High Frequency Response

Hybrid π model for a CE amplifier

Different parameters

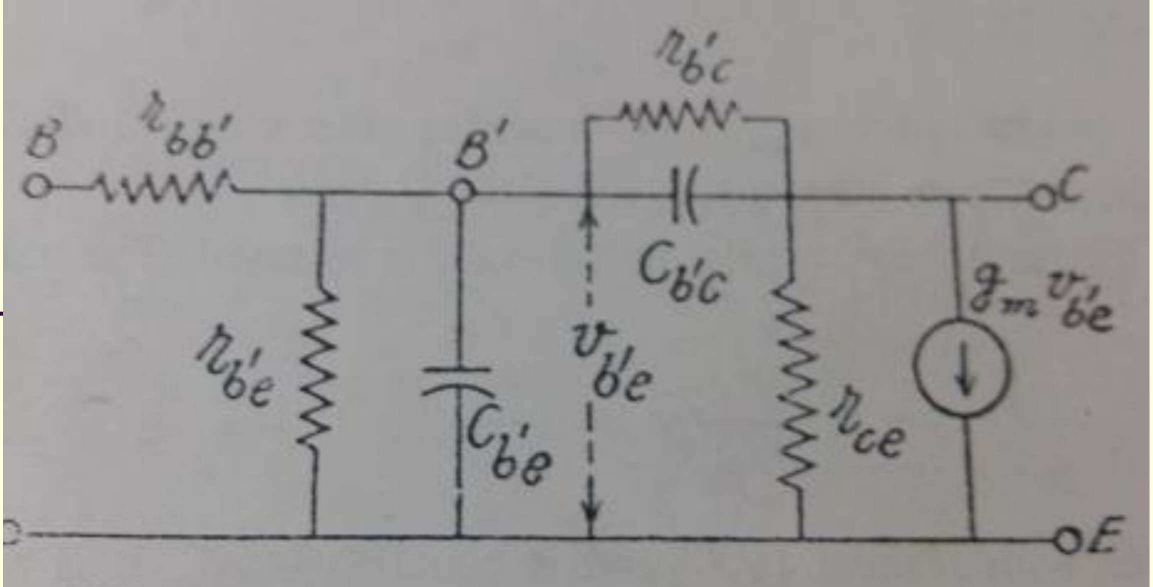


Fig. 45 (a). Hybrid π model for a CE amplifier.

The hybrid π capacitance:

The capacitance across a PN junction consists of two components:

1. depletion region capacitance C_T

Depends upon voltage across the junction
Predonimates for reverse biased junction

2. difusion capacitance C_D

Depends on current through the junction
Predonimates for forward biased junction

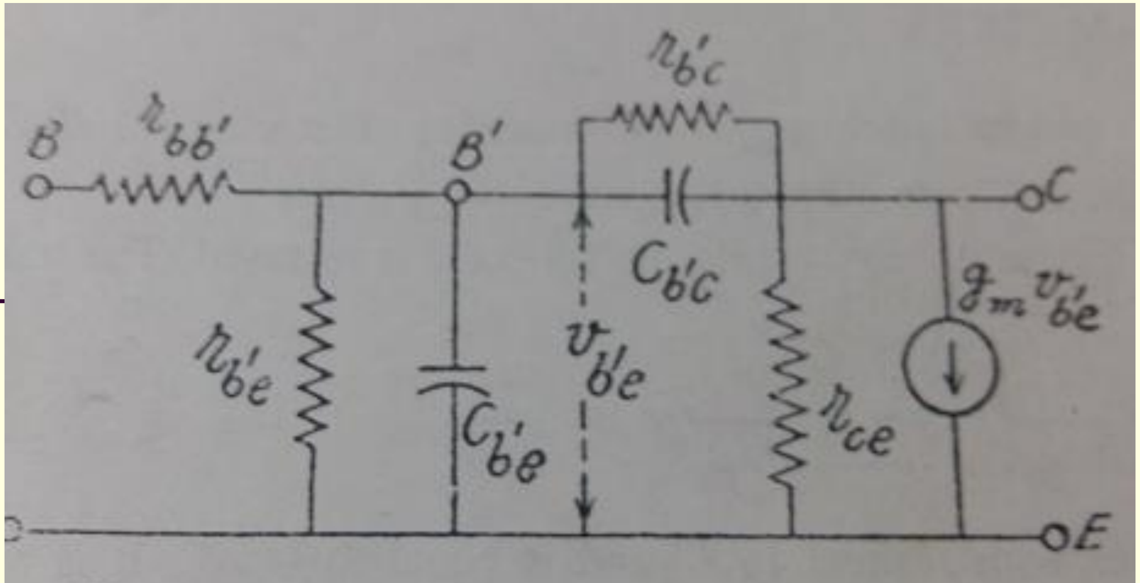


Fig. 45 (a). Hybrid π model for a CE amplifier.

For PNP transistor

When the input voltage increases → Hole density increases in the base

To maintain the charge neutrality → Electron enters the base

When the input voltage falls → The stored charge returns to the circuit

Thus, there is a charge flow or current which occurs with change in voltage

↓

To represent this charge effect, a capacitance $C_{b'e}$ (diffusion capacitance) is placed between B' and E

The depletion layer of collector-base reverse biased junction also act as a capacitance and is shown by $C_{b'c}$ between B' and C

