

102001208  
ENGINEERING GRAPHICS

**Engineering Curves**

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# Outline

- ◆ Introduction – Engineering Curves
- ◆ Application – Engineering Curves
- ◆ Different Methods of Drawing  
Engineering Curves
- ◆ Tangent and Normal to Engineering  
Curves

# USES OF ENGINEERING CURVES

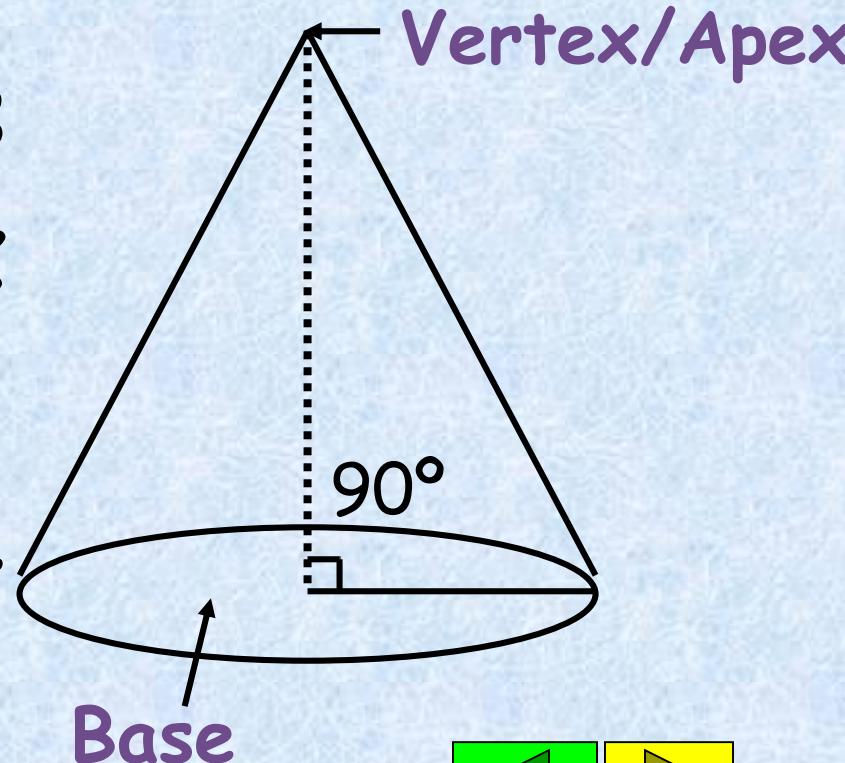
- Useful by their nature & characteristics.
- Laws of nature represented on graph.
- Useful in engineering in understanding laws, manufacturing of various items, designing mechanisms analysis of forces, construction of bridges, dams, water tanks etc.

# **CLASSIFICATION OF ENGG. CURVES**

1. CONICS
2. CYCLOIDAL CURVES
3. INVOLUTE
4. SPIRAL
5. HELIX
6. SINE & COSINE

# What is Cone ?

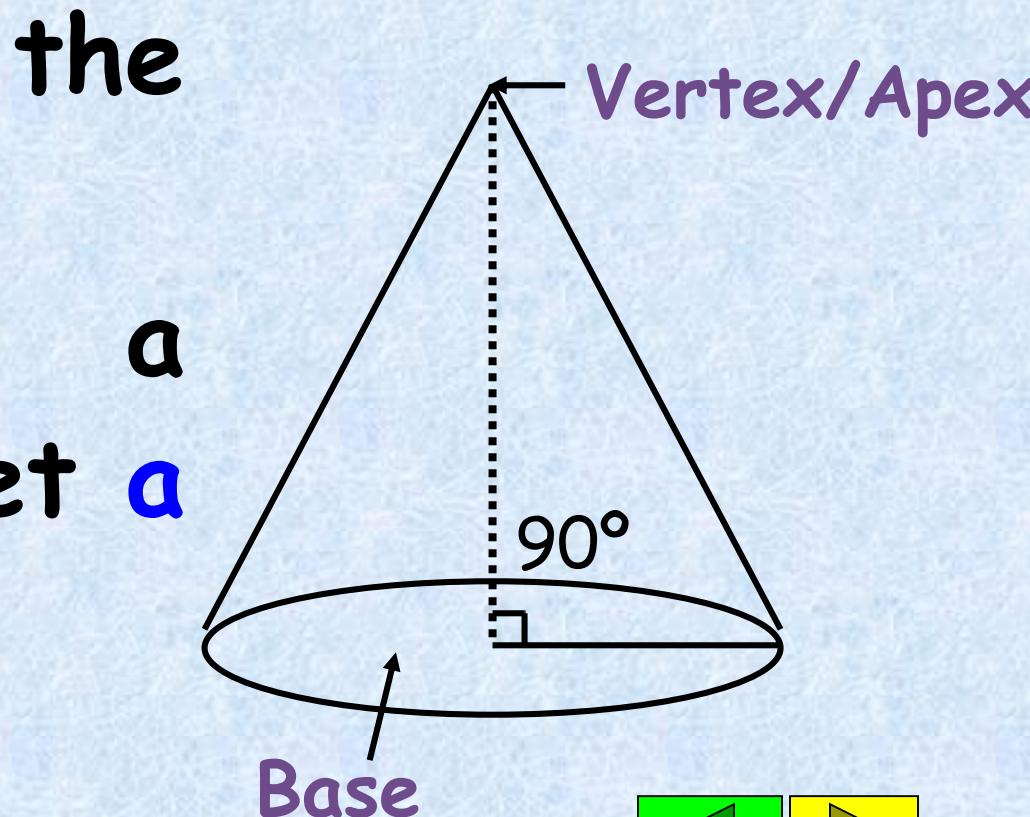
- ★ It is a surface generated by moving a Straight line keeping one of its end fixed & other end makes a closed curve.
- ★ The fixed point is known as vertex or apex.
- ★ The closed curve is known as base.



# What is Cone ?

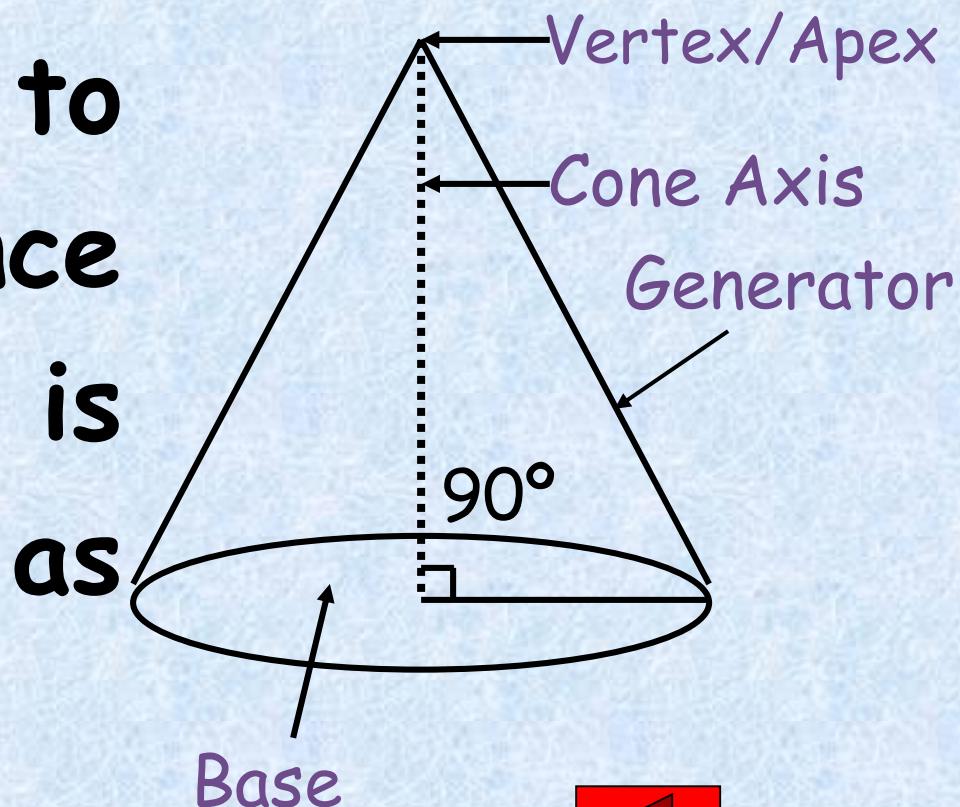
\* If the base/closed curve is a circle, we get a cone.

\* If the base/closed curve is a polygon, we get a pyramid.



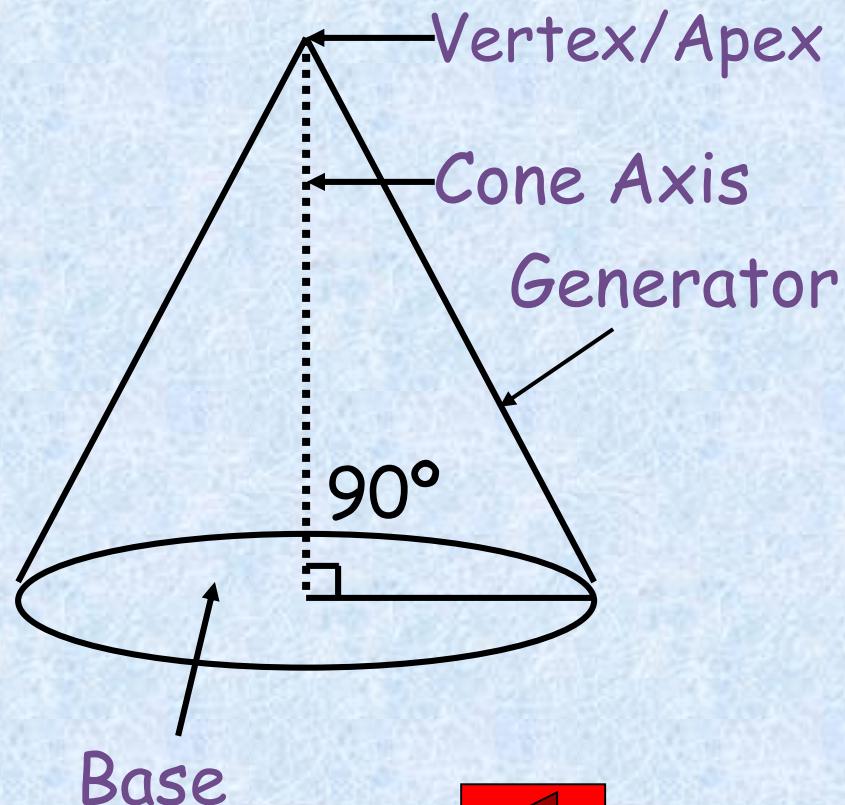
★ The line joins apex to the center of base is called **axis**.

★ The line joins vertex/ apex to the circumference of a cone is known as **generator**.



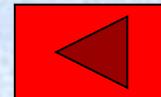
★ If axes is perpendicular to base, it is called as **right circular cone**.

★ If axis of cone is not perpendicular to base, it is called as **oblique cone**.



# **CONICS**

\* **Definition :-** The section obtained by the intersection of a right circular cone by a cutting plane in different position relative to the axis of the cone are called **CONICS.**



# CONICS

A - TRIANGLE

B - CIRCLE

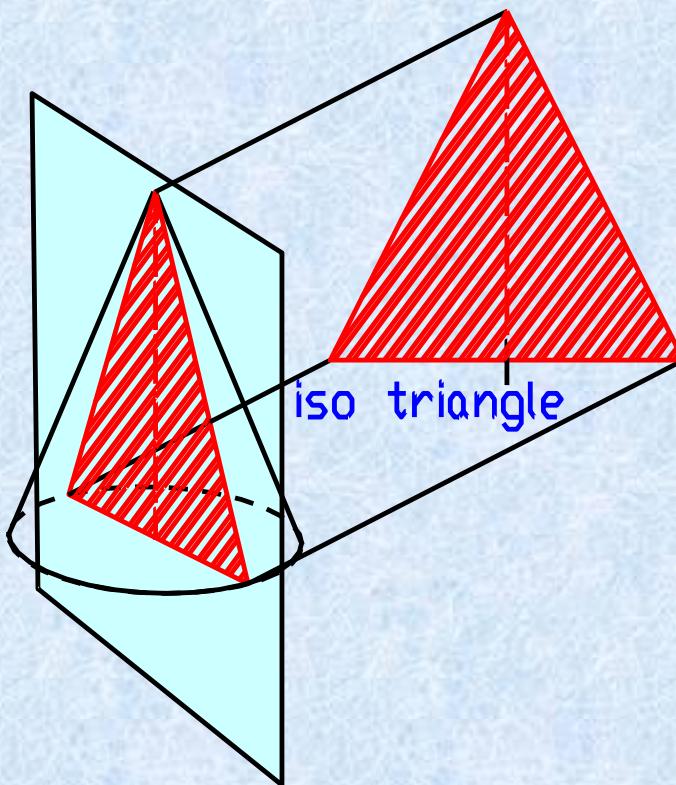
C - ELLIPSE

D - PARABOLA

E - HYPERBOLA

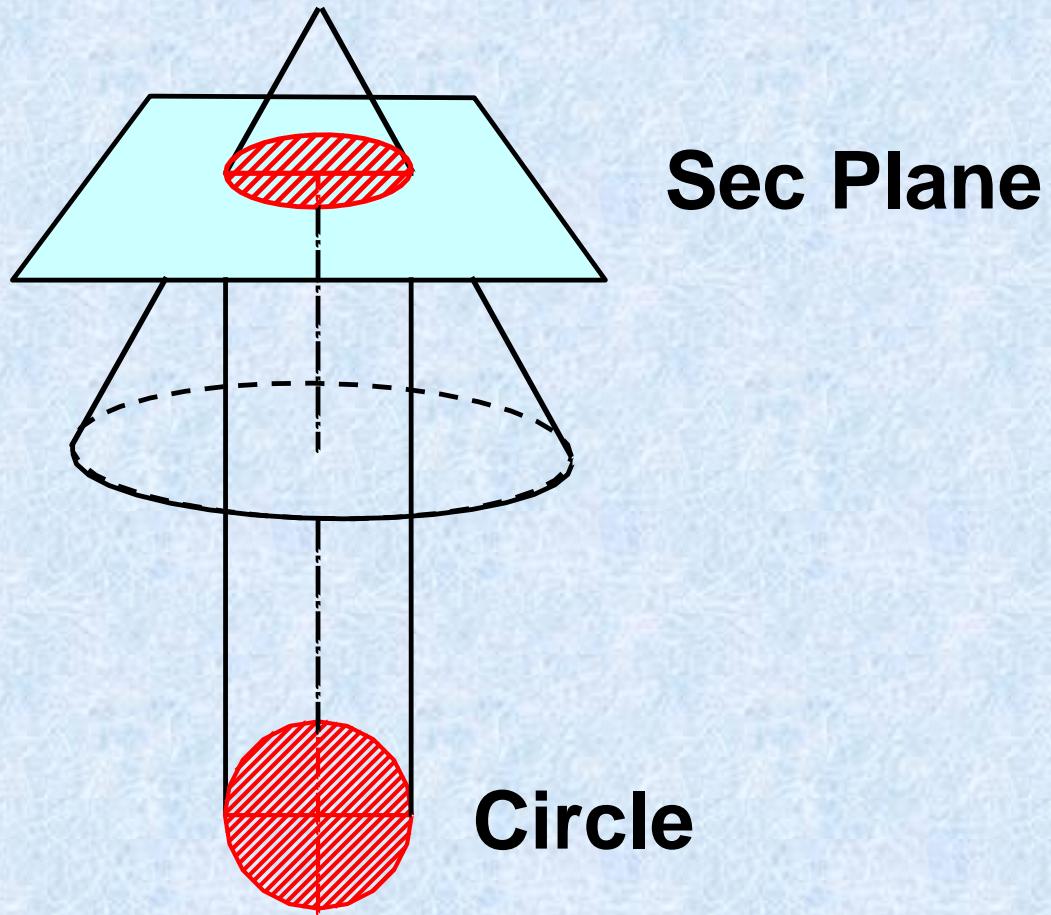
# TRIANGLE

- When the cutting plane contains the apex, we get a triangle as the section.



# CIRCLE

- When the cutting plane is perpendicular to the axis or parallel to the base in a right cone we get circle the section.

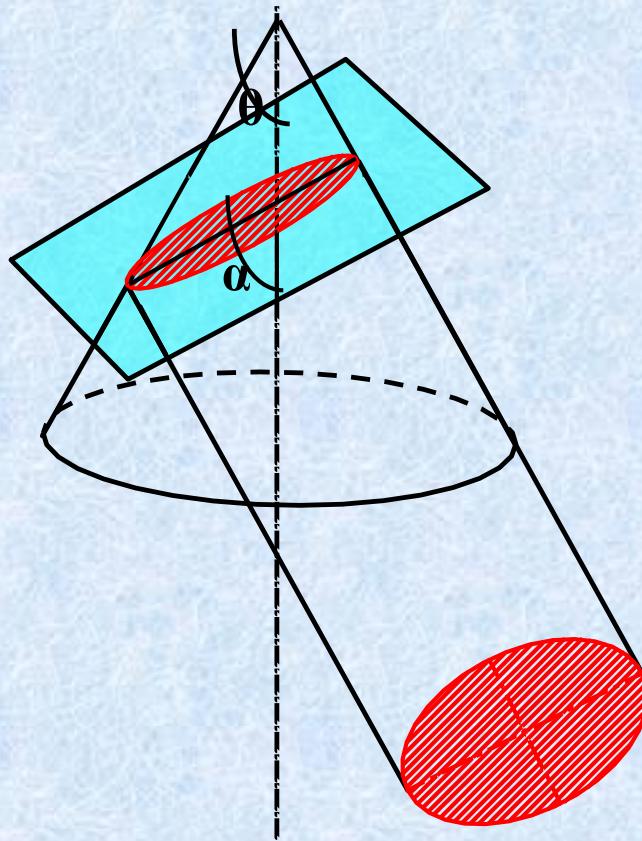


# ELLIPSE

Definition :-

→ When the cutting plane is inclined to the axis but not parallel to generator or the inclination of the cutting plane( $\alpha$ ) is greater than the semi cone angle( $\theta$ ), we get an ellipse as the section.

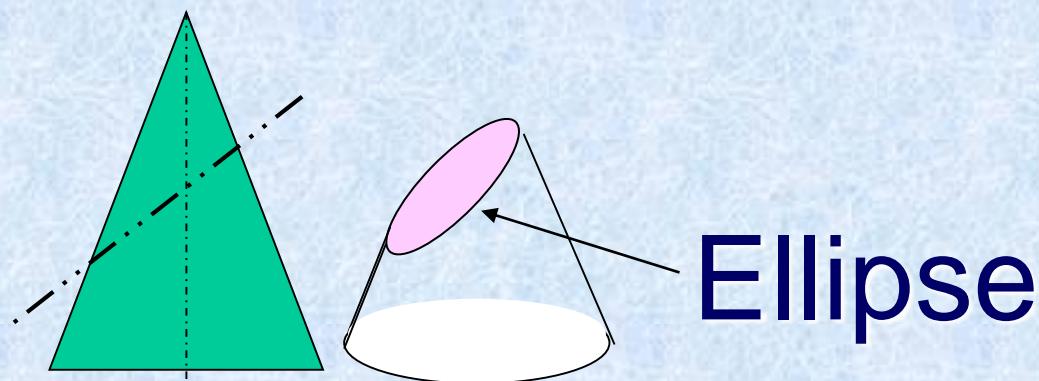
$$\alpha > \theta$$



# ELLIPSE

Definition :-

- When the cutting plane is inclined to the axis but not parallel to generator or the inclination of the cutting plane( $a$ ) is greater than the semi cone angle( $\theta$ ), we get an ellipse as the section.

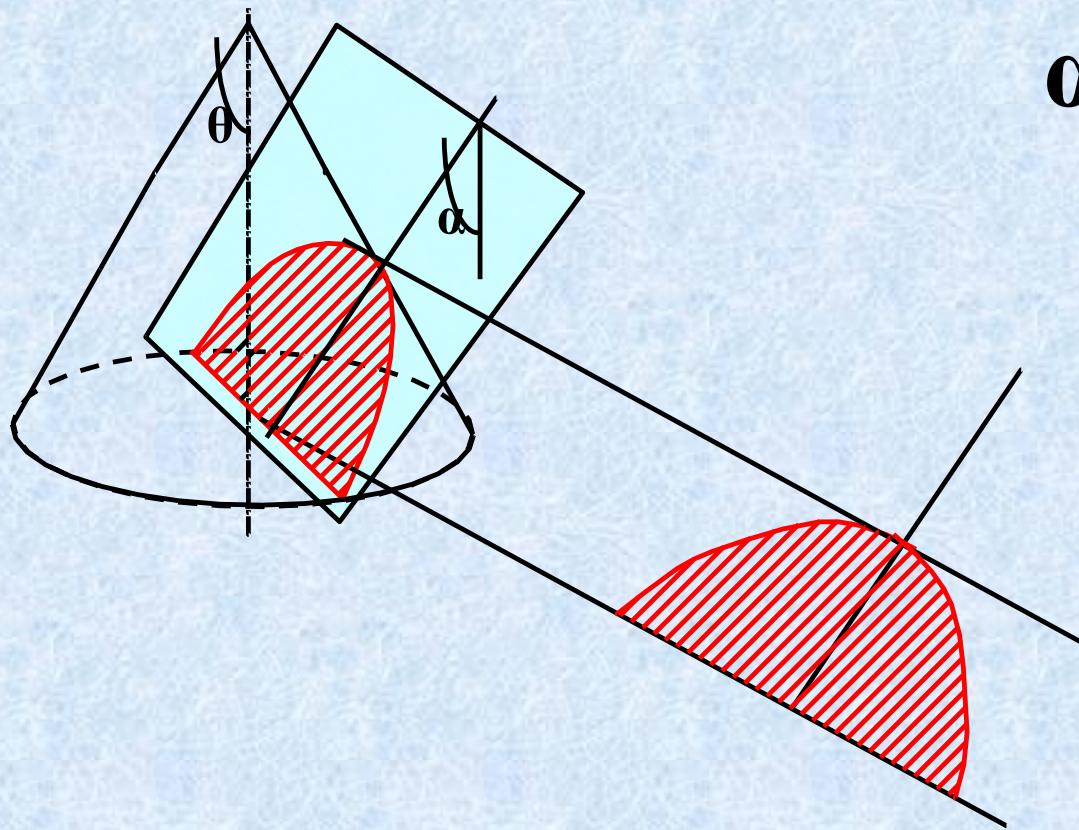


Section Plane

Through Generators

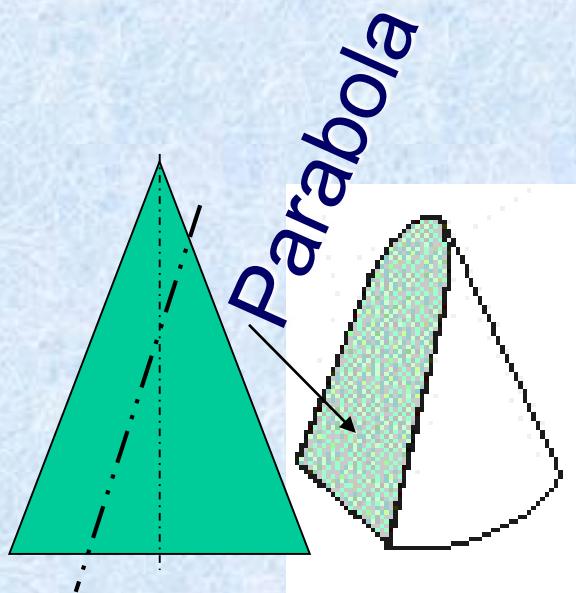
# PARABOLA

- When the cutting plane is inclined to the axis and parallel to one of the generators of the cone or the inclination of the plane( $\alpha$ ) is equal to semi cone angle( $\theta$ ), we get a parabola as the section.



# PARABOLA

- When the cutting plane is inclined to the axis and parallel to one of the generators of the cone or the inclination of the plane( $a$ ) is equal to semi cone angle( $\theta$ ), we get a parabola as the section.



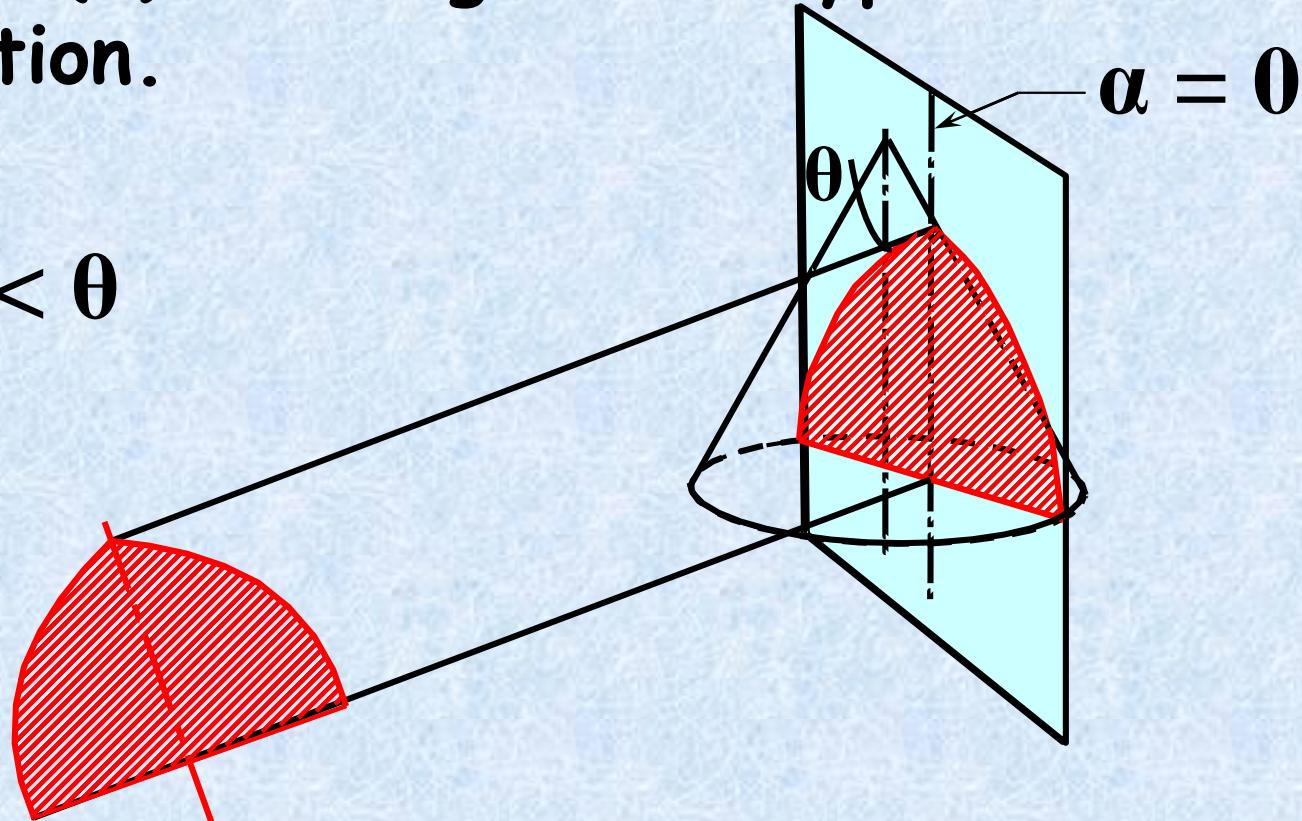
**Section Plane Parallel to  
end generator.**

# HYPERBOLA

Definition :-

- When the cutting plane is parallel to the axis or the inclination of the plane with cone axis( $\alpha$ ) is less than semi cone angle( $\theta$ ), we get a hyperbola as the section.

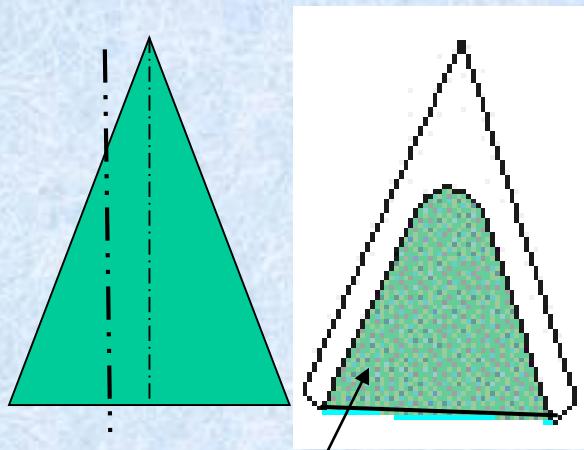
$$\alpha < \theta$$



# HYPERBOLA

**Definition :-**

- When the cutting plane is parallel to the axis or the inclination of the plane with cone axis( $a$ ) is less than semi cone angle( $\theta$ ), we get a hyperbola as the section.

