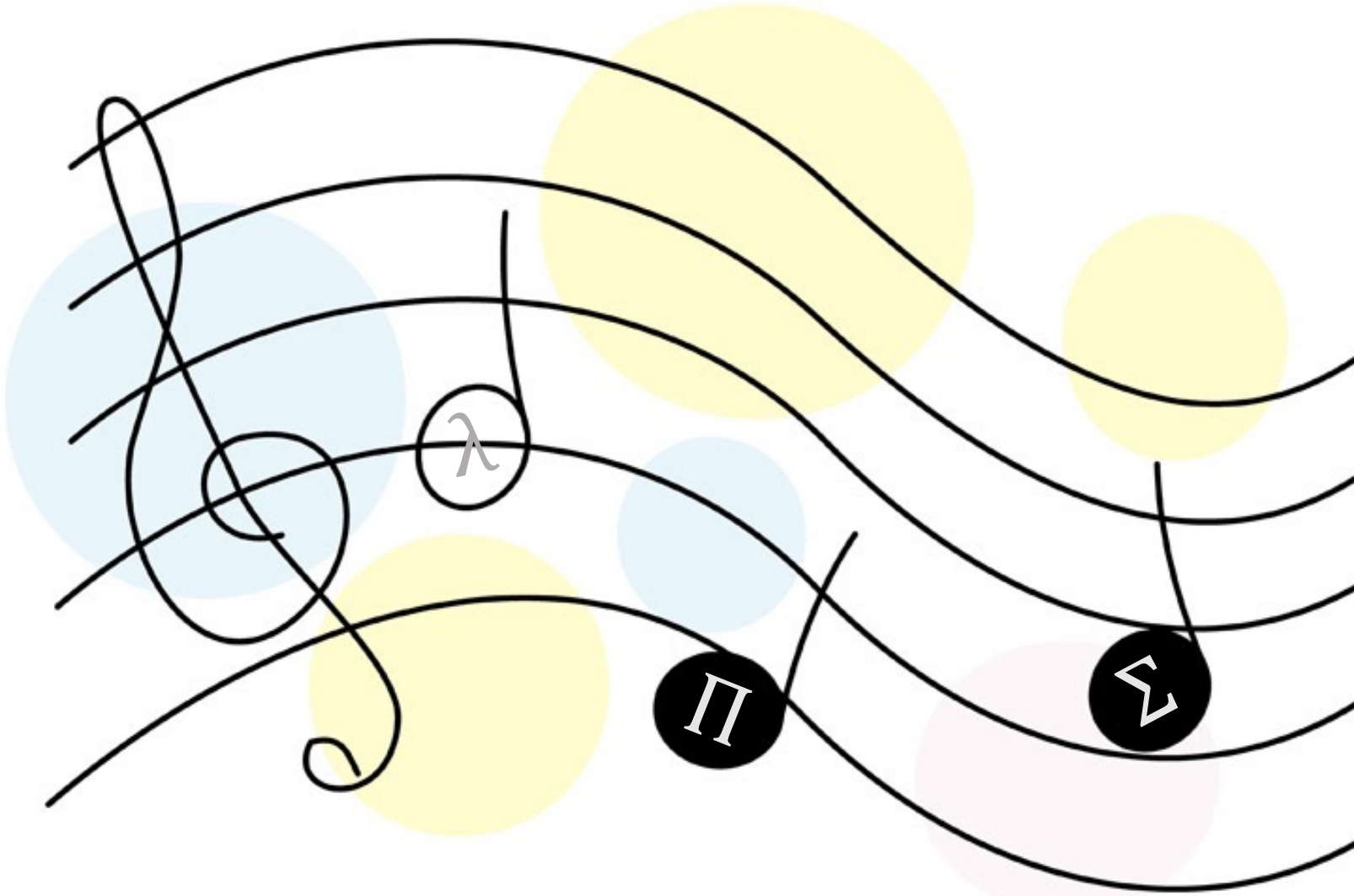
Composing Music from Types

Youyou Cong
Tokyo Institute of Technology
TYPES 2022



Bio