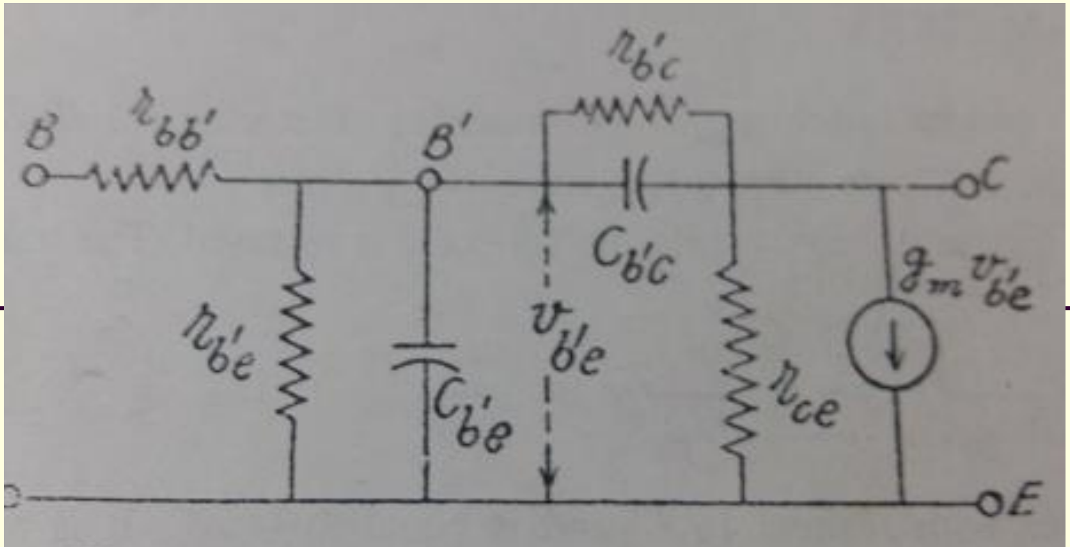
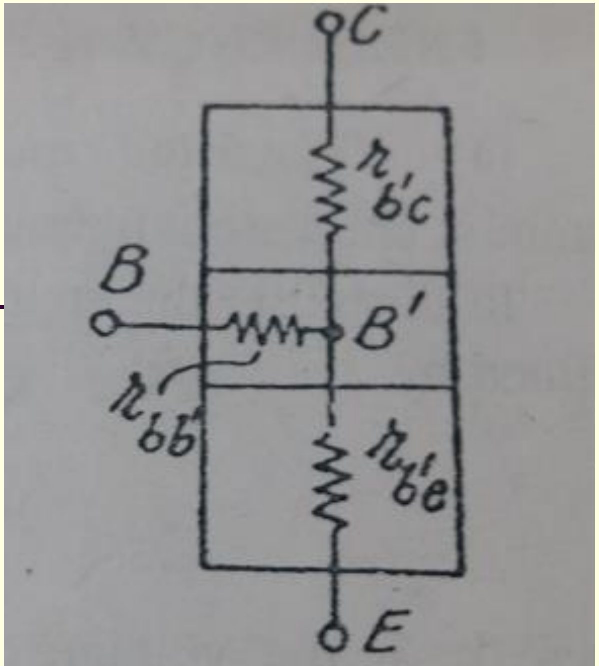


Fig. 45 (a). Hybrid π model for a CE amplifier.


(b). Showing virtual base.

Base spreading resistance :

- The two capacitances have a common terminal at B’ and not at the base terminal
- B’ is known as internal node and is not physically accessible
- The ohmic base-spreading resistance $r_{bb'}$ is represented as a lumped parameter between external base B and internal node B’
- This represents the bulk resistance of the base and is the portion of the base through which the proper base current flows.

The increase in minority carriers in the base results in increased recombination base current. This effect is taken into account by inserting a resistance $r_{b'e}$ between B’ and E.

At low frequencies, $h_{ie} = r_{bb'} + r_{b'e}$

$r_{b'c}$:

The varying voltage across the collector to emitter junction results in base-width modulation.

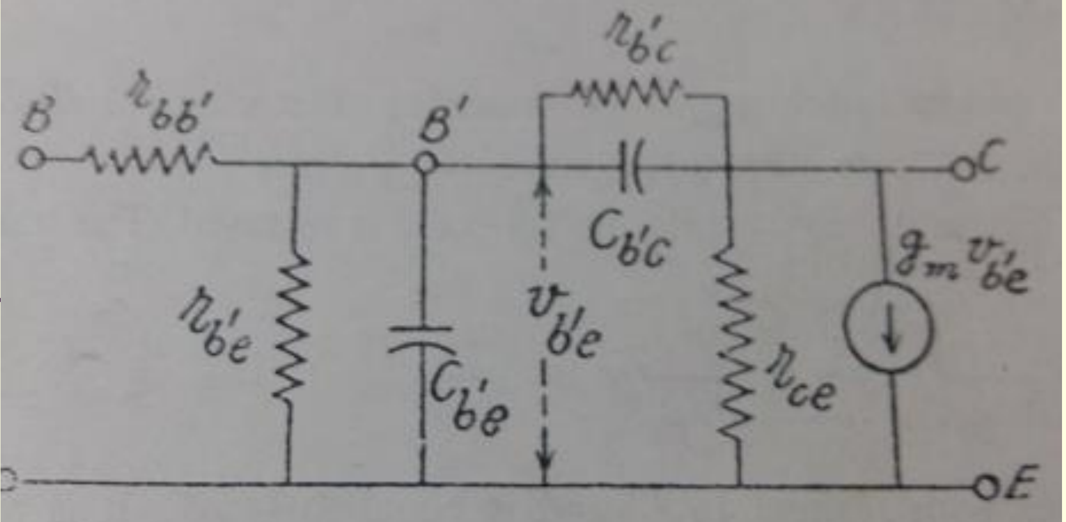


Fig. 45 (a). Hybrid π model for a CE amplifier.

A change in the effective base width causes the emitter and collector current to change.

This feedback effect between base and collector is taken into account by connecting $r_{b'c}$ between internal node B' and C.

