CLASSICAL CONTROL SYSTEMS ENGINEERING
CS 322

Lecture 5: System Modelling - Block Diagrams
Week 6 SMS II 2022/23

DESIRED OUTCOMES

- Understand relation between **Block Diagrams** and Transfer Function
- Understand **Block Diagrams** reduction techniques

BLOCK DIAGRAMS

Control systems modelling uses two forms of mathematical models

i. Transfer function models

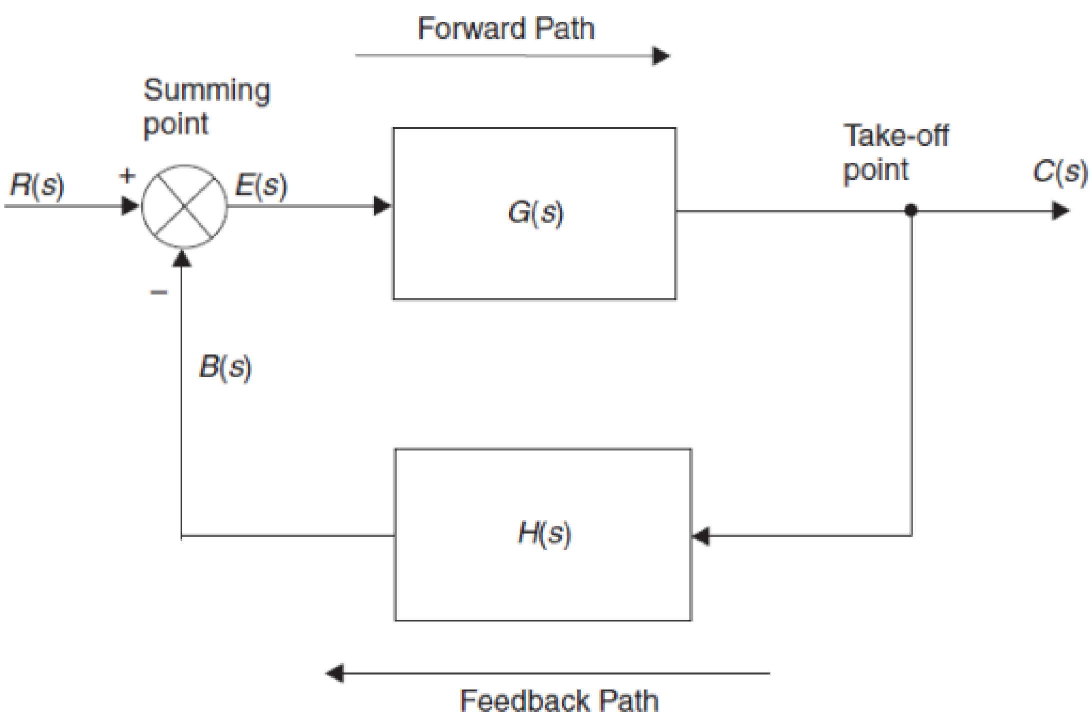
Express relationship between inputs and outputs based on Laplace Transform

ii. Block diagram models

- Block diagrams represent the **relationship** of **system variables** by **diagrammatic** means.
- In Block diagrams, blocks correspond to system elements
- Lines, signal flow; summing and measurement points

BLOCK DIAGRAMS

Transfer function of a block diagram



Where

R(s) is Laplace transform of reference input r(t)

C(s) is Laplace transform of controlled output c(t)

B(s) is primary feedback signal of value H(s)C(s)

E(s) is actuating or error signal of value R(s)-B(s)

G(s) is product of all transfer functions along the forward path

H(s) is product of all transfer functions along the feedback path

G(s)H(s) is **open loop** transfer function

BLOCK DIAGRAMS

Transfer function of a block diagram

From the diagram

$$C(s) = G(s)E(s) \tag{4.1}$$

$$B(s) = H(s)C(s) \tag{4.2}$$

$$E(s) = R(s) - B(s) \tag{4.3}$$

Substituting (4.2) and (4.3) into (4.1)