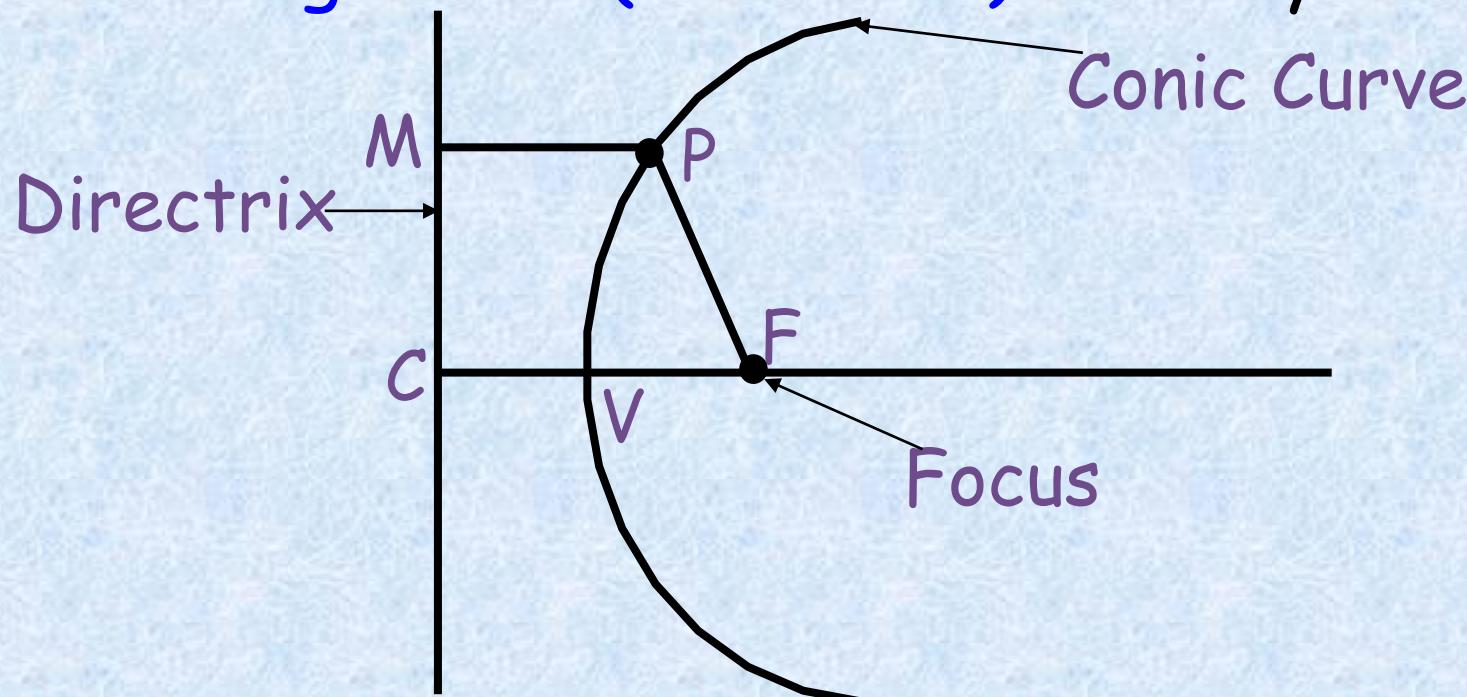


**Section Plane**      **Hyperbola**

**Parallel to Axis.**

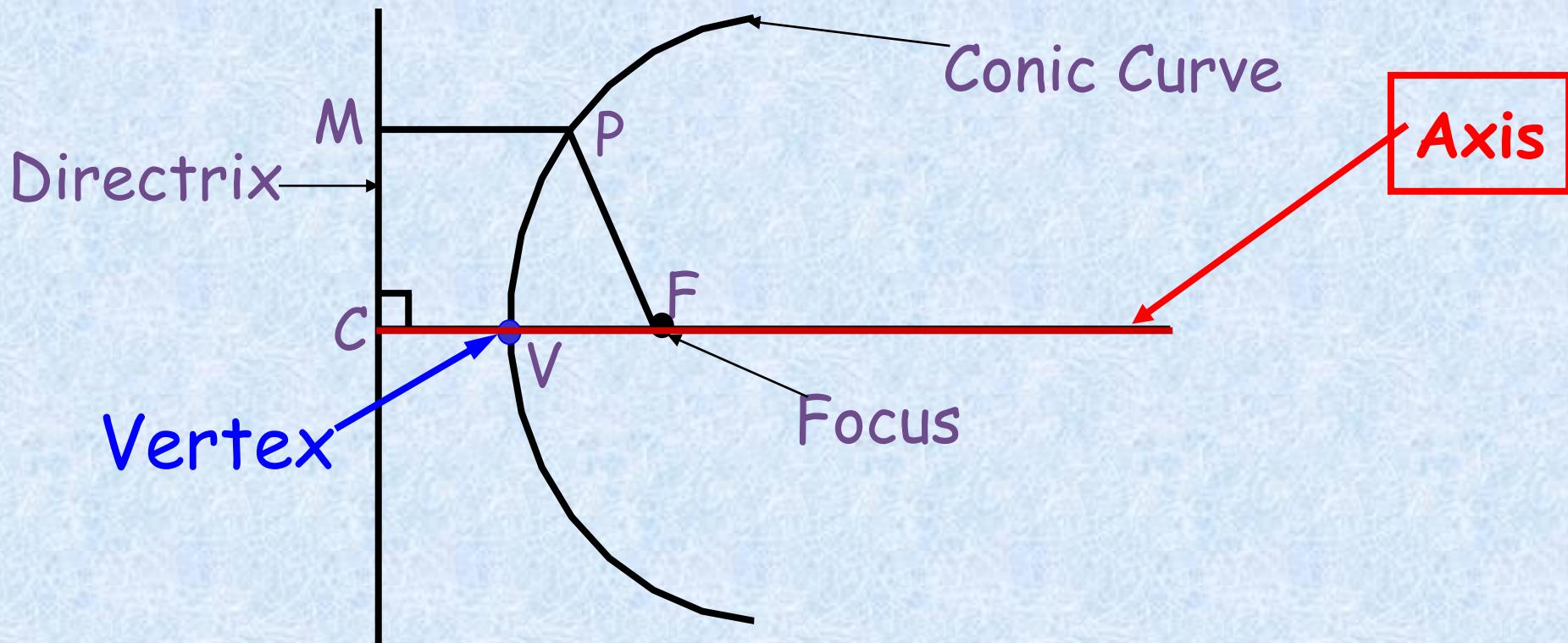
# CONICS

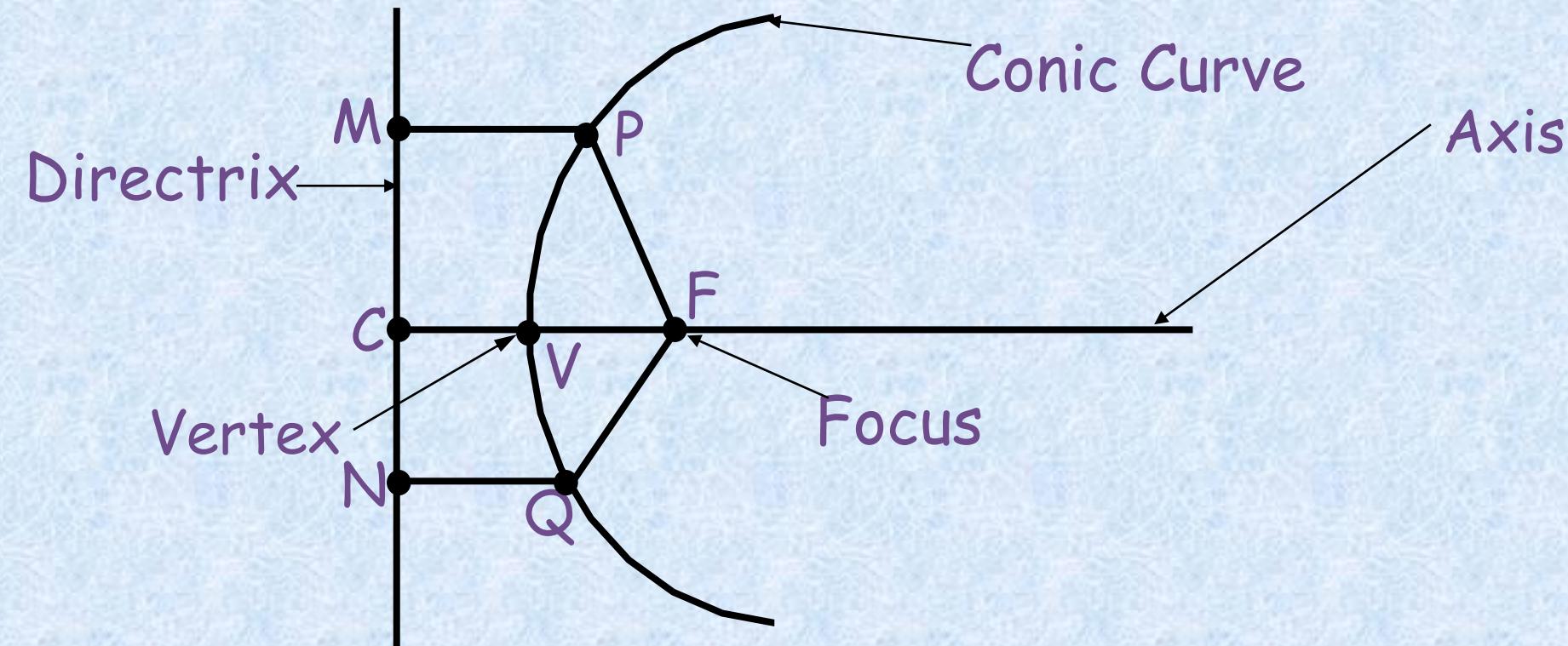
- ★ Definition :- The locus of point moves in a plane such a way that the ratio of its distance from **fixed point (focus)** to a **fixed Straight line (Directrix)** is always constant.



- ★ Fixed straight line is called as **directrix**.
- ★ Fixed point is called as **focus**.

- ★ The line passing through focus & perpendicular to directrix is called as **axis**.
- ★ The intersection of conic curve with axis is called as **vertex**.





Ratio =  $\frac{\text{Distance of a point from focus}}{\text{Distance of a point from directrix}}$

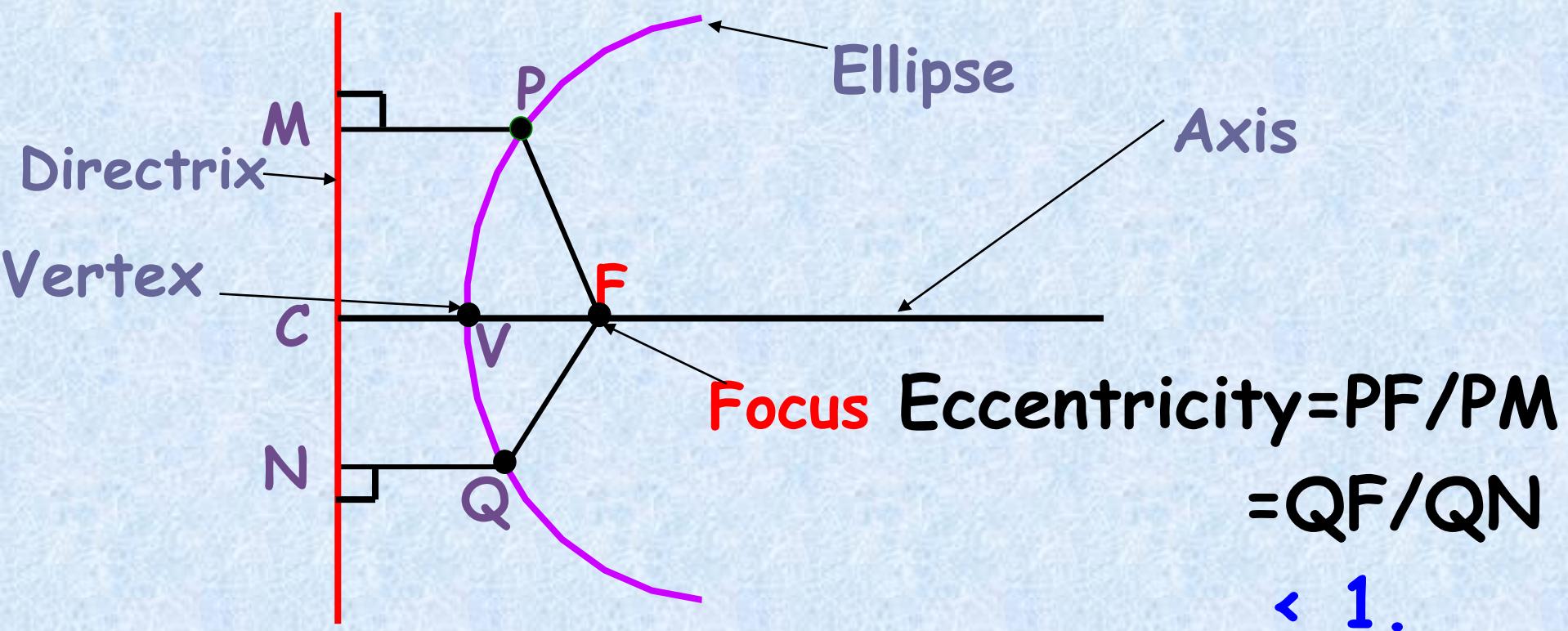
= **ECCENTRICITY**

=  $PF/PM = QF/QN = VF/VC$

= **e**

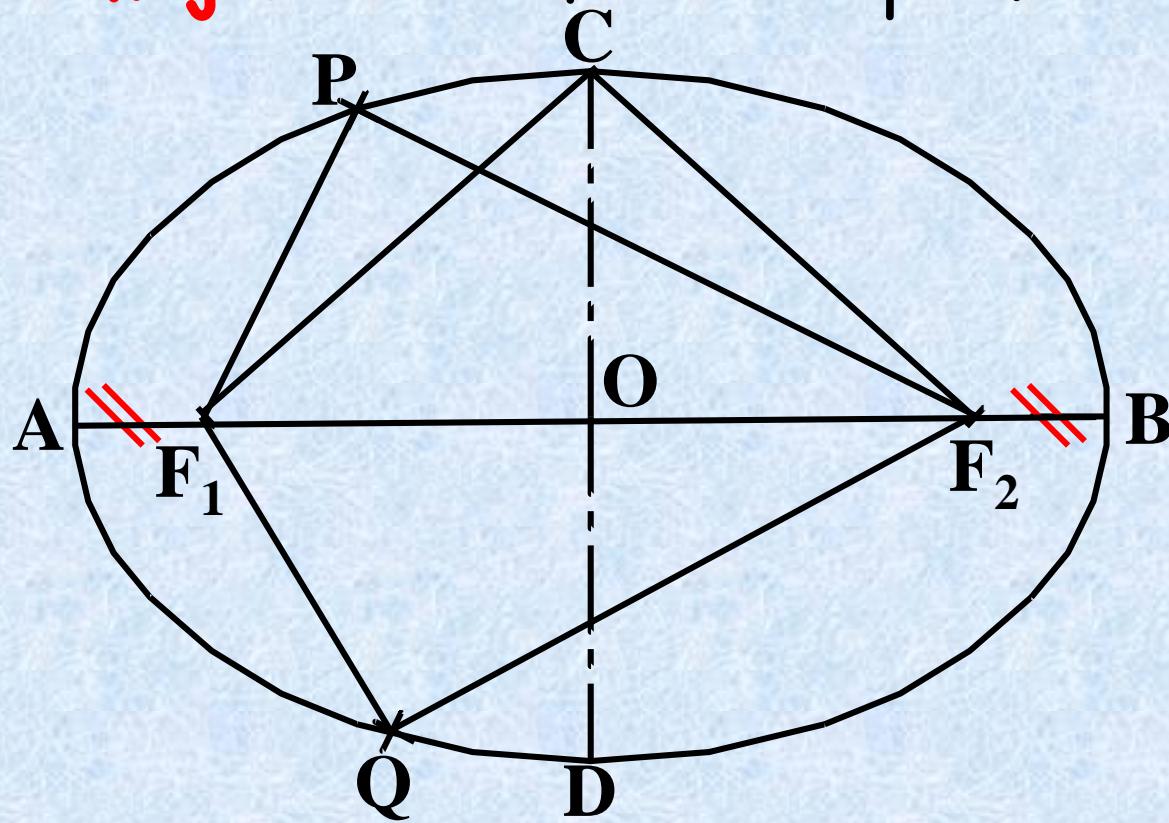
# ELLIPSE

→ Ellipse is the locus of a point which moves in a plane so that the ratio of its distance from a **fixed point (focus)** and a **fixed straight line (Directrix)** is a constant and **less than one.**



# ELLIPSE

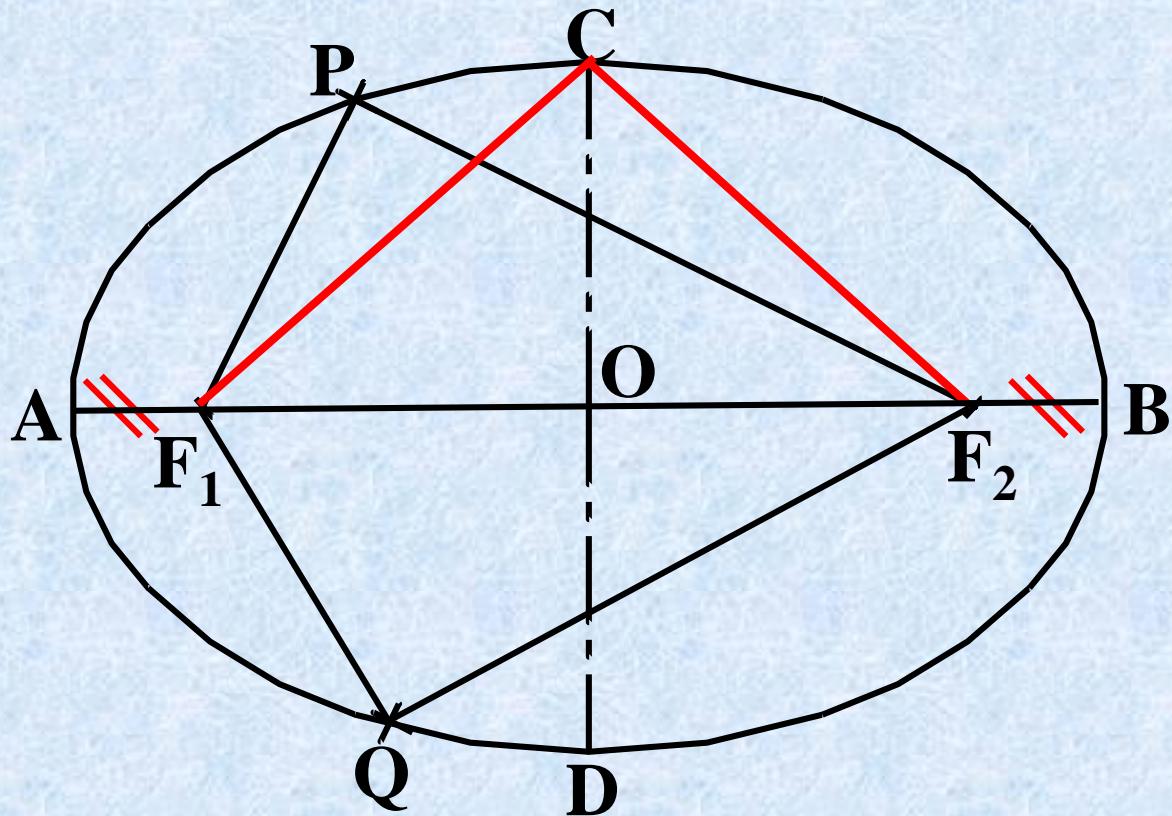
- Ellipse is the locus of a point, which moves in a plane so that the sum of its distance from two fixed points, called focal points or foci, is a constant. The **sum of distances** is equal to the **major axis** of the ellipse.



$$PF_1 + PF_2 = AB$$

$$QF_1 + QF_2 = AB$$

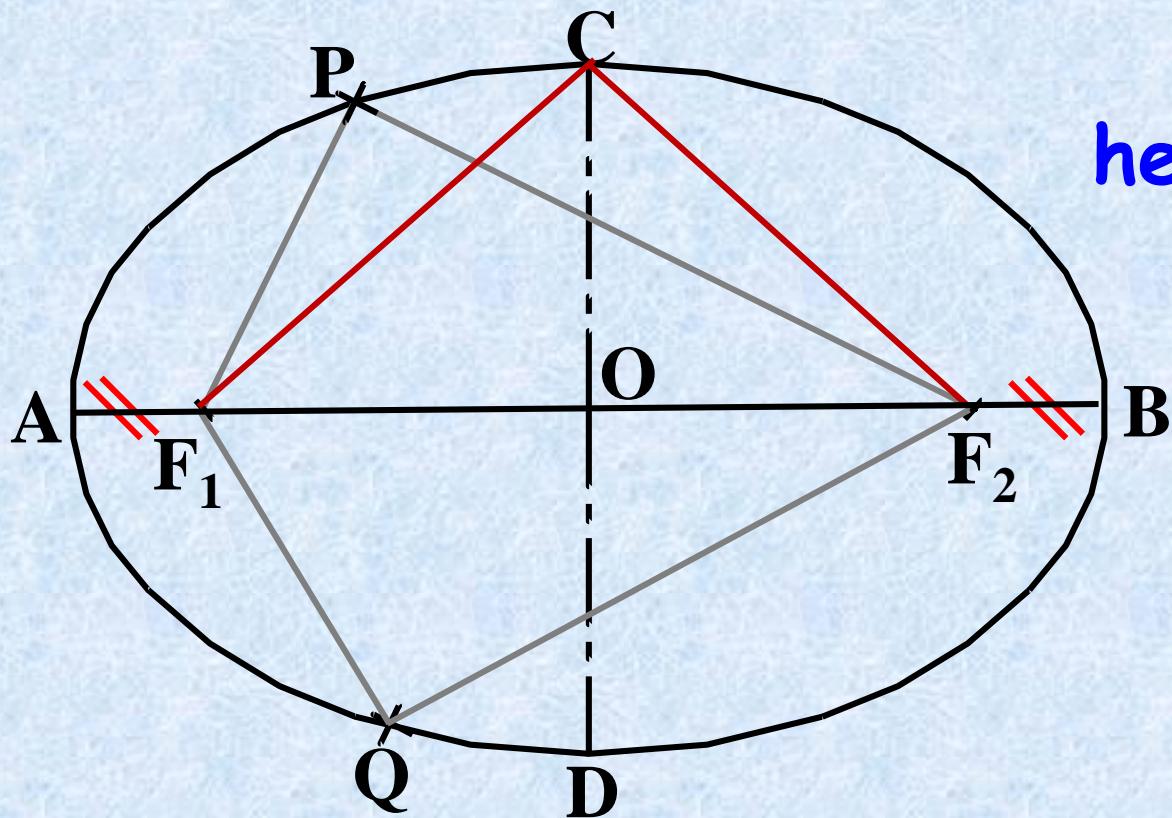
$$CF_1 + CF_2 = AB$$

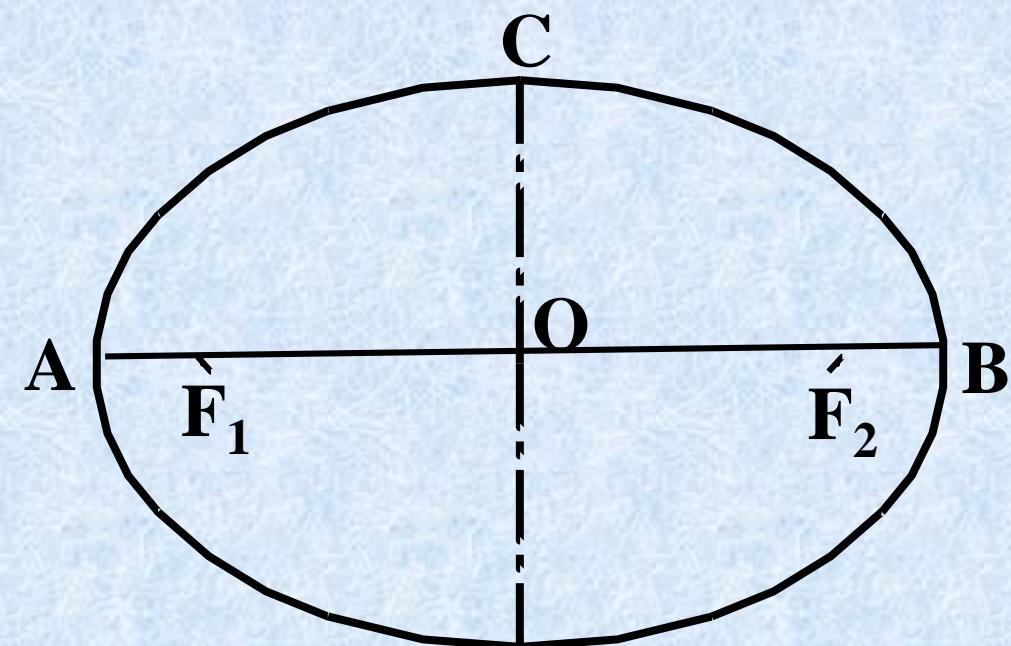


$$CF_1 + CF_2 = AB$$

$$\text{but } CF_1 = CF_2$$

hence,  $CF_1 = 1/2AB$

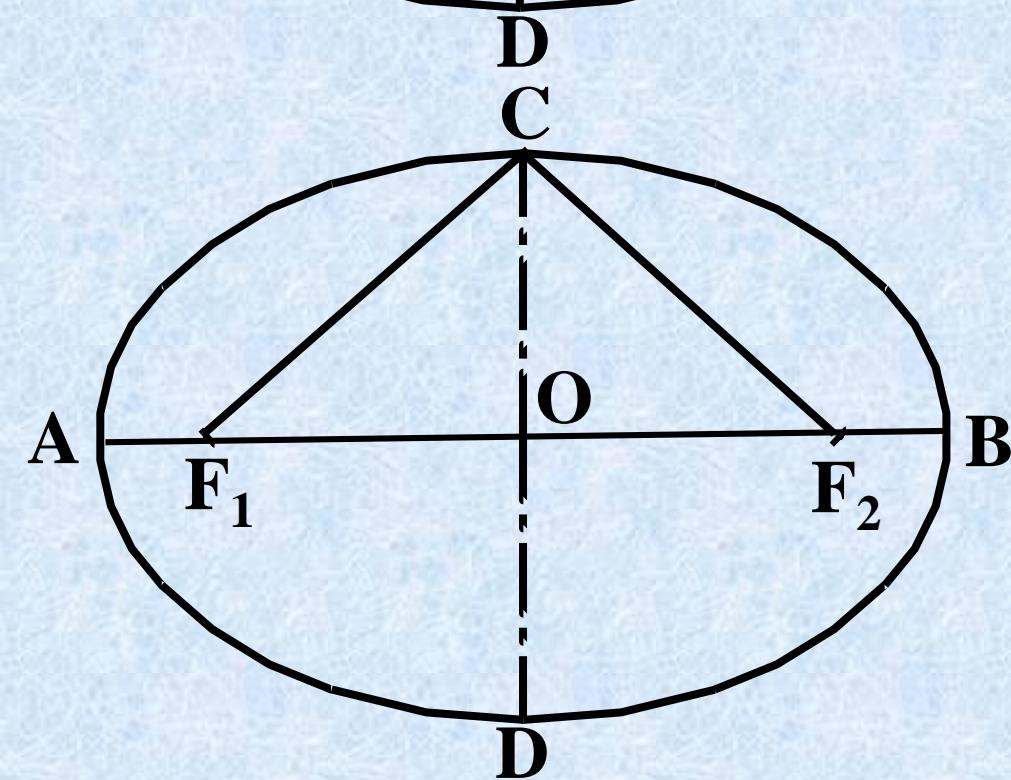




Major Axis = 100 mm

Minor Axis = 60 mm

$$CF_1 = \frac{1}{2} AB = AO$$



Major Axis = 100 mm

$F_1F_2 = 60 \text{ mm}$

$$CF_1 = \frac{1}{2} AB = AO$$

Uses :-

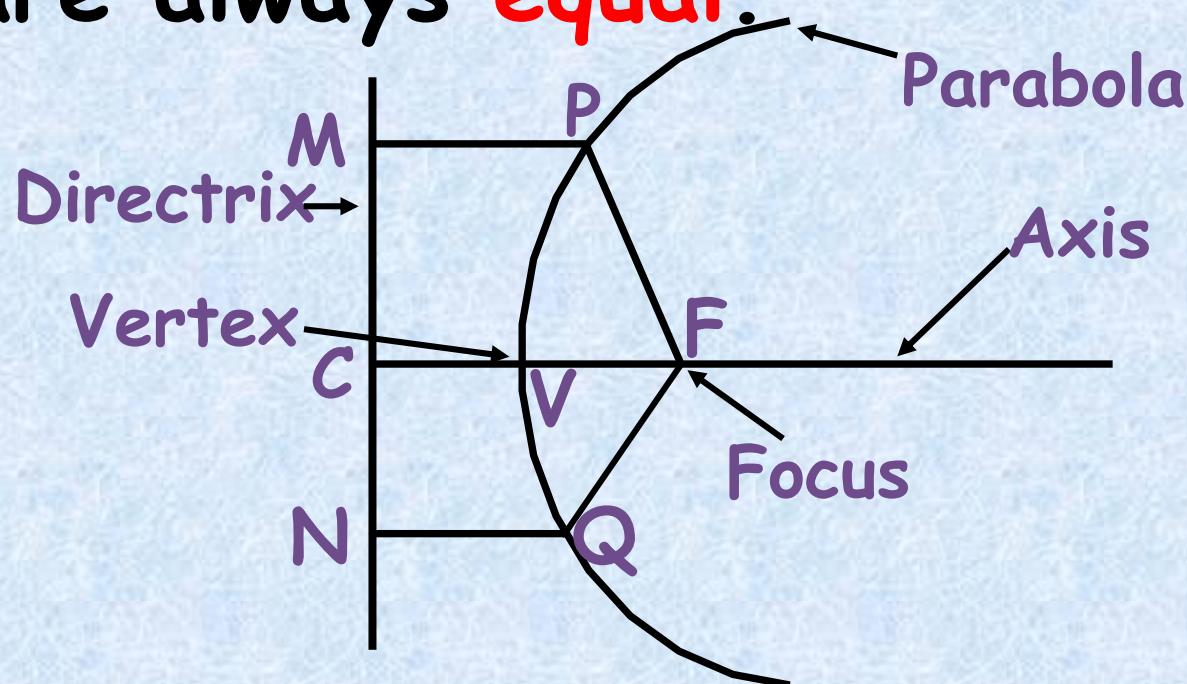
# ELLIPSE

- Shape of a man-hole.
- Shape of tank in a tanker.
- Flanges of pipes, glands and stuffing boxes.
- Shape used in bridges and arches.
- Monuments.
- Path of earth around the sun.
- Shape of trays etc.

# PARABOLA

Definition :-

► The parabola is the locus of a point, which moves in a plane so that its distance from a fixed point (focus) and a fixed straight line (directrix) are always **equal**.



# PARABOLA

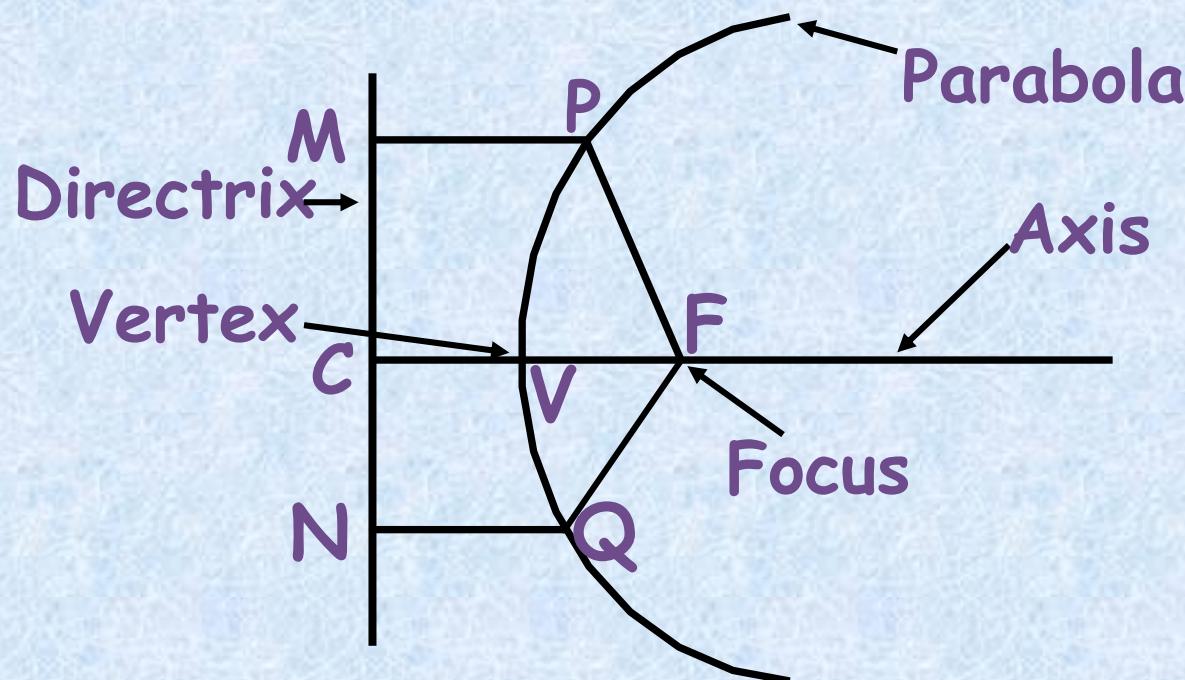
► Ratio (known as eccentricity) of its distances from **focus** to that of **directrix** is **constant** and **equal to one (1)**.