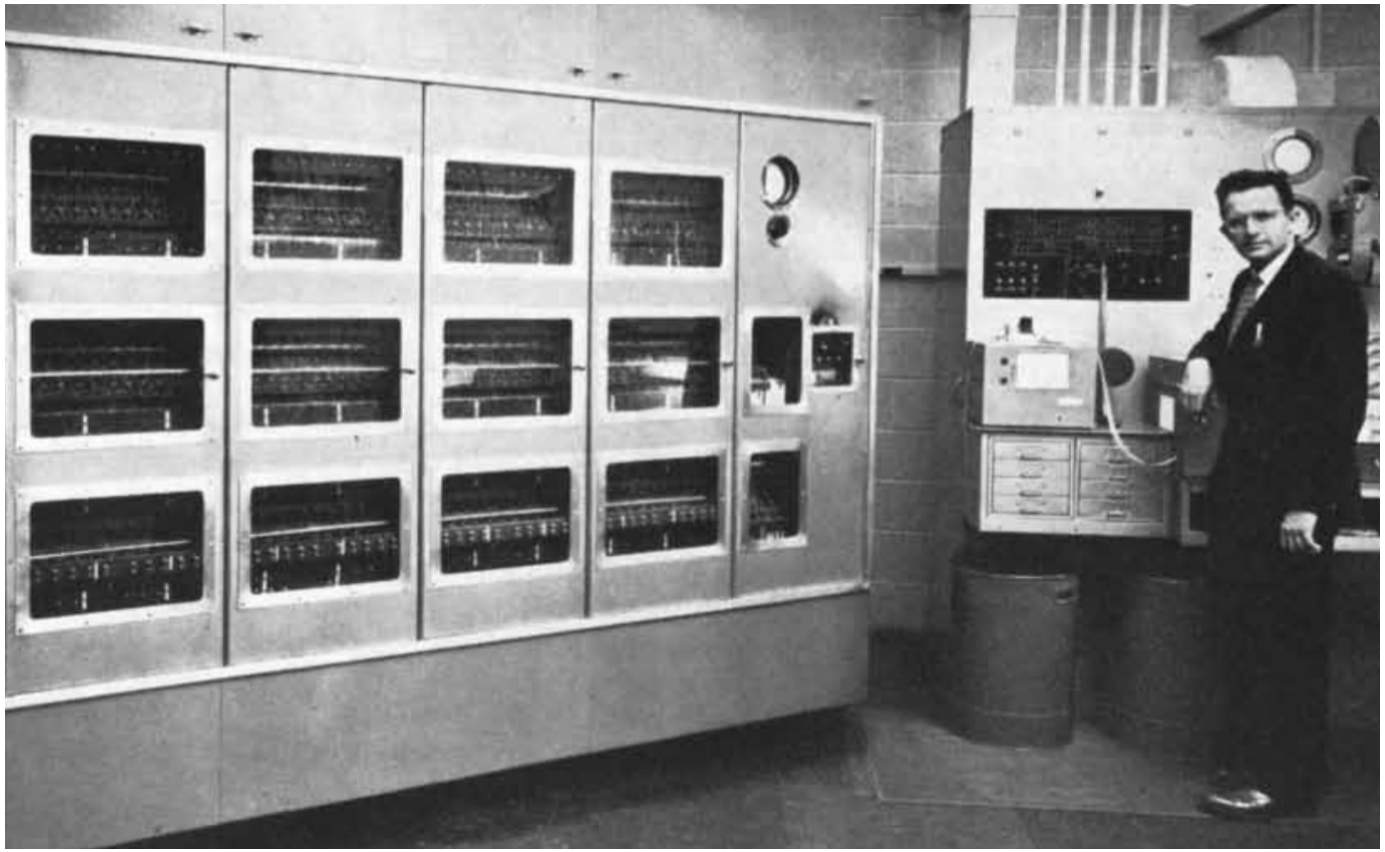
OCaml

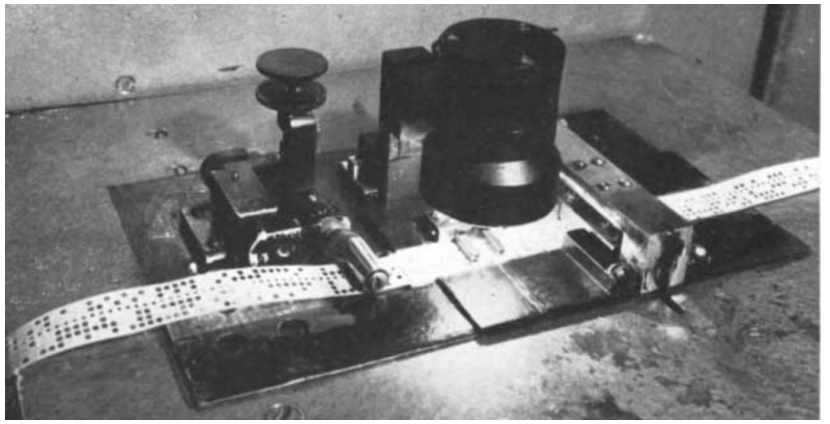
Agda

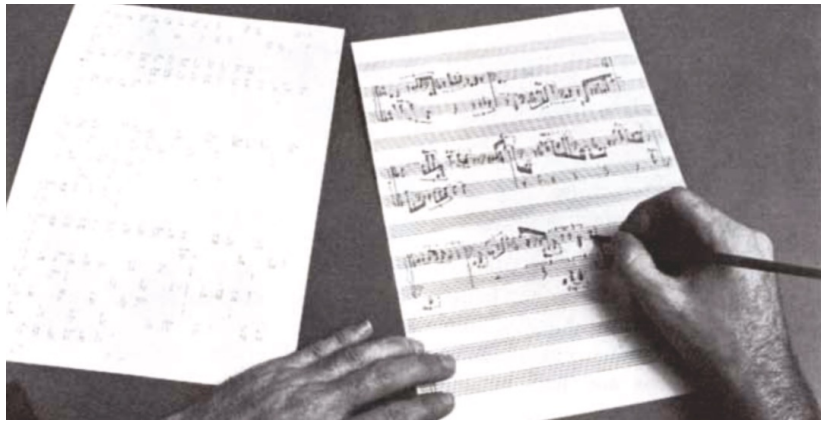
