```python
import pandas as pd
import numpy as np
import seaborn as sns
sns.set_theme(style='white')
from matplotlib import pyplot as plt
```

Singapore Population Prediction and Enhancement

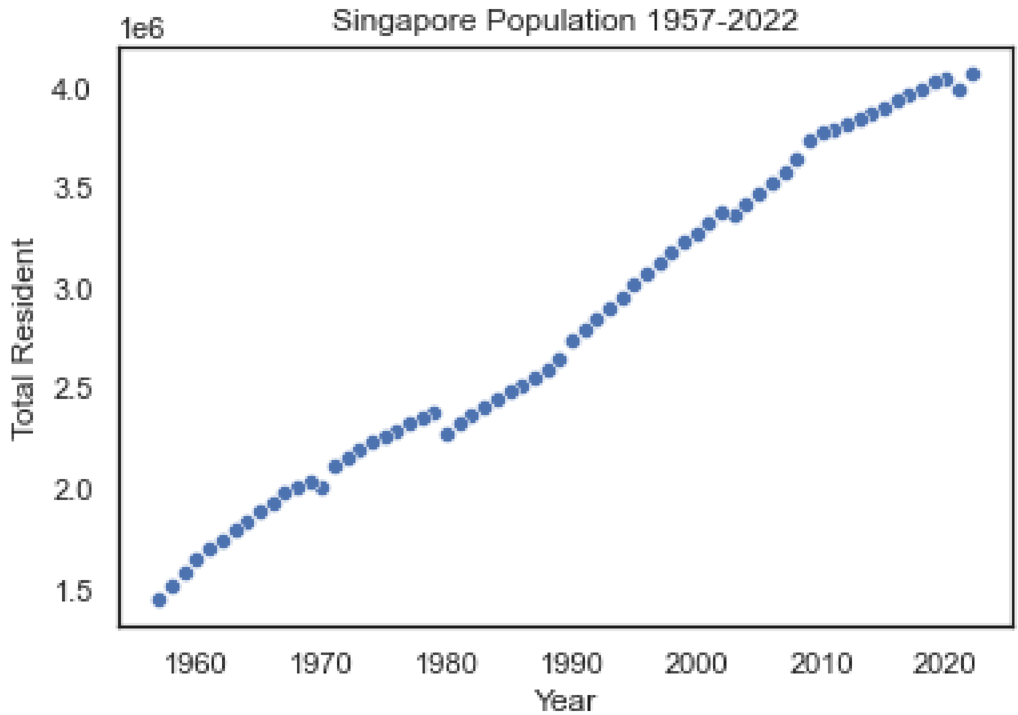
```python
SgData = pd.read_csv('SgData.csv')
SgData
```

```
    Year       TR      TMR      TFR    SR   MAR
0   2022  4073239  1990212  2083027   955  42.1
1   2021  3986842  1953114  2033728   960  41.8
2   2020  4044210  1977556  2066654   957  41.5
3   2019  4026209  1969382  2056827   957  41.1
4   2018  3994283  1955838  2038445   959  40.8
..   ...      ...      ...      ...   ...   ...
61  1961  1702400   886500   815900  1087  17.9
62  1960  1646400   859600   786800  1093  18.0
63  1959  1587200   830800   756400  1098  18.3
64  1958  1518800   797600   721200  1106  18.5
65  1957  1445929   762760   683169  1117  18.8

[66 rows x 6 columns]
```

TR = Total Residents TMR = Total Male Residents TFR = Total Female Residents SR = Sex Ratio MAR = Median Age of Residents

a. Year-Population Plot

```python
sns.scatterplot(x=SgData['Year'], y=SgData['TR'])
plt.title('Singapore Population 1957-2022')
plt.ylabel('Total Resident')
plt.xlabel('Year')
plt.show()
```