$$C(s) = G(s)\{R(s) - H(s)C(s)\}$$

$$C(s) = G(s)R(s) - G(s)H(s)C(s)$$

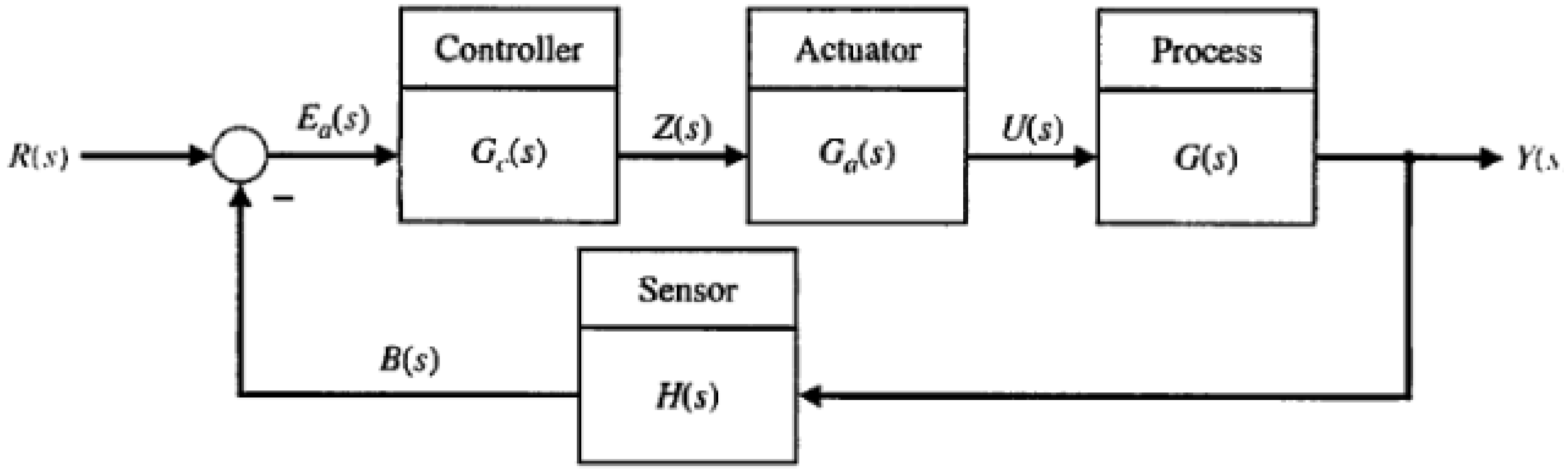
$$C(s)\{1 + G(s)H(s)\} = G(s)R(s)$$

$$\frac{C}{R}(s) = \frac{G(s)}{1 + G(s)H(s)} \tag{4.4}$$

The closed-loop transfer function is the forward-path transfer function divided by one plus the open-loop transfer function.

BLOCK DIAGRAMS

Example 5.1: Find the transfer function for the closed loop control system below



Solution

$$E_a(s) = R(s) - B(s) = R(s) - H(s)Y(s).$$

$$Y(s) = G(s)U(s) = G(s)G_a(s)Z(s) = G(s)G_a(s)G_c(s)E_a(s);$$

$$Y(s) = G(s)G_a(s)G_c(s)[R(s) - H(s)Y(s)].$$

$$Y(s)[1 + G(s)G_a(s)G_c(s)H(s)] = G(s)G_a(s)G_c(s)R(s).$$

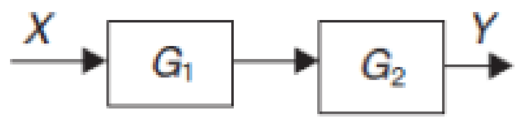
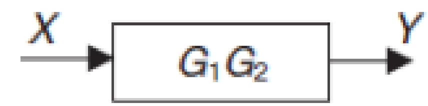
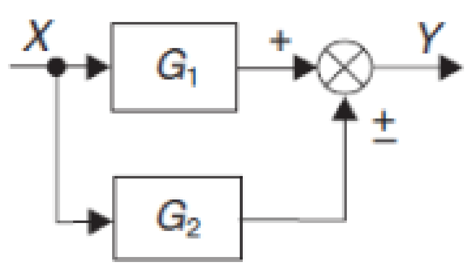
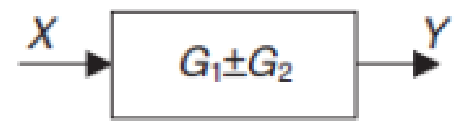
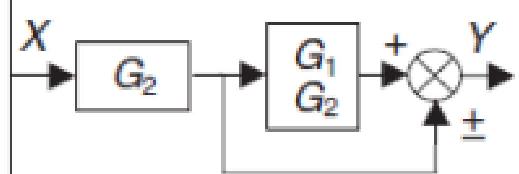
$$\boxed{\frac{Y(s)}{R(s)} = \frac{G(s)G_a(s)G_c(s)}{1 + G(s)G_a(s)G_c(s)H(s)}.}$$

BLOCK DIAGRAM REDUCTION

- A block diagram of a given system can be **reduced** to a **simplified** block diagram with **fewer blocks** than the original diagram.
- This is important especially for control systems with **multiple feedback loops**.
- When **reducing multiple loop** systems, the **minor loops** are considered first, until the system is reduced to a single overall closed loop transfer function.
- In some cases, **reduction** of a block diagram requires to **rearrange positions of elements** in the diagrams.
- Block diagrams **rearrangements** is **simplified** using **Block Diagram Transformation Theorems**.