## Eccentricity

$$= PF/PM$$

$$= QF/QN$$

$$= 1.$$



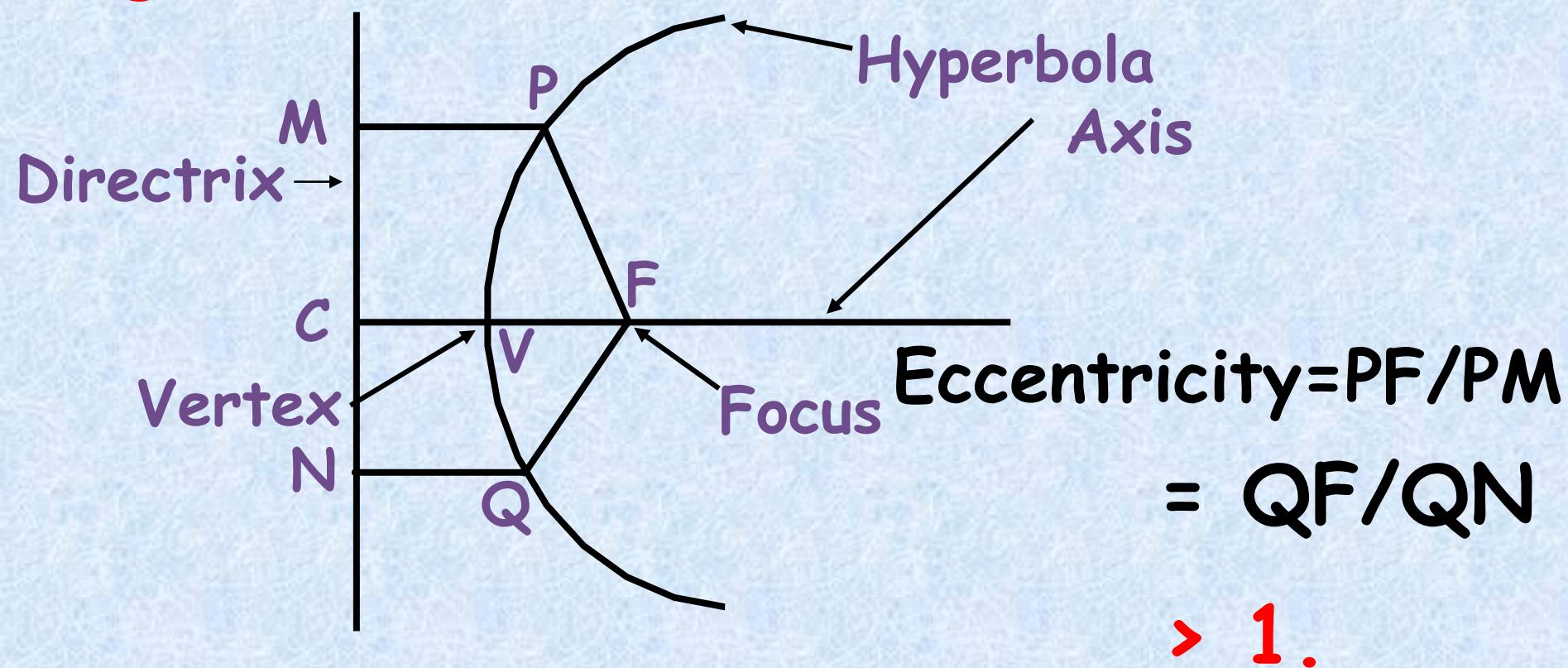
# PARABOLA

Uses :-

- Motor car head lamp reflector.
- Sound reflector and detector.
- Bridges and arches construction
- Shape of cooling towers.
- Path of particle thrown at any angle with earth, etc.

# HYPERBOLA

- It is the locus of a point which moves in a plane so that the ratio of its distances from a **fixed point (focus)** and a **fixed straight line (directrix)** is constant and greater than one.



# HYPERBOLA

Uses :-

- Nature of graph of Boyle's law
- Shape of overhead water tanks
- Shape of cooling towers etc.

# METHODS FOR DRAWING ELLIPSE

- 1. Arc of Circle's Method**
- 2. Concentric Circle Method**
- 3. Loop Method**
- 4. Oblong Method**
- 5. Ellipse in Parallelogram**
- 6. Trammel Method**
- 7. Parallel Ellipse**
- 8. Directrix Focus Method**

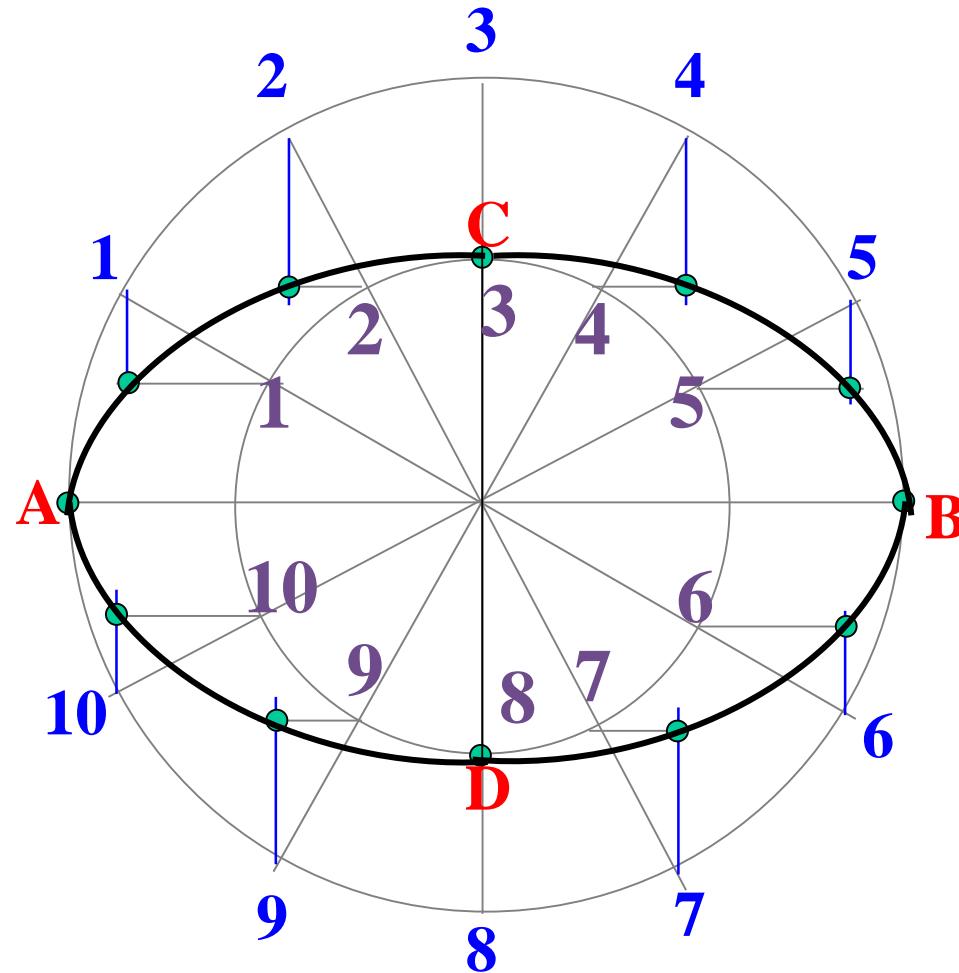
## ELLIPSE :

### ***BY CONCENTRIC CIRCLE METHOD***

*Draw ellipse by concentric circle method.*

*Take major axis 100 mm and minor axis 70 mm long.*

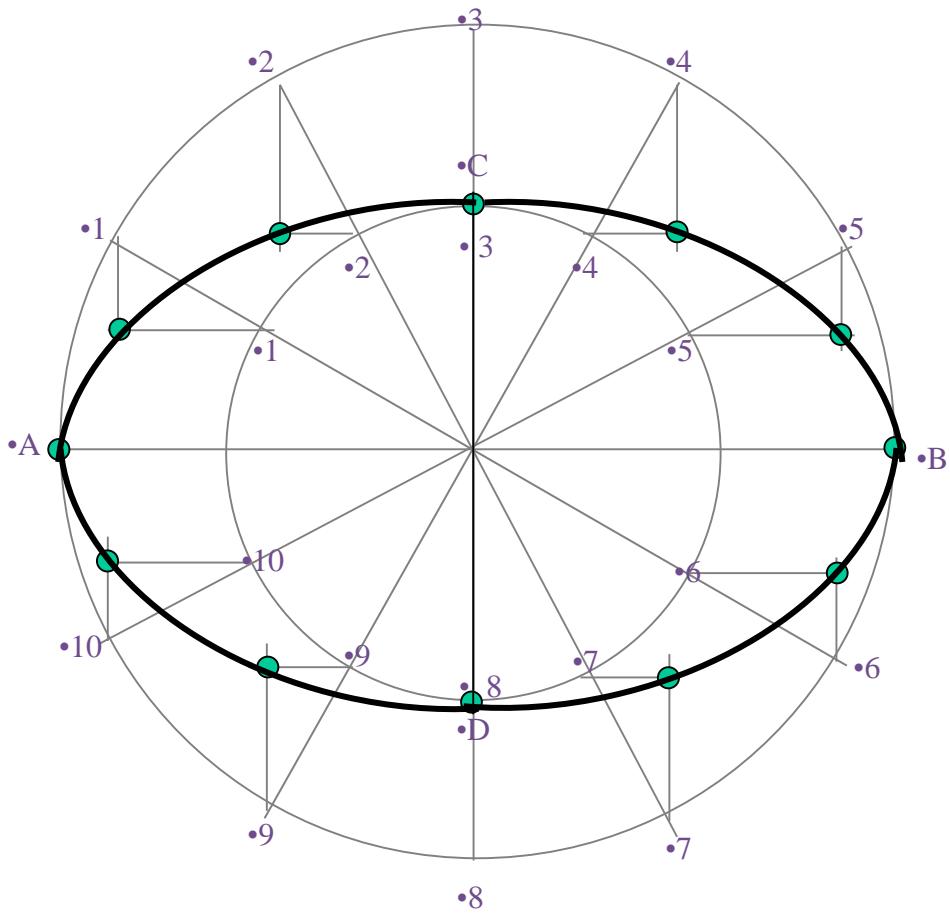
*Draw ellipse by concentric circle method. Take major axis 100 mm and minor axis 70 mm long.*



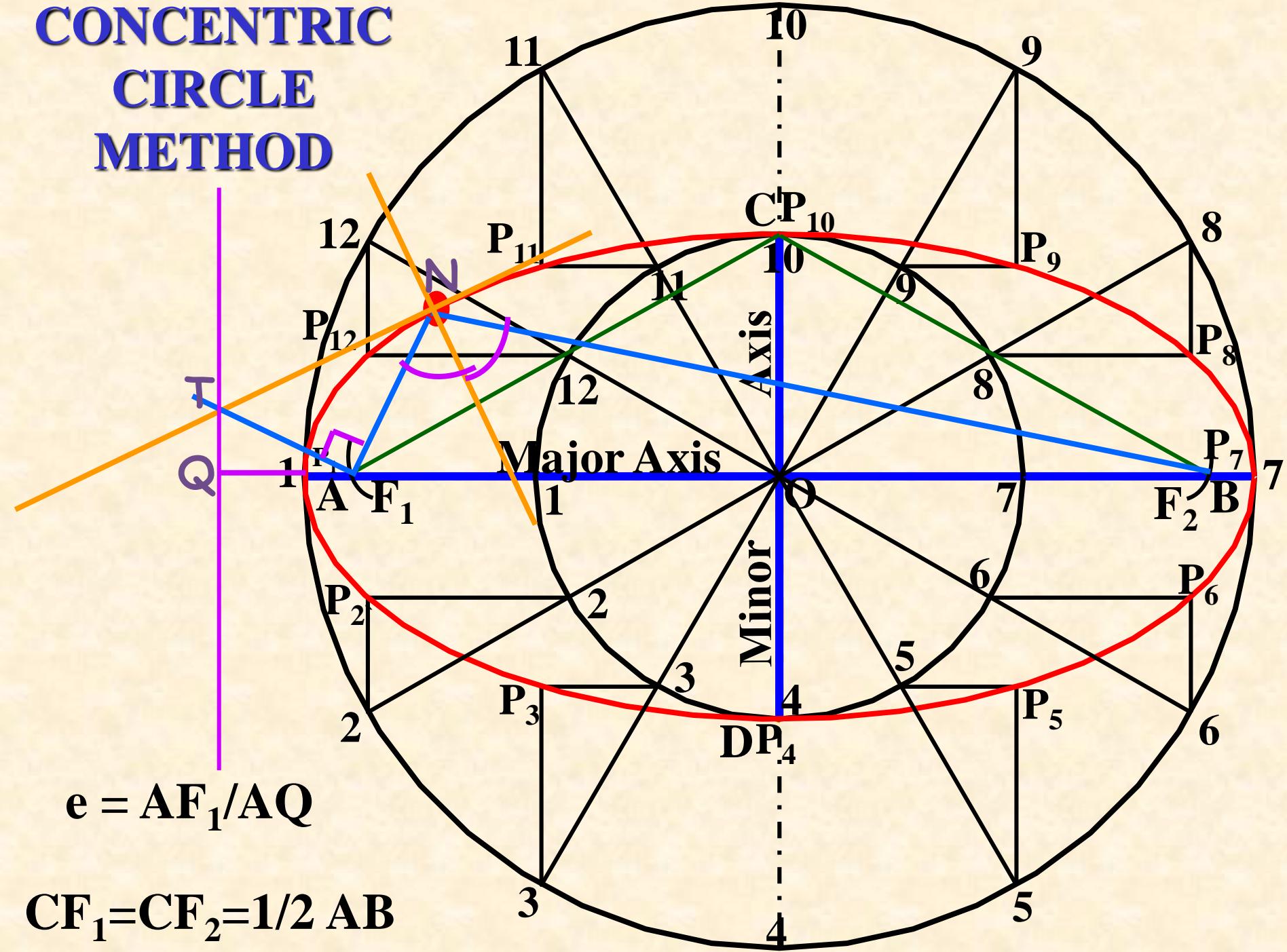
*Draw ellipse by concentric circle method. Take major axis 100 mm and minor axis 70 mm long.*

• Steps:

- 1. Draw both axes as perpendicular bisectors of each other & name their ends as shown.
- 2. Taking their intersecting point as a center, draw two concentric circles considering both as respective diameters.
- 3. Divide both circles in 12 equal parts & name as shown.
- 4. From all points of outer circle draw vertical lines downwards and upwards respectively.
- 5. From all points of inner circle draw horizontal lines to intersect those vertical lines.
- 6. Mark all intersecting points properly as those are the points on ellipse.
- 7. Join all these points along with the ends of both axes in smooth possible curve. It is required ellipse.



# CONCENTRIC CIRCLE METHOD



## **ELLIPSE :**

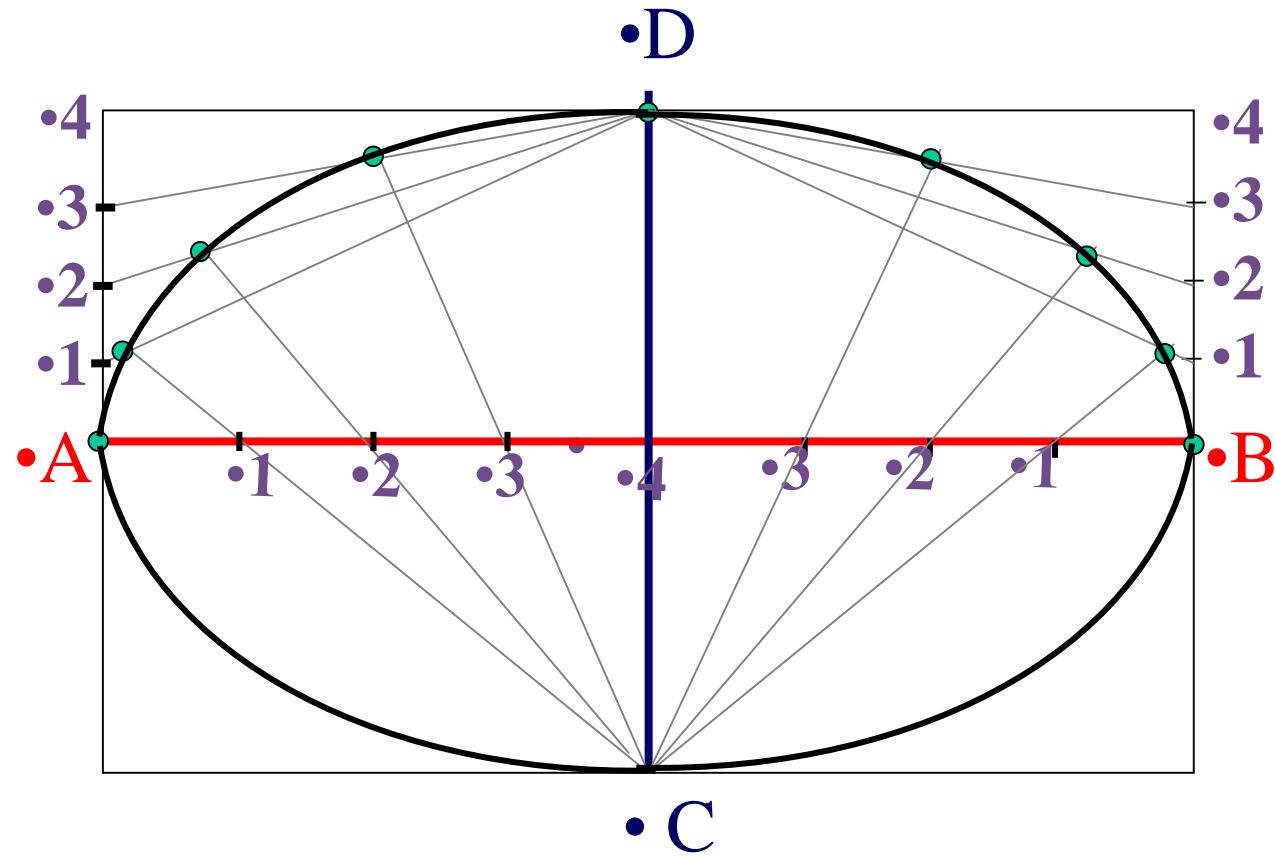
### ***BY RECTANGLE (OBLONG) METHOD***

- Draw ellipse by Rectangle method.

*Take major axis 100 mm and minor axis 70 mm long.*

Draw ellipse by **Rectangle method**.

*Take major axis 100 mm and minor axis 70 mm long.*

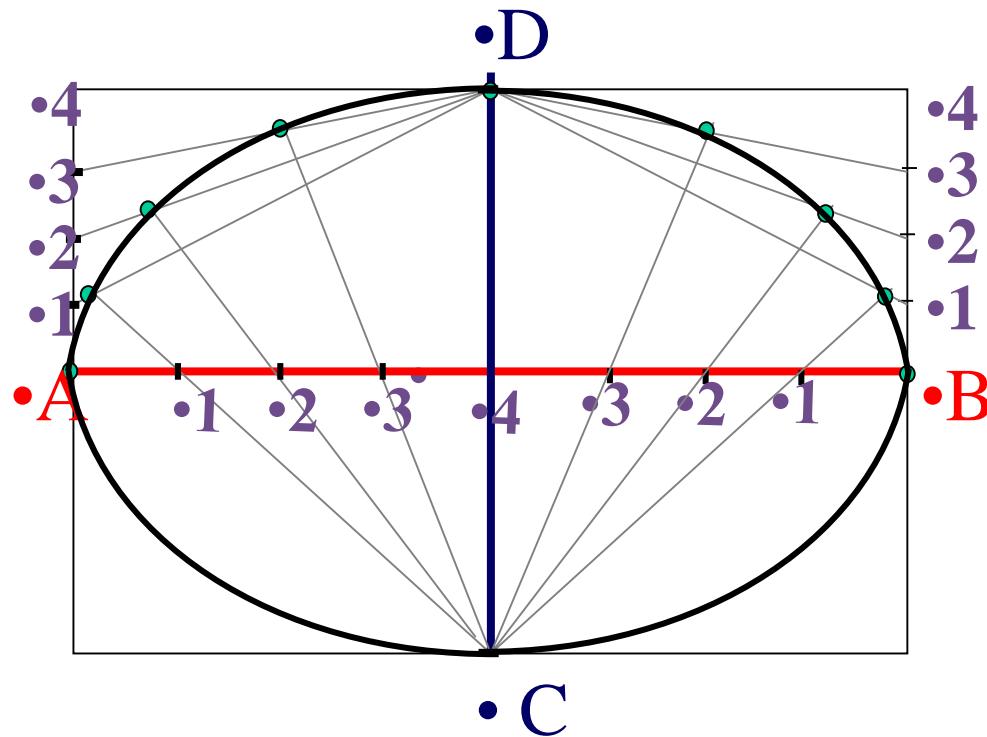


•**Steps:**

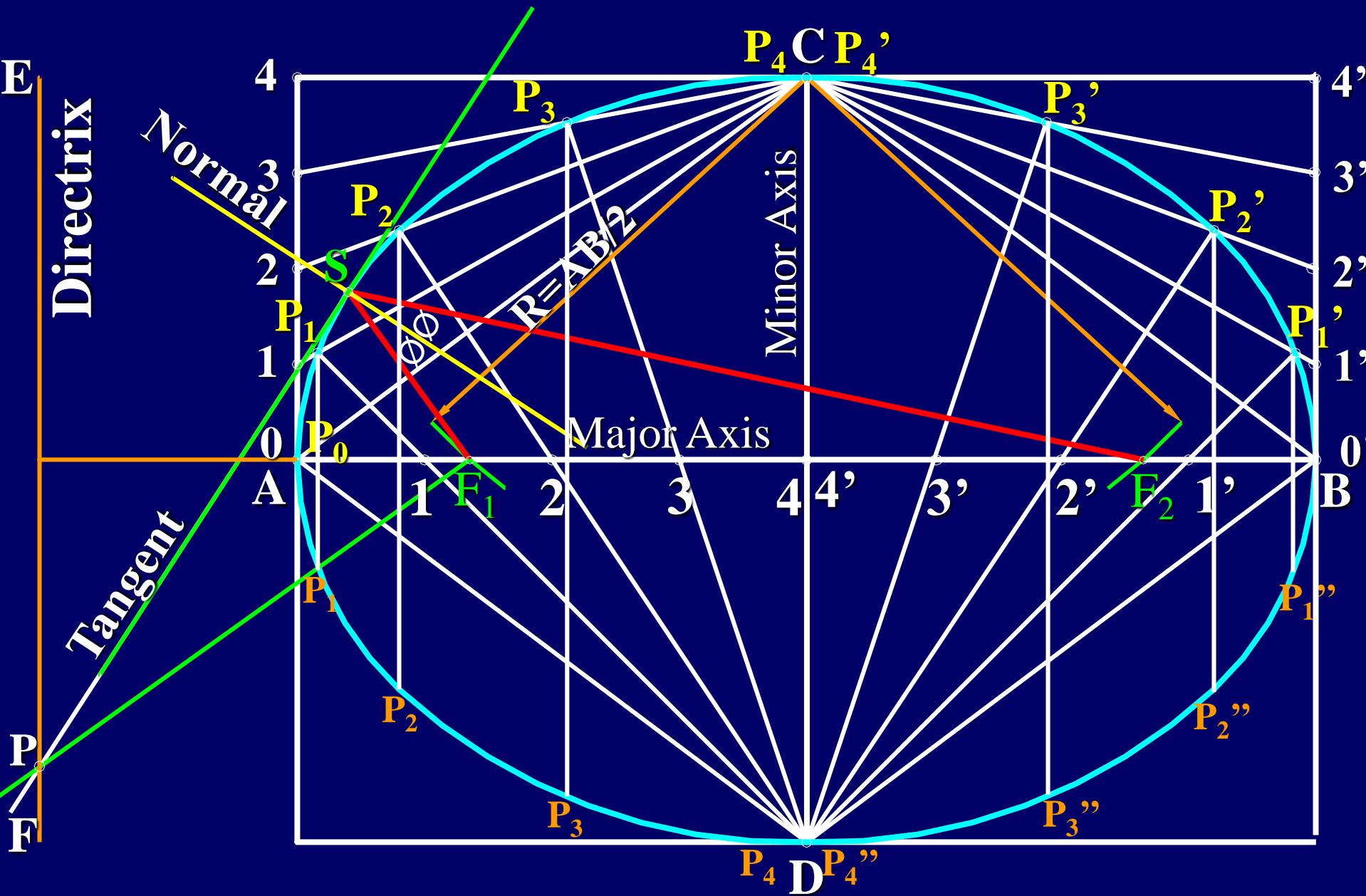
- 1 Draw a rectangle taking major and minor axes as sides.
  - 2. In this rectangle draw both axes as perpendicular bisectors of each other..
  - 3. For construction, select upper left part of rectangle. Divide vertical small side and horizontal long side into same number of equal parts.( here divided in four parts)
  - 4. Name those as shown..
  - 5. Now join all vertical points 1,2,3,4, to the upper end of minor axis. And all horizontal points i.e.1,2,3,4 to the lower end of minor axis.
  - 6. Then extend C-1 line upto D-1 and mark that point. Similarly extend C-2, C-3, C-4 lines up to D-2, D-3, & D-4 lines.
  - 7. Mark all these points properly and join all along with ends A and D in smooth possible curve. Do similar construction in right side part along with lower half of the rectangle. Join all points in smooth curve.
- It is required ellipse.

Draw ellipse by **Rectangle method.**

*Take major axis 100 mm and minor axis 70 mm long.*



# OBLONG METHOD



## ELLIPSE :

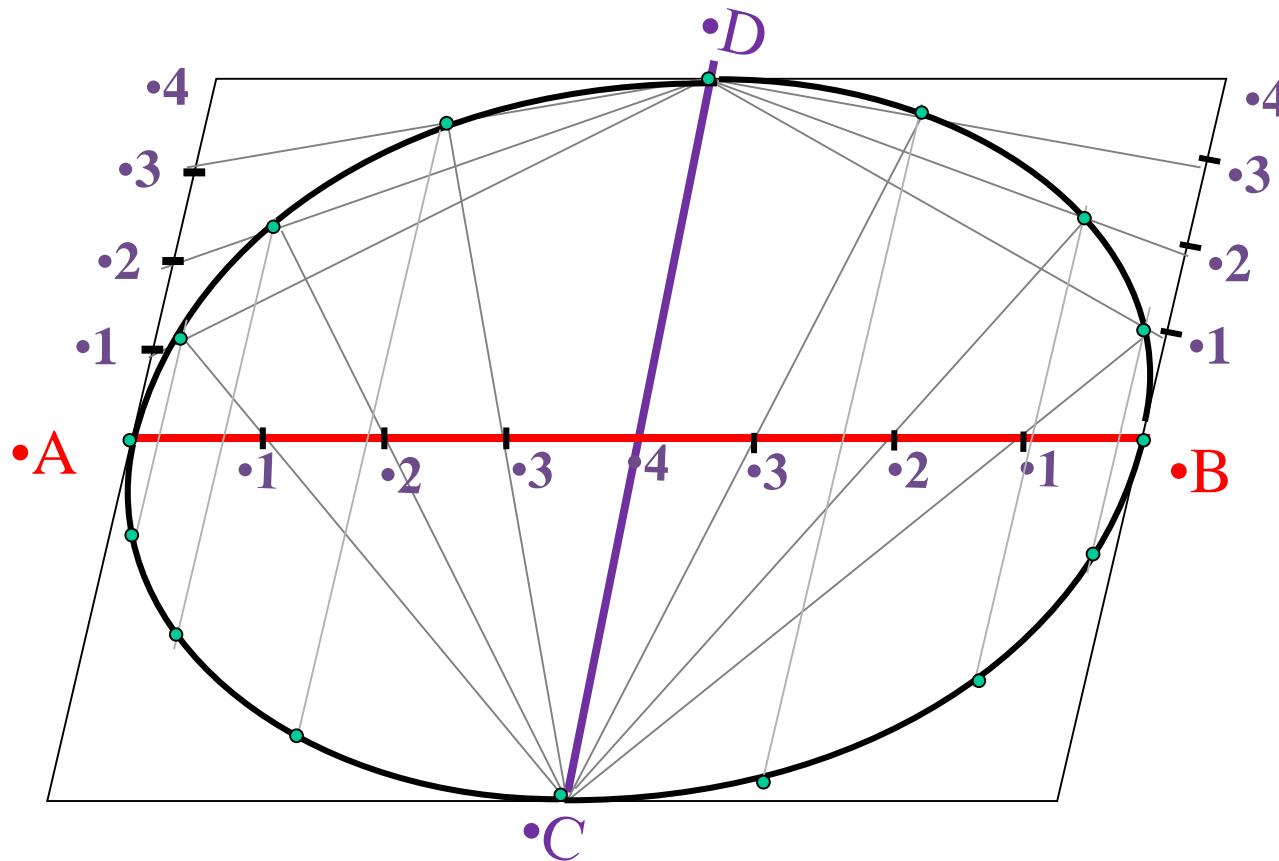
### ***BY PARALLELOGRAM METHOD***

Draw ellipse by **Parallelogram method.**

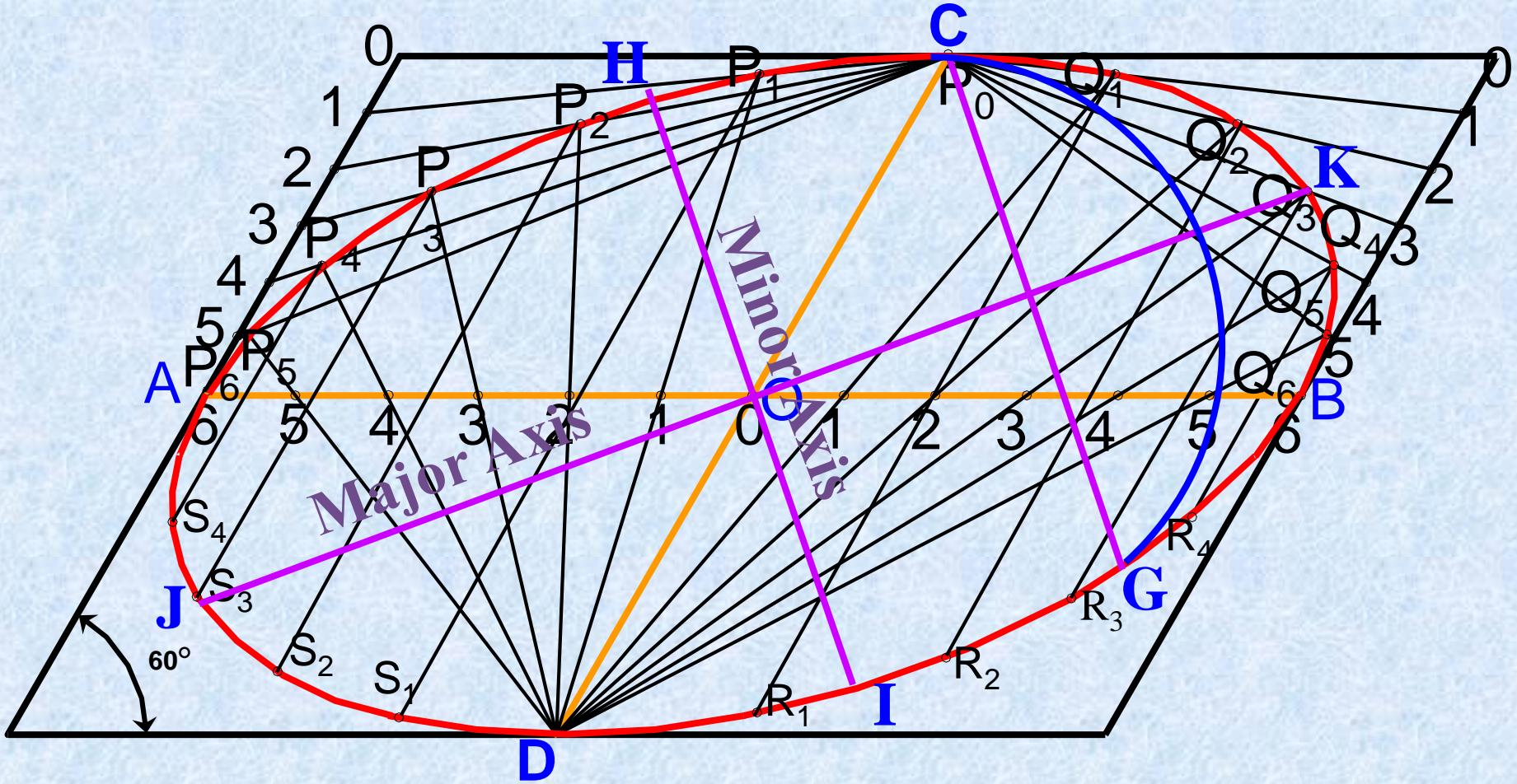
Take major axis 100 mm and minor axis 70 mm long sides with included angle of 75°. Inscribe Ellipse in it.

Draw ellipse by **Parallelogram method**.

Take major axis 100 mm and minor axis 70 mm long sides with included angle of  $75^0$ . Inscribe Ellipse in it.



