 + 1915 = 3101$ ampere-turns.

Current in magnetizing coil = 6·2 amperes.

Example 2.—A cast-iron magnet with a mild steel armature has the dimensions (in cm.) indicated in fig. 18.

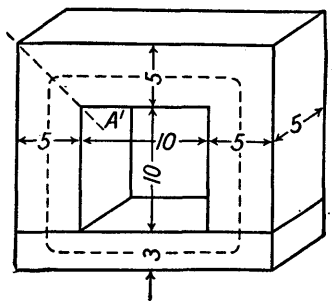


Fig. 18

Calculate (*a*) the flux density at the contact surface to produce a total pull on the armature of 160 lb.;

(*b*) The total M.M.F. required, if the joint at each contact surface is equivalent to an air gap of 0·1 mm.

(*a*) Total pull $= \dfrac{160}{0{\cdot}225} = 711$ newtons.

$$f = \frac{711}{2 \times 25 \times 10^{-4}} = 14{\cdot}22 \times 10^{4} \text{ newtons per sq. m.}$$

From equation (34), p. 185,

$$f = \tfrac{1}{2}\frac{B^2}{\mu_0}. \quad \therefore\ B = \sqrt{2\mu_0 f}.$$

$\therefore$ Flux density at contact surface $(B) = \sqrt{2 \times 4\pi \times 10^{-7} \times 14{\cdot}22 \times 10^{4}}$

$= 0{\cdot}597$ webers per sq. m. (5970 lines per sq. cm.).

Total flux $= 0{\cdot}597 \times 25 \times 10^{-4} = 1{\cdot}493 \times 10^{-3}$ weber.

(*b*) In many magnetic calculations it is necessary to assume the mean path of the flux. The length (l) is then determined either by drawing out to scale and measuring directly, or by estimation; the latter method is usually sufficiently accurate and is used here.

Since H is often measured in ampere-turns per cm. as in fig. 9, this unit is used in the calculations below.

Magnet Poles and Yoke.

Cross-section $= 5 \times 5 = 25$ sq. cm. and is assumed uniform throughout.*

$B = 0{\cdot}597$ weber per sq. m. (as at contact surface).

$H = 27$ ampere-turns per cm. (from B-H curve for cast iron (fig. 9, p. 220)).

Mean magnetic path is made up of three straight portions each 10 cm. long and two curved portions which are assumed to be quadrants of a circle of 2·5 cm. radius.

$$\therefore\ l = (3 \times 10) + 2{\cdot}5\pi = 37{\cdot}9 \text{ cm.}$$

M.M.F. for poles and yoke $= 27 \times 37{\cdot}9 = 1023$ ampere-turns.

Armature.

Cross-section $= 5 \times 3 = 15$ sq. cm.

$$B = \frac{1{\cdot}493 \times 10^{-3}}{15 \times 10^{-4}} = 0{\cdot}995 \text{ weber per sq. m.}$$

$H = 9$ ampere-turns per cm. (from B-H curve for mild steel (fig. 9, p. 220)).

Mean magnetic path consists of one straight portion 10 cm. long and two quadrants of 2 cm. radius.

$$\therefore\ l = 10 + 2\pi = 16{\cdot}3 \text{ cm.}$$

M.M.F. for armature $= 9 \times 16{\cdot}3 = 147$ ampere-turns.

Joints.

$B = 0{\cdot}597$ weber per sq. m.

$$H = \frac{0{\cdot}597}{4\pi \times 10^{-7}} \text{ ampere-turns per m.}$$

$l = 0{\cdot}01 \times 10^{-2} \times 2 = 2 \times 10^{-4}$ m. (two gaps in series).

$$\text{M.M.F. for joints} = \frac{0{\cdot}597}{4\pi \times 10^{-7}} \times 2 \times 10^{-4} = 95 \text{ ampere-turns.}$$

Total M.M.F. $= 1023 + 147 + 95 = 1265$ ampere-turns.

16. Economical Values of Flux Density

The B-H curves in fig. 2 (p. 212) and fig. 9 (p. 220) show that when the material is approaching saturation, a small increase in flux density (B) requires a large increase in magnetizing force (H), and therefore in the ampere-turns of the magnetizing coil. This necessitates an increase either in the current in the coil or in the number of turns; and, whichever alternative is adopted, the power loss in the coil will be increased unless the cross-section of the wire is made proportionately greater, i.e. the weight of copper in the coil is increased.

* The cross-section increases in the neighbourhood of the corners as at AA', but the length of path of those lines of force which follow the outer edge is considerably greater. The two effects tend to neutralize each other, and, in such cases, the section is usually assumed constant.

One of the factors on which the power output of an electrical machine depends is the *total flux* set up in the magnetic circuits; but the *flux density* in each portion of the circuit depends upon the cross-section of that portion. By reducing the section it is possible to decrease the weight of iron (which is a cheap material), but either the resistance losses in the magnetizing coils or the weight of copper (which is an expensive material) in the coil must be increased. At the same time, there is a rapid increase in the hysteresis and eddy-current losses in certain parts of the circuit.

As a general rule the power losses in a machine can be reduced only by increasing the quantities of iron and copper in it. Hence a machine with a very high efficiency, i.e. small losses, is expensive in first cost, and in the normal commercial machine the flux densities and current densities in each portion lie within limits which have been found, by experience, to give the best compromise between these conflicting factors.

The values of flux density used in practice generally lie around the knee of the magnetization curve, i.e. in the neighbourhood of the point b in fig 2.

EXAMPLES

1. A galvanometer for which $k_g = 0{\cdot}2$ micro-coulomb per division is connected to a coil of 100 turns, the total resistance being 1000 ohms. Calculate the change in the flux linked with the coil necessary to produce a deflection of one division.

2. A Hibbert standard magnet has a flux of 200×10^{-6} weber and a search coil of 10 turns. On dropping the coil a deflection of 20 divisions is produced in a ballistic galvanometer connected to it. What is the value of the galvanometer constant in weber-turns per division?

3. A ballistic galvanometer for which $k_g = 0{\cdot}5$ micro-coulomb per division is connected to a search coil of 300 turns and having a diameter of 3 cm., which is placed inside and at the centre of a solenoid 100 cm. long and containing 700 turns. The resistance of the galvanometer circuit is 935 ohms.

Calculate the deflection produced when a current of 5 amperes is reversed in the solenoid.

4. An iron specimen in the form of a ring having internal and external diameters of 18 and 20 cm. respectively and a cross-section of 2 sq. cm. is wound with a search coil of 10 turns and a magnetizing coil of 600 turns. The search coil is connected to a ballistic galvanometer in which a deflection of one division represents a change of 200×10^{-6} weber-turns.

Calculate the flux density produced in the specimen when the reversal of a current of 1 ampere in the magnetizing coil produced a deflection of 34 divisions. What is the relative permeability at this density?

5. The following data were obtained in the determination of the B-H curve of a sample of iron:

Standard solenoid—Length, 100 cm.
Number of turns, 800.
Diameter of search coil, 4 cm.
Number of turns on search coil, 250.

The reversal of a current of 5 amperes produces a deflection of 15 divisions.

Ring specimen—Mean diameter, 16 cm.
Net iron section, 2·5 sq. cm.
Number of turns in magnetizing coil, 600.
Number of turns in search coil, 10.

The resistance of the galvanometer circuit was constant during the calibration and subsequent magnetization test.

Determine the flux density and the relative permeability when the reversal of a current of 2 amperes in the magnetizing coil produces a deflection of 42·5 divisions on the ballistic galvanometer.

6. The following values were obtained in a magnetization test on a sample of iron:

H	20	16	12	8	4	0	-2	-4
B	1·5	1·495	1·48	1·46	1·41	1·3	1·2	0·5
H	-6	-8	-12	-16	-20 ampere-turns per cm.			
B	$-0·85$	$-1·1$	$-1·32$	$-1·42$	$-1·5$ webers per sq. m.			

Draw the hysteresis loop and find the hysteresis loss in joules per cu. m. per cycle.

7. A circular ring of mild steel having a mean diameter 20 cm. and a cross-sectional area of 2·5 sq. cm. is wound with 500 turns of wire.

Calculate the current necessary to produce a flux of 250 micro-webers. [$\mu_r = 800.$]

8. What will be the current necessary to maintain the same flux if a radial saw-cut 2 mm. wide is made in the ring in question (7)?

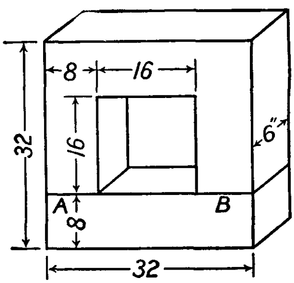


Fig. 19

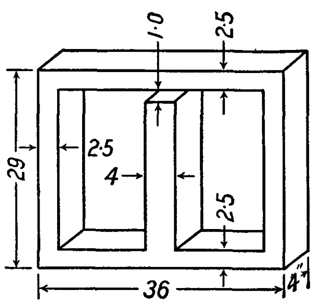


Fig. 20

9. A ring having a mean diameter of 15 cm. and a cross-section of 2 sq. cm. is made up of two semicircles of cast iron and mild steel respectively, separated by two brass distance pieces each 0·1 cm. thick.

Calculate (*a*) the total M.M.F. (ampere-turns) required to set up a flux of 160×10^{-6} weber.

(*b*) The force required to separate the two halves.

(For cast iron $\mu_r = 120$; for mild steel $\mu_r = 900.$)

10. A cast-steel magnet has an air gap of 2 mm., and the length of the iron path is 30 cm.

Calculate the M.M.F. to produce a density of 0·8 weber per sq. m. in the air gap.

11. The core of the electromagnet shown in fig. 19 is made of laminations of stalloy, the net iron area being 90 per cent of the gross iron area (dimensions in cm.).

Find the M.M.F. necessary to produce a pull of 1240 lb. on the armature. The joints at A and B are equivalent to air gaps each 0·05 mm. long.

12. A 500-turn coil is wound on the central core of the cast-steel magnet shown in fig. 20 (dimensions in cm.). Calculate the current to produce a flux of

$2{\cdot}4 \times 10^{-3}$ weber in the air gap. [Note that there are two magnetic circuits in parallel; the flux, after crossing the air gap, divides and half passes down each of the outer limbs.]

13. An iron ring, cut across a diameter, is wound with 500 turns of wire. When a current of 1 ampere is flowing through the coil, a force of 5 lb. is necessary to pull the two halves of the ring apart. If the mean diameter of the ring is 10 cm., and the cross-sectional area is 2 sq. cm., find the effective length of the air gaps between the two halves, on the assumption that the relative permeability of the iron is 1000. [Lond. B.Sc.(Eng.).]

14. A rectangular-shaped iron circuit is constructed from 500 laminations each 25 cm. long, 2 cm. wide, and 0·08 cm. thick. The four joints are butt-jointed each with an air gap of 0·01 cm. The magnetizing winding consists of 200 turns evenly distributed.

The *B-H* curve for the lamination material gives the following figures;

H	1·6	3·2	4·8	6·4	8	ampere-turns per cm.
B	0·55	0·92	1·14	1·26	1·30	webers per sq. m.

Determine the magnetizing current necessary to maintain total fluxes of 2, 2·4, and 2·6 milliwebers.

CHAPTER XII

Alternating Current Circuits

1. Direct and Alternating Current

In the electric circuits considered up to the present, the current has been due to an E.M.F. which, although possibly varying in magnitude, has been always in the same direction. Such a current is called a *direct current*; and since the E.M.F. produced by all types of primary and secondary cells is constant in direction, most of the early experimental investigations were carried out by means of direct currents. On the other hand, the E.M.F. induced in the conductors of a dynamo or generator reverses periodically as the armature rotates, so that the current which flows in these conductors and (unless some rectifying device is interposed) also in the external circuit, reverses periodically and is known as an *alternating current.*

For many purposes either type of current is equally suitable; and it was natural that in the early stages of development the more familiar direct current was generally employed, although it necessitated a more complicated form of generator. But, for the transmission and distribution of electrical energy on a large scale, alternating current has many advantages which have led to its general adoption; and at the present time, apart from existing systems which have not yet been converted, direct current is used only where it is essential, as in electrochemical processes, or where it has peculiar advantages, as in electric traction.

2. Production of Alternating E.M.F. in a Loop rotating in a Uniform Field

In fig. 1*a* is shown a simple loop of wire rotating with uniform angular velocity, in a uniform field, about an axis perpendicular to the direction of the field. E.M.F.s are induced in the sides *bc* and *de* which cut through the lines of force, but not in the end portions *ab*, *cd*, *ef*, the plane of rotation of which is parallel to the direction of the field. If the loop is in the position shown, and is rotating in a counter-clockwise direction, the conductor *bc* (represented by *A* in fig. 1*b*) is cutting the flux in a *downward* direction, so that the direction of the induced E.M.F. (§ 9, p. 150) is *out* of the paper, i.e. from *c* towards *b*; while at the same time the conductor *de* (represented by *B* in fig. 1*b*) is cutting the same flux in an *upward* direction, so that the induced E.M.F. is in the opposite direction, i.e. from e towards d. It is clear, however, that the two E.M.F.s, although opposite in direction, *assist* each other round the loop, so that the resultant E.M.F. is the sum of the two. Later, when the plane of the coil has passed through the vertical position, bc will cut the flux in an upward and de in a downward direction, so that the E.M.F. induced in each, and therefore the resultant E.M.F. of the loop, is reversed. Hence as the loop is rotated an *alternating E.M.F.* is induced, which reverses in direction once every half-revolution.

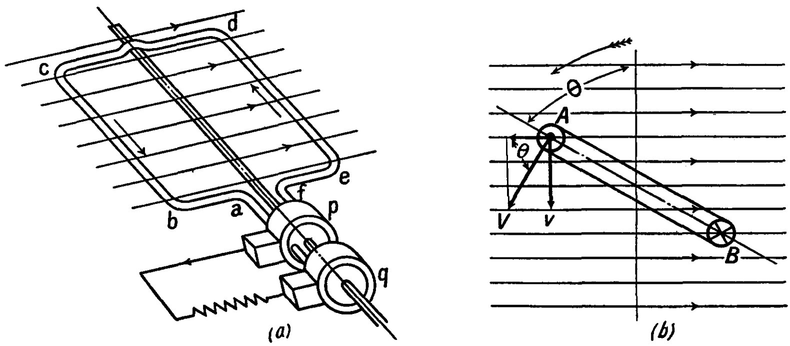


Fig. 1.—Simple loop alternator

The E.M.F. induced in each conductor is proportional to the rate at which it cuts through the field, i.e. proportional to v, the component of the tangential velocity V which is perpendicular to the direction of the field. This clearly varies between zero when the plane of the loop is vertical, and a maximum value $v = V$ when the plane of the loop is horizontal.

At any instant when the plane of the loop makes an angle θ (usually measured in radians) with the vertical

$$v = V \sin \theta \quad \text{(see fig. 1}b\text{)},$$

and if the effective length of each conductor is l and the density of the field B,

the E.M.F. induced in each conductor $= Blv$ (§ 8, p. 147).

The resultant E.M.F. in the loop at this instant is therefore

$$\begin{aligned} e &= 2Blv \\ &= 2BlV \sin \theta. \end{aligned}$$

If B is in webers per sq. m., V in m. per sec., and l in m., then

$$e = 2BlV \sin \theta \text{ volts.} \qquad \ldots \ldots \quad (1)$$

If the loop is replaced by a coil containing n turns, an E.M.F. of equal value is induced in each turn and the resultant E.M.F. of the coil is

$$e = 2nBlV \sin \theta \text{ volts.} \qquad (2)$$

The maximum value of the E.M.F. occurs when the coil is horizontal, i.e. when $\theta = \pi/2$ radians, or $\sin \theta = 1$. In this case, $v = V$, and

$$E_{max} = 2nBlV \text{ volts.} \qquad (3)$$

Hence from (2), the instantaneous value of the E.M.F. is

$$\boldsymbol{e = E_{max} \sin \theta}. \qquad (4)$$

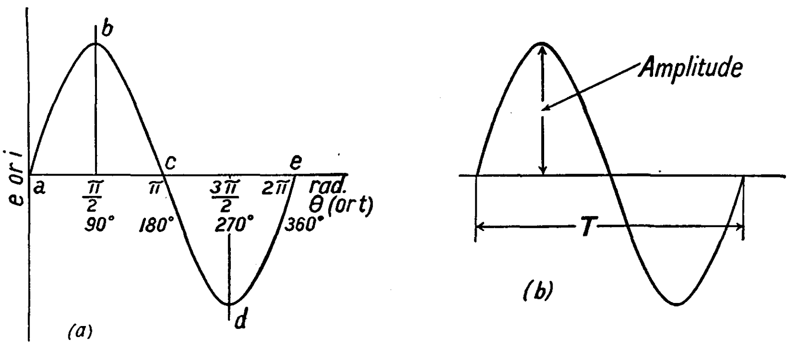


Fig. 2.—E.M.F. wave of simple alternator

If e is plotted vertically against θ, starting from the instant when $\theta = 0$, the curve in fig. 2a is obtained. The E.M.F. first of all increases to a positive maximum at b ($\theta=\pi/2$, $\sin \theta=1$); decreases to zero at c ($\theta = \pi$, $\sin \theta = 0$); reverses in direction and grows to an equal negative maximum at d ($\theta = 3\pi/2$, $\sin \theta = -1$); and finally decreases again to zero at e ($\theta = 2\pi$, $\sin \theta = 0$). The coil has now made one complete revolution, during which the alternating E.M.F. has passed through one complete *cycle*, which is repeated for each successive revolution. The curve by which this cycle is represented is known as a *sine wave*.

Since the angle is proportional to the time when the angular velocity is constant, the shape of the curve is the same whether it is plotted on an angular or a time base. If the angular velocity of the coil is ω radians per sec., then, if time is counted from the instant when $\theta = 0$,

$$\theta = \omega t.$$

Hence

$$\boldsymbol{e = E_{max} \sin \omega t}. \qquad (5)$$

If the ends of the coil are connected respectively to metal rings p, q, mounted on but insulated from the spindle, continuous connection may be made with a stationary external circuit by means of two conducting *brushes*, one rubbing on each ring (fig. 1a). The alternating E.M.F. induced in the coil will then cause an *alternating current* to flow round the coil and the external circuit, the value of which at any instant is

$$i = \frac{e}{R} = \frac{E_{\max} \sin \omega t}{R}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where R is the total resistance of the circuit.*

The maximum value of the current, which occurs when the E.M.F. reaches a maximum, is

$$I_{\max} = \frac{E_{\max}}{R}.$$

Therefore, from (6),

$$i = I_{\max} \sin \omega t. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In the above, time has been reckoned from the instant at which the coil passes through its vertical position, i.e. when $\theta = 0$, but this is a special condition and the coil, when first considered, may be inclined to the vertical at some angle α.

In this case, when

$$t = 0, \ \theta = \alpha,$$

and at the end of a time t, the coil will have rotated through a further angle ωt, so that

$$\theta = \omega t + \alpha.$$

Hence the general equation of the sine wave which represents any condition is

$$\boldsymbol{e = E_{\max} \sin (\omega t + \alpha)}. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

3. The Sine Wave

The rotation of a loop or coil in a uniform field produces an alternating E.M.F. which can be represented by a sine wave; and the apparatus is an *alternating current generator* in its simplest form.

In electrical engineering, the sine wave is taken as the standard of wave-form; it is the simplest of all wave-forms to treat mathematically, and it can be shown that all other waves of a similar nature may be regarded as made up of pure sine waves. It is impossible, however, to build a practical machine in this simple manner, and the E.M.F. waves occurring in practice are not pure sine waves, although the more closely this wave-form is approached the more satisfactory is the operation of electrical apparatus. The E.M.F. wave of a modern alternator is very nearly a sine wave, and fortunately in most calcu-

* For simplicity the circuit is assumed to be non-inductive.

lations it may be assumed with little error that the quantities are of this "sinusoidal" form; otherwise the mathematical treatment becomes more difficult.

Below are given definitions of some common terms used in connection with sine waves.

A *Periodic Function* is one which repeats the same series of values over and over again in the same time. The velocity or the displacement of a pendulum bob swinging at constant amplitude is a familiar example.

The sine wave is a particular example of a periodic function.

The *Cycle* is the series of values which is repeated.

The *Period* (T) is the time taken to complete the cycle (fig. 2*b*).

The *Frequency* (f) is the number of cycles completed in unit time ($f = 1/T$).

The frequencies used in power, telephone, and radio circuits extend over a very wide range. They may be divided roughly into three groups:

Power frequencies usually lie between 40 and 60 cycles per second (50 in Great Britain, 60 in the U.S.A.),

Audio-frequencies, i.e. those of sounds audible to the human ear, from about 50 to about 8000 cycles per second,

Radio frequencies from about 10,000 to many million cycles per second.

In the case of the loop in the previous section, 1 cycle is completed per revolution, so that in order to give a frequency of 50 cycles per sec. the loop must rotate at 3000 r.p.m.

The *Phase* is the point in the cycle reached at some particular instant; the term is most commonly used in connection with *difference in phase* (§ 6, p. 250).

The *Amplitude* is the maximum value reached during the cycle (fig. 2*b*).

Example.—A loop 10 cm. wide, and having active sides each 20 cm. long, rotates at 1500 r.p.m. in a uniform field of density 0·5 weber per sq. m. (5000 lines per sq. cm.) (see fig. 1).

Calculate

(*a*) the maximum value of the induced E.M.F.;
(*b*) the frequency and the period;
(*c*) the value of the E.M.F. 0·003 sec. and 0·032 sec. after passing through its zero value.

(*a*) The tangential velocity of each side of loop is:

$$V = \frac{10\pi}{100} \times \frac{1500}{60} = 7{\cdot}85 \text{ m. per sec.}$$

$$\therefore\ E_{\max} = 2BlV = 2 \times 0{\cdot}5 \times 0{\cdot}2 \times 7{\cdot}85$$
$$= 1{\cdot}57 \text{ volt.}$$

(b) 1500 r.p.m. = 25 r.p.s.

$$\therefore f = 25 \text{ cycles per sec.}$$

$$T = \frac{1}{f} = 0{\cdot}04 \text{ sec.}$$

(c) $\omega = 25 \times 2\pi = 157$ radians per sec.

When $t = 0{\cdot}003$ sec.,

$$\theta = \omega t = 157 \times 0{\cdot}003 = 0{\cdot}47 \text{ radian} = 27^\circ,$$

$$e = 1{\cdot}57 \sin 27^\circ = 0{\cdot}71 \text{ volt.}$$

Similarly when $t = 0{\cdot}032$ sec.,

$$e = 1{\cdot}57 \sin 288^\circ = 1{\cdot}57\,(-\sin 72^\circ) = -1{\cdot}49 \text{ volt.}$$

4. R.M.S. (Effective) and Average Values—Form Factor

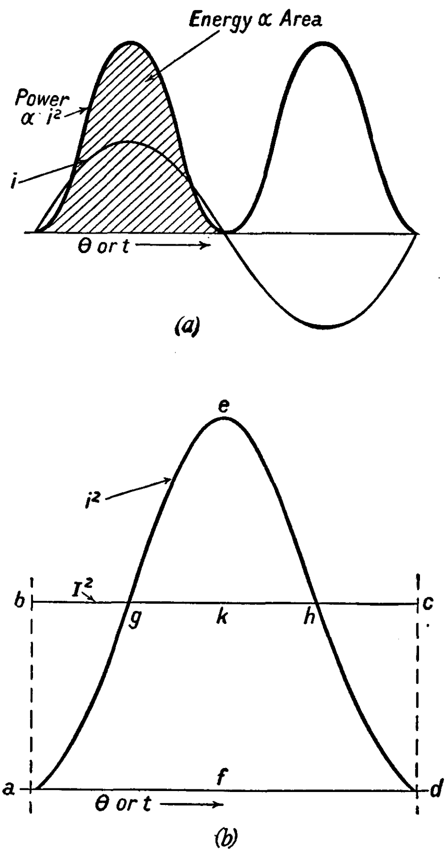


Fig. 3.—The r.m.s. value of a sine wave

In the case of an alternating current which is varying continuously between positive and negative maxima, the question at once arises as to what is to be taken as the effective value of the current in relation to a direct current.

The primary object of an electrical circuit is to take in energy at one point and convey it in the electrical form to another point (or points) and then convert it into other forms of energy. Alternating and direct currents are therefore compared on an energy basis. The simplest case is that of a current flowing through a resistance in which the energy is converted directly into heat; and from the point of view of energy transformation any piece of electrical apparatus can be replaced by an equivalent resistance. Hence:

If an alternating current, in flowing through a resistance, produces the same amount of heat in the same

time as a direct current having a value of I, then the effective value of the alternating current is I.

When a direct current I flows through a resistance R, the rate at which energy is converted into heat, i.e. the power expended, is I^2R (equation (16), p. 108). When an alternating current flows through the same resistance, the power varies throughout the cycle but is given at any instant by i^2R, where i is the value of the current at that instant, i.e. the instantaneous value of the power is proportional to the *square* of the instantaneous value of the current. Hence if one cycle of an alternating current is represented by the sine wave in fig. 3*a*, the curve obtained by plotting the *squares* of the current ordinates (not drawn to the same scale in the figure) on the same base represents the power during the cycle. Since the square of a quantity is always positive, this curve lies wholly on the positive side of the horizontal axis.* The latter represents either time or angular rotation which is proportional to it so that the heat produced, or in general terms the energy expended, during one cycle is proportional to the *area* under the curve; and since, as long as conditions are constant, all the half waves are similar, only one need be considered, and this is shown to a larger scale in fig. 3*b*.

In the case of a steady direct current I, the value of I^2 is constant and the power curve is a straight line parallel to the horizontal axis, as shown in fig. 3*b*, and the energy expenditure in a time equal to half the period of the a.c. wave is proportional to the area of the rectangle *abcd*, where $ab = I^2$. Hence *the effective value of the alternating current is that of a direct current which makes the area of the rectangle* abcd *equal to the area* agehd.

Hence,

$$ab \text{ is the } \textit{mean height} \text{ of the curve } agehd,$$

so that $$I^2 = \text{mean value of (alternating current)}^2$$

and Effective value of alternating current

$$= I = \sqrt{(\text{mean value of } i^2.)} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

It is therefore often called the *root-mean-square* or r.m.s. value.

In the case of the sinusoidal current shown in fig. 3 if the area *agehd* is calculated or the curve drawn to scale and the area measured directly, it will be found that when the rectangle *abcd* is of equal area

$$ab = \tfrac{1}{2}ef.$$

Hence $$I^2 = \tfrac{1}{2}I^2_{\max},$$

* This expresses mathematically the fact that the production of heat is independent of the direction of the current. Actually the curve is another sine wave of twice the frequency, and displaced from the current axis by a distance equal to its amplitude.

so that,

Effective value of a sinusoidal current

$$= I = \frac{1}{\sqrt{2}} I_{\max}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Equation (9) is true for any wave-form: the particular value given in equation (10) applies only to a sine wave.

In a similar way it can be shown that when a sinusoidal P.D. having a maximum value $V_{\max}$ is applied to a resistance, the mean value of the power is the same as with a steady P.D. having a value $\frac{1}{\sqrt{2}} V_{\max}$.

The r.m.s. or effective value of an alternating current or P.D. of sine wave form is

$$\frac{1}{\sqrt{2}} \text{(maximum value)}$$

$$= \mathbf{0{\cdot}707 \text{ maximum value.}}$$

Unless the contrary is expressly stated, r.m.s. values of alternating quantities are always implied, and are represented by capital letters without suffixes.

Instantaneous values, i.e. the value at some particular instant, are represented by small letters as in equations (7) and (8).

The mean or *average* value of an a.c. wave over a whole cycle is obviously zero. During a half cycle the average value is given by the mean height of the half wave. This can be calculated or determined graphically by actual measurement of the area under the curve, and is found to be,

$$\text{Mean height} = \frac{\text{Area under half wave}}{\text{Length of base}} = 0{\cdot}637 \text{ maximum height,}$$

$$\text{i.e. average value of } i = 0{\cdot}637\, I_{\max}.$$

Hence,

The average value of an alternating current or P.D. of sine wave form is

0·637 maximum value.

The ratio $\frac{\text{effective value}}{\text{average value}}$ is called the *form factor* of the wave. For a sine wave,

$$\text{Form factor} = \frac{0{\cdot}707 \text{ maximum value}}{0{\cdot}637 \text{ maximum value}} = 1{\cdot}11. \quad . \quad (11)$$

For non-sinusoidal waves the form factor is greater or less than this value according as the wave is more peaked or flatter than a sine wave. For example, the value for a triangular wave is 1·15 and for a rectangular wave 1·0.

5. Representation of Alternating Quantities by means of Rotating Vectors

The representation of alternating currents and E.M.F.s by drawing the actual sine waves is a very cumbersome method if used in any but the simplest cases. Such quantities can be represented much more easily and conveniently by means of *rotating vectors.*

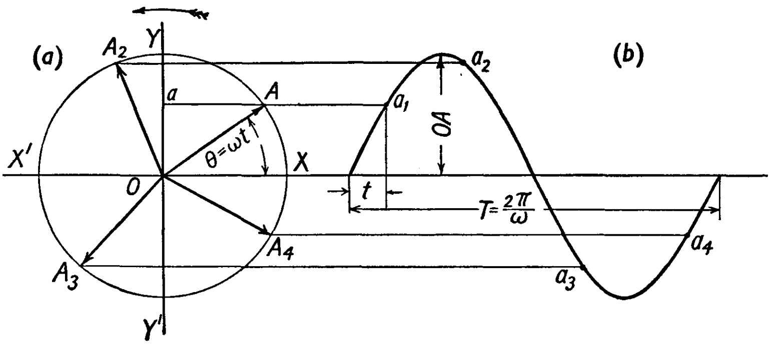


Fig. 4.—Representation of sine wave by rotating vectors

In fig. 4*a*, *OA* is a straight line capable of rotating about the point *O*; and from the extremity *A* a perpendicular *Aa* is drawn to the vertical axis through *O*, so that *Oa* is the projection of *OA* on this axis. If *OA*, starting from the position *OX*, rotates in a counter-clockwise direction, *a* will move from *O* towards *Y*, which it will reach simultaneously with *A*; and as the rotation continues *a* will return, passing through *O* as *A* passes through *X′*, reaching *Y′* simultaneously with *A*, and finally returning to *O* as *A* passes through *X*.

At any particular instant when *OA* is inclined at an angle θ to *OX*, as shown in the figure,

$$Oa = OA \sin\theta.$$

If the rotation is uniform, the angular velocity being ω and time being reckoned from the instant when *OA* is in the position *OX*,

$$\theta = \omega t$$

and

$$Oa = OA \sin\omega t,$$

so that, when Oa is plotted vertically against time, the curve is a sine wave having a maximum value or amplitude equal to OA (see fig. 4*b*). The figure also shows the positions of the vector OA corresponding with three other points a_2, a_3, and a_4 on the sine wave.

The line OA is known as a *rotating vector*, one complete revolution of which corresponds with one complete cycle, so that the frequency is

$$f = \frac{\omega}{2\pi}, \quad \ldots\ldots\ldots \quad (12)$$

and the period is

$$T = \frac{2\pi}{\omega}. \quad \ldots\ldots\ldots \quad (13)$$

Hence: **A quantity which varies sinusoidally with time can be represented by the projection on a vertical axis of a rotating vector having a length equal to the maximum value of the quantity, and making one revolution per cycle, i.e. rotating with an angular velocity $\omega = 2\pi f$ radians per second.**

The projection on any other axis through O might be used, but the vertical axis is obviously the most convenient. Further, the direction of rotation is immaterial, but for the sake of consistency and comparison, the *counter-clockwise* direction has been adopted as standard.

If the length of OA represents the maximum value (I_{max}) of an alternating current,

$$Oa = i = I_{max} \sin \omega t \quad \text{(equation 7, p. 244).}$$

Similarly, if OA represents the maximum value of an alternating E.M.F.,

$$Oa = e = E_{max} \sin \omega t \quad \text{(equation 5, p. 243).}$$

Vectors serve as a kind of *shorthand* for the representation of alternating quantities, and, as will be seen later, their use greatly simplifies the solution of a.c. problems.

6. Difference in Phase

In § 2, p. 241, it was shown that when the circuit is completed the alternating E.M.F. induced in the loop causes an alternating current to flow, which can be represented by a second sine wave. If the circuit contains resistance only, the variations in current and E.M.F. occur simultaneously, i.e. maximum and zero currents occur at the instants when the E.M.F. is a maximum and zero respectively.

When two alternating quantities attain corresponding values simultaneously, they are said to be *in phase*; and the condition is illustrated in fig. 5*a*. The two waves may represent an E.M.F. and a current, as in the example above, or two E.M.F.s or two currents; their amplitudes have been made different for clearness, as otherwise the two waves would coincide. The vectors representing the quantities coincide in direction, and are shown in a position which corresponds with the ordinate *ab*.

When two alternating quantities do not reach corresponding values simultaneously, they are said to be *out of phase*. Two sine waves which are out of phase are shown, with their corresponding vectors, in fig. 5*b*; these again may represent an E.M.F. and a current as in the example above, or two E.M.F.s or two currents. The *difference in phase* between the waves may be measured by the time

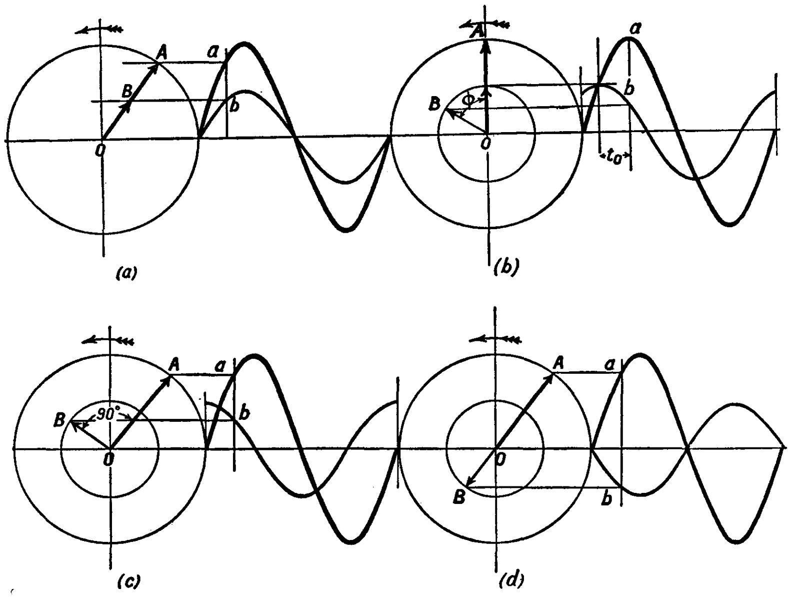


Fig. 5.—Difference in phase

t_0 elapsing between the instants at which they attain corresponding values (e.g. a positive maximum); or more usually, by the angle φ between the vectors by which they are represented. In fig. 5*b*, *B* is said to *lead A* or *A* is said to *lag behind B*. At first glance, the wave diagram is apt to give the impression that *A* is leading *B*, but it must be noted that the terms lead and lag are concerned with the *time* relationships and not the space relationship between the two waves; for instance, *B* reaches its positive maximum *earlier* than *A* does, and therefore *B leads A*. The vector diagram illustrates much more clearly the relation between the quantities; it can be seen at a glance that for a counter-clockwise direction of rotation B leads A by an angle φ or by a time $t_0 = \varphi/\omega$.

When the angle between the vectors is 90° (fig. 5*c*), one quantity attains its maximum value when the other is zero (and vice versa), and the two are said to be in *quadrature*. When the angle between the vectors is 180° (fig. 5*d*), so that one attains a positive maximum at the same instant at which the other attains a negative maximum, the two quantities are said to be *in phase opposition* or in *anti-phase*.

The angle φ between the vectors is called the *angle of phase difference* or the *phase angle*.

It is clear that these terms have no precise meaning unless the quantitites to which they refer are alternating at the same frequency.

7. The Sum and Difference of Two Sine Waves

Two sine waves A and B of the same frequency but differing in phase and amplitude are shown in fig. 6*a*; the wave C represents the sum of the waves A and B obtained by plotting the sum of the instantaneous values at a number of points along the time axis. The three circles have a common centre O and radii equal to the maximum values of A, B, and C respectively.

An ordinate pc, cutting the waves in a, b, and c, is set up at any point on the time axis at which the instantaneous values of A, B, and C are pa, pb, and pc respectively. At this instant A is still increasing positively, so that it can be represented by the vector OA in the first quadrant in such a position that its projection $Oa = pa$. On the other hand, at the same instant B is decreasing from a positive maximum, and the corresponding vector OB is in the second quadrant and in such a position that $Ob = pb$. Similarly, C is approaching its positive maximum, and if this curve also is a sine wave, it can be represented by the vector OC in the first quadrant and in such a position that $Oc = pc$.

Now if BC and AC are joined, it will be seen that $OBCA$ is a parallelogram and OC is the diagonal, and therefore the resultant or vector sum of OA and OB; and wherever the ordinate pc is drawn, it will be found that the vector representing C is the vector sum of the vectors representing A and B. Hence the curve C is also a sine wave of the same frequency as B and C.

Further, since the operation of subtraction consists simply of that of addition with the sign of one of the quantities reversed, the wave representing $A - B$ is obtained by adding the wave A to the wave $-B$ which is exactly opposite in phase to the wave B (fig. 6*b*). In this case the vector representing the resultant wave C is the vector difference of the vectors representing the waves A and B.

Hence:

The sum (or difference) of two sine waves of equal frequency is another sine wave of the same frequency, which can be represented

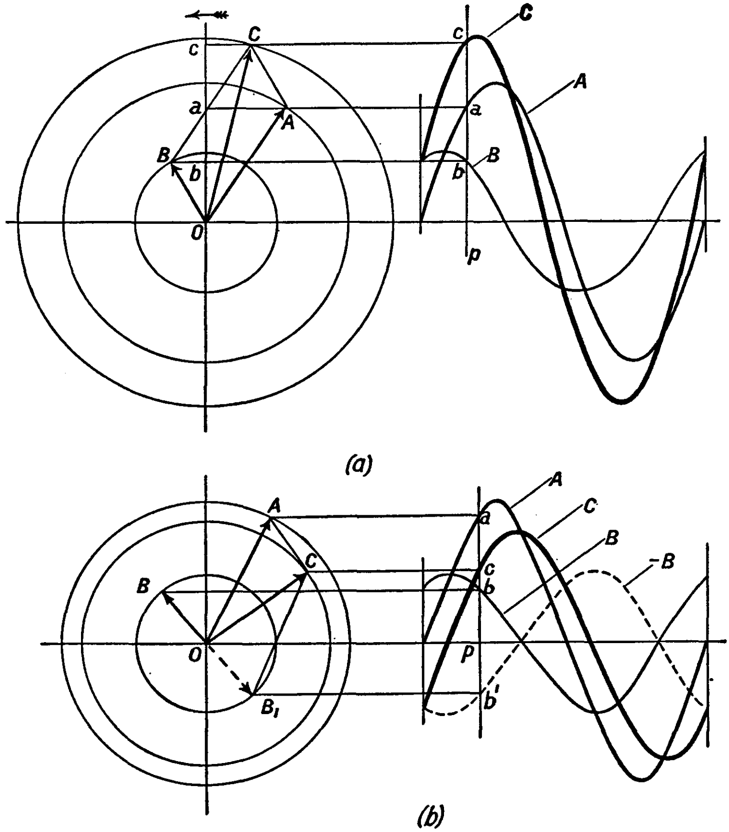


Fig. 6.—Sum and difference of two sine waves

by a vector which is the sum (or difference) of the vectors representing the component waves.

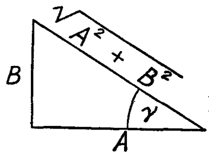
This can also be shown as follows:

If the two E.M.F.s are represented by

$$e_1 = E_1 \sin \omega t, \qquad e_2 = E_2 \sin (\omega t - \alpha),$$

then

$$\begin{aligned} e_r = e_1 + e_2 &= E_1 \sin \omega t + E_2 \sin (\omega t - \alpha) \\ &= E_1 \sin \omega t + E_2 \sin \omega t \cos \alpha - E_2 \cos \omega t \sin \alpha \\ &= (E_1 + E_2 \cos \alpha) \sin \omega t - E_2 \sin \alpha \cos \omega t. \end{aligned}$$



But, since E_1, E_2 and α are constant, we may put

$$(E_1 + E_2 \cos \alpha) = A, \quad \text{and} \quad E_2 \sin \alpha = B;$$

$$e_r = A \sin \omega t - B \cos \omega t.$$

Now $\frac{B}{A}$ is the tangent of some angle γ,* i.e. $\tan \gamma = \frac{B}{A}$.

$$\therefore \frac{B}{\sqrt{A^2 + B^2}} = \sin \gamma \quad \text{and} \quad \frac{A}{\sqrt{A^2 + B^2}} = \cos \gamma.$$

Hence $$e_r = \sqrt{A^2 + B^2} \cos \gamma \sin \omega t - \sqrt{A^2 + B^2} \sin \gamma \cos \omega t$$
$$= \sqrt{A^2 + B^2} \sin (\omega t - \gamma),$$

i.e. another sine wave having an amplitude $\sqrt{A^2 + B^2}$ lagging behind E_1 by an angle γ, and leading E_2 by an angle $\alpha - \gamma$.

The advantage of the vector method of representation is at once evident. The amplitude of the resultant wave and its phase relationship with both its components is shown far more clearly by the construction of a simple parallelogram than by the much longer and more difficult operation of drawing out the waves.

It is often necessary to find the sum of two or more alternating quantities of the same frequency but differing in amplitude and phase, or to illustrate the phase relationship between E.M.F.s and currents in a particular circuit. These operations are almost always carried out by the use of vectors, and the resulting figure is called a *vector diagram.* The diagram may be drawn to scale and the required results obtained by direct measurement, or it may be drawn simply to illustrate the relationship of the quantities, which are then obtained by calculation.

As has been seen, the length of the vector represents the maximum value of the quantity, but since the r.m.s. value is proportional to it $\left(I = \frac{1}{\sqrt{2}} I_{\max}\right)$, vectors are usually drawn to represent r.m.s. values. The use of the vector diagram is shown in the two simple cases which follow:

Case *I.*—Referring again to § 2, p. 241, suppose that a second coil BB_1, of the same dimensions and having the same number of turns, is mounted on the same spindle and with its plane inclined at an angle α to that of the first coil AA_1 (fig. 7*a*). The r.m.s. value of the E.M.F. in BB_1 is the same as that in AA_1, but corresponding points in the cycle, e.g. the positive maximum, are all attained later, so that there is a phase difference α between the two E.M.F. waves. If the two coils are connected in series, the resultant E.M.F. wave is either the sum or the difference (depending upon how they are connected) of the component waves, and is represented by a vector which is the sum or difference of the vectors representing the component waves.

* Since the value of a tangent extends between 0 and ∞, any number must be the tangent of some angle.

Example.—Two coils, each having 10 turns, are mounted on the same spindle with their planes inclined at 30°. The r.m.s. value of the E.M.F. in each turn is 1·5 volt. Determine the magnitude of the resultant E.M.F. when the two coils are connected in series, and its phase relationship with the E.M.F. in each coil.

The E.M.F. of each coil $= 1{\cdot}5 \times 10 = 15$ volts.

The value of the resultant E.M.F. depends upon how the coils are connected:

(*a*) If the end A_1 is connected to the end B, the E.M.F. between A and B_1 is the vector *sum* of the E.M.F.s in each coil.

(*b*) If the end A_1 is connected to the end B_1, the E.M.F. between A and B is the vector *difference* of the E.M.F.s in the two coils.

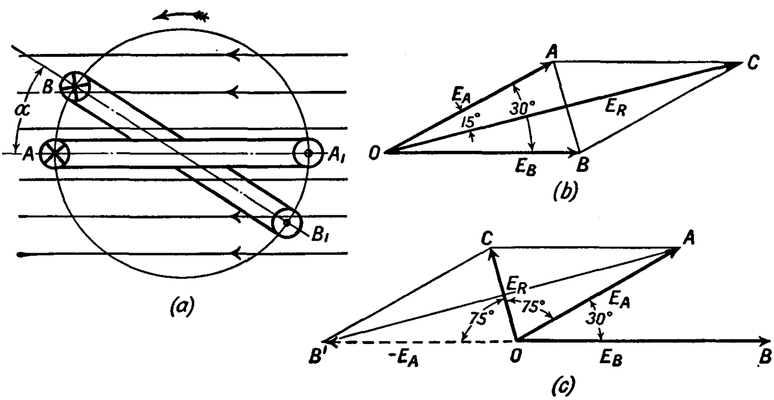


Fig. 7.—Use of vector diagrams

Consider the two E.M.F.s when that of coil B (E_B) is passing through its zero value and about to become positive, so that the corresponding vector OB is drawn horizontally and represents 15 volts (the r.m.s. value) to some convenient scale. Then the E.M.F. of coil A (E_A) is represented by OA, of equal length, but *leading* OB by 30° (fig. 7*b*).

(*a*) The parallelogram $OACB$ is completed and OC then represents the r.m.s. value of the resultant E.M.F. (E_R) in magnitude and phase.

By measurement $E_R = OC = 29$ volts, leading E_B and lagging behind E_A by 15°.

[Alternatively, since $OA = OB$, OC and AB are perpendicular and OC bisects $\angle AOB$.

$$\therefore \angle AOC = 15° \text{ and } E_R = OC = 2\,OA \cos 15° = 2 \times 15 \times 0{\cdot}966$$
$$= 29 \text{ volts.]}$$

(*b*) Since the connection of the loop B is reversed, E_B is reversed in direction relative to E_A, and is represented by OB', equal in length to OB but reversed in phase (fig. 7*c*). Then OC, the vector sum of OA and OB' (i.e. vector difference of OA and OB), represents E_R.

By measurement $E_R = OC = 7{\cdot}8$ volts, leading E_A by 75° and E_B by 105°.

[Alternatively, since OC and AB' are perpendicular and OC bisects $\angle AOB'$,

$$\angle AOC = 75° \text{ and } E_R = 2\,OA \cos 75° = 2 \times 15 \times 0{\cdot}259$$
$$= 7{\cdot}8 \text{ volts.]}$$

Case II.—The angle of phase difference between the E.M.F. and the current in a circuit depends upon the ratio of the inductance to the resistance. When part of a circuit consists of several branches in parallel (fig. 8*a*), it is probable that this ratio is different in each branch; the phase angle between the P.D. and the current in each branch is therefore different, and the total current is the *vector sum* of the branch currents.

Example.—A circuit splits up into three parallel branches across which is a P.D. of 100 volts. A contains resistance only and carries a current of 10 amperes; B and C are both inductive, B carrying 8 amperes lagging 60° behind the P.D., and C carrying 15 amperes lagging 45° behind the P.D. Determine the total current.

It is convenient, when dealing with parallel circuits, to choose the instant when the P.D. is passing through its zero value. The vector OF, representing the P.D., can then be set out horizontally (not to scale), and serves as a vector of reference (fig. 8*b*).

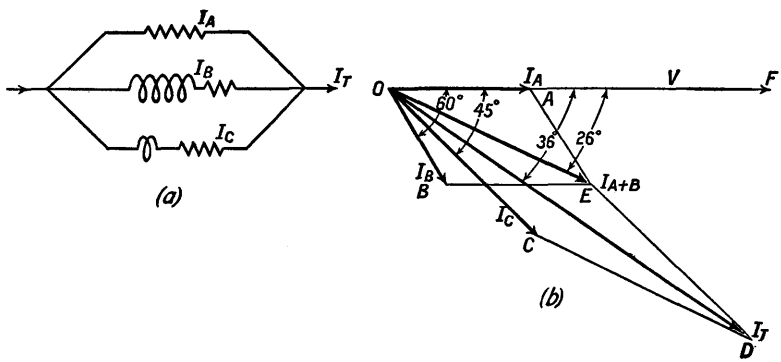


Fig. 8.—Use of vector diagrams

OA is then drawn, in phase with V, representing I_A (10 amperes) to a convenient scale; OB, representing I_B (8 amperes), lagging 60°; and OC, representing I_C (15 amperes), lagging by 45° behind V. Any pair, such as I_A and I_B, are then added vectorially, giving OE, and OE is then added to I_C, giving OD, which represents the total current (I_T) in magnitude and phase.

By measurement $OD = I_T = 30{\cdot}2$ amperes, lagging behind the P.D. by 36°.

[Alternatively, resolving horizontally and vertically:

Horizontal components: $10 + 8 \cos 60° + 15 \cos 45°$
$= 10 + 4 + 10{\cdot}6 = 24{\cdot}6.$

Vertical components: $0 + 8 \sin 60° + 15 \sin 45°$
$= 0 + 6{\cdot}93 + 10{\cdot}6 = 17{\cdot}5.$

$$I = \sqrt{(24{\cdot}6)^2 + (17{\cdot}5)^2} = 30{\cdot}2 \text{ amperes},$$

$$\angle DOF = \tan^{-1}\frac{17{\cdot}5}{24{\cdot}6} = \tan^{-1} 0{\cdot}711 = 35{\cdot}4°.]$$

8. Kirchhoff's Laws applied to a.c. Circuits (see also p. 83 *et seq.*)

Kirchhoff's laws in the form stated on pp. 83–4 are equally true for a.c. circuits when *instantaneous* values are being considered; but when, as is usually the case, r.m.s. values are used, the word *vector* must be substituted for *algebraic*.

The first law then becomes:

The vector sum of the currents in all conductors meeting at a point is zero.

The second law may be stated in the form:

In a closed circuit the vector sum of the E.M.F.s is equal to the vector sum of the potential drops due to resistance.

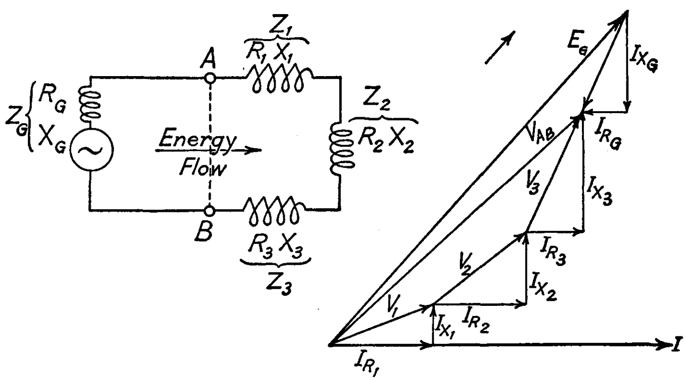


Fig. 9.—To illustrate Kirchhoff's laws

With regard to the P.D. between any two points in the circuit, it is simplest to consider the case where these two points are the supply terminals.

In the portion of the circuit on the supply side of the terminals, i.e. the portion *from* which energy is transferred, the terminal P.D. is given by the vector sum of the E.M.F.s in this portion of the circuit, *minus* the component required to overcome the resistance.

On the other hand, in the portion of the circuit which has been connected to the terminals, and *to* which energy is transferred, the terminal P.D. is given by the vector sum of components *equal and opposite* to the E.M.F.s in this portion of the circuit, *plus* the component required to overcome the resistance.

This is illustrated by the vector diagram in fig. 9 (which, however, will not be appreciated fully until the remainder of the chapter has been read), which shows an alternator with internal impedance Z_G supplying a circuit consisting of three impedances Z_1, Z_2, and Z_3 in series. Considering the left-hand portion of the circuit, the P.D. V_{AB} between the terminals A and B of the alternator is the vector sum of the generated E.M.F., E_G, and the reactance E.M.F., IX_G (90° behind the current), minus the component IR_G (in phase with the current) required to overcome the resistance. In the right-hand portion of the circuit, however, the terminal P.D. is the vector sum of IX_1, IX_2, and IX_3 equal and opposite to the respective reactance E.M.F.s (and therefore 90° ahead of the current) plus IR_1, IR_2, and IR_3, the components required to overcome the respective resistances of the three impedances.

These relationships have been treated at some length because, in many cases, the vector representing the terminal P.D. or generated E.M.F. is built up from a knowledge of the various components which it has to supply; and it is therefore necessary to be able to distinguish between vectors representing actual E.M.F.s and vectors representing components which are equal and opposite to E.M.F.s.

9. Circuit containing Resistance only

When the circuit is non-inductive* and contains resistance only, the current at any instant is

$$i = \frac{e}{R},$$

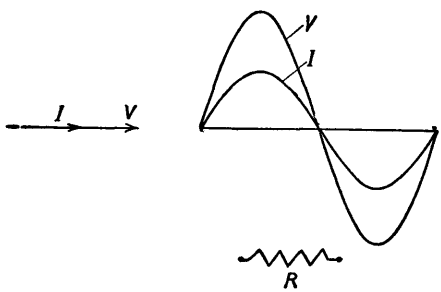


Fig. 10.—Circuit containing resistance only

where e is the value of the E.M.F. at that instant. Similarly, in any portion of the circuit

$$i = \frac{v}{R},$$

where v is the instantaneous value of the P.D. between the ends of that portion.

* All circuits are inductive to some extent; but when the inductance is negligibly small compared with the resistance, the circuit is termed non-inductive, e.g. lighting and heating circuits.

Hence the current is represented by a sine wave *in phase* with that representing the E.M.F. or P.D. as indicated in fig. 10, which also shows the corresponding vector relation.

Since P.D. and current attain their maximum values simultaneously,

$$I_m = \frac{V_m}{R}$$

and, dividing each side by $\sqrt{2}$,

$$\frac{I_m}{\sqrt{2}} = \frac{V_m}{\sqrt{2} \, . \, R}.$$

Hence $$I = \frac{V}{R} \quad \text{or} \quad V = IR, \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where V and I are r.m.s. values.

In a circuit containing resistance only, the current is

$$\boldsymbol{I = \frac{V}{R}}$$

and is in phase with the supply P.D.

It will be noticed that this expression is exactly the same as for the d.c. circuit.

10. Circuit containing Inductance only

When an inductive coil is connected to an a.c. supply an alternating current flows, setting up an alternating flux which induces in each turn an alternating E.M.F. of self inductance. If the resistance is negligibly small and there is no other counter-E.M.F. in the circuit, the total E.M.F. of self inductance must, at every instant, be equal and opposite to the supply P.D. The current and the flux, which (neglecting any saturation effects in an iron core) is proportional to and in phase with it, can be represented by the same sine wave as shown in fig. 11:

$$i = I_m \sin \omega t \text{ and } \Phi = \Phi_m \sin \omega t.$$

Now the E.M.F. of self-inductance (e_x) is proportional to the *rate of change* of the flux (equation 1, p. 146), i.e. to the *slope* of the flux wave. It is therefore a maximum when the flux is passing through zero, and zero when the flux is a maximum, and is, in fact, represented by a cosine wave, i.e. another sine wave displaced by $\frac{\pi}{2}$ radians or 90° from the flux wave. Further, since the E.M.F. of self inductance, in accordance with Lenz's law, always opposes the change in flux and current to which it is due, its *negative* maximum occurs at a when the current and flux are passing through zero and about to increase *positively*: it is therefore represented by a sine wave *lagging* 90° behind the flux and current waves. And since this is the only counter-E.M.F. in the circuit the supply P.D. is a sine wave of equal amplitude 90° *ahead* of the current.