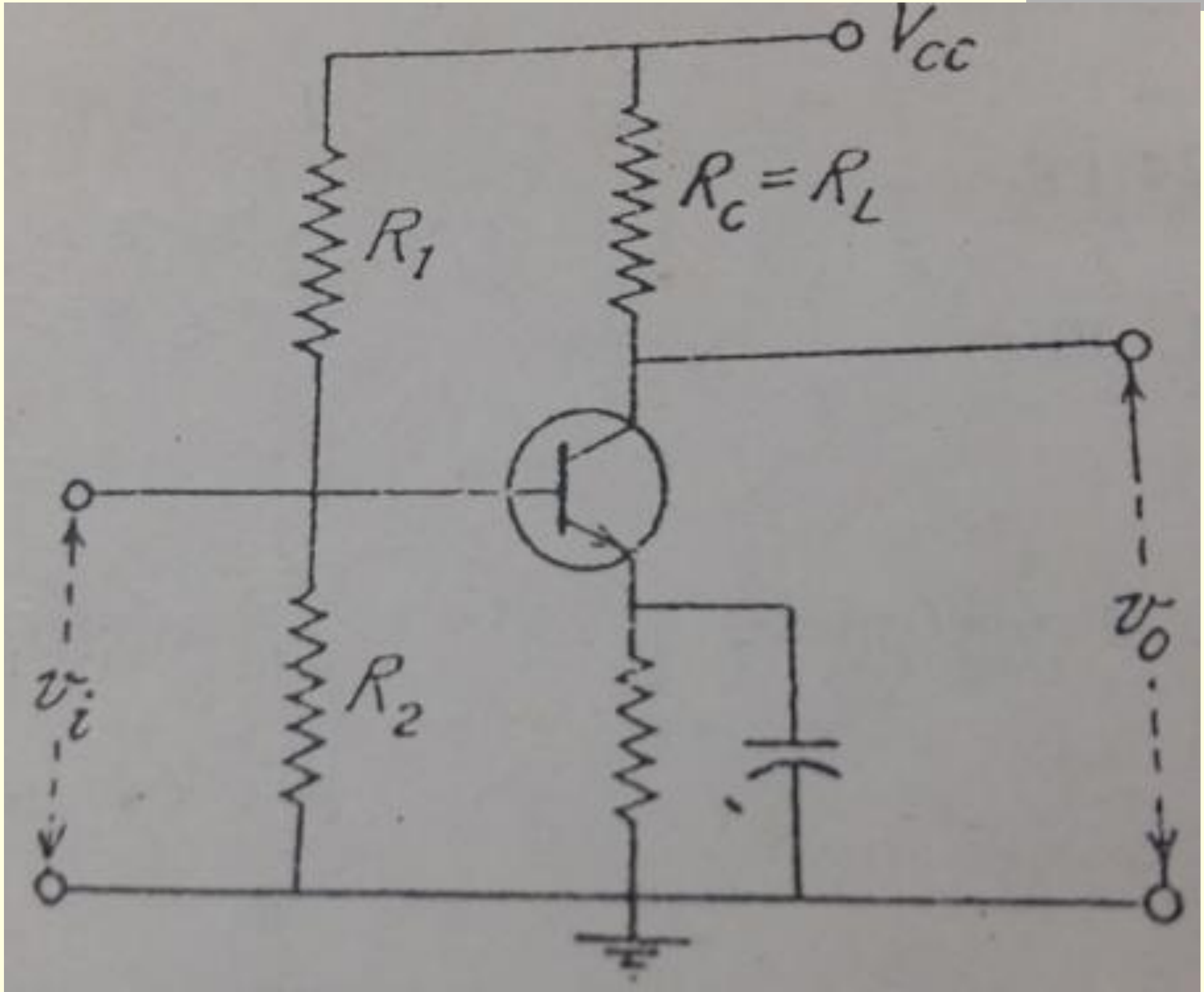
This resistance is typically 4M ohm and normally be omitted

r_{ce}:

Resistance r_{ce} is the output resistance.

When a load is connected between collector and emitter, this resistance may be omitted, because $r_{ce} \gg R_L$

Common Emitter Short Circuit Current Gain Using Hybrid PI model:



(a) Single stage *CE* amplifier

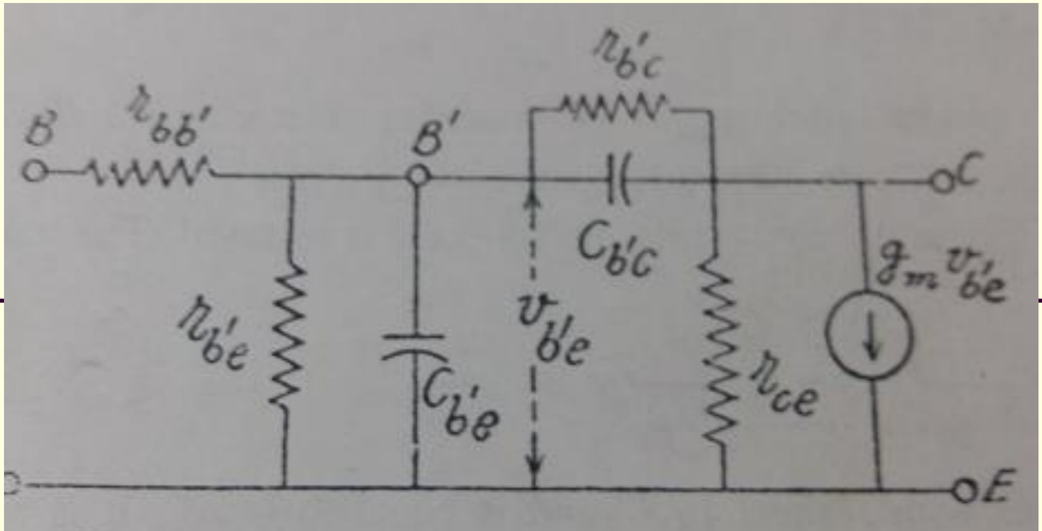
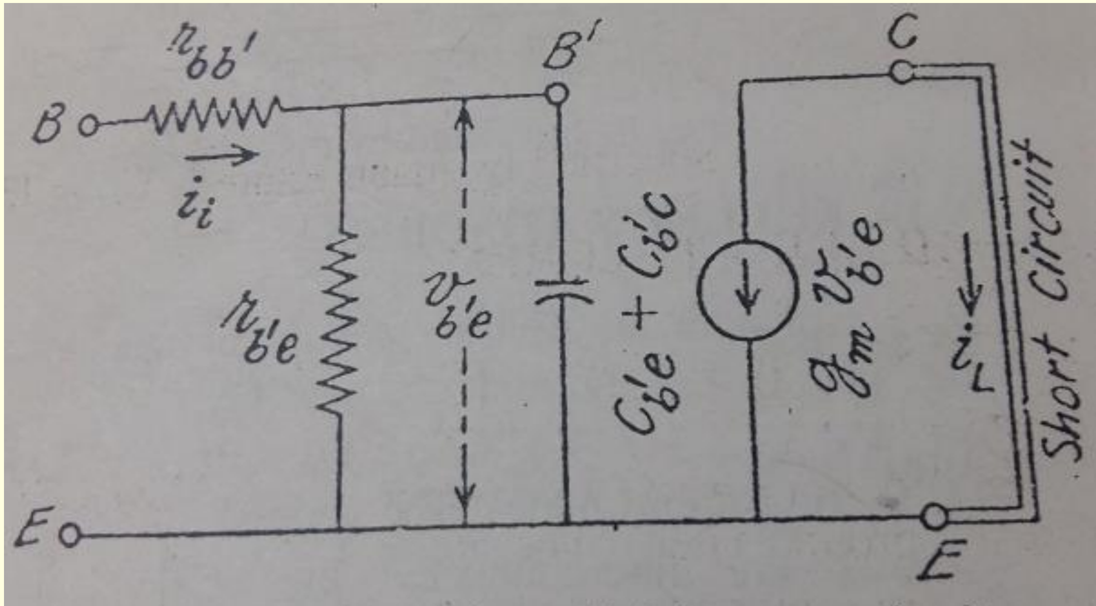


Fig. 45 (a). Hybrid π model for a CE amplifier.

The assumptions:

1. $r_{b'c}$ is neglected, as $r_{b'c} \gg r_{b'e}$
2. r_{ce} disapppears as it is in shunt with a short circuit
3. $R_c = R_L = 0$



(b) Approximate high frequency circuit

Current Gain:

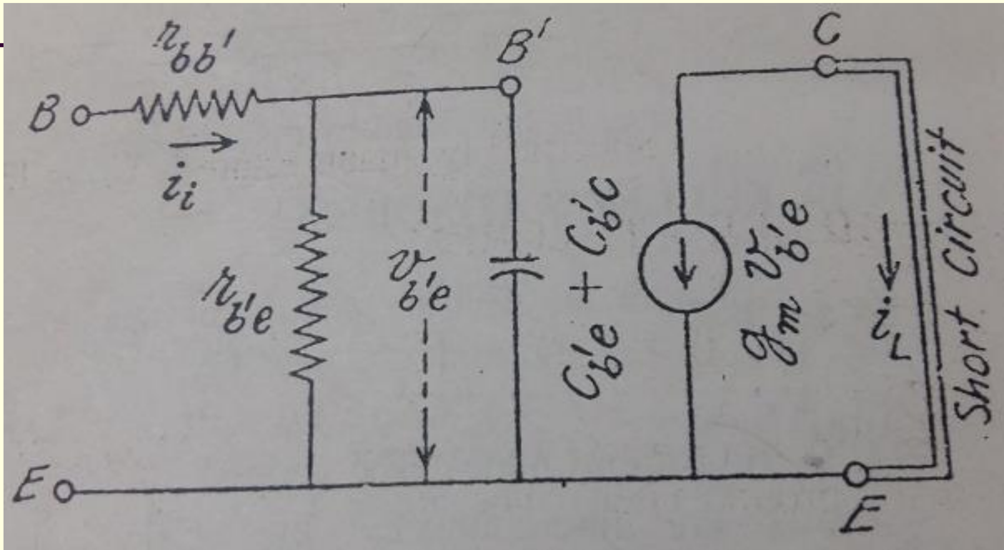
From the equivalent circuit, $i_L = -\, g_m\, v_{b'e}$

The resistance of parallel combination of $r_{b'e}$ and $(C_{b'e} + C_{b'c})$ is given by:

$$\frac{1}{R} = \left(\frac{1}{r_{b'e}}\right) + \frac{1}{[1/(C_{b'e}+C_{b'c})j\omega]}$$

$$= \frac{1}{r_{b'e}} + j\omega(C_{b'e} + C_{b'c})$$

$$R = \frac{1}{\frac{1}{r_{b'e}}+j\omega(C_{b'e}+C_{b'c})}$$



(b) Approximate high frequency circuit

Now $v_{b'e} = R \times i_i$

$$v_{b'e} = \frac{i_i}{\frac{1}{r_{b'e}}+j\omega(C_{b'e}+C_{b'c})}$$

But $i_L = \dfrac{-g_m\, i_i}{\frac{1}{r_{b'e}}+j\omega(C_{b'e}+C_{b'c})}$

Short circuit current gain $A_i = \dfrac{i_L}{i_i} = \dfrac{-g_m}{\frac{1}{r_{b'e}}+j\omega(C_{b'e}+C_{b'c})}$

Bandwidth of the amplifier f_β :

We know that From the equivalent circuit, $r_{b'e} = \frac{h_{fe}}{g_m}$

$$A_i = \frac{-g_m}{\frac{1}{r_{b'e}} + j\omega(C_{b'e} + C_{b'c})} = \frac{-g_m h_{fe}}{g_m + j\omega(C_{b'e} + C_{b'c})h_{fe}}$$

$$= \frac{-h_{fe}}{1 + j2\pi f(C_{b'e} + C_{b'c})\frac{h_{fe}}{g_m}} = \frac{-h_{fe}}{1 + j\left(\frac{f}{f_\beta}\right)}$$

$$where \quad f_\beta = \frac{g_m}{h_{fe}2\pi(C_{b'e} + C_{b'c})} = \frac{1}{2\pi r_{b'e}(C_{b'e} + C_{b'c})}$$

We consider two cases

(1) When $f=0$, $A_i = -h_{fe}$ which is in agreement with the defination of h_{fe} at low frequency short circuit CE current gain.