# ELLIPSE IN PARALLELOGRAM



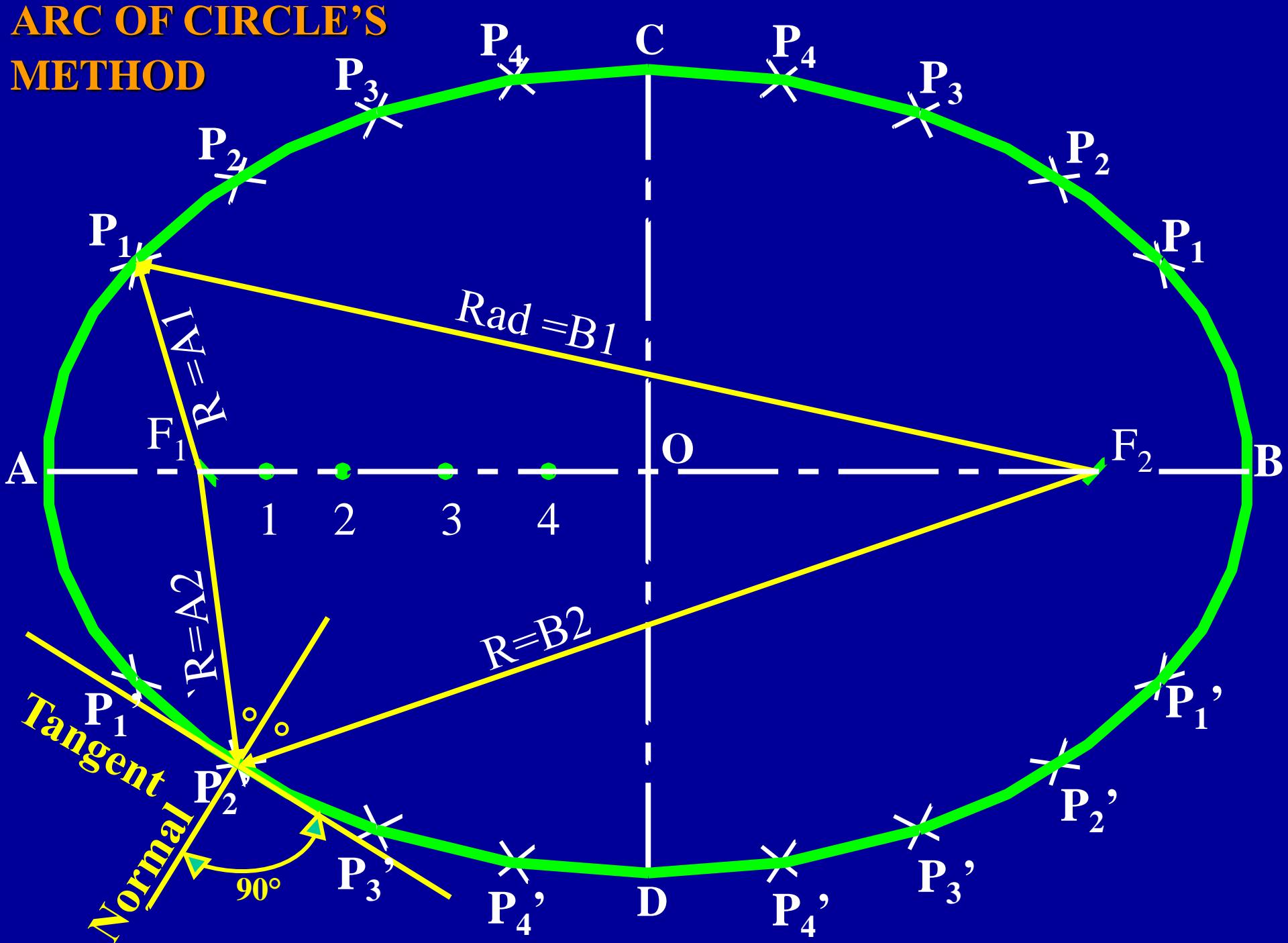
## ELLIPSE :

*BY ARC of CIRCLE METHOD*

Draw ellipse by ARC of CIRCLE method.

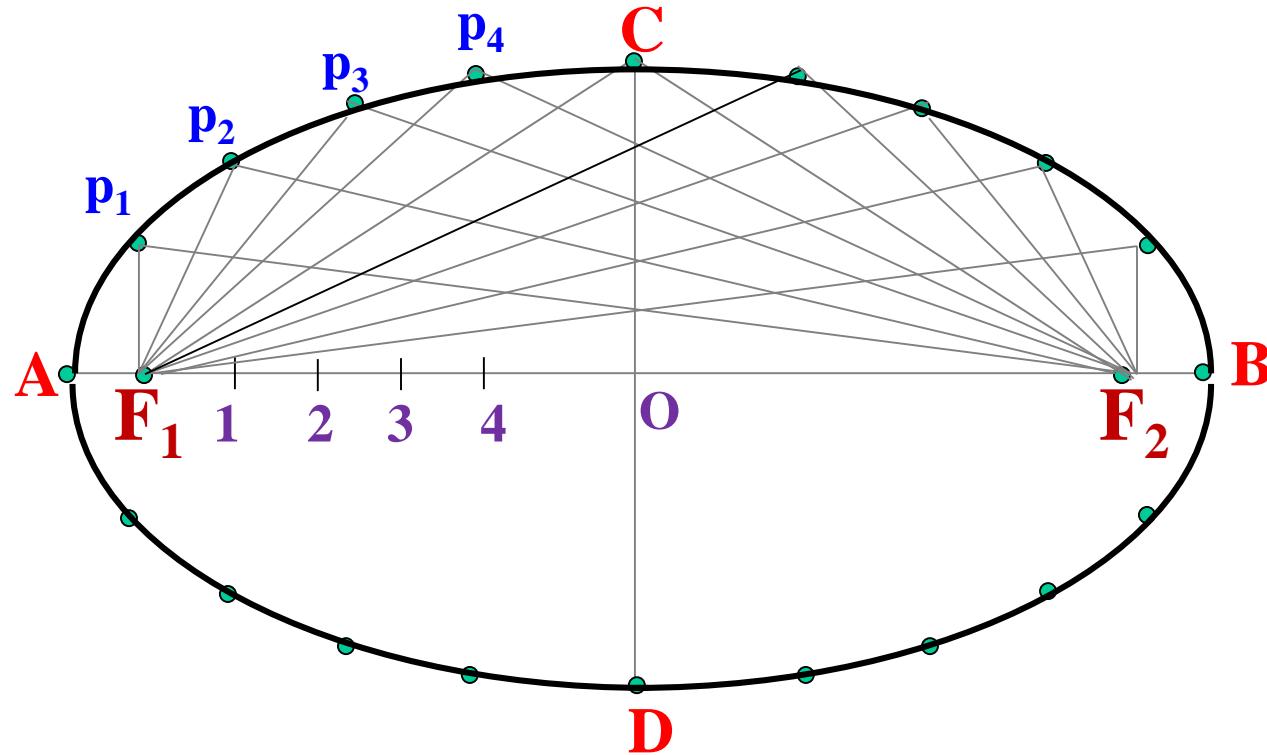
Take major axis 100 mm and minor axis 70 mm long sides.

# ARC OF CIRCLE'S METHOD



Draw ellipse by ARC of CIRCLE method.

Take major axis 100 mm and minor axis 70 mm long sides.

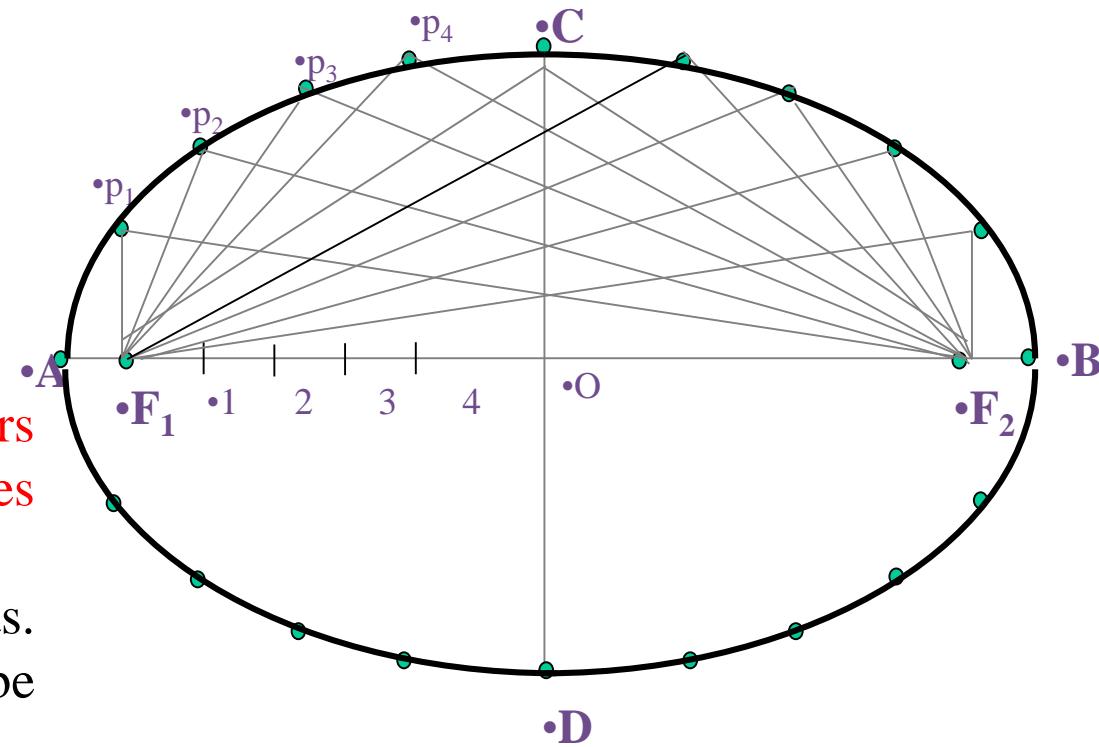


## STEPS:

1. Draw both axes as usual. Name the ends & intersecting point
2. Taking AO distance i.e. half major axis, from C, mark  $F_1$  &  $F_2$  On AB ( focus 1 and 2.)
3. On line  $F_1$ - O taking any distance, mark points 1,2,3, & 4
4. Taking  $F_1$  center, with distance A-1 draw an arc above AB and taking  $F_2$  center, with B-1 distance cut this arc. Name the point  $p_1$
5. Repeat this step with same centers but taking now A-2 & B-2 distances for drawing arcs. Name the point  $p_2$
6. Similarly get all other P points. With same steps positions of P can be located below AB.
7. Join all points by smooth curve to get an ellipse.

## Draw ellipse by ARC of CIRCLE method.

Take major axis 100 mm and minor axis 70 mm long sides.



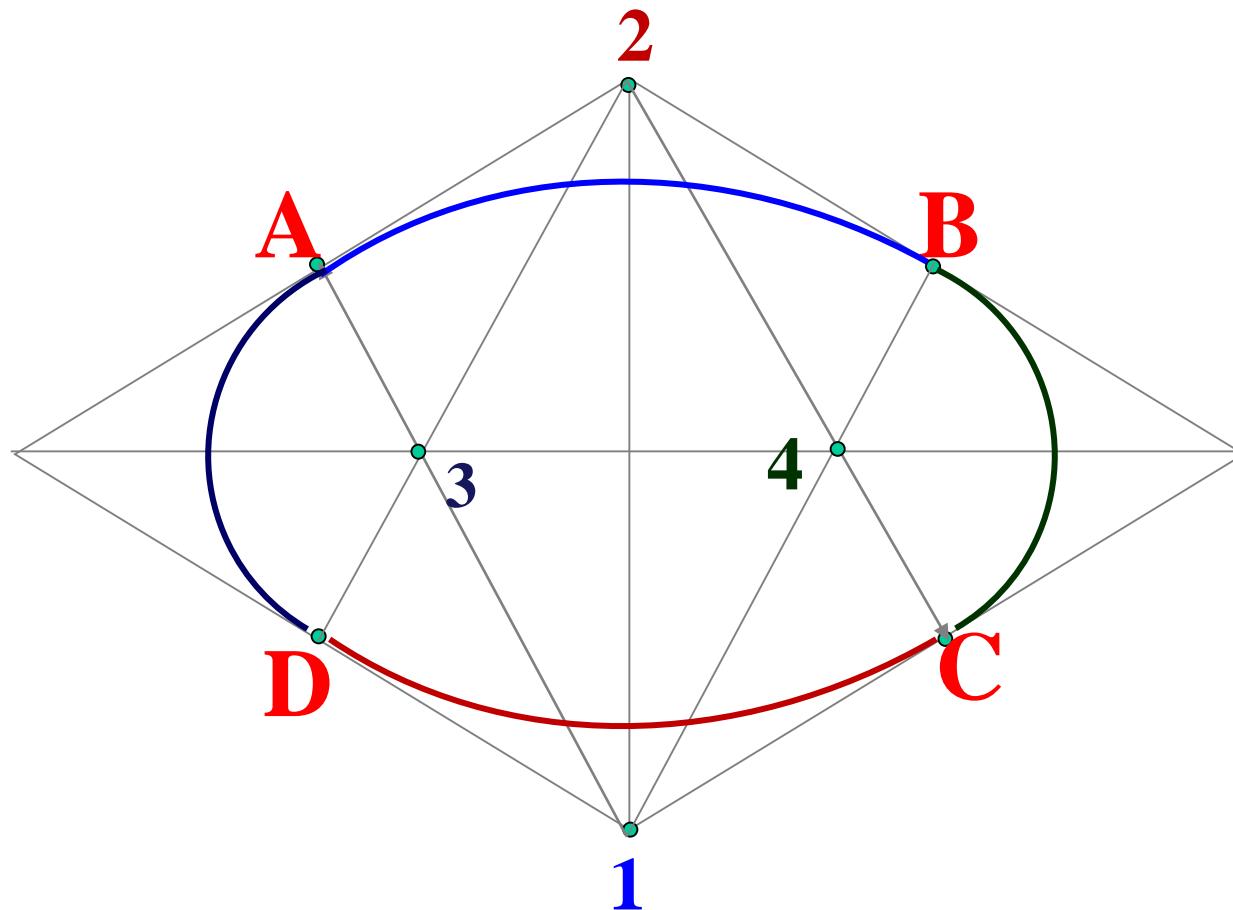
## ELLIPSE :

### *BY RHOMBUS METHOD*

Draw ellipse by **RHOMBUS** method.

Draw rhombus of 100 mm & 70 mm long diagonals and inscribe an ellipse in it.

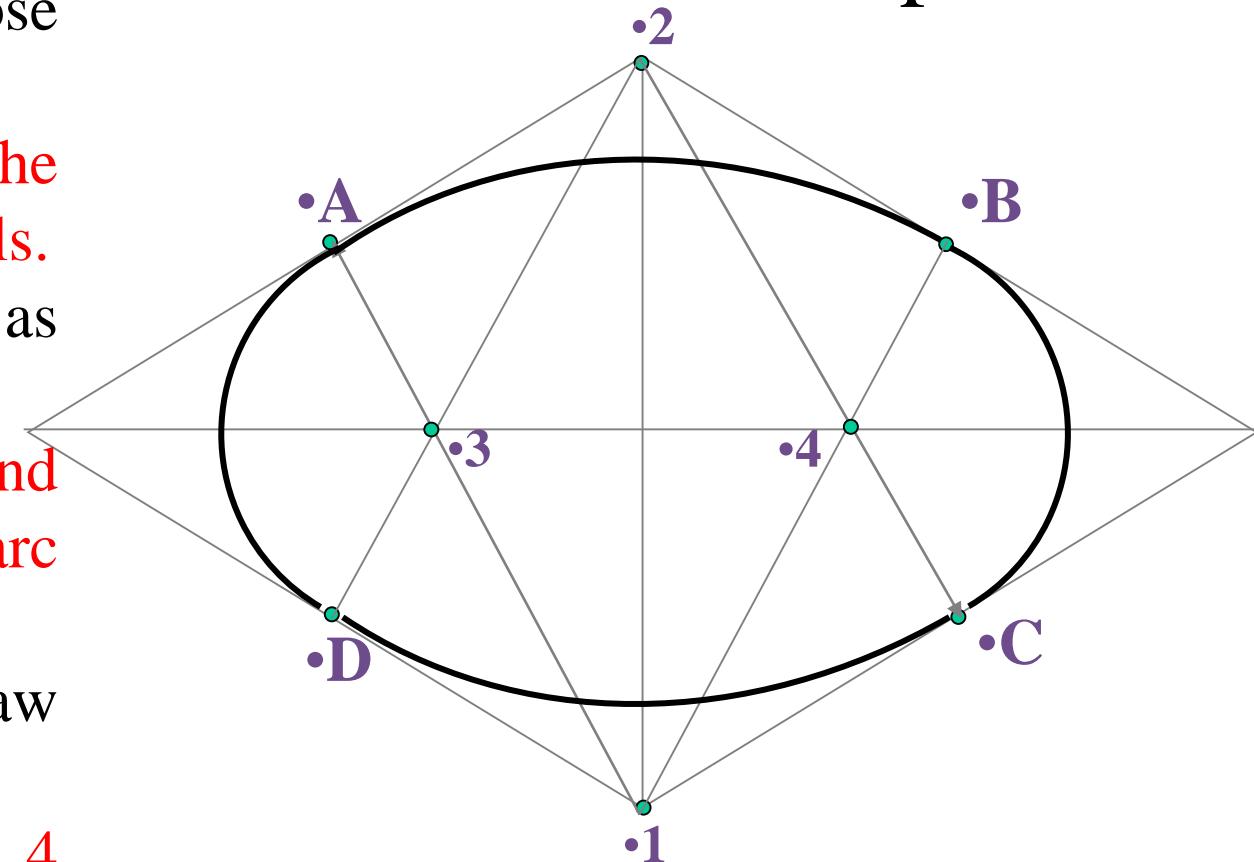
Draw rhombus of 100 mm & 70 mm long diagonals and inscribe an ellipse in it.



## STEPS:

1. Draw rhombus of given dimensions.
2. Mark mid points of all sides & name Those A,B,C,& D
3. Join these points to the ends of smaller diagonals.
4. Mark points 1,2,3,4 as four centers.
5. Taking 1 as center and 1-A radius draw an arc AB.
6. Take 2 as center draw an arc CD.
7. Similarly taking 3 & 4 as centers and 3-D radius draw arcs DA & BC.

Draw rhombus of 100 mm & 70 mm long diagonals and inscribe an ellipse in it.



## ELLIPSE :

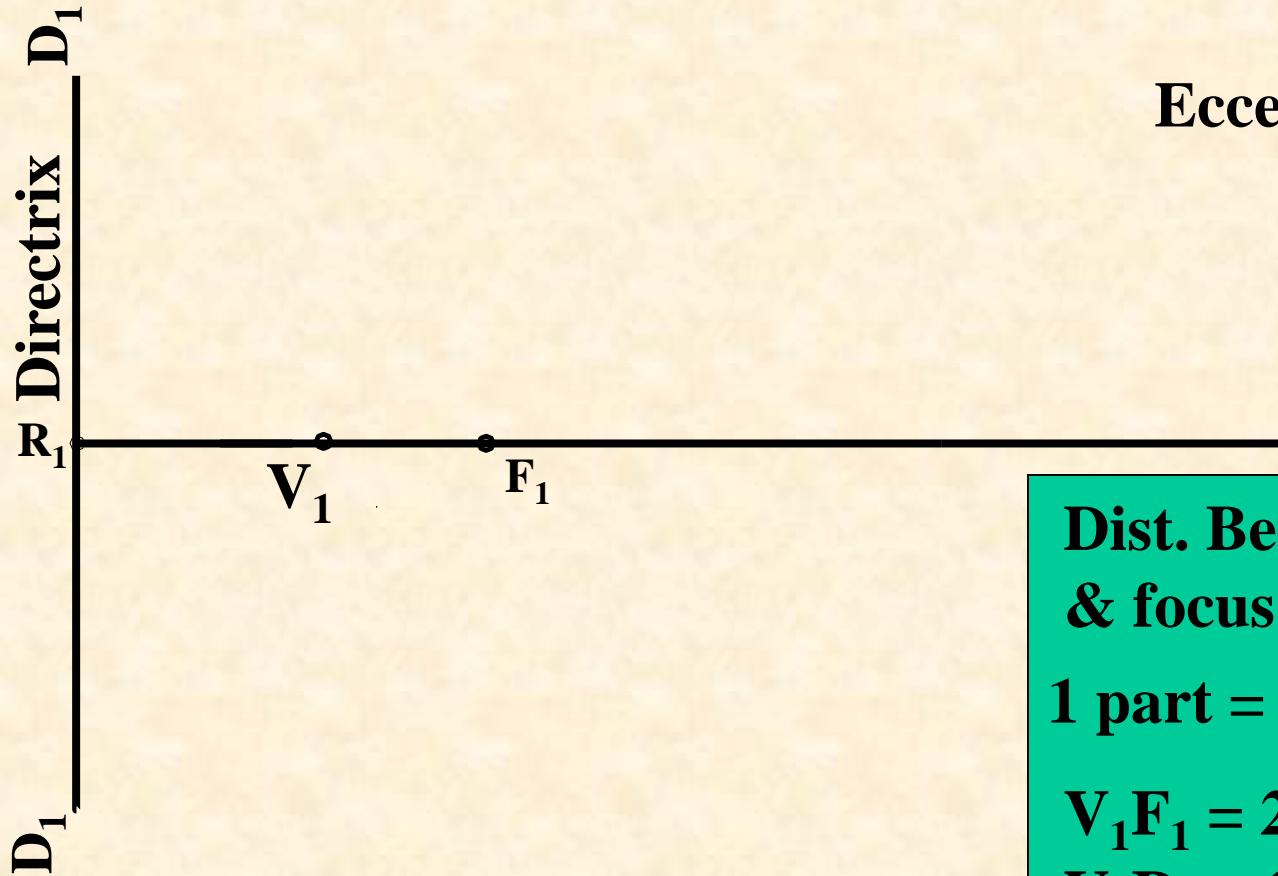
***BY DIRECTRIX - FOCUS METHOD***

***BY FOCUS - DIRECTRIX METHOD***

- Draw ellipse by Focus-Directrix method.

Distance between focus and directrix is 50 mm and eccentricity =  $2/3$ . Draw ellipse.