b. Model Fitting

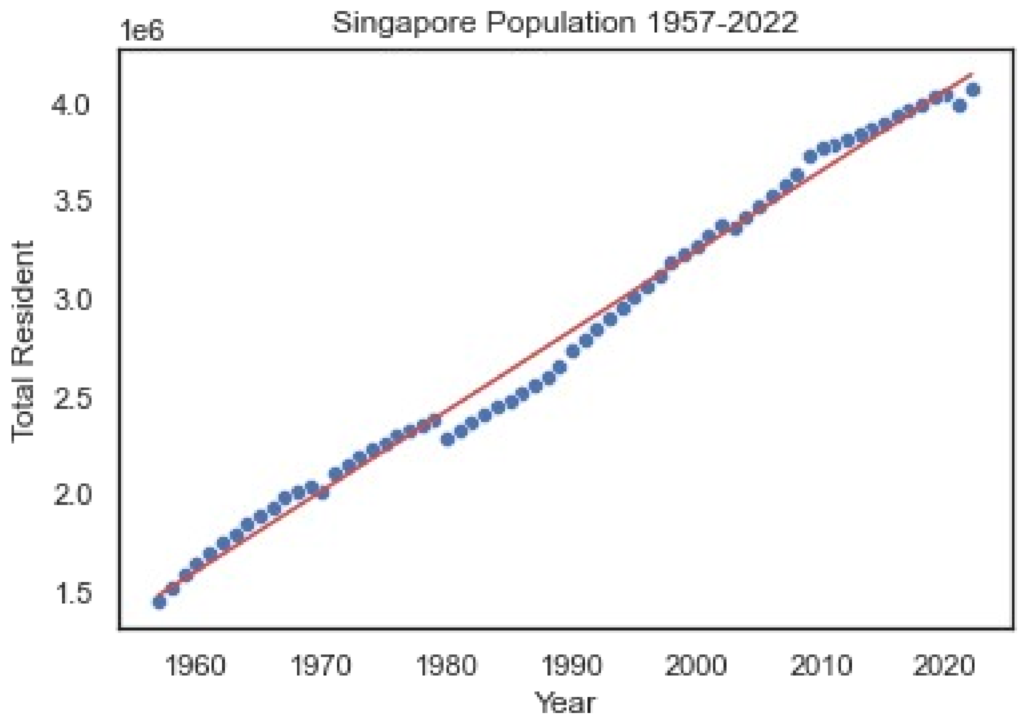
```
sg_train = SgData.loc[SgData['Year']<=2013, :]
sg_test = SgData.loc[SgData['Year']>2013, :]

from sklearn.linear_model import LinearRegression

x1 = sg_train['Year']
y1 = sg_train['TR']
x2 = sg_test['Year']
y2 = sg_test['TR']

x1 = np.array(x1).reshape(-1, 1)
x2 = np.array(x2).reshape(-1, 1)
y1 = np.array(y1).reshape(-1, 1)
y2 = np.array(y2).reshape(-1, 1)

rgs = LinearRegression()
rgs.fit(x1, y1)
y1_pred = rgs.predict(x1)
```

c. i. Best-Fit Line

```
print('Slope: ', rgs.coef_)
print('y-Intercept: ', rgs.intercept_)
```

```
Slope:  [[40975.96590614]]
y-Intercept:  [-78707770.39387262]
```

```
sns.scatterplot(x=SgData['Year'], y=SgData['TR'])
temp = SgData['Year'].map(lambda x: rgs.coef_*x+rgs.intercept_)
plt.plot(SgData['Year'], temp, color='r')
plt.title('Singapore Population 1957-2022')
plt.ylabel('Total Resident')
plt.xlabel('Year')
plt.show()
```



c. ii. R^2 Coefficient

```
from sklearn.metrics import mean_squared_error, r2_score

r2 = r2_score(y1, y1_pred)
print('R2: ', r2)

R2:  0.9857883020995728
```

c. iii. Training MSE

```
mse = mean_squared_error(y1, y1_pred)
print('Train MSE: ', mse)

Train MSE:  6551722294.557682
```

c. iv. Testing

```
y2_pred = rgs.predict(x2)
print(np.column_stack((x2, y2_pred)))
```

```
[[2.02200000e+03 4.14563267e+06]
 [2.02100000e+03 4.10465670e+06]
 [2.02000000e+03 4.06368074e+06]
 [2.01900000e+03 4.02270477e+06]
 [2.01800000e+03 3.98172880e+06]
 [2.01700000e+03 3.94075284e+06]
 [2.01600000e+03 3.89977687e+06]
 [2.01500000e+03 3.85880091e+06]
 [2.01400000e+03 3.81782494e+06]]
```