It is more usual to express the phase of the current relative to the supply P.D., i.e. the current lags 90° behind the supply P.D.

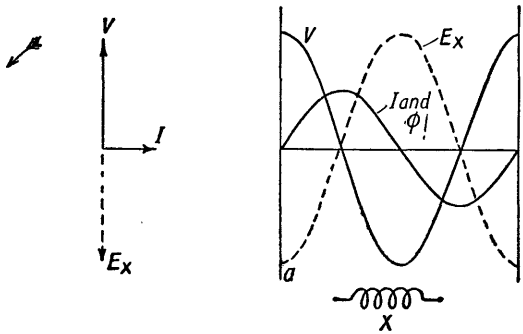


Fig. 11.—Circuit containing inductance only

If N is the number of turns with which the flux Φ_m is linked,

$$e_x = -N\frac{d\Phi}{dt} \text{ volts.*}$$

But $$N\frac{d\Phi}{dt} = L\frac{di}{dt}, \quad \text{from eq. (18), p. 168,}$$

$$\therefore\ e_x = -L\frac{di}{dt}$$

$$= -LI_m\frac{d\sin\omega t}{dt}$$

$$= -LI_m\,\omega\cos\omega t$$

$$= LI_m\,\omega\sin(\omega t - 90°); \quad . \quad . \quad (15)$$

i.e. the induced E.M.F. is a sine wave lagging 90° behind the current.

Hence $$(E_x)_m = LI_m\omega,$$

and, dividing by $\sqrt{2}$, to obtain r.m.s. values,

$$E_x = \omega LI.$$

* The negative sign, because the E.M.F. always opposes the change in flux by which it is produced.

The quantity ωL is called the *Inductive Reactance* (X) of the circuit,

$$X = \omega L = 2\pi f L, \quad \ldots \ldots \quad (16)$$

and the E.M.F. of self inductance is often called the Reactance E.M.F.

Since there is no other counter-E.M.F. in the circuit,

$$V = E_x = \omega LI = XI,$$

or

$$I = \frac{V}{X} = \frac{V}{2\pi f L}. \quad \ldots \ldots \quad (17)$$

This expression is of the same form as in the non-inductive or d.c. circuit, and therefore reactance has the same dimensions as resistance and is measured in ohms.

In a purely inductive circuit, the current is

$$\boldsymbol{I = \frac{V}{X} = \frac{V}{2\pi f L}}$$

and lags 90° behind the supply P.D.

It should be noted that while the inductance (L) of a coil or circuit is constant,* the reactance (X) is *proportional to the frequency*. The physical meaning of this is clear when it is remembered that if the frequency is doubled the E.M.F. of self inductance set up by the same flux is doubled, and therefore the supply P.D. to produce the same current and flux must also be doubled: in other words, the reactance $\left(X = \frac{V}{I}\right)$ is doubled.

Example 1.—Calculate the current taken by a coil of negligible resistance and having an inductance of 0·08 henry when connected to a 100-volt supply having a frequency of (*a*) 25 cycles, (*b*) 50 cycles per second.

(It is convenient to remember that the value of ω ($=2\pi f$) is 157 at 25 cycles, and 314 at 50 cycles per second).

(*a*) Inductive reactance $(X) = \omega L = 2\pi f L = 2\pi \times 25 \times 0{\cdot}08$

$$= 157 \times 0{\cdot}08 = 12{\cdot}56 \text{ ohms.}$$

$$I = \frac{V}{X} = \frac{100}{12{\cdot}56} = 7{\cdot}96 \text{ amperes.}$$

(*b*)

$$X = 314 \times 0{\cdot}08 = 25{\cdot}12 \text{ ohms.}$$

$$I = \frac{V}{X} = \frac{100}{25{\cdot}12} = 3{\cdot}98 \text{ amperes.}$$

* The effect of increasing saturation of an iron core is here neglected.

11. Circuit containing Inductance and Resistance in Series

This case covers the majority of series circuits which occur in practice, in which the capacitance is negligibly small.

The supply P.D. (V) can be looked upon as made up of two components, one, V_R ($=IR$), overcoming the resistance and therefore *in phase* with the current, and the other, V_X ($=IX$), equal and opposite to the reactance E.M.F. and therefore 90° *ahead* of the current. If the resistance of the inductive coil can be neglected, V_R and V_X are the voltages which would be read by a voltmeter connected across R and X respectively. The supply P.D. read by a voltmeter connected across the two in series is clearly the vector resultant of V_R and V_X, and leads the current by an angle φ, the value of which lies between 0° and 90°.*

The property possessed by a circuit in virtue of the combined effect of resistance (R) and reactance (X) is called its *Impedance* (Z); and the general equation of an a.c. circuit, corresponding with Ohm's law, for the d.c. circuit, is:

$$\boldsymbol{V = IZ}. \qquad \qquad (18)$$

From the vector diagram it is clear that, since the resistance and reactance components are always at right angles,

$$(IZ)^2 = (IR)^2 + (IX)^2;$$

and, dividing by I^2,

$$Z^2 = R^2 + X^2,$$

or

$$\boldsymbol{Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + (2\pi f L^2)}}, \qquad \qquad (19)$$

i.e. **Impedance = $\sqrt{\textbf{Resistance}^2 + \textbf{Reactance}^2}$.**

Impedance has the same dimensions as resistance and reactance, and is therefore measured in ohms.

Further, it is clear from the vector diagram that

$$\tan \varphi = \frac{V_X}{V_R} = \frac{IX}{IR} = \frac{X}{R}, \qquad \qquad (20)$$

i.e. the tangent of the phase angle by which the current lags behind the P.D. is given by the ratio of the reactance to the resistance.

Hence:

In a circuit containing resistance and inductance in series the current is given by

$$\boldsymbol{I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (2\pi f L)^2}}}, \qquad \qquad (21)$$

and lags behind the P.D. by an angle $\boldsymbol{\varphi = \tan^{-1} (X/R)}$.

* In practice the resistance of the inductive coil cannot be neglected. In such cases the supply P.D. is still the vector resultant of the P.D.s across the two portions, but the phase angle between them is less than a right angle (see § 12), and their values are not exactly those of V_R and V_X.

This general statement includes the particular cases considered in §§ 9 and 10. In the non-inductive circuit $X = 0$ so that $Z = R$ and $\varphi = \tan^{-1}(0/R) = 0°$. In the inductive circuit as the ratio X/R is increased either by increasing X or by decreasing R, the angle φ gradually increases until finally, when R is negligibly small, $Z=X$ and $\varphi = \tan^{-1}(X/0) = 90°$.

Although R, X, and Z are not themselves vector quantities, it follows from the vector diagram (fig. 12*a*) that their relationship may be represented as shown in fig. 12*b*. This is often known as the *impedance triangle*.

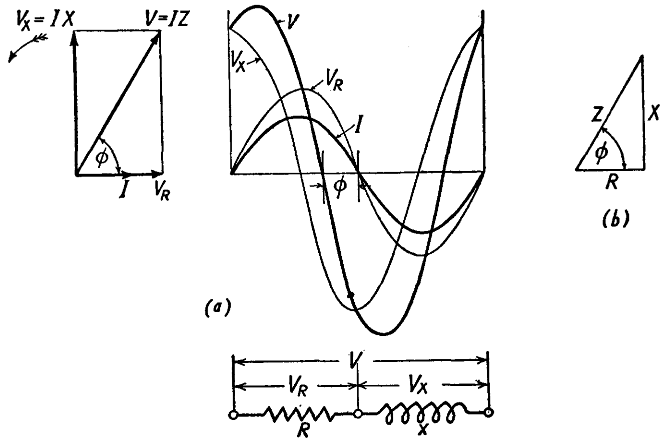


Fig. 12.—Circuit containing resistance and inductance in series

Example.—A non-inductive resistance of 80 ohms is connected in series with a coil of negligible resistance having an inductance of 0·3 henry. Calculate the value of the current, its phase relationship to the P.D., and the P.D. across the inductance, when the circuit is connected to a 100 volt supply having a frequency of (*a*) 25 cycles, (*b*) 50 cycles per second.

(*a*) $R = 80$ ohms, $X = 2\pi \times 25 \times 0{\cdot}3 = 47$ ohms,

$$Z = \sqrt{80^2 + 47^2} = 92{\cdot}8 \text{ ohms},$$

$$I = \frac{100}{92{\cdot}8} = 1{\cdot}08 \text{ amps}, \ \varphi = \tan^{-1}\frac{47}{80} = 30{\cdot}4° \text{ lag},$$

$$V_X = 1{\cdot}08 \times 47 = 50{\cdot}7 \text{ volts}.$$

(*b*) $R = 80$ ohms, $X = 47 \times \dfrac{50}{25} = 94$ ohms,

$$Z = \sqrt{80^2 + 94^2} = 123{\cdot}5 \text{ ohms},$$

$$I = \frac{100}{123{\cdot}5} = 0{\cdot}81 \text{ amp}, \ \varphi = \tan^{-1}\frac{94}{80} = 49{\cdot}6° \text{ lag},$$

$$V_X = 0{\cdot}81 \times 94 = 76{\cdot}1 \text{ volts}.$$

It will be noticed that increase in frequency, by increasing the inductive reactance, increases the impedance, so that the current decreases, and increases the angle of lag. At the same time the voltage distribution across the two portions of the circuit changes. At zero frequency (d.c.) the P.D. across the inductance is zero and V_R is equal to the supply voltage. If the frequency is raised, V_R decreases and V_X increases until at an infinitely high frequency the resistance would be negligible compared with the reactance and V_X would become equal to the supply P.D. This should be compared with the corresponding case of the circuit containing capacity reactance in § 14.

12. Impedances in Series

The circuit shown in fig. 13 contains two coils in series; in the first the reactance is small and in the second it is large compared with the resistance. Since the current is the same in all parts of a series circuit, the current vector is taken as the vector of reference and is usually drawn horizontally. The supply P.D. is the vector sum of the P.D.s across the two coils, and each of these consists of two components, one overcoming the resistance, and the other equal and opposite to the reactance E.M.F. The component IR_1, overcoming the resistance of the first coil, is in phase with the current vector, while the component IX_1, equal and opposite to the reactance E.M.F., is 90° ahead of the current vector. It is usually more convenient to use the vector triangle rather than the parallelogram, so that IX_1, is set up, not at O but at the end of IR_1. The resultant of these two, V_1 $(=IZ_1)$, is the P.D. across the first coil (which would be read by a voltmeter connected across its ends), and leads the current by an angle φ_1. From the end of V_1, IR_2 is set off horizontally, and at its extremity IX_2 is drawn vertically, the resultant being V_2 $(=IZ_2)$, the P.D. across the second coil, leading the current by a larger angle φ_2. The P.D. across the two coils in series, i.e. the supply voltage V_t $(=IZ_t)$, is the vector sum of V_1 and V_2 and leads the current by an angle φ_t.

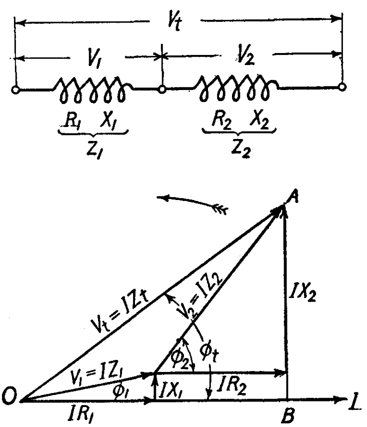


Fig. 13.—Impedances in series

It is clear from the diagram that

$$V_t^2 = (IZ_t)^2 = OA^2 = OB^2 + BA^2 = (IR_1 + IR_2)^2 + (IX_1 + IX_2)^2$$

or

$$Z_t^2 = (R_1 + R_2)^2 + (X_1 + X_2)^2,$$

and in the general case,

$$\boldsymbol{Z}_t = \sqrt{(\boldsymbol{R}_1 + \boldsymbol{R}_2 + \ldots)^2 + (\boldsymbol{X}_1 + \boldsymbol{X}_2 + \ldots)^2}. \qquad (22)$$

Further,
$$\tan \varphi_t = \frac{AB}{OB} = \frac{X_1 + X_2 + \ldots}{R_1 + R_2 + \ldots},$$

or
$$\boldsymbol{\varphi}_t = \tan^{-1} \frac{\boldsymbol{X}_1 + \boldsymbol{X}_2 + \ldots}{\boldsymbol{R}_1 + \boldsymbol{R}_2 + \ldots}. \qquad (23)$$

For the separate coils,

$$Z_1 = \sqrt{R_1^2 + X_1^2}, \qquad Z_2 = \sqrt{R_2^2 + X_2^2},$$

$$V_1 = IZ_1, \qquad V_2 = IZ_2,$$

$$\varphi_1 = \tan^{-1} \frac{X_1}{R_1}, \qquad \varphi_2 = \tan^{-1} \frac{X_2}{R_2}.$$

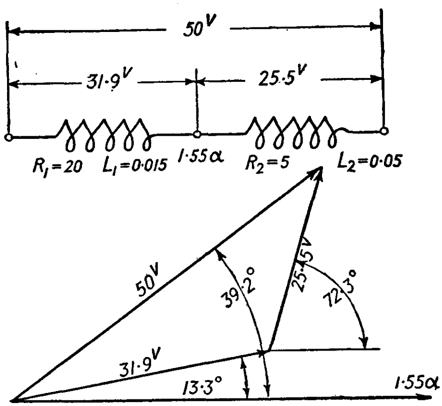


Fig. 13*a*

Example.—A circuit consists of two coils in series. In the first $R_1 = 20$ ohms $L_1 = 0{\cdot}015$ henry; in the second $R_2 = 5$ ohms, $L_2 = 0{\cdot}05$ henry. If the circuit (fig. 13*a*) is connected to a 50 volt 50 cycle supply, calculate

(*a*) the current and its phase relationship to the supply voltage;

(*b*) the P.D. and the phase angle for each coil.

$R_1 = 20$ ohms, $X_1 = 2\pi \times 50 \times 0{\cdot}015 = 4{\cdot}71$ ohms,

$$Z_1 = \sqrt{20^2 + 4{\cdot}71^2} = 20{\cdot}55 \text{ ohms}.$$

$R_2 = 5$ ohms, $X_2 = 2\pi \times 50 \times 0{\cdot}05 = 15{\cdot}7$ ohms,

$$Z_2 = \sqrt{5^2 + 15{\cdot}7^2} = 16{\cdot}45 \text{ ohms}.$$

$$Z_t = \sqrt{(20+5)^2 + (4{\cdot}71 + 15{\cdot}7)^2} = 32{\cdot}28 \text{ ohms.}$$

$$I = \frac{V_t}{Z_t} = \frac{50}{32{\cdot}28} = 1{\cdot}55 \text{ amps.}$$

$$\tan \varphi_t = \frac{15{\cdot}7 + 4{\cdot}71}{20+5} = 0{\cdot}816; \ \varphi_t = 39{\cdot}2^\circ \text{ lag.}$$

$$V_1 = IZ_1 = 1{\cdot}55 \times 20{\cdot}55 = 31{\cdot}9 \text{ volts.}$$

$$\tan \varphi_1 = \frac{4{\cdot}71}{20} = 0{\cdot}236; \ \varphi_1 = 13{\cdot}3^\circ \text{ lag.}$$

$$V_2 = IZ_2 = 1{\cdot}55 \times 16{\cdot}45 = 25{\cdot}5 \text{ volts.}$$

$$\tan \varphi_2 = \frac{15{\cdot}7}{5} = 3{\cdot}14; \ \varphi_2 = 72{\cdot}3^\circ \text{ lag.}$$

13. Circuit containing Capacitance only

The property of capacitance is treated in detail in Chapter XIV, which can be read with advantage before considering the present section. For the present purpose it is sufficient to note that:

(*a*) Any pair of conductors, separated by an insulating medium or dielectric, possess the property of *capacitance*, in virtue of which they are capable of receiving a quantity of electricity or *charge* which establishes a P.D. between the conductors, the work done during the charging process being stored in the electric field set up between the conductors.

(*b*) While the capacitance between the conductors in a circuit is usually extremely small, it is possible to dispose them, generally in the form of parallel plates separated by a thin layer of insulating material, so that the capacitance is large, and the arrangement is then known as a *capacitor.**

A capacitor may be looked upon as a storage reservoir analogous to a compressed air vessel. When such a vessel is connected to a compressed air supply, air flows into it at a rate which is a maximum at starting but continuously decreases as the pressure inside the vessel increases, and finally becomes zero when the internal pressure is equal and opposite to the supply pressure. The quantity (weight) of air held in such a reservoir can be increased only by increasing the pressure, to which it is proportional, and is limited only by the safe value of the stresses set up in the material of which it is made. The work done in the charging process remains stored up as potential energy until the valve is opened, when the direction of air flow is reversed and the stored energy is released and can be utilized in suitable apparatus.

When a capacitor is connected to a d.c. supply, the continuous flow of a current is impossible, since the circuit is broken by the

* *Capacitor* is the preferred term (B.S. 205), although the older term—*condenser*—is still in use.

dielectric; but a *displacement* of electrons takes place, consisting of their withdrawal from one plate, which is thereby left with a +ve charge, and their addition to the other, which receives an equal −ve charge. The dielectric is thereby put into a state of strain and becomes the seat of an increasing counter-E.M.F. (dielectric strain E.M.F.);* and the transient movement of electrons constitutes a *charging current* which continually decreases and finally ceases when the capacitor E.M.F. has become equal and opposite to the charging P.D. The energy stored in the dielectric by the flow of the charging current against this E.M.F. is retained after disconnection from the supply, until the plates are connected by a conductor: the capacitor E.M.F. then causes a *discharge* current to flow in the opposite direction by which the stored energy is converted into heat.

The *capacity* of the compressed air vessel could be expressed in terms of the quantity of air stored per unit pressure, e.g. in lb. per atmosphere.

Similarly, the *capacitance* (C) of a capacitor is measured in terms of the quantity of electricity stored per unit P.D. between the plates ($C = Q/V$). A capacitor in which one coulomb is stored when the P.D. is one volt has a capacitance of one farad; but such a capacitor would be extremely large, so that the common units are the microfarad and the micromicrofarad (or picofarad),

$$1 \text{ microfarad } (\mu\text{F}) = 10^{-6} \text{ farad (F)}$$

$$1 \text{ micromicrofarad } (\mu\mu\text{F}) = 10^{-12} \text{ farad } (= 1 \text{ picofarad}).$$

When a capacitor is connected to a source of alternating P.D. no such steady state as that described above occurs, since it is charged, discharged, charged in the opposite direction, and again discharged in consecutive quarters of a cycle, so that the charge and discharge currents constitute an alternating current which continues to flow as long as the alternating P.D. is maintained.

Superficially, the action of a capacitor in an a.c. circuit appears to be different from that in a d.c. circuit, and the statement is sometimes made that an alternating current can pass through a capacitor but a direct current cannot. That there is no difference in behaviour, and that in neither case does a current pass *through* the capacitor, can be shown by a hydraulic analogy. Fig. 14*a* shows a simple hydraulic circuit consisting of tubing, one portion of which is enlarged to form a chamber A, and in which a constant "E.M.F.", provided by a centrifugal pump, causes a steady current of water to flow right round the circuit. This is clearly analogous to the simple resistance circuit shown in fig. 14*b*. In fig. 14*c*, the chamber A is divided by a water-tight diaphragm of some elastic material. The pressure of the pump cannot now produce a continuous flow,

* Since a capacitor current is associated with the conversion of electrical energy into some other form (dielectric strain energy) or vice versa, it is permissible to refer to the P.D. between the plates as being due to the existence of an E.M.F.

but the diaphragm is deflected as shown, so that a transient displacement current flows, some water being forced out of the left-hand portion of A and an equal quantity entering the right-hand portion. The movement ceases when the restoring force set up by the deflection of the diaphragm is equal and opposite to the total force acting on it due to the pump, the work done being stored as potential strain energy in the diaphragm. This is, of course, analogous to the action of a capacitor in a d.c. circuit as shown in fig. 14*d*. If the valves at B and C are closed, and the pump removed and replaced by a length of tubing, the energy remains stored until the valves are opened, when the restoring force of the diaphragm restores it to its original shape and the stored energy is released and turned into heat by an equal displacement current in the opposite direction.

If the centrifugal pump is now replaced by a reciprocating pump, as shown in fig. 14*e*, the diaphragm is obviously deflected first in one direction and then in the other, and an alternating displacement of water occurs, so that at every point in the tubing there is an alternating flow or current of water, which continues as long as the pump is in operation. This is clearly analogous to conditions in the a.c. circuit containing a capacitor, shown in fig. 14*f*. It should be noted that in neither (*c*) nor (*e*) does water pass *through* the chamber as in (*a*), i.e. no current passes *through* the capacitor in either case.

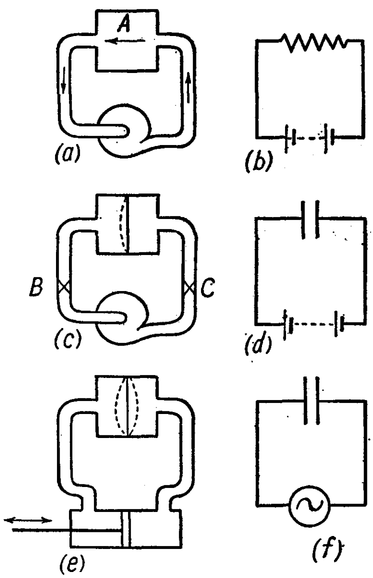


Fig. 14.—To illustrate the action of a capacitor in d.c. and a.c. circuits

The physical explanation of the phase relationship between the P.D. and current in a capacitor connected to an a.c. supply can be understood from the following (perhaps somewhat unacademic) reasoning. If a constant charging current is imagined to flow, the E.M.F. of the capacitor will rise uniformly, just as the pressure increases uniformly in a vessel into which air is passing at a constant rate. This is represented by the portion *ab* of the curves in fig. 15*a*. Suppose now that at *b* the current is reversed; discharge begins, the E.M.F. falls, and at the end of an equal time *bc*, the capacitor is fully discharged. If the current continues to flow in the same direction, at the end of a further equal period of time *cd*, the capacitor will have an equal charge in the opposite direction. If the current is finally reversed back to its original direction, after a further period *de*, equal to *cd*, the capacitor will again be discharged. It therefore appears that if the current is represented by a rectangular wave, the capacitor E.M.F. is a triangular wave which attains its maximum when the current is passing through zero, and is therefore displaced from it by 90°.

By rounding off the corners, the square wave can be made to approximate to a sine wave as shown by the broken line; and since this results in the diminution of current at the end of the charging and the beginning of the discharge period, the effect is to flatten and round off the E.M.F. wave. In fact, if the current wave is a sine wave, the capacitor E.M.F. wave is also a sine wave, leading by 90°; and the supply P.D., which is equal and opposite to the capacitor E.M.F., lags 90° behind the current as shown in fig. 15*b*.

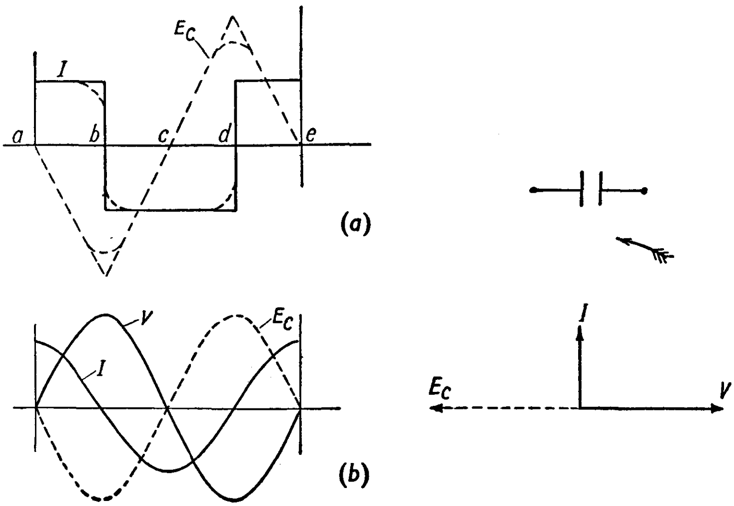


Fig. 15.—Circuit containing capacitance only

In practice, as in previous cases, the phase relationship is usually stated in relation to the supply P.D. Hence:

In a circuit containing capacitance only, the current leads the P.D. by 90°.

Alternatively:

The capacitance current (i) at any instant is equal to the rate of change of the charge (q) (just as the rate of flow of water in a pipe filling a tank is equal to the rate of increase of the quantity of water in the tank), i.e.

$$i = \frac{dq}{dt};$$

and since, from the definition of capacitance on p. 267, $q = Cv$, where C is the capacitance and v is the instantaneous value of the P.D.,

$$i = C\frac{dv}{dt}.$$

Further, in an a.c. supply, $v = V_m \sin \omega t$, so that

$$i = CV_m \frac{d \sin \omega t}{dt} = \omega C V_m \cos \omega t$$

$$= \omega C V_m \sin (\omega t + 90°),$$

i.e. the current is represented by a sine wave leading the P.D. by 90°.

Since the maximum value of a sine function is unity,

$$I_m = \omega C V_m,$$

and dividing by $\sqrt{2}$ to obtain r.m.s. values,

$$I = \omega C V.$$

In order to obtain a relationship similar in form to that in equation 17, p. 261, the quantity $1/\omega C$ is called the *Capacitive Reactance* (X) of the circuit, i.e.

$$X = \frac{1}{\omega C}\left(= \frac{1}{2\pi f C}\right), \quad . \quad . \quad . \quad . \quad . \quad (24)$$

so that
$$I = \frac{V}{X}(= V\omega C = V \,.\, 2\pi f C). \quad . \quad . \quad . \quad . \quad (25)$$

In a circuit containing capacitance only, the current is given by

$$I = \frac{V}{X} = V \,.\, 2\pi f C$$

and leads the P.D. by 90°.

It follows that, as in the case of inductive reactance, capacitive reactance is measured in ohms. When C is expressed in microfarads, as is more usual,

$$X = \frac{10^6}{\omega C} \text{ ohms.}$$

Where it is necessary to distinguish between the two kinds of reactance, inductive reactance is represented by X_L and capacitive reactance by X_C.

Two points should be noted:

(1) Whereas inductive reactance ($X_L = \omega L$) is *directly* proportional to both inductance and frequency, capacitive reactance $\left(X_C = \frac{1}{\omega C}\right)$ is *inversely* proportional to both capacitance and frequency (see fig. 16a). The latter is of course obvious from physical considerations. If, for instance, the capacitance is doubled while the P.D. and frequency remain constant, twice the quantity of electricity must flow into or out of the capacitor in the same time. If, on the other hand, the fre-

quency is doubled while the P.D. and the capacitance remain constant, the same quantity of electricity must flow into or out of the capacitor in half the time. In both cases, therefore, the rate of flow or current is doubled, i.e. the reactance $\left(X_C = \frac{V}{I}\right)$ is halved.

(2) Relative to the same current, the P.D. across a pure inductance (leading the current by 90°) and the P.D. across a pure capacitance

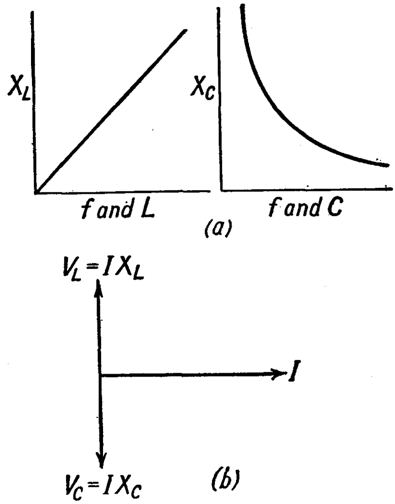


Fig. 16.—Characteristics of capacitive and inductive reactance

(lagging by 90°) are in phase opposition (see fig. 16*b*). The two reactances are therefore looked upon as opposite in sign, inductive reactance (X_L) being considered positive and capacitive reactance (X_C) negative.

Example.—Find (*a*) the reactance of a capacitor having a capacitance of 60 μF, at a frequency of 50 c/s;

(*b*) the frequency at which a capacitor of 100 μF has a reactance of 80 ohms;

(*c*) the capacitance of a capacitor which has a reactance of 200 ohms at a frequency of 25 c/s.

$$(a)\quad X = \frac{10^6}{2\pi f C} = \frac{10^6}{2\pi \,.\, 50 \,.\, 60} = 53{\cdot}1 \text{ ohms.}$$

$$(b)\quad f = \frac{10^6}{2\pi X C} = \frac{10^6}{2\pi \,.\, 80 \,.\, 100} = 19{\cdot}9 \text{ c/s.}$$

$$(c)\quad C = \frac{10^6}{2\pi f X} = \frac{10^6}{2\pi \,.\, 25 \,.\, 200} = 31{\cdot}8\ \mu\text{F.}$$

14. Resistance and Capacitance in Series

A circuit containing a resistance and a capacitor in series is shown in fig. 17. As in the case of the inductive circuit (§ 11), the property possessed by a circuit in virtue of the combined effect of resistance (R) and capacitive reactance (X_C) is called the Impedance (Z), so that

$$V = IZ.$$

The supply P.D., V, can be considered as made up of two components, one the P.D. across the resistance, V_R ($=IR$), in phase with the current, and the other the P.D. across the capacitor, V_X ($=IX_C$), lagging 90° behind the current. V_R and V_X are the voltages which would be read by a voltmeter connected across R and C respectively. The supply P.D., V, read by a voltmeter connected across the two in series is the vector resultant of V_R and V_X, and lags behind the current by an angle φ.

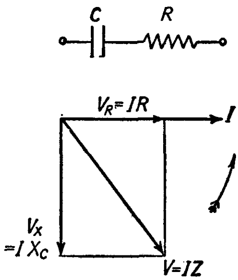


Fig. 17

From the vector diagram it is clear that

$$(IZ)^2 = (IR)^2 + (-IX_C)^2,$$

from which

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}, \quad . \quad . \quad (26)$$

which is similar to equation (19) for the inductive circuit, and

$$\varphi = \tan^{-1}\frac{-X_C}{R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

This is a negative angle, indicating that the P.D. *lags behind* the current or that the current *leads* the P.D.

Hence: **In a circuit containing resistance and capacitance in series, the current is**

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (1/2\pi fC)^2}}, \quad . \quad . \quad . \quad (28)$$

and leads the P.D. by an angle $\tan^{-1}\dfrac{X_C}{R}$.

Example.—A circuit contains a resistance of 30 ohms in series with a capacitor of 100 microfarads.

Calculate the value of the current, its phase relationship to the P.D., and the P.D. across the capacitor, when the circuit is connected to a 100 volt supply having a frequency of (a) 25 cycles, (b) 50 cycles per second.

(a) $R = 30$ ohms, $X = -\dfrac{10^6}{157 \times 100} = -63{\cdot}7$ ohms,

$Z = \sqrt{30^2 + 63{\cdot}7^2} = 70{\cdot}3$ ohms,

$I = \dfrac{100}{70{\cdot}3} = 1{\cdot}42$ amps, $\varphi = \tan^{-1} - \dfrac{63{\cdot}7}{30} = 64{\cdot}8°$ leading,

$V = 1{\cdot}42 \times 63{\cdot}7 = 90{\cdot}5$ volts.

(b) $R = 30$, $X = -63{\cdot}7 \times \dfrac{25}{50} = -31{\cdot}85$ ohms,

$Z = \sqrt{30^2 + 31{\cdot}85^2} = 43{\cdot}7$ ohms,

$I = \dfrac{100}{43{\cdot}7} = 2{\cdot}29$ amps, $\varphi = \tan^{-1} - \dfrac{31{\cdot}85}{30} = 46{\cdot}7°$ leading,

$V_X = 2{\cdot}29 \times 31{\cdot}85 = 72{\cdot}9$ volts.

It will be noticed that increase in frequency, by decreasing the capacitive reactance, decreases the impedance so that the current increases, and decreases the angle of lead. At the same time the voltage distribution across the two portions of the circuit changes. At zero frequency (d.c.) V_R is zero and V_X is equal to the supply voltage. If the frequency is raised, V_R increases and V_X decreases until at an infinitely high frequency the capacitive reactance would become zero and V_R would be equal to the supply P.D. This should be compared with the corresponding case of the inductive circuit in § 11.

15. Resistance, Inductance and Capacitance in Series

In this case, illustrated in fig. 18, V_R is in phase with the current, V_L is 90° ahead of the current (assuming the resistance of the coil is negligibly small), and V_C is 90° behind the current; the supply P.D., V, is the resultant of the three components.

$$V^2 = (V_L - V_C)^2 + V_R^2,$$

$$\text{i.e. } (IZ)^2 = (IX_L - IX_C)^2 + (IR)^2,$$

from which

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \ldots \quad (29)$$

and

$$\tan \varphi = \frac{X_L - X_C}{R}. \quad \ldots \quad (30)$$

It is clear from the diagram that if V_L is greater than V_C, V will lead, and if V_C is greater than V_L, V will lag behind the current. In other words, since the two types of reactance are opposite in characteristics, the effective reactance is $(X_L - X_C)$, and is either inductive or capacitive, depending upon which predominates. If X_L is greater than X_C, the effective reactance is inductive and the current lags;

while if X_C is greater than X_L, the effective reactance is capacitive and the current leads.

Example.—A resistance of 15 ohms is connected in series with an inductance of 0·07 henry and a capacitance of 200 μF to a 100 volt supply. Calculate the value and phase angle of the current when the frequency is (*a*) 25 cycles, (*b*) 50 cycles per second.

(*a*) $R = 15$ ohms, $X_L = 2\pi \times 25 \times 0{\cdot}07 = 10{\cdot}98$ ohms,

$$X_C = -\frac{10^6}{2\pi \times 25 \times 200} = -31{\cdot}9 \text{ ohms},$$

$$Z = \sqrt{15^2 + (10{\cdot}98 - 31{\cdot}9)^2} = 25{\cdot}8 \text{ ohms},$$

$$I = \frac{100}{25{\cdot}8} = 3{\cdot}88 \text{ amps}, \ \tan\varphi = -\frac{20{\cdot}92}{15} = -1{\cdot}39, \ \varphi = 54{\cdot}3^\circ \text{ leading}.$$

(*b*) $R = 15$ ohms, $X_L = 10{\cdot}98 \times \dfrac{50}{25} = 21{\cdot}96$ ohms,

$$X_C = -31{\cdot}9 \times \frac{25}{50} = -15{\cdot}95 \text{ ohms},$$

$$Z = \sqrt{15^2 + (21{\cdot}96 - 15{\cdot}95)^2} = 16{\cdot}18 \text{ ohms},$$

$$I = \frac{100}{16{\cdot}18} = 6{\cdot}19 \text{ amps}, \ \tan\varphi = \frac{6{\cdot}01}{15} = 0{\cdot}4, \ \varphi = 22^\circ \text{ lag}.$$

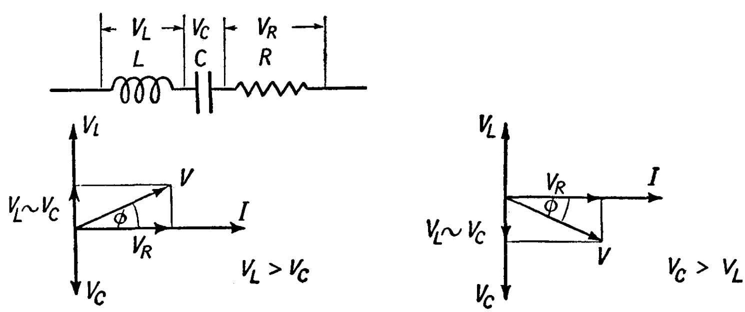


Fig. 18.—Circuit containing resistance, inductance and capacitance in series

Hence:

In a current containing resistance, inductance and capacitance in series, the current is given by

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}, \quad \ldots \ldots \quad (31)$$

and leads, lags behind, or is in phase with the P.D. according as X_C is greater than, less than, or equal to X_L.

Series or Voltage Resonance.

In the particular case where $X_L = X_C$ the effective reactance becomes zero, so that the impedance is equal to the resistance and

$$I = \frac{V}{Z} = \frac{V}{R} \quad \text{and} \quad \varphi = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{0}{R} = 0°,$$

i.e. the inductive and capacitive reactances completely neutralize one another as far as the circuit as a whole is concerned; the current is limited only by the resistance and is in phase with the P.D. This condition is known as *series* or *voltage resonance.*

Consider a circuit containing inductance and capacitance in series connected to a constant-voltage variable-frequency supply. At very low frequencies the inductive reactance is small but the capacitive reactance is large: hence the impedance is large and the current small and leading by a large angle. As the frequency is raised, X_L increases and X_C decreases, so that the effective reactance, and hence the impedance, decreases, the current increases and the angle of lead decreases. At the frequency for which $X_L = X_C$, the *resonant frequency,* the impedance reaches a minimum value—becoming equal to the resistance—so that the current is a maximum and is in phase with the voltage. If the frequency is increased above the resonant value, X_L continues to increase and X_C to decrease: the effective reactance again increases but is now inductive, so that the current diminishes and lags behind the voltage by an angle which increases as the frequency rises. The way in which the inductive and capacity reactances and the current vary as the frequency is raised from well below to well above the resonant frequency, is shown in fig. 19*a*.

Since the resonant frequency (f_r) is that at which $X_L = X_C$,

$$\omega_r L = \frac{1}{\omega_r C},$$

from which $\omega_r^2 LC = 1$,

hence $$\omega_r = \sqrt{\frac{1}{LC}} \quad \text{or} \quad f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}. \qquad (32)$$

The P.D.s across the inductance V_L ($=IX_L$) and the capacitance V_C ($=IX_C$), although having no resultant effect on the circuit as a whole, exist individually (see fig. 19*b*), and, since the current is large, may reach values far in excess of the supply voltage. This property, the *magnification* of the P.D.s across inductance and capacitance, while it makes resonance a condition to be avoided in industrial power circuits, is of fundamental importance in radio-engineering since the series resonant circuit or *acceptor* circuit is the basic circuit of the radio receiver.

By offering minimum impedance to currents of the resonant frequency, it is able to select or *accept* most readily currents of this one frequency from among those of many frequencies, set up by E.M.F.s induced in the aerial by electromagnetic waves of many different wave-lengths simultaneously impinging on it.

The process of *tuning* consists in adjusting the resonant frequency (by varying the value of the inductance or, more usually, the capacitance) to correspond with that of the wave-length it is desired to receive.

Example.—A circuit consists of an inductive coil, $R = 5$ ohms and $L = 0{\cdot}2$ henry, in series with a condenser $C = 50\ \mu$F. Calculate (*a*) the resonant frequency, (*b*) the current and the P.D. across the capacitor when the circuit is connected to a 50 volt supply of this frequency.

(a)
$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{0{\cdot}2 \times 50 \times 10^{-6}}}$$
$$= \frac{10^3}{2\pi}\sqrt{\frac{1}{10}} = 50{\cdot}4 \text{ c/s.}$$

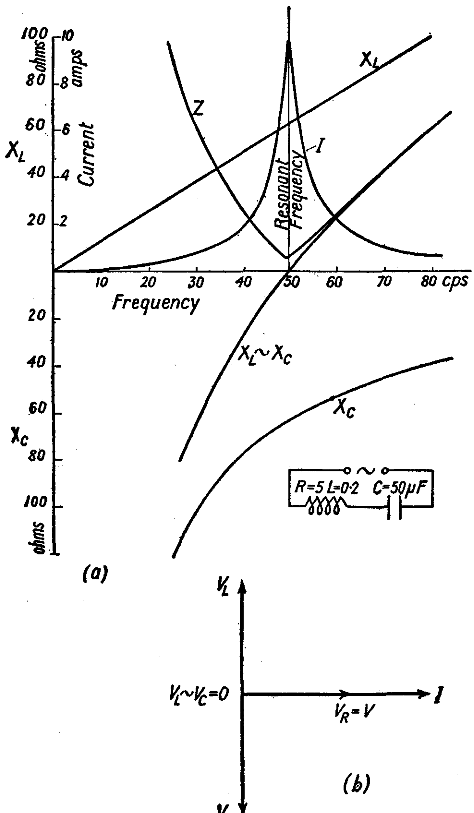


Fig. 19.—Series resonance

(b) At this frequency $Z = R$, so that

$$I = \frac{50}{5} = 10 \text{ amperes,}$$

$$X_C = 2\pi \times 50{\cdot}4 \times 0{\cdot}2 = 63{\cdot}4 \text{ ohms,}$$

$$V_C = IX_C = 634 \text{ volts.}$$

16. Power in a.c. Circuits

The power in an a.c. circuit at any instant, is given by the product of the values of P.D. and current at that instant,

$$p = vi,$$

just as in the d.c. case, but its value, and usually the direction of energy transfer, vary during the cycle; and it is the *average* value throughout the cycle which is of importance, and is indicated by a wattmeter (see § 10, p. 196).

Circuit containing Resistance only (Current in phase with P.D.).*

When the current and P.D. are in phase, the power wave, obtained by plotting the product of v and i at various points during the cycle, is shown in fig. 20a. Since both P.D. and current reverse simultaneously and remain negative throughout the second half of the cycle their product is always positive, so that the power wave, although falling to zero twice per cycle, lies wholly on the positive side of the axis: it is, of course, the same curve as in fig. 3.

As mentioned in § 4, the average height of the wave, obtained by direct measurement of the area under it, is half the maximum height, i.e.

$$\text{Average value of the power} = \frac{V_m I_m}{2}.$$

Alternatively, since $v = V_m \sin \theta$, and $i = I_m \sin \theta$,

$$\begin{aligned} p = vi &= V_m I_m \sin^2 \theta \\ &= \tfrac{1}{2} V_m I_m (1 - \cos 2\theta). \quad . \quad . \quad . \quad . \quad (33) \end{aligned}$$

This expression shows that the power at any instant can be considered as made up of a constant portion $\frac{1}{2} V_m I_m$ and a variable portion $\frac{1}{2} V_m I_m \cos 2\theta$, i.e. a cosine wave of double frequency (see fig. 20b). The average value of the cosine wave over one complete cycle is clearly zero, i.e. the average value of the variable portion of the power is zero, so that

$$\begin{aligned} \text{Average value of power} &= \text{constant portion of above expression for } p \\ &= \tfrac{1}{2} V_m I_m. \end{aligned}$$

This can be written, $$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = VI, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

where V and I are r.m.s. values.

* The expressions for the power obtained in this section are equally true when the circuit contains rotating electrical machines in which the phase of the current relative to the P.D. is not entirely determined by the values of the resistance and inductance in the circuit.

Hence, when the current is in phase with P.D.,

$$\text{Average power} = (\text{r.m.s. P.D.}) (\text{r.m.s. current}),$$

just as in the d.c. circuit.

Circuit containing Resistance and Inductance or Capacitance (Current lagging or leading).

The current lagging behind the P.D. wave by an angle φ is shown in fig. 20*c*; and the power wave is obtained as before by plotting a number of corresponding values of v and i. In this case, however, the

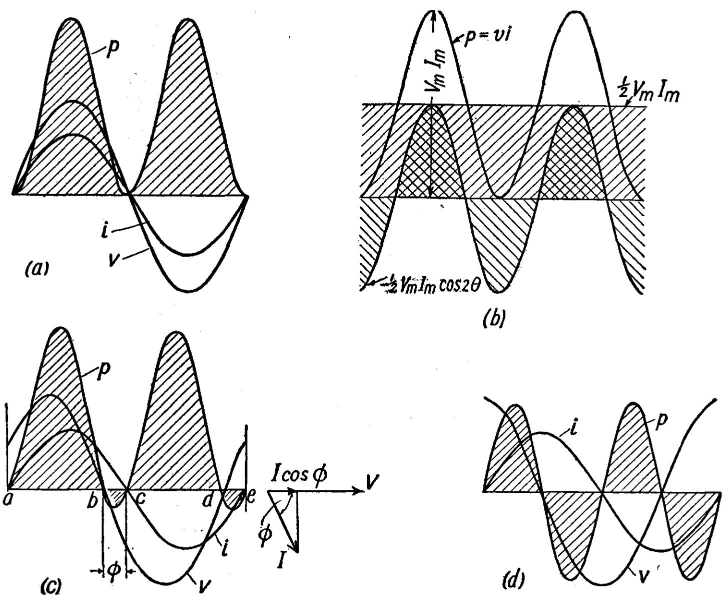


Fig. 20.—Power in a.c. circuits

power wave is positive between a and b when both v and i are positive, and between c and d when both are negative; but between b and c, and between d and e, when v and i are of opposite sign, the power wave is on the negative side of the axis.

Actually, for the same values of V and I, the power wave is the same as that in fig. 20*a* except that its horizontal axis has been lowered so that negative loops are formed. Hence, in obtaining the area under the power curve, that of the negative loops must be subtracted, so that the net area and therefore the average height, i.e. the average power, is less than in the previous case.

Since the circuit is inductive, part of the energy represented by the area under the power curve between a and b is stored in the magnetic field established by the increasing current. When the field collapses, this energy is *given back* to the circuit,* and is therefore represented by the area of the negative loop. This portion of the energy, which is alternately stored and released, is not available *outside* the circuit; so that for the same value of V and I, the average value of the power is less than in the non-inductive case.

If $$v = V_m \sin\theta,\ i = I_m \sin(\theta - \varphi),$$
$$p = vi = V_m I_m \sin\theta \sin(\theta - \varphi)$$
$$= \frac{V_m I_m}{2}\{\cos\varphi - \cos(2\theta - \varphi)\} \quad . \quad . \quad . \quad (35)$$

(expressing the product of the sines as the difference of two cosines, i.e. $\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$).

Here again, the power at any instant is made up of a constant portion $\frac{1}{2}V_m I_m \cos\varphi$ (since φ is constant), and a variable cosine term, of double frequency, $\frac{1}{2}V_m I_m \cos(2\theta - \varphi)$, the average value of which over a complete period is zero, so that

$$\text{Average value of power} = \tfrac{1}{2}V_m I_m \cos\varphi$$
$$= VI \cos\varphi.$$

Hence, when there is a difference of phase between the P.D. and the current, the power is less than the product of P.D. and current (VI): the latter is sometimes called the *apparent power*, and is expressed in *volt-amperes*. The ratio Watts/Voltamperes is called the **power factor** because it is the factor by which the volt-amperes must be multiplied in order to obtain the true power in watts.

For sinusoidal quantities, the power factor is equal to $\cos\varphi$, the cosine of the phase angle between P.D. and current.

The above is equally true for a circuit containing resistance and capacitance, in which the current *leads* the P.D. by an angle φ and part of the energy is alternately stored in and released from the *electrostatic field* between the plates of the condenser.

Therefore in any a.c. circuit:

$$\textbf{Power (watts)} = \textbf{Volt-amperes} \times \textbf{power factor}$$
$$= \boldsymbol{V.I.\cos\varphi}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

* Since energy cannot be stored *in* the circuit, the energy released by the collapsing magnetic or electrostatic field in the portion of the circuit under consideration is immediately stored up elsewhere, e.g. in the magnetic fields associated with the alternator supplying the circuit.

It should be noted that the expression for the power in a circuit containing resistance only, $P = VI$, is the particular case of the general expression above, when P.D. and current are in phase so that $\varphi = 0$ and $\cos \varphi = 1$.

It follows that in most a.c. circuits the value of the power cannot be obtained from the voltmeter and ammeter readings as in the d.c. circuit, but must be measured by means of a wattmeter (see § 10, Chapter X).

Circuit containing Inductance (or Capacitance) only.

An increase of the phase angle between P.D. and current causes a further lowering of the axis of the power wave, with consequent decrease in the area of the positive and increase in the area of the negative loops. In the extreme case where $\varphi = 90°$, the axis coincides with that of the P.D. and current waves, and the +ve and −ve loops are equal in area, as shown in fig. 20*d*. The average value of the power is therefore zero:

$$P = VI \cos \varphi = 0, \text{ since } \varphi = 90° \text{ and } \cos \varphi = 0.$$

This would be the condition in the case of a pure inductance, in which the *whole* of the energy is stored up in the magnetic field during one quarter cycle and returned to the circuit as the field collapses during the next quarter cycle.

In practice there is always some resistance present which requires the expenditure of a small amount of energy, so that the phase angle is slightly less than 90°, and the +ve areas somewhat greater than the −ve areas.

Conditions are similar in the case of a pure capacitance except that the phase of the P.D., which now lags 90° behind the current, is reversed. Here the energy is stored in the electric field during one quarter cycle and returned, as the field collapses, during the next quarter cycle. The dielectric loss in the capacitor requires the expenditure of a very small amount of energy; but in an air-capacitor ideal conditions are very nearly attained.

17. Power Factor: Active and Reactive Components of Current

In the circuits considered in this chapter in which there are no dynamic or generated E.M.F.s, such as occur in rotating electrical machines, the value of the power factor may be obtained from the constants of the circuit. Referring to the impedance triangle (fig. 12):

$$\text{Power Factor} = \cos \varphi = \cos \tan^{-1} (X/R),$$

$$\text{or alternatively} = R/Z. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

When there is a phase displacement between P.D. and current, as shown in fig. 21, the latter can be resolved into two components, one in phase and the other in quadrature with the P.D.

The in-phase component is $I \cos \varphi$, so that the power in the circuit can be expressed in the form:

$$\text{Power} = VI \cos \varphi = \text{P.D.} \times \text{Component of current in phase with it.}$$

Hence $I \cos \varphi$ is often called the *active*, *in-phase*, or *energy component*.*

The other component is $I \sin \varphi$, and is called the *reactive*, *quadrature*, *idle*, or *wattless component*.* The product $VI \sin \varphi$ is termed the *wattless power* or *reactive volt-amperes*, and represents the energy alternately stored in the magnetic or electric field and returned to the circuit, which cannot be utilized by conversion to other forms.

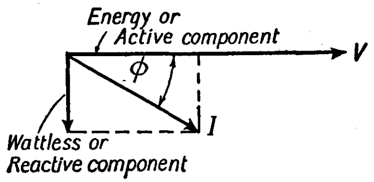


Fig. 21.—Active and reactive components of current

Inductive rather than capacitive reactance predominates in most industrial circuits, and a low lagging power factor decreases the efficiency and reduces the output of generators, transformers and the transmission lines and distribution systems to which they are connected, so that every endeavour is made to maintain the power factor of a system at a high value by the use of machines of the most suitable type or of corrective apparatus, a discussion of which is beyond the scope of the present volume.

18. Parallel Circuits

When a circuit, such as that shown in fig. 22*a*, consists of several parallel branches, the magnitude and phase of the current in each branch depend on its electrical characteristics, while the total current is the vector sum of the branch currents.

In a parallel circuit the P.D., which is the same for each branch, is taken as the reference vector in the vector diagram and is usually drawn horizontally. The first branch contains resistance and inductance in series, so that the current is (from equation 21)

$$I_1 = \frac{V}{Z_1} = \frac{V}{\sqrt{R_1^2 + X_1^2}} = \frac{V}{\sqrt{R_1^2 + (2\pi f L_1)^2}},$$

lagging by an angle $\qquad \varphi_1 = \tan^{-1} \dfrac{X_1}{R_1}.$

* Active and reactive are the preferred terms (B.S.I. Glossary of Terms).

The second branch contains resistance and capacitance in series, and the current is (from equation 26)

$$I_2 = \frac{V}{Z_2} = \frac{V}{\sqrt{R_2^2 + X_2^2}} = \frac{V}{\sqrt{R_2^2 + \left(\frac{1}{2\pi f C_2}\right)^2}},$$

leading by an angle $\qquad \varphi_2 = \tan^{-1} \frac{X_2}{R_2}.$

The total current and power factor can then be found graphically by setting out each branch current to scale in its proper phase relationship and adding vectorially as shown in fig. 22*b*.

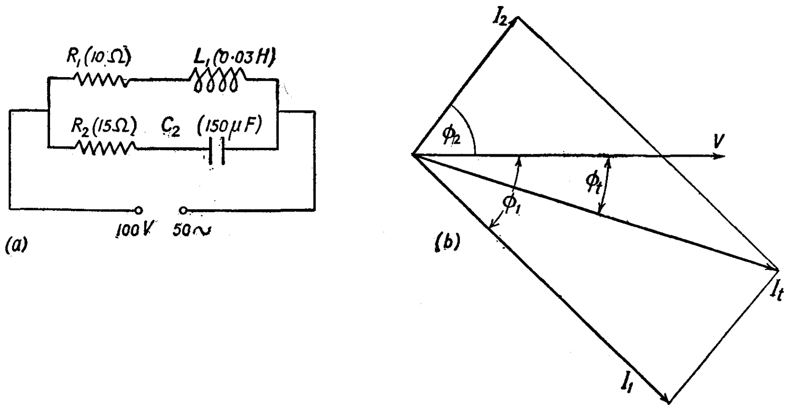


Fig. 22

Alternatively, the branch currents may be resolved into horizontal (active) and vertical (reactive) components:

$$\text{Active components} = I_1 \cos \varphi_1 + I_2 \cos \varphi_2,$$

$$\text{Reactive components} = -I_1 \sin \varphi_1 + I_2 \sin \varphi_2.*$$

$$I_t = \sqrt{(I_1 \cos \varphi_1 + I_2 \cos \varphi_2)^2 + (I_2 \sin \varphi_2 - I_1 \sin \varphi_1)^2}, \quad (38)$$

$$\varphi_t = \tan^{-1} \frac{I_2 \sin \varphi_2 - I_1 \sin \varphi_1}{I_1 \cos \varphi_1 + I_2 \cos \varphi_2}. \quad \ldots \ldots \ldots \quad (39)$$

It is obvious that both these methods can be used for any number of branches.

It will be noticed that the reactive leading component of the current in the capacitive branch neutralizes part of the reactive lagging component of the current in the inductive branch. The current

* Since the P.D. vector is horizontal, leading components of the current are +ve and lagging components −ve.

taken from the supply is therefore more nearly in phase with the voltage than the current taken by the inductive branch alone.

Capacitors are frequently used in a.c. circuits solely for this purpose. When connected across the terminals of inductive apparatus, such as induction motors, the combined power factor is raised and the current taken from the supply is reduced.

Example 1.—In the circuit shown in fig. 22*a*, using the values given in brackets,

$$X_1 = 2\pi \times 50 \times 0{\cdot}03 = 9{\cdot}42 \text{ ohms},$$

$$Z_1 = \sqrt{10^2 + 9{\cdot}42^2} = 13{\cdot}75 \text{ ohms},$$

$$I_1 = \frac{100}{13{\cdot}75} = 7{\cdot}28 \text{ amps}, \quad \varphi_1 = \tan^{-1}\frac{9{\cdot}42}{10} = 43{\cdot}3^\circ \text{ lagging};$$

$$X_2 = \frac{10^6}{2\pi \times 50 \times 150} = 21{\cdot}2 \text{ ohms},$$

$$Z_2 = \sqrt{15^2 + 21{\cdot}2^2} = 26 \text{ ohms},$$

$$I_2 = \frac{100}{26} = 3{\cdot}85 \text{ amps}, \quad \varphi_2 = \tan^{-1}\frac{21{\cdot}2}{15} = 54{\cdot}7^\circ \text{ leading}.$$

The vectors representing these currents are set out to scale in the vector diagram (fig. 22*b*), and added vectorially, giving a total current of 7·7 amps lagging 14° behind the P.D.