(2) When $f = f_\beta$, $A_i = \frac{-h_{fe}}{\sqrt{2}}$ i.e. f_β forms the frequency at which CE current gain falls to $\frac{1}{\sqrt{2}}$ of its low frequency value i.e falls by 3dB. The frequency range upto f_β forms the bandwidth of the amplifier.

Frequency parameter f_T :

$$A_i = \frac{-h_{fe}}{1 + j\left(\frac{f}{f_\beta}\right)}$$

The frequency f_T is defined as the frequency at which the magnitude of CE short cicuit current gain fall to unity

$$|\ A_i\ | = \frac{h_{fe}}{\sqrt{\left[1+\left(\frac{f}{f_\beta}\right)^2\right]}}$$

At $f = f_T$, $|\ A_i\ | = 1$

$$1 = \frac{h_{fe}}{\sqrt{\left[1+\left(\frac{f_T}{f_\beta}\right)^2\right]}} = \frac{h_{fe}}{\left(\frac{f_T}{f_\beta}\right)} \qquad As \left(\frac{f_T}{f_\beta}\right)^2 \gg 1$$

The frequency f_T is represents the short circuit current gain bandwidth product. Thus for CE configuration with the output shorted, f_T is the product of the low frequency current gain and the upper 3 dB frequency.