c. v. Testing MSE

```
mse = mean_squared_error(y2, y2_pred)
print('Testing MSE: ', mse)
```

```
Testing MSE:  2907187388.7105937
```

d. Prediction

```
X = []
for i in range(2030, 2051):
    X.append(i)
X = np.array(X).reshape(-1, 1)
```

```
Y = rgs.predict(X)
rs = np.column_stack((X, Y))
rs = pd.DataFrame(rs)
rs.columns = ['Year', 'Population']
rs
```

	Year	Population
0	2030.0	4.473440e+06
1	2031.0	4.514416e+06
2	2032.0	4.555392e+06
3	2033.0	4.596368e+06
4	2034.0	4.637344e+06
5	2035.0	4.678320e+06
6	2036.0	4.719296e+06
7	2037.0	4.760272e+06
8	2038.0	4.801248e+06
9	2039.0	4.842224e+06
10	2040.0	4.883200e+06
11	2041.0	4.924176e+06
12	2042.0	4.965152e+06
13	2043.0	5.006128e+06
14	2044.0	5.047104e+06
15	2045.0	5.088080e+06
16	2046.0	5.129056e+06
17	2047.0	5.170032e+06
18	2048.0	5.211008e+06
19	2049.0	5.251984e+06
20	2050.0	5.292960e+06

This result is partly reasonable. It is because, in natural population growth principles, there should be competition between samples and the growth rate might naturally decrease to 0. However, in the linear regression model, the growth rate is a constant value. Perhaps it fits the situation in a short term, but it might not precisely reflect the population growth tendency in long term (e.g., >100 years).

e. In the middle of the logistic pattern.

f. Enhancement In this case, we apply a multivariate linear regression. We selected some attributes from the Singapore official statistics data to increase the dimensions of the feature set. Before the formal multivariate linear regression, we conducted a correlation analysis to select the most influencing ones from these additional attributes. To prevent the imbalanced influences of data from different ranges, a normalization phase should be implemented.