Alternatively, resolving each current horizontally and vertically,

$$\begin{aligned}\text{Active components} &= 7{\cdot}28 \cos 43{\cdot}3^\circ + 3{\cdot}85 \cos 54{\cdot}7^\circ \\ &= (7{\cdot}28 \times 0{\cdot}73) + (3{\cdot}85 \times 0{\cdot}58) = 7{\cdot}53. \\ \text{Reactive components} &= -7{\cdot}28 \sin 43{\cdot}3^\circ + 3{\cdot}85 \sin 54{\cdot}7^\circ \\ &= (-7{\cdot}28 \times 0{\cdot}69) + (3{\cdot}85 \times 0{\cdot}82) = -1{\cdot}85. \\ \text{Total current } I_t &= \sqrt{7{\cdot}53^2 + 1{\cdot}85^2} = 7{\cdot}75 \text{ amps}, \\ \varphi_t &= \tan^{-1}\frac{-1{\cdot}85}{7{\cdot}53} = 14^\circ \text{ lagging}.\end{aligned}$$

Example 2.—The power input to a 240-volt, 50-c/s single-phase motor is 2 kW. at a power factor of 0·71 lagging. Find (*a*) the current, (*b*) the combined power factor and the current taken from the supply, if an 80-μF. capacitor is connected across its terminals.

(*a*) $\text{Current} = \dfrac{2000}{240 \times 0{\cdot}71} = 11{\cdot}7$ amps. $\varphi = \cos^{-1} 0{\cdot}71 = 45^\circ$.

Active component $= 11{\cdot}7 \times 0{\cdot}71 = 8{\cdot}3$ amps.

Reactive component $= 11{\cdot}7 \times \sin\varphi = 8{\cdot}3$ amps.

(*b*) $X_C = \dfrac{10^6}{314 \times 80} = 39{\cdot}8$ ohms, $I_C = \dfrac{240}{39{\cdot}8} = 6{\cdot}02$ amps.

Lagging component reduced to $8{\cdot}3 - 6{\cdot}02 = 2{\cdot}28$ amps.

Total current $= \sqrt{(8{\cdot}3^2 + 2{\cdot}28^2)} = 8{\cdot}6$ amps.

$\varphi = \tan^{-1}\dfrac{2{\cdot}28}{8{\cdot}3} = 15{\cdot}3^\circ$. $\cos\phi = 0{\cdot}96$ lagging.

Hence the addition of the capacitor has raised the power factor from 0·71 to 0·96 lagging and reduced the current from 11·7 to 8·6 amps.

Parallel or Current Resonance.

When an inductance of negligible resistance and a capacitor are connected in parallel to an a.c. supply (fig. 23), the currents in the two branches lag and lead respectively by 90°, and are therefore in phase opposition, so that the total current is the difference of the two ($I_T = I_L - I_C$) and is either lagging or leading, depending upon which of the two branches has the lower reactance.

If the frequency is gradually raised from zero, I_L decreases from an infinitely large value (since $R=0$), while I_C increases from zero. The result is that I_T gradually decreases, and becomes zero at the frequency at which $I_L = I_C$—the resonant frequency. Under these conditions the closed circuit consisting of the two branches continues to oscillate at this frequency—its *natural* frequency—and since there are no losses ($R = 0$) it requires no further supply of energy from the external circuit (see p. 379).

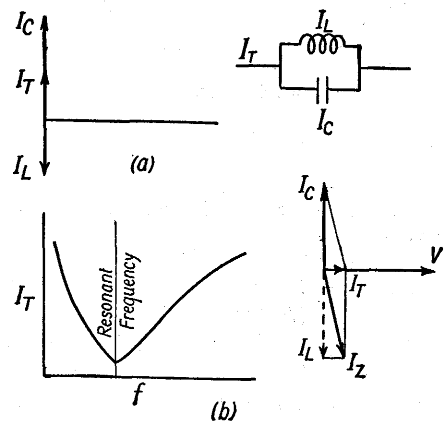


Fig. 23.—Parallel resonance

Since the resonant frequency is that at which $I_L = I_C$,

$$\frac{V}{\omega L} = V\omega C,$$

from which

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

i.e. the same value as for the series case.

In practice some resistance is always present, particularly in the inductive branch, and the oscillations cannot be maintained without some energy from the external circuit. Hence the current in the inductive branch I_Z lags behind the P.D. by an angle somewhat less than 90°. Resonance occurs when the reactive component of this current I_L is equal and opposite to I_C: under these conditions the resultant current I_T is a minimum (but never falls to zero), and in phase with the supply P.D., as shown in fig. 23*b*.

The value of the natural frequency is not entirely independent of the resistance. In fact, $f = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)}$. This shows that if the value of R exceeds $\sqrt{(L/C)}$, the circuit will not oscillate; but where the resistance is small, equation (40) holds very closely. The parallel resonant circuit is of great importance in radio engineering, because it forms the basic circuit of the wireless transmitter (see p. 380). Further, since the joint impedance of the two parallel branches, relative to the rest of the circuit, becomes very large (infinitely large if $R = 0$) at and near the resonant frequency, it may be used as a *filter* circuit to reject currents of the resonant frequency while offering a much lower impedance to currents having frequencies above or below this value: for this reason it is sometimes known as a *rejector* circuit.

19. Polyphase Circuits

The simple alternator of § 1, with its single coil, is essentially a *single-phase* alternator (fig. 24*a*). If a second coil is added, having the same number of turns, but with its plane perpendicular to that of the first, the E.M.F. generated is of the same frequency and amplitude as in the first coil, but owing to their relative displacement, the maximum value occurs in one when that in the other is passing through

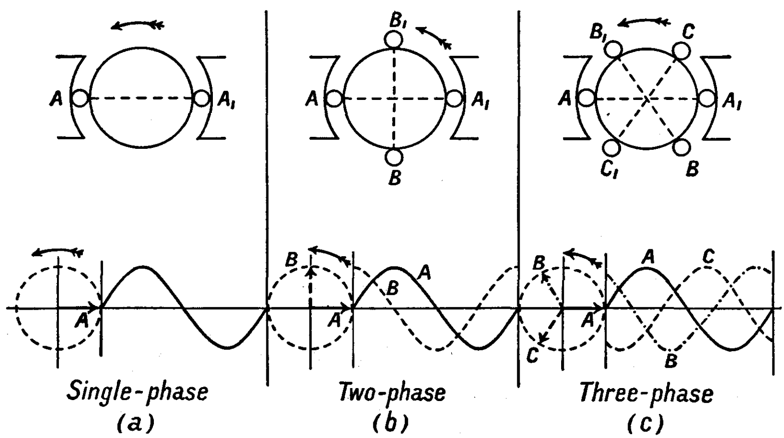


Fig. 24.—Single-, two- and three-phase alternators

zero and vice versa. The two E.M.F.s are therefore in quadrature and represented by two sine waves displaced by a quarter period,

$$e_A = E_m \sin \omega t, \; e_B = E_m \sin (\omega t + 90°), \quad . \quad . \quad (41)$$

or by two vectors of equal length displaced by 90° (fig. 24*b*).

Such an arrangement constitutes a simple *two-phase* alternator, and may be used either to supply two single-phase circuits or to feed a two-phase system supplying other apparatus having windings arranged in a similar manner.

If a third similar coil is added and the three arranged with their planes mutually inclined at 120°, the E.M.F.s are represented by three sine waves of equal frequency and amplitude, but displaced in phase by $\frac{1}{3}$ of a period,

$$e_A = E_m \sin \omega t,\ \ e_B = E_m \sin(\omega t + 120°),\ \ e_C = E_m \sin(\omega t + 240°); \quad (42)$$

or by three vectors of equal length mutually inclined at 120° as shown in fig. 24*c*.

Such an arrangement is essentially a *three-phase* alternator, and may be used to supply three separate single-phase circuits or to feed a three-phase system supplying other apparatus having windings arranged in a similar manner.

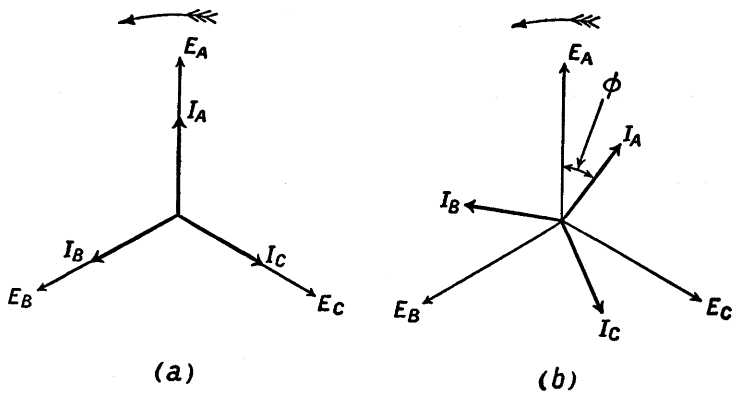


Fig. 25.—E.M.F. and current relations in (*a*) non-inductive and (*b*) inductive three-phase circuits

The value of the current in each winding (or phase) and its phase relationship to the E.M.F. depends upon the characteristics of the circuit of which it forms a part. If these are the same for all three phases (as is usually the case in three-phase machines), the currents have equal maximum values $\left(I_m = \dfrac{E_m}{Z}\right)$, and are represented by three vectors of equal length mutually inclined at 120°, and displaced relative to the E.M.F. vectors by an angle $\varphi = \tan^{-1}\dfrac{X}{R}$, where Z, X, and R are the impedance, reactance, and resistance of each phase circuit (fig. 25).

Assuming that the reactance is inductive,

$$i_A = I_m \sin(\omega t - \varphi),$$
$$i_B = I_m \sin(\omega t + 120° - \varphi),$$
$$i_C = I_m \sin(\omega t + 240° - \varphi).$$

The use of more than one phase (in polyphase systems and apparatus) has a number of advantages:

(1) It enables the active material of a machine to be used more efficiently, so that the output of a polyphase machine is greater than that of a single-phase machine of the same size.

(2) The total power output of a polyphase machine is constant, instead of fluctuating as in the single-phase machine, and the general operating characteristics are better.

The three-phase system is superior to the two-phase system in certain respects, particularly with regard to transmission, and since there is little advantage in any further increase in the number of phases, it has become the standard system for generation, transmission and distribution of electrical energy throughout the world.

When the E.M.F.s, currents and phase angles have the same values in each phase, the system is said to be *balanced.*

20. Star and Mesh Connections

If an ordinate is drawn to cut the three waves shown in fig. 24*c*, it will be found that the sum of the intercepts on the ordinate is always zero. These waves may represent either E.M.F.s or the currents which they produce, so that:

The sum of the instantaneous values of the E.M.F.s or currents in a balanced three-phase system is always zero, i.e.

$$e_A + e_B + e_C = 0, \quad \text{and} \quad i_A + i_B + i_C = 0. \quad . \quad . \quad (43)$$

This property enables the three-phase windings to be so interconnected inside the machine, that only three external leads are necessary.

Star Connection (Y).

A reference to the vector or wave diagram of fig. 24*c* shows that at any instant, in one or other of the three phases, there are E.M.F.s and currents in both +ve and −ve directions; so that some convention is clearly necessary as to which direction is to be considered positive. In an actual machine each phase winding is distributed in several slots occupying one-third of each pole pitch, and all the windings are similar, but with a phase displacement of 120°, so that the starting ends and the finishing ends are displaced by two-thirds of a pole pitch. Hence if the E.M.F. in one phase has just reached its maximum value in a direction from start to finish of the winding, then one-third of a period later the E.M.F. in the second phase winding has reached a similar value in the same direction, and one-third of a period later still the same thing happens in the third winding. It is therefore convenient to assume that the +ve direction of E.M.F. and current is from start to finish of the winding.

In fig. 26a the phase windings of a three-phase machine are shown with the +ve directions marked. If the three leads from the starting ends are grouped together, it is clear from what has been said above that the sum of the currents in this group is always zero: hence these three ends may be connected together and the leads dispensed with * (fig. 26b). This common point is called the *neutral point* or *star point.*

In this form of connection there are two phase windings between each pair of terminals, but since *similar* ends have been joined, they are connected in *opposition.* Hence the instantaneous value of the E.M.F. between terminals (or lines) is the difference of the phase E.M.F.s:

$$e_{AB} = e_B - e_A,$$

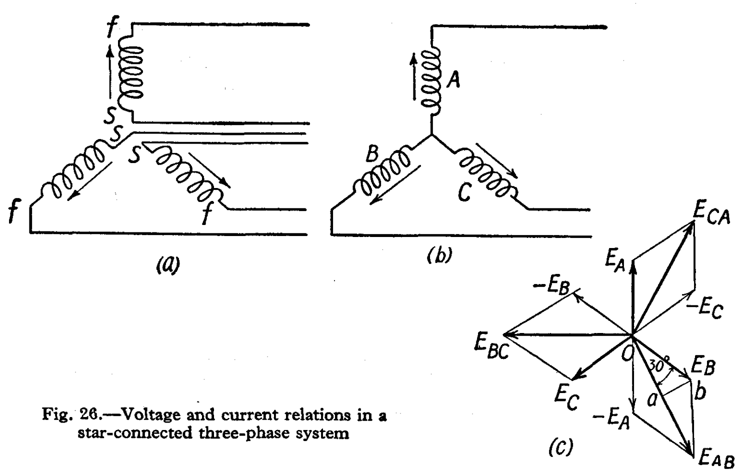


Fig. 26.—Voltage and current relations in a star-connected three-phase system

and the r.m.s. value is given by the *vector* difference of the r.m.s. values of the phase E.M.F.s, so that E_{AB} is obtained by subtracting E_A (i.e. adding $-E_A$) from E_B (fig. 26c),

$$E_{AB} = 2Oa = 2Ob \cos 30° = 2Ob \,.\, \sqrt{3}/2 = \sqrt{3}E_B,$$

and lags behind E_B by 30°.

Similarly, E_{BC} is obtained by subtracting E_B from E_C, and E_{CA} by subtracting E_C from E_A. Further, since each line is connected to one phase winding only, the line current is also the phase current. Therefore, in general terms:

In a star connection

$$\textbf{Line E.M.F.} = \sqrt{3}\ \textbf{Phase E.M.F.}$$

$$(E_l = \sqrt{3}E_{ph})$$

* Alternatively, since $i_A + i_B + i_C = 0$, $i_A = -i_B - i_C$, and each phase can be looked upon as carrying the return current of the other two.

and there is a phase displacement of 30° between them:

Line current = Phase current

$$(I_l = I_{ph})$$

This method of connection is the one usually adopted for alternator windings, and also for the secondary winding of a transformer supplying a mixed lighting and power load.

Mesh or Delta Connection (Δ).

In this form of connection the three windings are joined in series to form a closed mesh, and the lines are connected to the three junctions, as shown in fig. 27, where the +ve directions of E.M.F. and current are indicated as before. At first sight the connection appears to be a

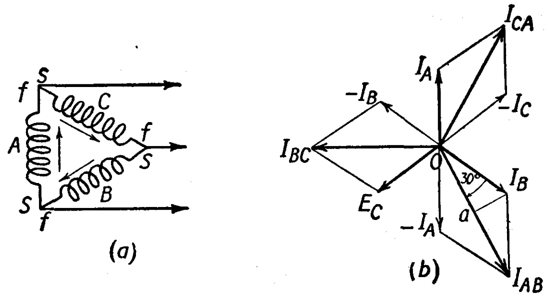


Fig. 27.—Voltage and current relations in a mesh-connected three-phase system

dead short-circuit, until it is remembered that, owing to the phase displacement between them, the sum of the instantaneous values of the three E.M.F.s is always zero, so that there is no resultant circulating current produced round the mesh.

In this case there is one phase winding between each pair of lines, so that the phase E.M.F. is also the line E.M.F. On the other hand, at the junction between the phases A and C (for instance), taking the positive directions of the currents to be that indicated by the arrows, it follows, from Kirchhoff's First Law, that

$$\dot{i}_A - \dot{i}_C - \dot{i}_{CA} = 0 \quad \text{or} \quad \dot{i}_{CA} = \dot{i}_A - \dot{i}_C,$$

i.e. the line current is the difference between the currents in the two phases to which it is connected. Hence the vector diagram of fig. 26*c* may now be used to represent currents (fig. 27*b*), from which it follows that:

In a mesh connection

Line current = $\sqrt{3}$ Phase current

$$(I_l = \sqrt{3}\, I_{ph})$$

and there is a displacement of 30° between them:

$$\textbf{Line E.M.F.} = \textbf{Phase E.M.F.}$$

$$(E_l = E_{ph})$$

21. Power in a 3-Phase System

Since a balanced three-phase system consists of three similar circuits,

$$\text{Total power} = 3\,(\text{Power in each phase}) = 3E_{ph}I_{ph}\cos\varphi, \quad (44)$$

where $\cos\varphi$ is the phase power factor, i.e. φ is the angle between *phase* E.M.F. and *phase* current.

In the star connection

$$E_{ph} = \frac{E_l}{\sqrt{3}} \quad \text{and} \quad I_{ph} = I_l;$$

therefore,

$$\text{Total power} = 3E_{ph}I_{ph}\cos\varphi = 3\frac{E_l}{\sqrt{3}}I_l\cos\varphi$$
$$= \sqrt{3}E_lI_l\cos\varphi.$$

In the mesh connection

$$E_{ph} = E_l \quad \text{and} \quad I_{ph} = \frac{I_l}{\sqrt{3}};$$

therefore,

$$\text{Total power} = 3E_{ph}I_{ph}\cos\varphi = 3E_l\frac{I_l}{\sqrt{3}}\cos\varphi$$
$$= \sqrt{3}E_lI_l\cos\varphi.$$

Hence for the same value of line E.M.F., line current, and power factor, the total power is the same whichever way the phase windings are connected. Similarly, when a piece of apparatus, such as a motor, is connected to a three-phase supply, in which the P.D. between lines (or terminals) is V_l,

$$\text{Total power input} = \sqrt{3}V_lI_l\cos\varphi, \quad . \quad . \quad . \quad (45)$$

whether the apparatus is star- or mesh-connected. Both methods are in common use, and each has particular advantages in certain cases.

In general:

The total power in any balanced three-phase system is

$$\sqrt{3}\,V_l\,I_l\cos\varphi,$$

where V_l and I_l are the P.D. between lines and the line current, respectively, and $\cos\varphi$ is the phase power factor.

Example 1.—In a star-connected alternator the E.M.F. generated in each phase is 1905 volts.

Calculate the E.M.F. between terminals, and the total power output, when the current in each phase is 250 amps and the power factor is 0·8.

$$\begin{aligned}\text{Total output} &= 3\ (\text{phase output})\\ &= 3 \times 1905 \times 250 \times 0{\cdot}8 \times 10^{-3}\\ &= 1143\ \text{kW}.\end{aligned}$$

Since the winding is star-connected,

$$\text{Terminal E.M.F. (or Line E.M.F.)} = E_l = \sqrt{3} \times 1905 = 3300 \text{ volts.}$$

Alternatively:

$$\text{Total output} = \sqrt{3}E_lI_l \cos\varphi = \sqrt{3} \times 3300 \times 250 \times 0{\cdot}8 \times 10^{-3} = 1143\ \text{kW}.$$

Example 2.—If the alternator in the above example supplies a mesh-connected induction motor, having an efficiency of 91 per cent and working at a power factor of 0·9, calculate (*a*) the phase current of the motor, (*b*) the motor output in horse-power, when the line current is 250 amps.

(*a*) Line current $= 250$ amps.

Phase current $= \dfrac{250}{\sqrt{3}} = 144$ amps.

(*b*) Input $= \sqrt{3} \times 3300 \times 250 \times 0{\cdot}9 \times 10^{-3} = 1285$ kW.

Output $= 1285 \times 0{\cdot}91 \div 0{\cdot}746 = 1568$ h.p.

22. Measurement of Power in a.c. Circuits

Except in the case of the non-inductive circuit, the power in an a.c. circuit cannot be obtained from readings of P.D. and current only, and must be measured by means of a wattmeter (see § 10, p. 196).

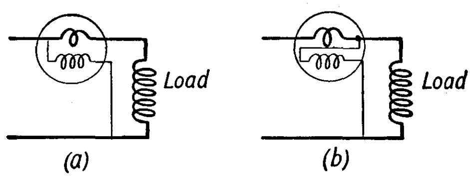


Fig. 28.—Measurement of power in single-phase circuit

Single-phase circuits.

The fixed or current coil is connected in series (fig. 28), and carries the circuit current, while the moving or potential coil is connected in parallel so that the P.D. applied to it is that between the ends of the portion of the circuit in which the power is to be measured.

There are two alternative ways in which this connection can be made, as shown in fig. 28. In (*a*) the P.D. across the potential coil includes that across the current coil, so that the wattmeter reading includes the losses in the current coil. In (*b*) this error is avoided by making the potential connection on the other side of the current coil, but in this case the potential coil current is added to that in the current coil, so that the reading now includes the potential coil losses. In most cases these losses are negligible compared with the power being measured, but when the latter is small one method may give more accurate results than the other; in any case, if the resistance of the coils is known, a correction may be made for the losses.

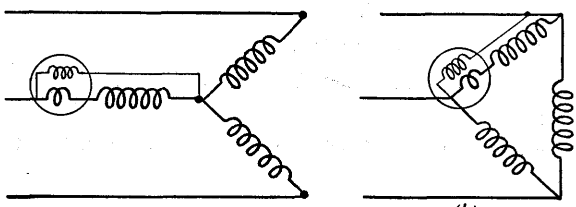


Fig. 29.—Measurement of power in balanced three-phase system with one wattmeter

Three-phase circuits.

When the load is balanced and the phase connections accessible, one wattmeter, inserted in one phase, may be used to measure the phase power, as shown for both star- and mesh-connected loads in fig. 29. Then,

$$\text{Total power} = 3\ (\text{Phase power}) = 3\ (\text{Wattmeter reading}).$$

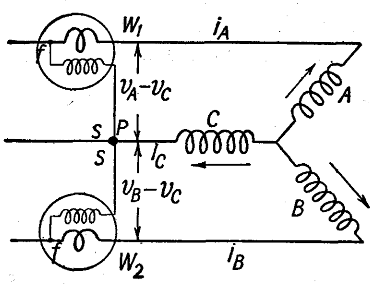


Fig. 30.—Two-wattmeter method of measuring power in three-phase system

A more generally applicable method, and the one most commonly employed, is the *Two-wattmeter Method* shown in fig. 30, in which the current coils of two wattmeters are connected in series with any two of the three lines, and the potential coil of each is connected between the line containing the current coil and the third line. The total power is then given by the *algebraic sum* of the wattmeter readings,

$$\text{Total power} = W_1 + W_2,$$

while the power factor of the circuit may be obtained from the relationship,

$$\tan \varphi = \sqrt{3}\,\frac{(W_1 - W_2)}{W_1 + W_2}.$$

A further discussion of this method is outside the scope of this book.

EXAMPLES

1. A rectangular coil, 20 cm. × 10 cm., containing 20 turns, is rotated at 3000 r.p.m. about an axis parallel with the longer side in a field of uniform density 0·5 weber per sq. m.

Calculate the maximum value of the induced E.M.F. and the frequency.

2. The speed of rotation of the coil in question (1) is reduced until the frequency is 40 cycles per sec.

Calculate (*a*) the maximum value of the E.M.F.;
(*b*) the value of the E.M.F., one-eighth of a revolution after it has attained a positive maximum.

3. A coil similar to that in questions (1) and (2), but having dimensions 30 cm. × 15 cm. and containing 50 turns, is required to generate an E.M.F. having a maximum value of 1000 volts.

If the density of the field is 0·5 weber per sq. m., what is the frequency of the E.M.F.?

4. Calculate the E.M.F. induced in the coil in the previous question at 30° intervals during one complete revolution, and use these to plot the complete sine wave.

5. Draw out the sine wave represented by $i = 10 \sin \theta$, and on the same base plot the (current)^2 wave. By finding the area under this wave, show that

$$\text{r.m.s. value} = 0{\cdot}707 \text{ max. value.}$$

6. An electric kettle, of which the heating element has a resistance of 53 ohms, is connected to an alternating current supply circuit in which the maximum value of the P.D. is 325 volts.

If the efficiency of the kettle is 90 per cent, how long will it take to boil 2 pints of water from 60° F.? [1 B.Th.U. = 1054 joules.]

7. Two coils mounted at right angles to each other on the same spindle generate E.M.F.s of which the maximum values are 10 volts and 20 volts. Draw the vectors representing these quantities, and hence determine the r.m.s. value of the resultant E.M.F. if they are connected in series.

8. Two sinusoidal currents separated by a phase angle of 60° and having amplitudes of 6 amperes and 8 amperes (the smaller current leading in phase) are fed into one conductor. Determine the r.m.s. value of the total current and its phase relationship with its components.

9. The frequency of a sinusoidal current is 50 cycles per sec. and its maximum value 5 amperes. A second current of the same frequency, but having an amplitude of 10 amperes and lagging by 30°, is fed into the same conductor. Draw the vectors in the positions which represent the currents 0·008 sec. after the first current has passed through its zero value.

Determine (*a*) the value of the two currents at this instant; (*b*) the r.m.s. value of the resultant current.

10. A coil has an inductance of 0·03 henry and a resistance of 10 ohms. Determine the current and its phase relationship to the P.D. when the coil is connected to a 25-volt supply at (*a*) 25 cycles, (*b*) 50 cycles.

11. An inductive coil is connected to a 230-volt (r.m.s.) a.c. supply and takes a lagging current of 10 amperes, the angle of phase difference being 60°.
Determine the power factor and the power input.

12. The current taken by an a.c. motor connected to a 200-volt supply is 20 amperes, but a wattmeter connected in the circuit shows that the power input is only 3300 watts. What is the angle of phase difference between the P.D. and the current?

13. A circuit consists of two coils in series. The first has a resistance of 8 ohms and is non-inductive. The second has a resistance of 4 ohms and an inductance of 0·01 henry. If the circuit is connected to a 50-volt 50-c/s supply, calculate (*a*) the current and power factor of the circuit, and (*b*) the P.D. across each coil and its phase relationship to the current.

14. A capacitor having a capacitance of 50 μF is connected in series with a non-inductive resistance of 100 ohms, to a 100-volt 50-cycle supply. Calculate (*a*) the current, (*b*) the phase difference between the current and the supply voltage, (*c*) the power.

15. A circuit consists of an inductive coil $R = 20$ ohms, $L = 0{\cdot}1$ henry, in series with a capacitor of 150 μF. Calculate the current, phase angle and P.D. across the capacitor when connected to a 100-volt supply at (*a*) 25 c/s, (*b*) 50 c/s.

16. Circuit A contains a non-inductive resistance of 40 ohms in series with an inductive coil having a resistance of 10 ohms and inductance of 0·1 henry. Circuit B consists of a resistance of 30 ohms in series with a variable capacitor. If an ammeter is inserted in each and the two are connected in parallel to a 50-c/s supply, what must be the value of the capacitance in order that the ammeter readings may be equal?

17. A circuit consists of two parallel branches. The first contains an inductive coil, $R = 20$ ohms, $L = 0{\cdot}06$ henry; the second contains a resistance of 30 ohms in series with a capacitor of 100 μF. Determine the total current and power factor when the circuit is connected to a 100-volt 50-c/s supply.

18. A circuit contains a 120 μF capacitor in series with an inductive coil in which the resistance is 10 ohms and the inductance 0·072 henry. Calculate the resonant frequency and the value of the current and the P.D. across the capacitor when connected to a 75-volt supply of this frequency.

19. A mesh-connected 30-h.p. 400-volt 3-phase induction motor is supplied from a star-connected alternator. The motor has a full-load efficiency and power factor of 90 per cent and 0·88 respectively, and the voltage drop in the cable connecting it to the alternator can be neglected.
Calculate (*a*) the phase P.D. of the alternator, (*b*) the line current, (*c*) the phase current of the induction motor.

20. Three 180 μF capacitors are connected in star across a 400-volt 3-phase supply. If the line current is 10·4 amperes, calculate the frequency. If the capacitors are re-connected in mesh to the same supply, calculate the line current.

CHAPTER XIII

Principles of Electrical Machines

1. Reversibility of Electrical Machines

There is no essential difference in construction between an electrical generator, by which mechanical energy is converted into electrical energy, and an electric motor, by which electrical energy is converted into mechanical energy (§ 9, p. 149). Both are devices which enable a number of conductors, suitably connected and known as a *winding*, to move continuously through a magnetic field,* and by which electrical connection is maintained with a stationary external circuit.

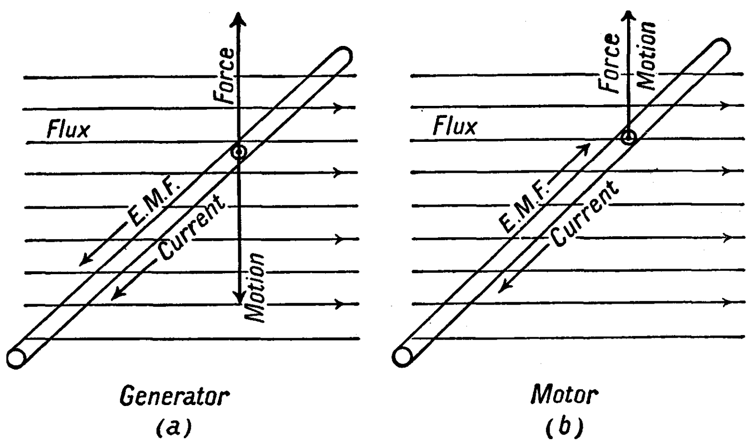


Fig. 1.—To illustrate generator and motor principles

If the circuit is complete, the E.M.F. induced in the conductors by their motion through the field produces a current which causes a force *opposing* the motion to act upon each conductor lying in the field; in overcoming this force work is done and the conversion of mechanical into electrical energy takes place (fig. 1*a*). The machine is then acting as a *generator* or *dynamo*.

* In some cases the magnetic field moves past stationary conductors (§ 3, p. 299), but the effect is the same.

On the other hand, if an external E.M.F. causes a current to flow through the winding in the same direction and the conductors move under the influence of and *in the direction of* the force exerted on them, the E.M.F. induced by this motion is in the *opposite* direction and opposes the current (fig. 1*b*). In order to maintain the current the external E.M.F. must be greater than the opposing E.M.F., and in overcoming the latter electrical energy is converted into mechanical energy, and the machine acts as a *motor*.

Hence electrical machines are, in general, reversible and are capable of acting in either capacity, although operation may be more efficient in that for which the machine was designed.

THE ALTERNATING CURRENT GENERATOR OR ALTERNATOR

2. The Two-pole Single-phase Alternator

The alternator in its simplest form has been described in § 2, p. 241, where a uniform field is assumed to exist without any inquiry as to how it was obtained. Apart from the earth's horizontal field, the strength of which is relatively small, such a field distribution is difficult to arrange even for experimental purposes. In machines of any practical size, the intense magnetic fields required can be obtained only by making most of the magnetic circuit of ferromagnetic materials; comparatively narrow air-gaps are then left at suitable points in the circuit, across which the magnetic flux passes and through which move the conductors in which the E.M.F. is induced.

The simple alternator, constructed in this manner, is shown diagrammatically in fig. 2*a*. The coil is mounted on a circular laminated iron core *A*, called the *armature*, which rotates between the poles *N*, *S* of an electromagnet; and the conductors forming the sides of the coil cut through the lines of flux passing across the air gap between the armature and the pole faces. The reluctance of the magnetic circuit is still further reduced by shaping the poles so that their faces are almost concentric with the armature, and by distributing the coil in a number of slots in the armature (fig. 2*b*) so that the air gap, except at that portion of the periphery actually occupied by conductors, is very short. Owing to the low reluctance of the circuit, a large flux is set up by the magnetizing windings or *field coils F*, which are supplied with direct current from some independent source; and a very intense field is produced in the air gap through which the conductors move.

Each end of the coil is connected to a brass or phosphor-bronze ring *R*, known as a *slip ring*, mounted on but insulated from the shaft (shown concentrically in fig. 2*a*, but actually side by side as in fig. 1,

p. 242), upon the outer surface of which stationary blocks of carbon B, known as *brushes*, are pressed by means of springs in order to provide continuous electrical connection between the rotating coil and the external circuit.

Owing to the annular shape and almost uniform length of the air gap, the flux crosses radially and the density under the pole face is almost uniform. The wave form of the E.M.F. in each conductor is therefore flat-topped, and other modifications have to be made in order that the E.M.F. wave of the coil may approach the sinusoidal form.

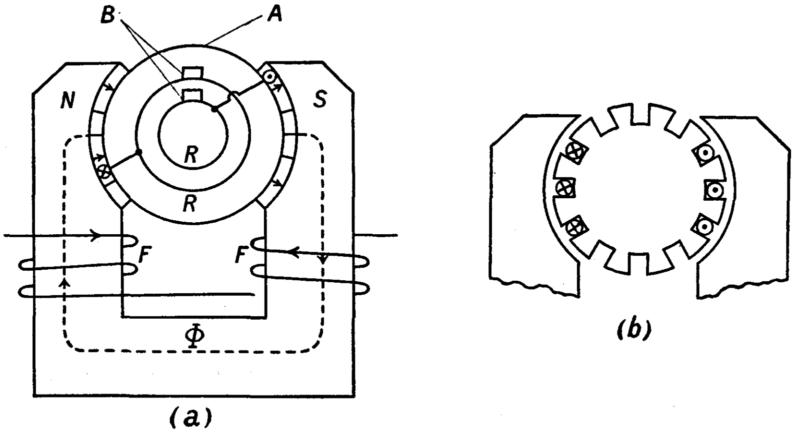


Fig. 2.—Simple single-phase alternator

3. Multipolar Machines—Relation between Speed, Poles and Frequency

In fig. 2*a* a two-pole machine has been illustrated for simplicity; but any number of pairs of poles may be used provided the conductors forming the winding are suitably connected. In multipolar machines the poles, which are alternately north and south, are connected at their outer ends by a yoke ring, and the path of the flux is as shown in fig. 15*d*, p. 232. In order that the E.M.F.s shall assist one another round the coil, it is necessary that at every instant the E.M.F. in the conductors forming one side of the coil shall be opposite in direction to that in the conductors forming the other side of the coil (fig. 1, p. 242). This is achieved by arranging that the coil sides occupy, simultaneously, corresponding positions under poles of opposite polarity, e.g. when one side is under the middle of a north pole, the other is under the middle of the adjacent south pole. In other words, the distance between the sides of the coil (the *coil span*) is made equal to that between the centre lines of adjacent poles (the *pole pitch*). It is a general rule, applicable to almost all machine windings, that the coil span should be equal or nearly equal to the pole pitch.

A four-pole alternator is illustrated diagrammatically in fig. 3, in which a and b are the sides of one coil, the back of which is represented by ab, while c and d are the sides of a similar coil occupying a similar position under the other pair of poles. The two coils are connected in series by the connector bc, and since the E.M.F.s of the two coils are in phase, the resultant E.M.F. between slip rings is the sum of the two.

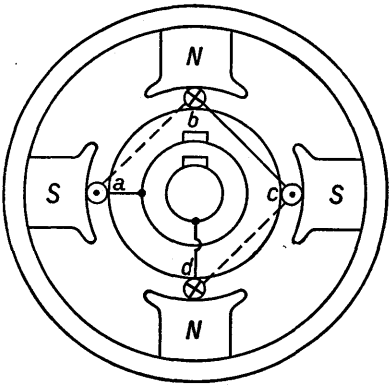


Fig. 3.—Four-pole alternator

Since the E.M.F. in each conductor goes through one complete cycle in passing under one pair of adjacent poles, which correspond with one complete revolution of a vector (§ 5, p. 249), each pair of pole pitches represents, in this sense, 2π radians or 360 *electrical degrees* or *degrees of phase*. If there are p pairs of poles, the E.M.F. goes through p cycles per revolution of the armature, and if the speed is n revolutions per sec., the frequency (f) is

$$f = np \text{ cycles per sec.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

Hence in an alternator, speed, frequency, and the number of poles are definitely related, the number of poles to produce a given frequency being inversely proportional to the speed. Alternators driven by reciprocating engines or water turbines, in which the speed is relatively low, require a large number of poles, e.g. at 250 r.p.m. 24 poles * (12 pairs) are necessary to give a frequency of 50 cycles per sec. On the other hand, the steam turbine is a high-speed machine, and as it is desirable to connect the alternator to it directly, modern turbo-alternators for this frequency usually have 2 poles and run at 3000 r.p.m. or occasionally 4 poles and run at 1500 r.p.m.; there is no speed available between these two.

Example 1.—A 6-pole alternator runs at 500 r.p.m. What is the frequency of the generated E.M.F.?

Here

$$n = \frac{500}{60} \text{ revs. per sec. and } p = \frac{6}{2} = 3.$$

$$\therefore f = np = \frac{500}{60} \times 3 = 25 \text{ cycles per sec.}$$

* It is common practice to specify the number of *poles* ($2p$) possessed by a machine although the *pair of poles* is the magnetic unit.

Example 2.—A number of 50-cycle alternators are to be built, having 6, 10, 14, 20 and 24 poles. At what speed must each run?

$n = \frac{f}{p}$. In a 6-pole machine, $p = 3$ and $n = 16{\cdot}67$ (1000 r.p.m.).
„ 10 „ $p = 5$ „ $n = 10$ (600 „).
„ 14 „ $p = 7$ „ $n = 7{\cdot}14$ (428·6 „).
„ 20 „ $p = 10$ „ $n = 5$ (300 „).
„ 24 „ $p = 12$ „ $n = 4{\cdot}17$ (250 „).

An E.M.F. is induced in a conductor when there is *relative* motion between the conductor and the field; which of the two actually moves is immaterial. In the elementary alternators described in the previous sections, the conductors moved in a stationary field, and this arrangement is adopted in practice in the case of small alternators and all d.c. machines. In larger alternators, however, it is usually more convenient to make the field move past stationary conductors. The armature winding is placed in slots on the inner periphery of a stationary

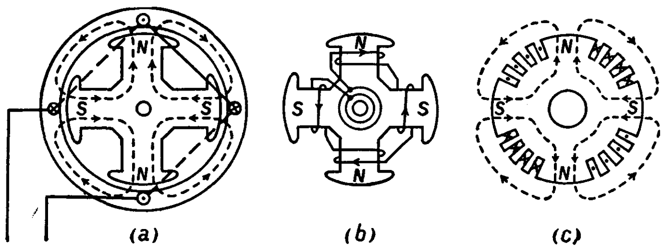


Fig. 4.—Alternator with rotating field

armature called the *stator*, inside which rotates the field system consisting of the poles, magnet yoke, and field coils, which is referred to as the *rotor*. The machine of fig. 3, reconstructed in this way, is shown in fig. 4*a*, details of the magnetizing or *field coil* connections being shown in fig. 4*b*. This form of construction is of particular advantage in large high-voltage machines; it is easier to insulate a stationary winding, while at the same time the large current produced has not to pass through moving contacts.* The field coils must, of course, be supplied through slip rings (see fig. 4*b*) but in this case the voltage is low and the current relatively small.

In the turbo-alternator, the length of which is great compared with its diameter (which is limited by centrifugal forces), salient (projecting) poles are eliminated, and the rotor is cylindrical with the field coils distributed in slots (fig. 4*c*). By this means a stronger construction is obtained, windage losses are reduced and the form of the E.M.F. wave can be made to approach more closely that of a sine wave.

* In a 60,000-kVA., 11,000-volt, 3-phase alternator the current in each circuit (or phase) is 3150 amperes.

4. The Three-phase Alternator

For the reasons given in § 19, p. 287, the three-phase system of generation, transmission and distribution has been adopted throughout the world, so that the three-phase alternator may be looked upon as the standard type of a.c. generator.

In a three-phase armature winding the conductors in any one phase are distributed in slots which occupy one-third of each pole pitch, the remaining two-thirds containing the conductors of the other two phases. This is illustrated in fig. 5, which shows a portion of the armature winding of a multipolar machine with 6 slots per pole, or 2 slots per pole per phase. The conductors in slots a and b form one

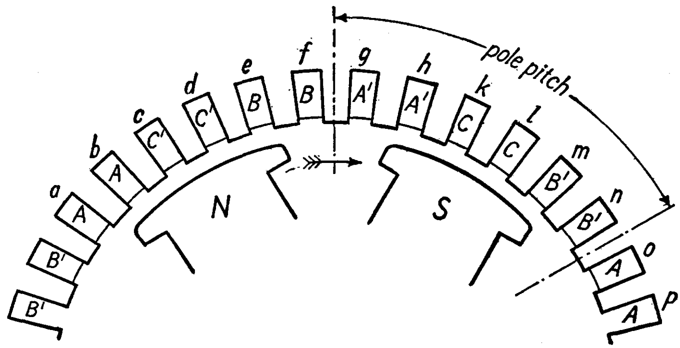


Fig. 5.—Three-phase alternator winding

side of a coil in phase A of which the other side is formed by the conductors $A'A'$ in slots g and h one pole pitch away, and this coil is connected in series with the next coil in the same phase which is similarly situated with regard to the next pair of poles. One side of this coil is shown in slots o and p. If it is assumed that one of the conductors in slot a forms the start of the winding of phase A, then the start of phase B is at slot e displaced from a by $\frac{2}{3}$ of a pole pitch or 120° of phase; and the start of phase C is in slot k displaced from e by another $\frac{2}{3}$ of a pole pitch. Hence if the poles rotate in a clockwise direction the E.M.F.s induced in the various phase windings will reach their maximum values in succession (when the respective phase conductors lie under the centre of the poles) separated by intervals of $\frac{1}{3}$ of a period, and in the order A, B, C.

The windings may be either single layer, in which the side of each coil fills the whole of the slot, or double layer as in d.c. machines (see p. 311), in which each coil occupies the upper half of the one slot and the lower half of the other, one pole pitch away.

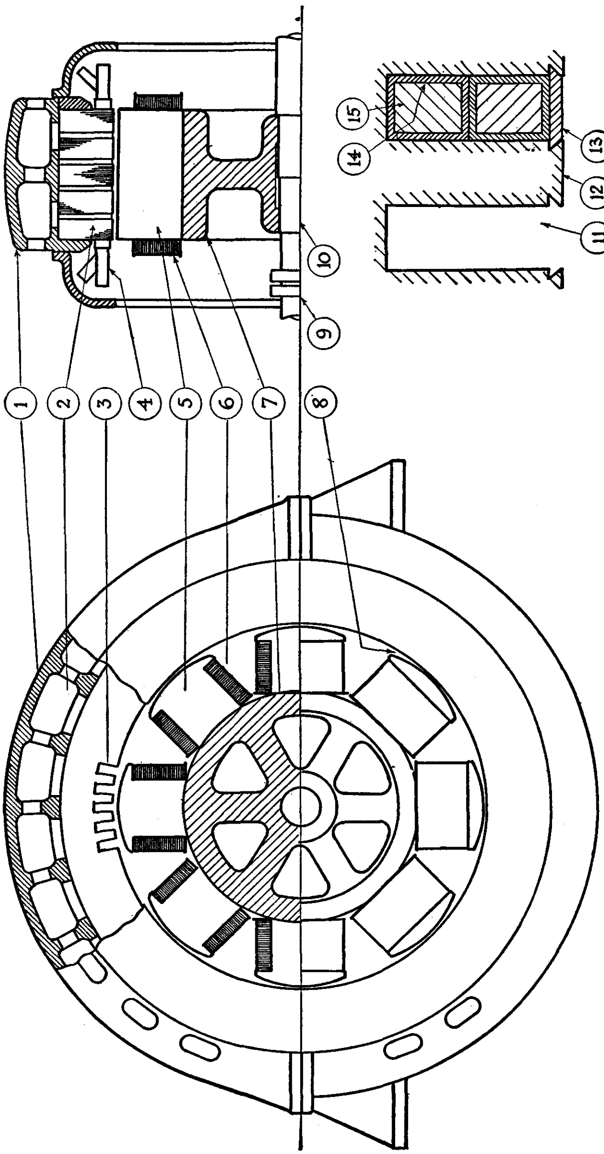
A section through an 8-pole 3-phase salient-pole alternator is shown in fig. 6, which is largely self-explanatory. The armature is built up of slotted steel laminations (2) (to reduce eddy currents, § 10, p. 151) which are assembled in groups separated by spaces, or ducts, through which passes the cooling air, and firmly keyed to and clamped in the stator frame (1). This is of box construction, which facilitates ventilation, and is either of cast iron or, in modern machines, fabricated of welded steelplates; it serves only as a mechanical support for the armature laminations and, unlike the magnet frame in a rotating armature alternator or d.c. machine, carries no flux.

The armature winding is double layer, and a section through two slots is shown in the figure. The conductors are, except in small machines, in the form of a copper strip, covered with either cotton or mica tape according to value of the generated E.M.F. Each coil is then insulated as a whole with some form of mica and placed in the slot, which is lined with a layer of some tough material such as horn fibre in order to protect the coil from abrasion. Both coil sides are held in position by a wedge of insulating material which is driven into the dovetail at the mouth of the slot.

The poles (5) are of solid mild steel or built up of steel laminations, and are secured to the magnet ring (7), which forms the rim of a steel wheel keyed to the shaft (10), either by bolts or by dovetails. The d.c. supply for the field coils (6) is obtained from a small d.c. generator known as an *exciter*, which is mounted on the same shaft and used solely for this purpose. In a machine of any size the field coils consist of copper strip wound on edge with mica or other insulating material between turns; the form of construction is mechanically strong and has good cooling properties. In small machines the coils are wound with cotton-covered copper wire of round or square section. All the field coils are connected in series, and the ends of the circuit brought to two phosphor-bronze or gunmetal slip rings (9), in contact with which are stationary carbon brushes connected to the exciter terminals.

For a frequency of 50 cycles per sec., the speed of an 8-pole machine would be 750 r.p.m.; such a machine would be suitable for being driven either by an electric motor (in which case the two machines form a *motor-generator* set) or through gearing by a steam turbine. A machine of similar type but having 24 poles would give the same frequency at 250 r.p.m., and would be suitable for drive by steam engine, diesel engine or water turbine; in the latter case the machine is often arranged so that the shaft is vertical and the plane of rotation horizontal.

In salient-pole alternators the length of the armature core is usually only a fraction of its diameter, whereas in the turbo-alternator, in consequence of the high speed, the core length is several times the diameter. The general arrangement of the turbo-alternator, however,



1, Stator frame. 2, Armature laminations. 3, Stator slots.
4, Stator coils. 5, Rotating field poles. 6, Field coils.
7, Magnet wheel. 8, Air gap.
9, Slip rings. 10, Shaft. 11, Slot.
12, Tooth. 13, Wedge. 14, Insulation.
15, Conductor.

Fig. 6.—Section of three-phase salient-pole alternator

is exactly the same as in the salient-pole machine, except for modifications due either to the difference in proportions, which makes ventilation difficult, or to the large centrifugal forces which are produced.

The shafts of medium-speed alternators usually run in plain journal bearings with ring lubrication; but in very large machines and in high-speed machines such as turbo-alternators, the bearings are forced-lubricated by oil under pressure.

5. The E.M.F. Equation of an Alternator

Let p = number of *pairs* of poles,

Φ = flux per pole, i.e. the flux issuing from each N pole, or entering each S pole, in webers,

n = speed in revolutions per sec.,

N = speed in revolutions per minute,

z = number of conductors in series per phase,

f = frequency in cycles per sec.

Assume for simplicity that the conductors are moving through a stationary field.

Time taken by each conductor to make one revolution

$$= \frac{1}{n} \text{ sec.}$$

Time taken by each conductor to move through one pole pitch is

$$t = \frac{1}{2pn} \text{ sec.}$$

and during this time it cuts a total flux Φ.

Average rate at which the conductor cuts the flux

$$= \frac{\Phi}{t} = \frac{\Phi}{1/(2pn)} = 2pn\Phi \text{ webers per sec.}$$

Average value of the E.M.F. induced in each conductor

$$= 2pn\Phi \text{ volts.}$$

Now the r.m.s. value can be obtained from the average value by multiplying by the *form factor* (equation 11, p. 248), which for a sine wave is 1·11. Hence, assuming that the E.M.F. wave is a sine wave,

$$\text{r.m.s. value of E.M.F. per conductor} = 2{\cdot}22pn\Phi \text{ volts,}$$

and the resultant E.M.F. per phase in which there are z conductors in series is

$$E_{ph} = 2{\cdot}22zpn\Phi \text{ volts}$$

$$= 2{\cdot}22zp\frac{N}{60}\Phi \text{ volts,*}$$

and since $pn = f$ (equation 1, p. 298),

$$E_{ph} = 2{\cdot}22z\Phi f \text{ volts.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In a three-phase machine the windings are almost invariably star-connected, so that the E.M.F. between terminals or line E.M.F. is

$$E_l = \sqrt{3}E_{ph}.$$

If I is the current in each phase, and φ the phase angle between current and E.M.F.,

$$\begin{aligned}\text{Total output} &= 3 \text{ (phase output)}\\ &= 3E_{ph}I_{ph} \text{ volt-amperes} = 3E_{ph}I_{ph} \,.\, 10^{-3} \text{ kVA.}\\ &= 3E_{ph}I_{ph}\cos\varphi \,.\, 10^{-3} \text{ kW.}\\ \text{or} \qquad &= \sqrt{3}E_lI_l\cos\varphi \,.\, 10^{-3} \text{ kW.} \quad . \quad . \quad . \quad . \quad . \quad (3)\end{aligned}$$

The output of a modern alternator is limited principally by heating. The heating of the stator windings depends chiefly on the current, i.e. on the kVA. rather than on the kW. output: that of the rotor, due principally to the losses in the field winding, depends also on the power factor of the load, since when the power factor is lagging the currents in the armature winding exert a demagnetizing effect (armature reaction) on the main field, so that the exciting current must be increased in order to maintain the voltage, thus increasing the rotor heating. For these reasons the rating of an alternator is expressed in kVA., usually at 0·8 power factor lagging, which is taken as a typical value.

High-speed alternators (turbo-alternators) are built for outputs up to (and sometimes exceeding) 100,000 kVA. from a single unit, but the sizes in most common use in modern power stations range from 25,000 to 60,000 kVA. The voltage is usually either 6600 or 11,000 volts between terminals, although there are now a number of machines working satisfactorily at 33,000 volts.

Example.—A three-phase star-connected alternator has 12 poles and runs at 500 r.p.m. There are 108 slots each containing 8 conductors, and the flux per pole is 0·0595 weber. Calculate (*a*) the frequency, (*b*) the E.M.F. between terminals.

* Actually the resultant E.M.F. is slightly less than this. Each coil is usually distributed in several slots, and since the E.M.F.s in each slot are not quite in phase, their *vector* sum is less than their algebraic sum.

(a) $$n = \frac{500}{60},\ p = 6,$$

$$f = np = \frac{500}{60} \cdot 6 = 50 \text{ cycles per sec.}$$

(b) $$z_{ph} = \frac{108 \times 8}{3} = 288,$$

$$E_{ph} = 2{\cdot}22 \times 288 \times 50 \times 0{\cdot}0595 = 1905 \text{ volts},$$

$$E_l = \sqrt{3} \times 1905 = 3300 \text{ volts (star connection).}$$

6. The Synchronous Motor

If an alternating current from some external source is passed through the armature of a stationary alternator which has its field excited, the conductors experience forces which alternate in direction at the frequency of the supply; and since the inertia of the rotor is too great to allow it to oscillate under the alternating torque produced, no motion at all takes place. If, on the other hand, the rotor is rotating at such a speed that, as the current in each conductor reverses, the conductor moves into a field in the reverse direction, i.e. under a pole of opposite polarity, the force exerted on the conductor will be in the *same* direction and a unidirectional driving torque will be produced. Thus at this particular speed the machine will act as a motor, but at any other speed the forces on the conductor alternate in direction so that there is no resultant driving torque.

At this speed, known as the *synchronous speed*, each conductor moves through one pole pitch while the current in it passes through one half cycle or *two pole pitches per cycle*. Hence the synchronous speed of the machine as a motor is that at which it would have to run as an alternator to produce a frequency equal to that of the supply. If p is the number of pole pairs and f the frequency of supply, the synchronous speed (n) in revolutions per sec. is, from equation (1), p. 298,

$$n = \frac{f}{p}. \qquad \qquad (4)$$

It is clear from the above that the synchronous motor is not self-starting but must be run up to synchronous speed by some other means. Although this is a disadvantage, it has other valuable characteristics. The most important of these is that the value of the power factor at which it operates can be controlled by means of the field excitation. When this is small the power factor is lagging, but as the field current is increased the power factor rises to unity, the stator current at the same time decreasing to a minimum. Further increase in field current causes the power factor to become *leading*, and the stator current increases again. Hence the leading current taken by an over-excited synchronous motor may be used to neutralize part of the lagging current taken by other apparatus, such as induction motors, and thus to raise the power factor of the load as a whole. For this reason methods of starting have been developed which have enabled the use of this type of machine to be greatly extended in recent years.

As a d.c. supply is seldom available, the field current is usually supplied by a direct coupled exciter.

This type of motor is also used extensively in very small sizes for driving gramophone turn-tables and electric clocks.

THE DIRECT-CURRENT GENERATOR

7. The Function of the Commutator

Referring once again to the simple loop of § 2, p. 241, suppose that one of the slip rings is divided along a diameter, the two halves being insulated from each other; and that one end of the loop is connected to each half or *segment* of the ring, the other ring being removed alto-

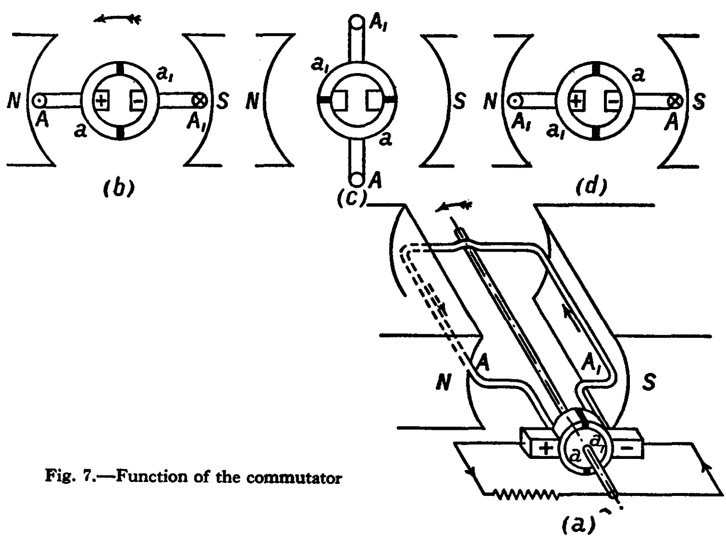


Fig. 7.—Function of the commutator

gether, as shown in fig. 7*a*. Connection with the external circuit is maintained, as before, by two brushes which bear on the outer surface of the split ring at opposite ends of the horizontal diameter (in the diagrams *b*, *c*, and *d* they are shown *inside* the ring for greater clearness).