$$\left(\frac{f_T}{f_\beta}\right) = h_{fe}$$

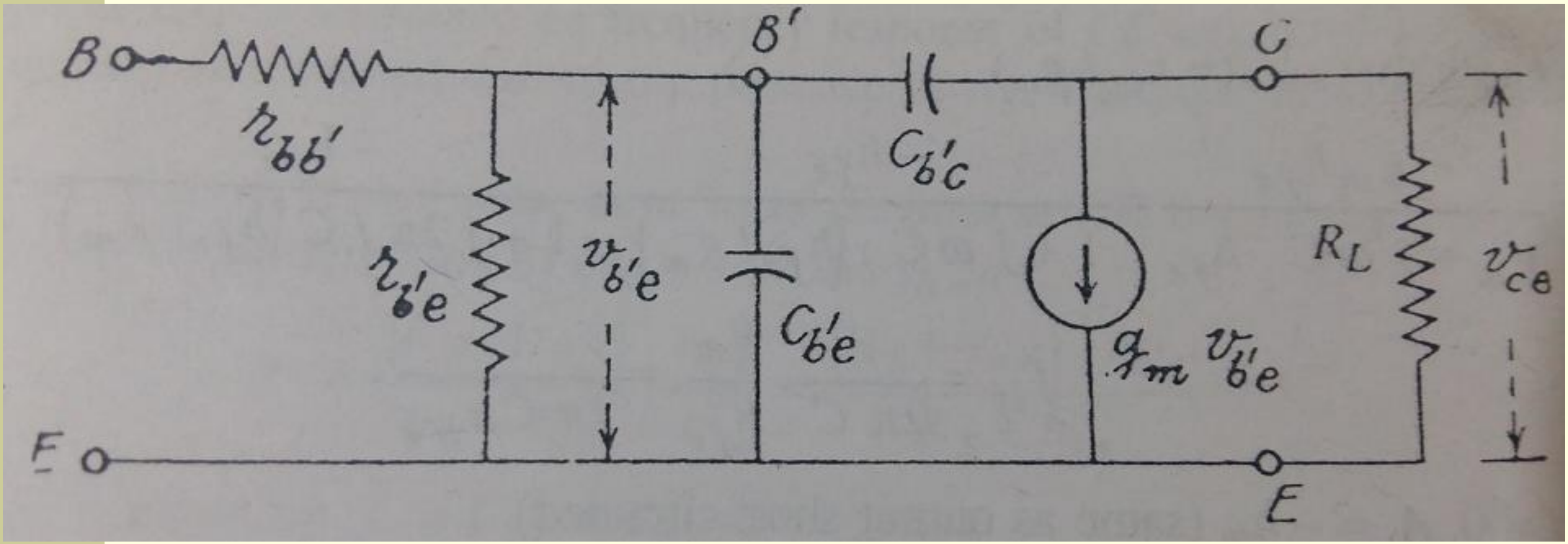
$$f_T = h_{fe} f_\beta$$

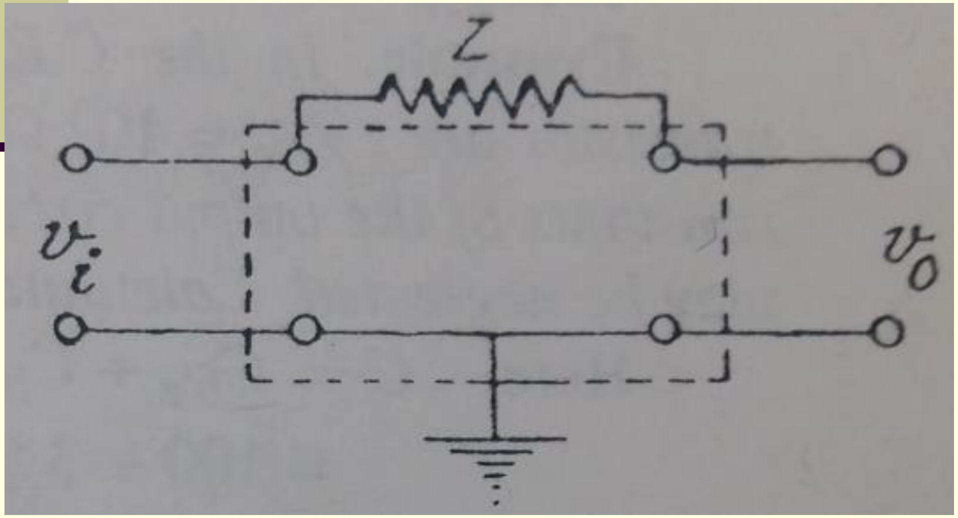
Putting the value of f_β

$$f_T = h_{fe}\frac{g_m}{h_{fe}2\pi(C_{b'e} + C_{b'c})} = \frac{g_m}{2\pi(C_{b'e} + C_{b'c})} = \frac{g_m}{2\pi C_{b'e}} \qquad (as\ \ C_{b'e} \gg C_{b'c})$$

$$A_i = \frac{h_{fe}}{1 + jh_{fe}\left(\frac{f}{f_T}\right)}$$

High Frequency Common Emitter Current Gain with Resistive Load:





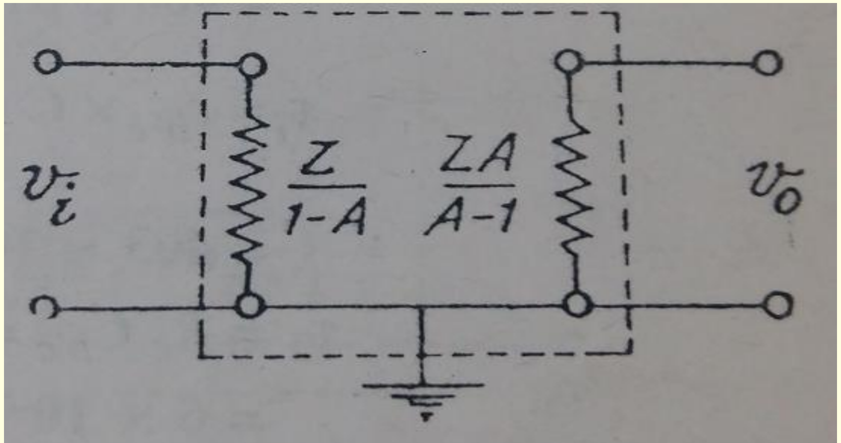
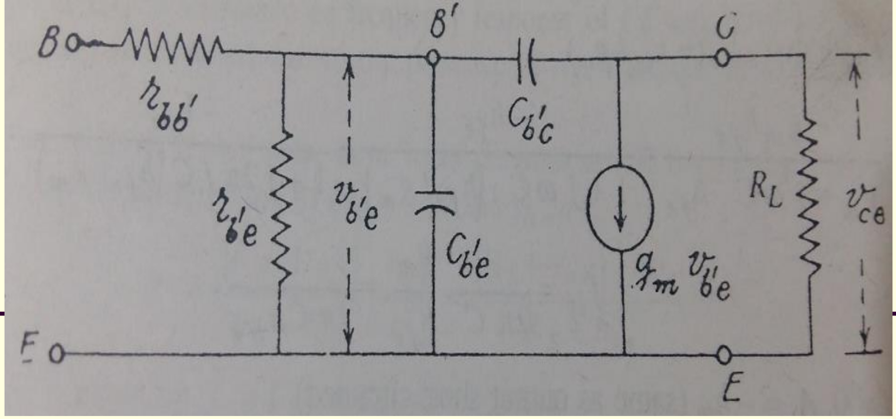


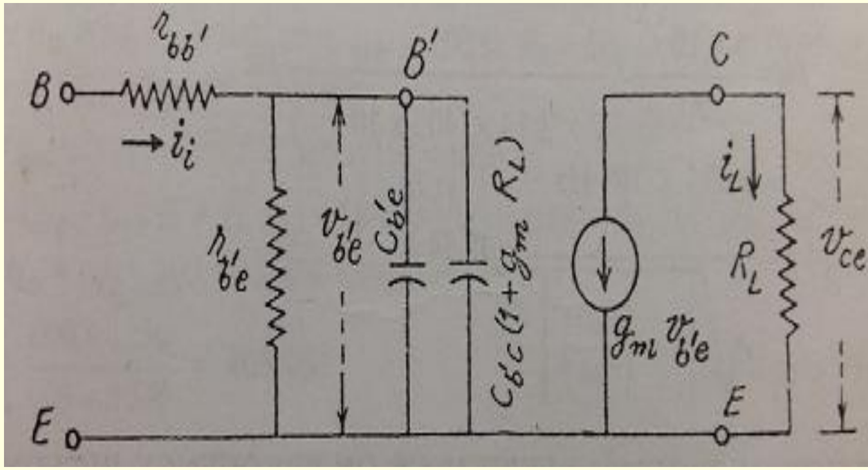
Fig. 53. Showing Millar's theorem.