# ELLIPSE – DIRECTRIX FOCUS METHOD



Eccentricity =  $2/3$

$$\frac{V_1F_1}{R_1V_1} = \frac{2}{3}$$

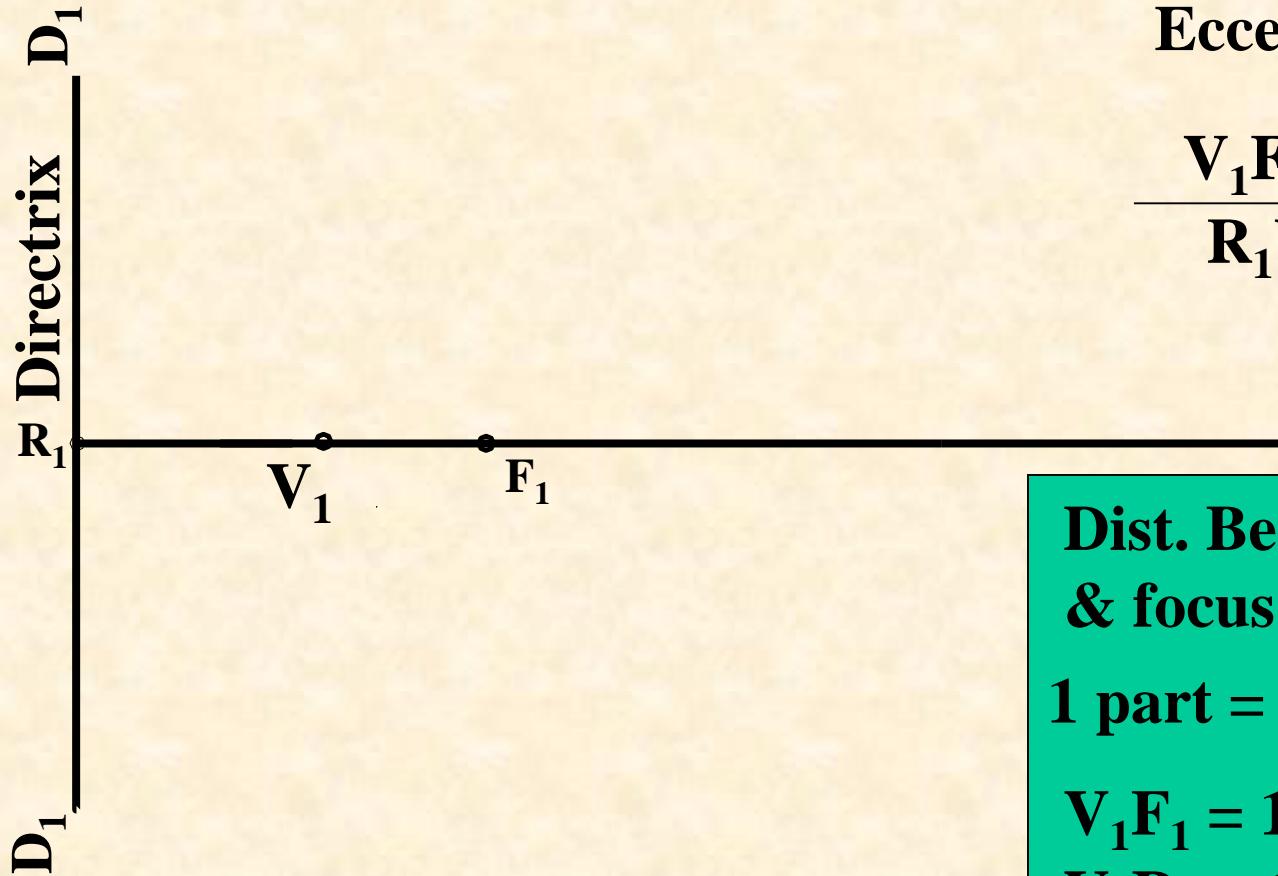
Dist. Between directrix & focus = 50 mm

1 part =  $50/(2+3)=10$  mm

$V_1F_1 = 2$  part = 20 mm

$V_1R_1 = 3$  part = 30 mm

# PARABOLA – DIRECTRIX FOCUS METHOD



Eccentricity = 1

$$\frac{V_1F_1}{R_1V_1} = 1 = \frac{1}{1}$$

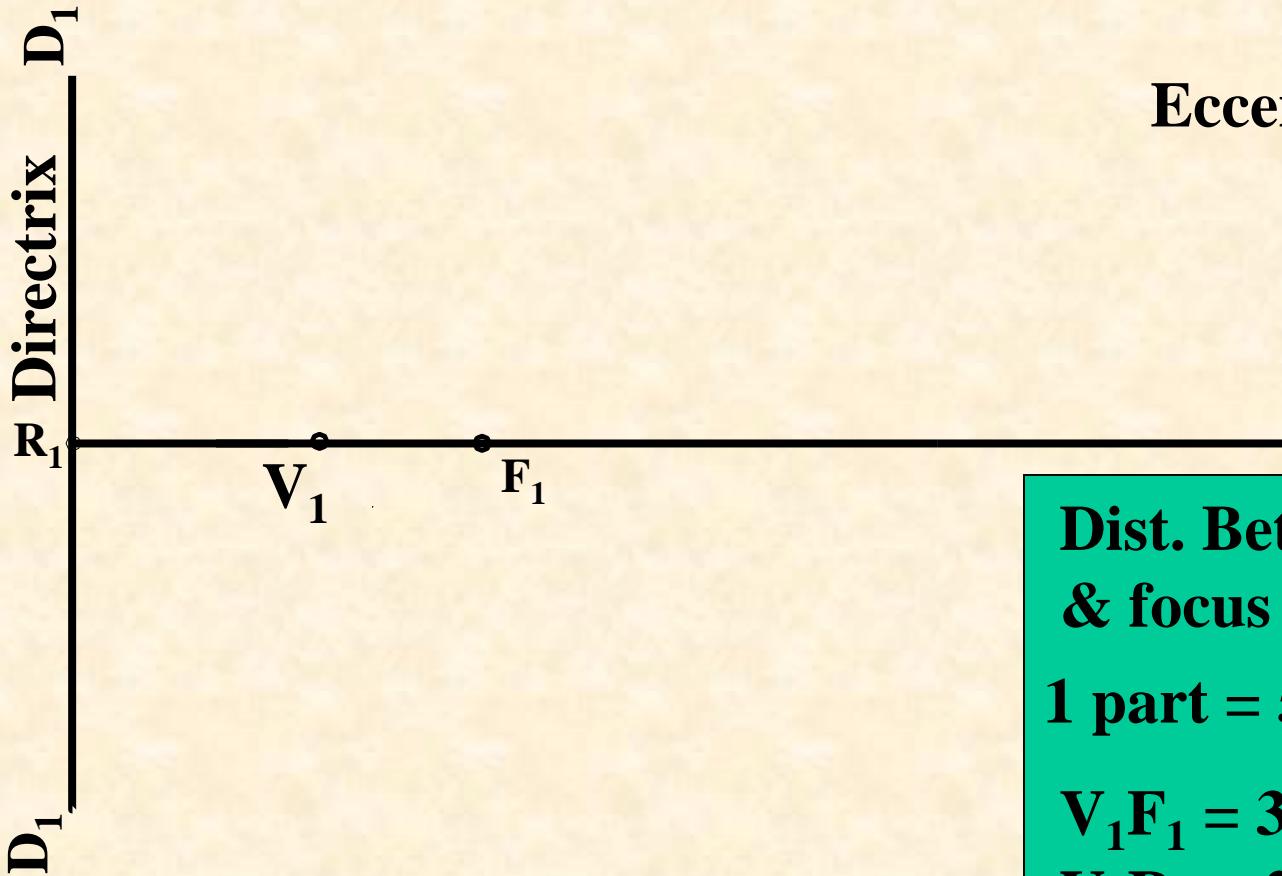
Dist. Between directrix & focus = 50 mm

1 part =  $50/(1+1) = 25$  mm

$V_1F_1 = 1$  part = 25 mm

$V_1R_1 = 1$  part = 25 mm

# HYPERBOLA – DIRECTRIX FOCUS METHOD



$$\text{Eccentricity} = 3/2$$

$$\frac{V_1F_1}{R_1V_1} = \frac{3}{2}$$

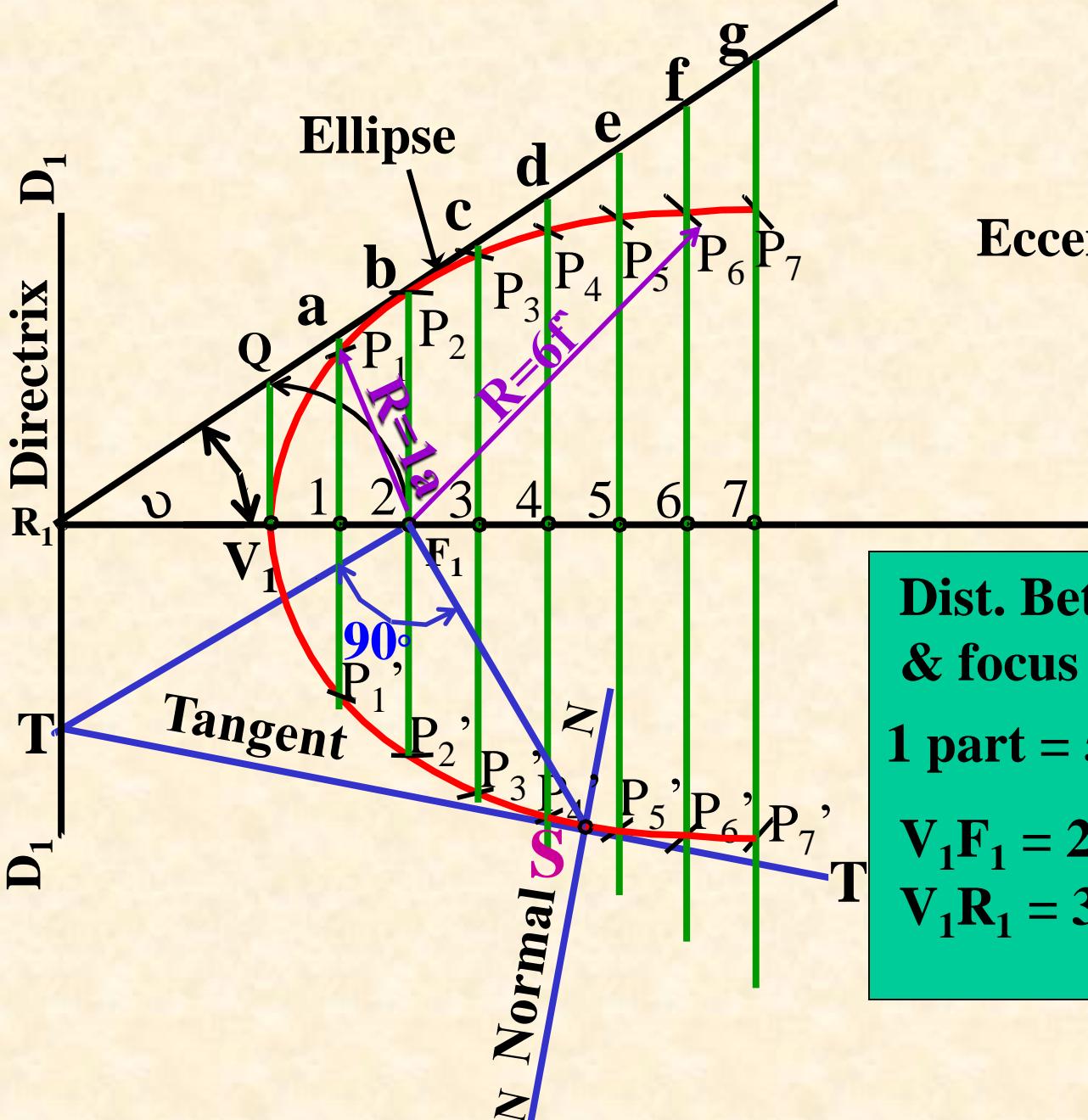
Dist. Between directrix & focus = 50 mm

$$1 \text{ part} = 50/(2+3) = 10 \text{ mm}$$

$$V_1F_1 = 3 \text{ part} = 30 \text{ mm}$$

$$V_1R_1 = 2 \text{ part} = 20 \text{ mm}$$

# ELLIPSE – DIRECTRIX FOCUS METHOD



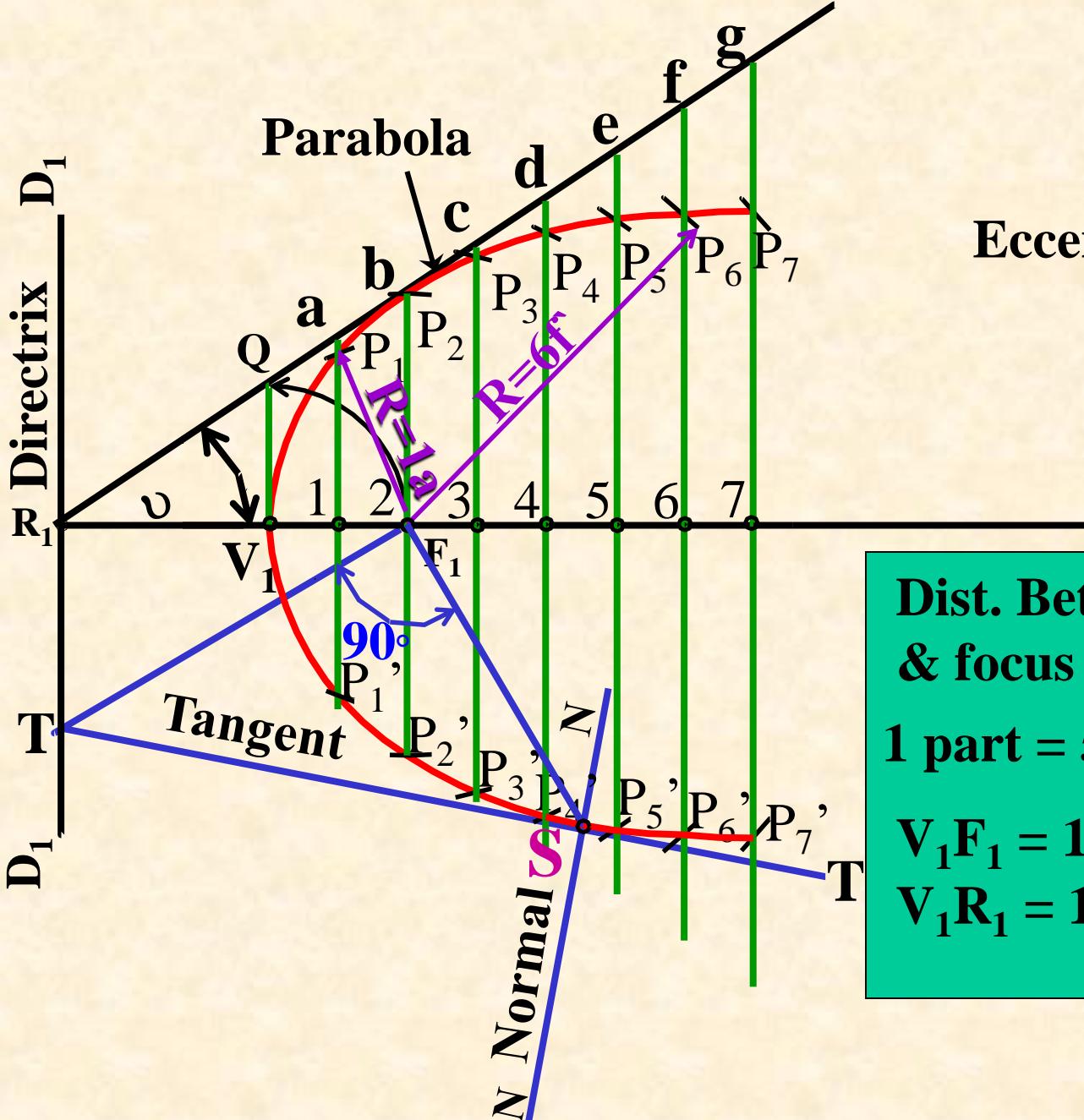
$$\theta < 45^\circ$$

$$\text{Eccentricity} = 2/3$$

$$\frac{V_1 F_1}{R_1 V_1} = \frac{2}{3}$$

Dist. Between directrix & focus = 50 mm  
 1 part =  $50/(2+3)=10$  mm  
 $V_1 F_1 = 2$  part = 20 mm  
 $V_1 R_1 = 3$  part = 30 mm

# PARABOLA – DIRECTRIX FOCUS METHOD



$$\theta = 45^\circ$$

# Eccentricity = 1

$$\frac{V_1 F_1}{R_1 V_1} = \frac{1}{1}$$

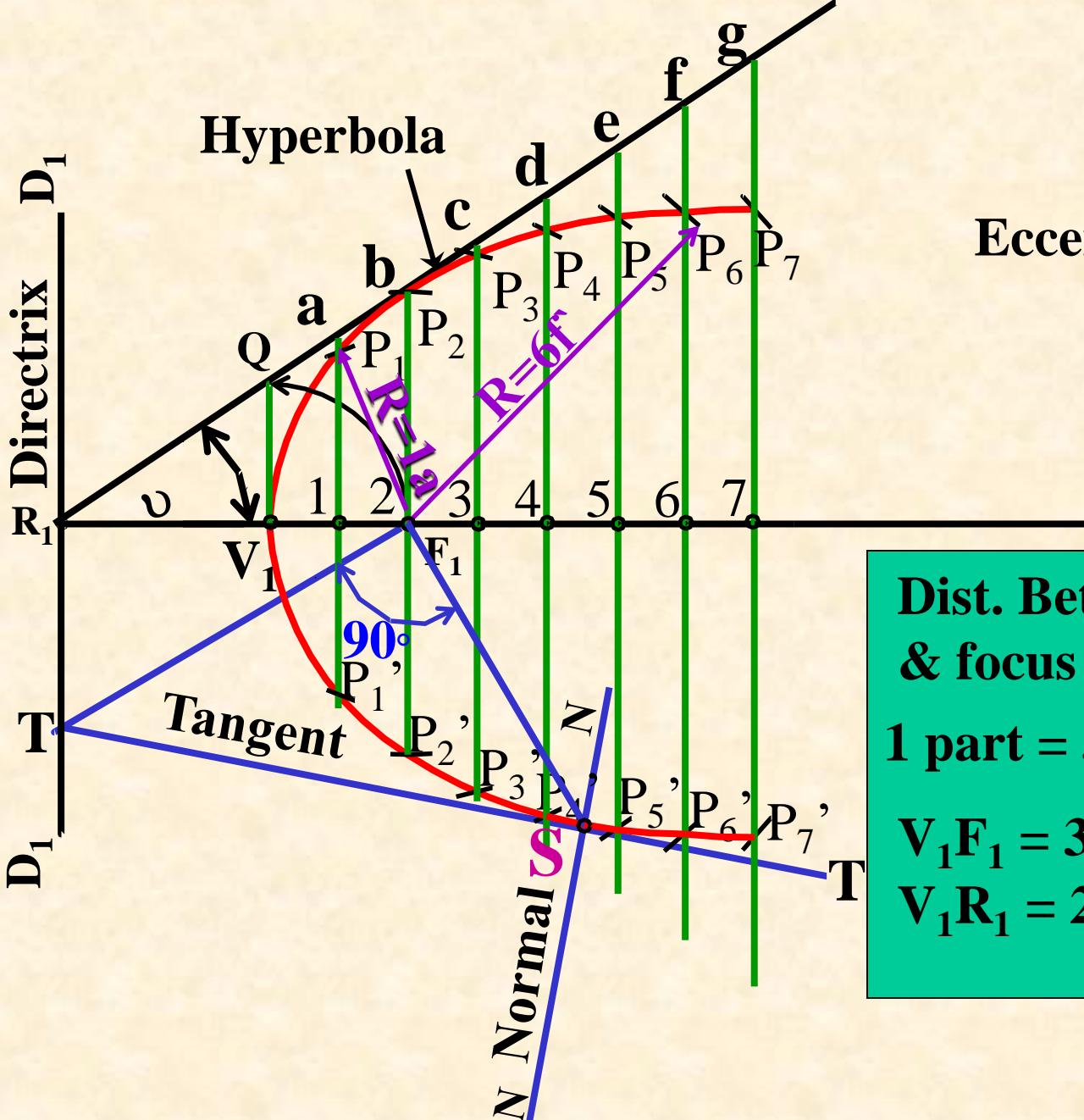
**Dist. Between directrix  
& focus = 50 mm**

**1 part =  $50/(1+1)=25$  mm**

**$V_1F_1 = 1$  part = 25 mm**

**$V_1R_1 = 1$  part = 25 mm**

# HYPERBOLA – DIRECTRIX FOCUS METHOD



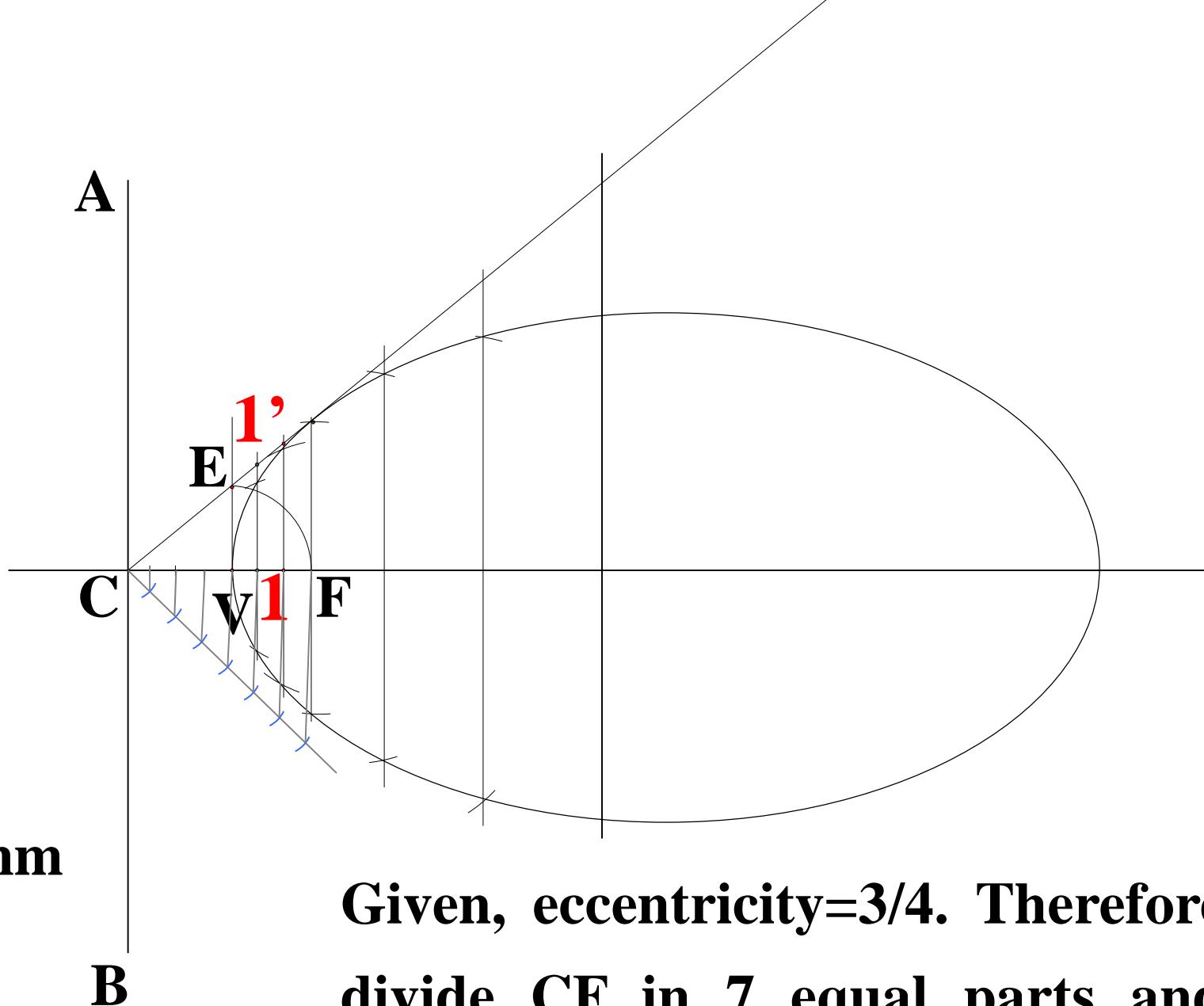
$$\theta > 45^\circ$$

$$\text{Eccentricity} = 3/2$$

$$\frac{V_1F_1}{R_1V_1} = \frac{3}{2}$$

Dist. Between directrix & focus = 50 mm  
 1 part =  $50/(3+2)=10$  mm  
 $V_1F_1 = 3$  part = 30 mm  
 $V_1R_1 = 2$  part = 20 mm

**Draw an ellipse by general method, given distance of focus from directrix 65mm and eccentricity  $\frac{3}{4}$ .**



Given

$CF=65\text{mm}$

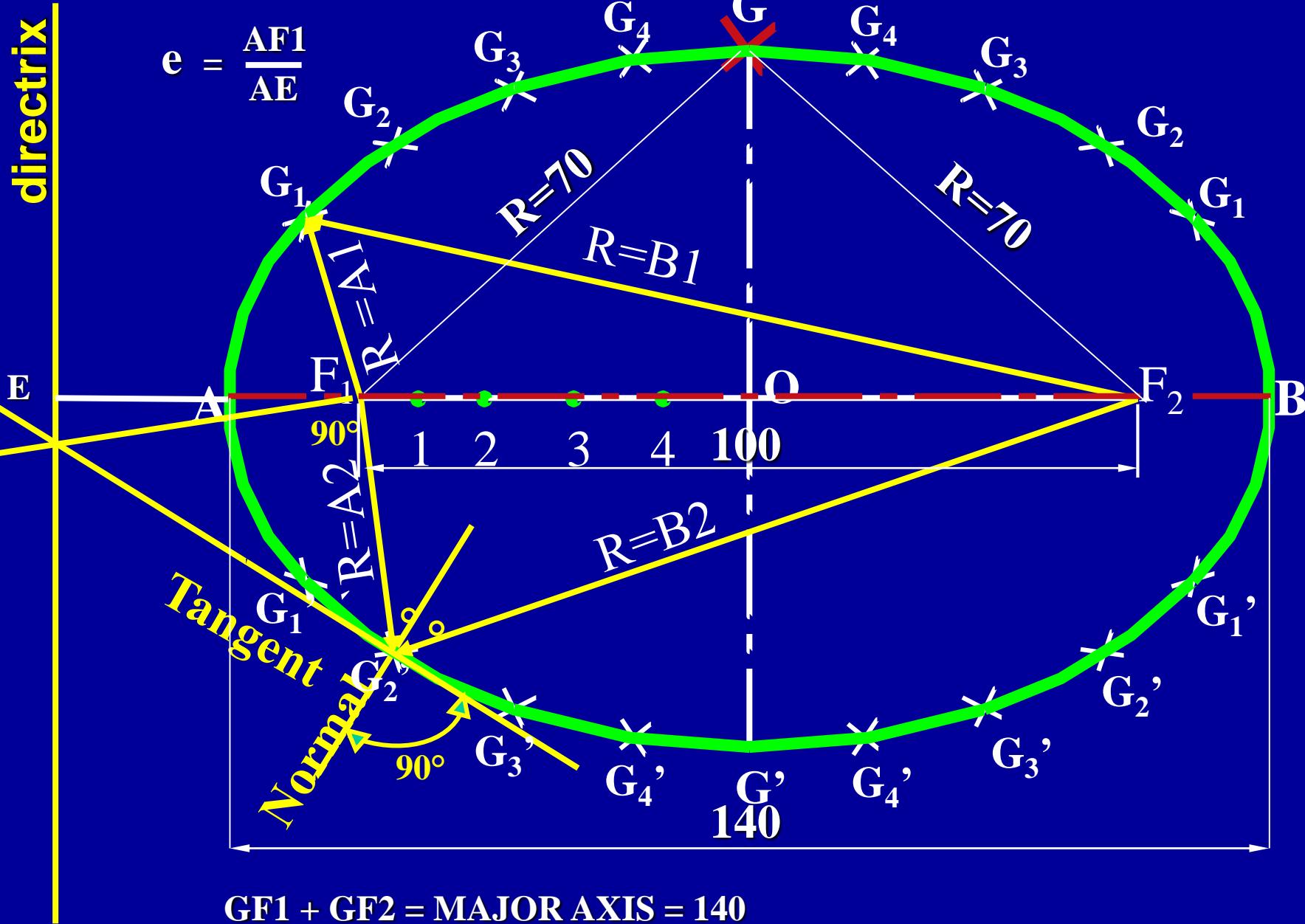
Given, eccentricity= $\frac{3}{4}$ . Therefore  
divide CF in 7 equal parts and  
mark V at 3rd division from F

# ***PROBLEM***

The distance between two coplanar fixed points is 100 mm. Trace the complete path of a point G moving in the same plane in such a way that the sum of the distance from the fixed points is always 140 mm.

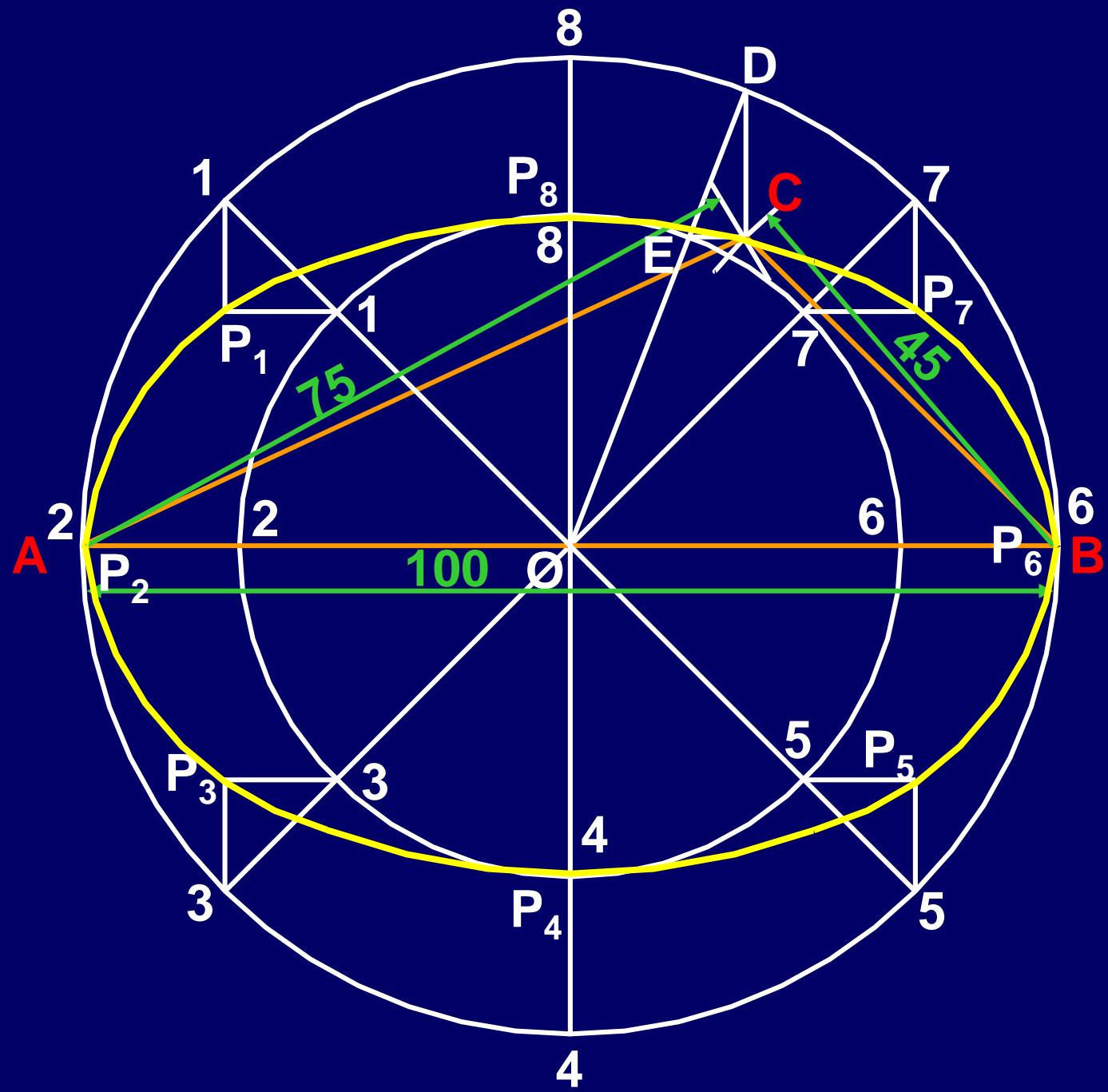
Name the curve & find its eccentricity.

# ARC OF CIRCLE'S METHOD



## ***PROBLEM***

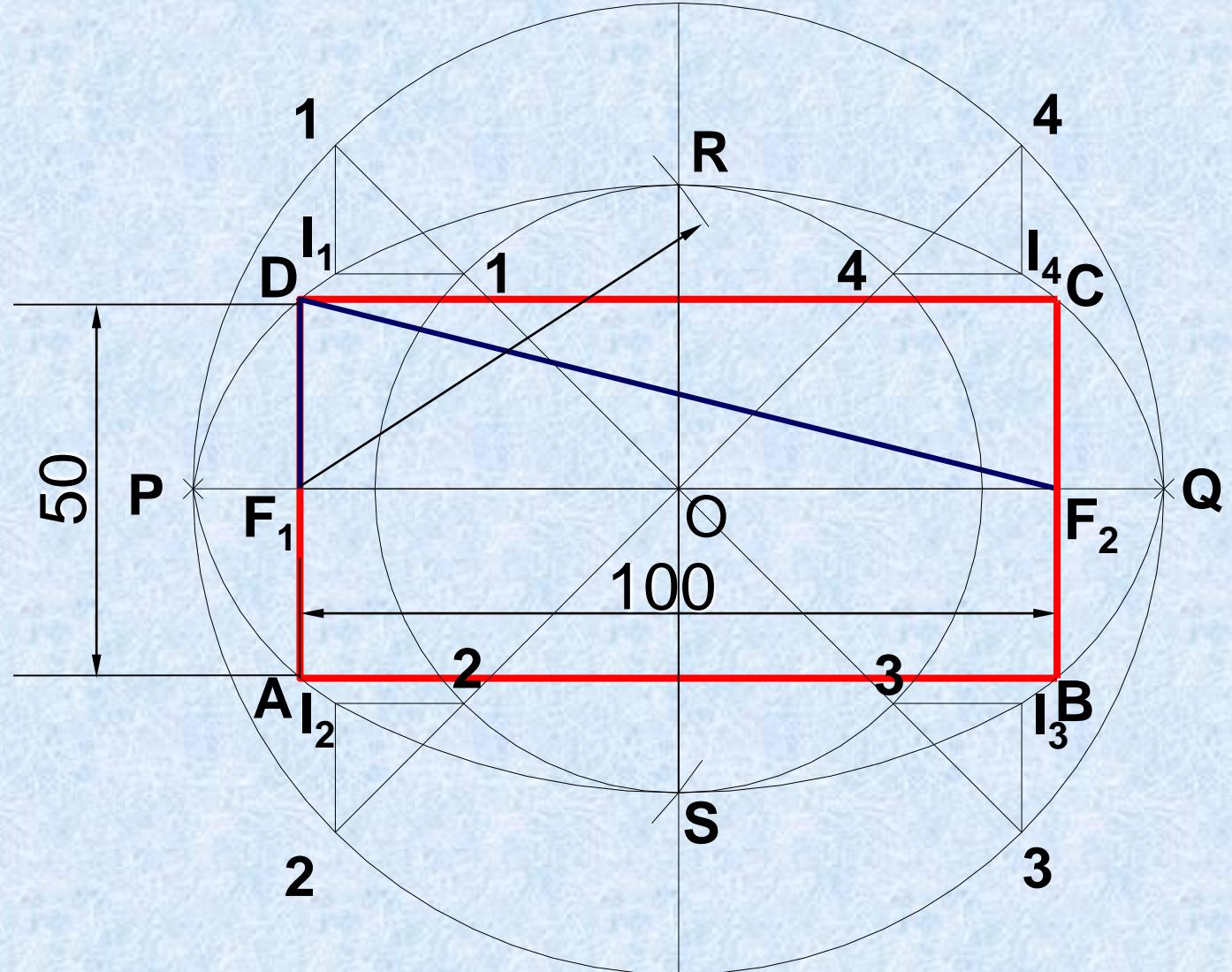
Two points A & B are 100 mm apart. A point C is 75 mm from A and 45 mm from B. Draw an ellipse passing through points A, B, and C so that AB is a major axis.



# ***PROBLEM***

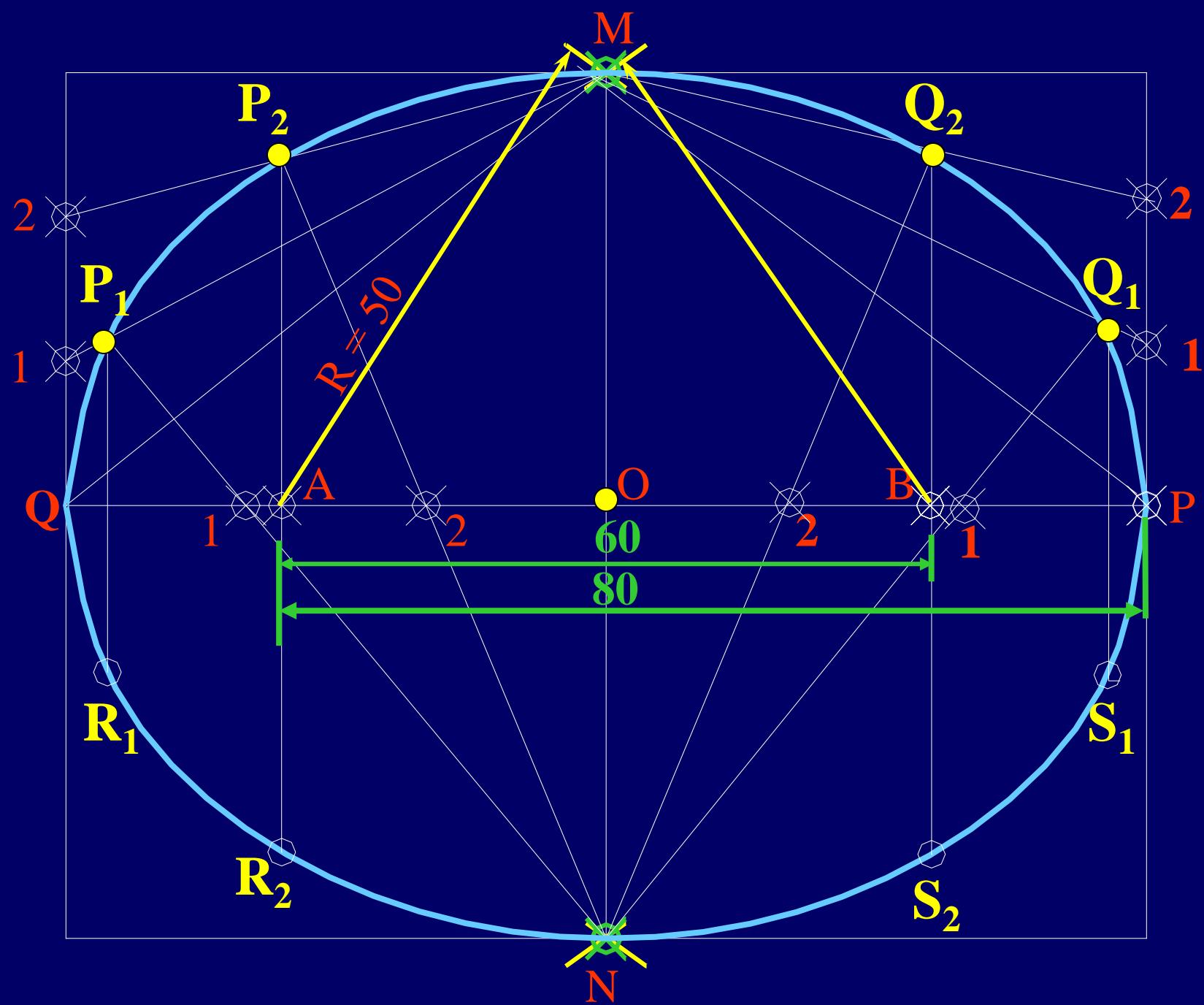
**ABCD** is a rectangle of 100mm x 60mm. Draw an ellipse passing through all the four corners A, B, C and D of the rectangle considering mid – points of the smaller sides as focal points.

**Use “Concentric circles” method and find its eccentricity.**



# **PROBLEM**

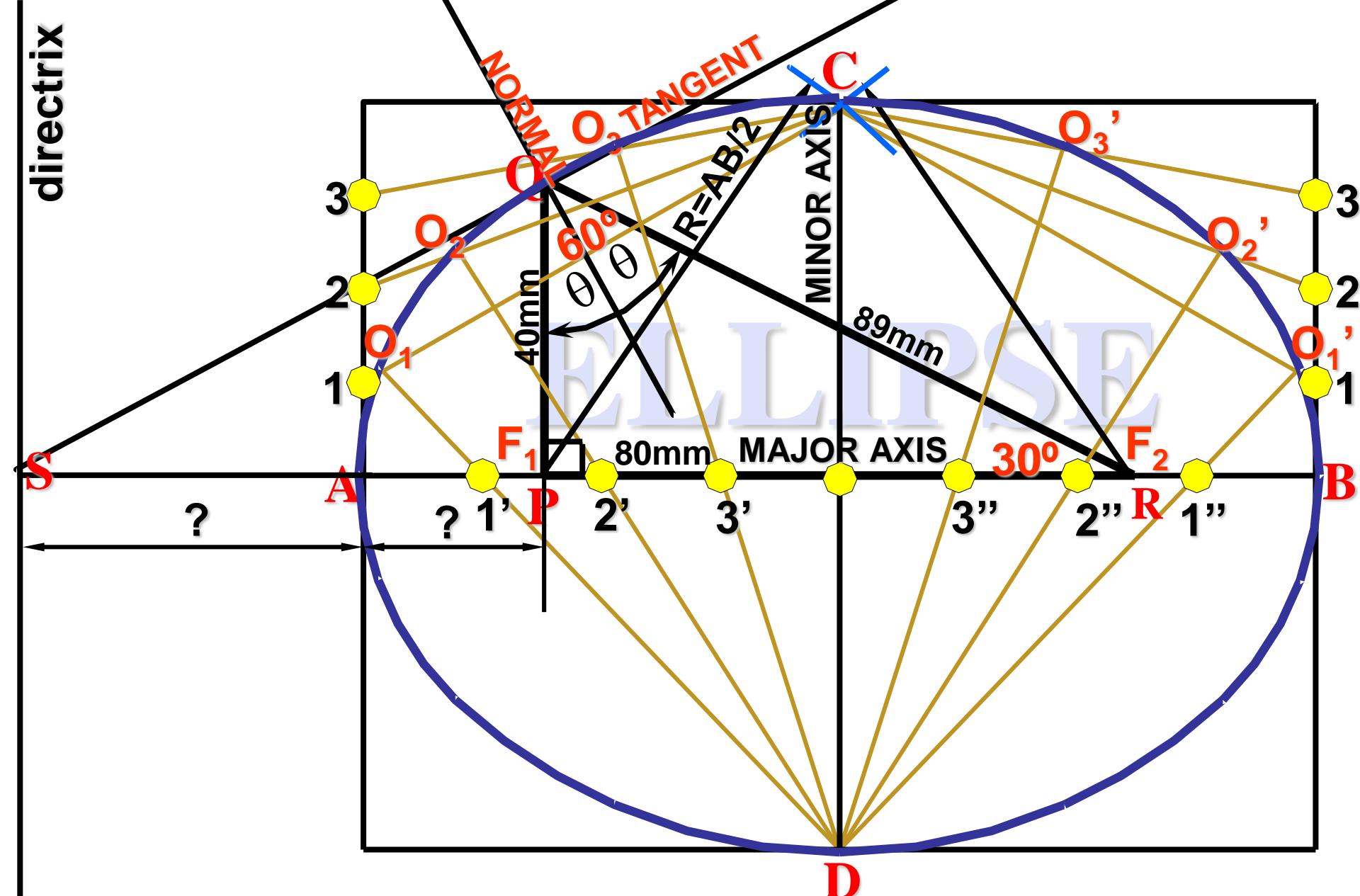
Three points A, B & P while lying along a horizontal line in order have  $AB = 60$  mm and  $AP = 80$  mm, while A & B are fixed points and P starts moving such a way that  $AP + BP$  remains always constant and when they form isosceles triangle,  $AP = BP = 50$  mm. Draw the path traced out by the point P from the commencement of its motion back to its initial position and name the path of P.