With the direction of flux and rotation as shown, when the loop is in the position shown in fig. 7, *a* and *b*, the direction of the E.M.F. is out of the paper in conductor A and into the paper in conductor A_1. If the circuit is completed, a current will flow up A and through segment a and the left-hand brush, to the external circuit, and will return through the right-hand brush and segment a_1 and down conductor A_1. Hence, relative to the external circuit, the left-hand brush is *positive* and the right-hand brush *negative* as shown. As the rotation continues the E.M.F. and current decrease, and have reached a very low value when eventually segment a_1 comes into contact with the positive brush, which segment a has not yet left. For a brief period both segments are in contact with the same brush and the loop is short-circuited (fig. 7*c*), but the E.M.F. and current in the coil are almost zero. Further rotation brings each conductor under the influence of the opposite pole and causes the E.M.F. and current in each to grow in the reverse direction, i.e. upwards in A_1 and downwards in A (fig. 7*d*);

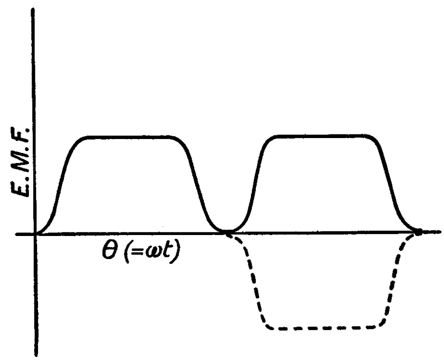


Fig. 8.—Wave-form of E.M.F. in d.c. armature

but A_1 is now connected through segment a_1 to the left-hand brush, which is therefore *still positive*, while A is connected through segment a to the right-hand brush, which is *still negative*.

Thus by means of this split ring the conductor under the north pole is always connected to the left-hand brush, and that under the south pole to the right-hand brush, so that the E.M.F. between brushes, although fluctuating, is unidirectional. The effect, relative to the external circuit, is that of reversing every alternate half wave; the split ring by which this reversal or rectification is brought about is known as a *commutator*, and the apparatus is essentially a d.c. generator. The form of the E.M.F. wave between brushes is shown in fig. 8; it is more flat-topped than a sine wave, for reasons given in § 2, p. 297.

8. D.C. Armature with Ring Winding

The general mechanical arrangement of the d.c. generator is similar to that of the alternator described in § 2, p. 296, and illustrated in fig. 2, except that the armature winding is distributed over the whole periphery and the slip rings are replaced by a commutator. The presence of the latter makes it necessary for the armature to be the rotating portion, so that the alternative rotating field construction is never adopted for d.c. machines.

The simple single-coil and two-part commutator is not a practicable arrangement since the E.M.F. and current fall to zero twice per revolution; but if a number of coils are used, distributed over the armature periphery, the resultant E.M.F. is reasonably steady. One method of doing this is shown diagrammatically in fig. 9. The armature consists of a hollow iron cylinder built up of ring-shaped laminations on which are wound eight equally spaced coils, the junction between

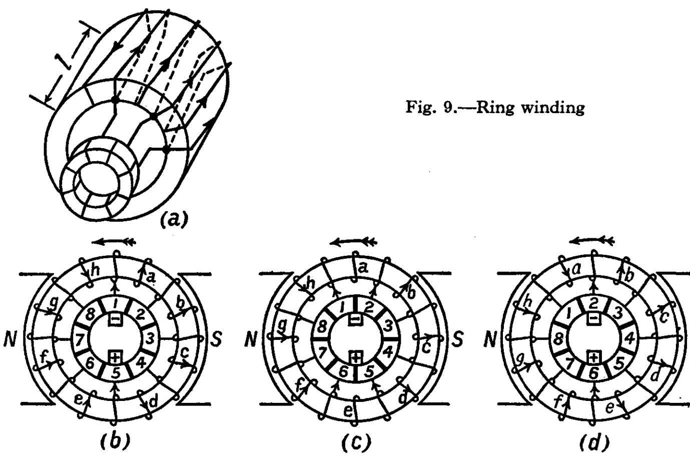


Fig. 9.—Ring winding

each pair being connected to one segment of an eight-part commutator (fig. 9, *a* and *b*). The coils therefore form a closed winding, and as there is one coil between each pair of segments the number of segments is equal to the number of coils; this is true of all d.c. armature windings. The brushes (shown inside the commutator for clearness) are at opposite ends of a diameter, and so placed that whenever two segments are in contact with the same brush the short-circuited coil between them is midway between the poles and is therefore cutting little or no flux. No flux crosses the space inside the armature (fig. 12*b*, p. 50), so that only that portion of each turn which lies on the outside surface is effective in producing an E.M.F.

If the direction of flux and rotation is the same as in fig. 7, a reference to that figure shows that the E.M.F. is upwards in the conductors under the north pole and downwards in those under the south pole, and its direction round each coil is indicated by the arrows. It will be seen that the brushes divide the winding into two parallel circuits, and at the instant shown in fig. 9*b* the right-hand circuit contains coils *a*, *b*, *c*, and *d* lying under the south pole, while the left-hand circuit contains coils *e*, *f*, *g*, *h* lying under the north pole. The E.M.F.s in coils *a*, *b*, *c*, *d* are therefore opposite in direction (relative to the coil) to those in coils *e*, *f*, *g*, *h*, but the general direction of the E.M.F.s in *both* circuits, relative to the brushes, is the same, i.e. from the upper brush to the lower brush; and the E.M.F. between brushes is equal to the sum of the E.M.F.s in the four coils in either circuit. Hence, if the external circuit is completed, a current flows downwards through each half of the winding and, uniting at segment 5, passes out at the lower or positive brush; after traversing the external circuit it enters at the upper or negative brush, and at segment 1 divides between the two circuits.

One-sixteenth of a revolution later, the armature has reached the position shown in fig. 9*c*. Coils *a* and *e*, which are now midway between the poles and in which there is therefore no E.M.F., are short-circuited by the brushes; and the E.M.F. between brushes is due to the three coils (*b*, *c*, *d*, and *f*, *g*, *h*) in series in each circuit.

After turning through a further one-sixteenth of a revolution the position of the armature is as shown in fig. 9*d*. The E.M.F. in coils *a* and *e*, which are now moving under poles of opposite polarity, are reversed in direction, but coil *a* has now been *transferred to the left-hand circuit* and coil *e to the right-hand circuit,* which has the effect of reversing the connection of each coil relative to the brush, so that the general direction of the E.M.F.s is still from the upper to the lower brush. Hence:

The function of the commutator is to transfer each coil from one armature circuit to the next (thereby reversing its connections relative to the brushes) during the time when the sides of the coil are passing between the poles, and the E.M.F. is about to reverse, so that the general direction of the E.M.F.s in the coils, relative to the brushes, remains the same.

In this simple case the resultant E.M.F., although constant in direction, will vary somewhat as the armature rotates. The E.M.F.s in each coil differ in phase by one-eighth of a period, and the resultant wave is obtained by plotting the individual E.M.F. waves of the four coils forming one circuit and adding them as shown in fig. 10. In a commercial machine, however, there is a much larger number of coils, often several hundreds, in the armature winding. As the armature rotates there is a continual transfer of coils from one circuit to another, but the amount of rotation during the transfer of a coil is so small that the resultant E.M.F. at any instant may be looked upon as always due to the same *number* of coils (but not the *same* coils) occupying the same positions in the field; this E.M.F. is therefore, for all practical purposes, constant in value.

The process of commutation is not as simple as might appear from the above description.

Up to the moment when the coil is short-circuited by the brush it is carrying full circuit current in one direction, while immediately it passes into the next circuit it must carry full circuit current in the opposite direction.

If during the period of short-circuit the current could be fully reversed, the coil would pass into the next armature circuit without disturbance. But the decay of the original current is delayed by the inductance of the coil, and since the coil sides are between the poles there is little E.M.F. to establish a current in the reverse direction. Hence, when the coil passes out of short-circuit, the

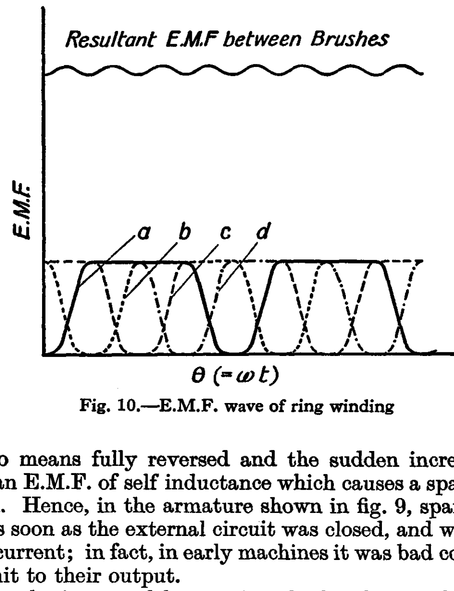


Fig. 10.—E.M.F. wave of ring winding

current is by no means fully reversed and the sudden increase to full circuit value produces an E.M.F. of self inductance which causes a spark as the segment leaves the brush. Hence, in the armature shown in fig. 9, sparking would occur at the brushes as soon as the external circuit was closed, and would increase with increase of load current; in fact, in early machines it was bad commutation which usually set a limit to their output.

Conditions can be improved by moving the brushes so that the coil during short-circuit lies under the fringe of the next pole. By this means an E.M.F. opposite in direction to that previously existing is induced in the coil, which, by neutralizing the E.M.F. of self inductance, assists the reversal of the current. This method was used for many years but had several disadvantages: among others, the displacement of the brushes caused the currents in the armature windings to exert a weakening effect on the field set up by the poles, and the position of the brushes had to be altered whenever there was any considerable change in load.

In modern machines small auxiliary poles, known as *inter-poles* or *commutating poles*, are provided midway between the main poles, by which an E.M.F. is induced in the short-circuited coil which neutralizes the E.M.F. of self inductance and ensures that the current is fully reversed by the time the coil passes into the next circuit. These poles are excited by coils in series with the armature so that their effect increases automatically as the load current increases; and by this means sparkless commutation is obtained, without moving the brushes, over the whole load range.

9. Drum Windings—Multipolar Machines

The type of winding shown in fig. 9 is known as a *ring winding*, and was chosen for its simplicity; but it is not used in modern machines because, among other reasons, it is difficult to wind, particulary with a slotted armature, and only a small portion of each turn is effective. If it is arranged that each turn, instead of passing through the centre of the armature, returns along a slot *one pole pitch* away so that it lies under a pole of opposite polarity, E.M.F.s are induced in *both* sides which, although opposite in direction, assist each other round the loop. Such a winding, in which the span of each coil is a pole pitch, is called a *drum winding*; it has the further advantage that all the coils can be wound and insulated before being placed in the slots. Drum windings are employed on all modern machines: it has already been used on the four-pole alternator of fig. 3, p. 298.

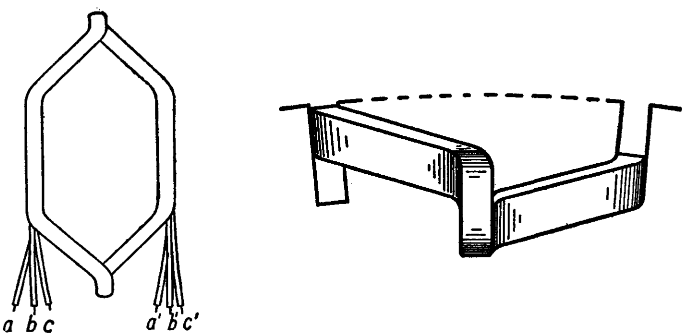


Fig. 11.—Armature coils for drum windings

Each slot usually contains the sides of several coils, which after individual insulation, are insulated as a unit; so that one coil from the winder's point of view may consist of several coils in the electrical sense as shown in fig. 11*a*.

The winding is in two layers, each coil having one side in the upper half of one slot and the other side in the lower half of a slot one pole pitch away, the transfer from one layer to the other taking place in a radial plane by means of a bend at the back end as shown in fig. 11*b*.

A section through a four-pole drum-wound armature is shown in fig. 12. The coils of which the upper sides lie in slot a have their lower sides in slot a_1, while those of which the lower sides are in slot a have their upper sides in slot a_2.

Any number of pairs of poles may be used, provided that the armature is suitably arranged. The size and output of a machine with only two poles is limited, and most machines, except in the smallest sizes, have at least four poles, the number increasing with the armature diameter to 16, 20, and 24 in the largest sizes. The E.M.F.s and currents in a d.c. armature winding are, of course, *alternating*, but

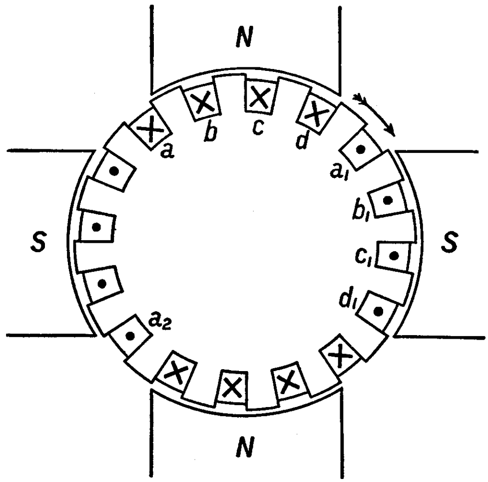


Fig. 12.—Section of four-pole drum-wound armature

since they are rectified by the commutator, the frequency, as far as the external circuit is concerned, is immaterial. Hence there is no fixed relationship between the number of poles and the speed, which, within limits, may have any convenient value.

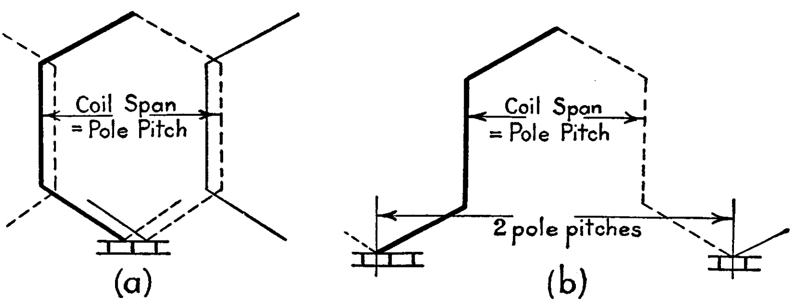


Fig. 13.—Lap and wave windings

There are two types of drum winding in common use in both of which the same type of coil is used, the difference being solely in the method of connecting them: (*a*) the *lap* or parallel winding (fig. 13*a*), in which the end of one coil is bent back and connected to the start of the next so that the two ends of any one coil are connected to adjacent segments. In this winding *there are as many parallel circuits as there are poles,* and for this reason it is the type generally used in large machines where the current is heavy. (*b*) The *wave* or series winding, in which the end connections are bent outward (fig. 13*b*) so that the end of a coil is connected to the start of one occupying an almost similar position under the next pair of poles, the two ends of any one coil being connected to segments two pole pitches apart. In this winding there are *only two* parallel circuits, whatever the number of poles. This is the type of winding generally used on small machines and is therefore the type most commonly met with.

10. Construction of Typical D.C. Machine

A section through a d.c. generator of medium size is shown in fig. 14; but the description applies equally well to a d.c. motor.

The poles (2), of solid mild steel or built up of steel laminations, are bolted to a circular yoke or magnet frame (1) of cast iron, cast steel, or rolled steel; each pole is surrounded by a magnetizing or field coil (3); all these coils are connected in series so as to produce alternate north and south poles and a flux distribution, as shown in fig. 15*d*, p. 232. Inside the circle of poles and separated from them by a small air-gap (13) (which varies in length from about 0·1 in. in a small machine to 0·3 in. in a large one) rotates the armature, built up of slotted ring-shaped steel laminations (4, 11), mounted on and keyed to the arms of a cast-iron *spider* (12), which itself is keyed to the shaft (14); as in the case of the alternator, the laminations are assembled in groups separated by air-ducts for cooling purposes. The construction of the armature coils (5) and their arrangement in the slots is similar to that shown in fig. 6 (p. 302). The commutator is built up of copper segments of the shape shown (8), separated by sheets of mica, and clamped between steel rings insulated with mica, which are supported on a separate spider. The armature coils (5) are connected to the commutator segments by thin strips of copper (9). The brushes (7) are mounted in brackets (6), which contain the springs by which they are pressed into contact with the commutator, and which are supported from the magnet frame (as shown) or (in the case of small machines) from the bearing.

Plain journal bearings with ring lubrication are still used for medium and large machines. The armature shown in the figure has one bearing of this type at the commutator end; the other end of the shaft terminates in a half coupling which is bolted directly to the engine crank shaft. On small machines it is now standard practice to fit ball bearings at both ends, although a roller bearing is sometimes substituted at the driving end.

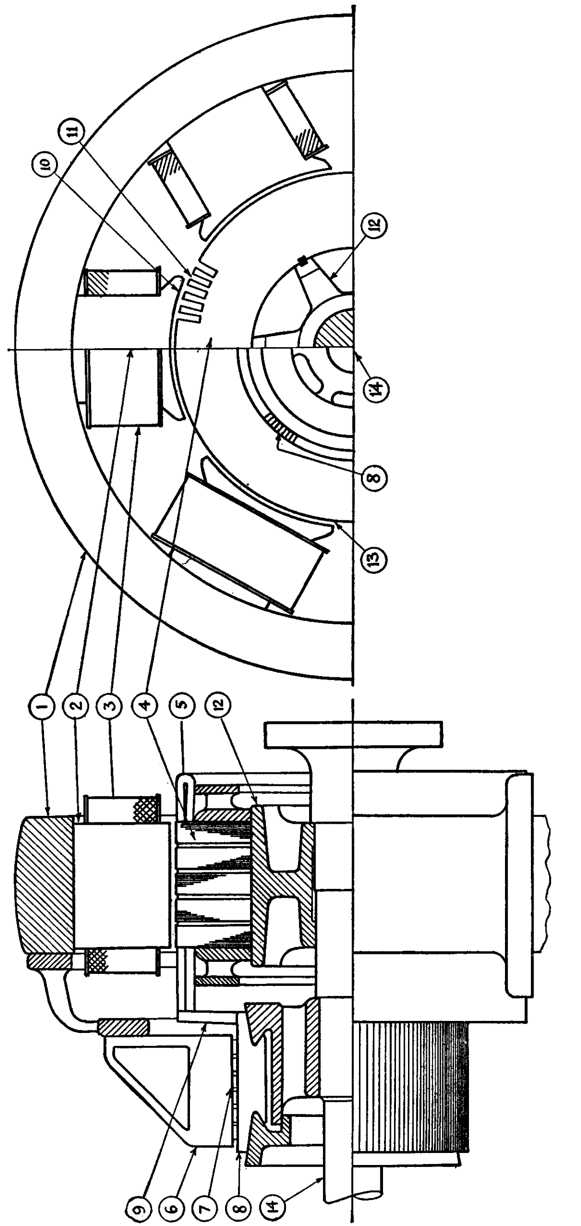


Fig. 14.—Section of multipolar d.c. generator

1, Magnet frame or yoke. 2, Field poles. 3, Field coils. 4, Armature laminations. 5, Armature coils. 6, Brush arm and boxes. 7, Brushes. 8, Commutator segment. 9, Commutator risers. 10, Pole face. 11, Armature slots. 12, Armature spider. 13, Air gap. 14, Shaft.

11. Methods of Field Excitation

The magnetic field, as in the case of the alternator, is produced by magnetizing coils surrounding the poles and supplied with direct current. This may be obtained from some entirely separate source, and the machine is then said to be *separately excited* (fig. 15*a*); but since the d.c. generator produces direct current, it may be arranged to supply its own field coils, and it is then said to be *self excited.*

There are three ways of producing *self excitation*:

(*a*) *Series excitation* (fig. 15*b*), in which the whole of the armature current is passed through the field coils. This current is comparatively

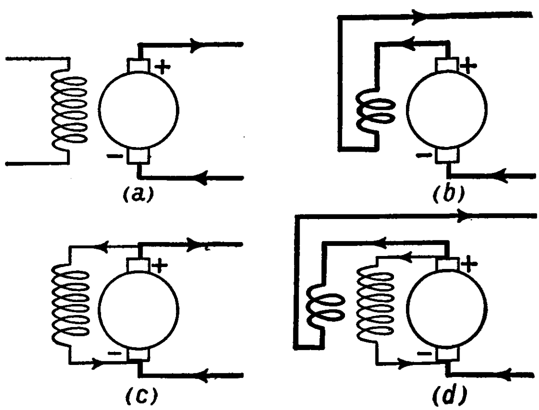


Fig. 15.—Methods of field excitation

large, and only a few turns are necessary to produce the required excitation; the section of conductor is large and the resistance low to reduce resistance losses.

(*b*) *Shunt excitation* (fig. 15*c*), in which the coils are connected directly across the armature; and a small current in parallel with the main current in the external circuit passes through the field coils, which contain a large number of turns and are of high resistance.

(*d*) *Compound excitation* (fig. 15*d*), which is a combination of (*a*) and (*b*).

The characteristics of the generator depend very largely on the method of excitation adopted.

12. The E.M.F. Equation of a D.C. Generator

It was pointed out at the end of § 8 that when the armature is rotating coils are continually being transferred by the commutator from one circuit to the other; but that, in a machine with a large number of coils, it can be assumed that there is always the same *number* of coils in each circuit between brushes although the *personnel* of each circuit is always changing. It follows that although the E.M.F. *in each coil* varies (from zero to a maximum value and back to zero) as it passes through a pole pitch, the E.M.F. *between brushes*, which is the sum of the E.M.F.s in all the coils in the circuit, has a constant value equal to the *average* E.M.F. in each coil multiplied by the number of coils in series. Thus, since each coil contains the same number of turns and therefore of conductors,

Resultant E.M.F. = (average E.M.F. per conductor) × (number of conductors in series).

Let

z = total number of conductors in the winding,

c = number of parallel circuits into which the winding is divided,

p = number of pairs of poles,

Φ = flux per pole in webers,

n = speed in revolutions per sec. $\left(= \frac{N}{60}\right)$,

N = speed in revolutions per min.

Time taken to traverse one pole pitch is $t = \frac{1}{2pn}$ seconds,

and during this time each conductor cuts a flux of Φ lines.

∴ Average E.M.F. per conductor is

$$\frac{\Phi}{t} = \frac{\Phi}{1/2pn} = 2pn\Phi \text{ volts.}$$

But there are z/c conductors in series in each circuit between brushes.

∴ Resultant E.M.F. of each circuit, i.e. of the machine, is

$$E = \frac{z}{c} \cdot 2p \cdot \Phi \cdot n$$

$$= \frac{z}{c} \cdot 2p\Phi \cdot \frac{N}{60} \text{ volts.} \quad . \quad . \quad . \quad . \quad (5)$$

It is clear that in any given machine, since z, c, and p are constant,

$$E \propto \Phi \cdot N,$$

so that if the flux is maintained constant by keeping a constant value of field current, the generated E.M.F. at no-load is proportional to the speed (fig. 16, *a* and *b*); similarly if the speed is maintained constant and the flux varied by varying the field current, the generated

E.M.F. is proportional to the flux. The relation between E.M.F. and field current rather than flux (which in any case is difficult to measure) is of practical importance, and the curve relating them (fig. 16c) is known as the *Open Circuit Characteristic* (because the machine is unloaded) or *Magnetization Curve*. At first the relation is almost linear, but later the curve bends over owing to the saturation of the magnetic circuit: it has, in fact, the shape of a *B-H* curve.

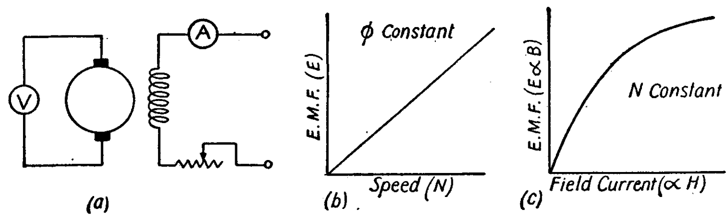


Fig. 16.—No-load characteristics

Example.—The armature of a 6-pole d.c. generator has 664 conductors arranged in 2 parallel circuits; and the flux per pole is 0·06 weber. What is the value of the generated E.M.F. when the armature is rotating at 250 r.p.m.?

Time taken by one conductor to move through one pole pitch

$$= \frac{60}{250 \times 6} = 0{\cdot}04 \text{ sec.}$$

$$\therefore \text{ Average E.M.F. per conductor} = \frac{0{\cdot}06}{0{\cdot}04}$$

$$= 1{\cdot}5 \text{ volt,}$$

$$\text{Number of conductors in series per circuit} = \frac{664}{2} = 332.$$

$$\text{Resultant E.M.F.} = 1{\cdot}5 \times 332 = 498 \text{ volts.}$$

Alternatively, by direct substitution in E.M.F. equation (5),

$$z = 664,\ c = 2,\ p = 3,$$

$$\Phi = 0{\cdot}06,\ N = 250,$$

$$\therefore\ E = \frac{664}{2} \times 2 \times 3 \times 0{\cdot}06 \times \frac{250}{6} = 498 \text{ volts}$$

13. Generator Load Characteristics

As the effective resistance of the load circuit of a d.c. generator is decreased, so that the current supplied increases, the terminal P.D. tends to fall, because

(1) the terminal P.D. is always less than the generated E.M.F. by the amount of the resistance drop in the armature winding, which is proportional to the load current;

and the generated E.M.F. itself falls as the load increases, because

(2) the magnetic effect of the current in the armature winding (known as *armature reaction*) is such as to distort and weaken the field produced by the poles;

(3) in most cases the speed of the prime mover, by which the generator is driven, falls as the load increases,

and this produces a further fall in the terminal P.D.

The generator-load characteristic is the graph showing the relation between the terminal P.D. and the load current. Typical curves for various methods of excitation are shown in fig. 17.

Separate Excitation.—In this case the terminal P.D. falls gradually as the load increases for the reasons given above (fig. 17*c*).

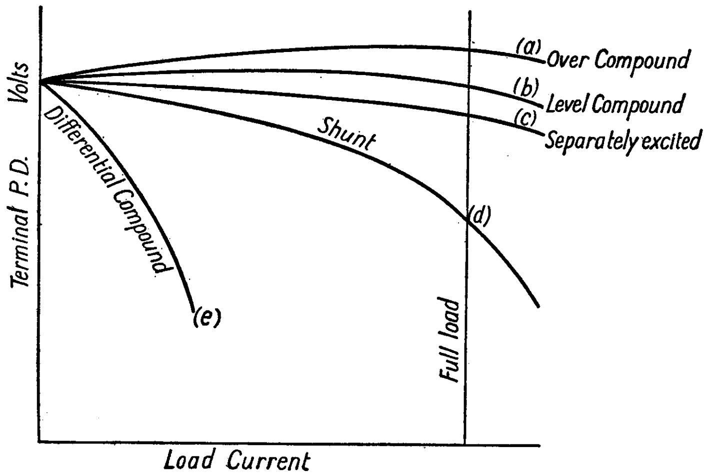


Fig. 17.—Generator-load characteristics

Series Excitation is not used for generators except in special cases.

Shunt Excitation.—In the case of any self-excited generator the initiation of the process by which the excitation is " built up " depends upon the retention of a small amount of residual flux in the magnetic circuit; otherwise, however long the armature were rotated there would be no E.M.F. generated, no field current and no flux. In practice, however, the small residual flux generates a small E.M.F. which produces a small field current; this causes an increase in flux and a further increase in E.M.F. and field current, and so on. If the flux continued to be proportional to the field current, conditions would be unstable and the E.M.F., flux and field current would continue to increase indefinitely; but since the magnetization curve bends over as the iron saturates, the increase in E.M.F. for a given increase in field current becomes less and less until finally a stable point is reached, beyond which any increase in E.M.F. requires a greater increase in field current than the increase in E.M.F. can supply. The position of this point depends upon the resistance of the field circuit, so that the value to which a shunt generator "builds up" its voltage can be controlled by a rheostat in series with the field coils.

When the shunt generator is loaded, there is another effect, in addition to those mentioned earlier in the section, which contributes to the fall in P.D. as the load increases.

Since the field circuit is connected across the armature terminals, any fall in terminal P.D. will cause a decrease in field current and a further fall in generated E.M.F. Hence the P.D. falls much more rapidly than in a separately excited generator, especially on heavy loads (fig. 17*d*).

In practice, in both these cases the rheostat included in the field circuit is gradually cut out as the load increases; by this means the field current is increased and the terminal voltage maintained at a constant value.

Compound Excitation.—Here each pole has two exciting coils—shunt and series. At no-load the machine builds up its voltage by means of the shunt-connected coils. When loaded, the series coils, which are traversed by the load current, produce additional ampere-turns which increase as the load increases. It is therefore possible to arrange that this increase not only neutralizes the weakening effect of armature reaction on the main field, but actually increases the flux so that the generated E.M.F. rises. If at full load this rise is equal to the amount of the resistance drop in the armature winding with full-load current, the terminal P.D. is maintained automatically at the same value at full load as at no load, and the machine is called *level compound* (fig. 17*b*). A level compound generator is the most suitable type for most services in which a constant terminal voltage must be maintained under varying load conditions. By increasing the number of turns on each series coil the generated E.M.F. can be increased still further, so that the terminal P.D. rises to a value which is, say, 10 per cent higher than its value at no-load: such a machine is said to be *over compound* (fig. 17*a*). An over compound generator may be used when it is necessary to maintain a constant voltage at the far end of a feeder, the increase in terminal voltage compensating for the resistance drop in the feeder.

Both these, in which the series coils assist the shunt, are examples of *cumulative compound* excitation. For certain special purposes, e.g. welding, the series coil is connected so as to oppose the shunt coil; this is called *differential compound* (fig. 17*e*), and produces a very rapid fall in terminal P.D.

Example.—In a 440-kW. 500/550-volt over-compound generator (see fig. 15*d*) the flux per pole required to produce 500 volts on no-load at 820 r.p.m. is 0·065 weber. The resistances of the armature, series field and shunt field are 0·00876, 0·002 and 55 ohms respectively. Calculate (*a*) the resistance losses in the windings, (*b*) the flux per pole, at full load, if the full load speed is 800 r.p.m.

(*a*) $$\text{Full load current} = \frac{440{,}000}{550} = 800 \text{ amperes.}$$

$$\text{Shunt field current} = \frac{550}{55} = 10 \text{ amperes.}$$

$$\text{Shunt field loss} = 550 \times 10 = 5500 \text{ watts.}$$

$$\text{Series field loss} = 800^2 \times 0{\cdot}002 = 1280 \text{ watts.}$$

$$\text{Armature current} = 800 + 10 = 810 \text{ amperes.}$$

$$\text{Armature loss} = 810^2 \times 0{\cdot}00876 = 5740 \text{ watts.}$$

(*b*) Generated E.M.F. at full load

$$= 550 + (810 \times 0{\cdot}00876) + (800 \times 0{\cdot}002)$$
$$= 558{\cdot}7 \text{ volts.}$$

$$\text{Flux per pole} = 0{\cdot}065 \times \frac{558{\cdot}7}{500} \times \frac{820}{800} = 0{\cdot}0743 \text{ weber}$$

THE DIRECT-CURRENT MOTOR

14. Principle of the D.C. Motor

Each side of the loop in fig. 7 (p. 306) is a conductor lying in and perpendicular to a magnetic field. When a current from some external source is passed round the loop, in the direction indicated, each conductor experiences a force in the direction shown in fig. 11 (p. 150), and the two forces produce a torque on the loop in a clockwise direction. If the loop moves under the influence of this torque, the connections relative to the external supply are reversed by the commutator as its plane is passing through the vertical position. Hence the current in conductor A, for instance, which has been under the north pole, is *reversed* just as it comes under the influence of the south pole, so that the direction of the force acting on it, relative to the direction of rotation, is unchanged; and similarly with conductor A_1. The loop will therefore continue to rotate, but the torque falls to zero twice per revolution when the loop is in a vertical position; and, anywhere in the neighbourhood of this position, the torque is insufficient to start rotation, so that this elementary form of d.c. motor is of little practical use.

The ring-wound armature of fig. 9 is shown again in fig. 18*b*, with the addition of circles to represent the section of each of the outer conductors; and if a current from an external source is passed through this winding in the same direction as in fig. 9, its direction in each of the conductors is as indicated in the section. Hence all the conductors which are under the poles experience tangential forces F in the direction shown, all of which produce a clockwise torque; since no flux passes across the interior of the armature, the inner conductors produce no torque. As was seen in § 8, p. 308, each coil as it passes

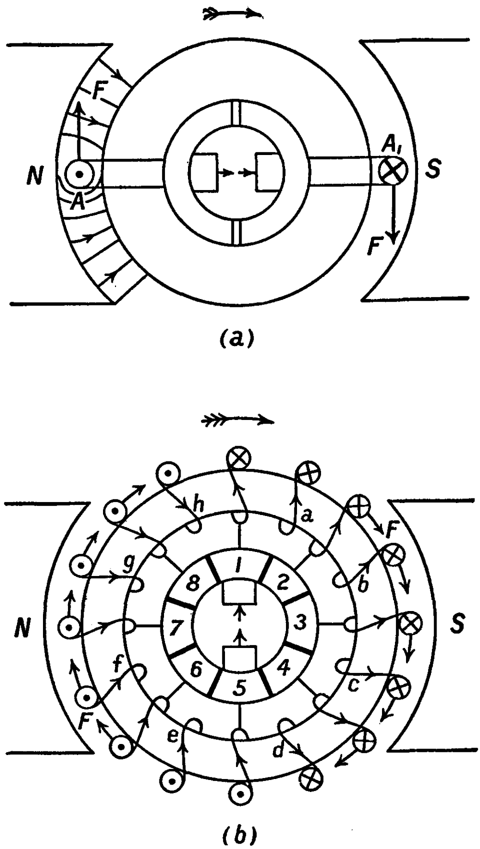


Fig. 18.—Principle of d.c. motor

between the poles is transferred by the commutator from one circuit to the other, and this results in the reversal of the current in it. For example, one quarter of a revolution later, coil g will occupy the position now occupied by coil a, and the current which is now flowing in the direction 8–7 (referring to the commutator segments) will then be flowing in the direction 7–8, so that coil a represents exactly coil g one quarter of a revolution later, not only in position but in the direction of the current and therefore of the force. Hence, as the armature rotates, a torque is produced by those conductors lying in the magnetic field at any particular instant, which is constant in direction although varying somewhat in magnitude.

A reference to fig. 12, which is a section through a four-pole drum armature, shows that the current distribution is such that the forces exerted on all the conductors lying under the poles produce torques in the same direction. In this type of winding, as used on all modern motors, the number of coils is large and the total torque produced maintains an almost constant value as the armature rotates, as long as the armature current and flux per pole are constant.

This torque of course exists also in a generator whenever the external circuit is completed so that a current flows through the armature winding, but in this case it is an *opposing* torque; in maintaining the armature rotation against this opposing torque mechanical energy is absorbed and converted into electrical energy. As long as the armature circuit is open, a large generator may require only a few horse-power to drive it at normal speed, but directly a current flows in the winding, all the conductors in the magnetic field experience forces which produce an opposing torque, and several hundred horse-power may be needed to maintain rotation at the same speed.

15. Calculation of Torque and Horse-power

In an armature winding, all the conductors are parallel with the shaft and therefore perpendicular to the magnetic field, their effective length being the axial length of the armature core (l, fig. 9a, p. 308).

In the case of the early machines in which the conductors were laid on the surface of a plain cylindrical armature without slots, the torque may be calculated directly from the equation (p. 148) for the force acting on a conductor in a magnetic field.

Let

z = total number of conductors,

B = average flux density under the poles (Wb/per sq. m.),

I_c = current in each conductor (amperes),

l = effective length of conductor (m.),

D = diameter of armature (m.),

$$k = \text{ratio } \frac{\text{conductors under poles}}{\text{total conductors}} = \frac{\text{pole arc}}{\text{pole pitch}},$$

n = speed (r.p.s.), N = speed (r.p.m.).

Then

Force acting on each conductor lying in field $= BlI_c$ newtons.

Torque due to this force $= BlI_c \cdot \frac{1}{2}D$ newton-metres.

Total torque $(T) = kzBlI_c \cdot \frac{1}{2}D$ newton-metres. . . . (6)

Total power output $= 2\pi Tn$ watts . . . . (7)

$$= \frac{1}{746} 2\pi T \frac{N}{60} \text{ horse-power.} \quad . \quad . \quad (8)$$

Alternatively, the torque (T_1) may be expressed in lb.-ft. (1 newton-metre $= 0{\cdot}738$ lb.-ft.),

$$T_1 = 0{\cdot}738\ T,$$

and

$$\text{Total power output} = \frac{1}{550} 2\pi T_1 \frac{N}{60} \text{ horse-power}$$

$$\left(= \frac{1}{746} 2\pi Tn\right).$$

This method of calculation also gives the correct result in the case of a slotted armature, but does not correspond with the physical facts. The actual flux density in the slots in which the conductors lie is very small, so that the torque is not exerted directly by the conductors but mainly through the medium of the armature teeth. It is therefore preferable to use the method given below, which involves no assumptions as regards flux density.

Work done by each conductor per revolution

$= (\text{current}) \times (\text{flux cut})$

$= I_c\Phi$ joules (eqn. 5, p. 149).

Total work done per revolution

$= zI_c \times 2p\Phi$ joules.

Total power output $= zI_c \times 2p\Phi \times n$ watts . . . . (9)

$$\left(= \frac{1}{746} zI_c \times 2p\Phi \times n \text{ h.p.}\right).$$

Total torque $= \dfrac{1}{2\pi} zI_c \times 2p\Phi$ newton-metres. . . . . (10)

The useful power output is somewhat less than this as the total power includes the friction, windage, and iron losses.

Example.—An armature of a 6-pole machine 75 cm. in diameter has 664 conductors, each having an effective length of 30 cm. and carrying a current of 100 amperes. If 70 per cent of the total conductors lie simultaneously in a field having an average density of 0·85 webers per sq. m., calculate (*a*) the torque in newton-metres, (*b*) the horse-power output at 250 r.p.m.

(*a*) Force acting on each conductor $= 0{\cdot}85 \times 0{\cdot}3 \times 100$ newtons

$= 25{\cdot}5$ newtons ($= 5{\cdot}73$ lb.).

Total number of active conductors $= 664 \times \dfrac{70}{100} = 465.$

Radius of armature $= 0{\cdot}375$ m.

Total torque $= 465 \times 0{\cdot}375 \times 25{\cdot}5 = 4450$ newton-metres

($= 3280$ lb.-ft.).

$$(b)\ \text{Power output} = \frac{2\pi \times 250 \times 3280}{33{,}000} = 156 \text{ h.p.}$$

Alternatively, the output and torque can be obtained from the flux per pole ($\Phi = 0{\cdot}07$ weber).

Work done per revolution (see equation 5, p. 149)

$= 664 \times 100 \times 6 \times 0{\cdot}07 = 27{,}900$ joules.

$$\text{Power output} = 27{,}900 \times \frac{250}{60} = 116{,}000 \text{ watts} = 156 \text{ h.p.}$$

$$\text{Torque} = \frac{27{,}900}{2\pi} = 4450 \text{ newton-metres } (= 3280 \text{ lb.-ft.})$$

16. Back-E.M.F. and Speed—Motor Startor *

When the armature rotates under the influence of the torque, the conductors move through the field and therefore an E.M.F. is induced in them; and since the direction of rotation is *opposite* to that of a generator for the same direction of flux and current (fig. 1), the induced E.M.F. *opposes* the current, and for this reason is often called the *back*-E.M.F. It must be realized, however, that there is no difference between the generated E.M.F. in a generator and the back-E.M.F. in a motor, except in their directions relative to the current, both are the *same* E.M.F. produced in the *same* way, i.e. by the motion of the conductors through the field.

It follows that the supply P.D. must always be greater than the back-E.M.F., and in overcoming and causing the current to flow against this E.M.F., electrical energy is converted into mechanical energy. It must, in fact, exceed the back-E.M.F. by an amount sufficient to overcome the resistance of the armature winding, so that

$$V = E_b + IR_a,$$

* The British Standards Institution recommend the termination *-or* for the names of pieces of apparatus by which certain operations are accomplished, e.g. *startor*, *resistor*, *convertor*, &c.

but as the resistance is always small, the value of the back-E.M.F. is never much less than that of the supply P.D.

The presence of the back-E.M.F. makes the d.c. motor a self-regulating machine. When running unloaded, the torque required is only that necessary to overcome friction and windage, so that the current is small and the back-E.M.F. almost equal to the supply P.D. If the motor is suddenly loaded, the first effect is to cause the armature to slow down. The speed at which the conductors move through the field is therefore decreased, the back-E.M.F. falls and allows an increase in armature current, which produces an increase in torque. On the other hand, if the load is removed, the torque is momentarily in excess of requirements, so that the armature is accelerated; this increases the back-E.M.F., which cuts down the current until a value is reached corresponding with the reduced torque required. The motor, therefore, always runs at the speed at which the back-E.M.F. generated is such as to allow the armature current to reach a value sufficient to produce the torque required:

$$E_b = V - IR_a. \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Multiplying equation (11) by I gives

$$\begin{aligned} E_bI &= VI - I^2R_a \\ &= \text{total amature input} - \text{resistance losses} \\ &= \text{total armature output.} \end{aligned}$$

Hence the product (back-E.M.F. $\times$ armature current) represents the total armature output.

Useful or brake output

= total output — friction, windage and iron losses.

Since the back-E.M.F. is a generated E.M.F., the E.M.F. equation obtained on p. 316 applies, from which it can be seen that in any given machine,

$$E_b \propto N \,.\, \Phi, \text{ or } N \propto \frac{E_b}{\Phi}, \quad . \quad . \quad . \quad . \quad (12)$$

and as E_b is never much less than V, even at full load,

$$N \propto \frac{1}{\Phi} \text{ approximately,}$$

when the supply voltage is constant.

Hence weakening the field of a motor increases its speed and vice versa. The first effect of a decrease in flux is a fall in back-E.M.F. Because the armature resistance is low, a small decrease in back-

E.M.F. allows a relatively large increase in armature current, and the increased torque accelerates the armature. The back-E.M.F. therefore rises and cuts down the armature current almost to its initial value, but the motor is now running at a higher speed, generating the same back-E.M.F. in a weaker field.

When the armature is stationary, there is of course no back-E.M.F., and since the resistance is small, a very large current would flow if the motor were connected directly to the supply. Hence it is necessary to use a *startor*, which consists of a variable resistance placed in series with the armature during the starting period. The maximum value of the resistance is sufficient to limit the current to a safe value when there is no back-E.M.F., and it is gradually reduced as the speed and back-E.M.F. increase, until, when these have attained the normal values, it is cut out altogether and the motor is connected directly to the supply. The startor for a d.c. motor consists essentially of a number of resistance coils connected to contact studs and controlled by a rotary switch arm; but several safety devices are usually included, which automatically disconnect the machine from the mains if the supply fails or in case of sustained overloading.

17. Motor Characteristics

The characteristic curves of a d.c. motor are those connecting (1) speed and armature current, (2) torque and armature current, and (3) speed and torque, which can be obtained from (1) and (2). They depend upon the method of excitation employed, and are shown in fig. 19.

Shunt Excitation.—In the shunt excited motor connected to a constant voltage supply, the field current is independent of the load, so that the machine is virtually separately excited.

An increase in load torque causes the speed to fall until the back-E.M.F. has decreased sufficiently to allow the necessary increase in armature current, but since the armature resistance is low, a small decrease in speed and back-E.M.F. causes a large increase in armature current. Hence the speed characteristic is only slightly drooping, so that the shunt motor is an almost constant speed machine (fig. 19*a*).

Since $T \propto I\Phi$ and the field current is constant, the curve relating torque and armature current is a straight line through the origin, although with large currents it falls away a little because the flux is slightly reduced by the effect of armature reaction (fig. 19*f*). This is the total torque, some of which is used in overcoming friction and windage, so that the useful torque, available at the shaft, is somewhat less.

Shunt motors may be used for any drive in which an almost constant speed is required and which is not subject to heavy overloads.

Series Excitation.—Here the armature current is also the field current, and the field strength therefore depends on the load. At small loads, since the current is small, the field is weak and the speed high,* but it falls rapidly as the load current increases and the field is strengthened (fig. 19*c*). In fact, while the magnetic circuit is unsaturated, so that $\Phi \propto I$, the speed is inversely proportional to the current.

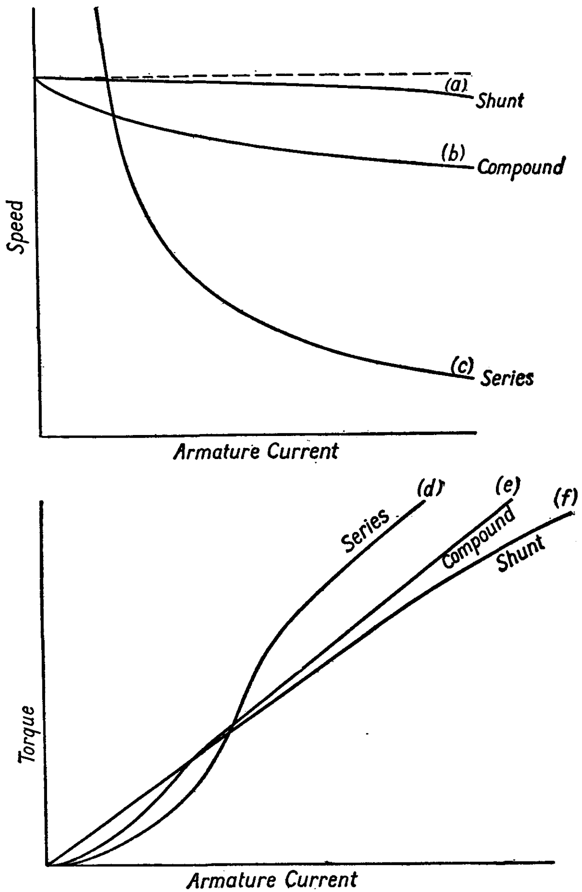


Fig. 19.—Motor characteristics

For the same reason the torque at first increases rapidly (fig. 19*d*) ($T \propto I\Phi \propto I^2$), but more slowly later on as approaching saturation causes the flux to increase more slowly.

* A series motor should never be used with a belt drive, because, if the belt comes off or breaks, the no-load speed is dangerously high.

Series motors exert large torques at low speeds and are therefore used principally for cranes, lifts, traction, etc.

Compound Excitation.—In this case the characteristics are intermediate between shunt and series, and approach either the one type or the other according to the relative proportions of the ampere-turns supplied by the shunt and series coils. The curves shown (fig. 19, *b* and *e*) are for a case in which the series field is sufficiently strong to cause a drop in speed of between 15 and 20 per cent between no-load and full-load. Such motors are used in drives where considerable overloads occur, where a flywheel is used to assist in fluctuating loads, or where a large starting torque is required.

18. Speed Control

From equation (12),

$$\text{Speed} \propto \frac{E_b}{\Phi} \propto \frac{V}{\Phi} \text{ approximately,}$$

so that the speed may be varied by varying either the armature P.D. or the flux per pole.

If the armature P.D. is decreased, the armature current falls and the torque becomes insufficient, so that the speed must fall until the back-E.M.F. has decreased sufficiently to restore the current to its

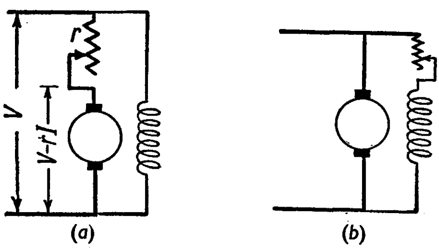


Fig. 20.—Speed control of d.c. motors

previous value. By this means any speed below normal can be obtained, but as the armature P.D. must usually be reduced by means of a series resistance (fig. 20*a*), the method is very inefficient; although it is sometimes used for short periods.

The most common method of speed control of shunt and compound motors is by the insertion of a variable resistance in the shunt field circuit (fig. 20*b*), so that the flux may be reduced, which has the effect or raising the speed as shown in § 16. By this means a considerable range of speed above normal may be obtained economically as the losses in the rheostat, which carries only the shunt field current, are small. Occasionally the same method is applied to the series motor by placing a resistance, know as a *divertor*, in parallel with the field circuit, which, by diverting some of the current from the field coils, produces the reduction in the flux required.

Example.—A 500-volt shunt motor has armature and shunt field resistances of 1 ohm and 500 ohms respectively. When running unloaded the total current taken is 2 amperes and the speed is 1000 r.p.m. Calculate the speed when the motor is fully loaded and the total current taken is 20 amperes.

Estimate the speed at this load current if (*a*) a resistance of 5 ohms is connected in series with the armature, (*b*) the field current is reduced so that the flux is reduced by 10 per cent.

At no-load, Field current $= \frac{500}{500} = 1$ ampere (fig. 21*a*),

Armature current $= 2 - 1 = 1$ ampere,

$E_b = 500 - (1 \times 1) = 499$ volts.

At full-load, Field current $= 1$ ampere,

Armature current $= 20 - 1 = 19$ amperes,

$E_b = 500 - (19 \times 1) = 481$ volts.

$N \propto \frac{E_b}{\Phi}$, and since in this case Φ can be assumed constant,

$$N \propto E_b.$$

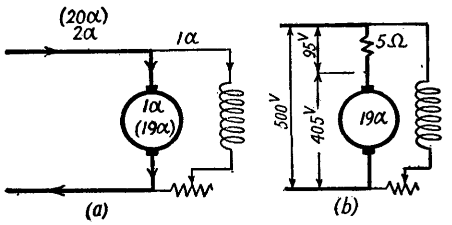


Fig. 21.—To illustrate Example

In order that the back-E.M.F. may fall to 481 volts the speed must fall to

$$1000 \times \frac{481}{499} = 965 \text{ r.p.m.}$$

(*a*) Armature P.D. reduced to $500 - (19 \times 5) = 405$ volts (fig. 21*b*),

$$E_b = 405 - (19 \times 1) = 386 \text{ volts},$$

$$\text{Speed} = 1000 \times \frac{386}{499} = 774 \text{ r.p.m.}$$

(*b*) Here E_b remains unchanged but Φ is reduced. Hence

$$\frac{N_1}{N_2} = \frac{\Phi_2}{\Phi_1}. \quad \therefore N_2 = N_1 \cdot \frac{\Phi_1}{\Phi_2} = 965 \cdot \frac{100}{90} = 1072 \text{ r.p.m.}$$

THE INDUCTION MOTOR

19. General Principle—Production of a Rotating Field

In the induction motor there is no electrical connection between the stator and the rotor, the energy being transferred entirely magnetically, by means of the E.M.F. induced (as the name implies) in the rotor conductors by the rotating field set up by the stator windings. This principle of operation enables the motor to be manufactured as a simple, robust and efficient machine, and for those reasons it is the most extensively used of all types of electric motor.

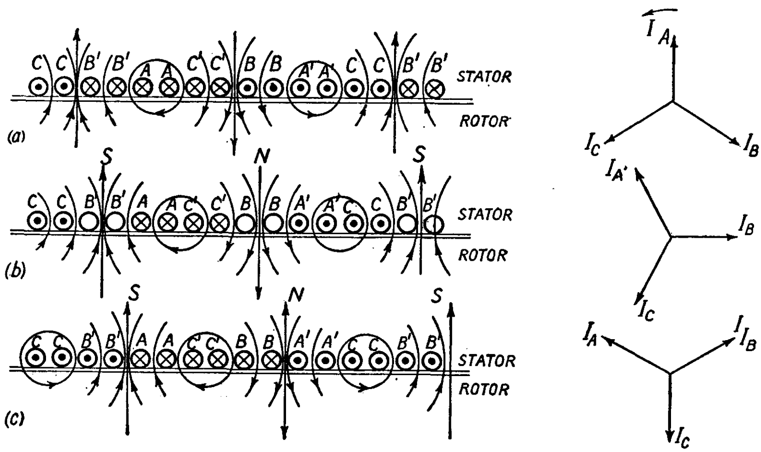


Fig. 22.—Production of rotating field in three-phase induction motor

The stator winding of a three-phase induction motor is similar to that of a three-phase alternator (fig. 5). A portion of such a winding, *developed** for simplicity, is shown in fig. 22; there are 6 slots per pole and the starts of the three windings are assumed to be at A, B and C, $\frac{2}{3}$ of a pole pitch (or 120° of phase) apart. At the instant represented by the vector diagram in fig. 22*a*, the current in phase A is a positive maximum and those in phases B and C are half their maximum value in a negative direction. If it is assumed that the positive direction is into the paper, then the current in slots AA is inwards, while that in slots BB and CC is outwards; and in the slots containing the other sides of the coils the directions are outwards in $A'A'$ and inwards in $B'B'$ and $C'C'$. Hence the current and flux distribution at this instant is as shown, alternate N and S poles being formed of which the centre lines are indicated in the figure. One-twelfth of a period later I_A has fallen to 86·6 per cent of its positive

* i.e. assumed to be opened out flat.

maximum, I_B has become zero, and I_C has risen to 86·6 per cent of its negative maximum. The current and flux distribution corresponding with this condition is shown in fig. 22*b*, from which it can be seen that the centre lines of the poles have moved one slot pitch to the right, which, since there are 6 slots per pole, is one-twelfth of two pole pitches or 30° of phase. One-twelfth of a cycle later still, I_A has fallen to 50 per cent of its positive maximum, I_C has reached its negative maximum, while I_B has risen to 50 per cent of its *positive* maximum, so that the direction of the current in *BB* and *B′B′* is reversed. This gives the distribution shown in fig. 22*c*, in which the centre lines of the poles have moved another slot pitch to the right.

It appears, therefore, that the sequence of current changes in a stationary 3-phase winding sets up a magnetic field which moves along, or in the actual case, *round* the stator, rotating through 30° of phase in each one-twelfth of a period, i.e. 360° or two pole pitches per cycle. For example, in an 8-pole stator, connected to a 50-cycle supply, the field moves once round the stator in the time taken for the stator currents to pass through 4 cycles. In other words, the field makes $12\frac{1}{2}$ revolutions per sec. This is known as the *synchronous* speed (n_0), and is, of course, the speed of an alternator having the same number of poles and generating an E.M.F. of the same frequency: it is given by

$$n_0 = \frac{f}{p},$$

where f is the frequency and p the number of pairs of poles.

20. Principle of Operation

If a conductor is placed near the inner surface of the stator shown in fig. 22, so that it is cut by the rotating field, an alternating E.M.F. is generated in it having the same frequency as that of the supply.