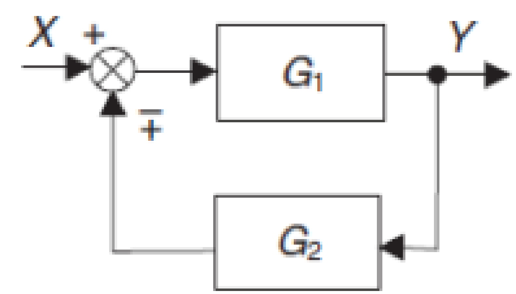
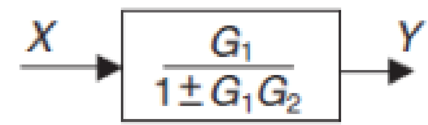
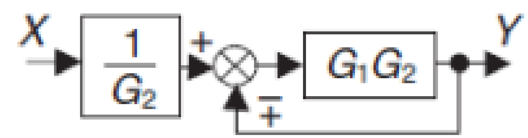
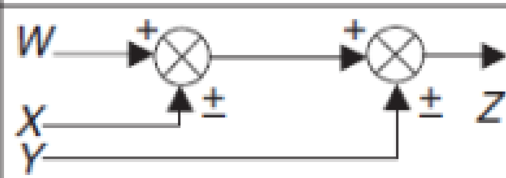
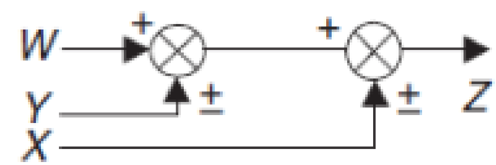
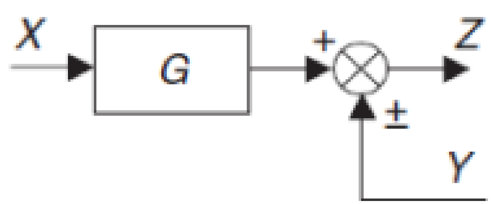
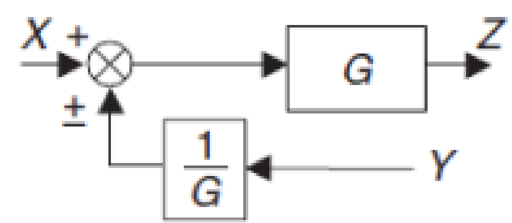
BLOCK DIAGRAM REDUCTION

Block diagram transformation theorems

Transformation	Equation	Block diagram	Equivalent block diagram
1. Combining blocks in cascade	$Y=(G_1G_2)X$		
2. Combining blocks in parallel; or eliminating a forward loop	$Y=G_1X \pm G_2X$		
3. Removing a block from a forward path	$Y=G_1X \pm G_2X$		

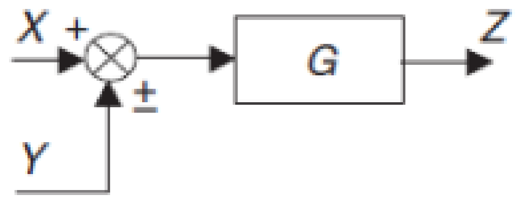
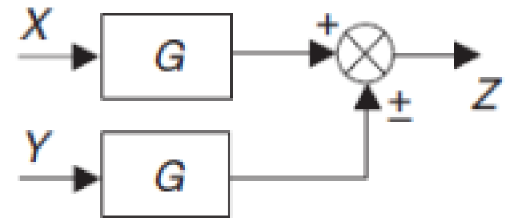
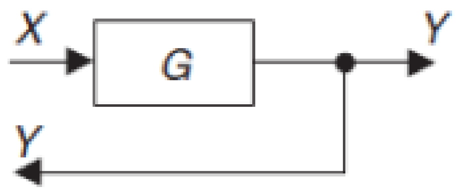
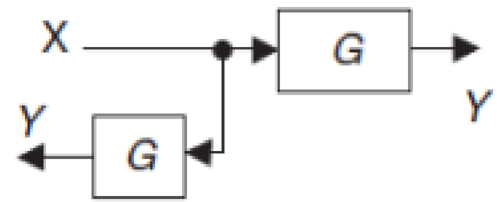
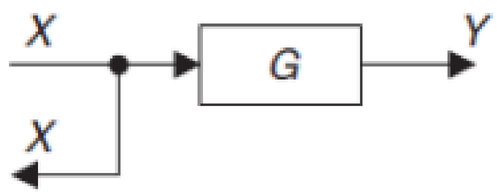
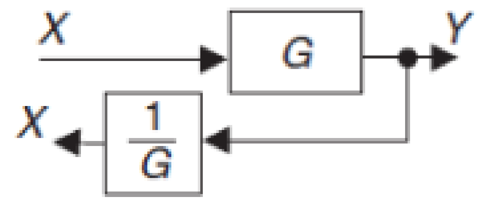
BLOCK DIAGRAM REDUCTION