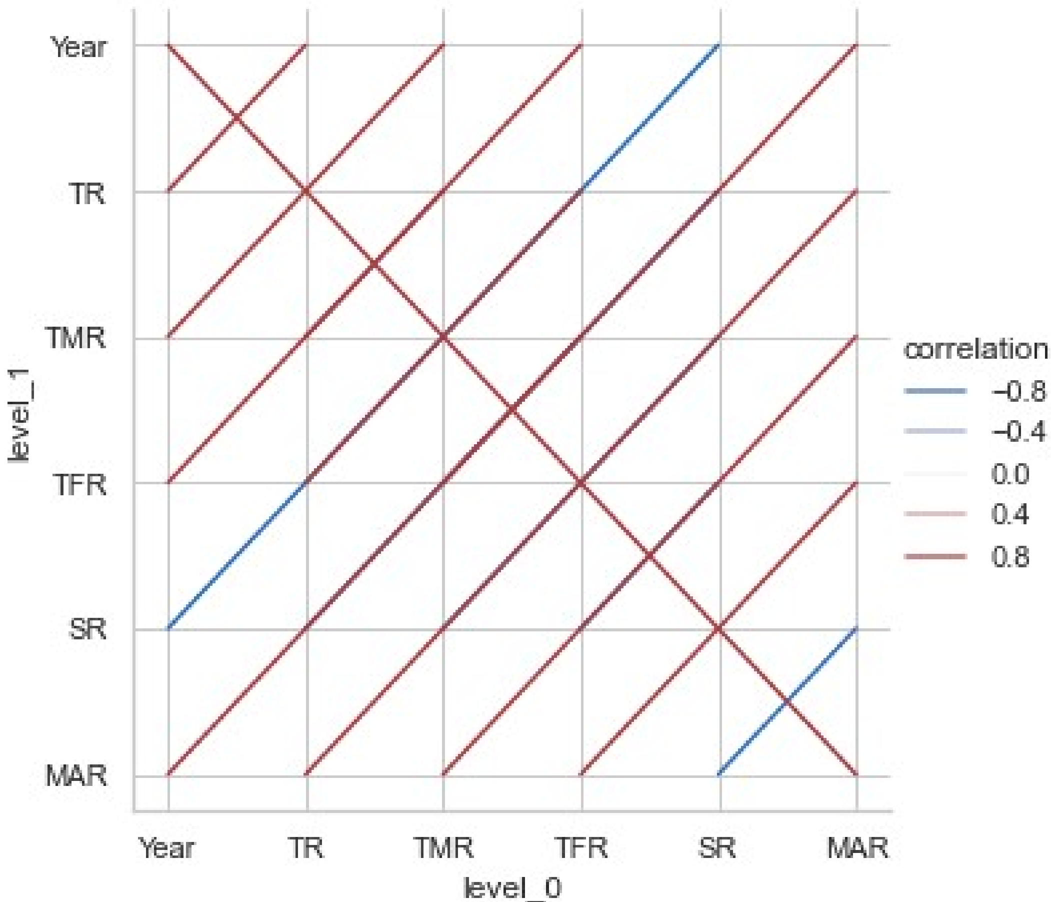
```
from sklearn.preprocessing import MinMaxScaler

sc = MinMaxScaler()
sc = sc.fit(SgData)
tmp = sc.transform(SgData)
tmp = pd.DataFrame(tmp)
tmp.columns = SgData.columns
tmp
```

```
        Year        TR       TMR       TFR        SR       MAR
0   1.000000  1.000000  1.000000  1.000000  0.000000  1.000000
1   0.984615  0.967116  0.969776  0.964783  0.030864  0.987705
2   0.969231  0.988951  0.989689  0.988304  0.012346  0.975410
3   0.953846  0.982100  0.983030  0.981284  0.012346  0.959016
4   0.938462  0.969948  0.971996  0.968152  0.024691  0.946721
..       ...       ...       ...       ...       ...       ...
61  0.061538  0.097617  0.100810  0.094817  0.814815  0.008197
62  0.046154  0.076303  0.078895  0.074030  0.851852  0.012295
63  0.030769  0.053770  0.055432  0.052313  0.882716  0.024590
64  0.015385  0.027736  0.028384  0.027168  0.932099  0.032787
65  0.000000  0.000000  0.000000  0.000000  1.000000  0.045082

[66 rows x 6 columns]
```

```
corr_mat = tmp.corr().stack().reset_index(name='correlation')
print(corr_mat)
sns.set_theme(style='whitegrid')
sns.relplot(data=corr_mat,
            x='level_0',
            y='level_1',
            palette="vlag",
            hue_norm=(-1, 1),
            hue='correlation',
            kind='line'
            )
```

```
   level_0 level_1  correlation
0     Year    Year     1.000000
1     Year      TR     0.995049
2     Year     TMR     0.995224
3     Year     TFR     0.994672
4     Year      SR    -0.986310
5     Year     MAR     0.993768
6       TR    Year     0.995049
7       TR      TR     1.000000
8       TR     TMR     0.999865
9       TR     TFR     0.999895
10      TR      SR    -0.985545
11      TR     MAR     0.986491
12     TMR    Year     0.995224
13     TMR      TR     0.999865
14     TMR     TMR     1.000000
15     TMR     TFR     0.999521
16     TMR      SR    -0.985694
17     TMR     MAR     0.986876
18     TFR    Year     0.994672
19     TFR      TR     0.999895
20     TFR     TMR     0.999521
21     TFR     TFR     1.000000
22     TFR      SR    -0.985193
23     TFR     MAR     0.985931
24      SR    Year    -0.986310
25      SR      TR    -0.985545
26      SR     TMR    -0.985694
27      SR     TFR    -0.985193
28      SR      SR     1.000000
29      SR     MAR    -0.963656
30     MAR    Year     0.993768
31     MAR      TR     0.986491
32     MAR     TMR     0.986876
33     MAR     TFR     0.985931
34     MAR      SR    -0.963656
35     MAR     MAR     1.000000
```

```
<seaborn.axisgrid.FacetGrid at 0x128d3f730>
```