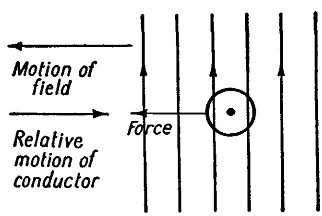


Fig. 23.—Direction of force acting on rotor conductors

If the conductor is replaced by a *short-circuited* turn having a span equal to the pole pitch, the E.M.F. generated in the conductors forming the sides of the turn causes a current to flow; and each conductor experiences a force, the relative direction of flux, force and current being as shown in fig. 23. If the field is moving past the conductor from right to left, the direction of the generated E.M.F. is the same as if the conductor were moved from *left* to *right* in a *stationary* field, and

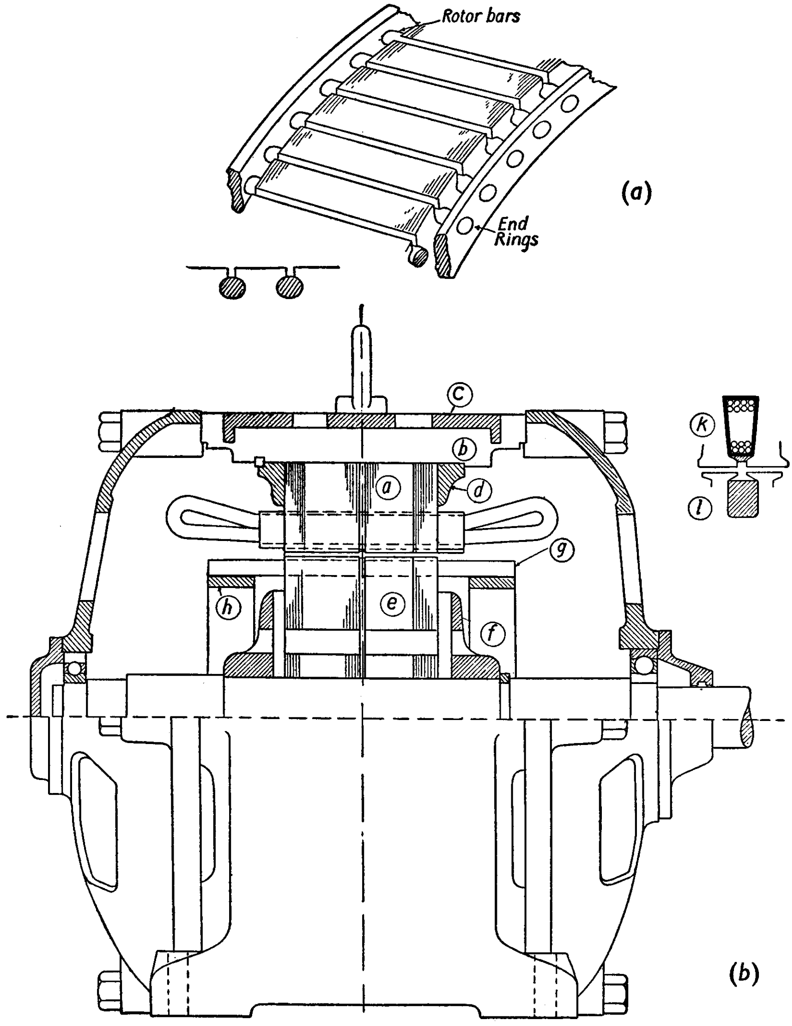


Fig. 24.—Construction of squirrel-cage induction motor

is therefore such that the force produced by a current due to the E.M.F. opposes the motion (§ 1, p. 295), i.e. is from right to left. Hence, when the E.M.F. is induced by the motion of the field past the conductor, the force due to the current set up urges the conductor *in the direction* of the field.*

In the simplest form of rotor the single turn is replaced by a laminated iron rotor having slots containing bare copper (or aluminium) bars, the ends of which are short-circuited by heavy copper (or aluminium) rings (fig. 24*a*): this is known as a *squirrel-cage* (or simply a *cage*) rotor. The E.M.F. induced in the conductors by the stator field causes currents to circulate, and the forces thereby exerted on the bars produce a torque which causes the rotor to rotate in the direction of the field. As the speed increases, the relative motion and hence the value and frequency of the rotor E.M.F.s and currents continually decrease, so that the rotor can never run right up to synchronous speed at which the relative motion, and therefore the E.M.F. and the torque, would be zero. At no-load, the retarding torque is only that of friction and windage, and the speed very nearly reaches synchronous value. The application of a load causes the speed to decrease, and thus produces an increase in rotor E.M.F. and current; and the speed falls to a value at which the rotor current is sufficient to give the increased torque required. The resistance of the rotor is low, and the E.M.F. required to circulate full-load current is not large, so that the fall in speed between no-load and full-load is not more than about 5 per cent in a small motor, and may be as little as 2 per cent in a large machine: hence the speed characteristic is similar to that of a shunt motor.

The difference between the synchronous speed (n_0) and the actual speed (n) is called the *slip* (s) (i.e. the amount by which the rotor slips back in the field), and it is usually expressed as a fraction or percentage of the synchronous speed,

$$s = \frac{n_0 - n}{n_0}.$$

Over the normal load range the slip is almost proportional to the load.

Example.—A 10-h.p., six-pole, three-phase, 50-c/s induction motor has a full-load slip of 5 per cent. Calculate the speed at full-load and estimate its value at half-load.

$$\text{Synchronous speed} = n_0 = \frac{f}{p} = \frac{50}{3} \text{ r.p.s.}$$
$$= 1000 \text{ r.p.m.}$$

$$\text{Speed at full-load} = 1000(1 - 0{\cdot}05) = 950 \text{ r.p.m.}$$

$$\text{Slip at half-load} = \frac{5}{2} = 2{\cdot}5 \text{ per cent approx.}$$

$$\text{Speed at half-load} = 1000(1 - 0{\cdot}025) = 975 \text{ r.p.m. approx.}$$

* This is, of course, in accordance with the general principle laid down by Lenz, that the direction of an induced E.M.F. is always such as to oppose the change to which it is due: in this case, by moving in the direction of the field, the relative motion between the conductor and flux, to which the E.M.F. is due, is reduced.

21. Types of Rotor

The torque developed by the rotor is porportional to the flux set up by the stator windings, the current in the rotor windings, and the rotor power factor. If the power factor is unity, i.e. if the rotor current is in phase with the induced E.M.F., the space distribution of current and flux are such that the forces acting on all the rotor conductors produce torques acting in the same direction; whereas if there is a phase difference between E.M.F. and current, the relative distribution is altered so that some of the conductors produce a torque in the opposite direction, with a consequent reduction in the resultant torque. The proportion of such conductors increases as the angle of phase difference increases, until at zero power factor (phase difference of 90°) half the conductors would produce a backward torque and the resultant torque would be zero.

In the squirrel-cage rotor the resistance is inherently low, but since each bar is largely surrounded by iron, its inductance is considerable, and, at starting, the reactance is much higher than the resistance. Hence the rotor current lags behind the E.M.F. by a large angle, i.e. the rotor power factor is low, and, if the starting current is limited to a reasonable value, the starting torque is poor.

As stated above, the torque is improved by bringing the current more into phase with the E.M.F., and, since it is not possible to reduce the reactance, this can be done only by increasing the resistance of the rotor circuit (if R is increased, $\phi = \tan^{-1} (X/R)$ is decreased). This, of course, increases the impedance and therefore decreases the current for a given slip, but, provided the increase in resistance is not excessive,* the effect of the improvement in power factor is greater than that of the decrease in current, and the starting torque is increased. This could be done most simply by making the bars and rings of a resistance alloy, but the result would be to increase the rotor losses under normal running conditions. Hence, when the starting torque of the squirrel-cage rotor is insufficient, a *slip-ring* or *wound rotor* is used, in which there is an insulated winding, brought out to slip-rings, across which are connected external resistances, which can be gradually cut out as the motor runs up, the rings being finally short-circuited.

The starting resistances, if continuously rated and retained in the circuit, can also be used for speed control; by this means any speed below normal can be obtained, but the method (which corresponds with the insertion of resistance in the armature circuit of a d.c. motor—§ 18) is very inefficient and is used only when low speeds are required for short periods.

* Maximum starting torque occurs when the rotor resistance is equal to the rotor reactance at starting.

The slip-ring rotor is more expensive and slightly less efficient than the squirrel-cage rotor, which is always used when possible.

The general construction of a squirrel-cage induction motor is shown in fig. 24*b*. The stator laminations *a* are supported by ribs *b* on the inside of a cast-iron or welded-steel frame *c*, and clamped between end rings *d*. The stator winding is housed in semi-closed slots as shown in detail at *k*. The rotor laminations *e* are keyed directly to the shaft, and clamped between similar end rings *f*. The bars *g* of the squirrel cage are laid in semi-closed slots *l*, and short-circuited by end rings *h* to which they are welded. In small machines the squirrel cage is often of aluminium, the bars being cast into the slots, together with the end rings, in one operation.

22. The Heating of Electrical Machines

Whenever mechanical energy is converted into electrical energy in a generator or, vice versa, in a motor, certain losses occur which appear in the form of heat and raise the temperature of the machine; and in general it is the temperature attained which sets a limit to the power output of which the machine is capable.

The efficiency of a rotating electrical machine depends upon its size, speed, load, and various other factors. Broadly speaking, the full-load value ranges from about 75 per cent for small machines (excluding fractional horse-power motors) to about 94 per cent for very large machines, and for the great majority of industrial motors is round about 90 per cent, i.e. the losses are about 10 per cent of the input.

These losses may be divided into three groups, of which (*a*) is common to all types of machines, while (*b*) and (*c*) occur only in electrical machines:

(*a*) Losses due to friction between shaft and bearing and between brushes and commutator or slip rings; and windage losses due to the disturbance, by the rotating portion, of the surrounding air.

(*b*) Resistance losses due to the resistance of the armature and field windings.

(*c*) Iron losses which occur in all parts of the magnetic circuit which are subjected to an alternating magnetic field and are made up of losses due to the hysteresis effect in the iron (§ 4, p. 217), and to eddy currents induced in the iron (§ 10, p. 151).

When an electrical machine, all the parts of which are at the temperature of the surrounding air, is started up and put on load, i.e. used to supply either electrical or mechanical energy, the heat produced is at first entirely absorbed by the machine, and its temperature begins to rise. As soon as this exceeds that of the surrounding air, heat begins to pass from the machine to the air at a rate which depends upon the *difference* between the two temperatures. Hence as the temperature rises, the rate at which the heat produced is absorbed by the machine becomes less, and that at which it is dissipated to the surrounding air increases. Ultimately a value is reached at which the heat is lost at the same rate as that at which it is produced; hence no heat is absorbed and the temperature of the machine remains steady at this value until either the load or the temperature of the surrounding air changes.

If the temperature of the surrounding air rises, then the rate of dissipation decreases, some of the heat is absorbed, and the temperature of the machine again rises until once more heat is lost at the same rate as it is produced. Similarly, if the load is increased, the rate of heat production increases, and heat is absorbed by the machine until its temperature has risen so that the rate of heat loss is correspondingly increased.

The rate of heat production, and also the ease with which it is conveyed to the surrounding air, vary in different parts of the machine. Further, the effectiveness of a cooling surface depends largely upon how rapidly the heated air is removed, so that cooling conditions are best for the rotating portion and those stationary parts which are exposed to the draught from it. In the case of the armature winding, and more particularly in that of the field coils, the heat produced in the middle of the coil, in order to reach the surface, has to pass through a path consisting largely of insulating materials of which the heat conductivity is very poor; in such cases the internal temperature of the winding is considerably higher than that measured by a thermometer at the surface.

It is the temperature attained by these windings which usually sets a limit to the power output of the machine. The insulating material in most cases contains organic fibrous materials such as cotton, linen or paper; in low-voltage machines these may serve as the insulation itself; in other cases they are used as a support or mechanical protection for the insulating material, which is usually some form of mica. Fibrous materials deteriorate if subjected for long periods to high temperatures, so that the output of the machine must be such that a safe temperature is not exceeded. The power output which a machine is capable of giving continuously or for a given period without attaining a dangerously high temperature is know as its *rating*; and in order to secure uniformity in the rating of machines, standard specifications have been drawn up by the British Standards Institution for all types of commercial machines and for various conditions of service.

In designing an electrical machine, one of the chief problems is that of limiting the rate at which heat is produced, and of providing adequate cooling facilities. The former is accomplished by making the sections of the electric and magnetic circuits sufficient to carry the necessary current and flux; the latter by arranging the machine in such a way that the rotating portion, acting as a fan, directs a stream of cooling air on to the most effective cooling surfaces of the stationary portion.

The temperature may always be kept low by increasing the amount of iron and copper and so decreasing the losses, but such a machine, though very efficient, is high in first cost. On the other hand, the amounts of copper and iron can be reduced to a minimum and the temperature kept down by increasing the volume of cooling air, but such a machine, though lower in first cost, is inefficient. Hence the design of a machine which shall be satisfactory in operation and efficiency and reasonable in first cost entails a series of compromises between conflicting factors, which can be made only by experience.

THE TRANSFORMER

23. Principle of the Transformer

The principle underlying the operation of the transformer has been described in § 23, p. 171. When two coils A and B are so placed that some of the flux set up by A is linked with B (and vice versa), the two coils possess *mutual inductance*. Any change in the flux produced by coil A induces an E.M.F. in coil B, and if the latter forms a complete circuit, a change in the current in A causes a current to flow in B and a *momentary* transfer of energy from A to B. If coil A is connected to an a.c. supply, the alternating current flowing sets up an alternating flux which induces an alternating E.M.F. and current in B, and a *continued* transfer of energy from A to B takes place. Since the two circuits are not connected electrically, the transfer of energy is entirely magnetic; and the arrangement constitutes a transformer in its simplest form.

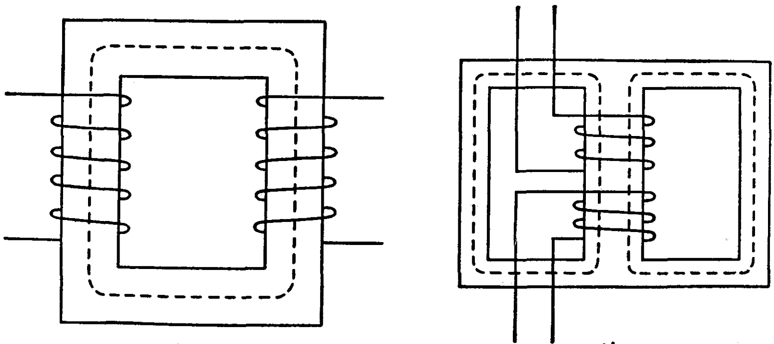
There would be no particular object in using such a piece of apparatus were it not for the fact that it is possible in this way to make the values of E.M.F. and current in the two circuits to differ.

It is usually convenient to generate electrical energy on a large scale, at voltages between 6600 and 11,000 volts, but in order to transmit the energy efficiently over long distances, a higher voltage and correspondingly smaller current is desirable; while for reasons of safety, distribution of energy to lighting and power circuits is usually carried out at a few hundred volts.* These voltage transformations can be made very easily by means of the transformer, which, owing to the absence of moving parts, is a piece of apparatus of great reliability and high efficiency; and the possibility of using the transformer in this way is one of the chief reasons which have led to the universal adoption of the a.c. system.

* The transmission voltage depends upon the length of line: that used on the main transmission lines of the National Grid System is 132,000 volts, and in the Super Grid now under construction, 275,000 volts.

The standard distribution voltage in this country is 415 volts for power and 240 volts for lighting circuits.

A transformer consists essentially of two coils linked by a common magnetic circuit and arranged so that their *mutual inductance* is as large as possible; in order to increase this mutual inductance, the reluctance of the magnetic circuit is made very low by placing both coils on a common iron core (fig. 25).



(a) (b)

Fig. 25.—(*a*) Core type, (*b*) shell type transformers

The coil *to* which energy is supplied is called the *primary*, and that *from* which energy is supplied to the load circuit the *secondary*; but the transformer is completely reversible, and either winding may be used as the primary, provided it is connected to a supply of the correct voltage.

The Transformer on No-load—Voltage Ratio.

Since the reluctance of the magnetic circuit is low, the self inductance of the primary winding is large compared with its resistance. When it is connected to an a.c. supply with the secondary winding open circuited, it acts simply as a highly inductive coil, and the current taken lags by almost 90° and is just sufficient to set up a flux which (if the resistance is neglected) induces an E.M.F. of self inductance (E_L) equal and opposite to the supply P.D. Under this no-load condition it may be assumed that all the flux set up by the primary is linked with the secondary coil, so that an E.M.F. of mutual inductance (E_2) is induced in each secondary turn of the same value as the E.M.F. of self inductance induced in each primary turn by the same flux. Hence, if the secondary winding has twice as many turns as the primary winding, the total E.M.F. induced in the secondary winding will be twice the total E.M.F. induced in the primary winding; and in general terms, if N_1 and N_2 are the numbers of turns in the two windings,

$$\frac{E_L}{E_2} = \frac{N_1}{N_2}.$$

At no-load E_L is numerically almost equal to the primary P.D., V_1, while the secondary P.D., V_2, is equal to the secondary E.M.F., so that

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

i.e. the ratio of the terminal P.D.s at no-load is that of the number of turns in the respective windings; this ratio is known as the *transformation ratio.* The impedance drops in each winding are always small, so that this relationship remains almost true when the transformer is loaded. If V_2 is greater than V_1, the transformer is known as a *step-up* transformer; if V_2 is less than V_1, as a *step-down* transformer.

E.M.F. Equation of a Transformer.

Let N_1, N_2 = number of turns in the primary and secondary windings respectively,

Φ_m = maximum value of the flux linking both windings,

f = frequency of supply.

Since the flux grows from zero to its maximum in $\frac{1}{4}$ cycle, i.e. in a time $t = 1/4f$ sec.,

Average rate of change of flux is

$$\frac{\Phi_m}{t} = 4\Phi_m f \text{ webers per second;}$$

Average E.M.F. induced in each turn $= 4\Phi_m f$ volts.

Now in any a.c. wave $\dfrac{\text{r.m.s. value}}{\text{average value}}$ = form factor (= 1·11 for a sine wave).

Hence, assuming that the flux wave is sinusoidal,

$$\begin{aligned}\text{r.m.s. E.M.F. induced in each turn} &= 1{\cdot}11 \text{ (average value)} \\ &= 4{\cdot}44\ \Phi_m f \text{ volts.}\end{aligned}$$

Hence:

E.M.F. of self inductance induced in primary winding is

$$E_1 = 4{\cdot}44 N_1 \Phi_m f \text{ volts;} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

E.M.F. induced in secondary winding is

$$E_2 = 4{\cdot}44 N_2 \Phi_m f \text{ volts.} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Example 2.—A 6600/230-volt single-phase transformer has a primary winding of 1650 turns. Calculate the maximum value of the flux induced in the core when this winding is connected to a 6600-volt 50-c/s supply. Neglect the impedance of the primary winding.

From equation (14)

$$\Phi_m = \frac{6600}{4{\cdot}44 \times 1650 \times 50} = 0{\cdot}018 \text{ weber.}$$

Transformer on Load.

If the secondary circuit is closed the secondary E.M.F. causes a current to flow, and, in accordance with Lenz's law, the ampere-turns set up by this current, being linked with the same magnetic circuit, tend to reduce the flux set up by the primary ampere-turns. But a decrease in the flux causes a decrease in the primary E.M.F. of self inductance, and this allows the primary current to increase by an amount which almost completely compensates for the demagnetizing effect of the secondary current. Hence any change in the value or phase of the current in the secondary circuit is accompanied by a corresponding change in the value or phase of the primary current.

Although it is clearly impossible to obtain a secondary output quite as great as the primary input, the losses in a transformer are so small that there is little error in assuming that the two are equal, i.e.

$$\text{Output} = \text{Input},$$

$$V_2I_2\cos\varphi_2 = V_1I_1\cos\varphi_1.$$

φ_1 is always slightly greater than φ_2, but the difference is small, so that

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1} \text{ (very nearly)}. \quad . \quad . \quad . \quad . \quad (16)$$

Hence it is almost true to say,

The terminal P.D.s are directly proportional to, and the currents inversely proportional to, the number of turns in each winding.

The current relationship also follows from magnetic considerations. When the secondary circuit is closed a current I_2 flows and sets up N_2I_2 ampere-turns opposing the primary ampere-turns which produce the flux. This effect is neutralized by an increase in the primary current by an amount I'_1 such that

$$N_1I'_1 = N_2I_2,$$

and since the no-load component (which sets up the flux) is small, $I_1 = I'_1$ very nearly, and

$$N_1I_1 = N_2I_2 \text{ or } \frac{I_1}{I_2} = \frac{N_2}{N_1}.$$

24. Construction of Transformers

There are two common types of transformers, which differ in the relative arrangement of the magnetic and electric circuits. In one, known as the *core type* and shown diagrammatically in fig. 25*a*, the iron core is largely surrounded by the coils; while in the other, known as the *shell type* (fig. 25*b*), the coils are very largely surrounded by the iron core, which provides two magnetic circuits in parallel.

Most transformers in this country are of the core type, although the shell type has advantages for certain conditions of service. Both types, except in small sizes, are usually completely immersed in oil, for purposes of cooling and insulation.

Core Type.

A section through a small core-type transformer is shown in fig. 26. The iron core (1) is built up of laminations (in order to reduce eddy currents) which are held at the top and bottom between steel clamps

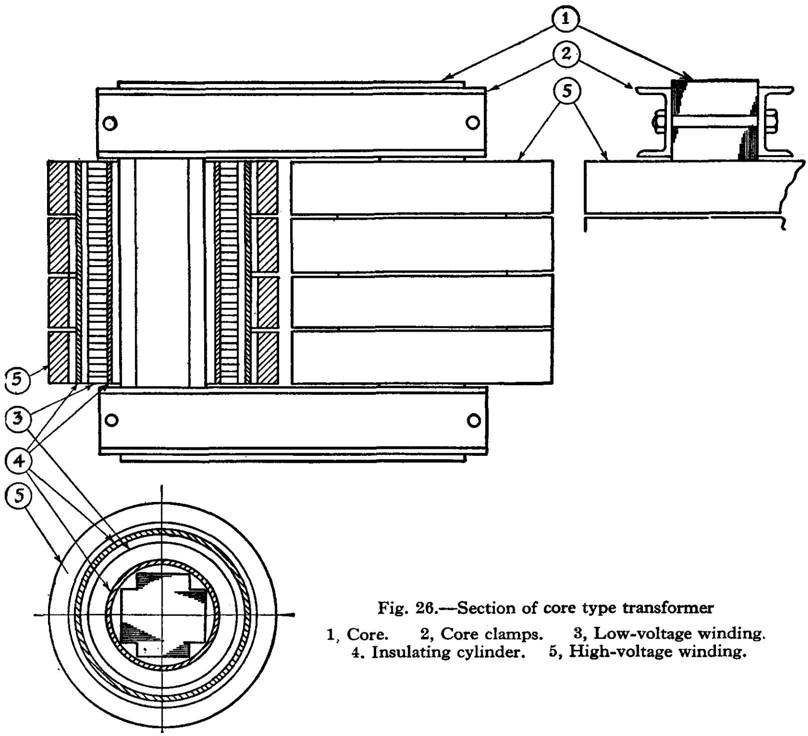


Fig. 26.—Section of core type transformer

1, Core. 2, Core clamps. 3, Low-voltage winding. 4. Insulating cylinder. 5, High-voltage winding.

of angle or channel section (2); for economy in copper, the ideal core section is circular, but since this is not practicable, a cruciform section is usually adopted.

In order to improve the performance of the transformer, each winding is distributed between the two limbs and not as shown diagrammatically in fig. 25*a*. The lower voltage, and therefore heavier current coil, generally consisting of copper strip insulated with cotton or paper, is wound in a single or double layer helix (3), and placed next to the core from which it is separated by a thin cylinder of insulating material (4). Outside this is placed the high-voltage winding (5), and between the two coils and separated from each to allow the oil to circulate is a second insulating cylinder. This winding, in which the current is small, is of cotton-covered wire and consists of a number of short coils each containing several layers, which are spaced axially from each other and connected in series; by this arrangement, the maximum value of the P.D. between adjacent layers is reduced and the circulation of the cooling oil is facilitated.

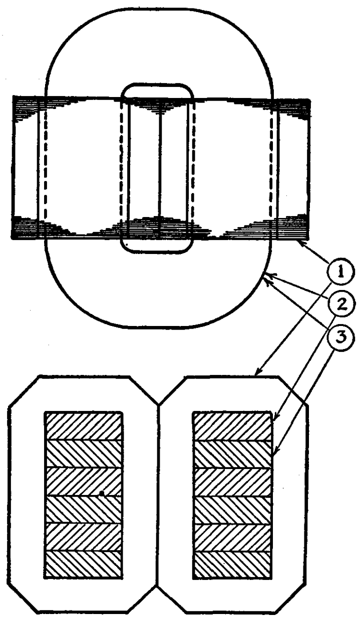


Fig. 27.—Section of shell-type transformer
1, Core. 2, Low-voltage winding. 3, High-voltage winding.

Shell Type.

A section through a shell type transformer is shown in fig. 27. In this case the core (1), in which the plane of the laminations is horizontal, is built up round the coils. Each winding is made up of a number of flat rectangular coils (2, 3) connected in series. The high-voltage coils (3) contain only a few turns per layer, while the low-voltage coils (2) often consist of copper strip wound radially, i.e. only one turn per layer. The high-voltage and low-voltage coils are assembled alternately side by side, spaces being left for oil circulation.

25. Losses and Efficiency—Cooling

The losses in a transformer, since there are no rotating parts, consist only of iron losses in the core (eddy current and hysteresis losses) and resistance losses in the windings, and are consequently much lower than in a rotating machine of equal output. The efficiency is therefore high: that of a small transformer of only a few kVA. is of the order of 95 per cent at full-load, while that of a large unit, e.g. 20,000 kVA., may exceed 99 per cent. For the same reason the natural cooling properties are poor, since there is nothing but convection currents to produce air circulation; thus it is necessary, in all but very small sizes, to increase the cooling surface artificially. This is usually

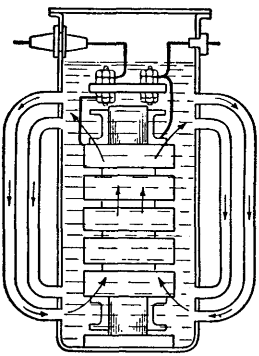


Fig. 28.—Oil-immersed transformer in tank

done by immersing the whole transformer in a tank containing mineral oil, by which heat is rapidly transferred to the exterior of the tank, the effective cooling surface of which is often increased by cooling tubes or radiators (fig. 28); in the case of very large transformers the oil is often circulated by a pump through external coolers.

Of equal importance are the insulating properties of the oil, which enable clearances between parts at different potentials to be much less than would be necessary in air; in fact the construction of high-voltage transformers of the size and efficiency of those in use at the present time would be impossible without the use of oil as an insulating medium.

Measurement of Losses.

The iron losses can be measured by means of the *open-circuit test.* One winding (whichever is convenient) is open-circuited. The other is supplied at normal voltage and frequency, and the power input measured. Under these conditions the normal flux is set up in the core; hence normal iron losses occur and are given by the wattmeter reading, since the resistance losses, due only to the no-load current, are usually negligibly small.

The resistance losses are measured by the *short-circuit test*. One winding is short-circuited and a P.D. applied to the other sufficient to circulate full-load current in both windings. Since the P.D. is small, the flux is small and the iron losses negligible, so that the wattmeter reading gives the total resistance loss at full load.

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{Losses}}{\text{Input}}$$

$$= 1 - \frac{\text{Losses}}{\text{Input}} = 1 - \frac{\text{Losses}}{\text{Output} + \text{Losses}}. \qquad (17)$$

The latter form of expression enables greater accuracy to be obtained from slide-rule calculations. This is of particular importance when the efficiency is high, as in the case of the transformer.

Like other a.c. apparatus, the transformer is rated in kVA., but efficiency is the ratio of *power* output to *power* input, hence it depends on the power factor as well as the load current. For example, the output of a 100 kVA. transformer at unity power factor is 100 kW.: at 0·8 power factor the output is only 80 kW., but the losses are the same, so that the efficiency is lower.

Example.—A 150-kVA. 6600/240-volt single-phase transformer gave the following test results when tested on the low-voltage side:

6600-volt winding, open-circuited: 240 volts, 60 amperes, 1100 watts. 6600-volt winding, short-circuited: 7 volts, 625 amperes, 1500 watts. Determine the efficiency at (*a*) full load, unity power factor; (*b*) full load, 0·8 power factor; (*c*) 1/2 full load, 0·8 power factor.

Full-load secondary current $= 150 \times 10^3/240 = 625$ amps.

(*a*) Iron loss = 1100 watts. Resistance loss = 1500 watts.
Total loss = 2600 watts. Output = 150 kW.
Efficiency $= 1 - 2{\cdot}6/152{\cdot}6 = 1 - 0{\cdot}0170 = 0{\cdot}9830 = 98{\cdot}30$ per cent (eqn. 17).

(*b*) Total loss = 2600 watts. Output $= 150 \times 0{\cdot}8 = 120$ kW.
Efficiency $= 1 - 2{\cdot}6/122{\cdot}6 = 1 - 0{\cdot}0212 = 0{\cdot}9788 = 97{\cdot}88$ per cent.

(*c*) Secondary current 313 amperes.
Iron loss = 1100 watts. Resistance loss $= 1500 \times (313/625)^2 = 375$ watts.
Total loss = 1475 watts. Output $= 75 \times 0{\cdot}8 = 60$ kW.
Efficiency $= 1 - 1{\cdot}475/61{\cdot}475 = 1 - 0{\cdot}0240 = 0{\cdot}9760 = 97{\cdot}60$ per cent.

EXAMPLES

1. The armature of a 6-pole d.c. generator has 468 conductors arranged in a lap winding. If the flux per pole is 0·064 weber, and the speed is 1000 r.p.m., calculate (*a*) the generated E.M.F., (*b*) the total power output (in kW.) of the armature when the current in each circuit is 50 amperes.

2. The armature of a 4-pole d.c. generator has 774 conductors connected to form a wave winding which has a resistance of 0·5 ohm. If the terminal P.D. is 500 volts, when the speed is 700 r.p.m. and the armature current is 50 amperes, what is the flux per pole?

3. In a 500-kW. 500/550-volt over-compound generator the flux per pole required to produce 500 volts at no-load, 1030 r.p.m., is 0·06 weber. The resistances of the armature and series fields are, respectively, 0·006 and 0·0016 ohm, and the shunt field resistance is 110 ohms. Calculate the value of the flux per pole at full-load, 1000 r.p.m.

4. The armature of a 6-pole shunt generator contains 664 conductors arranged to form a wave winding, having a resistance of 0·15 ohm. Calculate (*a*) the generated E.M.F. when running unloaded at 260 r.p.m., the flux per pole being 0·06 weber; (*b*) the terminal P.D. when the armature current is 200 amperes and the speed 250 r.p.m., assuming that the effect of armature reaction and decrease in field current is to reduce the flux by 10 per cent.

5. Find the power output of a 6-pole armature, having 468 conductors and running at 750 r.p.m., if the current in each conductor is 130 amps and the flux per pole is 0·06 weber.

6. A 4-pole 200-volt motor has 660 conductors forming a wave winding, with a resistance of 0·15 ohm, and runs at 250 r.p.m. when the armature current is 200 amperes. Find the value of the flux per pole.

7. A 250-volt shunt motor has an armature resistance of 0·4 ohm and runs at 750 r.p.m. when taking an armature current of 25 amperes. Estimate its speed at no-load when the armature current is 3 amperes, assuming the flux per pole is unchanged.

8. A 500-volt compound motor has armature and series field resistances of 0·3 and 0·15 ohm respectively, and runs at 500 r.p.m. when unloaded and taking an armature current of 5 amperes. Estimate the speed at which it will run when taking an armature current of 100 amperes, assuming that the series field increases the flux per pole by 10 per cent.

9. A 6-pole 3-phase induction motor is supplied from a 16-pole 3-phase alternator running at 375 r.p.m. What is the speed of the induction motor when it is running with a slip of 4 per cent?

10. The primary and secondary windings of a transformer contain, respectively, 1105 turns and 77 turns. Find the value of the E.M.F. induced in the secondary when the primary is connected to a 3300-volt supply.

11. The number of turns in the primary and secondary windings of a transformer are 990 and 60 respectively. If the primary is connected to a 6600-volt supply, (*a*) calculate the secondary P.D. at no-load; (*b*) estimate the primary current when the secondary current is 100 amperes.

12. A 100-kVA. transformer, suitable for converting from a primary voltage of 3800 to a secondary voltage of 230, was tested on the low-voltage side when the high-voltage winding was (*a*) open-circuited, (*b*) short-circuited. The results were:

	Volts	Amperes	Watts
(*a*)	230	42	615
(*b*)	7	435	1120

Calculate the input and efficiency of this transformer when the primary voltage is 3800, and the secondary load takes 300 amperes at 0·8 power factor.

The power factor is low in both tests; explain the reason for the low value in the short-circuit test. [J.S.A., 1948.]

CHAPTER XIV

Principles of Electrostatics

1. Importance of Electrostatics

The earliest experimental researches, since they dealt with the effects of electricity at rest (see Chapter II), were made in the realm of electrostatics. But, with the discovery of the principle of electromagnetic induction and the consequent development of the electric generator and industrial applications of the effects of an electric current, the study of current electricity and electromagnetism became of prime importance to the electrical engineer, and that of electrostatics, as being of less practical use, tended to be neglected.

During recent years, however, the continual increase in the magnitude of the voltages employed, not only for engineering purposes in long-distance transmission lines, but also in the laboratory in connection with the study of atomic physics, and the development of radio-telegraphy and telephony, have produced many problems, particularly in the design of insulators and cable insulation, the calculation of capacitance, &c., which demand a considerable knowledge of the principles of electrostatics.

Several elementary experiments in electrostatics have been described briefly in Chapter II, and are summarized in § 3 below, but since further information can be obtained from them by means of the *electroscope*, this instrument will first be described.

2. The Electroscope

An electroscope is a simple piece of apparatus by which small electric charges can be detected or compared. In its simplest form, it consists of a metal rod from the lower end of which two strips or leaves of thin metal foil hang parallel with each other,* as shown in fig. 1. The rod passes through an insulating stopper of paraffin-wax or sulphur into a glass jar which protects the leaves from injury or draughts; to the upper end of the rod is attached a metal disc.

The leaves, if electrically charged, repel each other and diverge; in this way small charges may be detected and compared roughly by

* One leaf is often replaced by a fixed plate; the divergence is then restricted to the single remaining leaf and is easier to observe and measure.

the amount of divergence. Further, if the electroscope carries a charge of which the sign is known, the sign of an unknown charge can be determined from the behaviour of the leaves when it is brought near.

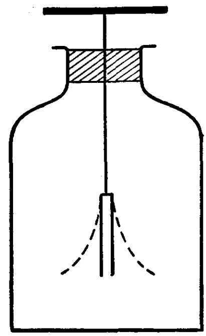


Fig. 1.—Simple electroscope

In many cases the charge would be sufficient to damage the leaves, if the body were connected directly or even brought too near the electroscope; to prevent this a small metal disc mounted on an insulating handle and known as a *proof plane* is used to convey to the electroscope a sample of the charge, obtained by touching the body. If, when the body, or a proof plane which has been in contact with it, is brought near to the disc of the charged electroscope, the divergence of the leaves *increases*, the charge on the body is of the *same* kind as that on the electroscope; but if the divergence *decreases*, then either the body is *uncharged* or the charge is of *opposite* kind (§ 5, p. 350). As in the case of magnetism, repulsion is the only definite evidence of electrification. The reason for this will be evident in § 4, p. 349.

3. Summary of Chapter II

(1) When certain pairs of substances such as ebonite and flannel, or glass and silk, are rubbed together, they are found to exert forces on each other and upon other bodies; for example, when brought near an uncharged electroscope, the leaves are caused to diverge. Such effects are attributed to the fact that they have become electrified or electrically charged.

(2) Bodies which are *similarly* charged, e.g. the leaves of an electroscope, *repel* one another; but since there are also cases in which attraction takes place, it is assumed that there are two kinds of electrical charge and that *attraction* takes place between bodies which are *dissimilarly* or *oppositely* charged.

(3) When two substances are rubbed together, *both* become electrically charged, and the two charges can be shown by means of an electroscope to be *opposite* in kind but *equal* in magnitude. If both are placed, before separation, inside a metal jar standing on the disc of an uncharged electroscope, no effect can be observed; but if either is removed from the jar, the leaves diverge to an equal extent. From this it is assumed that an uncharged body contains equal quantities of both kinds of electricity, and that electrification, by friction or other means, is due to a disturbance of this equality by a partial separation of the two kinds.

(4) The kind of electricity which appears on glass when rubbed with silk is called *positive* electricity; and that which appears on

ebonite when rubbed with flannel is called *negative* electricity. These names, when given, were allocated quite arbitrarily, and served merely to indicate the *oppositeness* of their properties.

In accordance with (3), a negative charge appears on the silk, equal to the positive charge on the glass; and a positive charge on the flannel, equal to the negative charge on the ebonite.

(5) Later investigations have shown that electric charge is subatomic in structure. The particle of negative electricity is called the *electron*; while that of positive electricity, of equal charge but much greater mass, is called the *proton*. Owing to the relative mobility of the electron, the process of electrification consists in the transfer of *negative* electricity or electrons; a negative charge on a body is produced by a surplus, and a positive charge by a deficiency, of electrons.

(6) The atoms of which all matter is composed are assumed to be built up of a central nucleus consisting of protons, and uncharged particles called neutrons, around which rotate in one or more orbits an equal number of electrons; the different properties of the elements are attributed to differences in the number and arrangement of the protons and electrons in the atom.

(7) In the atoms of certain materials, some of the outer electrons are easily detached, so that a solid consisting of such atoms contains a large number of *free* or *conduction electrons*; the directed motion of these electrons constitutes an electric current. Such materials are called *conductors*.

In other materials, electrons can be detached from their atoms only with difficulty, and there are few if any free electrons; these materials are called *insulators*.

(8) Electrification by friction consists in the transfer of electrons, torn from some of the surface atoms of the one body, which is left positively charged, to the other body, which becomes negatively charged.

All substances can be electrified by friction, but in the case of conductors held in the hand, electrons immediately pass through the body either from or to the earth in order to remedy the surplus or deficiency which exists in the conductor; so that no sign of electrification can be detected. When the conductor is supported on an insulating stand, the charge produced spreads all over the conductor through the motion of the free electrons; the charge on an insulator, on the other hand, remains in the neighbourhood of the spot where it is produced.

4. Electrification by Induction

An uncharged cylindrical conductor is supported on an insulating stand, and a positively charged rod is brought up near to but not actually touching one end of the cylinder (fig. 2*a*). By touching each end of the cylinder in turn with a proof plane which is then brought near to a charged electroscope, it will be found that the end B *nearest* the charged rod is *negatively* charged while the remote end A is positively charged; but if the charged rod is removed, no part of the cylinder shows any sign of electrification.

An uncharged conductor, when brought near a charged body, undergoes electrification by induction, a charge of opposite sign appearing on the nearer portion and of the same sign on the farther portion. These induced charges disappear when the inducing charge is removed.

The explanation of these observations is that the positively charged rod exerts attractive forces on the electrons in the conductor and the free electrons move towards it, so that the end B possesses an excess of electrons and is therefore negatively charged, while the end A is left with a deficiency and thus exhibits a positive charge. When the rod is removed, the disturbing force disappears and the electrons return to their original distribution.

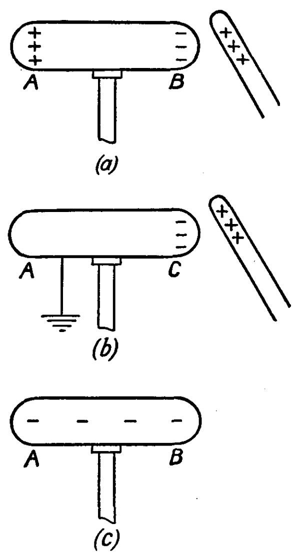


Fig. 2.—Electrification by induction

The attractive force between the positively charged rod and the negative charge at the near end of the cylinder is much greater than the repulsive force between the rod and the positive charge at the far end; hence, if either rod or cylinder is free to move, attraction takes place. The attraction of uncharged bodies referred to in § 3 (1) is now explained; the previously uncharged body, when brought near the charged body, becomes charged by induction, and attraction then takes place as the result of the force between the inducing charge and the nearer induced charge of opposite sign. (Compare with magnetization by induction, § 10, p. 44.)

If the cylinder is of insulating material, there are no free electrons, but the charge on the rod causes a *displacement* of the electrons in the atom which produces the effect of a negative charge at B and a positive charge at A.

5. Production of Permanent Charge by Induction

If while the charged rod is in position the conducting cylinder is momentarily connected to earth by touching it with the finger (fig. 2*b*), electrons pass from the earth to supply the deficiency at A. The attractive force between the rod and the surplus electrons at B prevents the latter from moving, i.e. the negative charge is *bound*; but if the rod is now removed (fig. 2*c*), these electrons redistribute themselves over the cylinder, which is left with a surplus of electrons and therefore negatively charged.

It is thus possible by induction to give a conductor a permanent charge of *opposite* sign to that of the inducing charge.

If the conducting cylinder is replaced by two insulated metal spheres in contact, both charges may be isolated. When the charged rod is brought near, charges are induced as shown in fig. 3, electrons passing from A to B across the small area of contact. If the spheres are separated before the rod is removed, A is left with a positive and B with a negative charge.

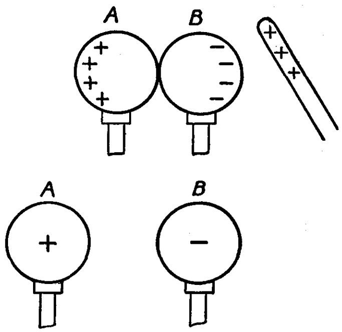


Fig. 3.—Electrification by induction

This process may be repeated indefinitely without recharging the rod; the energy represented by the charges is obtained by a conversion of the work done in separating the spheres against the attractive force produced by the unlike charges. Electrostatic machines of the induction type, such as the Wimshurst machine, are pieces of apparatus in which this process is repeated very rapidly.

The process of charging an electroscope by induction and of using it to test the nature of a charge can now be explained (fig. 4). If a positively charged body is brought near the disc of an electroscope, a negative charge is induced in the disc and a positive charge appears on the leaves which diverge (*a*). On touching the disc with the finger the positive charge is released and the leaves collapse (*b*); but when the inducing charge is removed, the leaves again diverge as the negative charge on the disc distributes itself and leaves the whole electroscope negatively charged (*c*).

When a negatively charged body is now brought near (without touching), an induced negative charge is added to the already negatively charged leaves and the divergence *increases*. If, on the other hand, the body has a positive charge, then the positive charge induced on the leaves reduces the effective value of the negative charge and the divergence *decreases*. If the positively charged body is brought too near, the induced positive charge may entirely neutralize the negative charge on the leaves and produce a positive charge. Under these conditions the leaves first collapse and then diverge again; hence the charged body or proof plane should be brought near *slowly* and the *first* movement of the leaves noted.

If the body is uncharged, a positive charge is induced on the nearer side of it by the negative charge on the electroscope, and the proximity of this charge decreases the divergence of the leaves. Hence, as stated in § 2, p. 347, an *increase* in divergence is the only definite indication that a body is charged.

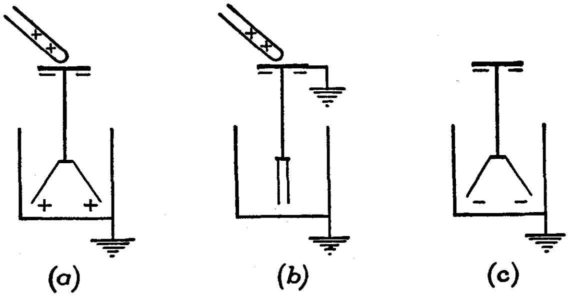


Fig. 4.—Charging of electroscope by induction

6. In a Hollow Charged Conductor the Charge resides wholly on the Surface

This fact can be demonstrated very simply by experiment, and it also follows as a consequence of Coulomb's law (see next section).

A hollow metal sphere with a hole in it, sufficiently large to admit a proof plane, is mounted on an insulating stand. If the sphere is charged, a proof plane touched on the *external* surface, when brought near to an uncharged electroscope, indicates a state of charge; but no such indication is given when the proof plane is passed through the hole so as to touch the *internal* surface.

Another experiment due to Cavendish is shown in fig. 5. *A* is a charged metal ball suspended by a silk fibre; *B* and *C* are two uncharged metal hemispheres each with an insulating handle. The hemispheres are brought together so as to close completely round the ball, which is then allowed to touch them at some point on the interior surface. When the hemispheres are removed, they are found to possess charges of the same kind as that originally possessed by the ball; while the ball itself is *completely discharged*. This shows that immediately contact was made, by which the enclosing sphere and the ball became virtually one conductor, the charge passed *completely* to the external surface.*

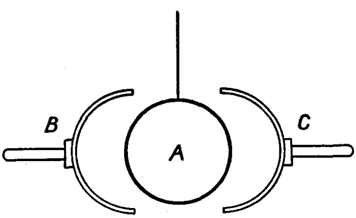


Fig. 5.—Cavendish's experiment

* A charge may of course be *induced* on the inner surface of a *hollow* conductor by means of a charged body suspended inside it without touching it.

Faraday demonstrated this fact by taking his most delicate instruments inside a large cube of sheet metal on an insulating stand; however strongly the cube was electrified, no effect whatever was observable inside.

Delicate electrostatic instruments are often shielded from the effects of external electric fields by enclosing them in a sheet-metal or wire casing; and in some cases readings are taken by an operator also inside the cage.

7. Distribution of the Charge on a Conductor—Surface Density (σ)

Although the charge on a conductor resides wholly on the surface, in most cases the distribution is not uniform; the *surface density*, i.e. the quantity per unit area, is not constant over the whole surface. The distribution is affected by the presence of neighbouring charges, but even supposing that the conductor in question is far removed from all other bodies, only in the case of a sphere is the distribution uniform.

In bodies of other shapes, the surface density *increases* as the radius of curvature *decreases* and is therefore greatest at all edges, corners and points, as indicated in fig. 6. The distribution at various points can be compared roughly by touching the conductor at these points with a proof plane, and observing the relative divergence of the leaves of an electroscope when the charge is conveyed to it directly.

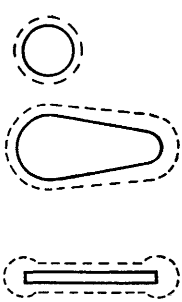


Fig. 6.—Influence of shape of conductor on distribution of charge.

At a *point* the concentration may become so great that the air in the neighbourhood becomes *ionized*,* and a stream of electrified air particles is repelled from it; such a point acts therefore as a leak and is usually to be avoided. On the other hand, a lightning conductor *should* terminate in a sharp point so that the charge induced on it by a neighbouring electrified cloud may leak away; the cloud is thus gradually and quietly discharged so that a disruptive discharge or lightning stroke may be prevented. The action of a lightning conductor is therefore preventive rather than protective; there is no method by which a building may be adequately protected from the effects of a direct lightning stroke.

* In § 3, p. 119, ions were defined as molecules, or groups of atoms, which, having temporarily gained or lost one or more electrons, are electrically charged. The rays from radioactive substances have the power of producing ionization, so that there are always small numbers of ions present in the air. Such ions in the neighbourhood of a charged conductor move under the electric forces experienced, and if these forces are large, the velocity of the ions may reach a value so high that an electron is ejected from any neutral molecule with which they may collide. Under these conditions, the number of ions rapidly increases, and the air in the neighbourhood becomes ionized.

8. Coulomb's Law—Unit Charge

The quantitive relation connecting the force between two small charged bodies, the amount of charge, and their distance apart, and the effect of the medium between them, was investigated originally by Coulomb with the aid of the torsion balance.

As a result of his experiments, Coulomb showed that:

The force between two charges is proportional to the product of the charges, inversely proportional to the square of distance between them, and inversely proportional to a property of the medium separating them, known as the permittivity (ϵ),

$$F \propto \frac{q_1 \cdot q_2}{\epsilon \, . \, d^2},$$

and from this relationship the electrostatic C.G.S. unit of quantity was defined. It is an extremely small unit equal only to $\frac{1}{3 \times 10^{10}}$ of the electromagnetic unit of quantity. In the M.K.S. system there is, of course, only one unit of quantity (or charge), namely the *coulomb*.

One coulomb $= 3 \times 10^9$ electrostatic C.G.S. units.

9. The Electric Field—Electric Flux

Any portion of space in which electric forces exist is called an *electric field*; in practice the term is applied to the comparatively restricted region in the neighbourhood of a charged body, in which these forces can be detected. The field is bounded by charged surfaces, the charges being such that on the boundary surfaces taken as a whole there are always equal amounts of the two kinds of electricity. It may have definite boundaries, as in the case of a charged body suspended in the middle of a room in which an equal and opposite charge is induced on the floor, walls and ceiling, or it may extend indefinitely, as when the charged body is imagined suspended in space and remote from all other bodies.

The medium in which the field exists, which must of necessity be an insulator, is called the *dielectric*.

Since the charges themselves are confined to the surface of the conductor, it is convenient to attribute the electric forces which are set up in the intervening dielectric to the establishment of an *electric* flux (Ψ), the distribution of which is visualized in terms of lines of electric flux, as that of a magnetic field is visualized in terms of lines of magnetic flux.

Since the direction of the force experienced by one charged body, when brought into the electric field due to another charged body, depends upon the *relative signs* of the two charges, some convention is necessary before the term "direction of field" has any precise meaning.

The direction of the electric field at any point is assumed to be that of the force acting on a positive charge placed at that point.

Since a positive charge is repelled by another positive charge and attracted by a negative charge, the direction of the lines of electric flux is always from a positive charge to a negative charge;* and every line of flux which issues from a positive charge terminates on a negative charge.

10. Examples of Electric Field Distribution

There are many similarities between the distribution of the electric field surrounding an electric charge and that of the magnetic field surrounding a magnet; and lines of electric flux are assumed to possess properties similar to those assigned to lines of magnetic flux. But there are also fundamental differences between them; it is possible, for instance, to isolate an electric charge of either kind, whereas every magnet has two poles of opposite kind. Again, lines of magnetic flux always form closed loops, while, since there is no electric field inside a charged conductor, lines of electric force *terminate* on the surface of the charged body.

The experimental mapping out of electric fields is not so simple as is that of magnetic fields, and it is often carried out indirectly, by means of an electrical analogue, in an *electrolytic tank*. This is a shallow tank containing a weak electrolyte—often tap water—in which are placed sections of the boundary surfaces of the charged conductors, cut from sheet metal.

If a P.D. is maintained between them, the distribution of the current-flow lines is similar to that of the lines of electric flux. By means of a suitable detector, a number of equipotential lines (§ 13) are plotted, which enables the electric flux lines to be drawn.

The electrolyte tank is also used in a similar manner for the plotting of magnetic fields.

If a small positively charged sphere is isolated from all other bodies, e.g. suspended in the centre of a large room, negative charges are induced on the walls, floor and ceiling of the room, the sum of which is equal to the positive charge. This can be represented as in fig. 7, in which lines of flux issue uniformly† in all directions from the sphere and terminate on negative charges induced on the walls.

The field due to two small equal and oppositely charged spheres is shown in fig. 8*a*. If the spheres are far from all other bodies, all

* It is often more convenient to consider the supposed motions of a *positive* charge notwithstanding the fact that it is probable that motion of electricity consists almost entirely of the movement of negative electricity (electrons).

† If the room is very large compared with the sphere.

the lines of flux issuing from the positive sphere terminate on the negative sphere, but the distribution is not uniform, the greater portion issuing and terminating on those portions of the sphere which face each other. This is due to the attractive force between the two charges which causes them to move as far as possible towards each other, so that the surface density of charge is greatest on the nearer portion of the spheres.

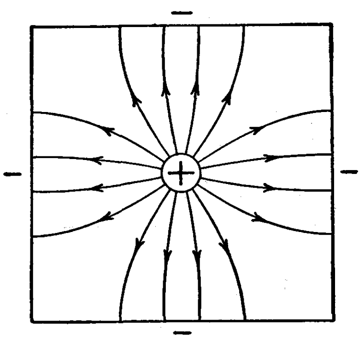


Fig. 7.—Electric field due to isolated charged sphere

The attractive force known to exist between opposite charges and the characteristic curvature of the lines, are explained by endowing the lines with properties similar to those possessed by lines of *magnetic* flux (§ 15, p. 47), i.e. by supposing that there exists a ten-

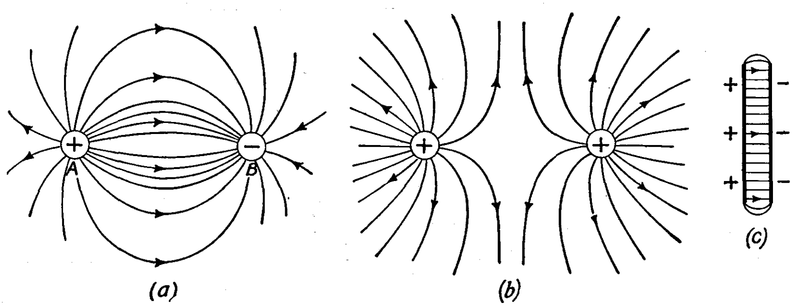


Fig. 8.—Examples of electric fields

sion along their length and a lateral repulsive force between them. A necessary consequence of the former property is that each line must meet the surface of the conductor *perpendicularly*; otherwise the tangential component of the tensile force would cause the charge to move until the component disappeared, i.e. until its direction at the surface is perpendicular to the surface. The tendency of the lines of flux to shorten produces the attractive force between the charges; but their ends cannot leave the surface of the conductor. If, however, the two oppositely charged conductors in fig. 8*a* are connected by another uncharged conductor such as a wire, a path is offered along which the ends of the lines can approach each other. Each line is thus able to shorten and disappear as the charges associated with its ends neutralize each other; and the motion of the electrons along the wire constitutes a momentary current.

When the two charges are equal and of the *same* kind, the field distribution is as in fig. 8*b*. No lines can pass between the bodies, and all terminate on negative charges induced on the nearest uncharged objects. The repulsive force between the charges causes the density of the charge and of the lines of flux to be greater on the portions of the spheres facing away from each other; and this force can be ex-

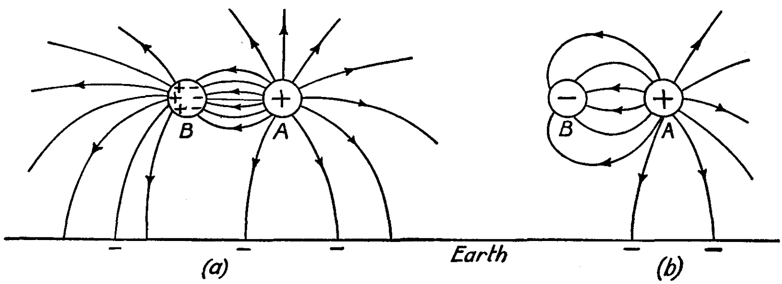


Fig. 9.—Electric fields surrounding induced charges

plained as due partly to the lateral repulsion between the two sets of lines and partly to the attractive force produced by the tension along the lines, between each charge and its induced negative charge.