Block diagram transformation theorems

4. Eliminating a feedback loop	$Y = G_1(X \pm G_2 Y)$		
5. Removing a block from a feedback loop	$Y = G_1(X \pm G_2 Y)$		
6. Rearranging summing points	$Z = W \pm X \pm Y$		
7. Moving a summing point ahead of a block	$Z = GX \pm Y$		

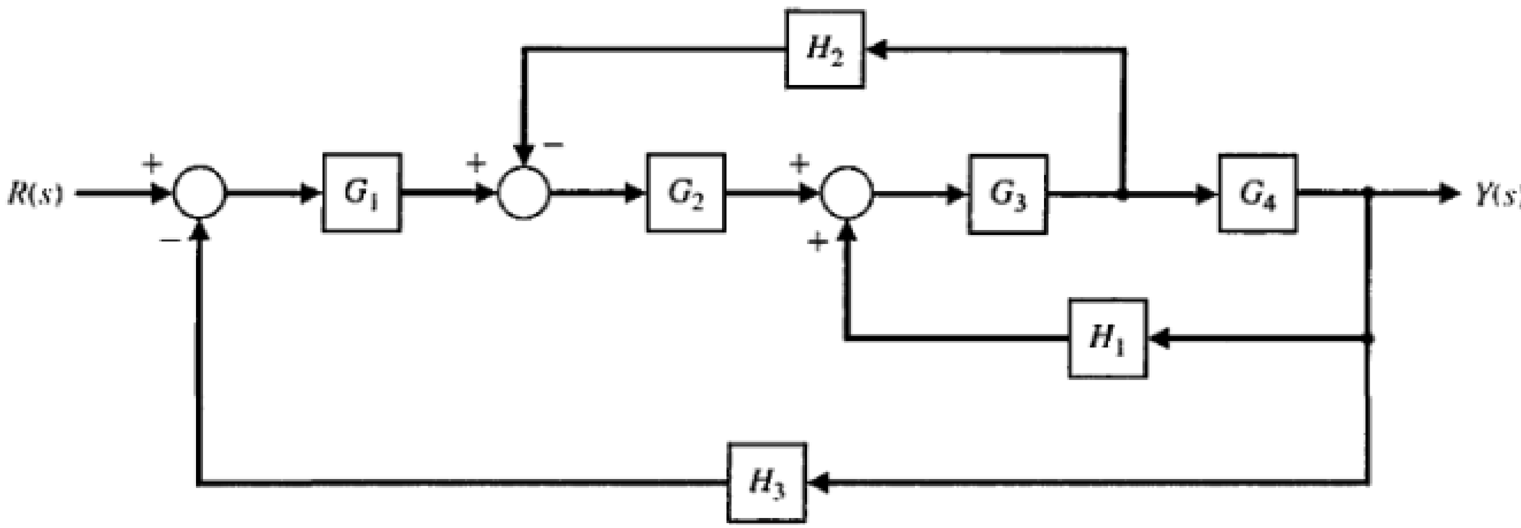
BLOCK DIAGRAM REDUCTION

Block diagram transformation theorems

8. Moving a summing point beyond a block	$Z = G(X \pm Y)$		
9. Moving a take-off point ahead of a block	$Y = GX$		
10. Moving a take-off point beyond a block	$Y = GX$		

BLOCK DIAGRAM REDUCTION

Example 5.2: Find the reduce block diagram for the given figure below



Solution:

1st step: Move H_2 behind block G_4

2nd step: Eliminate the loop $G_3G_4H_1$

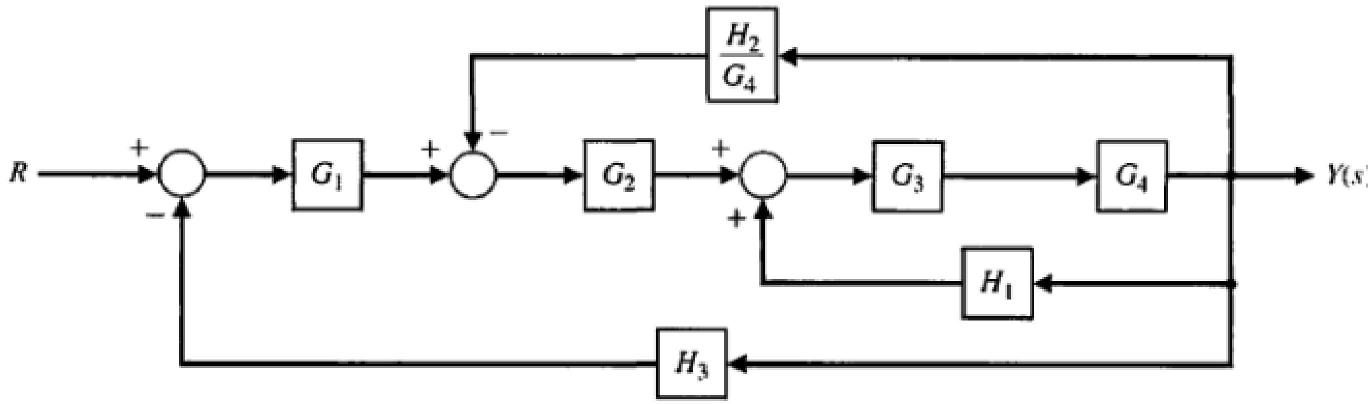
3rd step: Eliminate the inner loop H_2/G_4