TR = Total Residents TMR = Total Male Residents TFR = Total Female Residents SR = Sex Ratio MAR = Median Age of Residents

From the correlation plot above, we can conclude that TMR and SR have the greatest positive influence on TR because the correlation coefficients between these attributes and TR are equal to 1. Therefore, we can add these two attributes to our X matrix to improve the accuracy of the prediction.

```
df = SgData[['Year', 'TR', 'TMR', 'SR']]
df
```

```
    Year       TR      TMR    SR
0   2022  4073239  1990212   955
1   2021  3986842  1953114   960
2   2020  4044210  1977556   957
3   2019  4026209  1969382   957
4   2018  3994283  1955838   959
..   ...      ...      ...   ...
61  1961  1702400   886500  1087
62  1960  1646400   859600  1093
63  1959  1587200   830800  1098
64  1958  1518800   797600  1106
```

```
65  1957  1445929   762760  1117

[66 rows x 4 columns]
```

```
sc = MinMaxScaler()
sc = sc.fit(df)
train = df.loc[df['Year']<=2013, :]
test = df.loc[df['Year']>2013, :]
tmp_1 = sc.transform(train)
tmp_2 = sc.transform(test)
tmp_1 = pd.DataFrame(tmp_1)
tmp_2 = pd.DataFrame(tmp_2)
tmp_1.columns = df.columns
tmp_2.columns = df.columns

# training
X = tmp_1[['Year', 'TMR', 'SR']]
Y = tmp_1['TR']

rgs = LinearRegression()
rgs.fit(X, Y)
Y_pred = rgs.predict(X)

rs = np.column_stack((np.array(X['Year']),
                      np.array(Y_pred),
                      np.array(X[['TMR', 'SR']])))
rs = sc.inverse_transform(rs)
rs = pd.DataFrame(rs)
rs.columns = df.columns
rs
```

```
      Year            TR        TMR      SR
0   2013.0  3.824059e+06  1891504.0   968.0
1   2012.0  3.800450e+06  1880046.0   970.0
2   2011.0  3.775927e+06  1868170.0   972.0
3   2010.0  3.761988e+06  1861133.0   974.0
4   2009.0  3.727567e+06  1844732.0   976.0
5   2008.0  3.637403e+06  1802992.0   980.0
6   2007.0  3.578670e+06  1775477.0   982.0
7   2006.0  3.520707e+06  1748242.0   983.0
8   2005.0  3.462875e+06  1721139.0   985.0
9   2004.0  3.407377e+06  1695031.0   986.0
10  2003.0  3.361518e+06  1673401.0   988.0
11  2002.0  3.386488e+06  1684295.0   992.0
12  2001.0  3.331487e+06  1658558.0   995.0
13  2000.0  3.280524e+06  1634667.0   998.0
14  1999.0  3.238530e+06  1614804.0  1000.0
15  1998.0  3.189700e+06  1591816.0  1002.0
16  1997.0  3.133979e+06  1565750.0  1005.0
17  1996.0  3.078989e+06  1540018.0  1008.0
18  1995.0  3.023564e+06  1514015.0  1010.0
```

```
19  1994.0  2.970536e+06  1489180.0  1013.0
20  1993.0  2.917084e+06  1464223.0  1017.0
21  1992.0  2.863346e+06  1439063.0  1020.0
22  1991.0  2.810176e+06  1414307.0  1025.0
23  1990.0  2.750348e+06  1386291.0  1027.0