When the charged conductors are in the form of parallel plates separated by a distance which is small compared with their dimensions, the field distribution is as shown in fig. 8*c*. Except near the edges of the plates, where the density is increased, the distribution of charge is uniform and the lines of flux are parallel.

The case of induced charges is shown in fig. 9*a*. If an uncharged body *B* is brought into the electric field due to a positive charge *A*, it will intercept some of the lines of force. But since *B* has no inherent charge, the total electric flux associated with it must be zero, i.e. the number of lines leaving it must be the same as the number which terminate on it. Hence a number of lines enter it on the side nearer *A* where a negative charge is induced, and an equal number of lines leave it on the farther side where there is an equal induced positive charge.

If B is connected to earth by a conductor, the lines issuing from B and terminating on surrounding objects are able to shorten indefinitely and the positive charge disappears, but the lines entering B cannot do so since their farther ends are attached to A, which is separated from B by a dielectric. The negative charge on B is thus *bound*, and the field distribution becomes as shown in fig. 9*b*.

11. Electric Force or Intensity (E)

The electric force or intensity at any point in the electric field is measured in a similar way to that of the magnetic field by the force experienced by a unit charge when placed at that particular point, assuming that the original field distribution is unaffected by the introduction of the charge. It should be noticed that this is entirely in the nature of a concept since the introduction of so large a charge would, in practice, completely upset the original field distribution.

The electric force or intensity at any point in the electric field is measured in magnitude and direction by the force (in newtons) acting on a unit positive charge placed at that point.

Hence electric force or intensity is measured in newtons per coulomb (= joules per metre/coulomb = volts/metre).

12. Difference of Potential (V)

If a positive charge is moved in an electric field in a direction opposite to that of the electric force, from a point A to a point B, work is done, which is stored as potential energy. The energy associated with the charge is therefore greater when it reaches B than it was at A, so that the point B is said to be at a higher potential than the point A.

The difference in potential between two points is measured in terms of the work done in joules in moving a unit positive charge from the point of lower potential to the point of higher potential.

If the work done is W joules, the P.D. between A and B is W joules per coulomb (or volts), i.e. the potential of B is W volts higher than that of A.

If the field between the two points is uniform so that the electric force has a constant value E newtons per coulomb, and their distance apart is d metres, the P.D. is

$$V = W = E.d \text{ joules per coulomb or volts.}$$

Usually, however, E varies from point to point so that the total work done must be found by integration (fig. 10):

$$V = W = \int_0^d E\,dx \text{ volts.} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It follows that the work done in moving a charge Q coulombs from A to B when the difference of potential is V volts is

$$W = QV \text{ joules.}$$

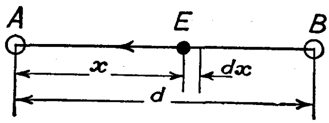


Fig. 10.—To illustrate difference of potential

Conversely, if the charge moves under the influence of the electric force from B to A, an equal amount of work is done *by* the charge.

The *absolute potential* of a point is measured by the work done in moving a unit positive charge from a point of *zero potential* to the point in question. An uncharged body at an infinite distance from all charged bodies is assumed to be at zero potential, so that

The absolute potential of a point is measured by the work done in moving a unit positive charge from an infinite distance up to the point in question.

13. Equipotential Surfaces

In every electric field a number of points can be found which are at the same potential. A surface drawn so as to include only such points is known as an *equipotential surface*, and is represented in any section of the electric field by an *equipotential line*. It follows that no work is done in moving a charge between any two points on such a surface; hence *the surface must be perpendicular to every line of electric force which passes through it*; for otherwise there would be a tangential component of the electric force, and a charge could not be moved without work being done.

The surface of a charged conductor must necessarily be an equipotential surface, since any difference of potential between points would obviously cause a movement of the charge, which would equalize the potential; therefore, as stated in § 10, p. 355, the lines of flux always meet the surface of the conductor perpendicularly.

A general consideration of the equipotential surfaces also throws light on the variation in surface density which exists on any isolated charged conductor which is not a sphere. At distances from it which are very large compared with its size, the effect of a charged conductor of any shape is the same as if the charge were concentrated at a point inside the conductor. The equipotential surface at this distance is therefore a sphere and the distribution of the flux passing through it is uniform; while at the conductor the equipotential surface is the external surface of the conductor. Hence, in approaching the conductor, the shape of the equipotential surface gradually changes from the spherical to that of the conductor. In fig. 11 is shown the section of an isolated charged rod and of several equipotential surfaces in which the rate of change of shape has been exaggerated. The outer surface is almost spherical and the distribution of the lines of flux is uniform, but as the shape approaches more closely that of the conductor, the density of the lines, which must cross the surfaces perpendicularly, increases in the neighbourhood of the ends, i.e. the surface density is greatest where the radius of curvature of the surface is least (§ 7, p. 352).

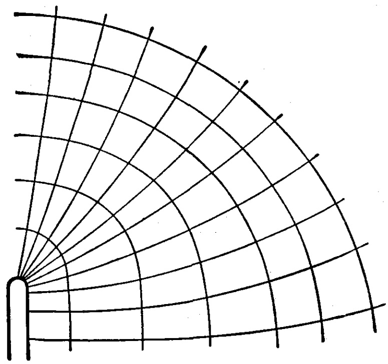


Fig. 11.—Equipotential surfaces

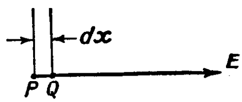


Fig. 12.—Concept of potential gradient

14. Electric Force and Potential Gradient

Consider a point P in the electric field at which the electric force E is in the direction shown (fig. 12). If Q is a second point separated from P by a very small distance dx, on the line of action of the force at P, the work done by a unit positive charge in moving under the force from P to Q is $E\,dx$; and, by definition, this is the difference of potential between the two points P and Q. Hence

$$E\,dx = -dV$$

or

$$E = -\frac{dV}{dx}, \qquad (2)$$

the minus sign being used because the charge has moved *in the direction of* the electric force, so that the potential of Q is lower than that of P, i.e. V decreases as x increases.

The quantity dV/dx, which is the rate of change of potential with regard to distance, is called the *potential gradient.* Hence:

The electric force at any point is equal to the potential gradient at that point, measured in the direction of the force.

15. Electric Stress—Dielectric Strength

When an electric field is set up in a dielectric, there is no movement of free electrons as in a conductor, but a relative displacement of the electrons in the molecules takes places (§ 4, p. 349 and § 7, p. 22). In other words, the dielectric is subjected to an *electric stress* by which a state of strain is produced. If the stress exceeds a certain value, which differs in different materials, mechanical rupture occurs at some point in the dielectric, which is said to have " broken down "; and a spark passes between the two charges to which the field is due. When the insulator is a gas or a liquid, the insulating properties are restored as soon as the discharge ceases, but in the case of a solid, the material is actually punctured and permanently damaged.

The stress produced at any point in the dielectric depends upon the electric force and therefore upon the *potential gradient* at that point, and this is the term in more frequent use in connection with the properties of insulating materials and the design of insulators.

The value of the potential gradient at which an insulating material breaks down is called the *dielectric strength* of the material, and is usually expressed in volts or kilovolts per cm. Figures for some common materials are given below, but these are approximate only, as values obtained by experiment depend upon many factors, such as the shape of the electrodes and the thickness of the sample.

Air	30,000	volts per cm.
Mica	600,000	,, ,,
Manila paper	100,000	,, ,,
Empire cloth	125,000	,, ,,
Oil	50,000	,, ,,

16. Capacitance

The transfer of a charge to an uncharged body raises or lowers its potential according as the charge is positive or negative.

Since electrification consists of a separation of charges, the isolation of a positive charge $+q$ involves the creation of a negative charge $-q$ on the body from which the positive charge was taken. Hence in transferring a positive charge from body A to body B work is done in overcoming (*a*) the attractive force exerted by the negative charge on A, and (*b*) the repulsive force exerted by the positive charge on B. This work is stored as potential energy in virtue of the difference in potential between A and B created by the raising of the potential of B and the lowering of that of A.

As the transfer proceeds the work done in transferring each element of charge increases, e.g. when the charge on each body is q the force acting on each element of charge at any point during its transfer is twice as great as when the charge on each is $\frac{1}{2}q$. Hence the work done is twice as great, and therefore, by definition, the P.D. is twice as great, so that the P.D. at any instant is proportional to the charge, i.e.

$$v \propto q.$$

The amount of charge required to produce a given P.D. depends on the shape and dimensions of the bodies, their relative positions, and the nature of the medium between them. The ratio of the charge to the P.D. is called the *capacitance* (C) of the system.

$$C = \frac{Q}{V}. \quad \ldots \ldots \ldots \quad (3)$$

The unit of capacitance is that of a system in which unit charge (1 coulomb) produces unit difference of potential (1 volt), and is called the *farad.*

This unit is inconveniently large and capacitances are usually measured in microfarads,

$$1 \text{ microfarad } (\mu\text{F.}) = 10^{-6} \text{ farad (F.)}$$

while in the case of very small capacitances a still smaller unit is used.

$$1 \text{ micro-microfarad } (\mu\mu\text{F.}) = 10^{-6} \text{ microfarad} = 10^{-12} \text{ farad.}$$

The micro-microfarad is sometimes called the *picofarad* (pF.).

Two conducting surfaces separated by a dielectric arranged so that the capacitance between them is large, form what is known as a *capacitor*. (The older term *condenser* is still widely used.)

17. Charge and Discharge of a Capacitor

The charging process described above is largely in the nature of a concept.

In considering the actual charge and discharge of a capacitor the engineering student may find it of assistance to make use of the mechanical analogy offered by a spring. An open spring, neither expanded nor compressed, with its ends fixed at A and B is shown in fig. 13a. If a horizontal force is applied at P, a displacement takes place in the direction of the force, and immediately a restoring force proportional to the displacement is set up, due to the extension of the portion AP and the compression of the portion PB. The movement ceases when the restoring force becomes equal to the applied force; and the work done in overcoming the restoring force is stored up as potential strain energy in the spring. It remains so stored until the strain is relieved by allowing the point P to return to its initial position. (Incidentally, if P is suddenly released, the spring, owing to its inertia, will execute a series of oscillations before coming to rest, and this is the mechanical counterpart of the oscillatory circuit on p. 379.)

Now consider the arrangement shown in fig. 13*b*, which corresponds in form most closely to the spring of fig. 13*a*, the conductors *AP* and *PB* being in the form of metal rods.

The E.M.F. of the cell produces a general displacement of electrons from left to right, with the result that the rod *AP* exhibits a +ve charge due to a deficiency, and the rod *PB* exhibits a −ve charge due to a surplus, of electrons. But this displacement is immediately opposed by the attraction exerted by the +ve charge on *AP*, on the electrons leaving it, and the repulsion exerted by the −ve charge on *PB*, on the electrons approaching it; and this joint action has the effect of a

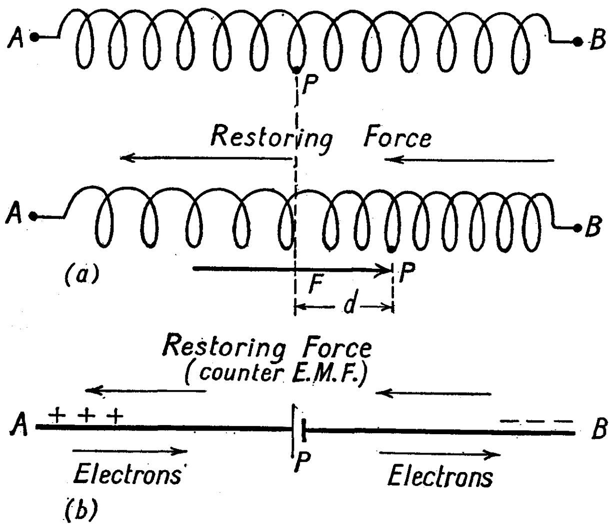


Fig. 13.—To illustrate charge and discharge of a capacitor

counter-E.M.F. (sometimes called the dielectric strain E.M.F.). The movement of electrons constitutes a *charging current* (i) which continually decreases as the restoring force (or counter-E.M.F.) increases, and finally becomes zero when the counter-E.M.F. becomes equal to the E.M.F. of the cell. The arrangement therefore acts as a capacitor having a charge $Q = \int i\,dt$, although the capacitance is extremely small.

If the cell is removed, the rods, if supported on insulators, retain their charges and the energy remains stored in the electric field set up between them, until they are connected by a conductor. This is equivalent to releasing the spring. The restoring force or counter-E.M.F. then causes a movement of the electrons through the conductor in the reverse direction, which constitutes a discharge current, the stored energy being released and converted into heat.

(If the connecting conductor is of low resistance, the discharge, because of the inductance of the rods, is actually an oscillatory one (see p. 379).)

If, however, the same two rods are placed as shown in fig. 14, parallel and close to each other, a greater displacement of electrons will take place before the counter-E.M.F. becomes equal to that of the cell. This is because the attraction exerted by the +ve rod A on electrons leaving it is partially neutralized by the repulsion exerted by the neighbouring −ve charge on B; and similarly the repulsion exerted by B on electrons approaching it is partially neutralized by the attraction of the neighbouring +ve charge on A. The charge has therefore

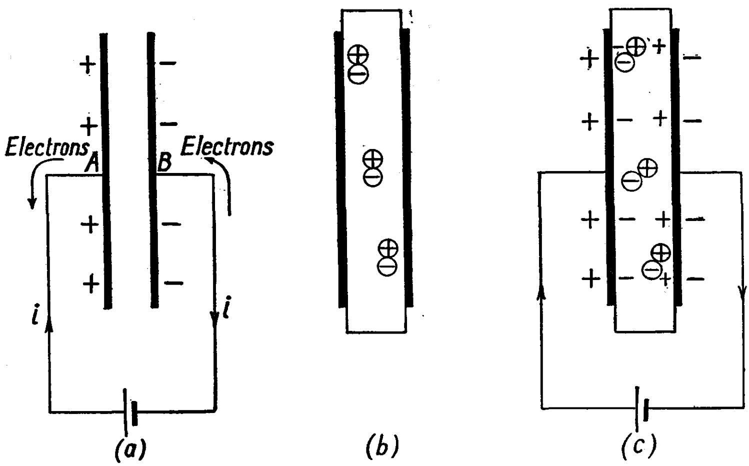


Fig. 14.—To illustrate the charging of a capacitor

been increased by adopting the new arrangement, and as the E.M.F. of the cell has remained the same, the capacitance has been increased. Reducing the distance (d) between the rods will increase the neutralizing effect and hence the capacitance. In fact,

$$\text{capacitance} \propto \frac{1}{\text{distance apart}}, \quad C \propto \frac{1}{d}.$$

The capacitance will clearly be increased still further by replacing the rods by metal plates: in fact,

$$\text{capacitance} \propto \text{surface area of plates}, \quad C \propto a.$$

All the above holds true when the plates are separated by air or even in a vacuum. Suppose, however, that a solid dielectric is inserted. Since there are no free electrons in such a material, there can be no continuous movement of electrons: nevertheless some relative displacement takes place in each atom between the orbital electrons and the protons in the nucleus, as represented diagrammatically in fig. 14, *b* and *c*. Consequently, each atom becomes what is known as a dipole (behaving somewhat like the molecular magnets in a magnetic material, postulated in § 9, p. 41), and is oriented by the electric field so that the dielectric is put in a state of electric strain (polarized), and charges are developed on the surfaces of the dielectric in contact with the capacitor plates (fig. 14*c*). These charges, being of opposite sign to those on the plates with which they are in contact, still further neutralize the restoring forces opposing the movement of electrons, with the result that a still greater displacement takes place before the back-E.M.F. becomes equal to that of the cell, i.e. the capacitance has been further increased. The extent to which the polarization of the dielectric takes place depends upon the nature of the material. Hence in a parallel-plate capacitor, the capacitance can be expressed in the form

$$C = \varepsilon \frac{a}{d} \text{ farads}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where ε is a constant of the material known as the permittivity.

18. Permittivity—Relative Permittivity

Permittivity is an electric property of a medium which corresponds with the magnetic property of permeability. As has been stated previously, in the C.G.S. system both these properties, in the case of free space, were arbitrarily assigned the value unity; and this gave rise to the two systems of C.G.S. units—the electromagnetic and the electrostatic systems.

From equation (4)

$$\varepsilon = c \frac{d}{a} = \frac{Qd}{Va}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In the M.K.S. system, where Q is measured in coulombs, d in metres, a in square metres, and V in volts, the value of ε for free space, usually denoted by ε_0 (and sometimes called the *electric space constant*) is

$$\varepsilon_0 = 8{\cdot}85 \times 10^{-12} \text{ farads per metre.}$$

This can be obtained experimentally by careful measurement of the capacitance of a capacitor of known dimensions *in vacuo*; but it is also known, from other considerations, that the permeability and the permittivity of free space are related by an expression of the form,

$$\frac{1}{\sqrt{(\mu_0 \varepsilon_0)}} = c, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where c is the velocity of light (300×10^6 m. per sec.). Since in the rationalized M.K.S. system μ_0 has been assigned the value of $4\pi \times 10^{-7}$, the corresponding value of ε_0 is

$$\varepsilon_0 = \frac{1}{c^2\mu_0} = \frac{1}{(300 \times 10^6)^2 \times 4\pi \times 10^{-7}} \quad . \quad . \quad . \quad (7)$$

$$= 8{\cdot}85 \times 10^{-12} \text{ farads per metre.}$$

This can also be taken as the value for air, unless extreme accuracy is required.

The permittivity of most solid dielectrics is greater than that of free space, although the range of values is much less than in the case of permeability. The *relative permittivity* of a given dielectric can be expressed as the ratio of the capacitance of a capacitor with the given dielectric to the capacitance when this dielectric is replaced by a vacuum (or air).

As in the case of permeability,

Absolute permittivity = relative permittivity × permittivity of free space

$$\varepsilon = \varepsilon_r\varepsilon_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The relative permittivities of some common dielectrics are given below:

Air	1·0006
Transformer oil	2
Waxed paper	2·5–3
Bakelite	4–6
Mica	4–8
Glass	3–12

19. Growth and Decay of Charge in a Capacitor

Consider a capacitor of capacitance C connected in series with a non-inductive resistance R to a source of supply of which the P.D. is V.

During the charging period the charging P.D. can be looked upon as made up of two portions, one equal and opposite to the capacitor E.M.F. (e) due to the charge already received, and the other overcoming the resistance of the circuit and causing the charging current (i) to flow, i.e.

$$V = iR + e. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In other words, the current is due to the excess of the charging P.D. over the capacitor E.M.F. $\left(i = \frac{V - e}{R}\right)$ and therefore continuously decreases as the latter rises.

Now $i = dq/dt$ and $e = q/C$, so that equation (9) becomes

$$V = R\frac{dq}{dt} + \frac{q}{C}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

When the charge is complete, q reaches a constant value Q and $i = dq/dt = 0$, so that $V = Q/C$. The last equation therefore becomes

$$\frac{Q}{C} = R\frac{dq}{dt} + \frac{q}{C}.$$

Multiplying by C,

$$Q = CR\frac{dq}{dt} + q,$$

from which

$$\frac{dq}{Q-q} = \frac{1}{CR}\,dt.$$

Integrating,

$$-\log_e(Q-q) = \frac{t}{CR} + k \text{ (where } k \text{ is a constant).}$$

But when

$$t = 0,\ q = 0. \quad \therefore\ k = -\log_e Q.$$

$$\therefore\ \log_e\frac{Q-q}{Q} = -\frac{t}{CR}.$$

$$\therefore\ \frac{Q-q}{Q} = e^{-\frac{t}{CR}}.$$

$$\therefore\ q = Q(1 - e^{-\frac{t}{CR}}). \qquad \text{(11)}$$

Now the current at any instant is

$$i = \frac{dq}{dt} = -Q\left(-\frac{1}{CR}\cdot e^{-t/CR}\right)$$

or

$$i = \frac{Q}{CR}e^{-t/CR}. \qquad \text{(12)}$$

But

$$\frac{Q}{CR} = \frac{V}{R} = I$$

where I is the initial value of the current, and therefore

$$i = Ie^{-t/CR}. \qquad \text{(13)}$$

Further, from equation (11) since $q = Ce$ and $Q = CE$

$$e = E(1 - e^{-t/CR})$$

and since the P.D. across the capacitor is always equal and opposite to the E.M.F.,

$$v = V(1 - e^{-t/CR}). \qquad \text{(14)}$$

Hence during charge the rise in the P.D. across the capacitor and the decay of the charging current are both represented by exponential curves, as shown in fig. 15. The broken curves show the effect of increasing the resistance.

If the charging P.D. is removed, and the capacitor allowed to discharge through the resistance R, since $V = 0$, equation (10) becomes

$$R\frac{dq}{dt} + \frac{q}{C} = 0. \qquad \text{(15)}$$

$$\therefore\ q = -CR\frac{dq}{dt}$$

or

$$\frac{1}{q}\,dq = -\frac{1}{CR}\,dt.$$

Integrating,

$$\log_e q = -\frac{t}{CR} + k.$$

When

$$t = 0,\ q = Q. \quad \therefore\ k = \log_e Q.$$

$$\therefore\ \log_e q = -\frac{t}{CR} + \log_e Q.$$

$$\therefore\ \frac{q}{Q} = e^{-t/CR},$$

$$q = Qe^{-t/CR}, \quad \ldots\ldots\ldots \quad (16)$$

and the current at any instant during discharge is

$$i = \frac{dq}{dt} = -\frac{Q}{CR}e^{-t/CR}, \quad \ldots\ldots\ldots \quad (17)$$

the negative sign indicating that the current is such as to *decrease* the charge.

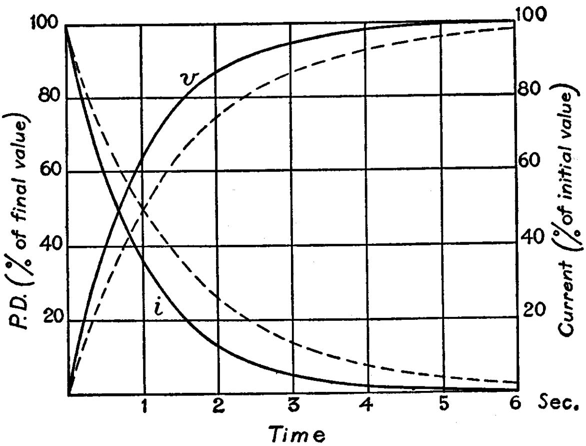


Fig. 15.—Charging current of capacitor

The initial value of the current, obtained by putting $t = 0$ in this equation, is

$$I = \frac{Q}{CR},$$

and therefore

$$i = Ie^{-t/CR}. \quad \ldots\ldots\ldots \quad (18)$$

It will be noticed that this equation is identical with (13) above: hence the discharge-current curve is similar in shape to the charging-current curve shown in fig. 15.

It follows from equation (16) that

$$v = Ve^{-t/CR}. \quad \ldots\ldots\ldots \quad (19)$$

Hence in discharge, both P.D. and current decay exponentially.

The quantity CR is known as the *time constant*, and has a meaning similar to that in § 27, p. 179. It is:

(*a*) The time in which the capacitor would be fully charged (or discharged) if the initial rate of charge (or discharge) were maintained.

(*b*) The time in which the charge actually reaches 63·2 per cent of its final value (or is reduced to 36·8 per cent of its original value); and in which the charging (or discharging) current falls to 36·8 per cent of its initial value.

Example.—A capacitor of 20 microfarads is connected in series with a non-inductive resistance of 1000 ohms to a 500-volt supply. Calculate

(*a*) the initial value of the current;
(*b*) the current and charge 0·02 sec. after connection;
(*c*) the time taken for the charge to reach 99 per cent of its final value.

(*a*) The initial value of the current, since $\frac{q}{C} = 0$, is

$$I = \frac{V}{R} = \frac{500}{1000} = 0{\cdot}5 \text{ ampere.}$$

(*b*) Since the time constant is 0·02 sec., the charge has reached 63·2 per cent of its final value.

$$\begin{aligned} q &= 0{\cdot}632Q = 0{\cdot}632VC = 0{\cdot}632 \times 500 \times 20 \\ &= 6320 \text{ microcoulombs,} \\ i &= 0{\cdot}368I = 0{\cdot}184 \text{ ampere.} \end{aligned}$$

(*c*) $q = 9900$ microcoulombs.

$$\therefore\ 9900 = 10{,}000(1 - e^{-t/{\cdot}02}).$$

$$\therefore\ e^{-50t} = 0{\cdot}01.$$

$$\therefore\ -50t \log_{10} e = \log_{10} 0{\cdot}01,$$

from which

$$t = \frac{2}{21{\cdot}7} = 0{\cdot}092 \text{ sec.}$$

20. Fundamental Relationships

From equation (4)

$$C = \varepsilon \frac{a}{d} \text{ farads,}$$

where ε is a constant the value of which depends upon the dielectric and is called the *permittivity*.

Hence

$$Q = V\varepsilon \frac{a}{d} \text{ coulombs,}$$

and since the electric flux (Ψ) is numerically equal to the quantity (Q),

$$\Psi = V\varepsilon \frac{a}{d}. \qquad \qquad (20)$$

The quantity $\varepsilon(a/d)$ is sometimes called the *permittance*: it is a property which corresponds with the permeance of a magnetic circuit and the conductance of an electric circuit. Hence the relationship between the fundamental quantities in what may be termed the 'dielectric' circuit is of the same form as in the magnetic and electric circuits, i.e.

electric current = electromotive force × conductance,
magnetic flux = magnetomotive force × permeance,
electric flux = potential difference × permittance.

Further, equation (20) can be written,

$$\frac{\Psi}{a} = \varepsilon \frac{V}{d}.$$

Electric flux density (D) = permittivity × potential gradient (electric force),

$$D = \varepsilon E. \qquad (21)$$

(Compare $B = \mu H$, § 2, p. 212.)

21. Calculation of Capacitance

The capacitance of a system of conductors can be calculated only in a few simple cases.

Capacitance of a parallel-plate capacitor

Fig. 16 shows two parallel plates, each of area a square metres, having charges of $+Q$ and $-Q$ coulombs and separated by a dielectric of permittivity ε and thickness d metres. In such an arrangement the electric field between the plates is uniform except at the edges, and if these small effects are neglected, the electric force can be assumed to have a constant value E throughout.

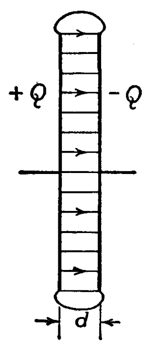


Fig. 16.—Parallel-plate capacitor

Hence, by definition,

$$V = \int E\,dx = E\,d \text{ volts}.$$

Further, the flux density $= D = \frac{\Psi}{a} = \frac{Q}{a}$ coulombs per square metre, and from equation (21)

$$E = \frac{D}{\varepsilon} = \frac{Q}{\varepsilon a}.$$

Therefore, $$V = \frac{Qd}{\varepsilon a} \text{ volts}$$

and $$C = \frac{Q}{V} = \varepsilon \frac{a}{d}$$

$$= \varepsilon_r \varepsilon_0 \frac{a}{d} \text{ farads}. \qquad (22)$$

Strictly, since edge effects are neglected, this expression holds only for infinitely large plates, but it is very nearly true when the dimensions of the plates are large compared with the distance between them.

Example 1.—A parallel-plate capacitor contains a total of 61 sheets of tin-foil, each 30 cm. × 25 cm., separated by paraffin-waxed paper 0·015 cm. thick ($\varepsilon_r = 3$). Neglecting edge effects, calculate the capacitance in microfarads.

One plate consists of 30 sheets in parallel, interleaved between 31 sheets forming the other plate; hence *both* sides of each of the 30 sheets are effective.

$$a = 2 \times 30 \times 30 \times 25 \times 10^{-4} = 45{,}000 \times 10^{-4} = 4{\cdot}5 \text{ sq. m.}$$
$$d = 0{\cdot}015 \times 10^{-2} \text{ m.}$$

Hence, by equation (22),

$$C = 3 \times 8{\cdot}85 \times 10^{-12} \times \frac{4{\cdot}5}{0{\cdot}015 \times 10^{-2}}$$
$$= 0{\cdot}8 \times 10^{-6} \text{ farad}$$
$$= 0{\cdot}8 \text{ microfarad } (\mu\text{F.}).$$

Capacitance between two long concentric cylinders

This case is of importance in connection with single-core cables in which the outer cylinder or sheath is usually earthed. The radii of the inner and outer cylinders are (in metres) r and R respectively, and the charge is q coulombs per metre (fig. 17).

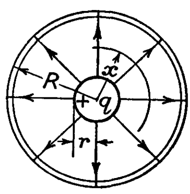


Fig. 17.—Capacitance of single-core cable

Here the electric field between the cylinders is radial so that at a distance x metres from the centre, the flux per metre, ψ, passes through a cylinder x metres in diameter and one metre long. Hence the flux density is

$$D = \frac{\psi}{2\pi x} = \frac{q}{2\pi x} \text{ coulombs per square metre}$$

and from equation (21)

$$E = \frac{D}{\varepsilon} = \frac{q}{2\pi\varepsilon x} \text{ volts per metre.} \qquad (23)$$

Hence $$V = \int E\,dx = \frac{q}{2\pi\varepsilon}\int_r^R \frac{1}{x}\,dx = \frac{q}{2\pi\varepsilon}\log_e\frac{R}{r} \text{ volts,} \qquad (24)$$

so that $$C = \frac{q}{V} = \frac{2\pi\varepsilon}{\log_e(R/r)} = \frac{2\pi\varepsilon_r\varepsilon_0}{\log_e(R/r)} \text{ farads per metre}$$
$$= \frac{5280}{3{\cdot}28} \times 2\pi \times 8{\cdot}85 \times 10^{-12} \cdot \frac{\varepsilon_r}{2{\cdot}3 \times \log_{10} R/r} \times 10^6$$
$$= 0{\cdot}0388 \frac{\varepsilon_r}{\log_{10} R/r} \text{ microfarads per mile.} \qquad (25)$$

Again this is strictly correct for infinitely long cylinders, but is very nearly true in all cases where the length is great compared with the radii.

Example.—Find the capacitance of 5 miles of single-core cable in which the conductor is 1·25 cm. in diameter, the radial thickness of the dielectric 0·6 cm., and the relative permittivity 2·5.

Since only the *ratio* of the radii is involved, the values may remain in cm.

$$r = 0{\cdot}625 \text{ cm.}, \quad R = 1{\cdot}225 \text{ cm.}, \quad \varepsilon_r = 2{\cdot}5.$$

$$C = 5 \times \frac{0{\cdot}0388 \times 2{\cdot}5}{\log_{10} 1{\cdot}225/0{\cdot}625}$$
$$= 1{\cdot}66 \text{ microfarads.}$$

One of the most important quantities in connection with high-voltage cables is the electric stress or potential gradient ($E = -dV/dx$) set up in the dielectric, which, from equation (23), is

$$E = \frac{q}{2\pi\varepsilon x}$$

and from eqn. (24)

$$= \frac{V}{x \,.\, \log_e R/r} \text{ volts per metre.} \qquad . \quad . \quad (26)$$

From this it can be seen that the stress is inversely proportional to the distance from the centre and is therefore a maximum at the conductor surface. It is usually expressed in kilovolts per centimetre, and in modern high-voltage cables may have a maximum value as high as 90–100 kV. per cm.

Example.—If, in the previous example, the r.m.s. value of the P.D. between core and sheath is 19,000 volts, find the maximum value of the stress in the dielectric.

$$V_{max} = 19{,}000 \times \sqrt{2} = 26{,}800 \text{ volts}, \quad x = r = 0{\cdot}625 \times 10^{-2} \text{ m.}$$

$$E = \frac{26{,}800}{0{\cdot}625 \times 10^{-2} \times 2{\cdot}3 \log_{10} 1{\cdot}225/0{\cdot}625}$$
$$= 64 \times 10^5 \text{ volts per metre} = 64 \text{ kV. per cm.}$$

Capacitance between two long parallel conductors

This case is of importance in connection with overhead lines.

In such cases the spacing of the conductors is so large compared with their diameter that the charge may be considered uniformly distributed over the surface, as in the previous case.

Hence, the electric force due to each conductor considered separately is radial, and at a distance x from the centre of the conductor has a value (from equation 23)

$$E = \frac{1}{\varepsilon} \cdot \frac{q}{2\pi x}$$

where q is the charge per unit length.

If each conductor has a charge of q coulombs per metre, and considering a length of one metre, then at a point P on the line joining the centres of the conductors, and distant x metres from A (fig. 18)

Electric force due to A is $E_A = \frac{1}{\varepsilon}\frac{q}{2\pi x}$

Electric force due to B is $E_B = \frac{1}{\varepsilon}\frac{q}{2\pi(D-x)}$.

$$E = E_A + E_B = \frac{1}{2\pi} \cdot \frac{q}{\varepsilon}\left(\frac{1}{x} + \frac{1}{D-x}\right) \text{ volts per metre}$$

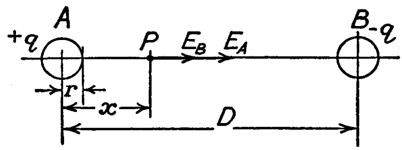


Fig. 18.—Capacitance of single-phase line

and therefore
$$V = \int E\,dx = \frac{1}{2\pi} \cdot \frac{q}{\varepsilon}\int_r^{D-r}\left(\frac{1}{x} + \frac{1}{D-x}\right)dx$$
$$= \frac{1}{2\pi} \cdot \frac{q}{\varepsilon}\Big[\log_e x - \log_e (D-x)\Big]_r^{D-r}$$
$$= \frac{1}{2\pi} \cdot \frac{q}{\varepsilon} \cdot \left[\log_e \frac{x}{D-x}\right]_r^{D-r}$$
$$= \frac{1}{2\pi} \cdot \frac{q}{\varepsilon} \cdot \left[\log_e \frac{D-r}{r} - \log_e \frac{r}{D-r}\right]$$
$$= \frac{1}{\pi} \cdot \frac{q}{\varepsilon} \cdot \log_e \frac{D-r}{r}.$$

Hence
$$C = \frac{q}{V} = \frac{\pi\varepsilon}{\log_e \frac{D-r}{r}}.$$

Since r is very small

$$C = \frac{\pi\varepsilon}{\log_e \frac{D}{r}} \text{ farads per metre.} \quad \ldots\ldots \quad (27)$$

Putting $\varepsilon = \varepsilon_r\varepsilon_0$, and using common logarithms,

$$C = \frac{5280}{3{\cdot}28} \times \frac{\pi \times \varepsilon_r \times 8{\cdot}85 \times 10^{-12}}{2{\cdot}3 \log_{10} \frac{D}{r}} \times 10^6$$

$$= 0{\cdot}0194 \frac{\varepsilon_r}{\log_{10} \frac{D}{r}} \text{ microfarads per mile.} \quad . \quad . \quad (28)$$

Example.—Calculate the capacitance of an overhead line 10 miles long, consisting of two conductors each 2 cm. in diameter spaced 3 m. apart.

Since only the ratio of D and r is involved they may be expressed in either metres or centimetres.

$$C = \frac{10 \times 0{\cdot}0194}{\log_{10} \frac{300}{1}} \qquad [\varepsilon_r = 1]$$

$$= 0{\cdot}0785 \text{ microfarads.}$$

22. Energy Stored in the Electric Field

Considering some instant during this charging period (§ 17) when the charging current is i and the counter-E.M.F. of the capacitor is e,

Power input to capacitor is $p = ei$.
Energy input in a time dt is $p \,.\, dt = ei\, dt$.

$$\text{Since } e = \frac{q}{C} \text{ and } i = \frac{dq}{dt}$$

$$p\, dt = \frac{q}{C} \cdot \frac{dq}{dt}\, dt = \frac{1}{C}\, q\, dq.$$

In establishing a total charge Q,

$$\text{Energy input} = \frac{1}{C}\int_0^Q q\, dq = \frac{1}{C} \cdot \tfrac{1}{2}Q^2$$

$$= \tfrac{1}{2}\frac{C^2E^2}{C} = \tfrac{1}{2}CE^2 \text{ joules}$$

when C is in farads and E in volts. When fully charged from a source of which the P.D. is V, $E = V$, and

$$\text{Stored energy} = \tfrac{1}{2}CV^2 \text{ joules.} \quad . \quad . \quad . \quad (29)$$

(This expression should be compared with that for the energy stored in a magnetic field, $\frac{1}{2}LI^2$ joules.) (§ 29, p. 182.)

This energy remains stored when the capacitor is removed from the supply; but, if the plates are connected by a conductor, discharge takes place, the electric field collapses, and the stored energy is converted into heat by the discharge current.

In many cases, however, the process of discharge is more complicated and is described in § 25, p. 378.

23. Types of Capacitor (Condenser)

The capacitor, in virtue of its property of storing energy, is used in many different ways. All circuits possess a certain amount of capacitance; in some cases it is desirable to reduce this, by suitable arrangement, to a minimum, but in others it is necessary to increase it locally, and this is done by means of capacitors. Capacitors are used extensively in small sizes in telephone and wireless receiving circuits; and in large sizes in wireless transmitters, and for improving the power factor of industrial power circuits.

A capacitor has been defined as consisting of two conductors so arranged that an intense electric field can be produced between them. This requires that the distance between the conductors shall be small, and that the dielectric shall have a high permittivity and also a high dielectric strength, so that a thin layer may be able to withstand a high P.D. without breakdown.

The earliest form of capacitor, known from its place of origin as a Leyden jar, consisted of a glass jar coated with tin-foil on both inner and outer surfaces; the two plates were therefore in the form of concentric cylinders, with glass ($\varepsilon_r = 4$ approx.) as the dielectric between them. In a more modern form of this type, long glass tubes are used, the inner and outer surfaces of which are coated with silver deposited by chemical means.

A more compact and portable type is the flat plate capacitor, in which the plates are sheets of tin-foil separated by a dielectric of paraffin-waxed paper ($\varepsilon_r = 3$ approx.) or mica ($\varepsilon_r = 5$ approx.). In order to obtain large capacitances, each plate consists of a number of sheets connected in parallel and interleaved as shown in fig. 19*a*. The capacitor connected across the contact breaker in the induction coil described in § 25, p. 175, and the small fixed capacitors used in wireless receivers and for general laboratory purposes, are of this type.

Still greater capacitance for a given size can be obtained in the Mansbridge type, in which two long strips of tin-foil or metallized paper separated by strips of paper are rolled up on a mandrel, like ribbon on a reel (fig. 19*b*); the whole roll is then impregnated with paraffin-wax, or, in the case of capacitors for use in industrial power circuits, permanently immersed in oil.

In standard capacitors, the dielectric is usually air, since its permittivity is constant and known with great accuracy. The dielectric strength, however, is small, so the spacing must be wide and the capacitor is large for its capacitance. Such capacitors are usually divided into a number of sections which can be connected in parallel as required.

In variable capacitors, such as those used in tuning radio circuits, both sets of plates are of brass or aluminium sheet; and one set can be rotated about an axis so as to interleave with the other set, the capacitance depending upon the extent of the overlap. The plates may be semicircular (19*c*) in which case $C \propto \theta$ where θ is the angle of rotation, but for tuning purposes it is often an advantage to use a *square-law* capacitor (19*d*) in which the plates are shaped so that $C \propto \theta^2$; then, since the wave-length (λ) to which the circuit is tuned is proportional to $\sqrt{\text{capacitance}}$,

$$\text{Wave-length} \propto \sqrt{C} \propto \theta,$$

so that the dial may be engraved uniformly in wave-lengths.

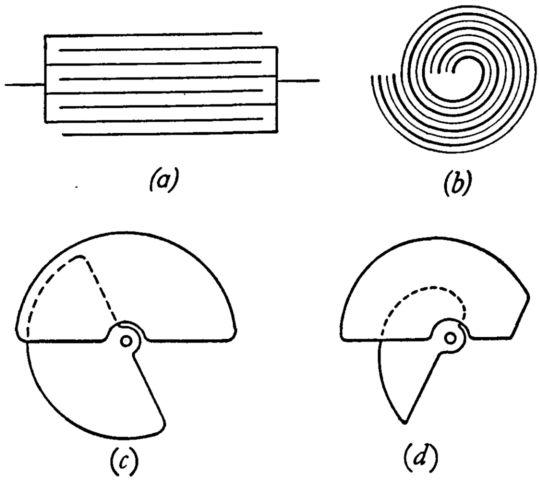


Fig 19.—Types of capacitor

Electrolytic capacitors are now extensively used for certain purposes. If an aluminium plate is immersed in a suitable electrolyte such as ammonium borate, and a current passed from the aluminium to the electrolyte, an insulating film of aluminium oxide is formed on the surface of the metal and gradually reduces the current to an extremely small value. Raising the P.D. between plate and electrolyte produces a temporary increase in current which increases the thickness of the oxide film. Hence as long as the polarity of the aluminium electrode remains positive, the arrangement can be used as a capacitor, the plates of which are the electrode and the electrolyte respectively, while the film is the dielectric; and since it is only a few molecules thick (the thickness depending on the value of the " forming voltage "), the capacitance is very large in relation to the area of plate.

In the earlier wet type the electrode, in a form which presented a large surface, was immersed in the electrolyte contained in a metal can through which it was connected with the negative side of the circuit. This has been replaced by the so-called " dry " type in which both plates consist of long strips of aluminium foil, one of which is " formed ", separated by an absorbent paper containing the electrolyte, and rolled up in cylindrical form as in the Mansbridge type (fig. 19*b*).

Such a *polarized* capacitor cannot be used on an a.c. circuit as the film ceases to act as a dielectric when the polarity of the aluminium electrode is reversed: its chief application is as a smoothing capacitor in the output circuit of a rectifier (see p. 387) where the output voltage is d.c. with an a.c. ripple which is considerable, but not of sufficient amplitude to reverse the polarity. For use on a.c. circuits a non-polarized dry type can be employed, in which a film is formed on both strips, or alternatively, two capacitors of the polarized type may be used in series with their +ve electrodes connected together, i.e. back to back.

The losses, however, are considerable, and heating limits the use of capacitors of this type for continuous duty on a.c. circuits.

24. Grouping of Capacitors

Capacitors in Parallel.

It is obvious that in a parallel grouping, as shown in fig. 20*a*, in which one plate of each capacitor is connected to a common point, so that the P.D. across each is the same, the effect is that of one large capacitor having a plate area equal to the sum of the plate areas of the separate capacitors.

If C_1, C_2, C_3, etc., are the capacitances and q_1, q_2, q_3, etc., the charges on each capacitor, then

$$q_1 = C_1V,\ q_2 = C_2V,\ q_3 = C_3V,\ \text{etc.},$$

and if C is the total equivalent capacitance,

$$Q = CV.$$

But

$$Q = q_1 + q_2 + q_3 + \dots .$$

$$\therefore\ CV = C_1V + C_2V + C_3V + \dots .$$

$$\therefore\ C = C_1 + C_2 + C_3 + \dots . \qquad . \quad . \quad . \quad . \quad (30)$$

The total capacitance of a number of capacitors in parallel is the sum of the individual capacitances.

Capacitors in Series.

Consider a number of capacitors of capacitance C_1, C_2, C_3, etc., connected in series as shown in fig. 20*b*, the P.D.s across them being v_1, v_2, v_3, etc., and the total P.D. being V.

If a charge $+Q$ is given to plate a of the first capacitor, a charge of $-Q$ is induced on the opposite plate b, and since b and c are in metallic connection and therefore, virtually, one conductor, a charge

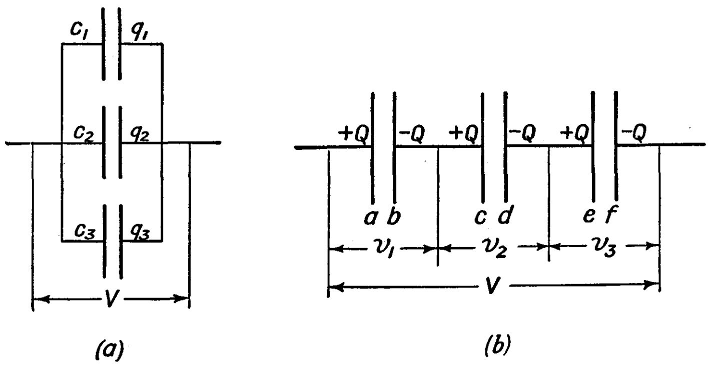


Fig. 20.—Capacitors (*a*) in parallel, and (*b*) in series

of $+Q$ is induced on plate c. This in turn induces a charge of $-Q$ on d, $+Q$ on e, and $-Q$ on f. Hence each capacitor has an equal charge Q.

$$\therefore\ v_1 = \frac{Q}{C_1},\quad v_2 = \frac{Q}{C_2},\quad v_3 = \frac{Q}{C_3},\ \text{etc.,}$$

and if the equivalent capacitance is C,

$$\frac{Q}{C} = V;$$

but

$$V = v_1 + v_2 + v_3 + \dots .$$

$$\therefore\ \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \dots$$

or

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots . \qquad (31)$$

The reciprocal of the equivalent capacitance of a number of capacitors in series is equal to the sum of the reciprocals of the capacitances of the separate capacitors.

It should be noticed that the P.D. across each capacitor is *inversely proportional* to the capacitance; and that the equivalent capacitance is *less than the least* of the separate capacitances. It is convenient to remember that the expression for capacitances in *parallel* is similar to that for resistances in *series*, and vice versa.

Example 1.—Three capacitors have capacitances of 10, 20 and 30 microfarads respectively.

Calculate (*a*) the charge on each and the total capacitance when connected in parallel to a 200-volt d.c. supply; (*b*) the resultant capacitance and the P.D. across each when connected in series to the same supply.

(*a*) $q = cV. \quad \therefore\ q_1 = 10 \times 200 = 2000$ microcoulombs,
$q_2 = 20 \times 200 = 4000$ microcoulombs,
$q_3 = 30 \times 200 = 6000$ microcoulombs.

$\therefore\ Q = 12{,}000$ microcoulombs ($= 0{\cdot}012$ coulomb).

$$\text{Hence } C = \frac{Q}{V} = \frac{12{,}000}{200} = 60 \text{ microfarads.}$$

Alternatively

$$C = 10 + 20 + 30 = 60 \text{ microfarads.}$$

(*b*)
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots = \frac{1}{10} + \frac{1}{20} + \frac{1}{30} = \frac{11}{60}.$$

$$\therefore\ C = 5{\cdot}45 \text{ microfarads.}$$

$$\therefore\ Q = CV = 5{\cdot}45 \times 200 = 1090 \text{ microcoulombs,}$$

and this is the charge on each capacitor.

$$\therefore\ v_1 = \frac{Q}{C_1} = \frac{1090}{10} = 109{\cdot}0 \text{ volts,}$$

$$v_2 = \frac{1090}{20} = 54{\cdot}5 \text{ volts,}$$

$$v_3 = \frac{1090}{30} = 36{\cdot}3 \text{ volts.}$$

Example 2.—Two capacitors A and B, each having a dielectric for which $\varepsilon_r = 2$, are connected in series. When they are connected across a 230-volt d.c. supply, it is found that the P.D. across A is 130 volts and that across B is 100 volts. If the dielectric in the smaller capacitor is replaced by one for which $\varepsilon_r = 5$, what will be the new values of the P.D. across each?

Since the capacitors are in series, the charge on each is the same, i.e.

$$Q = C_A v_A = C_B v_B \quad \text{and} \quad \therefore\ \frac{C_B}{C_A} = \frac{v_A}{v_B} = 1{\cdot}3.$$

Hence

$$C_B = 1{\cdot}3 C_A, \text{ i.e. } C_A \text{ is the smaller.}$$

Now the capacitance of a given capacitor is proportional to the relative permittivity of the dielectric, so that when this has been changed in A, the new capacitance C_A' is

$$C_A' = \frac{5}{2} C_A \quad \text{or} \quad C_A = \frac{2}{5} C_A'.$$

Now

$$C_B = 1{\cdot}3 C_A = 1{\cdot}3 \times \frac{2}{5} C_A' = 0{\cdot}52 C_A'.$$

When connected to the supply,

$$\frac{v_A'}{v_B} = \frac{C_B}{C_A'} = \frac{0{\cdot}52C_A'}{C_A'} = 0{\cdot}52.$$

$$\therefore\ v_A' = 230 \times \frac{0{\cdot}52}{1{\cdot}52} = 79 \text{ volts.}$$

$$\therefore\ v_B = 230 - 79 = 151 \text{ volts.}$$

25. Oscillatory Discharge

Consider a charged capacitor, the two plates of which are connected by a conductor in the form of a coil possessing considerable inductance, as shown in fig. 21*a*. The capacitor begins to discharge, and a current flows through the coil; but owing to the inductance of the latter, the current grows comparatively slowly, and as the electric field between the plates collapses, a magnetic field linked with the coil is set up (fig. 21*b*). By the time the electric field has disappeared completely, the current in the coil has reached a maximum value, and neglecting, for the moment, any losses which may have occurred, all the energy which was originally stored in the electric field is at this stage

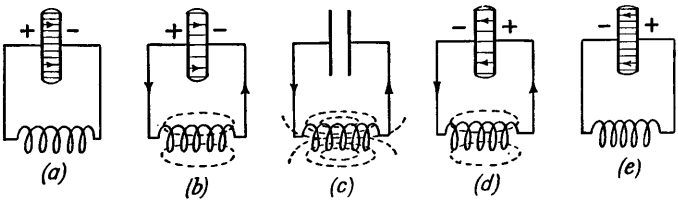


Fig. 21.—Oscillatory circuit

stored in the magnetic field (fig. 21*c*). The capacitor is now fully discharged, but the current cannot cease immediately since the collapse of the magnetic flux induces in the coil an E.M.F. which tends to maintain the current. Hence as the magnetic field decreases, the current, though decreasing, continues to flow, and the capacitor is recharged in the opposite direction (fig. 21*d*); and if losses are again neglected, by the time the current has ceased and the magnetic field has disappeared, the whole of the original energy is again stored in the electric field (fig. 21*e*).

In the ideal case, this process, once started, would continue indefinitely at the *natural frequency* of the circuit (see p. 284), which depends upon the values of resistance, inductance and capacitance, and may reach several million cycles per second if L and C are small. The energy is stored at one instant wholly in the electric field, one-quarter of a period later wholly in the magnetic field, and at intermediate instants partly in both. Such a discharge is called an *oscillatory discharge*, and the circuit is known as an *oscillating circuit*.

A very close mechanical analogy is provided by the pendulum of a clock, in which the potential energy of the bob at the end of a swing is converted into kinetic energy during the first half of the swing (i.e. the first quarter period), the conversion being complete at the mid-point when the velocity is a maximum; and the energy is reconverted into potential energy during the second half. Under ideal conditions these oscillations would continue indefinitely, but in practice losses occur, and at every swing some of the energy is converted into heat by air friction and the formation of eddies, so that the amplitude gradually decreases.

Similarly, in the oscillating circuit, some of the energy is converted into heat owing to the resistance of the circuit, and unless more energy is supplied the oscillations gradually decrease in amplitude and finally disappear. In addition, if the frequency is high enough (above about 10,000 cycles per second), and particularly if the plates of the capacitor are opened out in the form of an aerial-earth system, the rapid changes in the electric field produce electromagnetic waves, by which some of the energy is radiated into space in all directions. These waves, which are of the same nature as light waves but of greater wave-length, can be detected by means of suitable apparatus at great distances, and are employed in radio-communication; and the oscillating circuit is the basic circuit of the radio transmitter (see also p. 285).

EXAMPLES

1. What must be the capacitance of a capacitor which is to store a charge of 0·01 cmb. when the P.D. is 500 volts?

2. What is the P.D. between the plates when a capacitor of capacitance 10 μF. has a charge of 0·02 cmb.?

3. Calculate the energy (in joules) stored in a capacitor of 20 μF. when charged to a P.D. of 200 volts.

4 The energy stored by a capacitor, when the dielectric is air and the P.D. is 1000 volts, is 0·04 joules. What is the energy stored if the capacitor is immersed in oil ($\varepsilon_r = 2{\cdot}5$) and the P.D. is raised to 2000 volts?

5. What must be the capacitance of a capacitor which, when connected in series with a capacitor of 20 μF., reduces the effective capacitance to 5 μF.?

6. Find an expression for the joint capacitance of two condensers having capacitances C_1 and C_2 when connected (*a*) in series, (*b*) in parallel.

A 2 μF. condenser is charged to a P.D. of 200 volts. Its terminals are then connected to those of a 0·5 μF. condenser. Find the charge of electricity on each condenser, stating the units in which each is expressed. [Lond. Inter. B.Sc.(Eng.).]

7. Three capacitors of 10, 15, and 50 μF. are connected in series across a 500-volt supply. Calculate the P.D. across each capacitor.

8. Three air capacitors A, B, and C, of which A has a capacitance of 0·1 μF., are connected in series across a 500-volt supply, and the P.D.s across A and C are 250 volts and 100 volts respectively.

If capacitor B is now immersed in oil ($\varepsilon_r = 2{\cdot}5$), calculate the P.D. across each capacitor.

9. A capacitor of capacitance 20 μF. and charged to a P.D. of 500 volts is connected to a coil having an inductance of 0·002 henry. Neglecting losses, calculate the current in the inductance at the instant when the capacitor is fully discharged.

10. Deduce from first principles a formula for the capacitance of a concentric cable in terms of the radii of the conductors and the permittivity of the dielectric. Determine the capacitance per mile of such a cable if the diameter of the inner conductor is 1·5 cm., the relative permittivity of the dielectric 2·5 and the radial thickness of the dielectric 0·5 cm. [Lond. B.Sc.(Eng.).]

11. Two parallel wires, each 1 mile long and 2 cm. diameter, are spaced 350 cm. apart in air, and have equal and opposite charges of 0·6 μcmb. per m.

(*a*) Calculate the P.D., in volts, between the conductors.

(*b*) Use (*a*) to determine the capacitance in microfarads.

12. Calculate the capacitance of a parallel-plate capacitor in which the two plates have an effective area of 10,000 sq. cm. and are separated by a dielectric 0·1 cm. thick having a relative permittivity of 3.

13. Calculate the capacitance of a plate capacitor containing 73 sheets of tin-foil 30 cm. square separated by sheets of paraffin-waxed paper 0·035 cm. thick ($\varepsilon_r = 3$).

14. A capacitor of 10 μF. is charged from a 200-volt supply through a non-inductive resistance of 500 ohms. Calculate:

(*a*) the time taken for the charge to attain a value of 1264 μcmb.;

(*b*) the current at the end of 0·01 sec.

15. A capacitor of 30 μF. charged to a P.D. of 1000 volts is allowed to discharge through a non-inductive resistance of 2×10^6 ohms. Calculate:

(*a*) the charge 30 sec. after discharge has started;

(*b*) the time taken for the charge to be reduced to 1 per cent of its initial value.