4th step: Reduce the loop containing H_3

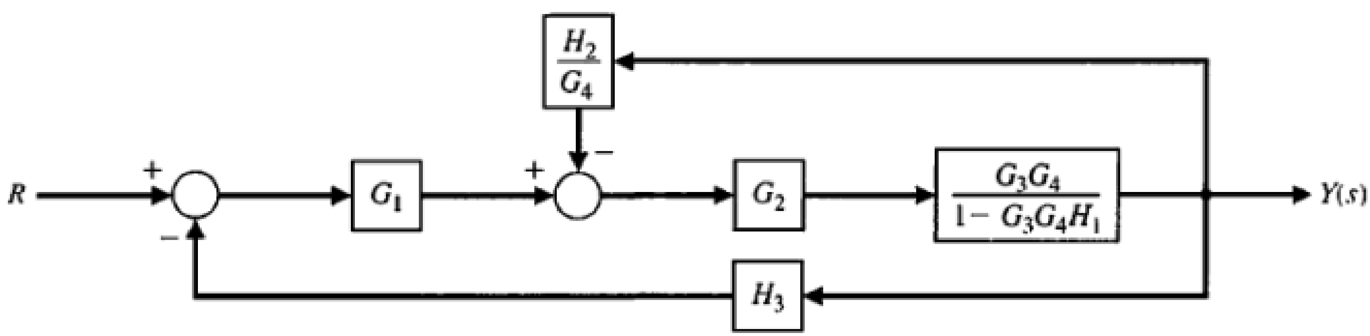
BLOCK DIAGRAM REDUCTION

Example 5.2: Solution

1st : Move H_2 behind block G_4 using transformation 10



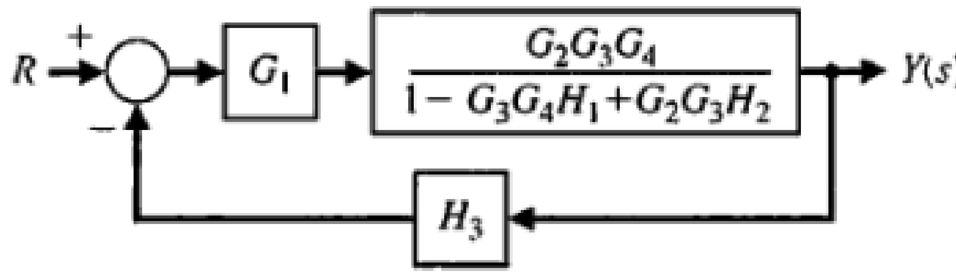
2nd : Eliminate the loop $G_3G_4H_1$ using transformation 4