$$A = \frac{v_{ce}}{v_{b'e}}$$

$$A = \frac{v_{ce}}{v_{b'e}} = \frac{-g_m v_{b'e} R_L}{v_{b'e}} = -g_m R_L$$

$$(1\text{-}A) = 1 - (-g_m R_L) = 1 + g_m R_L$$



Millers effect of $C_{b'c}$ at input side is

$$C_{b'c}\,(1+g_m R_L)$$

Total capacitance C at input side is

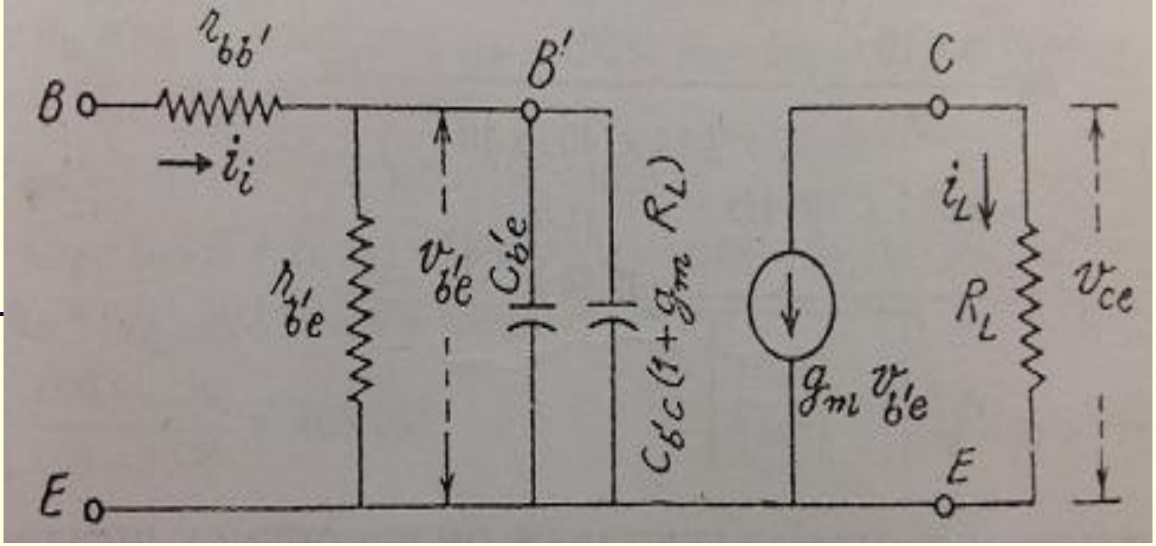
$$C = C_{b'e} + C_{b'c}\,(1+g_m R_L)$$

Millers effect of $C_{b'c}$ at output side is

$$C_{b'c}\,(1+g_m R_L)/g_m R_L \approx C_{b'c} \qquad (As\ \ A \gg 1)$$

This small value of capacitance in parallel with a low resistance R_L has a negligible time constant with input time constant and therefore can be omitted

This represents a considerable increase in capacitance at input side



From the equivalent circuit, the load current

$$i_L = -g_m v_{b'e}$$

$$\frac{1}{R} = \left(\frac{1}{r_{b'e}}\right) + \frac{1}{[1/\{C_{b'e}+C_{b'c}(1+g_m R_L)\}j\omega]}$$

$$= \frac{1}{r_{b'e}} + j\omega\{C_{b'e} + C_{b'c}(1+g_m R_L)\}$$

$$R = \frac{1}{\frac{1}{r_{b'e}}+j\omega C}$$

Now $v_{b'e} = R \times i_i$ $\quad\therefore\ v_{b'e} = \dfrac{i_i}{\frac{1}{r_{b'e}}+j\omega C}$ $\quad$ But $i_L = \dfrac{-g_m\, i_i}{\frac{1}{r_{b'e}}+j\omega C}$

Current gain $A_i = \dfrac{i_L}{i_i} = \dfrac{-g_m}{\frac{1}{r_{b'e}}+j\omega C} = \dfrac{-g_m}{\frac{1}{r_{b'e}}+j\omega\{C_{b'e}+C_{b'c}(1+g_m R_L)\}}$

Bandwidth of the amplifier f_H :

We know that from the equivalent circuit, $r_{b'e} = \frac{h_{fe}}{g_m}$

$$A_i = \frac{-g_m}{\frac{1}{r_{b'e}} + j\omega C} = \frac{-g_m h_{fe}}{g_m + j\omega C h_{fe}} = \frac{-h_{fe}}{1 + j2\pi f C \frac{h_{fe}}{g_m}}$$