24  1989.0  2.653647e+06  1341275.0  1027.0
25  1988.0  2.601390e+06  1316577.0  1027.0
26  1987.0  2.554182e+06  1294187.0  1027.0
27  1986.0  2.517153e+06  1276522.0  1028.0
28  1985.0  2.479421e+06  1258464.0  1028.0
29  1984.0  2.438695e+06  1239109.0  1029.0
30  1983.0  2.399695e+06  1220471.0  1029.0
31  1982.0  2.357230e+06  1200321.0  1030.0
32  1981.0  2.313951e+06  1179799.0  1031.0
33  1980.0  2.270090e+06  1159011.0  1032.0
34  1979.0  2.395604e+06  1216300.0  1042.0
35  1978.0  2.364622e+06  1201400.0  1043.0
36  1977.0  2.335549e+06  1187300.0  1043.0
37  1976.0  2.302161e+06  1171300.0  1044.0
38  1975.0  2.270524e+06  1156100.0  1045.0
39  1974.0  2.236262e+06  1139700.0  1046.0
40  1973.0  2.198001e+06  1121400.0  1046.0
41  1972.0  2.155207e+06  1101100.0  1047.0
42  1971.0  2.113289e+06  1081200.0  1048.0
43  1970.0  2.004674e+06  1030809.0  1049.0
44  1969.0  2.041169e+06  1046900.0  1052.0
45  1968.0  2.014153e+06  1034100.0  1057.0
46  1967.0  1.978920e+06  1017400.0  1060.0
47  1966.0  1.933283e+06   995800.0  1061.0
48  1965.0  1.886142e+06   973800.0  1066.0
49  1964.0  1.838660e+06   951500.0  1069.0
50  1963.0  1.791239e+06   929300.0  1073.0
51  1962.0  1.748282e+06   909500.0  1082.0
52  1961.0  1.698954e+06   886500.0  1087.0
53  1960.0  1.640937e+06   859600.0  1093.0
54  1959.0  1.578922e+06   830800.0  1098.0
55  1958.0  1.506810e+06   797600.0  1106.0
56  1957.0  1.430639e+06   762760.0  1117.0
```

```
r2 = r2_score(rs['Year'], train['Year'])
mse = mean_squared_error(rs['Year'], train['Year'])
print('Training MSE: ', mse)
print('Training R2: ', r2)
```

```
Training MSE:  0.0
Training R2:  1.0
```

```
# testing
X = tmp_2[['Year', 'TMR', 'SR']]
Y = tmp_2['TR']
```

```
rgs = LinearRegression()
rgs.fit(X, Y)
Y_pred = rgs.predict(X)

rs = np.column_stack((np.array(X['Year']),
                      np.array(Y_pred),
                      np.array(X[['TMR', 'SR']])))
rs = sc.inverse_transform(rs)
rs = pd.DataFrame(rs)
rs.columns = df.columns
rs
```

```
     Year            TR        TMR     SR
0  2022.0  4.073281e+06  1990212.0  955.0
1  2021.0  3.986846e+06  1953114.0  960.0
2  2020.0  4.044114e+06  1977556.0  957.0
3  2019.0  4.025991e+06  1969382.0  957.0
4  2018.0  3.994519e+06  1955838.0  959.0
5  2017.0  3.965867e+06  1943545.0  961.0
6  2016.0  3.933324e+06  1929526.0  963.0
7  2015.0  3.903308e+06  1916628.0  965.0
8  2014.0  3.870317e+06  1902410.0  967.0
```

```
r2 = r2_score(rs['Year'], test['Year'])
mse = mean_squared_error(rs['Year'], test['Year'])
print('Training MSE: ', mse)
print('Training R2: ', r2)
```

```
Training MSE:  5.744309809396025e-27
Training R2:  1.0
```