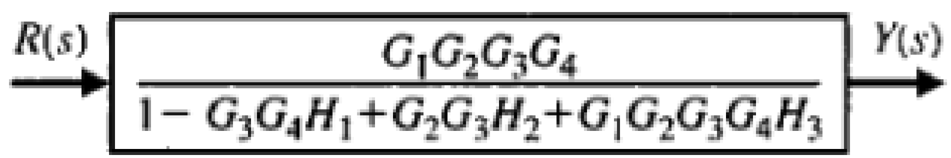
BLOCK DIAGRAM REDUCTION

Example 5.2: Solution

3rd : Eliminate the inner loop H_2/G_4

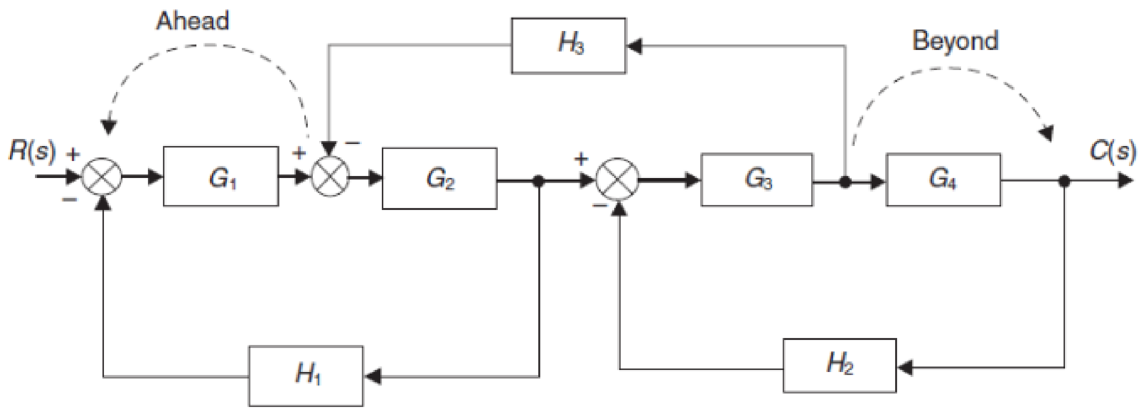


4th : Reduce the loop containing H_3



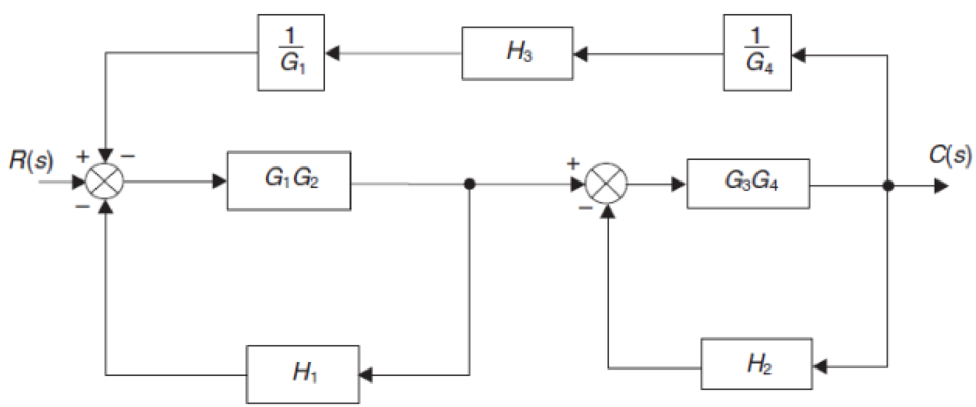
BLOCK DIAGRAM REDUCTION

Example 5.3: Find the overall closed loop transfer function for the system below



Solution:

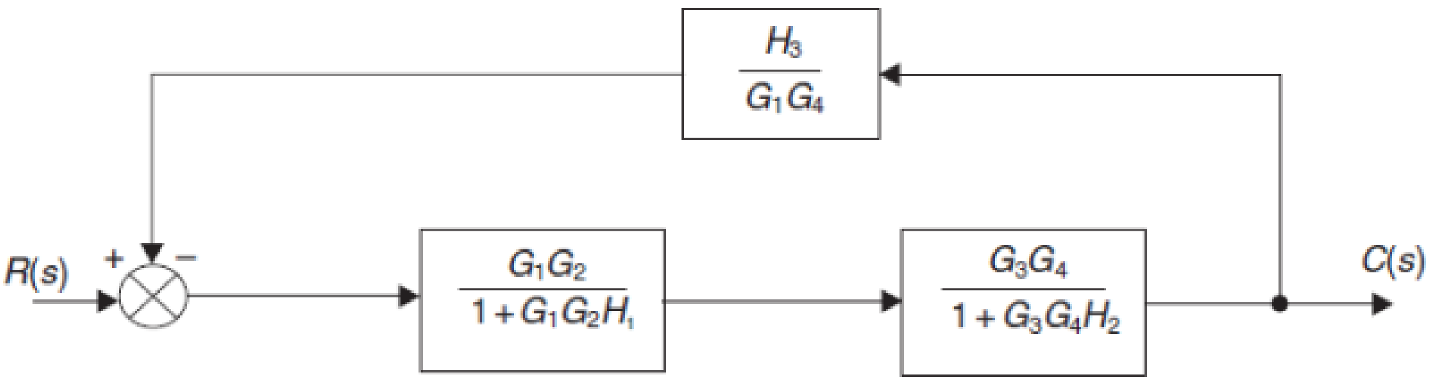
1st : Move H_3 ahead of G_1 and beyond G_4 , combine G_1G_2 and G_3G_4