AIM XXIX

1995 1996 2011 2014 2019

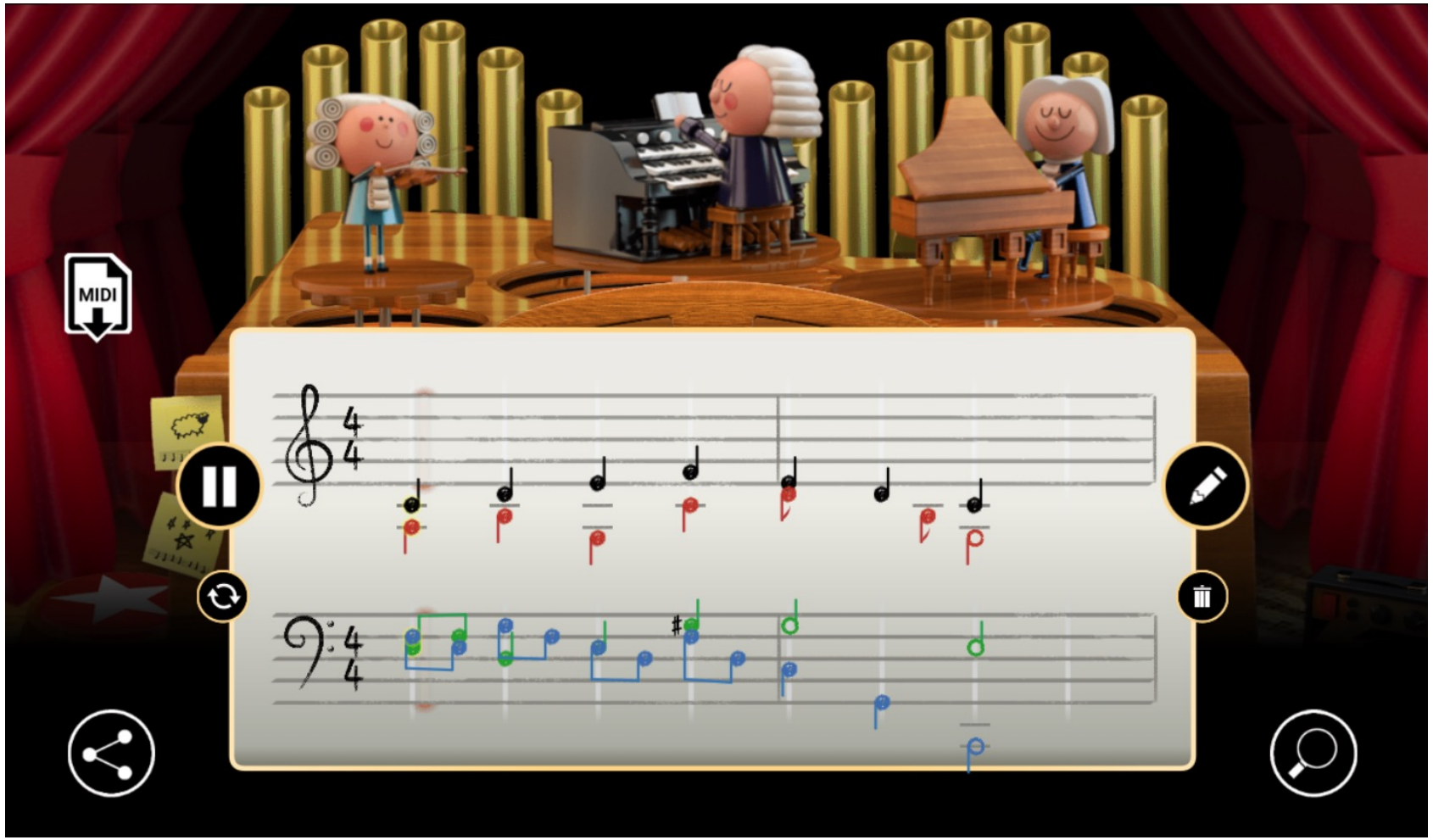
Illiac Suite (Hiller & Isaacson ‘57)