The testing result of multivariate regression has a 0-like MSE and its R^2 coefficient equals to 1, which is much more precise than the previous simple linear regression with [Testing MSE, R^2] = [2907187388.7105937, 0.9857883020995728]. Therefore, multivariate linear regression can be considered an effective enhancement method.

Chinese Population Prediction

a. Load Dataset

```
CNData = pd.read_csv('CNData.csv')
CNData
```

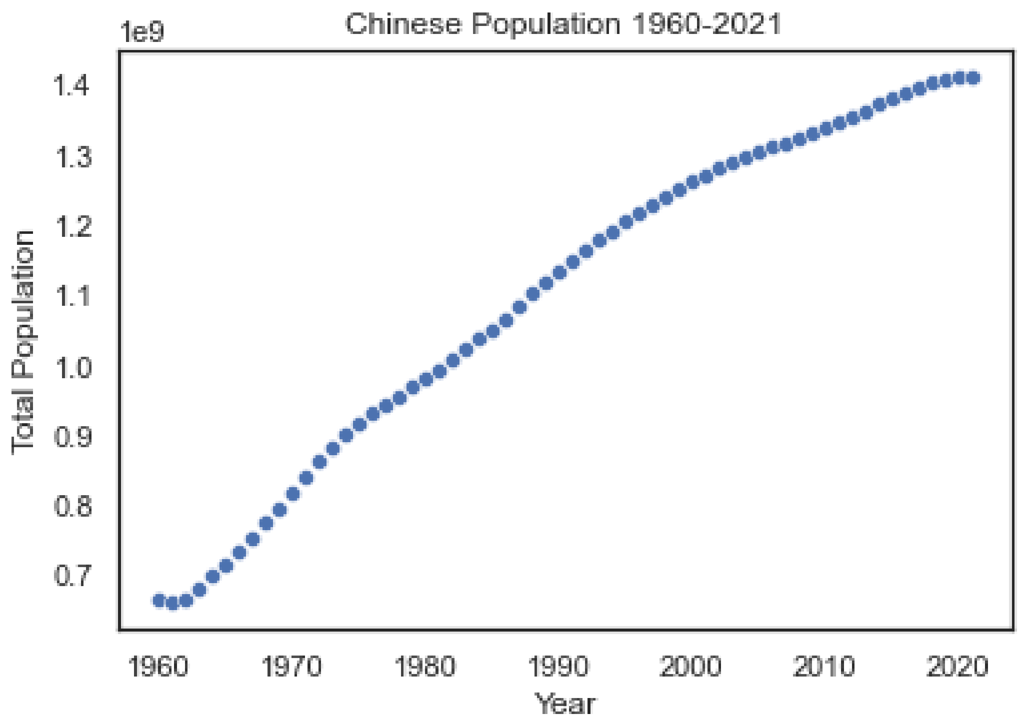
```
    Year         PPL
0   1960   667070000
1   1961   660330000
2   1962   665770000
3   1963   682335000
4   1964   698355000
..   ...         ...
57  2017  1396215000
58  2018  1402760000
```

```
59  2019  1407745000
60  2020  1411100000
61  2021  1412360000

[62 rows x 2 columns]
```

PPL = Population

b. Year-Population Plot

```
sns.set_theme(style='white')
sns.scatterplot(x=CNData['Year'], y=CNData['PPL'])
plt.title('Chinese Population 1960-2021')
plt.ylabel('Total Population')
plt.xlabel('Year')
plt.show()
```



c. Model Training

```
cn_train = CNData.loc[CNData['Year']<=2013, :]
cn_test = CNData.loc[CNData['Year']>2013, :]

x1 = np.array(cn_train['Year']).reshape(-1, 1)
x2 = np.array(cn_test['Year']).reshape(-1, 1)
y1 = np.array(cn_train['PPL']).reshape(-1, 1)
y2 = np.array(cn_test['PPL']).reshape(-1, 1)
```