# ***PROBLEM***

Draw an ellipse passing through  $60^\circ$  corner Q of a  $30^\circ$  -  $60^\circ$  set square having smallest side PQ vertical & 40 mm long while the foci of the ellipse coincide with corners P & R of the set square.

Use “OBLONG METHOD”. Find its eccentricity.



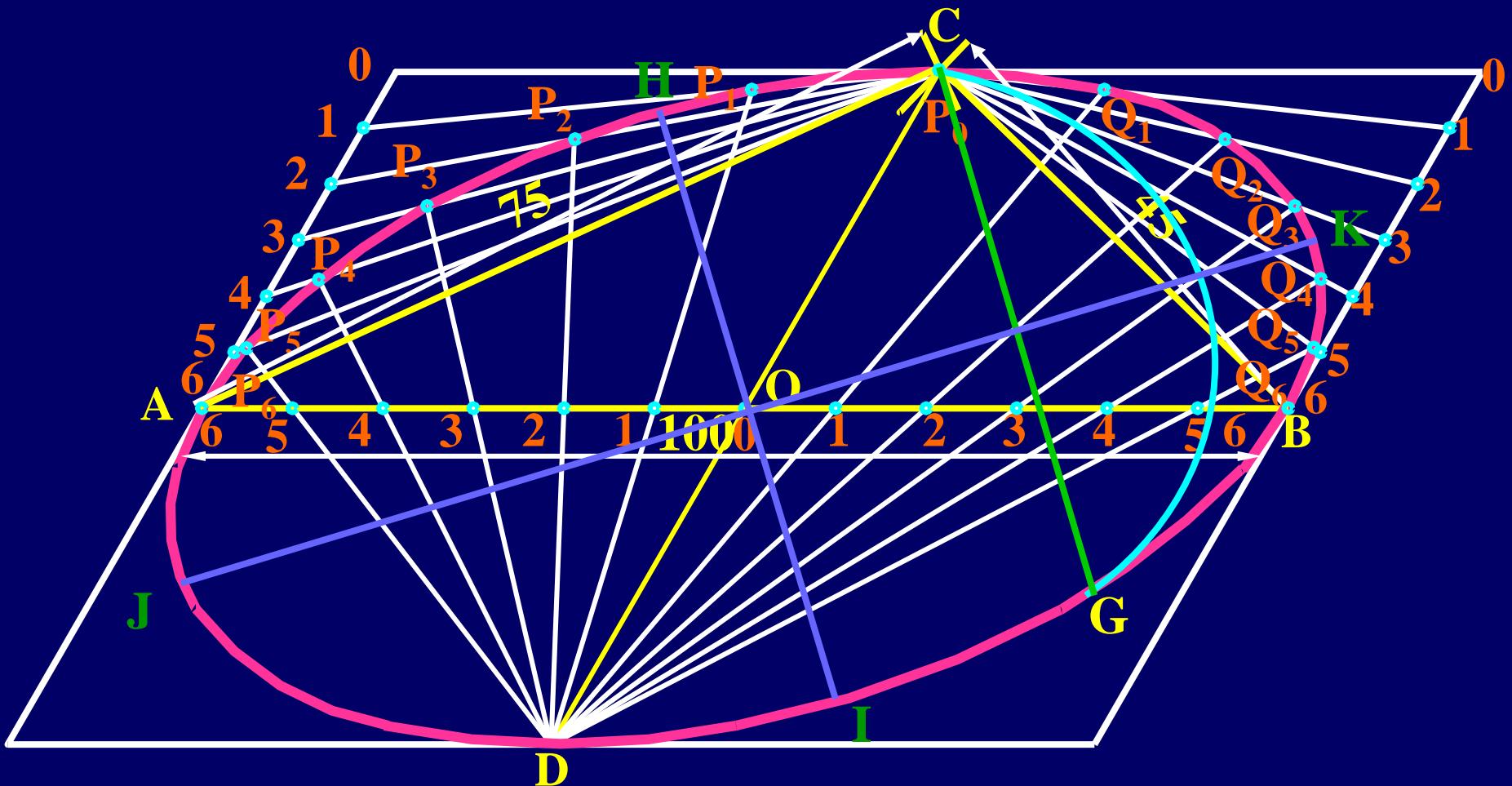
$$\text{MAJOR AXIS} = PQ + QR = 129\text{mm}$$

$$\text{ECCENTRICITY} = AP / AS$$

## ***PROBLEM***

Two points A & B are 100 mm apart. A point C is 75 mm from A and 45 mm from B. Draw an ellipse passing through points A, B, and C so that AB is not a major axis.

# ELLIPSE



# ***PROBLEM***

Draw an ellipse passing through A & B of an equilateral triangle of ABC of 50 mm edges with side AB as vertical and the corner C coincides with the focus of an ellipse. Assume eccentricity of the curve as  $2/3$ . Draw tangent & normal at point A.

# ***PROBLEM***

**Draw an ellipse passing through all the four corners A, B, C & D of a rhombus having diagonals AC=110mm and BD=70mm.**

**Use “Arcs of circles” Method and find its eccentricity.**

# METHODS FOR DRAWING PARABOLA

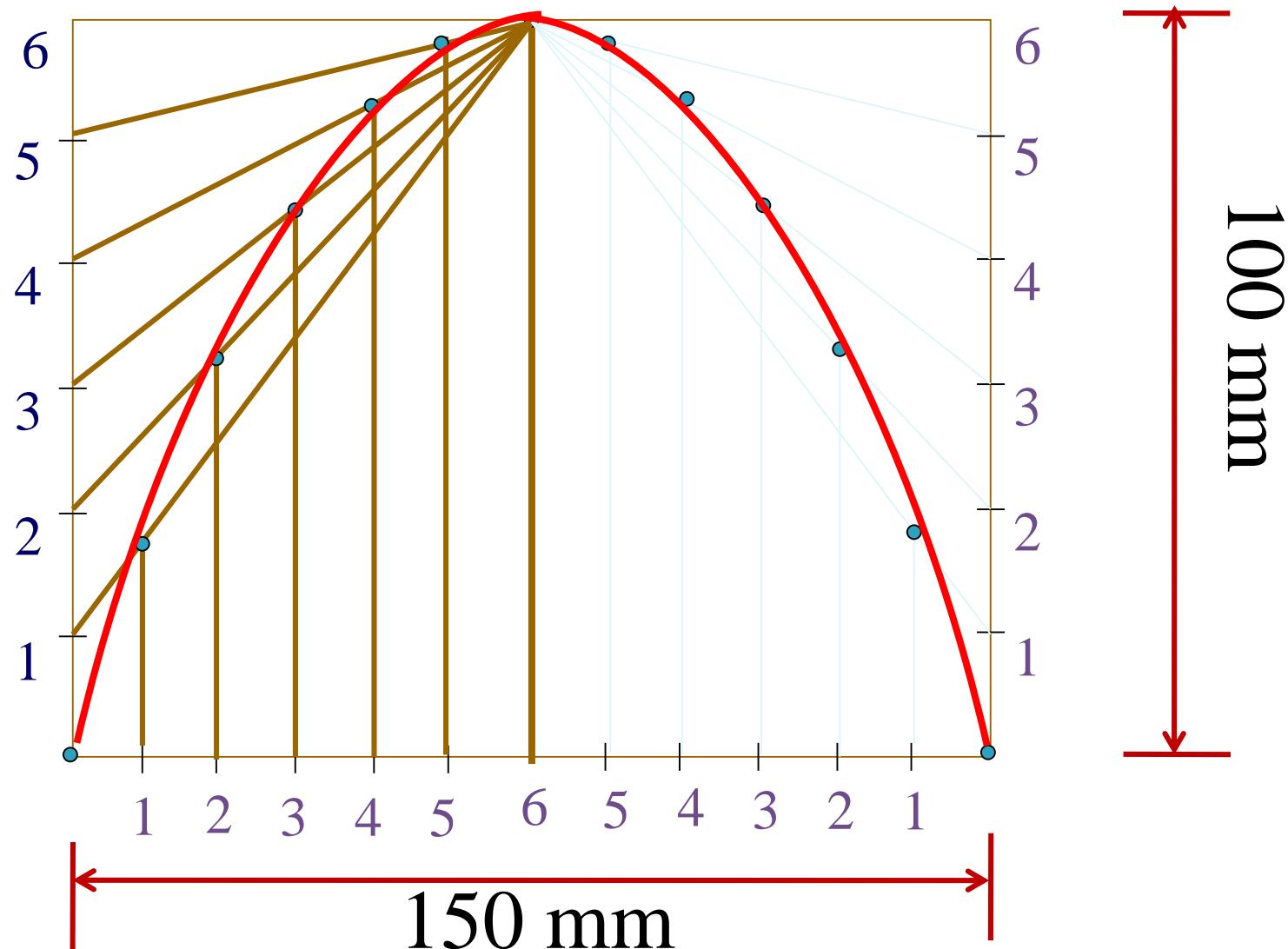
1. Rectangle Method
2. Parabola in Parallelogram
3. Tangent Method
4. Directrix Focus Method

## **PARABOLA :**

### ***BY RECTANGLE METHOD***

A ball thrown in air attains 100 m height and covers horizontal distance 150 m on ground. Draw the path of the ball (projectile).

A ball thrown in air attains 100 m height and covers horizontal distance 150 m on ground. Draw the path of the ball (projectile). **Scale 1 m = 1mm**

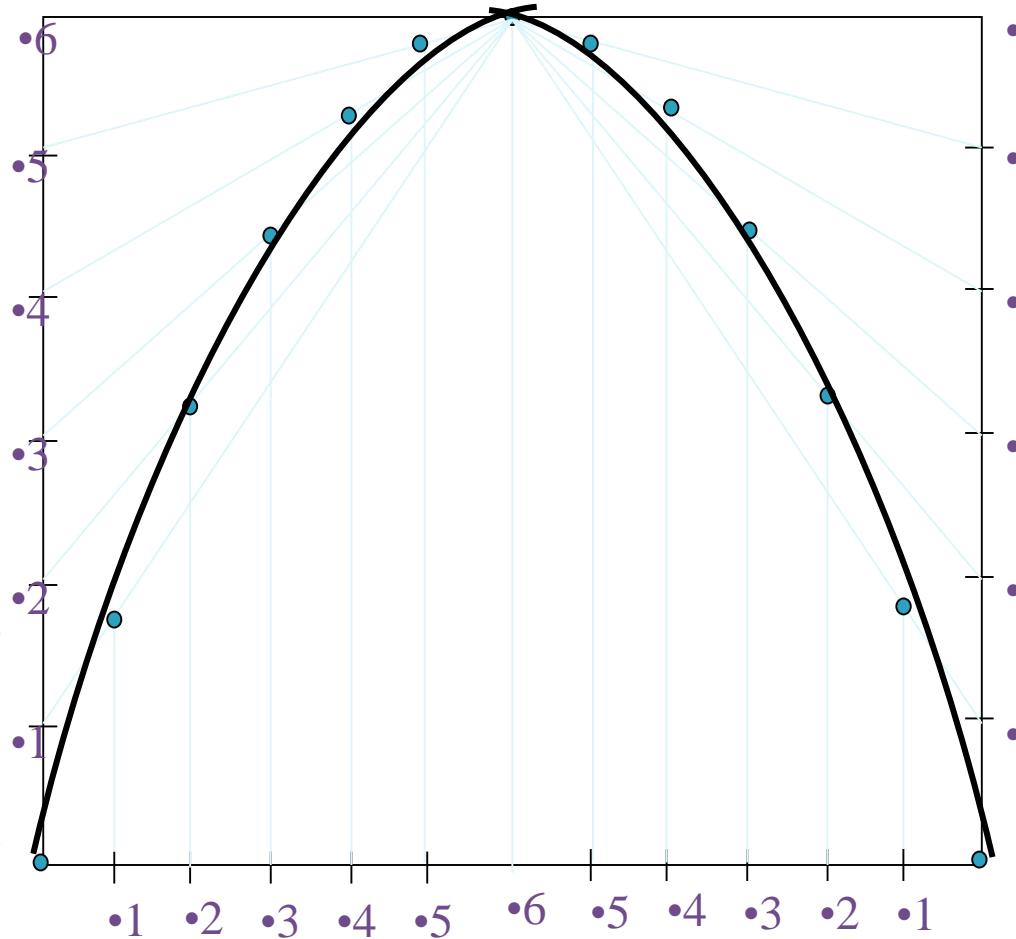


## STEPS:

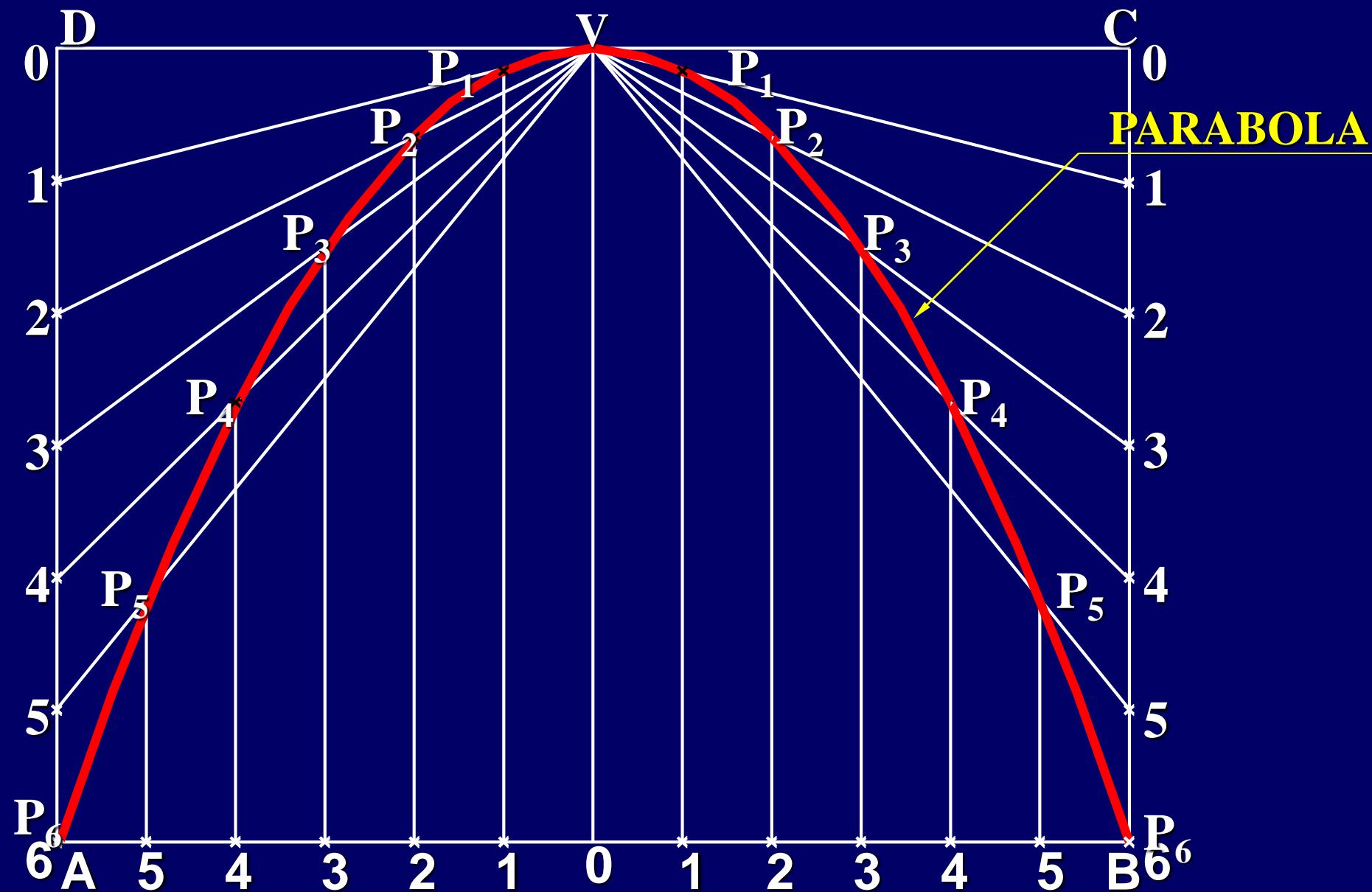
1. Draw rectangle of above size and divide it in two equal vertical parts
2. Consider left part for construction. Divide height and length in equal number of parts and name those 1,2,3,4,5& 6
3. Join vertical 1,2,3,4,5 & 6 to the top center of rectangle
4. Similarly draw upward vertical lines from horizontal 1,2,3,4,5 and wherever these lines intersect previously drawn inclined lines in sequence Mark those points and further join in smooth possible curve.
5. Repeat the construction on right side rectangle also. Join all in sequence.

This locus is Parabola.

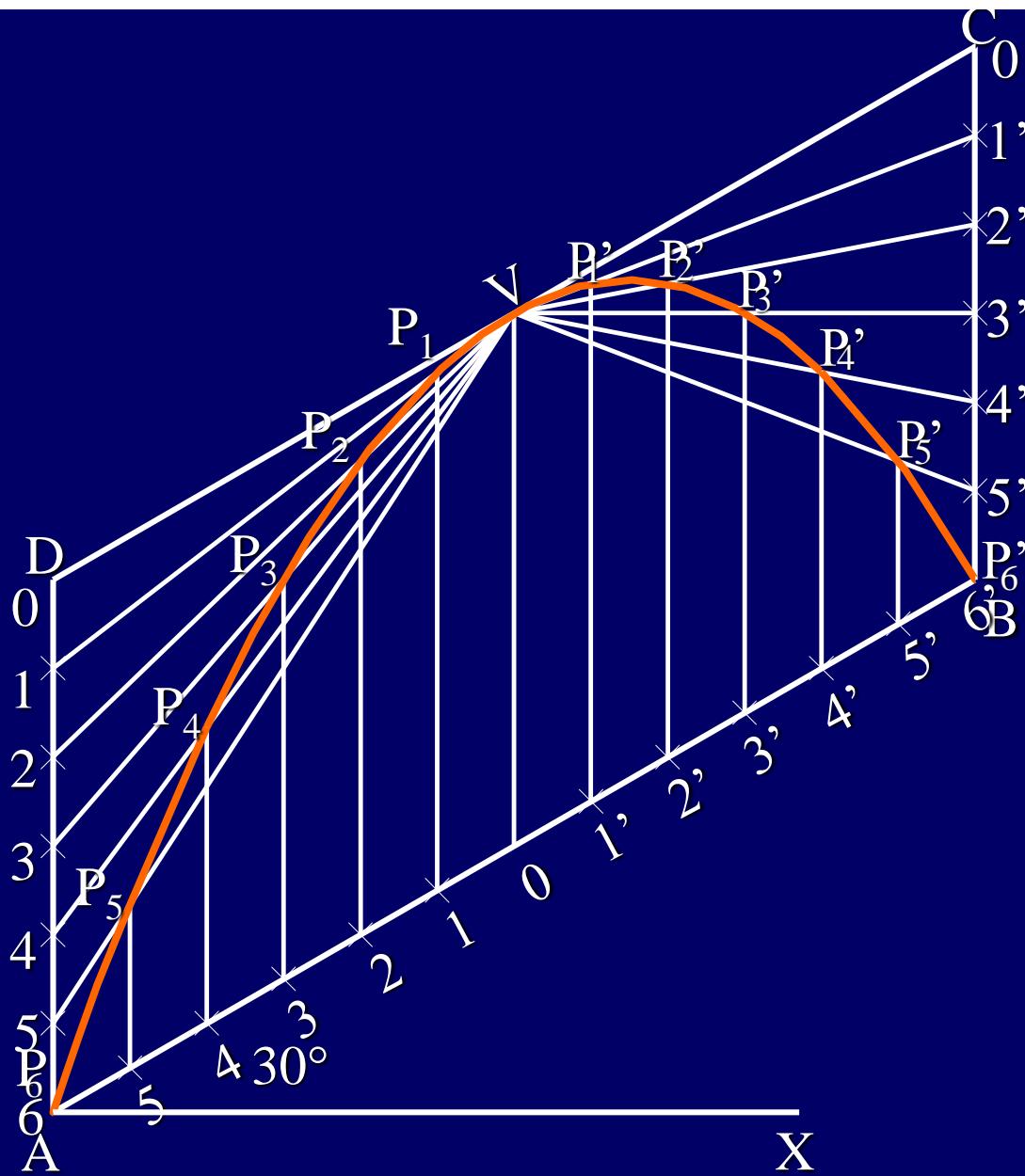
A ball thrown in air attains 100 m height and covers horizontal distance 150 m on ground. Draw the path of the ball (projectile).



# PARABOLA –RECTANGLE METHOD



# PARABOLA – IN PARALLELOGRAM

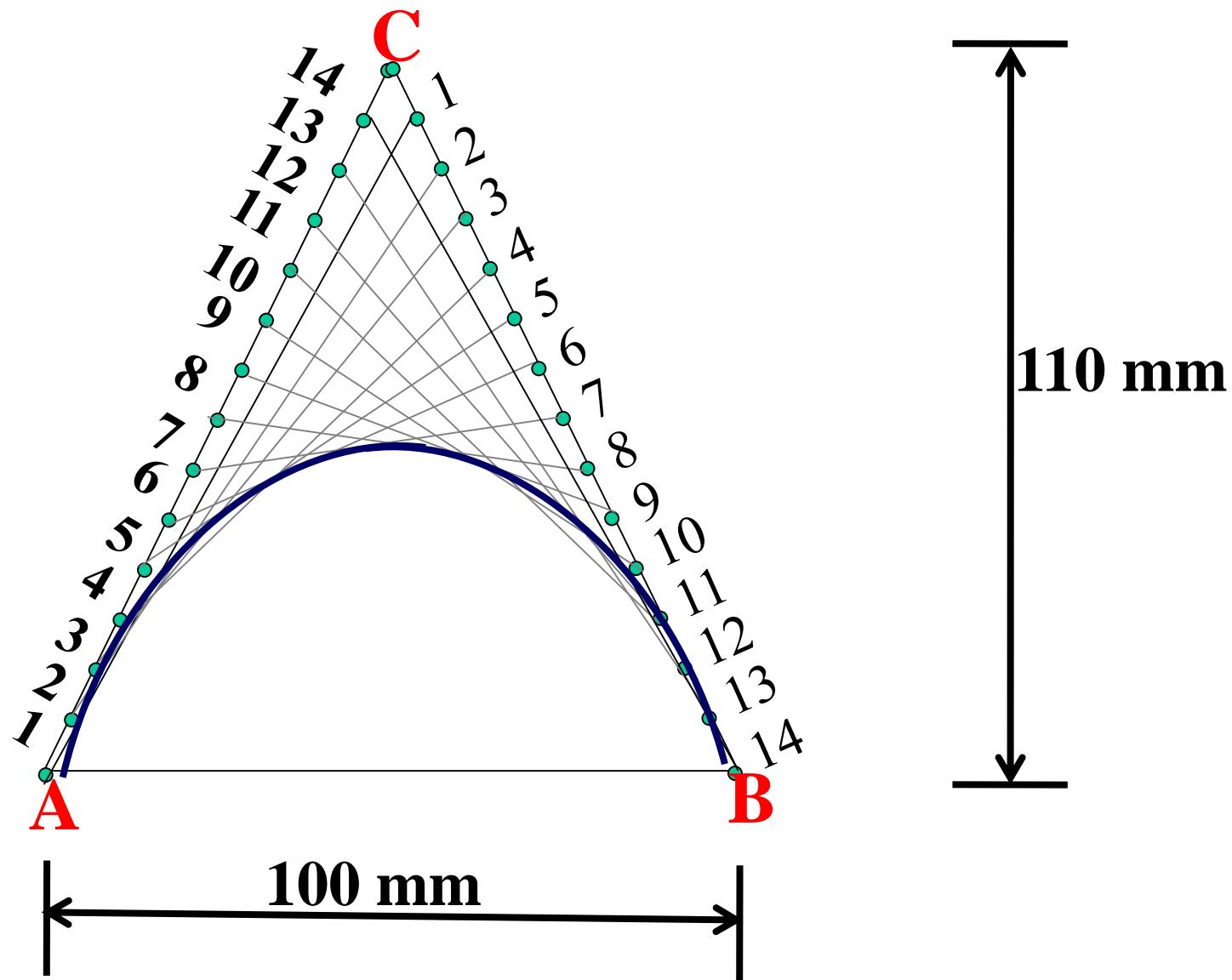


# PARABOLA :

*BY TANGENT METHOD*

Draw a parabola with base length of 100 mm and height of 55 mm by tangent method.

Draw a parabola with base length of 100 mm and height of 55 mm by tangent method.

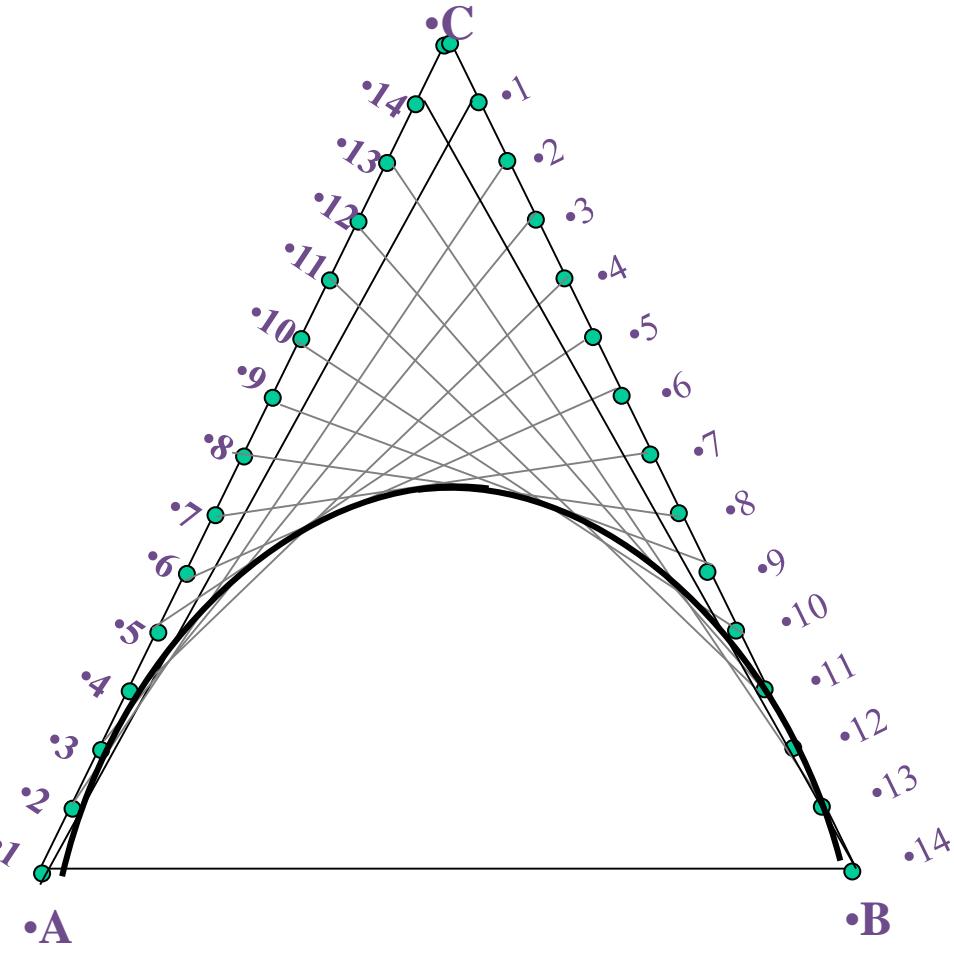


## Solution Steps:

1. Construct triangle as per the given dimensions.
2. Divide it's both sides in to same no. of equal parts.
3. Name the parts in ascending and descending manner, as shown.
4. Join 1-1, 2-2, 3-3 and so on.
5. Draw the curve as shown i.e. tangent to all these lines.

The above all lines being tangents to the curve, it is called method of tangents.

Draw a parabola with base length of 100 mm and height of 55 mm by tangent method.



# PARABOLA :

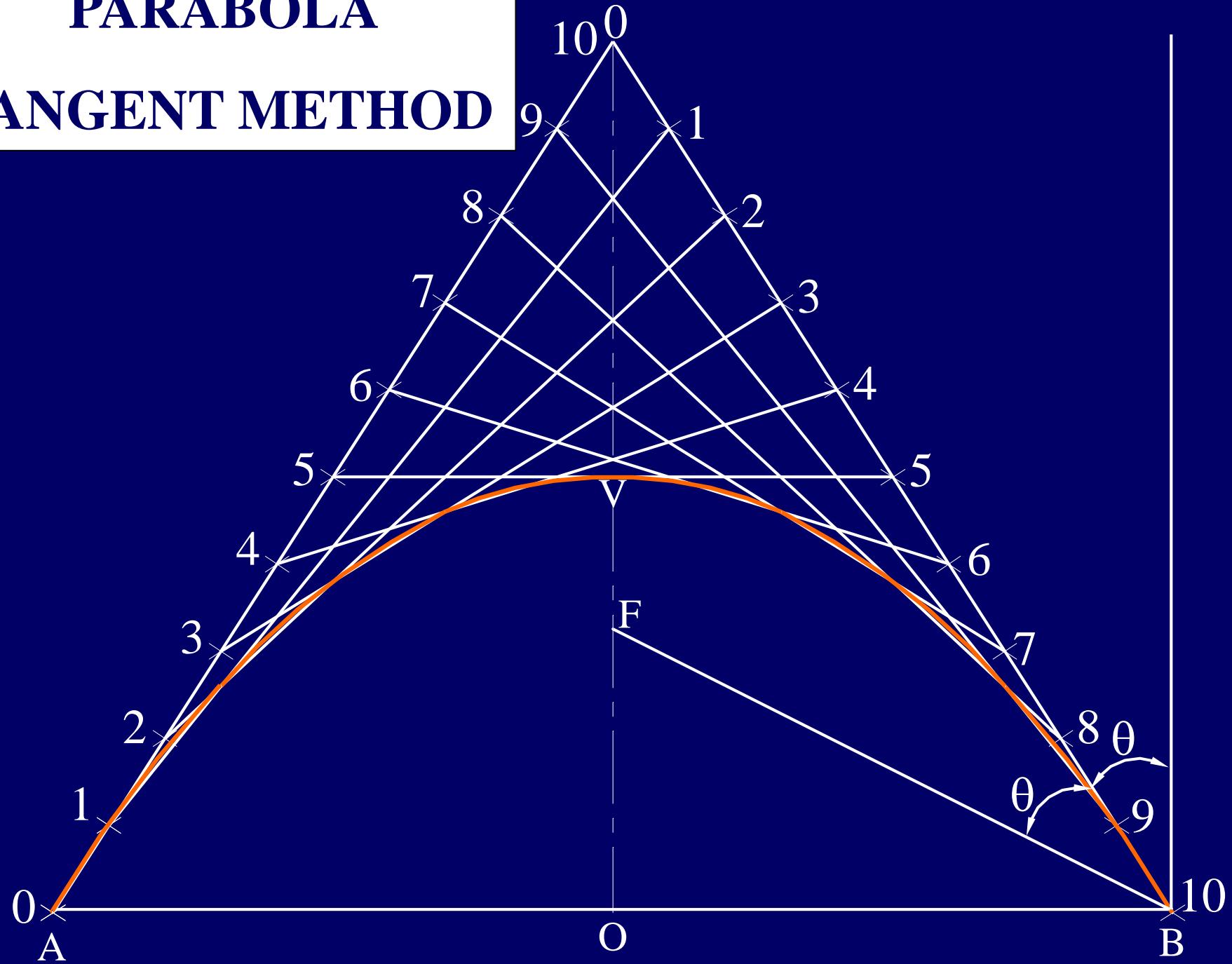
## *BY TANGENT METHOD*

Draw a parabola with base length of 100 mm and height of 55 mm by tangent method.