BLOCK DIAGRAM REDUCTION

Example 5.3: Solution

2nd : Reduce the loops $G_1G_2H_1$ and $G_3G_4H_2$



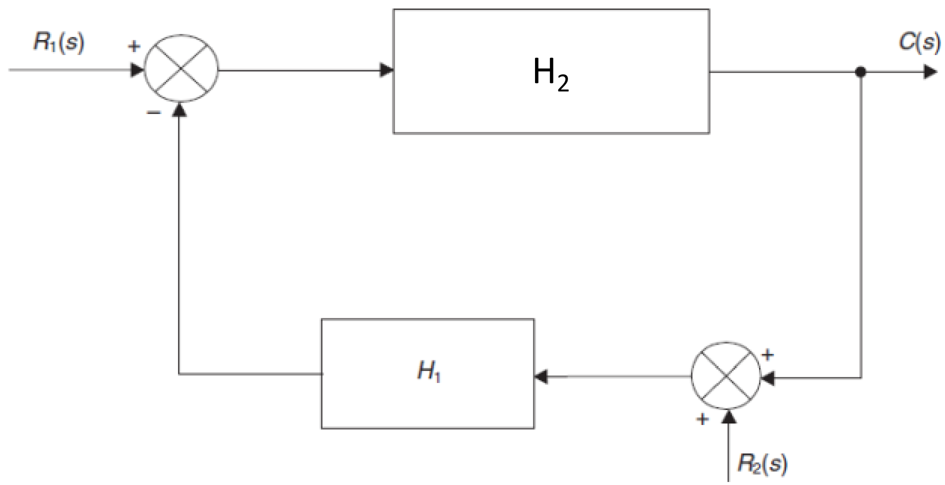
The overall transfer function

$$\frac{C}{R}(s) = \frac{G_1(s)G_2(s)G_3(s)G_4(s)}{(1+G_1(s)G_2(s)H_1(s))(1+G_3(s)G_4(s)H_2(s)) + G_2(s)G_3(s)H_3(s)}$$

BLOCK DIAGRAM REDUCTION

Systems with multiple inputs

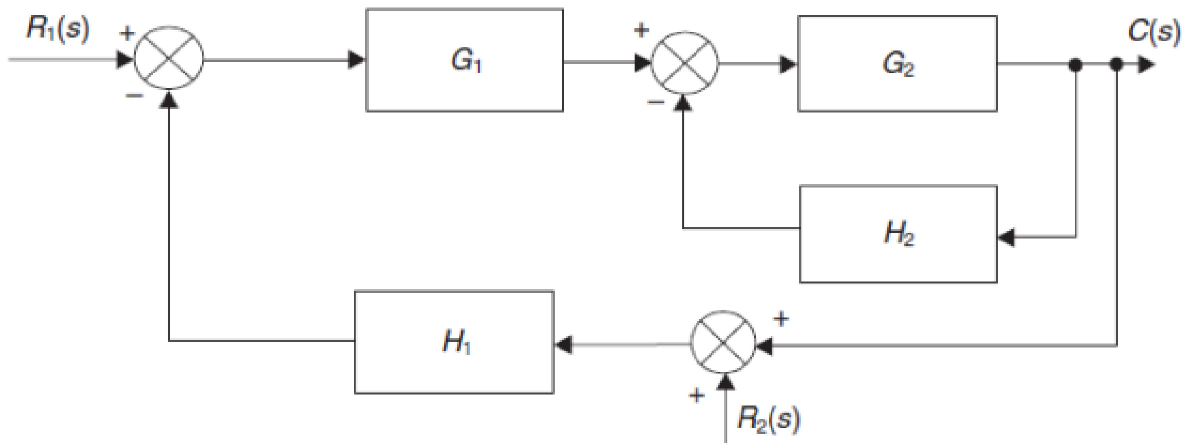
We use the **Principle of Superposition** for analysis of systems with **multiple inputs**.



The principle states that "The **response** c(t) of a **linear system** due to **several inputs** $r_1(t)$, $r_2(t)$,….,$r_n(t)$ acting **simultaneously** is equal to the **sum** of the responses of **each input** acting alone."