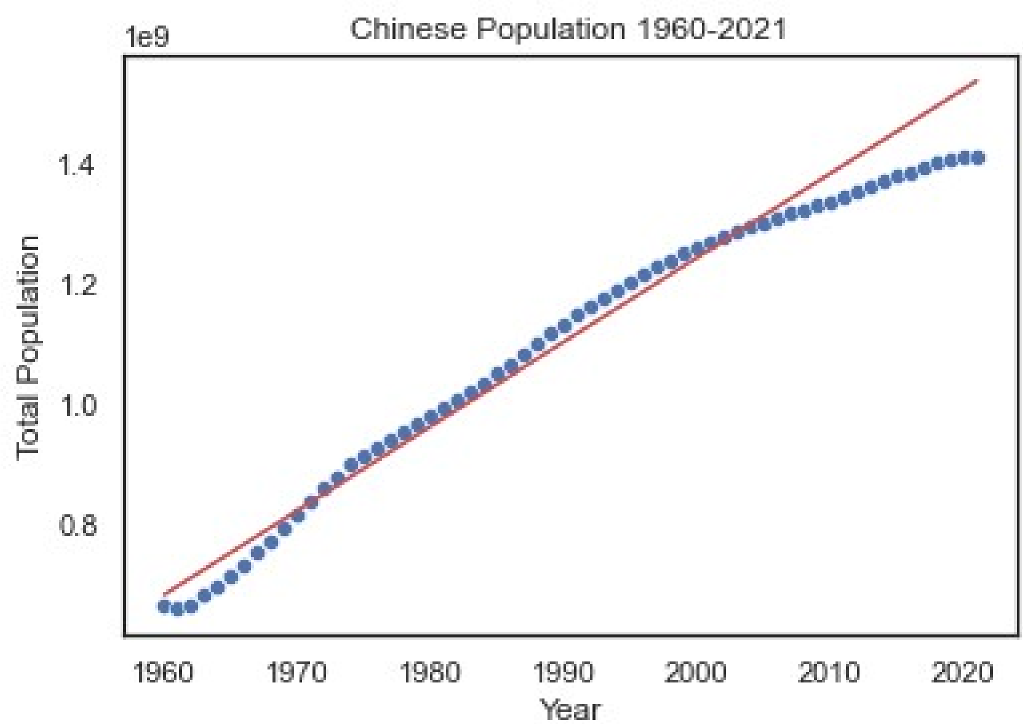
```
rgs = LinearRegression()
rgs.fit(x1, y1)
y1_pred = rgs.predict(x1)
```

d. i. Best-Fit Line

```
print('Slope: ', rgs.coef_)
print('y-Intercept: ', rgs.intercept_)
```

```
Slope:  [[14006789.21288355]]
y-Intercept:  [-2.67690896e+10]
```

```
temp = CNData['Year'].map(lambda x: rgs.coef_*x+rgs.intercept_)
sns.scatterplot(x=CNData['Year'], y=CNData['PPL'])
plt.plot(CNData['Year'], temp, color='r')
plt.title('Chinese Population 1960-2021')
plt.ylabel('Total Population')
plt.xlabel('Year')
plt.show()
```



d. ii. R^2 Coefficient

```
r2 = r2_score(y1, y1_pred)
print('R2: ', r2)
```

```
R2:  0.9832476129933141
```

d. iii. Training MSE

```
mse = mean_squared_error(y1, y1_pred)
print('Training MSE: ', mse)
```

```
Training MSE:  811985540736408.5
```

d. iv. Testing

```
y2_pred = rgs.predict(x2)
```

d. v. Testing MSE

```
mse = mean_squared_error(y2, y2_pred)
print('Testing MSE: ', mse)
```

```
Testing MSE:  9065457889543732.0
```

e. Prediction of 2030-2050

```
X = []
for i in range(2030, 2051):
    X.append(i)
X = np.array(X).reshape(-1, 1)
```

```
Y = rgs.predict(X)
rs = np.column_stack((X, Y))
rs = pd.DataFrame(rs)
rs.columns = ['Year', 'Population']
rs
```

	Year	Population
0	2030.0	1.664692e+09
1	2031.0	1.678699e+09
2	2032.0	1.692706e+09
3	2033.0	1.706713e+09
4	2034.0	1.720720e+09
5	2035.0	1.734726e+09
6	2036.0	1.748733e+09
7	2037.0	1.762740e+09
8	2038.0	1.776747e+09
9	2039.0	1.790754e+09
10	2040.0	1.804760e+09
11	2041.0	1.818767e+09
12	2042.0	1.832774e+09
13	2043.0	1.846781e+09
14	2044.0	1.860788e+09
15	2045.0	1.874794e+09
16	2046.0	1.888801e+09
17	2047.0	1.902808e+09
18	2048.0	1.916815e+09
19	2049.0	1.930821e+09
20	2050.0	1.944828e+09

This prediction is not reasonable. From the figure plotted in the previous stage, it can be easily found that the growth rate of the Chinese population is descending. The predicted growth rate (slope) is a large constant because the dataset used includes the phases of acceleration of Chinese population growth. It does not match the current situation in China.

f. Logistic pattern.

g. As mentioned earlier, multivariate linear regression can be used to improve the precision of the prediction. However, due to the lack of further information, we did not verify this enhancement method by predicting the Chinese population.