16. A capacitor of nominal capacitance 0·03 microfarad is constructed of alternate sheets of tin-foil and mica. Each metal sheet has an area of 10 sq. cm. and each mica sheet is 0·2 mm. thick and has a relative permittivity of 4. Find the number of mica sheets required. Deduce any formula used. [Grad. I.E.E.]

17. Prove that the energy stored in a condenser of capacitance C farads, charged to a potential difference of V volts, is $\frac{1}{2}CV^2$ joules.

A 10-μF. condenser is charged from a 200-volt battery 250 times per second, and completely discharged through a 5-ohm resistor during the interval between charges. Determine: (*a*) the power taken from the battery; (*b*) the average value of the current in the 5-ohm resistor. [J.S.A., 1945.]

18. A variable condenser, of the type commonly used in radio receivers, consists of metal plates 0·5 mm. thick separated by air. The plates are interleaved, and the moving system has 13 plates, all connected to the spindle and mounted so as to be parallel, with spaces of 1·38 mm. between them; the fixed system has 12 plates in parallel similarly spaced.

When the systems are fully meshed and all the air-gaps are equal, the effective area bounding each gap is 11 cm.^{2} Show that the marking "0·00055 μF." corresponds fairly closely to the calculated capacitance. (One farad = 9×10^{11} e.s.u. of capacitance.) [J.S.A., 1948.]

19. A capacitor is formed by two co-axial cylinders, each 75 cm. long. The outside diameter of the inner cylinder is 5 mm., the inside diameter of the outer cylinder is 12 mm., and the dielectric is air.

Find the capacitance and the position of an equipotential surface which has a potential midway between those of the two cylinders.

20. A parallel-plate capacitor with air dielectric has two plates each having effective area of 200 sq. cm. and spaced 2 mm. apart. If the capacitor is charged to a P.D. of 200 volts, find the change in the stored energy if the separation of the plates is reduced to 1 mm. and (*a*) the charge remains constant, (*b*) the P.D. is maintained constant.

CHAPTER XV

Elements of Thermionics

1. Introduction

The applications of thermionic devices, at first confined chiefly to radio receivers and transmitters, have now become so numerous and so varied that there are few branches of engineering or industry in which they are not used in one form or another.

This chapter is concerned mainly with the consideration of the fundamental properties and characteristics of the vacuum diode and triode. Many modern valves have more than three electrodes, but in general these are either developed from the triode by the addition of one or more grids with special functions, or consist of two or more separate electrode systems enclosed in the same bulb. Hence a knowledge of the properties of the triode forms a basis for the study of most other types of valve.

2. Thermionic Emission

Some seventy years ago it was noticed by Edison that an electric current could be made to pass across the empty space between the filament of an electric lamp and another electrode sealed into the same evacuated bulb, and that this was possible *in one direction only*, namely from the electrode to the filament. The cause of the phenomenon remained obscure until the discovery of the electron about 1896 offered a satisfactory explanation; and almost at once a large amount of research on the subject was carried out by O. Richardson and others, and also by Fleming, who realized the practical importance of the one-way conductivity and gave to the device the name of *valve*.

Conditions in the surface layers of a metal differ from those in layers well inside. Conduction electrons in the inner layers are in equilibrium under the forces exerted by the atomic nuclei by which they are surrounded, and can therefore move freely. At the surface, however, electrons experience a resultant force which at normal temperatures prevents them from leaving the surface.

If the temperature is raised, the increase in energy causes the thermal agitation of the atoms to increase. Part of this energy is imparted, by collision, to the electrons, and some of the electrons in the outer layer attain a velocity sufficient to break through the surface and are ejected. This phenomenon is known as *thermionic emission,* and is somewhat similar to the process of evaporation in the case of a liquid, although the latter is concerned with molecules instead of electrons; it can also be pictured in terms of the emission from a pan of rapidly boiling water of small droplets, which immediately fall back into the water. In the absence of any electric field, e.g. due to some neighbouring charged body, these ejected electrons return because of (*a*) the attraction exerted by the parent body which, owing to the loss of electrons, exhibits a positive charge, and (*b*) the repulsive effect of electrons already emitted, which form a cloud surrounding the hot body, known as the *space charge.*

3. The Diode

If a positively charged electrode is brought near the hot body it exerts an attractive force on the electrons which decreases the effect of the space charge, and causes some of them to move towards it. In air or any other gas at normal pressure, both the emission and the movement are extremely small unless the potential gradient (see p. 359) is very high, because of the dense cloud of gas molecules by which both the hot body and the electrode are surrounded. If, however, most of these are removed by reducing the gas pressure to a very low value, the motion can take place more freely, and a stream of electrons flows to the positive electrode (anode) as long as it is maintained positive relative to the hot body (cathode). This constitutes a current flowing *from* the anode *to* the cathode, since the conventional direction of current flow, chosen arbitrarily long before the discovery of the electron, is *opposite* to that of the motion of the electrons.

In the vacuum thermionic diode, the cathode consists of an electrically heated filament of tungsten (as in a filament lamp), surrounded by a cylindrical nickel plate or anode, and enclosed in a highly exhausted glass bulb as shown diagrammatically in fig. 1*a*. In the early types it was necessary to heat the filament to bright redness to obtain a sufficiently copious emission of electrons, but by coating it with certain oxides such as those of strontium and barium, which emit electrons freely at comparatively low temperatures, the necessary filament temperature may be reduced to a dull red with consequent economy in the power required for heating.

Indirectly Heated Cathode.

When the development of the mains-operated receiver made it necessary to use A.C. for heating the filament cathode, it was found that the slight variations, during the cycle, of (*a*) the P.D. between anode and cathode, and (*b*) the filament temperature (due to its small heat capacity), caused variations in the valve characteristic which set up a hum of mains frequency which was emitted by the loudspeaker. In order to get over this difficulty, the indirectly heated cathode was introduced, in which the electron emission takes place from an oxide-coated metal cylinder, up the centre of which, but insulated from it, passes a tungsten heating filament of hairpin form, as shown in fig. 1*b*. The cathode is thus entirely isolated from the heating circuit and the heat

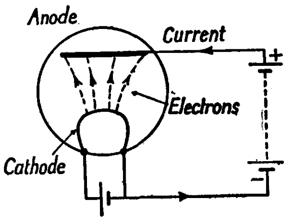


Fig. 1*a*.—Diode with filament cathode

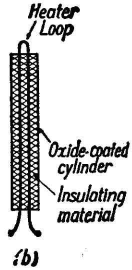


Fig. 1*b*.—Indirectly heated cathode

capacity of the cylinder is sufficient to make variations in temperature due to the alternating heating current negligible; but for the same reason the cathode takes some 30 seconds to attain its operating temperature. This is the reason for the time-lag after switching on which is a familiar characteristic of the A.C. mains set. The indirectly heated cathode is represented diagrammatically as shown in fig. 9*b*.

Diode Characteristics.

In the circuit shown in fig. 2*a* the diode is connected so that the anode is positive relative to the cathode, and the P.D. between anode and cathode (often called the anode voltage) can be varied by means of a potentiometer across the battery. The anode current is measured by a milliammeter in the anode lead so connected that it does not include the current taken by the voltmeter reading the anode voltage. If the filament current is maintained constant and the anode voltage gradually increased, it will be found that the anode current increases almost uniformly for a time, but ultimately the curve bends over and flattens (fig. 2*b*), so that further increase in anode voltage produces no increase in current. This is known as the *saturation current*, and occurs when *all* the available electrons are being drawn to the anode. Further increase in anode current can be obtained only by increasing the electron emission: this requires an increase in filament temperature, obtained by increasing the heating current. At higher temperatures similar characteristies are obtained, having higher values of saturation current as shown in fig. 2*b*. With oxide-coated filaments (cathodes), however, although the curve bends over, the saturation current does not maintain a steady value but continues slowly to increase.

Usually the saturation value lies well beyond the normal working range of the diode.

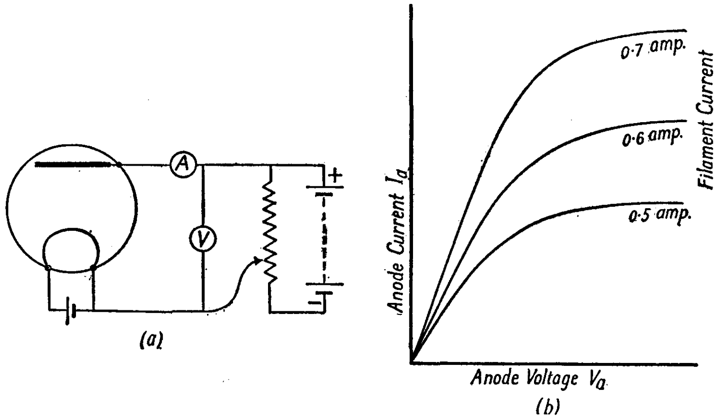


Fig. 2.—Diode characteristic

4. The Diode as a Rectifier

If the potential of the anode is made *negative* relative to the cathode, by reversing the H.T. battery for instance, the repulsive force set up will prevent any movement of the electrons towards it, i.e. no current will flow between anode and cathode. Hence the diode acts

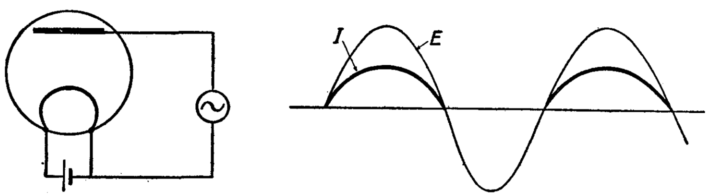


Fig. 3.—Diode as rectifier

as a non-return *valve*; and if an *alternating* P.D. exists between anode and cathode, current will flow during the half cycle when the anode is positive relative to the cathode, but not during the next half cycle when the P.D. is reversed. The current therefore flows only during alternate half cycles (fig. 3), but is always in the same direction, so that the device acts as a *rectifier*. The diode is frequently used in this capacity for providing the H.T.-d.c. supply from a.c. mains in a radio receiver or transmitter.

Half-wave Rectifier.

The arrangement shown in fig. 3 is known as a half-wave rectifier, since only one half of the a.c. wave is utilized, the other being entirely suppressed. The output P.D. and current, although uni-directional, are violently fluctuating and remain zero during alternate half cycles. Fig. 4*a* shows a typical half-wave rectifier unit. The a.c. supply is obtained from the mains through a step-down transformer of suitable ratio, the secondary of which is connected between anode and cathode, and has a low-voltage tapping for providing the filament heating current. Such a rectifier may be used to charge a battery connected to the output terminals, but where a steady d.c. supply is required, as in a radio receiver, a *smoothing circuit* must be interposed between the output terminals and the load.

Full-wave Rectifier.

By the use of two diodes (which are often enclosed in the same bulb with a common cathode, and known as a double diode), it is possible to make use of both half waves. This is clearly a more efficient arrangement in which the wave-form is improved and the average value of the d.c. output voltage increased (fig. 4*c*). The two anodes are connected to the ends of the secondary winding of the transformer, which is provided with a centre tapping (fig. 4*b*). During the half cycles when the upper end of the winding is positive, current passes via the upper anode to the cathode, through the d.c. load circuit, and back to the centre point of the secondary. During the next half cycle the upper anode becomes negative and the current through it ceases, but the lower end of the winding is now positive, so that the current now flows through the lower anode to the cathode, through the d.c. load circuit in the *same* direction, and back to the centre point of the secondary winding. Hence each half of this winding is used during alternate half cycles, and must be wound to give the required voltage, i.e. the total voltage between the ends must be *twice* the value required.

Most rectifier valves in mains-operated radio receiving sets are of this type, the working value of the anode voltage being between 200 and 300 volts and of the output current between 60 and 120 milliamps.

Smoothing Circuit.

A common form of smoothing circuit is shown in fig. 4*b* and consists of two capacitors connected across the output terminals, one on each side of a series iron-cored inductance or choke. Capacitor C_1 acts as a *reservoir* capacitor, since it serves to maintain the current in the load circuit during those portions of the cycle when the rectifier output is either zero or falls below the mean value. Assuming, first of all, that there is no load, C_1 is charged to the maximum value of the output voltage, and the P.D. then remains constant, since it cannot discharge back through the rectifier. When the load circuit is connected, the capacitor will begin to discharge immediately the rectifier P.D. falls below its maximum value, and will continue to act as a source of supply, with gradually falling P.D., until the rectifier P.D., on rising again, replaces the charge which has been lost. As shown in

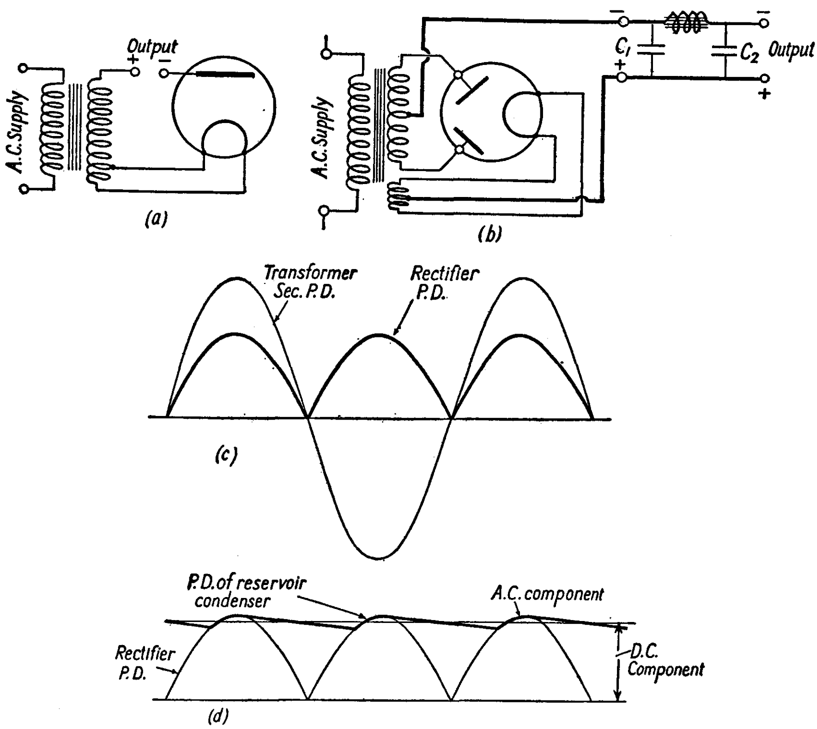


Fig. 4.—(*a*) Half-wave rectifier circuit; (*b*) Double diode and smoothing circuit; (*c*) Full-wave rectification; (*d*) Effect of smoothing circuit

fig. 4*d*, the size of the capacitor, relative to the rectifier output and the load, is so chosen that the fluctuation of the capacitor P.D. about the mean value is small.

The output voltage from the reservoir capacitor therefore consists mainly of a steady d.c. component with a small alternating component superimposed. The series choke, which has a high reactance but a comparatively low resistance, passes the d.c. component of the current but reduces the a.c. component to a ripple. The second capacitor C_2, acting as a reservoir relative to the ripple, reduces it still farther, so that the final output is almost pure d.c.

Vacuum diode rectifiers are used for current outputs of a few milliamps at voltages ranging from about 200 to 300 volts in wireless receivers to about 50,000 volts in certain forms of cable-testing apparatus.

The Gas-filled Diode.

If a small amount of mercury vapour is included in the bulb, electrons on their way to the anode will frequently collide with mercury atoms, and sometimes the collisions will be sufficiently violent to knock an electron out of the atom. This results in an additional free electron, which proceeds towards the anode and leaves a mercury atom. This atom, having lost an electron, exhibits a positive charge and is called a positive *ion*.

The positive ions move in the opposite direction and are eventually neutralized by an electron extracted from the cathode. Hence they assist in conduction and thus increase the current output. At the same time their presence neutralizes much of the effect of the space charge and thereby reduces the internal resistance of the valve, so that comparatively large currents (up to about 200 amperes) can be passed with a voltage drop in the valve of only 15 to 30 volts.

5. Metal Rectifiers

The metal rectifier is not a thermionic device, but as it is often used as an alternative to a diode rectifier, a short description is not out of place.

The operation of rectifiers of this type is due to the barrier layer produced at the contact surface between a good conductor and a semi-conductor. When a P.D. is applied an electric field is set up across the layer, and because it is very thin the potential gradient is high. During the half-cycle when the conductor is negative, electrons (of which there is a plentiful supply in the metal) are accelerated and pass through the barrier layer to the semi-conductor; but during the next half-cycle, since there are few electrons available in the semi-conductor, the reverse current is very small.

Copper Oxide and Selenium Rectifiers

If a copper plate is covered with a film of cuprous oxide (which is a semi-conductor), the resistance is very small when a current is flowing from oxide to copper (electron flow from copper to oxide) but very large when the direction of flow is reversed.

The copper oxide rectifier is built up of units consisting of copper discs or plates on which a hard film of oxide has been produced by heat treatment. The P.D. across each disc in the reverse direction required

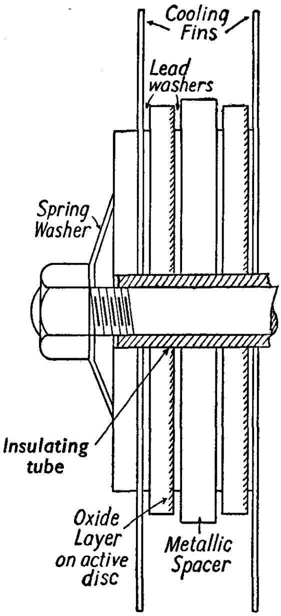


Fig. 5.—Construction of copper oxide rectifier

to break down the film is comparatively small, so that it is usually necessary to connect a number of units in series: the current output may then be increased by joining several of these sections in parallel.

For small currents copper discs are used, and contact with the oxide film is made by means of lead washers, the unit, consisting of oxide discs, lead washers and cooling fins, separated by metallic spacers, being mounted on an insulating rod and clamped under pressure, as shown in fig. 5. For larger current outputs, the discs are replaced by copper plates having an oxide film on both sides which is sprayed with metal in order to give a good contact: contact with the copper is made through a small area from which the oxide has been removed.

In the selenium rectifier, which is now very widely used, a disc of nickel-plated steel or of aluminium is covered with a thin layer of crystalline selenium, which acts as the semi-conductor. This layer is then sprayed with a low melting-point alloy of tin and cadmium which serves as the conductor. A low resistance is offered to a current flowing from the selenium to the alloy, and a very high resistance to a current in the reverse direction.

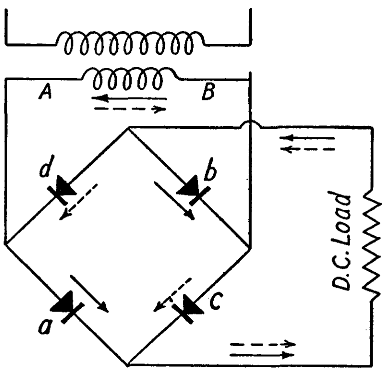


Fig. 6.—Oxide rectifier—bridge connection for full-wave rectification

The allowable current density is considerably less than in the copper oxide rectifier so that the discs are large and act also as cooling fins. The maximum permissible operating temperature is higher—about 80° C. as compared with about 60° C. for the copper oxide rectifier.

On a single-phase supply full-wave rectification can be obtained by using a centre tapped transformer as shown in fig. 4*b*, but more usually the bridge arrangement shown in fig. 6 is employed. During one half-cycle when the end *A* of the transformer secondary is +ve, the current flows to the d.c. circuit via rectifier *a* and returns via rectifier *b*; while during the next half-cycle, when the end B is +ve, the current passes through c and d.

Owing to their simplicity, rectifiers of this type are used for a variety of purposes, with outputs ranging from currents of a few milliamperes at several hundred thousand volts for use in X-ray work to currents of several thousand amperes at 5 or 6 volts, for electro-plating plants: other common uses include battery charging and the supply of cinema arcs. The efficiency is between 70 and 80 per cent, depending on the size and type.

The characteristics of both types are similar in shape to that shown in fig. 8.

The Germanium Rectifier

Within the last few years a new type of contact rectifier has been developed. This employs the metal germanium, forecast by Mendeljeff

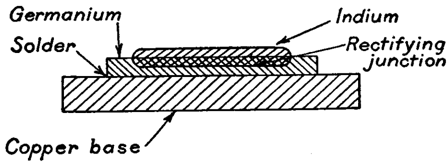


Fig. 7.—The germanium rectifier

in 1870 and discovered by Winkler in 1886. Germanium occurs widely in nature but in very low concentrations. It is absorbed by trees in the moisture from the soil, and the principal source is the soot or flue dust produced by burning coal. After extraction and exhaustive purification, long crystals are grown in the molten metal and afterwards cut into thin slices. A layer of indium is fused to the upper surface of one of these slices, and the lower surface is soldered to a copper base as shown in fig. 7.

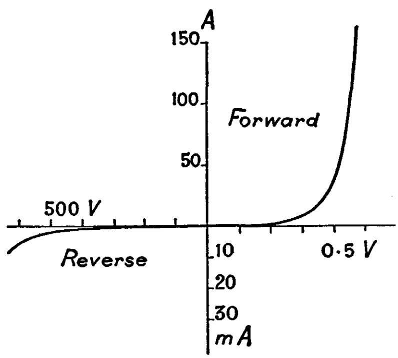


Fig. 8.—Characteristics of metal rectifiers

Rectification takes place inside the germanium slice at the junction between the pure germanium and that portion into which the indium has penetrated. To pass a relatively large current from the indium to the germanium requires a P.D. of only about half a volt, while a P.D. of several hundred volts applied in the opposite direction produces a current of only a few milliamperes. The characteristic curve of all metal rectifiers has the form shown in fig. 8, but because of the very low voltage drop, the germanium rectifier has a higher efficiency than other types, particularly at low voltages. Rectifiers with outputs of several hundred kilowatts have been built and are now in service, and it is probable that in the future rectifiers of this type will be used for a wide variety of purposes, including traction service where they may enable the advantages of d.c. traction motors to be retained in combination with a high voltage a.c. distribution system.

6. The Triode

The triode is a diode with the addition of a third electrode, situated between the anode and cathode as shown diagrammatically in fig. 9*a*. This usually consists of an open spiral of wire surrounding the cathode and, from its open construction, is known as the *grid*. The open construction allows the electrons to pass through on their way to the anode, but since it is in the direct path of the electron stream the grid can exert a very sensitive and powerful control over the anode current.

If the potential of the grid is made negative relative to the cathode,

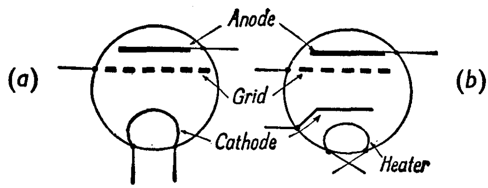


Fig. 9.—The triode

it will assist the space charge in impeding the electrons leaving the cathode, and the anode current will thereby be decreased. As the negative potential of the grid is increased, so the anode current will continuously decrease until it ultimately becomes zero.

If, on the other hand, the grid is made positive relative to the cathode, it will assist the action of the anode by partially neutralizing the effect of the space charge, and cause the anode current to increase.

Since the grid is much nearer to the cathode than is the anode, a given change in grid potential causes a change in the anode current which it would require a much greater change in anode voltage to produce; and it is due to this that the triode possesses the valuable property of *amplification* (see below).

Triode Characteristics.

The characteristic curves of the triode may be obtained by means of the testing circuit shown in fig. 10*a*, which is similar to fig. 2*a*, with the addition of a battery and potentiometer by which the potential of the grid can be varied over a considerable range in either direction. Two sets of characteristic curves may be obtained, from which are determined certain ratios which serve as criteria of the valve performance.

(i) Curves showing the relation between anode current and anode voltage when the grid potential is maintained at some constant value (fig. 10*b*). These are similar in shape to the diode characteristic and would eventually show the same kind of saturation effect, but as this lies beyond the part of the characteristic which is normally used, it

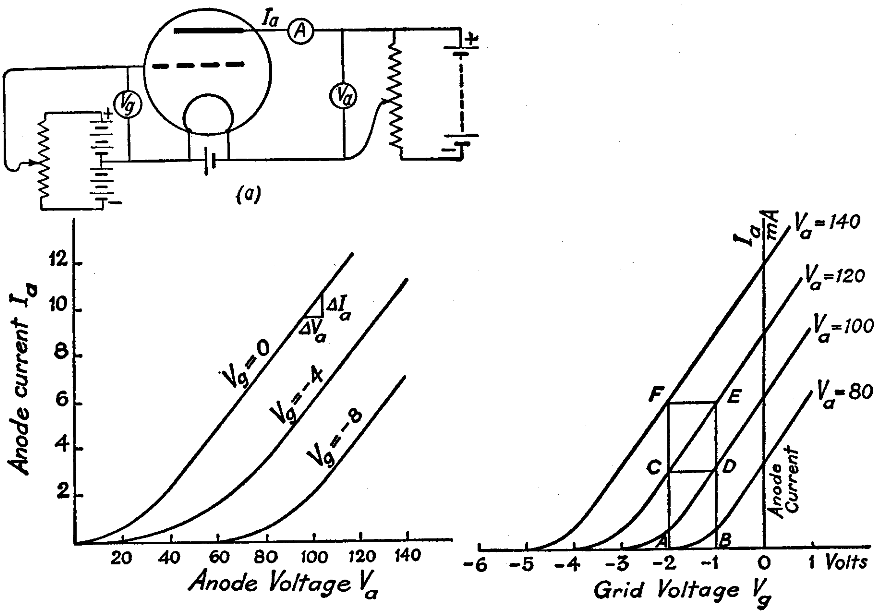


Fig. 10.—Triode characteristics

is not included in the figure. It will be seen that increased values of anode voltage are required to produce a given value of anode current, as the negative grid potential is increased.

The ratio of the change in anode voltage, ΔV_a, to the corresponding change in anode current, ΔI_a, the grid voltage being constant, is called the *a.c. resistance* or *anode slope resistance*, r_a, i.e.

$$r_a = \frac{\Delta V_a}{\Delta I_a}. \qquad \qquad (1)$$

It is, of course, the slope of the V_a/I_a curve, and this depends, to some extent, upon the point chosen and the value of the grid voltage, so that it is often specified for particular conditions, e.g. 100 volts and zero grid voltage. Since the upper part of the curve, before any saturation effect occurs, is almost linear, ΔV_a and ΔI_a can be relatively large, and for the triode of fig. 10*b*, when $\Delta V_a = 8$ volts, $\Delta I_a = 1$ mA., so that $r_a = 8/10^{-3} = 8000$ ohms.

(ii) Curves showing the relation between the anode current and the grid potential for various constant values of anode voltage (fig. 10*c*). These clearly show the effect of increasing negative grid potential upon the anode current which is ultimately reduced to zero in each case.

In general the grid is never allowed to become *positive* relative to the cathode so that curves for +ve grid potentials are omitted from fig. 10*b* and *c*.

When the anode voltage is $V_a = 120$, the anode current for a grid voltage OA is AC. If the grid voltage is reduced to OB the anode current rises to BE.

The ratio of the change in anode current, ΔI_a, to the change in grid voltage, ΔV_g, the anode voltage being constant, is called the *mutual conductance*, g_m, i.e.

$$g_m = \frac{\Delta I_a}{\Delta V_g}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and is expressed in mhos or more usually in mA. per volt. For the triode of fig. 10*c*, when the anode voltage is maintained at 120 volts,

$$\Delta I_a = DE = 2{\cdot}5 \text{ mA. and } \Delta V_g = AB = 1 \text{ volt.}$$

Hence $$g_m = \frac{\Delta I_a}{\Delta V_g} = 2{\cdot}5 \text{ mA. per volt } (2{\cdot}5 \times 10^{-3} \text{ mho}).$$

The anode current can be restored to its original value AC (= BD) by reducing the anode voltage to 100 volts, i.e. a change of 1 volt in the grid potential is neutralized by, or is equivalent to, a change of 20 volts in the anode voltage. In other words, in this particular valve the grid potential is 20 times as effective in controlling the anode current as the anode potential.

The ratio of the change in the anode voltage, ΔV_a, to the change in grid voltage ΔV_g, the anode current remaining constant, is called the *amplification factor*, μ, i.e.

$$\mu = -\frac{\Delta V_a}{\Delta V_g}.^* \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In this case $\mu = 20$. The value for a triode may vary over a wide range depending upon the construction, but usually lies between 10 and 100.

* The minus sign occurs because an increase in the negative potential of the grid requires an increase in the positive potential of the anode. Hence when the changes are of an alternating character, the two quantities are in *anti*-phase. Where the relative phase is not being specifically considered the sign is often omitted.

It also follows from equations (1) and (2) that

$$\mu = -\frac{\Delta V_a}{\Delta V_g} = -\frac{\Delta V_a}{\Delta I_a} \cdot \frac{\Delta I_a}{\Delta V_g} = -r_a \cdot g_m, \quad . \quad . \quad (4)$$

where r_a is in ohms and g_m is in amperes per volt, and provided that the values of both quantities correspond with the same point on the characteristics.

A detailed discussion of the applications of the triode lies quite outside the scope of this volume, but it may be stated that when used simply as an *amplifier* the mean value of the grid voltage (the grid *bias*) is adjusted so that the range of variation of grid potential produced by the incoming signals lies wholly on the straight portion of the characteristic (fig. 10*c*): hence the variation in anode current, and therefore the variation in potential passed on to the grid of the next valve, are exact copies of those produced by the signals. On the other hand, if the grid bias is adjusted so that the working range lies on the curved portion of the characteristic, a certain amount of rectification takes place, and the valve can then be used as a *detector* amplifier.

7. The Oscillograph

The oscillograph is an instrument used chiefly for the observation and recording of transient phenomena such as the train of oscillations set up by the discharge of a capacitor through an inductance, and the wave-form of periodic quantities.

The *electromagnetic* oscillograph is essentially a moving-coil reflecting galvanometer, having a moving system in which the inertia is very small so that it can respond instantly to rapid changes in the current passing through it, and of which the natural frequency is well above the highest frequency to be observed, so that there is no likelihood of mechanical resonance taking place. A beam of light is deflected by the mirror in a horizontal direction and in order to obtain a photographic record, it is brought to a focus on the surface of a sensitized film or bromide paper moving with uniform velocity in a vertical direction; and by this means a graph of the quantity, on a time base, is traced on the film. For the visual observation of periodic waves, the beam is projected on to an oscillating or rotating mirror, which superimposes a uniform vertical motion, so that the wave may be seen in the mirror; or alternatively the beam may be brought to a focus on a ground-glass screen. In either case the image may be made to appear stationary by adjusting the speed of the mirror.

The range of frequency is limited to about 5000 cycles per second, and one of its chief uses is in the examination of phenomena connected with power circuits, for which purpose it can be built to work at voltages up to 50 kilovolts above earth; and by providing additional moving-coil units (or *vibrators*) a number of variables can be recorded simultaneously on the same film.

In the *cathode ray* oscillograph the moving system and beam of light of the electromagnetic oscillograph are replaced by a narrow stream or beam of electrons which, impinging on a fluorescent screen, produce on it a luminous spot, and which can be deflected by the application of either a magnetic or an electrostatic field—usually the latter. In practice the beam is deflected in a horizontal direction by a field which increases uniformly with time, and having reached a certain value, is suddenly reduced to zero. The luminous spot therefore moves horizontally across the screen and then quickly returns to the zero point, the motion being repeated at a frequency which can be varied between wide limits. At the same time the beam is deflected in a vertical direction by a field which is proportional to the value of the quantity being examined. The luminous spot therefore produces a trace which is a wave representing the variation of the quantity plotted on a time base, i.e. the wave-form, which, owing to retentivity of vision, appears as a wave on the screen; and since the inertia of the electron beam is negligibly small, it is capable of following faithfully variations of extremely high frequency.

The simple cathode ray tube, generally used as an oscilloscope for the qualitative examination of wave-forms of power and audio-frequencies, is cheaper, more robust and more suitable for unskilled use than the electromagnetic type. In the simple form only one variable can be examined at a time, but double beam tubes have been developed in which two variables can be shown simultaneously.

8. The Cathode Ray Tube

The Beam Methods of Focusing.

The cathode ray tube may be considered as a development of the diode already described, in which the anode is in the form of a flat disc pierced with a small central hole (fig. 11*a*). The acceleration of the electrons emitted by the cathode depends upon the P.D. between the anode and the cathode, and if this is sufficienty high the velocity of some of those approaching the aperture will be great enough to cause them to pass through and continue moving almost in a straight line until they strike the far end of the tube, where they cause fluorescence of the salts with which it is coated. In order to increase the effect, the cathode is surrounded by a negatively charged cylinder known as the Wehnelt shield, which by its repulsive action on the electrons compresses the beam so that a greater proportion of the electrons pass through the aperture (fig. 11*b*).

But, because of the mutually repulsive forces which exist between the electrons, the beam is divergent, and a large area of the end of the tube is caused to fluoresce to some extent, decreasing in intensity from the centre outwards. If the beam is to be effective, the luminous area must be reduced to a small bright spot, and this requires the divergence to be eliminated and the beam to be concentrated or focused.

One way of doing this is to introduce a little gas into the tube so that collisions occur between the electrons and the gas atoms. Some of those which occur in the middle of the beam, where the electron velocity is highest, are sufficiently violent for ionization to take place (see p. 389). The attractive forces exerted on the electrons by the positive ions thus formed in the middle of the beam, not only neutralize the mutual repulsive forces which cause the divergence, but

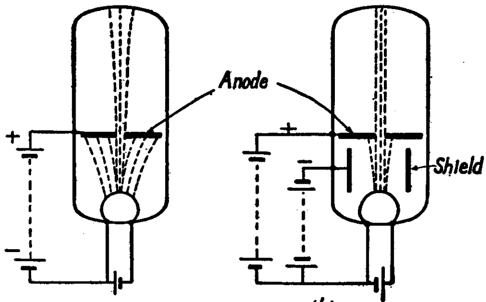


Fig. 11.—The cathode ray tube

can be made to produce convergence and bring the beam to a focus at the end of the tube, which is flattened so that its radius is approximately equal to the length of the beam. A tube of this kind is known as a " soft " tube with gas focusing.

In most modern oscillographs a " hard " tube, containing no gas, is used, and the focusing is carried out by means of an additional anode. The two anodes are maintained at different positive potentials relative to the cathode, and have an effect on the electron beam similar to that of a lens on a beam of light, so that it is brought to a relatively sharp focus on the end of the tube. This is known as electrostatic focusing.

The hard tube is more expensive and less sensitive than the soft tube but can respond to higher frequencies.

Deflection—The Time Base.

The focused beam, after issuing from the anode, passes between two pairs of plates (fig. 12), one pair X being vertical and the other Y horizontal. When a P.D. exists between these pairs, the beam, since it is composed of electrons, is deflected away from the —ve and towards the +ve of each pair of plates, to an extent which is proportional to the P.D. The vertical pair are used to produce the uniform horizontal sweep which serves as the time base. By means of a special time-base circuit, the P.D. between them is increased uniformly to a value which

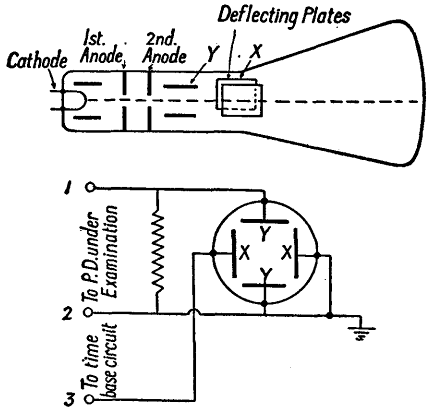


Fig. 12.—Focusing and deflecting systems in cathode ray tube

produces a deflection of the beam sufficient to cause the spot to travel right across the tube: the P.D. is then suddenly reduced so that the spot returns rapidly to its initial position. In the simplest form of time-base circuit, the X plates are connected to a capacitor (fig. 13a) which is charged from a constant voltage supply through a high resistance.

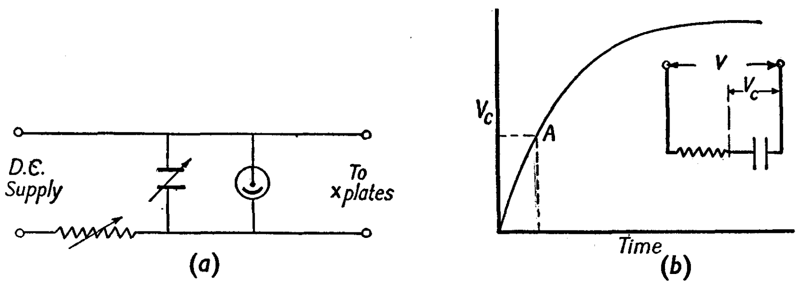


Fig. 13.—Time base in cathode ray tube

The capacitor P.D., and therefore the P.D. between the X plates, rises according to an exponential curve (see p. 365) of which the initial portion is almost a straight line (fig. 13b). Across the capacitor is connected a neon lamp. A lamp of this type remains non-conducting until the P.D. has risen to a value sufficient to cause the gas to be ionized; the resistance then falls to a low value and the capacitor discharges very rapidly until its P.D. has fallen to some lower value at which ionization ceases and the lamp again becomes non-conducting. This process is repeated at a frequency which depends on the value of the resistance and the capacitance of the capacitor. This circuit has a number of disadvantages and is seldom now used in its simple form, although it forms the basis of other more satisfactory time-base circuits.

Between the other pair of plates Y (fig. 12) a P.D. is established which is proportional to the quantity being examined, and which produces the vertical deflection. The cathode ray oscillograph is a potential operated device (in contrast with the electromagnetic oscillograph which is current operated), so that the Y plates are connected either to the two points between which the P.D. is to be examined, or across the ends of a non-inductive resistance carrying the current to be examined.

When examining periodic quantities the frequency of the time base is adjusted so that it is equal to that of the quantity under examination; the trace is then repeated again and again in the same position and appears stationary on the screen.

9. Transistors

The unilateral conductivity at the junction between a fine wire and the surface of certain crystals was used for high-frequency rectification in early radio receivers; but this "cat's whisker" detector was later entirely superseded by the thermionic valve which is capable of amplification in addition to rectification.

During the Second World War the valve was found to have certain limitations when used in connection with the increasingly high frequencies employed in radar equipment, and the "crystal valve" was resurrected. Much research was carried out in order to find the most suitable of a large class of substances known as *semi-conductors*, and one result of this was the later development of the germanium rectifier mentioned on p. 391. Further research revealed the possibility of using a crystal valve as an amplifier, and to this was given the name of *transistor*.

The full theory of the action of a transistor, which depends upon the atomic and crystal structure, is very complex and not yet fully understood, and what follows must be taken as a much simplified version.

The atomic structure of germanium is such that in the absolutely pure state there are no free electrons to act as conductors. If however a small but controlled amount of certain "impurities" is added, it becomes partially conducting in one of two ways.

The addition of antimony or arsenic, which have one electron in the outer orbit of each atom in excess of the number required to bond with the germanium atom, gives an *n-type* germanium in which conduction is due to these excess electrons. Alternatively, an impurity, e.g. indium or boron, may be added which contains one less electron in the outer orbit than is required for bonding with the germanium atom. In this case the impurity may rob a neighbouring germanium atom of an electron. The space thus vacated exhibits a localized positive charge and is called a *positive hole*, and the material is known as *p-type* germanium. The hole may then be filled by an electron from a neighbouring germanium atom, which leaves a hole in another position, and this process may continue, the hole appearing to move in the opposite direction to the motion of the electrons. Hence, in a semi-conductor conduction may take place by the movement of either free electrons or positive holes.

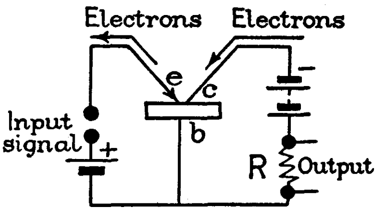


Fig. 14

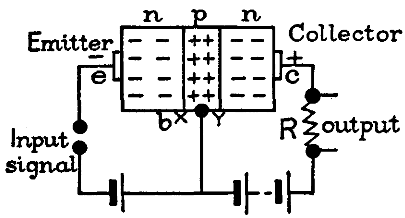


Fig. 15

A point-contact transistor (fig. 14) consists of two fine wire points, a few thousandths of an inch apart, in contact with a thin slice of germanium. When a positive potential is applied to the *emitter* e, electrons pass out of the crystal, leaving positive holes which move towards the *collector* c. This lowers the potential barrier at the surface and allows a much larger number of electrons to pass in through c. A small portion of these are used in filling the holes and the remainder pass into the crystal b and round the collector circuit. Hence a small change in the input current at e produces a much larger change in the output current through c and in the P.D. across the load R, i.e. the transistor acts as a triode.

The point-contact type is being rapidly replaced by the junction type which is more stable and capable of greater amplification. It consists of a thin slice of p-type germanium sandwiched between two slices of n-type germanium (or vice versa), as shown in fig. 15. The positive potential of the collector c tends to attract electrons, but since the central section contains mainly positive holes a potential barrier is set up at the p-n junction Y, and very little current flows. If however a negative potential is applied to the n-type emitter e, electrons cross the n-p junction X, and this lowers the potential barrier and allows electrons to pass across the n-p junction Y to the collector. Hence small changes in signal current produce large changes in collector current.

When connected as shown there is no actual amplification of current—the collector current is, in fact, rather less than the emitter current—but voltage and power amplification take place because of the high value of R. Current amplification can be obtained by using alternative connections.

While it is unlikely that the transistor will completely replace the thermionic valve, its small dimensions—often little larger than a glass bead—and extremely small power requirements (since there is no heater circuit) are great advantages, and it is being used in large numbers in hearing aids, portable radio receivers, computers, etc.

Appendix

THE C.G.S. SYSTEM OF UNITS

The C.G.S. System of Units

The rationalized M.K.S. system of units was adopted by the International Electro-technical Commission in 1950 and officially recognized by the Institution of Electrical Engineers in 1952, and is now used almost universally in the electrical engineering courses in universities and technical colleges throughout the world.

Nevertheless, because this system has not yet been wholly adopted by physicists and because of the wealth of technical literature in which C.G.S. units are employed, the electrical engineering student must also be familiar with the C.G.S. system and with the relation between corresponding units in the two systems.

Coulomb's Law

The electromagnetic C.G.S. system of units is based upon the concept of a *unit pole*, i.e. a pole of unit strength.

Every magnet has two poles, of equal strength* but opposite polarity. An isolated pole, therefore, cannot exist, although it can be realized approximately by using or imagining a very long thin magnet in which the other pole is so far away that it has no appreciable effect. However this does not affect the value and usefulness of the concept.

As stated on p. 46, Coulomb showed that,

The force between two magnet poles is proportional to the product of their pole strengths, inversely proportional to the square of the distance between them, and also depends upon a property of the surrounding medium, so that,

$$F \propto \frac{m_1 m_2}{\mu d^2},$$

where m_1, m_2 are the pole strengths in units yet to be defined, d is the distance between them, and μ is a constant depending upon the medium and called the *permeability*.

* A magnet freely supported, e.g. floated on a liquid, in a magnetic field, turns so that its axis is in line with the field but exhibits no tendency to *move bodily*, from which it is inferred that the poles are of equal strength.

Unit Pole Strength

The constant μ was, in the case of free space or a vacuum, arbitrarily assigned a value of unity, and unit pole strength was then defined by giving unit values to the other terms in the above expression, so that,

A pole of unit strength repels a similar pole at unit distance (1 cm.) from it in a vacuum, with unit force (1 dyne).

The value of μ for air is very nearly equal to that for a vacuum. Hence, in general, in a vacuum (or in air)

$$F = \frac{m_1 m_2}{d^2} \text{ dynes}, \quad \ldots \ldots \quad (1)$$

where m_1 and m_2 are the pole strengths in units defined above and d is the distance apart in centimetres.

Magnetic Field Strength (Magnetizing Force) (H)

The strength and direction of a magnetic field at any point is measured by the magnitude and direction of the force acting on a unit north pole placed at that point.

Unit field strength is that of a field in which a unit pole experiences unit force (1 dyne), and is represented by one line of force drawn through each square centimetre of area perpendicular to the force.

The unit of field strength is called the *oersted.*

At a point where a unit pole experiences a force of n dynes the field strength is

$$H = n \text{ oersteds}$$

and is represented by n lines per square centimetre.

It follows that a pole of strength m units when placed at the same point experiences a force

$$F = mH \text{ dynes}. \quad \ldots \ldots \quad (2)$$

It is assumed here that the introduction of the unit pole does not disturb the original field distribution as would in fact happen in practice.

Flux Associated with Unit Pole—Unit of Flux

If a unit pole is assumed to be placed at the centre of a sphere of unit radius (1 cm.), a second equal and similar pole placed at any point on the surface experiences unit force (1 dyne) directed radially outward. Hence, by definition, the field strength at every point on the sphere is of unit value and represented by one line passing radially outward through each unit area (1 sq. cm.) of the sphere. But the area of a sphere of 1 cm. radius is 4π sq. cm. Hence,

The total flux issuing from a unit north pole (or entering a unit south pole) is 4π lines.

The C.G.S. unit of flux is called the *Maxwell*, but the name has never come into common use among engineers.

Total flux of a magnet pole of strength m units $= 4\pi m$ maxwells.

Lines of Magnetic Induction—Flux Density (B)

When a piece of iron is placed in a magnetic field, the total flux is the sum of the lines of force due to the original field and the lines of magnetic induction due to the induced poles. If the section of the iron is a sq. cm. and the pole strength is m units,

$$\Phi = Ha + 4\pi m$$

and the flux density in the iron is

$$B = \frac{\Phi}{a} = H + \frac{4\pi m}{a} = H + 4\pi \mathscr{I}, \quad . \quad . \quad . \quad (3)$$

where $\mathscr{I}$ is the pole strength per unit area and is called the *intensity of magnetization.*

The total number of lines are referred to, collectively, as the *total flux of magnetic induction.*

The quantity $4\pi\mathscr{I}$, which is the flux density due to the induced poles only, is sometimes called the *ferric induction.* If H is continuously increased the portion Ha of the total flux increases uniformly, but the rate of increase of $\mathscr{I}$ decreases, and it finally reaches a constant value when the iron is fully saturated.

Engineers are usually only interested in the total flux density (B) which is sometimes called the *induction density.*

The unit of flux density is called the *gauss.* A flux density of n gauss is represented by n lines per square centimetre, and this was (and still is) the term most frequently used by engineers.

$$1 \text{ gauss} = 10^{-4} \text{ weber per sq. m.}$$

The ratio of the flux density (gauss) to the magnetizing force (oersteds) is the permeability.

$$\mu = \frac{B}{H} \quad \text{or} \quad B = \mu H. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In air and other non-magnetic materials for which μ was assigned a value of unity, B and H are numerically equal in the C.G.S. system.

Demagnetizing Effect of the Poles

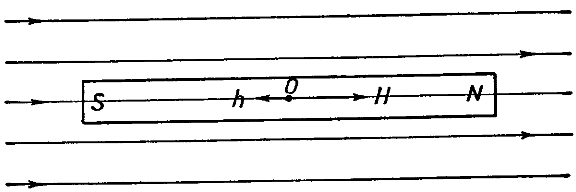
In the last section it was shown that the total flux entering or leaving a piece of iron placed in a magnetic field is the sum of the lines due to the original field and those set up by the induced poles, i.e.

$$\Phi = Ha + 4\pi m,$$

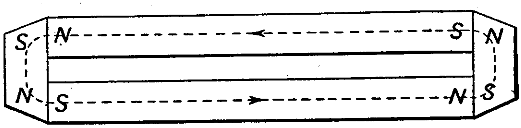
and therefore

$$B = H + 4\pi \mathscr{I}.$$

In most cases, however, this is not quite true, the value of H inside the iron being somewhat less than when the iron is removed, owing to the demagnetizing effect of the induced poles.



(a)



(b)

Fig. 1.—Demagnetizing effect of the poles of a magnet

Consider a short iron bar placed in and with its axis parallel to a uniform field of strength H, the polarity of the induced poles being as shown in fig. 25*a*. A unit N pole, imagined to be *immersed* in the iron at O, experiences a force due not only to the original field, but also, since the bar is short, to the induced poles at its ends. That due to the original field is from left to right, while that due to the induced poles is clearly from right to left, since the N pole is at the right-hand end. Hence the resultant force on the unit pole, by which the field strength at O is measured, has a value

$$H_0 = H - h,*$$

i.e. the field strength at O is reduced by the presence of the induced poles.

* Although, as has been seen, the magnetic forces produced in the surrounding air (for which $\mu = 1$) by the induced poles of a magnet are much greater than those due to the original field by which they are induced, in this case h is much smaller than H since the unit pole is imagined to be *immersed* in the iron, i.e. in a medium in which μ is large, and therefore the forces produced are correspondingly small. [$F \propto 1/\mu$, equation (1), p. 400.]

This weakening or demagnetizing effect clearly depends upon the pole strength and also decreases as the distance between the poles increases, so that in the case of a long thin rod its value is very small. For this reason it is necessary, in certain methods of magnetic testing, to use specimens in the form of long thin rods or wires.

The strength of an isolated bar magnet tends to decrease owing to the demagnetizing effect of its own poles. Such magnets, when not in use, are usually arranged side by side in pairs with opposite poles adjacent (fig. 1*b*), their ends being connected by a soft iron "keeper"; a similar keeper is placed across the poles of a horseshoe magnet. Poles of opposite polarity are induced in adjacent portions of the keeper, the effect of which almost neutralizes that of the magnet poles.

Self-demagnetization occurs only when there are free poles. In the above case, by providing keepers, these have been very nearly eliminated, and the flux lies almost entirely inside the iron. As has been seen (§ 18, p. 165), when it is necessary to calculate exactly the magnetic field inside the iron, a ring magnet is often used, in which the flux is entirely inside the iron so that there are no free poles and therefore no demagnetizing effects.

The Absolute Unit of Current

Ampère's researches showed that the magnetizing force produced at any point by a current element can be expressed in the form

$$dH \propto \frac{I \, . \, dl}{d^2} \sin \theta$$

where d is the distance of the element from the point in question, and θ is the angle between the direction of the current and the line joining the element to the point (see p. 158).

If a conductor is bent into a circular arc, each element is the same distance from the centre and perpendicular to the radius passing through it, i.e. $\theta = 90°$, so that every element produces the same magnetic effect at the centre, and

$$H \propto \frac{Il}{d^2} \quad (\sin \theta = 1).$$

This magnetically simple arrangement is used in order to define unit current.

Considering a conductor of unit length (1 cm.) forming part of a circle of unit radius (1 cm.), unit current flows in this conductor when unit force (1 dyne) is exerted on a unit pole placed at the centre of the circle, i.e. when unit field strength (1 oersted) is produced at the centre.

From this definition it follows that, in the general case, if a conductor of length l cm., forming part of a circle of radius d cm., carries a current of I_a C.G.S. units, the field strength at the centre is:—

$$H = \frac{I_a l}{d^2} \text{ oersteds.}$$

Ampère's law now becomes,

$$dH = I_a dl \frac{\sin \theta}{d^2} \text{ oersteds.}$$

At a time when the chief source of electrical energy was the primary cell, the unit of current, as defined above, was felt to be inconveniently large, and for general use a unit having one-tenth of this value was chosen which was called the *ampere.*

1 ampere = 10^{-1} electromagnetic C.G.S. unit of current.

Hence when the current is measured in amperes, Ampère's law becomes

$$dH = \frac{I}{10} dl \frac{\sin \theta}{d^2} \text{ oersteds.} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Field Strength at Centre of a Circular Loop or Short Coil

It follows from the above that if a current I amperes flows in a circular loop of unit radius, the field strength at the centre is

$$H = 2\pi \frac{I}{10} \text{ oersteds} \quad (\text{since } l = 2\pi \text{ cm.}). \quad . \quad . \quad (6)$$

Now if the radius is increased to r cm., the magnetic effect of each element at the centre will be reduced to $1/r^2$ of its previous value, but there will be r times as many elements, so that at the centre

$$H = \frac{2\pi}{10} \frac{I}{r^2} r = \frac{2\pi}{10} \frac{I}{r} \text{ oersteds,}$$

and if the circular loop is replaced by a short coil of N turns,

$$H = \frac{2\pi}{10} \frac{NI}{r} \text{ oersteds.} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Field Strength on Axis of Short Coil

It will be seen that equation (7) above can be obtained from the corresponding M.K.S. relation (equation. 11, p. 160) by multiplying by $4\pi/10$.

Similarly, the field strength at any point on the axis of a short coil is obtained by multiplying equation (12), p. 162, by $4\pi/10$ which gives

$$H = \frac{2\pi}{10} NI \frac{r}{x^2} \sin \theta \text{ oersteds,} \quad . \quad . \quad . \quad . \quad (8)$$

r and x being measured in cm.

Field Strength at Midpoint of Long Solenoid

Referring to p. 162 and replacing $dH = \frac{1}{2}n\,dl\,I\,\frac{r}{x^2}\sin\theta$ by $dH = \frac{2\pi}{10}n\,dl\,I\,\frac{r}{x^2}\sin\theta$, it follows from equations (12*a*) and 13, p. 163, that at the midpoint of a very long solenoid,

$$H = \frac{4\pi}{10}\frac{NI}{l} \text{ oersteds,} \qquad \qquad (9)$$

l being measured in cm.

1 oersted = 79·6 ampere-turns per metre.

The value of H at any other point on the axis is obtained as on p. 163.

The total flux passing through the solenoid at the midpoint is

$$\Phi = Ba = \mu Ha = \frac{4\pi}{10}\frac{NI}{l}\mu a \text{ maxwells or C.G.S. lines.} \qquad (10)$$

Field Strength in Neighbourhood of Long Straight Conductor

Referring to pp. 159 and 160 and substituting $dH = \frac{I}{10}\frac{dl\sin\theta}{x^2}$ for $dH = \frac{1}{4\pi}\frac{I\,dl\sin\theta}{x^2}$, i.e. multiplying by $\frac{4\pi}{10}$,

$$H = \frac{2}{10}\frac{I}{r} \text{ oersteds,} \qquad \qquad (11)$$

r being measured in cm.

The corresponding flux density is

$$B = \mu H = \frac{2}{10}\frac{I}{r}\mu \text{ gauss (or lines per sq. cm.).} \qquad (12)$$

Force on a Current-carrying Conductor lying in a Magnetic Field

It follows from the definition of unit current that if a conductor l cm. long forming an arc of a circle of 1 cm. radius carries a current I_a C.G.S. units, the force acting on a unit pole placed at the centre of the circle is

$$F = I_a l \text{ dynes.}$$

There is, therefore, an equal and opposite reaction force on the conductor, and, by definition, each element of conductor is lying in and perpendicular to a field of density 1 gauss (or 1 line per sq. cm.).

The essential conditions are the constancy of the field strength and the perpendicular relationship between the directions of current and flux; and these are fulfilled if a *straight* conductor lies in and *perpendicular to* a *uniform* field. Hence,

A conductor of unit length (1 cm.) lying in and perpendicular to a field of unit strength (1 gauss) and carrying unit current (10 amperes), experiences unit force (1 dyne) in a direction perpendicular to that of both current and flux.