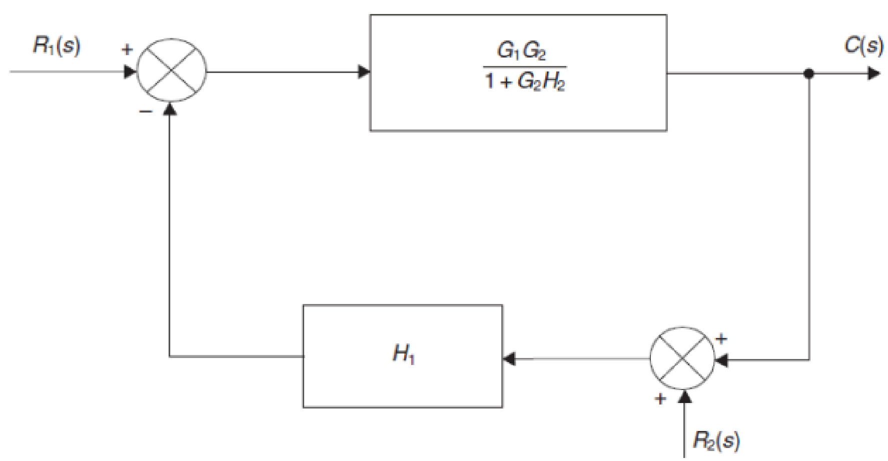
BLOCK DIAGRAM REDUCTION

Example 5.4: Find the overall output for the system in the figure below



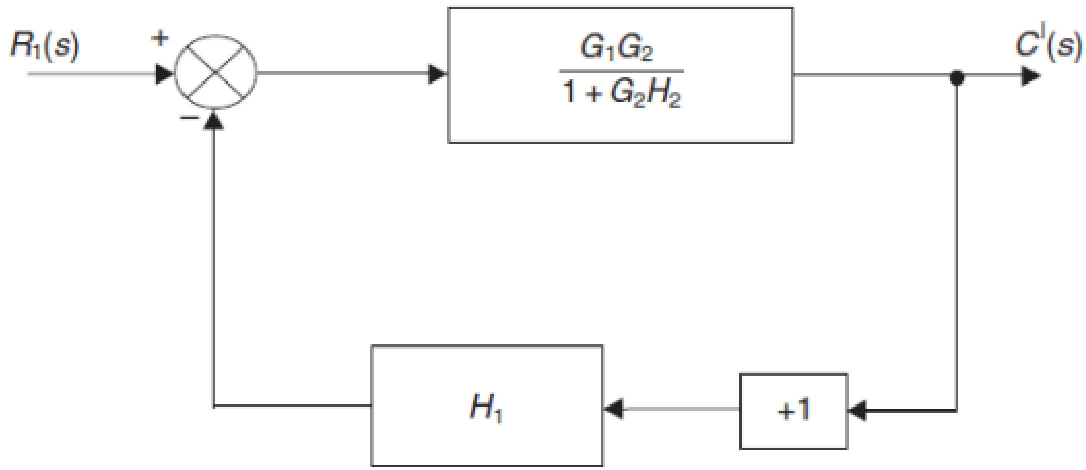
Solution:

1st : Remove the inner loop G_2H_2 and sum with G_1



BLOCK DIAGRAM REDUCTION

2nd : Consider input $R_1(s)$ alone

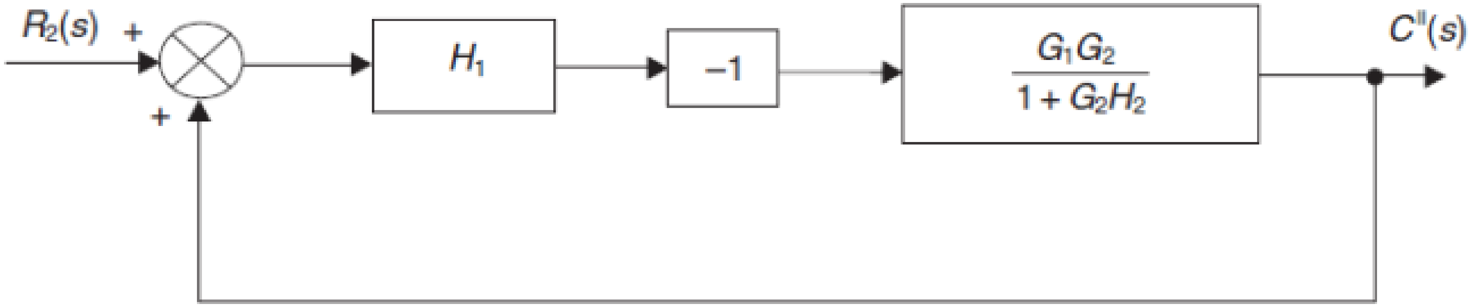


$$\frac{C^{\mathrm{I}}}{R_1}(s) = \frac{\frac{G_1G_2}{1+G_2H_2}}{1+\frac{G_1G_2H_1}{1+G_2H_2}}$$

$$C^{\mathrm{I}}(s) = \frac{G_1(s)G_2(s)R_1(s)}{1+G_2(s)H_2(s)+G_1(s)G_2(s)H_1(s)}$$

BLOCK DIAGRAM REDUCTION

3rd : Considering input $R_2(s)$ alone



$$C^{II}(s) = \frac{-G_1(s)G_2(s)H_1(s)R_2(s)}{1 + G_2(s)H_2(s) + G_1(s)G_2(s)H_1(s)}$$

Using Principle of Superposition $C(s) = C^{I}(s) + C^{II}(s)$

$$C(s) = \frac{(G_1(s)G_2(s))R_1(s) - (G_1(s)G_2(s)H_1(s))R_2(s)}{1 + G_2(s)H_2(s) + G_1(s)G_2(s)H_1(s)}$$

BLOCK DIAGRAM REDUCTION

Example 5.5 : Find the overall closed loop transfer function for the system below

(Class work 10 mins)