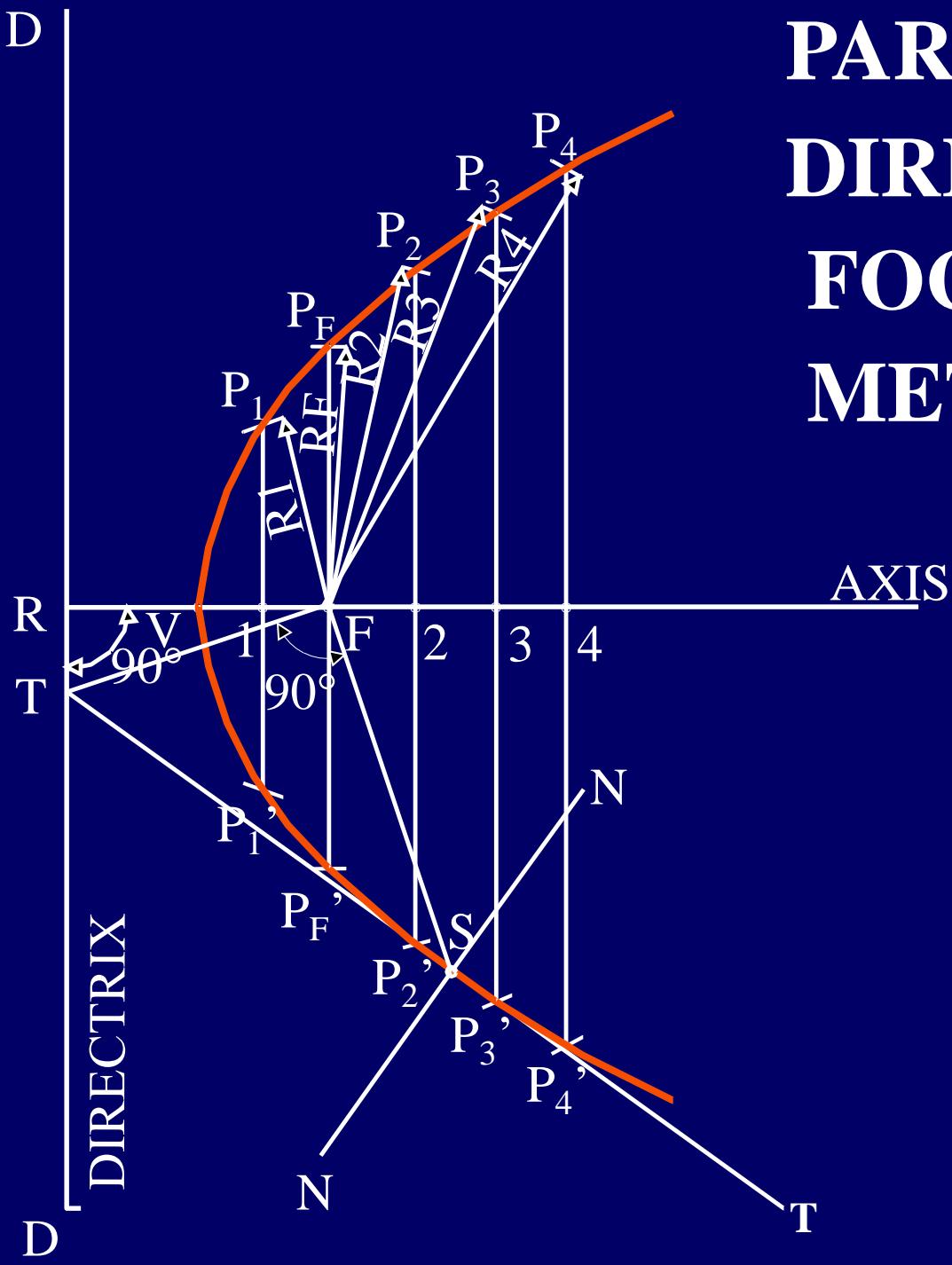
Draw an isosceles triangle of 100 mm long base and 110 mm long altitude. Inscribe a parabola in it by method of tangents.

# PARABOLA

## TANGENT METHOD

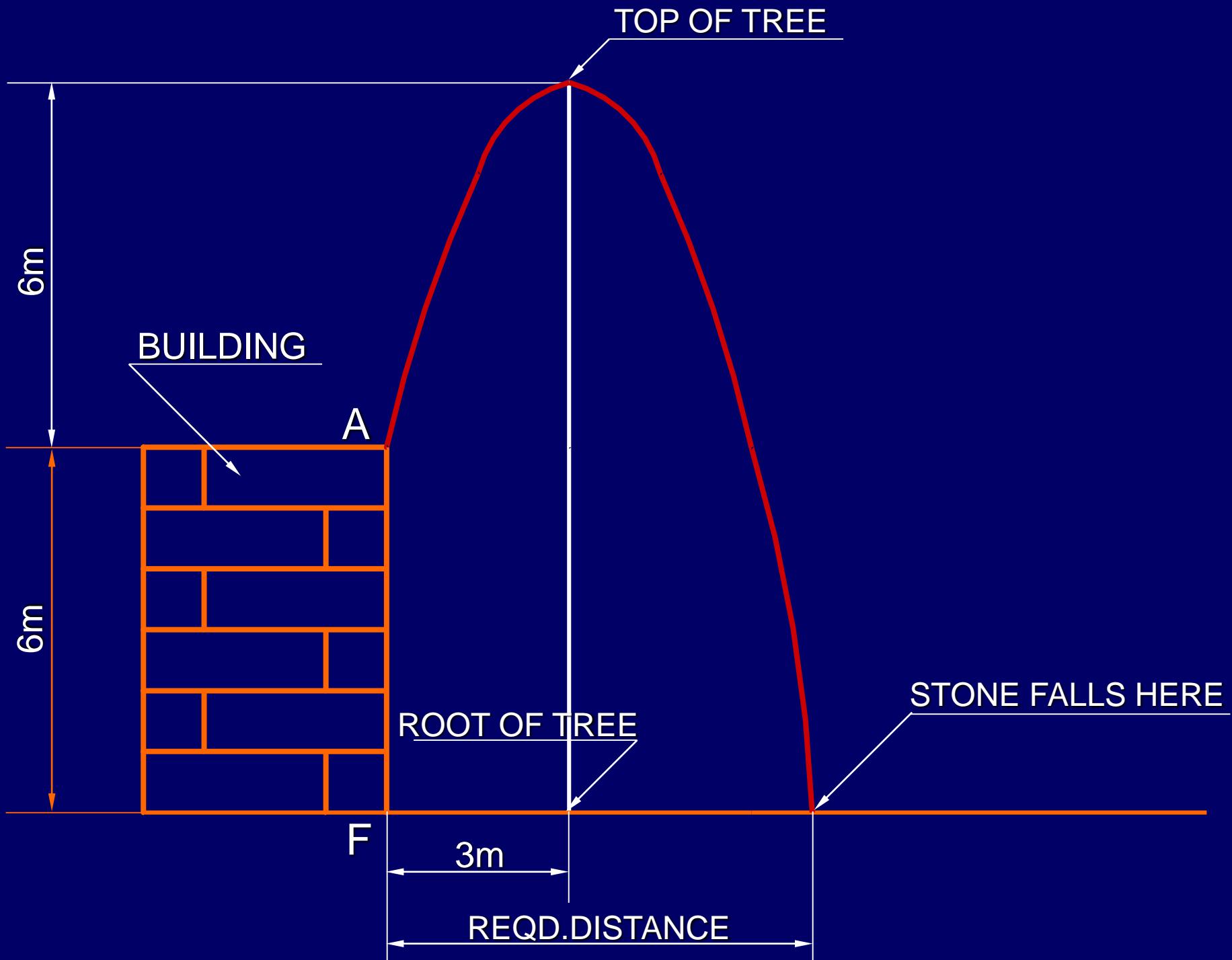


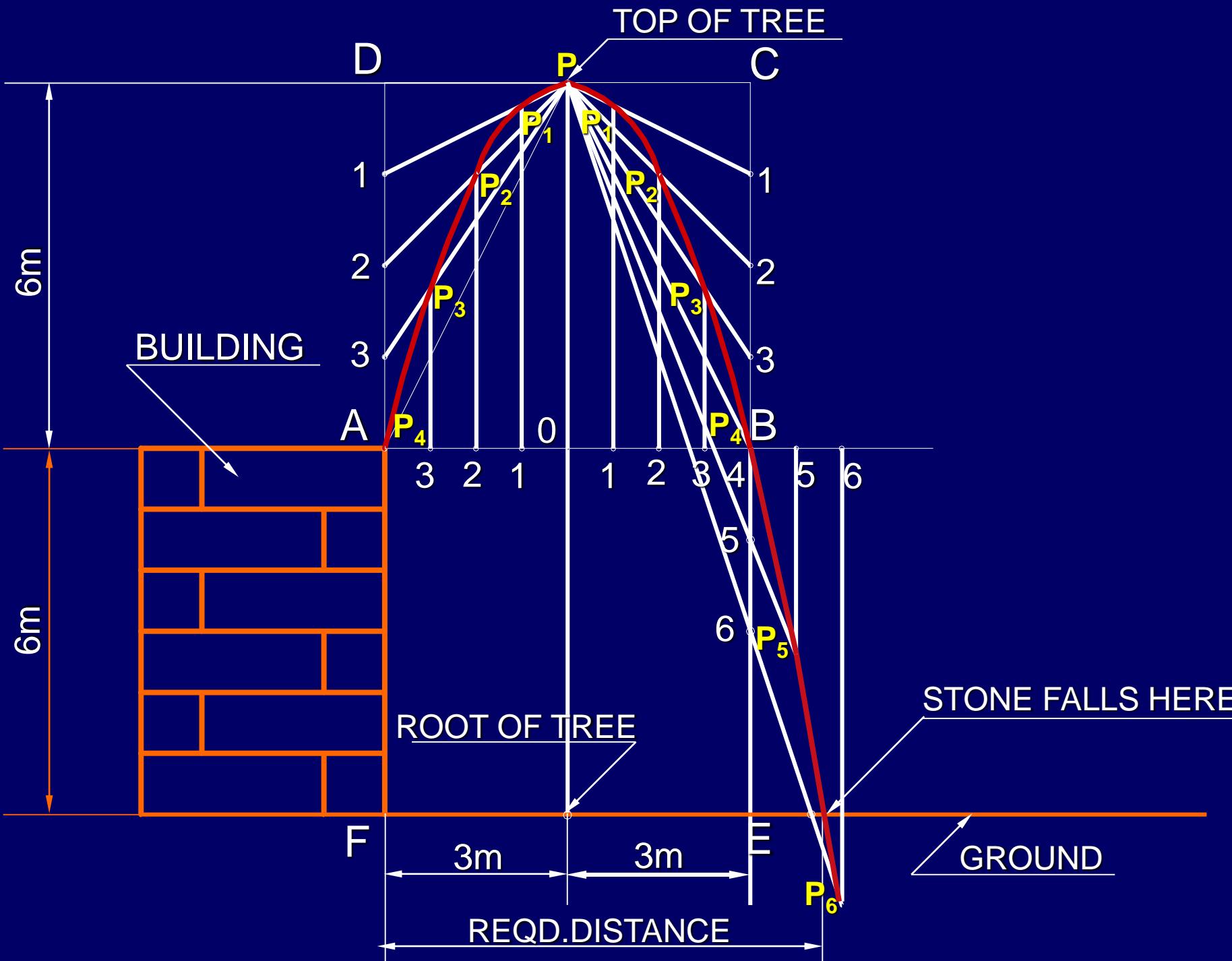
# PARABOLA DIRECTRIX FOCUS METHOD



# PROBLEM

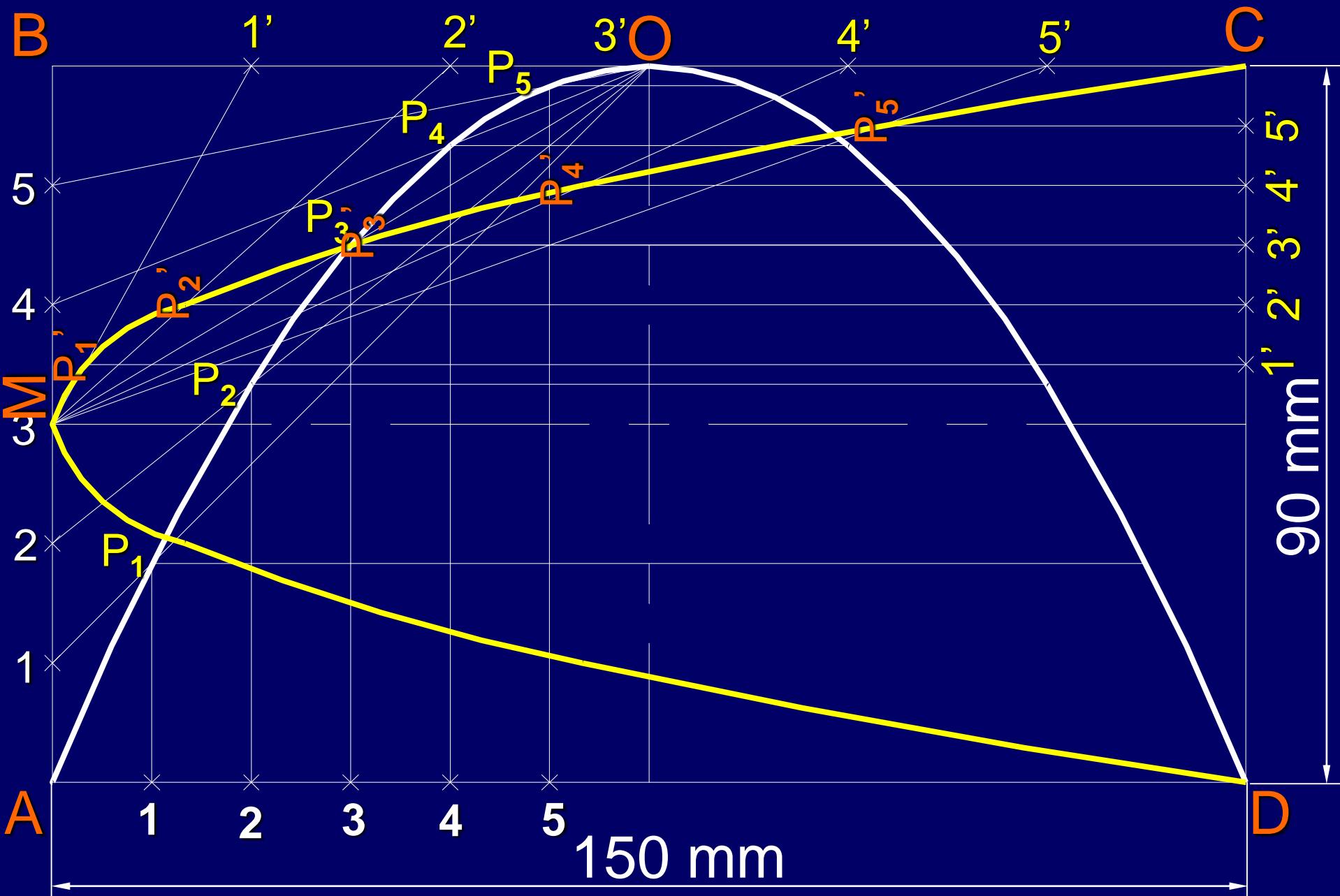
A stone is thrown from a building 6 m high. It just crosses the top of a palm tree 12 m high. Trace the path of the projectile if the horizontal distance between the building and the palm tree is 3 m. Also find the distance of the point from the building where the stone falls on the ground.





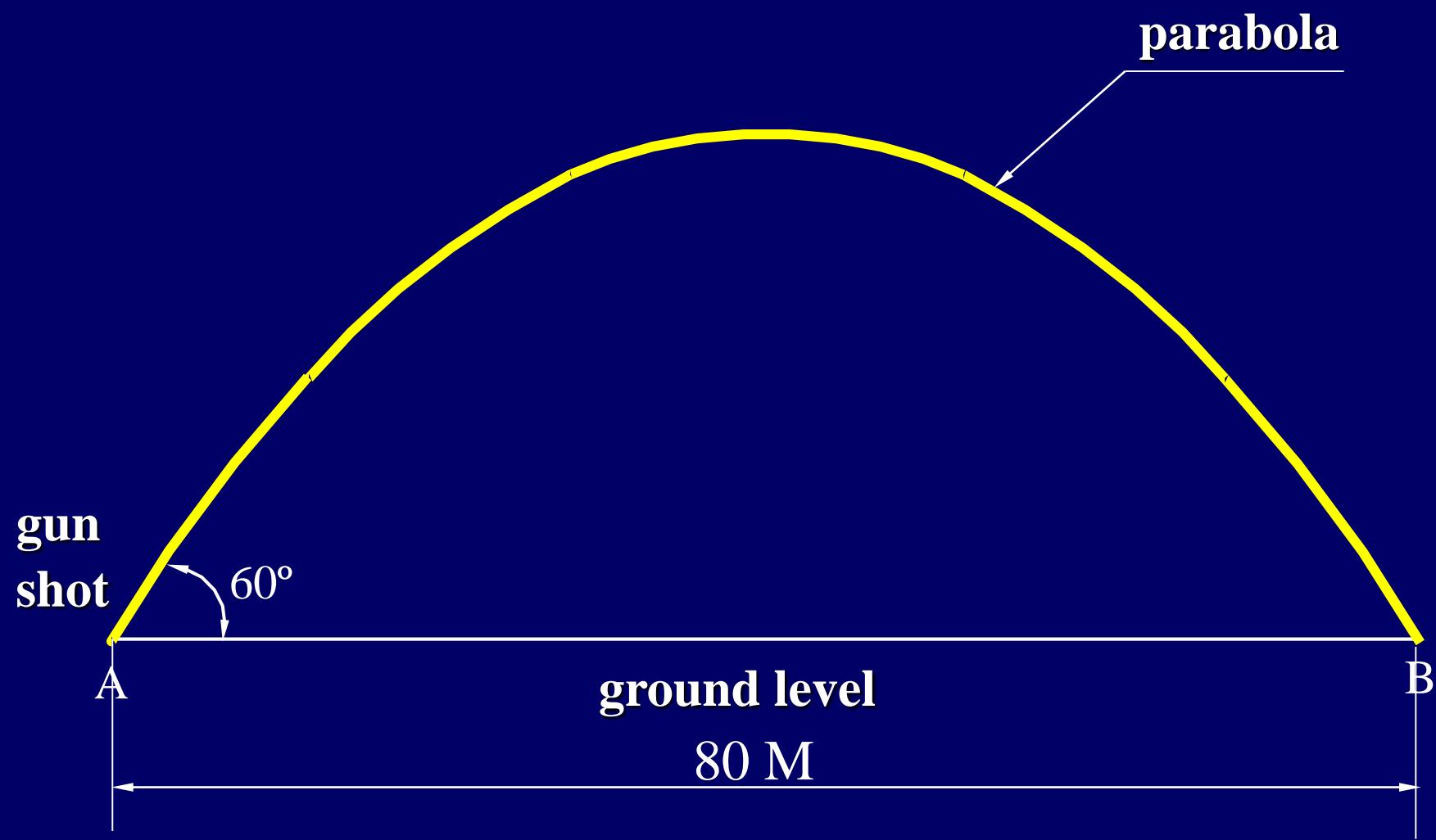
# PROBLEM

In a rectangle of sides 150 mm and 90 mm, inscribe two parabola such that their axis bisect each other. Find out their focus points & positions of directrix.

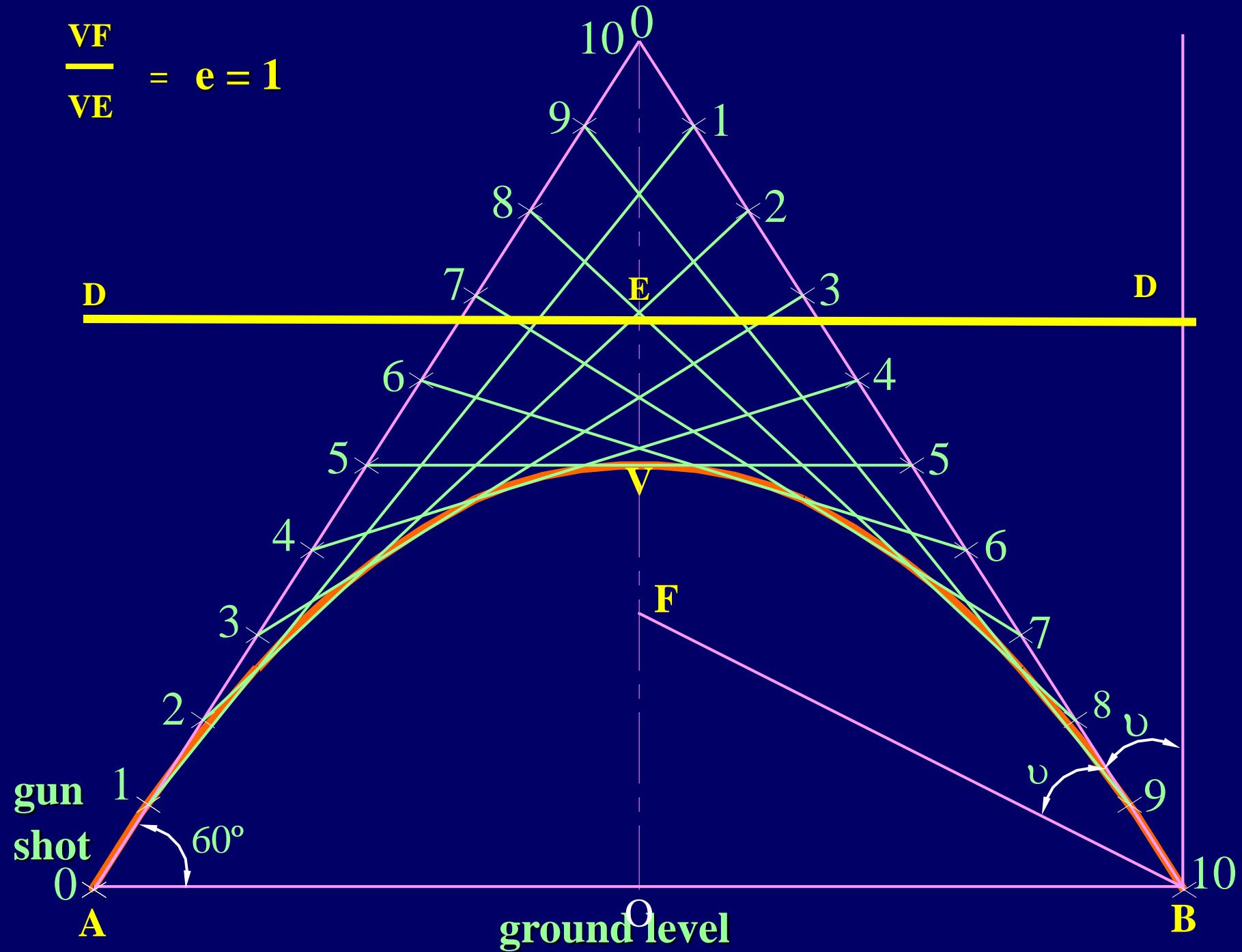


# **EXAMPLE**

**A shot is discharge from the ground level at an angle 60 to the horizontal at a point 80m away from the point of discharge. Draw the path trace by the shot. Use a scale 1:100**



$$\frac{VF}{VE} = e = 1$$

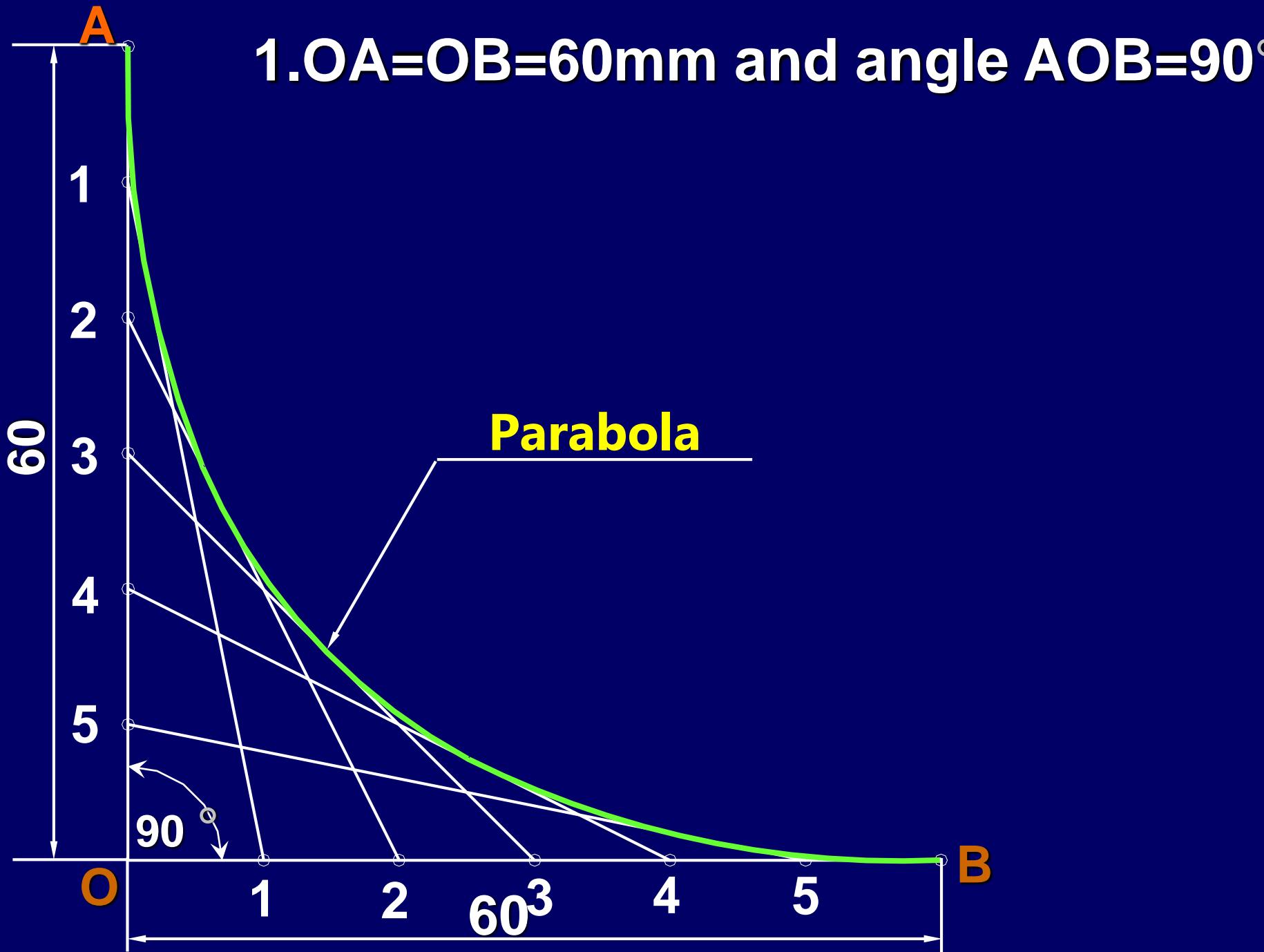


**Connect two given points A and B by a Parabolic curve, when**

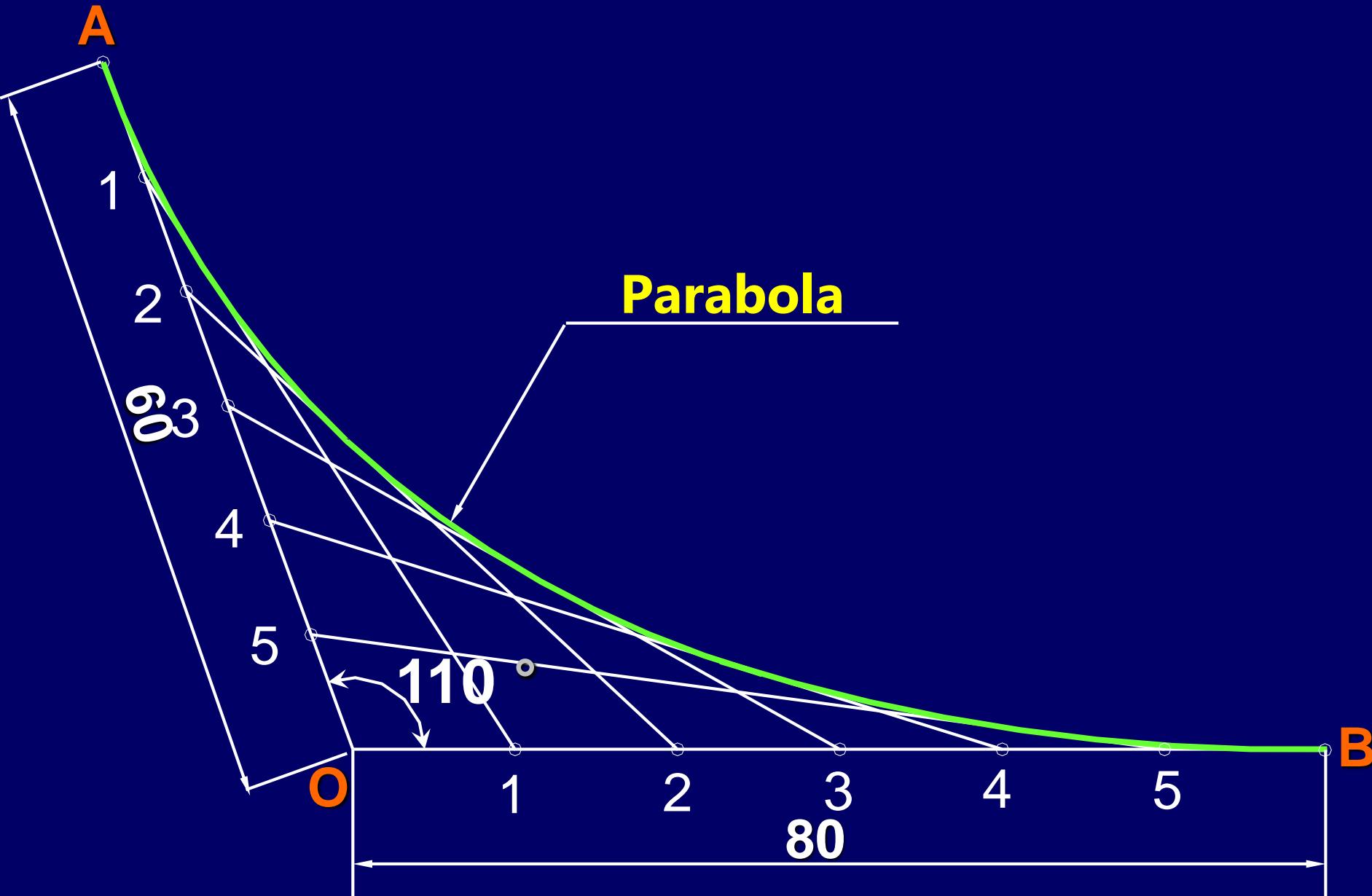
**1.  $OA=OB=60\text{mm}$  and angle  $AOB=90^\circ$**