Therefore in the general case

$$F = BlI_a = Bl\frac{I}{10} \text{ dynes,} \qquad . \quad . \quad . \quad . \quad (13)$$

where $B =$ flux density (lines per sq. cm.),

$l =$ length (cm.),

$I =$ current (amperes).

Work done by a Conductor carrying a Current and moving through a Field

Referring to p. 148 it follows that if a conductor carrying a current I_a C.G.S. units in moving through a magnetic field cuts a total flux of Φ maxwells, the total work done is

$$I_a\Phi \text{ ergs} = \frac{I}{10}\Phi \text{ ergs.} \qquad . \quad . \quad . \quad . \quad . \quad . \quad (13a)$$

The Magnetomotive Force Theorem

It has been seen (p. 231) how the solution of magnetic problems—particularly those concerned with electrical machines—is simplified by the concept of the magnetic circuit.

In the electric circuit the E.M.F. is measured in terms of the energy (in ergs) received by each unit quantity of electricity as it passes the seat of the E.M.F., and later given out as it passes round the circuit. It is therefore equal to the work done by the unit quantity in passing round the circuit, and its value is independent of the length and shape of the circuit.

Similarly the analogous quantity known as the *magnetomotive force* (F) is measured by the work done (in ergs) in taking " unit quantity of magnetism ", i.e. a unit pole, once round a magnetic path linked with the electric circuit; and its value is independent of the shape or length of the path, provided it is closed.

Fig. 2 shows a single turn in which the current is I_a C.G.S. units. If a unit pole is taken once round any path linked with the turn, each of the 4π lines associated with the pole cuts some part of the circuit once. Hence from equation (13*a*)

$$\text{Work done} = 4\pi I_a = 4\pi \frac{I}{10} \text{ ergs.}$$

If the direction of motion is the same as that of the magnetizing force, work is done by the pole: if in the opposite direction an equal amount of work is done on the pole. If there are N turns,

$$\text{Work done} = \frac{4\pi}{10} NI \text{ ergs,}$$

which, by definition, is a measure of the magnetomotive force.

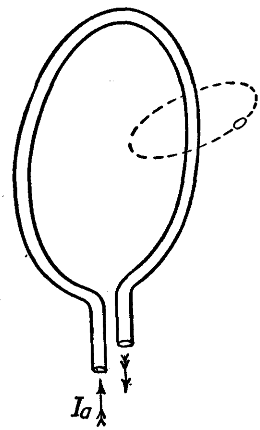


Fig. 2.—Magnetomotive force

Hence

The total M.M.F. acting on any magnetic path is

$$\frac{4\pi}{10} \text{ (ampere-turns linked with the path).}$$

The unit of M.M.F. is the *gilbert* so that

$$F = \frac{4\pi}{10} NI \text{ gilberts.}$$

$$1 \text{ gilbert} = 0{\cdot}795 \text{ ampere-turn.}$$

Note that the measurement in the M.K.S. system of M.M.F. directly in ampere-turns, although taken almost as fundamental, is quite consistent with the definition given above. It is in fact measured by the work done in joules, in taking "unit quantity of magnetism"—in this case one weber—once round any closed path linked with the circuit. If the circuit has N turns and carries a current of I amperes,

$$\begin{aligned}\text{Work done} &= \text{(total current)} \times \text{(flux cut)}\\ &= NI \times 1 = NI \text{ joules.}\end{aligned}$$

Hence, by definition the M.M.F. is equal to the ampere-turns.

Since the magnetizing force is measured by the force in dynes acting on a unit pole, the work done in taking the pole once round the circuit can be expressed as $\int H\,dl$, i.e.

$$F = \int H\,dl \text{ gilberts.}$$

The application of the M.M.F. theorem enables the value of the magnetizing force to be obtained very simply in those cases in which for reasons of symmetry it can be assumed to have a constant value throughout the path. For example, a long straight conductor carrying a current I (since it forms part of one turn, however distant the return conductor may be) produces I ampere-turns, so that the M.M.F. is

$$F = \frac{4\pi}{10} I \text{ gilberts.}$$

If the magnetizing force at a distance r cm. from the conductor is H oersteds, the force acting on a unit pole is H dynes and the work done in carrying it round a circular path of radius r (for which H has a constant value) is $2\pi rH$ ergs.
Hence

$$2\pi rH = \frac{4\pi}{10} I$$

or

$$H = \frac{2}{10}\frac{I}{r} \text{ oersteds,}$$

which was the result obtained on p. 405.

Magnetizing Force due to Toroidal Coil

The magnetizing force due to a toroidal coil is obtained very simply by means of the M.M.F. theorem. Considerations of symmetry show that the value of H is constant along any concentric path inside the coil. Hence at any radius r (fig. 26, p. 165)

$$\text{M.M.F.} = F = 2\pi rH.$$

$$\therefore\ H = \frac{F}{2\pi r} = \frac{\frac{4\pi}{10} NI}{2\pi r} = \frac{2}{10}\frac{NI}{r} \text{ oersteds.}$$

It is, however, more usually expressed in the form,

$$H = \frac{4\pi}{10}\frac{NI}{l} \text{ oersteds,} \quad \ldots\ldots \quad (14)$$

where $l = 2\pi r$ and is measured in cm.

The mean value of H is given when $l = 2\pi R$ where R is the mean radius.

When the core is non-magnetic the mean flux density is

$$B = \frac{4\pi}{10}\frac{NI}{l}\mu \text{ gauss (where } \mu = 1), \quad . \quad . \quad . \quad (15)$$

and the total flux is

$$\Phi = \frac{4\pi}{10}\frac{NI}{l}\mu a \text{ maxwells (lines).} \quad . \quad . \quad . \quad (16)$$

This expression is very nearly true for a magnetic core if the radial thickness is small compared with the mean radius (see p. 166).

Note that equation (16) can be written in the form

$$\Phi = \frac{\frac{4\pi}{10}NI}{\frac{1}{\mu}\frac{l}{a}}$$

or

$$\text{flux} = \frac{\text{magnetomotive force}}{\text{reluctance}},$$

which is the general law of the magnetic circuit.

Energy stored in a Magnetic Field

Following the reasoning on p. 182 and substituting $H = 4\pi\frac{Ni_a}{l}$ oersteds for $H = \frac{Ni}{l}$ ampere-turns per metre,

$$i_a = \frac{1}{4\pi}\frac{Hl}{N} \text{ C.G.S. units} \quad \text{and} \quad e = Na\frac{dB}{dt} \text{ C.G.S. units,}$$

B being measured in gauss, l in cm. and a in sq. cm.

Energy stored in a time $dt = p\,dt = ei_a\,dt$

$$= Na\,dB\,\frac{1}{4\pi}\frac{Hl}{N}$$

$$= \frac{1}{4\pi}al\,HdB \text{ ergs,}$$

and since al is the volume of the magnetic circuit in cubic centimetres, in establishing a flux density of B gauss,

$$\text{Energy stored in ergs per c.c.} = \frac{1}{4\pi}\int_0^B H\,dB. \quad . \quad (17)$$

In an air gap or other non-magnetic part of the circuit, since $\mu = 1$, B is numerically equal to H, so that

Energy stored in an air gap, in ergs per c.c.,

$$= \frac{1}{4\pi}\int_0^B B\,dB = \frac{B^2}{8\pi}. \quad . \quad . \quad . \quad . \quad . \quad (18)$$

In that portion of the circuit which consists of magnetic materials, the integral can only be evaluated graphically by measurement of the area under the B-H curve. If the scale of the curve is such that 1 cm. $= p$ gauss and 1 cm. $= q$ oersteds,

Energy stored in ergs per c.c.

$$= \frac{1}{4\pi}\text{ (area under } B\text{-}H \text{ curve in cm.}^2) \times p \times q. \quad . \quad . \quad (19)$$

Force of Attraction between Magnetized Surfaces

Following the reasoning of p. 184, if the lower pole is moved down through a distance dx against an attractive force of F dynes,

$$\text{Work done} = F\,dx \text{ ergs.}$$

Increase in the energy stored in the gap

$$= \frac{B^2}{8\pi}\,ab\,dx \text{ ergs.}$$

The only source of this energy is the work done in moving the pole. Hence

$$F\,dx = \frac{B^2}{8\pi}\,ab\,dx,$$

$$F = \frac{B^2}{8\pi}\,ab \text{ dynes.}$$

$$f = \frac{F}{ab} = \frac{B^2}{8\pi} \text{ dynes per sq. cm.} \quad . \quad . \quad . \quad . \quad (20)$$

Comparative Magnetic Calculations

In order to illustrate the relation between the two systems the magnetic calculations below are worked in both C.G.S. and M.K.S. units.

Example 1.—The figure shows an electromagnet of cast steel with an armature of mild steel, separated by brass distance pieces. The dimensions and the mean lengths of magnetic path are shown in cm. Find the number of ampere-turns required to set up a flux of 1 milliweber (100,000 C.G.S. lines). Leakage can be neglected.

At the flux density required the value of H for cast steel, obtained from the appropriate B-H curve, is 400 ampere-turns per metre (5 oersteds) and the relative permeability of the mild steel is 620.

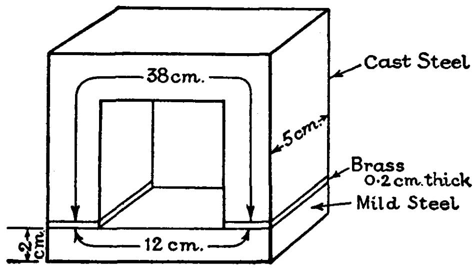


Fig. 3

M.K.S. units	*C.G.S. units*
Magnet	
Flux density (B) $= \dfrac{10^{-3}}{12\cdot5 \times 10^{-4}}$ $= 0\cdot8$ Wb. per sq. m.	Flux density (B) $= \dfrac{100{,}000}{12\cdot5}$ $= 8000$ gauss.
Magnetizing force (H) $= 400$ AT per m.	Magnetizing force (H) $= 5$ oersteds.
Length of path $= 0\cdot38$ m.	Length of path $= 38$ cm.
Magnetomotive force (F) $= 400 \times 0\cdot38$ $= 152$ ampere-turns.	Magnetomotive force (F) $= 5 \times 38 = 190$ gilberts. $= 190 \times \dfrac{10}{4\pi} = 152$ ampere-turns.
Air Gaps	
$B = 0\cdot8$ Wb. per sq. m.	$B = 8000$ gauss.
$H = \dfrac{B}{\mu_0} = \dfrac{0\cdot8}{4\pi \times 10^{-7}}$ $= 637{,}000$ AT per m.	$H = \dfrac{B}{\mu} = 8000$ oersteds.
$l = 0\cdot4 \times 10^{-2}$ m.	$l = 0\cdot4$ cm.
$F = Hl = 637{,}000 \times 0\cdot4 \times 10^{-2}$ $= 2548$ ampere-turns.	$F = Hl = 8000 \times 0\cdot4 = 3200$ gilberts $= 3200 \times \dfrac{10}{4\pi} = 2548$ ampere-turns.
Armature	
$B = \dfrac{10^{-3}}{10 \times 10^{-4}} = 1$ Wb. per sq. m.	$B = \dfrac{100{,}000}{10} = 10{,}000$ gauss.
$H = \dfrac{B}{\mu_r\mu_0} = \dfrac{1}{620 \times 4\pi \times 10^{-7}}$ $= 1280$ AT per m.	$H = \dfrac{B}{\mu} = \dfrac{10{,}000}{620}$ $= 16\cdot14$ oersteds.
$l = 0\cdot12$ m.	$l = 12$ cm.
$F = 1280 \times 0\cdot12$ $= 154$ ampere-turns.	$F = 16\cdot1 \times 12 = 193$ gilberts $= 193 \times \dfrac{10}{4\pi} = 154$ ampere-turns.
Total ampere-turns $= 2854$.	Total ampere-turns $= 2854$.

Example 2.—Find the total attractive force exerted by the magnet on the armature in the previous example.

M.K.S. units	*C.G.S. units*
At contact surface,	At contact surface,
$B = 0{\cdot}8$ Wb. per sq. m.	$B = 8000$ gauss.
$f = \frac{1}{2} \cdot \frac{B^2}{\mu_0} = \frac{1}{2} \cdot \frac{0{\cdot}8^2}{4\pi \times 10^{-7}}$	$f = \frac{B^2}{8\pi} = \frac{8000^2}{8\pi}$
$= 254{,}800$ newtons per sq. m.	$= 2{\cdot}548 \times 10^6$ dynes per sq. cm.
	$a = 2 \times 12{\cdot}5$ sq. cm.
$a = 2 \times 12{\cdot}5 \times 10^{-4}$ sq. m.	$F = 25 \times 2{\cdot}548 \times 10^6$ dynes
$F = 254{,}800 \times 25 \times 10^{-4}$	$= 63{\cdot}7 \times 10^6$ dynes
$= 637$ newtons (143·4 lb.).	$= 637$ newtons (143·4 lb.).

The Electrostatic Unit of Charge

The electrostatic C.G.S. system of units is based on the concept of *unit charge.*

Coulomb's researches showed that,

The force between two small charged spheres is proportional to the product of the charges, inversely proportional to (the distance between them)^2, and also depends on a property of the surrounding medium, so that

$$F \propto \frac{q_1 q_2}{\varepsilon d^2}$$

where q_1, q_2 are the charges, in units yet to be defined, d is the distance between them, and ε is a constant depending on the nature of the medium, and called the *permittivity* (formerly also called the *specific inductive capacity* or the *dielectric constant*).

This constant, in the case of free space, was arbitrarily assigned the value unity, and unit charge was then defined by giving unit values to the other terms in the above expression, so that,

A unit charge repels a similar unit charge placed at unit distance (1 cm.) from it, in a vacuum, with unit force (1 dyne).

The value of ε for air is so near unity (1·006) that it follows that, in general, in a vacuum or in air,

$$F = \frac{q_1 q_2}{\varepsilon d^2} \text{ dynes,} \qquad \qquad (21)$$

where q_1 and q_2 are measured in the unit defined above and d is in cm.

This unit of quantity is very much smaller than the electromagnetic unit of quantity.

1 e.m. unit $= 3 \times 10^{10}$ e.s. units of charge or quantity.

1 coulomb $= 3 \times 10^9$ e.s. units of charge or quantity.

$$1 \text{ e.s. unit} = \frac{1}{3 \times 10^9} = 0{\cdot}33 \times 10^{-9} \text{ cmb.}$$

Electric Field Strength—Electric Force

The strength and direction of an electric field at any point is measured in terms of the value and direction of the force acting on a unit positive charge placed at that point.

Unit field strength is that of a field in which a unit charge experiences unit force (1 dyne), and is represented by one line of electric flux drawn through each square centimetre of area perpendicular to the force.

It is, of course, assumed that the introduction of the charge does not upset the original field distribution.

Flux due to Unit Charge (Ψ)

In classical theory the electric flux was assumed to be numerically equal to the charge, and this is the assumption made in the M.K.S. system. But many writers, in order to preserve the parallel between electrostatic and electromagnetic concepts, have treated the unit charge on the same lines as a unit pole.

Adopting this treatment, since a unit charge placed at any point on a sphere of unit radius, having another unit charge at its centre, experiences unit force in a radial direction, the field at any point on the surface is of unit value and is therefore represented by a flux of one line per square centimetre. Hence the total flux associated with unit charge is 4π lines, and in the general case the flux due to a charge of q units is

$$\Psi = 4\pi q \text{ lines.}$$

Difference of Potential

Unit P.D. exists between two points when unit work (1 erg) is done in transferring unit charge from one to the other.

Hence, in general terms

$$V = \frac{W}{Q}.$$

In the M.K.S. system, when the work done in transferring one coulomb is one joule, the P.D. is one volt.

Since one e.s. unit of charge $= 0{\cdot}33 \times 10^{-9}$ cmb.

and one erg $= 10^{-7}$ joule,

$$\text{one e.s. unit of P.D.} = \frac{10^{-7}}{0{\cdot}33 \times 10^{-9}} = 300 \text{ volts.}$$

Capacitance

Since the e.s. unit of quantity $= 0{\cdot}33 \times 10^{-9}$ cmb.

and the e.s. unit of P.D. $= 300$ volts,

the e.s. unit of capacitance $\left(C = \frac{Q}{V}\right) = \frac{0{\cdot}33 \times 10^{-9}}{300}$

$= 1{\cdot}1 \times 10^{-12}$ farad:

or 1 farad $= 9 \times 10^{11}$ e.s. units of capacitance,

1 microfarad $= 9 \times 10^{5}$ e.s. units of capacitance.

Capacitance of Parallel-plate Capacitor.

Following the general lines of § 21, p. 369, since the field between the plates (except at the edges) is uniform, the flux density is

$$D = \frac{4\pi q}{a},$$

where q is the charge in e.s. units and a is the area of each plate in square centimetres. The electric force has, therefore, a constant value

$$E = \frac{D}{\varepsilon} = \frac{4\pi q}{\varepsilon a} \text{ e.s. units.}$$

Hence, the P.D. between the plates is

$$V = Ed = \frac{4\pi q d}{\varepsilon a} \text{ e.s. units,}$$

where d is the distance between the plates in cm., and the capacitance is therefore

$$C = \frac{q}{V} = \frac{1}{4\pi} \varepsilon \frac{a}{d} \text{ e.s. units} \quad . \; . \; . \; . \; . \; . \; . \quad (22)$$

$$= \frac{1}{9 \times 10^{5}} \frac{1}{4\pi} \varepsilon \frac{a}{d} \text{ microfarads.} \quad . \; . \; . \quad (23)$$

Capacitance between Two Long Concentric Cylinders (single-core cable).

Following the general lines of § 21, p. 370, the distribution of the charge is uniform except just near the ends, and the electric flux passes radially between the cylinders.

Hence, if the charge is q e.s. units per cm., the flux density at a distance x cm. from the centre is

$$D = \frac{4\pi q}{2\pi x} = \frac{2q}{x}$$

and the electric force is $E = \dfrac{D}{\varepsilon} = \dfrac{2q}{\varepsilon x}$ e.s. units. . . . . (24)

The P.D. between the inner and outer cylinders is therefore

$$V = \int_r^R E\,dx = \frac{2q}{\varepsilon}\int_r^R \frac{1}{x}\,dx = \frac{2q}{\varepsilon}\log_e\frac{R}{r} \text{ e.s. units,}$$

and the capacitance is

$$C = \frac{q}{V} = \frac{\varepsilon}{2\log_e\dfrac{R}{r}} \text{ e.s. units per cm.} \quad . \quad . \quad . \quad (25)$$

$$= \frac{2{\cdot}54 \times 12 \times 5280}{9 \times 10^5 \times 2{\cdot}3}\;\frac{\varepsilon}{2\log_{10}\dfrac{R}{r}}$$

$$= \frac{0{\cdot}0388}{\log_{10}\dfrac{R}{r}} \text{ microfarads per mile.} \quad . \quad . \quad (26)$$

Capacitance between Parallel Conductors.

Following the lines of § 21, p. 371, if the spacing of the conductors is large compared with their diameter, as in the case of overhead lines, and the charge on each conductor is q e.s. units per cm., then at a point P on the line joining the centres of the conductors, distant x cm. from A (from equation. 24),

$$\text{Electric force due to } A = E_A = \frac{2q}{\varepsilon x} \text{ e.s. units,}$$

$$\text{Electric force due to } B = E_B = \frac{2q}{\varepsilon(D - x)} \text{ e.s. units.}$$

Resultant electric force at P is

$$E = E_A + E_B = \frac{2q}{\varepsilon}\left(\frac{1}{x} + \frac{1}{D - x}\right)$$

and therefore

$$V = \int_r^{D-r} E\,dx = \frac{2q}{\varepsilon}\int_r^{D-r}\left(\frac{1}{x} + \frac{1}{D - x}\right)dx$$

$$= \frac{2q}{\varepsilon}\Big[\log_e x - \log_e (D-x)\Big]_r^{D-r}$$

$$= \frac{4q}{\varepsilon}\log_e\frac{D - r}{r} \text{ e.s. units.}$$

Therefore

$$C = \frac{q}{V} = \frac{\varepsilon}{4 \log_e \dfrac{D - r}{r}}$$

$$= \frac{\varepsilon}{4 \log_e \dfrac{D}{r}} \text{ e.s. units per cm.,} \quad . \quad . \quad (27)$$

since r is small compared with D.

Converting to common logarithms,

$$C = \frac{5280 \times 12 \times 2{\cdot}54}{9 \times 10^5 \times 2{\cdot}3} \; \frac{\varepsilon}{4 \log_{10} \dfrac{D}{r}}$$

(for air $\varepsilon = 1$) $$= \frac{0{\cdot}0194}{\log_{10} \dfrac{D}{r}} \text{ microfarads per mile.} \quad . \quad . \quad . \quad (28)$$

RATIONALIZATION

The (unrationalized) M.K.S. system, as originally suggested by Giorgi (in Italy) and David Robertson (in England), in which the permeability of free space (and of air very nearly) was assigned the value of 10^{-7}, includes all the so-called Practical Units, the ampere, volt, ohm, etc., which therefore become absolute units in their own right instead of multiples or sub-multiples of the C.G.S. units. The adoption of this system has no effect on ordinary circuit calculations, although it produces changes in magnetic expressions, among which is the appearance of a new unit of flux—the weber.

Since the ampere is now an absolute unit, the flux density at the centre of a long air-cored solenoid is

$$B = 4\pi \frac{NI}{l} \mu = 4\pi \frac{NI}{l} 10^{-7} \text{ Wb. per sq. m.}$$

If, however, the permeability of free space is assigned a value $\mu_0 = 4\pi \times 10^{-7}$, this expression becomes

$$B = \frac{NI}{l} \mu_0 \text{ Wb. per sq. m.}$$

and

$$H = \frac{B}{\mu_0} = \frac{NI}{l} \text{ ampere-turns per m.,}$$

i.e. magnetizing force is measured in ampere-turns per metre and magnetomotive force in ampere-turns, as has been assumed in this book.

If, in addition, the permittivity of free space is given the corresponding value

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} = \frac{1}{4\pi \times 10^{-7}\, c^2},$$

the electromagnetic and electrostatic systems are merged into one system, so that the unit of P.D., for example, has the same value (1 volt) whether obtained from electromagnetic or electrostatic relationships.

At the same time the factor π (which cannot be *entirely* eliminated) is suppressed in those expressions relating to rectilinear systems, at the expense of its appearance in other expressions where, since they relate to circular or spherical systems, its presence might be expected.

This process is called *rationalization*, and it is the *rationalized* M.K.S. system which has been adopted internationally and is normally implied when M.K.S. units are stated.

Corresponding expressions in the C.G.S. and the rationalized M.K.S. systems are given below for a number of electromagnetic and electrostatic relationships.

	C.G.S.	M.K.S.
a. Flux density due to long straight conductor	$\frac{2I}{10r}\mu$ gauss	$\frac{I}{2\pi r}\mu$ Wb. per sq. m.
b. Flux density at centre of circular loop	$\frac{2\pi}{10}\frac{I}{r}\mu$ gauss	$\frac{I}{2r}\mu$ Wb. per sq. m.
c. Flux density on axis of long coil	$\frac{4\pi}{10}\frac{NI}{l}\mu$ gauss	$\frac{NI}{l}\mu$ Wb. per sq. m.
d. Capacitance of parallel-plate capacitor	$\frac{1}{4\pi}\varepsilon\frac{a}{d}$ e.s. units	$\varepsilon\frac{a}{d}$ farads
e. Capacitance of single-core cable	$\frac{\varepsilon}{2\log_e\frac{R}{r}}$ e.s. units per cm.	$\frac{2\pi\varepsilon}{\log_e\frac{R}{r}}$ farads per m.
f. Capacitance between parallel conductors	$\frac{\varepsilon}{4\log_e\frac{D}{r}}$ e.s. units per cm.	$\frac{\pi\varepsilon}{\log_e\frac{D}{r}}$ farads per m.

Note that in the M.K.S. expressions, $\mu = \mu_r\mu_0$ and $\varepsilon = \varepsilon_r\varepsilon_0$.

On comparing these expressions it will be noticed that, (1) in each pair one contains the factor π and the other does not, (2) in the C.G.S. expressions π occurs in expressions connected with rectilinear fields, e.g. in (*b*) and (*c*) in which, at any point on the axis the magnetizing force is directed along the axis, and in (*d*) in which the electric force between the plates is uniform and perpendicular to them. Since π is normally connected with a circle or sphere, it seems to be out of place in these expressions.

On the other hand, π is absent from the corresponding M.K.S. expressions, although it occurs in cases (*c*), (*e*) and (*f*). But in these the field is circular, so that the presence of π seems more natural.

Solutions to Examples

CHAPTER VI (p. 87)

1. (*a*) 6·67 ohms.; (*b*) 1·5 kW.; (*c*) 18,000 watt-hours.

2. Input $= 500 \times 15 = 7500$ watts.

   Output $= 7500 \times \dfrac{90}{100} = 6750$ watts $= \dfrac{6750}{746} = 9{\cdot}04$ h.p.

3. (*a*) Input $= 100 \times \dfrac{100}{92} \times \dfrac{746}{1000} = 81{\cdot}1$ kW.

   (*b*) Energy input in one hour = 81·1 kW.-hr. = 81·1 B.O.T. units.
   Cost = 3*s*. 4½*d*.

4. Total power input $= (20 \times 100) + (40 \times 60) + (30 \times 40)$ watts.
   $= 5600$ watts.

   Weekly energy consumption $= \dfrac{5600}{1000} \times 3 \times 7 = 117{\cdot}6$ kW.-hr. (units).

5. $I = \dfrac{5{\cdot}5}{20} = 0{\cdot}275$ ampere.

   $r = \dfrac{E - V}{I} = \dfrac{6 - 5{\cdot}5}{0{\cdot}275} = 1{\cdot}82$ ohm.

6. P.D. at lamp $= \dfrac{2}{0{\cdot}75} = 2{\cdot}67$ volts = battery P.D.

   $r = \dfrac{E - V}{I} = \dfrac{3 - 2{\cdot}67}{0{\cdot}75} = 0{\cdot}44$ ohm.

   $\dfrac{\text{Energy input to lamp}}{\text{Energy output of battery}} = \dfrac{2{\cdot}67}{3} = 0{\cdot}89$, i.e. 89 per cent.

7. P.D. at motor terminals = 92·5 volts.
   Input to motor $= 92{\cdot}5 \times 50 = 4625$ watts.
   Output of motor = 3930 watts = 5·27 h.p.

8. (*a*) 45 ohms.; (*b*) 4·62 ohms.

9. $I_1 = \dfrac{1{\cdot}35}{100} = 0{\cdot}0135$ ampere, $I_2 = \dfrac{0{\cdot}93}{\dfrac{1}{\dfrac{1}{10} + \dfrac{1}{100}}} = 0{\cdot}102$ ampere.

   $V_1 = E - I_1 r.$
   $V_2 = E - I_2 r.$ Hence $r = 4{\cdot}75$ ohms, $E = 1{\cdot}415$ volt.

10. When current in 2-ohm resistance is 1 ampere,
P.D. across group $= 2 \times 1 = 2$ volts.
Hence total current $= \frac{2}{1} + \frac{2}{2} + \frac{2}{3} = 3{\cdot}67$ amperes.
Battery P.D. $= 20 - (3{\cdot}67 \times 1) = 16{\cdot}33$ volts.
$\therefore$ P.D. across variable resistance $= 16{\cdot}33 - 2 = 14{\cdot}33$ volts.
$\therefore\ R = \frac{14{\cdot}33}{3{\cdot}67} = 3{\cdot}91$ ohms.

11. Voltmeter current may be neglected.
(a) 0·74 volt; (b) the E.M.F., i.e. 1·08 volt.

12. (a) 0·086 ampere; (b) 0·15 ampere.

13. 0·87 pence.

14. 4·18 amperes.

15. In the mesh ABD $10I_1 + 10i - 20I_2 = 0$.
In the mesh BCD $15(I_1 - i) - 25(I_2 + i) - 10i = 0$,
from which $i = \frac{I_2}{13}$.
Also in ADC, $20I_2 + 25(I_2 + i) = 20$,
and substituting value of i above,
$I_2 = 0{\cdot}425$ amps, $I_1 = 0{\cdot}817$ amps, $i = 0{\cdot}033$ amps.
Hence, $I_{AB} = 0{\cdot}817$, $I_{BC} = 0{\cdot}784$, $I_{AD} = 0{\cdot}425$,
$I_{DC} = 0{\cdot}458$ and $I_{BC} = 0{\cdot}003$ amps.

16. 9 micro-amperes.

17. 12,375 ohms. 83·5 per cent.

18. $200 = 5(I_A + I_B) + 80 + 0{\cdot}2I_A = 5(I_A + I_B) + 90 + 0{\cdot}22I_B$.
$I_A = 35{\cdot}64$ amps, $I_B = -13{\cdot}05$ amps, $I_T = 22{\cdot}6$ amps.
Power in resistor = 2550 watts.

19. Each coulomb dissolves $3{\cdot}37 \times 10^{-4}$ gm. of zinc from *each* cell.
Time = 15·12 hours; 147 : 1.

CHAPTER VII (p. 116)

1. Assuming that the 100-ohm coil (R_s) is at left-hand end:
(a) $R_x = 100 \times \frac{25}{75} = 33{\cdot}33$ ohms;
(b) $R_s = 50$, $\frac{l_a}{l_b} = \frac{R_s}{R_x}$. $\therefore\ l_a = 60$ cm.

2. To balance the bridge: $R_s = 501 + \frac{6}{10} = 501{\cdot}6$ ohms.
$$\therefore\ R_x = 501{\cdot}6 \times \frac{10}{100} = 50{\cdot}16 \text{ ohms.}$$

3. 1st determination: $R_x = 50 \times \dfrac{11{\cdot}78}{38{\cdot}22} = 15{\cdot}41$ ohms.

2nd determination: $R_x = 50 \times \dfrac{11{\cdot}68}{38{\cdot}32} = 15{\cdot}24$ ohms.

Mean value of $R_x = 15{\cdot}33$ ohms.

4. $R = \dfrac{\rho l}{a} = \dfrac{0{\cdot}68 \times 10^{-6} \times 1500 \times 6\pi}{0{\cdot}7854 \times (0{\cdot}028)^2} = 31{\cdot}2$ ohms.

5. (a) $R = 20$ ohms.

(b) $l = \dfrac{a}{\rho} R = 496$ cm.

6. 0·093 cm.

7. 2690 ohms.

8. $\rho_{40} = 1{\cdot}31 \times 10^{-6}$ ohm-in. Hence $R = 9{\cdot}77$ ohms.

9. $R_{20} = 500$ ohms, $R_t = 589$ ohms. Hence $t = 64{\cdot}9^\circ$ C.

10. Copper 52·5 per cent; iron 47·5 per cent.

11. 83·7 amperes.

12. Electrical input to coil $= 2 \times 5 \times 10 \times 60 = 6000$ joules.
Heat output to water $= 100 \times 14{\cdot}2 = 1420$ calories.

Joule's equivalent $= \dfrac{6000}{1420} = 4{\cdot}22$ joules per calorie.

13. Electrical energy input $= 50I^2 \times 600$ joules.

Heat output $= 1000 \times 80 \times \dfrac{100}{80} \times 4{\cdot}18 = 418{,}000$ joules.

Hence $I = 3{\cdot}73$ amperes.

14. 3·35 ohms.

15. 79 per cent.

16. 1732 cm.

17. Total length of cable = 1000 yd. (a) 222 volts; (b) 3·6 per cent.

18. Input to motor = 41·4 kW. Line loss = 4140 watts. Section = 0·237 sq. in.

19. 39·1 volts.

20. 0·0304 ohms; 72·8 calories per second.

CHAPTER VIII (p. 136)

1. 4·22 amperes.

2. 0·001118 gm. per cmb.

3. 50·97 gm.

4. 100 gm. of water contain $\dfrac{100}{9} = 11{\cdot}11$ gm. of hydrogen.

$$\therefore\ Q = \frac{11{\cdot}11}{0{\cdot}0000104} = 1{,}070{,}000 \text{ cmb.};\quad I = 14{\cdot}87 \text{ amperes.}$$

5. 0·275 ampere, 10 per cent low; 0·507 ampere, 1·4 per cent low; 0·735 ampere, 2 per cent high; 0·997 ampere, 0·3 per cent high.

6. 1·858 ampere, i.e. 7·1 per cent high.

7. (*a*) 20 amperes; (*b*) 2·3 per cent.

8. 384 to 504 volts.

9. 0·001 ohm.

10. 4·1 ohms, 17·4 amperes.

CHAPTER IX (p. 186)

1. (*a*) 0·01 volt; (*b*) 0·2 volt.

2. 20 volts.

3. 0·0667 weber per sec.

4. 0·432 volt.

5. 125 AT/m.

6. (*a*) 100; (*b*) 2500 AT/m.

7. $H = 10{,}000$ AT/m, $\Phi = 63{\cdot}3 \times 10^{-6}$ weber.

8. 3000 AT/m.

9. 188·5 amperes.

10. 5 joules.

11. At a distance of 1 in. from one conductor

$$B = \frac{5000}{2 \times 0{\cdot}0254\pi} \times \frac{4\pi}{10^7} = 0{\cdot}0394 \text{ Wb/m}^2.$$

Force on second conductor lying in the field

$$= 0{\cdot}0394 \times 12 \times 0{\cdot}0254 \times 5000 \times 0{\cdot}225$$
$$= 13{\cdot}5 \text{ lb. per ft. (repulsion).}$$

12. Since diameter of coil is large, each element of length can be assumed to be part of a straight conductor. Force = 1·02 kg. per cm. (attraction).

13. 3980 AT/m.

14. 1·44 Wb/m^2, 288×10^{-6} Wb.

15. 0·0015 henry.

16. 0·00135 henry.

17. 125 volts.

18. (*a*) 0·675 henry; (*b*) 338 volts.

19. 0·494 millihenry.

20. (*a*) 96 henrys; (*b*) 1200 joules (885 ft.-lb.); (*c*) 4800 volts.

21. (*a*) 0·3 sec.; (*b*) 8·1 amperes.

22. (*a*) 1·35 amperes; (*b*) 3·77 sec.; (*c*) 1·77 ampere (in this case $R = 200$ ohms).

23. (*a*) 31·1 henrys; (*b*) current attains 80 per cent of steady value in 1 sec. Hence steady value $= 10 \times \dfrac{9{\cdot}6}{8} = 12$ amperes; P.D. $= 500 \times \dfrac{9{\cdot}6}{8} = 600$ volts.

24. $\dfrac{R}{L} = 13{\cdot}89.$

Initial rate of growth $= \dfrac{V}{L} = \dfrac{IR}{L} = 2 \times 13{\cdot}89 = 27{\cdot}78$ amp./sec.

25. 519 kg.

26. $1{\cdot}2$ Wb/m^2.

27. 202 lb.

28. $0{\cdot}78$ ohm. $0{\cdot}202$ millihenry.

CHAPTER X (p. 208)

1. For full deflection 300 ampere-turns are necessary, hence
(a) $300/20 = 15$ turns; (b) 300 turns.

2. Current at 200 volts $= 200/4000 = 0{\cdot}05$ ampere.
No. of turns $= 300/0{\cdot}05 = 6000$.

3. Total resistance at 25° C. $= 2008{\cdot}1$ ohms.

Reading $= 100 \times \dfrac{2000}{2008{\cdot}1} = 99{\cdot}59$ volts. Error $= 0{\cdot}41$ per cent low.

4. (a) $0{\cdot}0015005$; (b) $6661{\cdot}7$ ohms.

5. (a) 1000; (b) 2000.

6. Total resistance must be such that a P.D. of 25 volts produces a current of $0{\cdot}032$ ampere.

$R_T = \dfrac{25}{0{\cdot}032} = 781{\cdot}3$ ohms; $R = 781{\cdot}3 - 15 = 766{\cdot}3$ ohms.

7. $0{\cdot}66$ gm.-cm.

8. $14{\cdot}7$ ohms. $2{\cdot}33$ volts.

9. $0{\cdot}10001$ ohm.

CHAPTER XI (p. 238)

1. For a deflection of one division, $q = 0{\cdot}2$ micro-cmb. Hence

$$0{\cdot}2 \times 10^{-6} = \varDelta\Phi \times \frac{100}{1000}. \quad \varDelta\Phi = 2 \times 10^{-6} \text{ weber.}$$

2. 100×10^{-6} weber-turn per division.

3. $\varphi = 3{\cdot}12 \times 10^{-6}$ weber; $q = 2$ micro-cmb.; $\delta = 4$ divisions.

4. $\varphi = 340 \times 10^{-6}$ weber; $B = 1{\cdot}7$ Wb/m^2;
$H = 1005$ AT/m; $\mu_r = 1347$.

5. Calibration test: $H = 4000$ AT/m; $\varphi = 6{\cdot}33 \times 10^{-6}$ weber;
$k_g = 210 \times 10^6$ weber-turns per division.
Magnetization test: $H = 2388$ AT/m; $B = 1{\cdot}795$ Wb/m^2.
$\mu_r = 598$.

6. 2880 joules per cu. m. per cycle (approx.).

7. $B = 1$ Wb/m^2.

$$H = \frac{1}{800 \times 4\pi \times 10^{-7}} = 995 \text{ AT/m}; \quad I = \frac{995 \times 20\pi}{500 \quad 10^2} = 1{\cdot}25 \text{ ampere.}$$

8. Reduction in length of iron path is negligible.
For air gap: $B = 1$ Wb/m^2. $\therefore H = 79{,}700$ AT/m.
$NI = 1590$.
Total excitation $= 1590 + 623 = 2213$. $\therefore I = 4{\cdot}43$ amperes.

9. (a) $B = 0{\cdot}8$ Wb/m².

Cast iron:	$l = 23{\cdot}5$ cm., $NI =$	1245 ampere-turns.
Mild steel:	$l = 23{\cdot}5$ cm., $NI =$	166 ampere-turns.
Brass:	$l = 0{\cdot}2$ cm., $NI =$	1272 ampere-turns.
		2683 ampere-turns.

(b) $F = \dfrac{0{\cdot}8^2 \times 10^{-7}}{2 \times 4\pi} \times 4 \times 10^{-4} = 102$ newtons $= 22{\cdot}9$ lb.

10. Steel portion: From curve (fig. 2, p. 212): $H = 380$ AT/m.
$NI = 380 \times 0{\cdot}3 = 114$ ampere-turns.

Air gap: $NI = \dfrac{0{\cdot}8 \times 10^7}{4\pi} \times 0{\cdot}2 \times 10^{-2} = 1274$ ampere-turns.

Total excitation = 1388 ampere-turns.

11. $f = 0{\cdot}575 \times 10^6$ newtons per sq. m. (pull is due to both poles).
In air gap, $B = \sqrt{(0{\cdot}575 \times 8\pi \times 10^6/10^7)} = 1{\cdot}2$ Wb/m².
In iron core, $B = 1{\cdot}33$ Wb/m².

Air gap: $l = 0{\cdot}1$ cm., $NI = \dfrac{1{\cdot}2 \times 10^7 \times 0{\cdot}1 \times 10^{-2}}{4\pi} = 955$ AT.

Iron core: $l = (4 + 16) + 8\pi = 89$ cm., $H = 500$ AT/m.
(from curve, p. 220).

$NI = 445$ ampere-turns.
Total excitation = 1400 ampere-turns.

12. Two magnetic circuits in parallel:

	B webers per sq. m.	l m.	H ampere-turns per m.	NI ampere-turns
Air gap	1·5	0·001	1,200,000	1200
Central core	1·5	0·24	1650	396
Side core and yokes	1·2	0·58	800	464
			Total	2060

13. $f = 55{,}600$ newtons per sq. m. $B = 0{\cdot}374$ Wb/m².
Excitation for iron = 93 ampere-turns.
Excitation for air gap = 500 − 93 = 407 ampere-turns.
Length of each air gap = 0·69 mm.

14. 3·38; 4·56; 5·96 amperes.

CHAPTER XII (p. 293)

1. Peripheral velocity of coil side = 15·7 m. per sec.
$E_{max} = 2 \times 0{\cdot}5 \times 20 \times 0{\cdot}2 \times 15{\cdot}7 = 62{\cdot}8$ volts.
$f = 50$ cycles per sec.

2. Speed $= 3000 \times \dfrac{40}{50} = 2400$ r.p.m.
(a) $E_{max} = 50{\cdot}2$ volts; (b) $\theta = 90 + 45 = 135°$,
$e = 50 \sin 135° = 35{\cdot}6$ volts.

3. 14·17 cycles per sec.

6. Power input = 1000 watts. Time = 7·4 min.

7. $E_R = \dfrac{1}{\sqrt{2}} \sqrt{(10)^2 + (20)^2} = 15{\cdot}8$ volts.

8. Either graphically or by trigonometry:
$E_R = 8{\cdot}6$ amperes, 34·5° behind smaller current,
25·5° ahead of larger current.

9. Period $= 0{\cdot}02$ sec.; $\theta_1 = 360° \times \dfrac{0{\cdot}008}{0{\cdot}02} = 144°$.

(*a*) $i_1 = 5 \sin 144° = 2{\cdot}94$ amperes; $i_2 = 10 \sin(144° - 30°) = 9{\cdot}14$ amperes.
(*b*) Either graphically or by trigonometry: $I_R = 10{\cdot}28$ amperes.

10. (*a*) $X = 2\pi \times f \times 25 \times 0{\cdot}03 = 4{\cdot}71$ ohms, $Z = \sqrt{10^2 + 4{\cdot}71^2} = 11{\cdot}07$ ohms.

$$I = \frac{25}{11{\cdot}07} = 2{\cdot}26 \text{ amps}, \ \varphi = \tan^{-1}\frac{4{\cdot}71}{10} = \tan^{-1} 0{\cdot}471 = 25° \text{ lag.}$$

(*b*) $X = 4{\cdot}71 \times \dfrac{50}{25} = 9{\cdot}42$ ohms, $Z = 13{\cdot}74$ ohms,

$I = 1{\cdot}82$ amps, $\varphi = 43°$ lag.

11. Power factor $= \cos 60° = 0{\cdot}5$. Power input $= 230 \times 10 \times 0{\cdot}5 = 1150$ watts.

12. $\cos\varphi = \dfrac{3300}{4000} = 0{\cdot}825$; $\varphi = 34°$.

13. $X_2 = 2\pi \times 50 \times 0{\cdot}01 = 3{\cdot}14$ ohms, $Z_2 = \sqrt{4^2 + 3{\cdot}14^2} = 5{\cdot}09$ ohms,

$Z_t = \sqrt{(8+4)^2 + 3{\cdot}14^2} = 12{\cdot}4$ ohms, $I = \dfrac{50}{12{\cdot}4} = 4{\cdot}03$ amps,

$$\cos\varphi = \frac{R}{Z} = \frac{12}{12{\cdot}4} = 0{\cdot}97.$$

$V_1 = IR_1 = 4{\cdot}03 \times 8 = 32{\cdot}24$ volts, $\varphi_1 = 0°$.
$V_2 = IZ_2 = 4{\cdot}03 \times 5{\cdot}09 = 20{\cdot}5$ volts, $\varphi_2 = 38°$ leading the current.

14. $X_c = \dfrac{10^6}{2\pi \times 50} \times 50 = 63{\cdot}7$ ohms, $Z = 118{\cdot}5$ ohms.

(*a*) $I = 0{\cdot}84$ amp, (*b*) $\varphi = 32{\cdot}5°$ lead,
(*c*) $100 \times 0{\cdot}84 \times \cos 32{\cdot}5 = 70{\cdot}5$ watts.

15. (*a*) $Z = \sqrt{20^2 + (15{\cdot}7 - 42{\cdot}5)^2} = 33{\cdot}5$ ohms, $I = 2{\cdot}98$ amps,
$\varphi = 53{\cdot}3°$ lead.
(*b*) $Z = \sqrt{20^2 + (31{\cdot}4 - 21{\cdot}25)^2} = 22{\cdot}4$ ohms, $I = 4{\cdot}46$ amps,
$\varphi = 27°$ lag.

16. $Z_A = \sqrt{50^2 + 31{\cdot}4^2} = 59$ ohms $= Z_B$,

$X_B = \sqrt{59^2 - 30^2} = 50{\cdot}75$ ohms, $C = \dfrac{10^6}{314 \times 50{\cdot}75} = 62{\cdot}9\ \mu$F.

17. $I_1 = 3{\cdot}64$ amps, $\varphi = 43{\cdot}3°$ lagging,
$I_2 = 2{\cdot}29$ amps, $\phi = 46{\cdot}7°$ leading.
Active components: $3{\cdot}64 \times \cos 43{\cdot}3 + 2{\cdot}29 \times \cos 46{\cdot}7$
$= 2{\cdot}65 + 1{\cdot}57 = 4{\cdot}22$.
Reactive components: $3{\cdot}64 \times \sin 43{\cdot}3 + 2{\cdot}29 \times \sin 46{\cdot}7$
$= -2{\cdot}53 + 1{\cdot}66 = -0{\cdot}87$.

$I_t = \sqrt{4{\cdot}22^2 + 0{\cdot}87^2} = 4{\cdot}31$ amps, $\varphi = \tan^{-1} - \dfrac{0{\cdot}87}{4{\cdot}22} = 11{\cdot}6°$ lag,

$\cos\varphi = 0{\cdot}98$.

Alternatively: solution by graphical method.

18. $f = \dfrac{1}{2\pi}\sqrt{\dfrac{10^6}{0{\cdot}072 \times 120}} = 54{\cdot}2$ c/s.

$I = \dfrac{75}{10} = 7{\cdot}5$ amps, $X_c = \dfrac{10^6}{2\pi \times 54{\cdot}2 \times 120} = 24{\cdot}5$ ohms,

$V_c = 7{\cdot}5 \times 24{\cdot}5 = 184$ volts.

19. $V_{ph} = \frac{400}{\sqrt{3}} = 230$ volts, $I_l = \frac{30 \times 746}{\sqrt{3} \times 400 \times 0{\cdot}9 \times 0{\cdot}88} = 40{\cdot}8$ amps,

$I_{ph} = \frac{40{\cdot}8}{\sqrt{3}} = 23{\cdot}6$ amps.

20. $V_{ph} = 230$ volts, $X_c = \frac{230}{10{\cdot}4} = 22{\cdot}1$ ohms,

$$f = \frac{10^6}{2\pi \times 22{\cdot}1 \times 180} = 40 \text{ c/s},$$

when connected in mesh, $I_c = \frac{400}{22{\cdot}1} = 18{\cdot}1$ amps.

$$I_l = \sqrt{3} \times 18{\cdot}1 = 31{\cdot}4 \text{ amps.}$$

CHAPTER XIII (p. 344)

1. (*a*) $E = \frac{468}{6} \times 6 \times 0{\cdot}064 \times \frac{1000}{60} = 500$ volts.

   (*b*) Total current $= 6 \times 50 = 300$ amps.
   Power output $= 500 \times 300 \times 10^{-3} = 150$ kW.

2. $E = 500 + (50 \times 0{\cdot}5) = 525$ volts.

   $\Phi = \frac{525 \times 2 \times 60}{774 \times 4 \times 700} = 0{\cdot}0291$ weber.

3. Shunt field current $= \frac{550}{110} = 5$ amps.

   At full-load,

   $$\text{Armature current} = \frac{500{,}000}{550} + 5 = 915 \text{ amps.}$$

   $$E = 550 + 915(0{\cdot}006 + 0{\cdot}0016) = 557 \text{ volts.}$$

   $$\Phi = 0{\cdot}06 \times \frac{557}{500} \times \frac{1030}{1000} = 0{\cdot}0689 \text{ weber.}$$

4. At no-load,

   $$E = \frac{664}{2} \times 6 \times 0{\cdot}06 \times \frac{260}{60} = 518 \text{ volts.}$$

   At full-load,

   $$E = 518 \times \frac{250}{260} \times \frac{0{\cdot}054}{0{\cdot}06} = 449 \text{ volts,}$$

   $$V = 449 - (200 \times 0{\cdot}15) = 419 \text{ volts.}$$

5. Work done per revolution $= 468 \times 130 \times 6 \times 0{\cdot}06$
   $= 21{,}900$ joules.

   Work done per sec. $= 21{,}900 \times \frac{750}{60}$ joules per sec. or watts.

   Power output $= \frac{21{,}900}{746} \times \frac{750}{60} = 367$ horse-power.

6. $E_b = 200 - (200 \times 0{\cdot}15) = 170$ volts.

   $\Phi = \frac{170 \times 2 \times 60}{660 \times 4 \times 250} = 0{\cdot}0309$ weber.

7. When $I = 25$ amps, $E_b = 240$ volts.
When $I = 3$ amps, $E_b = 248{\cdot}8$ volts.

$$\text{No-load speed} = 750 \times \frac{248{\cdot}8}{240} = 778 \text{ r.p.m.}$$

8. At no-load, $E_b = 497{\cdot}75$ volts.
At full-load, $E_b = 455$ volts.

$$\text{Full-load speed} = 500 \times \frac{455}{497{\cdot}75} \times \frac{100}{110} = 416 \text{ r.p.m.}$$

9. $p = 8,\ n = \frac{375}{60},\ f = 8 \times \frac{375}{60} = 50$ c/s.

Synchronous speed of motor $= \frac{50}{3} \times 60 = 1000$ r.p.m.

Full-load speed $= 1000(1 - 0{\cdot}04) = 960$ r.p.m.

10. $E_2 = 3300 \times \frac{77}{1105} = 230$ volts.

11. (a) $V_2 = 6600 \times \frac{60}{990} = 400$ volts,

$$I_1 = 100 \times \frac{60}{990} = 6{\cdot}06 \text{ amps.}$$

12. Output $= 55$ kW. Iron loss $= 615$ watts.
Resistance loss $= 1120 \times (300/435)^2 = 533$ watts.

$$\text{Efficiency} = 1 - \frac{1148}{56148} = 97{\cdot}96 \text{ per cent.}$$

CHAPTER XIV (p. 380)

1. $C = \frac{Q}{V} = \frac{0{\cdot}01 \times 10^6}{500} = 20\ \mu$F.

2. 2000 volts.

3. $W = \frac{1}{2}CV^2 = \frac{1}{2} \times 20 \times 10^{-6} \times (200)^2 = 0{\cdot}4$ joules.

4. 0·4 joules.

5. 6·67 μF.

6. 320 micro-cmbs. in 2 μF. capacitor.
80 micro-cmbs. in 0·5 μF. capacitor.

7. $V = q\left(\frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}\right)$. $\therefore\ q = 2675$ micro-cmbs.

$$\therefore\ v_1 = \frac{q}{c_1} = 268 \text{ volts},\quad v_2 = 178 \text{ volts},\quad v_3 = 54 \text{ volts.}$$

8. $v_B = 150,\ q = 25$ micro-cmbs., $C_A = 0{\cdot}1,\ C_B = 0{\cdot}167,\ C_C = 0{\cdot}25\ \mu$F.
When B is immersed in oil,

$$C_A = 0{\cdot}1,\ C_B = 0{\cdot}417,\ C_C = 0{\cdot}25\ \mu\text{F.},\ q = 30{\cdot}5 \text{ micro-cmb.}$$

Hence

$$v_A = 305,\ v_B = 73,\ v_C = 122.$$

9. Energy stored in capacitor $= 2{\cdot}5$ joules.

$$\therefore\ \tfrac{1}{2}LI^2 = 2{\cdot}5,\ I = \sqrt{\frac{5}{L}} = 50 \text{ amperes.}$$

10. From equation (25), p. 370, $C = \dfrac{0\cdot0388 \times 2\cdot5}{\log_{10} \dfrac{1\cdot25}{0\cdot75}} = 0\cdot437\ \mu F.$

11. (*a*) From Section 21, p. 372, $V = \dfrac{1}{\pi} \cdot \dfrac{q}{\varepsilon} \cdot \log_e \dfrac{D}{r}$ (when r is small compared with D)

$$= \frac{0\cdot6 \times 10^{-6}}{\pi \times 8\cdot85 \times 10^{-12}} \log_e 350$$

$$= 126{,}500 \text{ volts.}$$

Total charge $= 0\cdot6 \times 10^{-6} \times \dfrac{5280}{3\cdot28} = 966\ \mu$cmb.

$\therefore$ Capacitance $= \dfrac{Q}{V} = \dfrac{966}{126{,}500} = 0\cdot00763\ \mu F.$

12. $a = 1$ sq. m., $d = 0\cdot1 \times 10^{-2}$m.,
from equation (22), p. 369,

$$C = 3 \times 8\cdot85 \times 10^{-12} \times \frac{1}{0\cdot1 \times 10^{-2}} = 0\cdot0266\ \mu F.$$

13. The effective area of each plate is due to *both* sides of 36 sheets.

$$\therefore C = 0\cdot491\ \mu F.$$

14. (*a*) 0·005 sec., i.e. the time constant.
(*b*) From equation (12), p. 366, $i = 0\cdot4\ \varepsilon^{-\cdot01/\cdot005} = 0\cdot054$ ampere.

15. (*a*) 0·0182 cmb.; (*b*) 277 sec.

16. $a = 1700$ sq. cm., i.e. both sides of 85 sheets.
Total number of sheets of tin-foil $= 85 + 86 = 171$.
Total number of sheets of mica $= 170$.

17. (*a*) Stored energy $= 0\cdot2$ joules; power $= 50$ watts.
(*b*) Charge $= 2000 \times 10^{-6}$ coulomb; current $= 0\cdot5$ ampere.

18. 0·000532 μF.

19. $C = \dfrac{2\pi \times 8\cdot85 \times 10^{-12}}{2\cdot3 \log_{10} \dfrac{6}{2\cdot5}} \times 0\cdot75 = 47\cdot9\ \mu\mu F.$

Equipotential surfaces are concentric cylinders.

If V_0 is the P.D. between the outer cylinder and an equipotential surface of radius x, from equation (24),

$$V_0 = \frac{q}{2\pi\varepsilon} \log_e \frac{R}{x} = \frac{V}{2} = \frac{1}{2} \cdot \frac{q}{2\pi\varepsilon} \cdot \log_e \frac{R}{r}.$$

$$\log_e \frac{R}{x} = \frac{1}{2} \log_e \frac{R}{r}.$$

$$\frac{R}{x} = 1\cdot55,\ x = 3\cdot87 \text{ mm.},$$

i.e. $3\cdot87 - 2\cdot5 = 1\cdot37$ mm. from the surface of the inner cylinder.

20. $C = 88\cdot5 \times 10^{-12}$ F.

Stored energy $= \dfrac{1}{2} \times 88\cdot5 \times 10^{-12} \times 200^2 = 177 \times 10^{-8}$ joules.

(*a*) Capacitance doubled, P.D. halved, stored energy halved, i.e. decrease of $88\cdot5 \times 10^{-8}$ joules.

(*b*) Capacitance doubled, charge doubled, stored energy doubled, i.e. increase of 177×10^{-8} joules.

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