$$= \frac{-h_{fe}}{1 + j\left(\frac{f}{f_H}\right)}$$

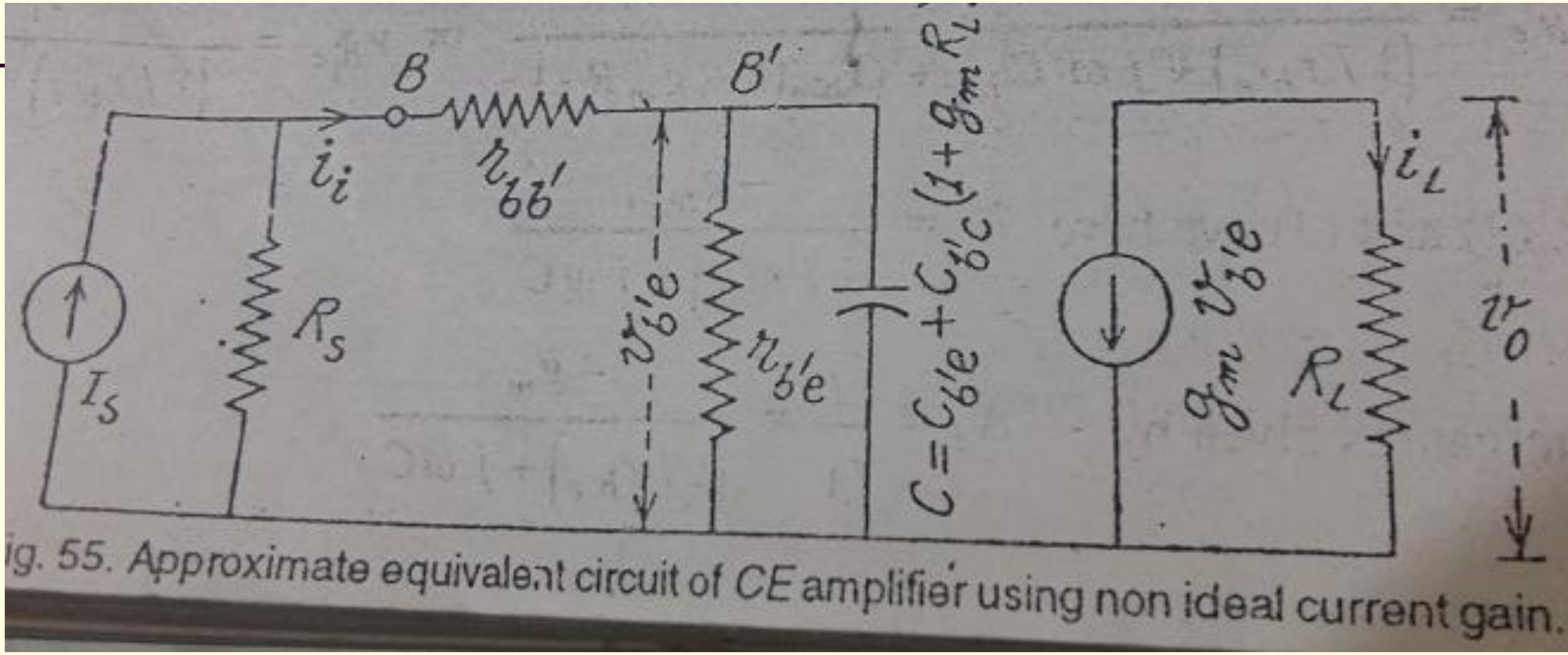
$$where\ f_H = \frac{g_m}{2\pi C h_{fe}} = \frac{1}{2\pi C r_{b'e}} = \frac{1}{2\pi r_{b'e}\{C_{b'e} + C_{b'c}(1+g_m R_L)\}}$$

We consider two cases

(1) When *f=0*, $A_i = -h_{fe}$ which is in agreement with the defination of h_{fe} at low frequency short circuit CE current gain.

(2) When *f=* f_H , $A_i = \frac{-h_{fe}}{\sqrt{2}}$ i.e. f_H forms the frequency at which CE current gain falls to $\frac{1}{\sqrt{2}}$ of its low frequency value i.e falls by 3dB. The frequency range upto f_H forms the bandwidth of the amplifier.

Effect of Source Resistance on Frequency Response of CE Amplifier at High Frequencies:



ig. 55. Approximate equivalent circuit of CE amplifier using non ideal current gain.

$$R = \frac{(R_s + r_{b'b})r_{b'e}}{R_s + r_{b'b} + r_{b'e}} = \frac{(R_s + r_{b'b})r_{b'e}}{R_s + h_{ie}}$$

$$C = C_{b'e} + C_{b'c}(1 + g_m R_L)$$

$$f_2 = \frac{1}{2\pi RC}$$

The following expressions can be derived

(i) The current gain taken into account the source resistance is given by

From the equivalent circuit, the load current

$$i_L = -\,g_m\, v_{b'e}$$

Now $v_{b'e} = R \times i_i$ $\quad \therefore\ v_{b'e} = \frac{i_i\,(R_s + r_{b'b})\,r_{b'e}}{R_s + h_{ie}}$

But $i_L = -g_m \frac{i_i\,(R_s + r_{b'b})\,r_{b'e}}{R_s + h_{ie}}$

$$i_L = \frac{-i_i\,(R_s + r_{b'b})\,h_{fe}}{R_s + h_{ie}} \qquad r_{b'e} = \frac{h_{fe}}{g_m}$$

Current gain $(A_i)_s = \frac{i_L}{i_i} = \frac{-(R_s + r_{b'b})\,h_{fe}}{R_s + h_{ie}} = \frac{-h_{fe} R_s}{R_s + h_{ie}}$

The following expressions can be derived

(i) The current gain taken into account the source resistance is given by

$$(A_i)_s = \frac{-h_{fe} R_s}{R_s + h_{ie}}$$

(ii) The voltage gain is given by

$$(A_v)_s = (A_i)_s \frac{R_L}{R_s} = \frac{-h_{fe} R_L}{R_s + h_{ie}}$$

(iii) Gain bandwidth product

$$|(A_v)_s\ f_2| = \frac{h_{fe}R_L}{R_s+h_{ie}} \times \frac{1}{2\pi RC}$$

$$= \frac{h_{fe}R_L}{R_s+h_{ie}} \times \frac{R_s+h_{ie}}{2\pi C(R_s+r_{b'b})r_{b'e}} \quad = \frac{g_m R_L}{2\pi C(R_s+r_{b'b})}$$

$$= \frac{g_m R_L}{2\pi\{C_{b'e}+C_{b'c}\ (1+g_m R_L)\}(R_s+r_{b'b})}$$

$$= \frac{g_m R_L}{2\pi\{C_{b'e}+C_{b'c}g_m R_L\}(R_s+r_{b'b})}$$

$$= \frac{g_m R_L}{2\pi C_{b'e}\left\{1+\frac{C_{b'c}g_m R_L}{C_{b'e}}\right\}(R_s+r_{b'b})}$$

$$= \frac{f_T R_L}{\left\{1+\frac{2\pi g_m C_{b'c}R_L}{2\pi C_{b'e}}\right\}(R_s+r_{b'b})} \qquad As\ \ f_T = \frac{g_m}{2\pi C_{b'e}}$$

$$= \frac{f_T R_L}{\{1+2\pi f_T C_{b'c}R_L\}(R_s+r_{b'b})}$$

Current-gain bandwidth product is

$$|(A_i)_s f_2| = \frac{f_T R_L}{(1 + 2\pi f_T C_{b'c} R_L)(R_s + r_{b'b})}$$