**2.  $OA=60\text{mm}, OB=80\text{mm}$  & angle  $AOB=110^\circ$**

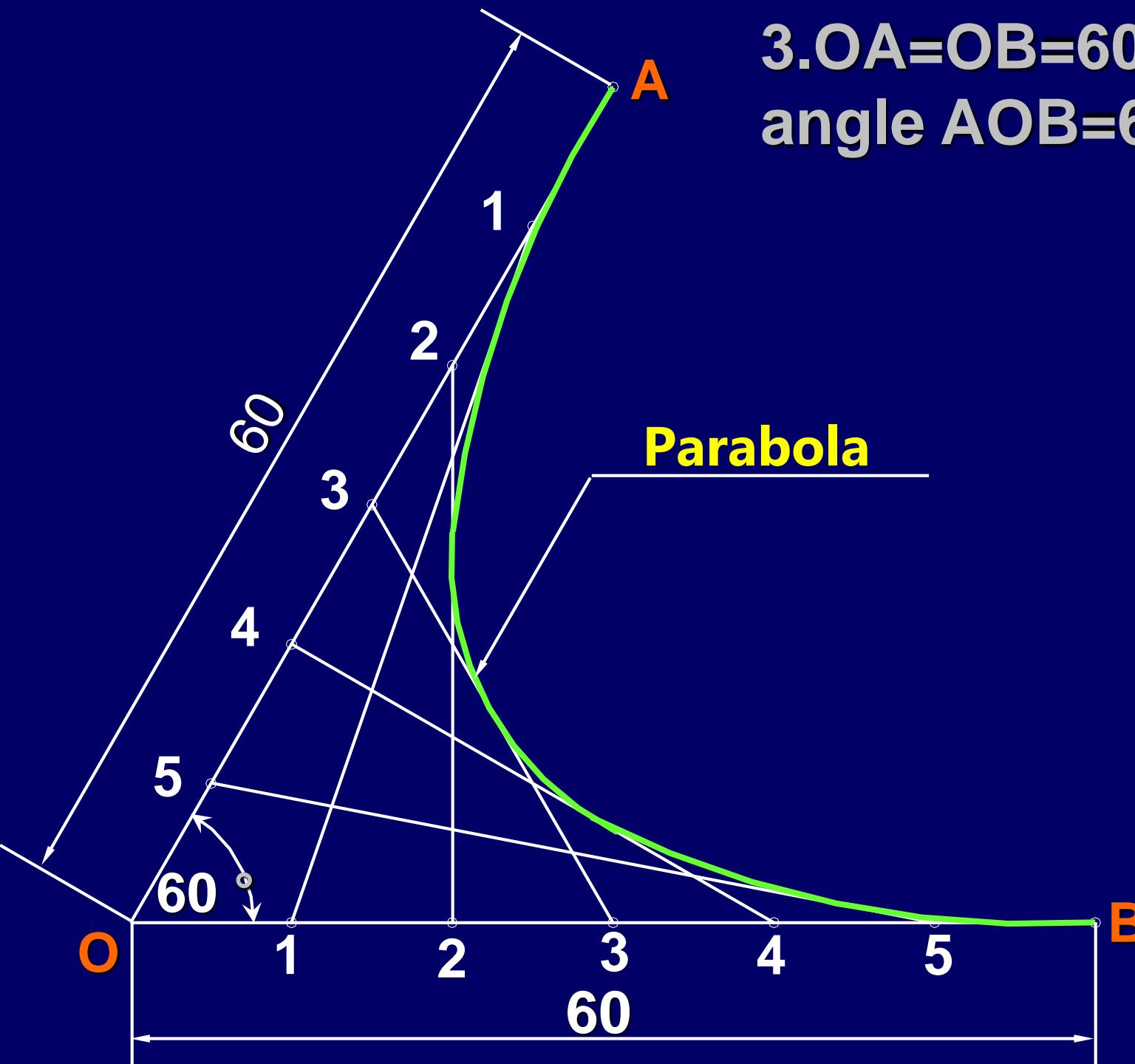
**3.  $OA=OB=60\text{mm}$  and angle  $AOB=60^\circ$**



**2.OA=60mm,OB=80mm and angle AOB=110°**

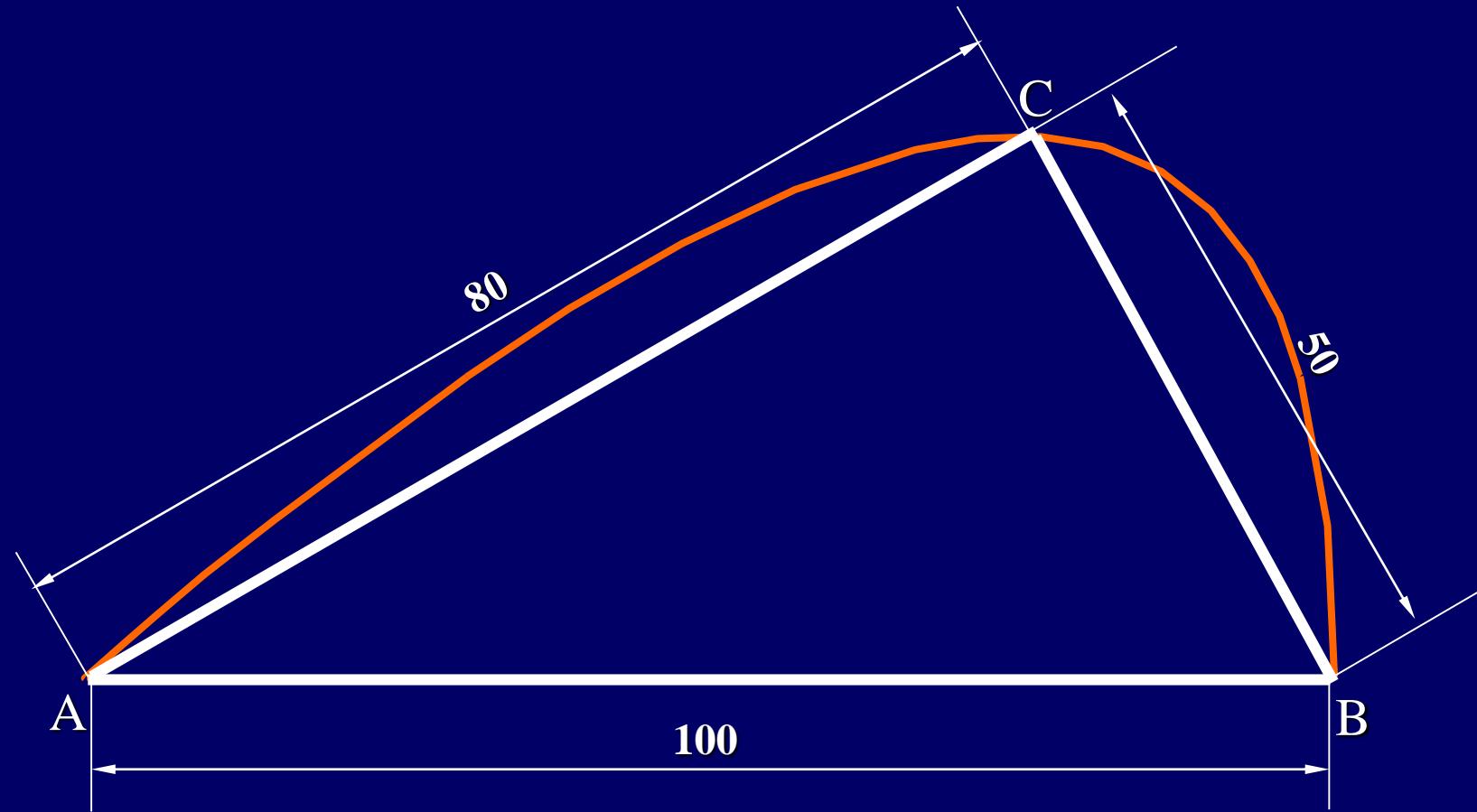


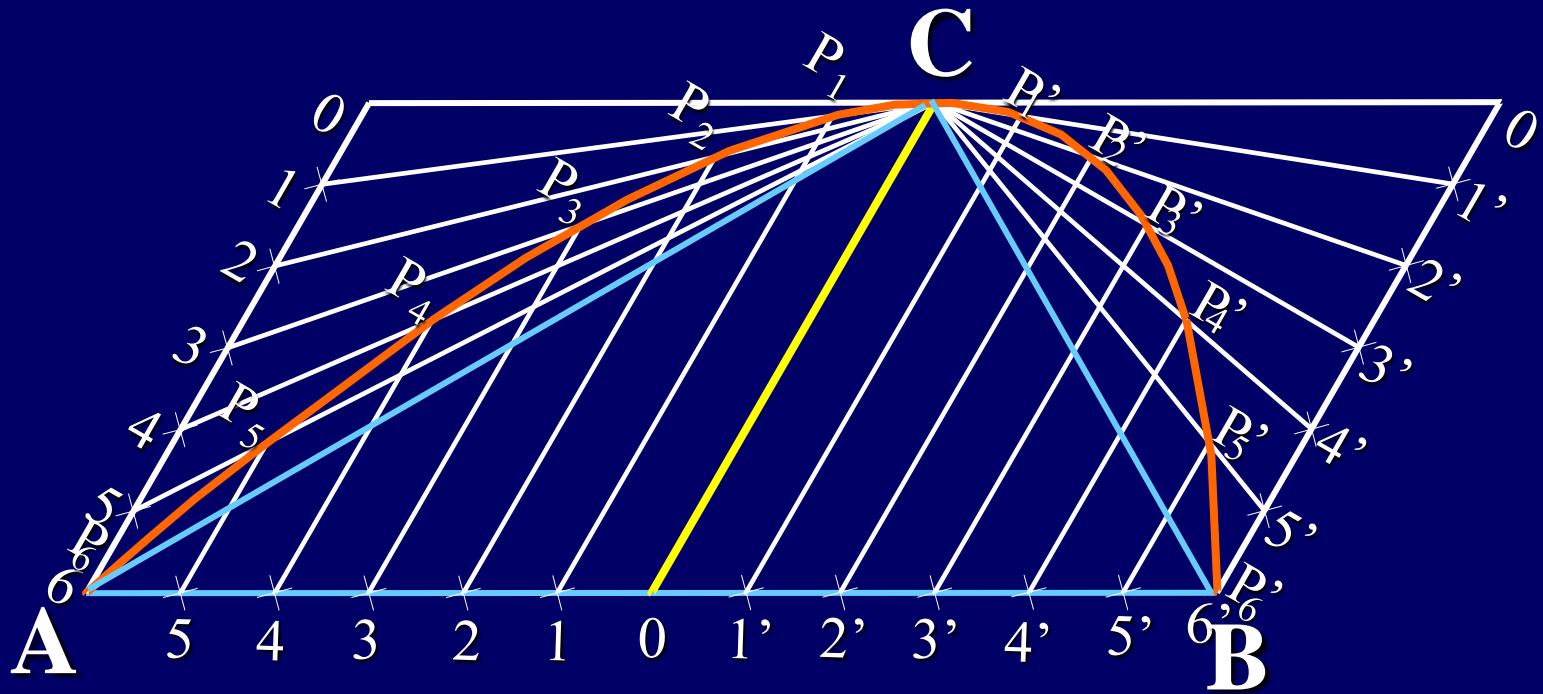
$OA = OB = 60\text{mm}$  and  
angle  $AOB = 60^\circ$



# example

Draw a parabola passing through three different points A, B and C such that  $AB = 100\text{mm}$ ,  $BC=50\text{mm}$  and  $CA=80\text{mm}$  respectively.





# **METHODS FOR DRAWING HYPERBOLA**

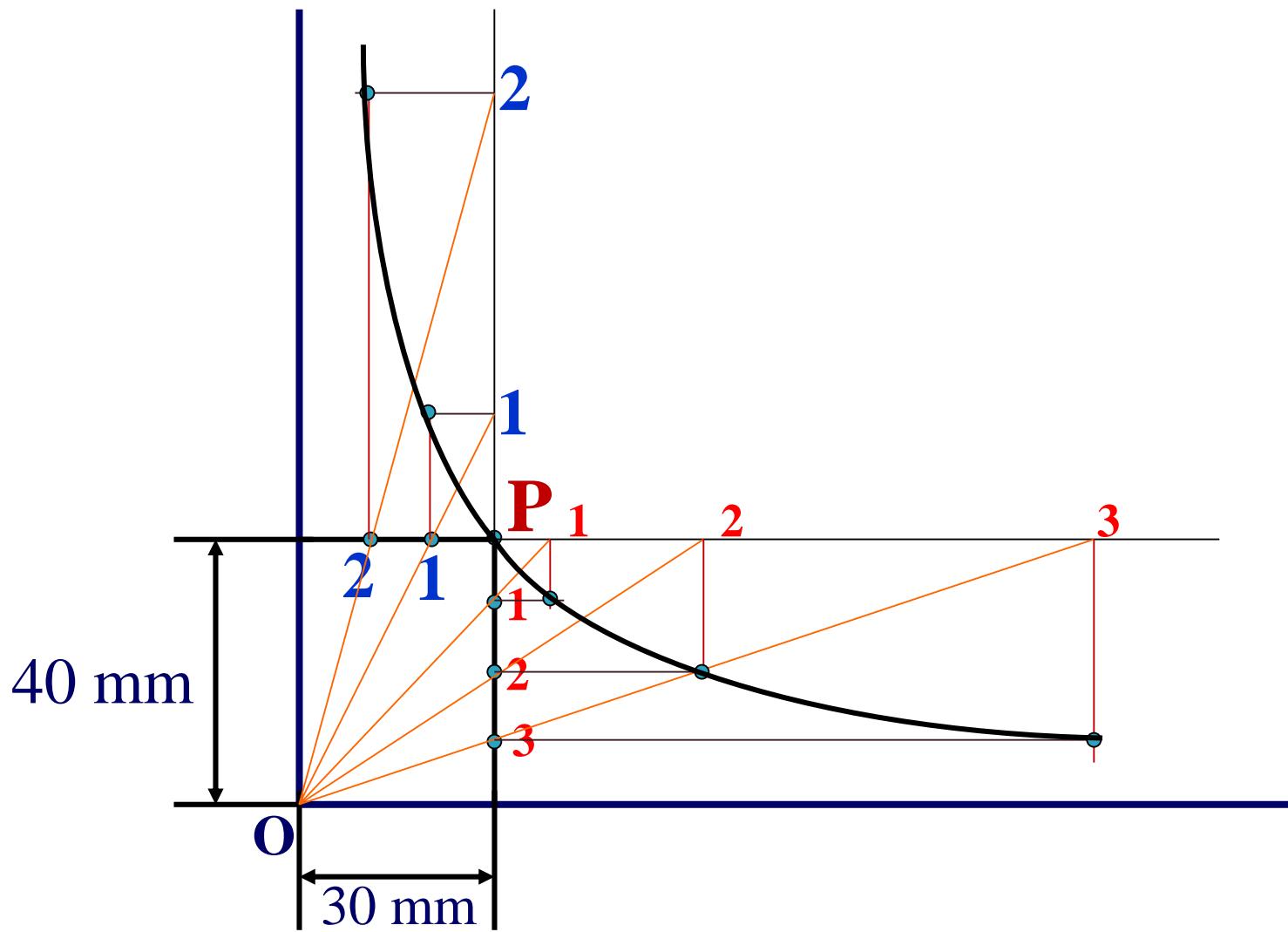
- 1. Rectangle Method**
- 2. Oblique Method**
- 3. Directrix Focus Method**

## HYPERBOLA :

*Through a Point of Known Coordinate*

Point P is 40 mm and 30 mm from horizontal and vertical axes respectively. Draw Hyperbola through it.

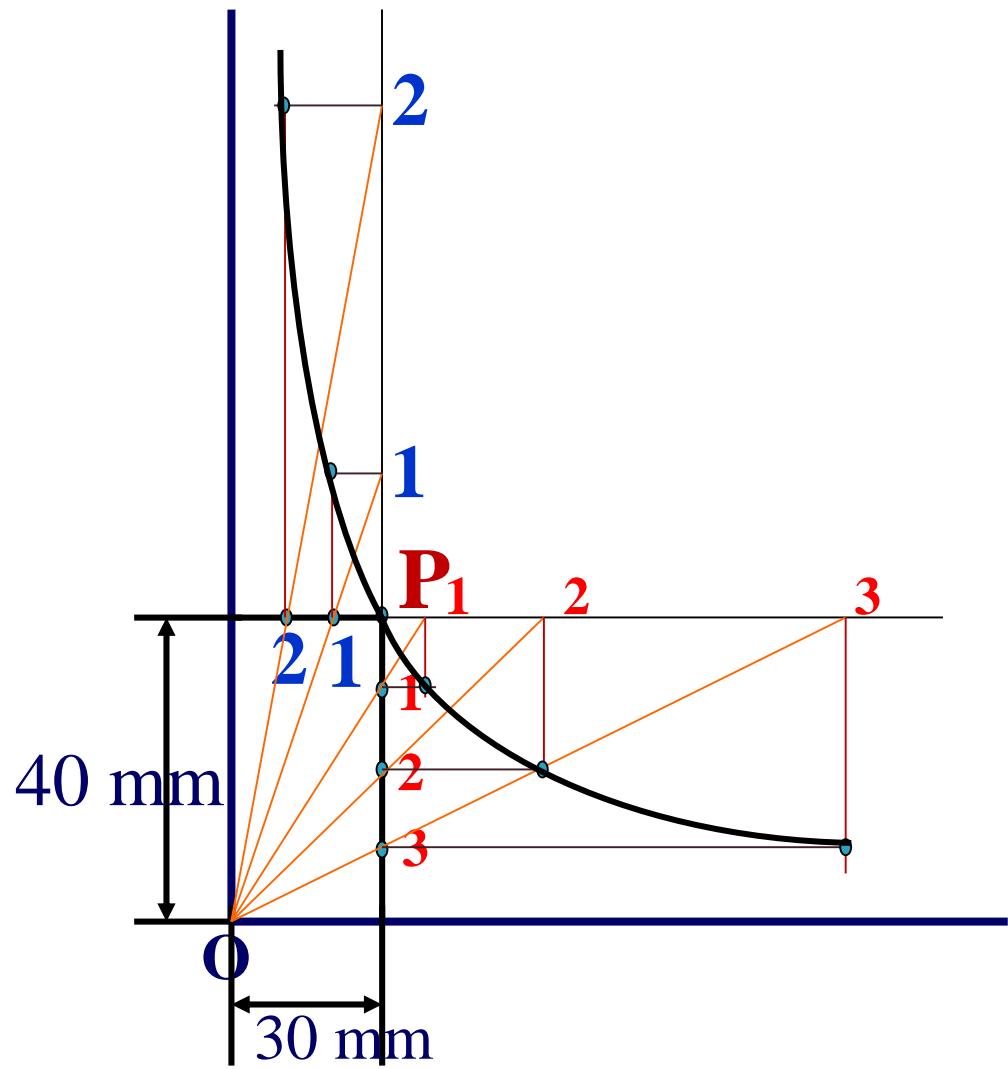
Point P is 40 mm and 30 mm from horizontal and vertical axes respectively. Draw Hyperbola through it.



## Solution Steps:

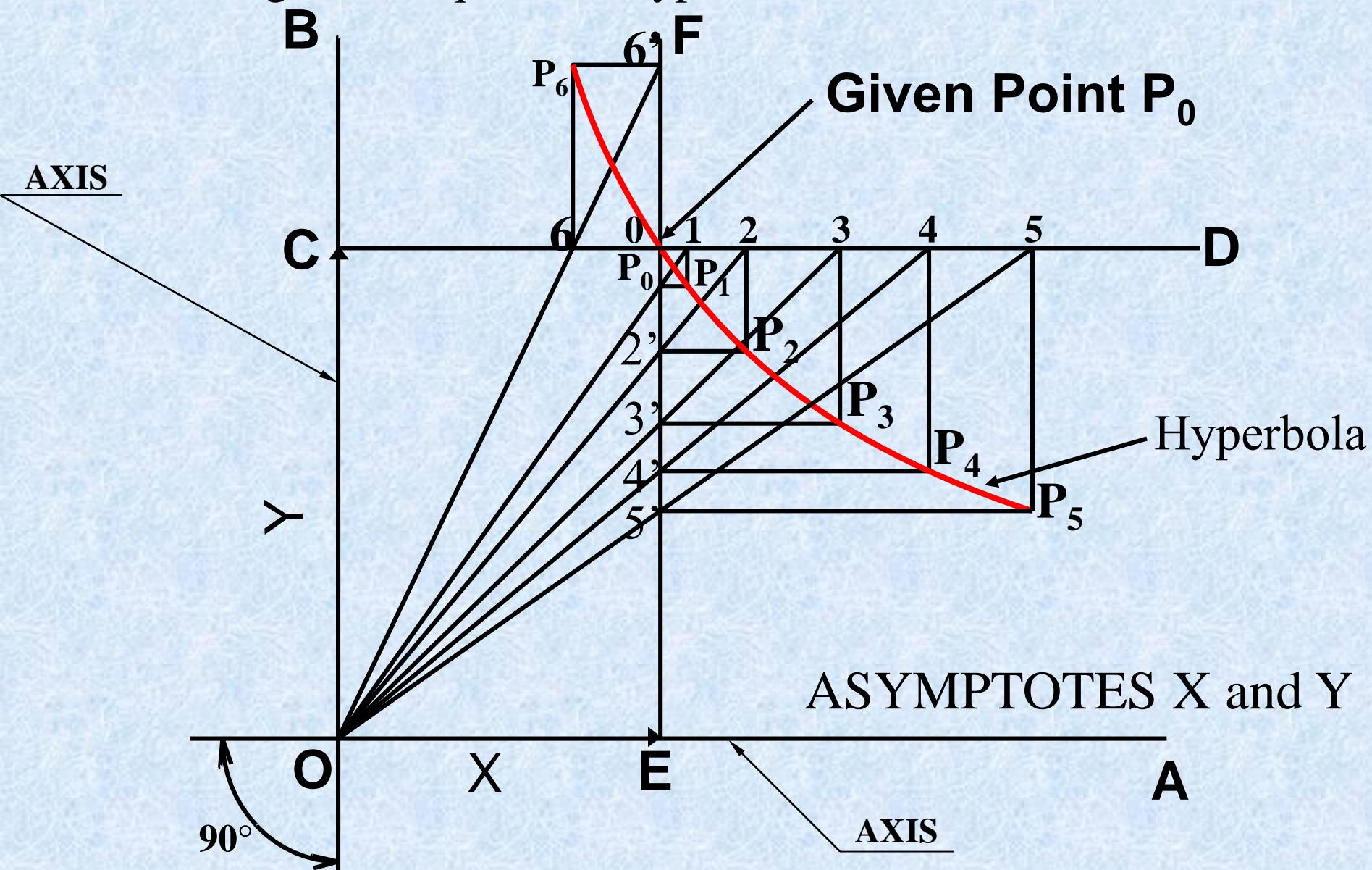
- 1) Extend horizontal line from P to right side.
- 2) Extend vertical line from P upward.
- 3) On horizontal line from P, mark some points taking any distance and name them after P-1, 2, 3, 4 etc.
- 4) Join 1-2-3-4 points to pole O. Let them cut part [P-B] also at 1, 2, 3, 4 points.
- 5) From horizontal 1, 2, 3, 4 draw vertical lines downwards and
- 6) From vertical 1, 2, 3, 4 points [from P-B] draw horizontal lines.
- 7) Line from 1 horizontal and line from 1 vertical will meet at  $P_1$ . Similarly mark  $P_2$ ,  $P_3$ ,  $P_4$  points.
- 8) Repeat the procedure by marking four points on upward vertical line from P and joining all those to pole O. Name this points  $P_6$ ,  $P_7$ ,  $P_8$  etc. and join them by smooth curve.

Point P is 40 mm and 30 mm from horizontal and vertical axes respectively. Draw Hyperbola through it.



# RECTANGULAR HYPERBOLA

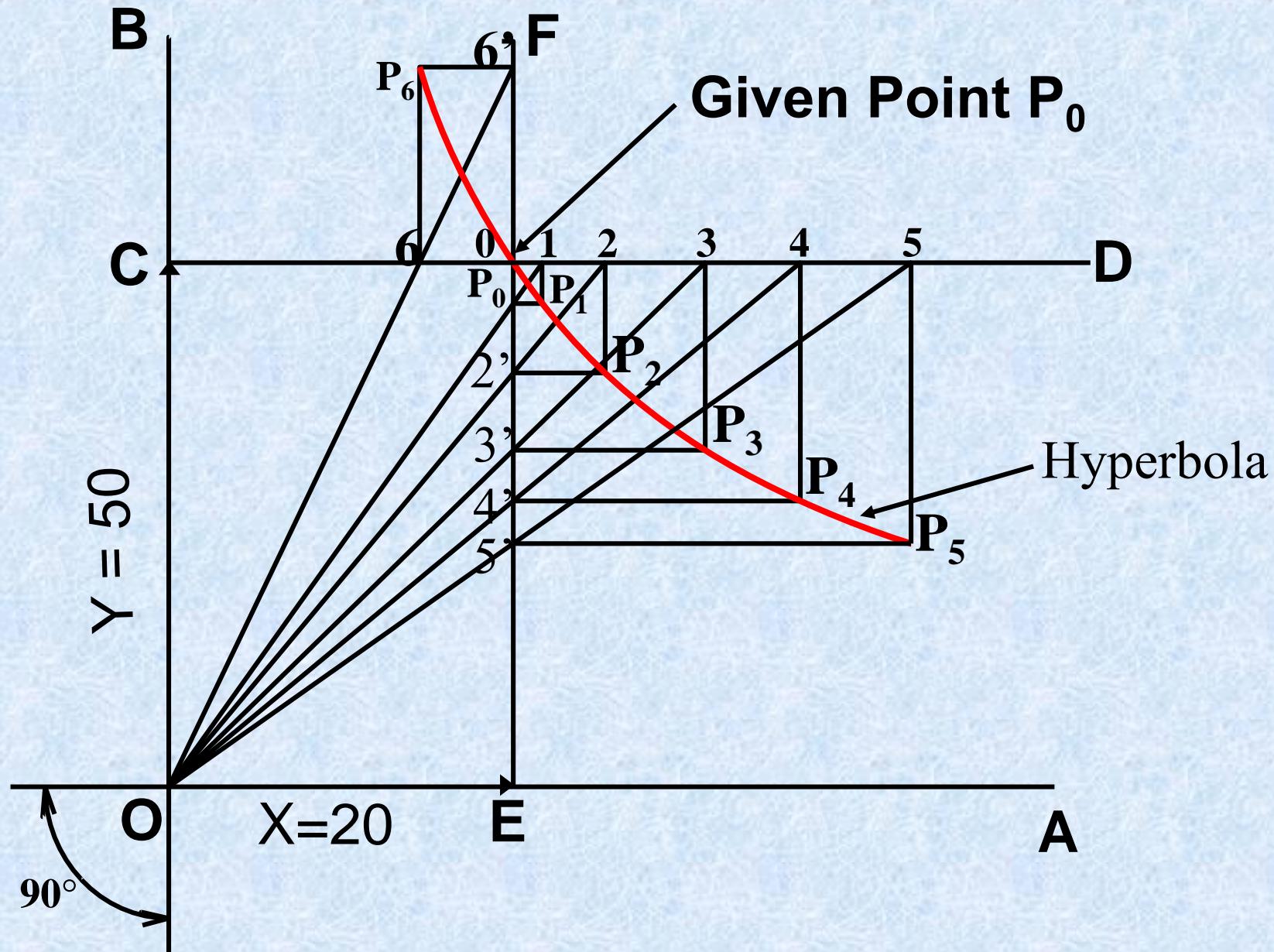
When the asymptotes are at right angles to each other, the hyperbola is called rectangular or equilateral hyperbola



## Problem:-

Two fixed straight lines OA and OB are at right angle to each other. A point “P” is at a distance of 20 mm from OA and 50 mm from OB. Draw a rectangular hyperbola passing through point “P”.

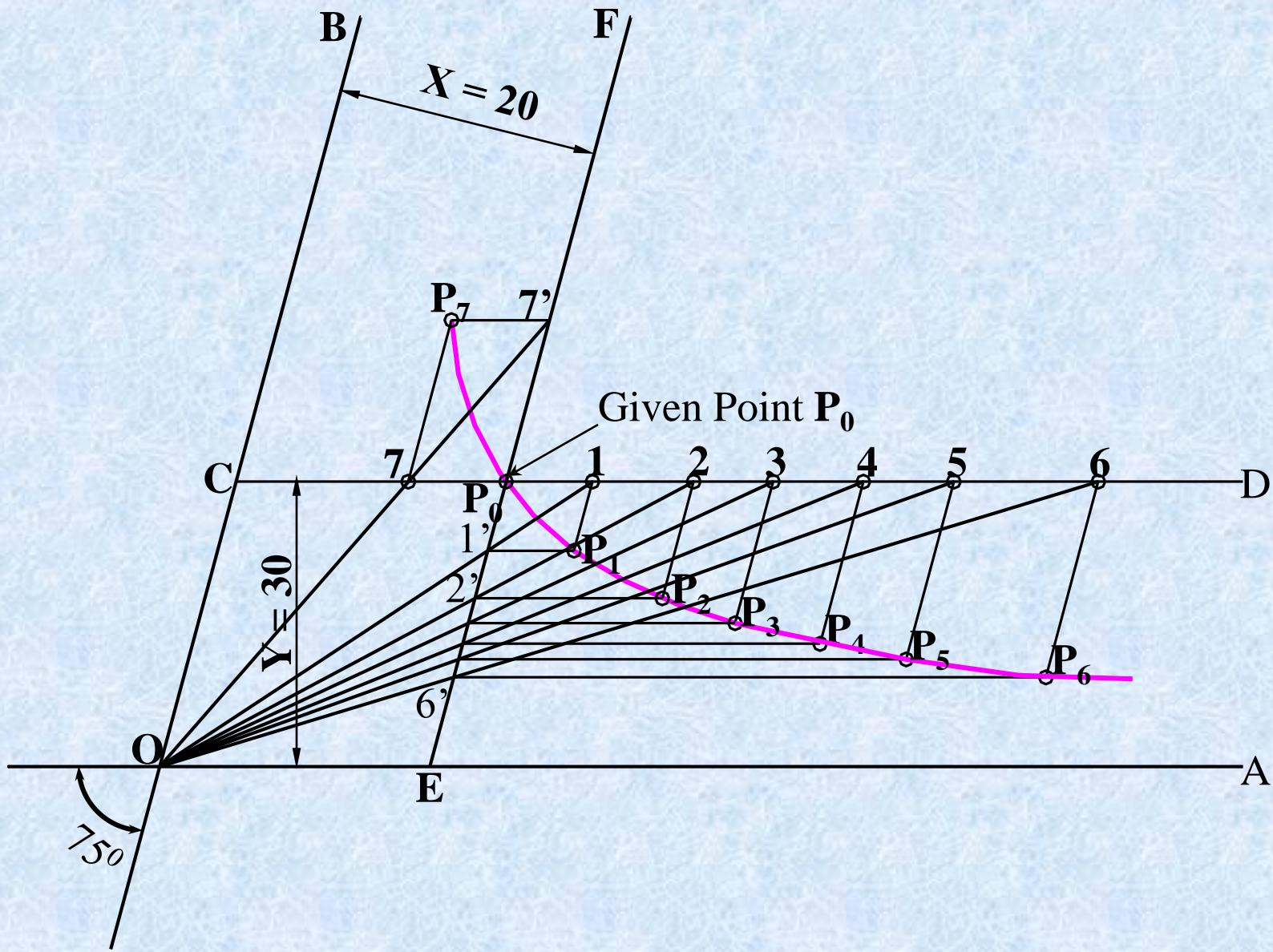
# RECTANGULAR HYPERBOLA



## PROBLEM:-

Two straight lines OA and OB are at  $75^\circ$  to each other. A point P is at a distance of 20 mm from OA and 30 mm from OB. Draw a hyperbola passing through the point “P”.

# Oblique Hyperbola



# Directrix and focus method