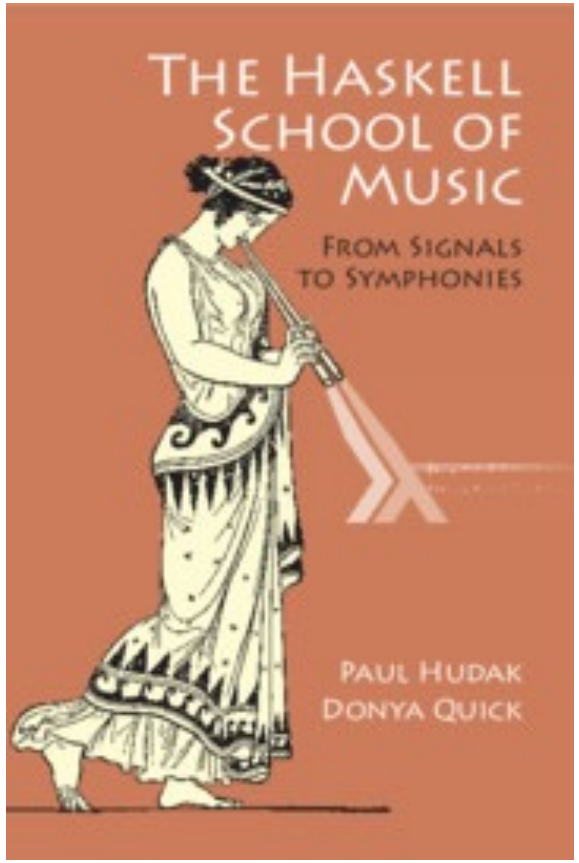




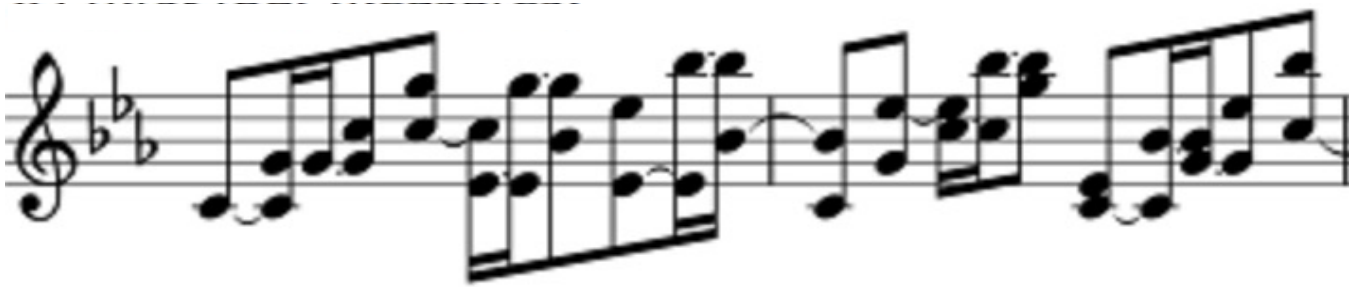
Bach Doodle (Huang et al. ‘19)



Euterpea (Hudak & Quick '18)




```
x1 = c 4 en :+: g 4 en :+: c 5 en :+: g 5 en
x2 = x1 :+: transpose 3 x1
x3 = x2 :+: x2 :+: invert x2 :+: retro x2
x4 = forever x3 :=: forever (tempo (2/3) x3)
```



Mezzo (Szamozvancev & Gale ‘17)

Well-Typed Music Does Not Sound Wrong (Experience Report)

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```
v1 = d qn :|: g qn :|: fs qn :|: g en :|:
     bf qn :|: a qn :|: g hn
v2 = d qn :|: ef qn :|: d qn :|: bf_ en :|:
     b_ qn :|: a_ qn :|: g_ hn
comp = defScore (v1 :-: v2)
```

```
Can't have major sevenths
in chords: Bb - B_.

Parallel octaves are
forbidden: A - A_, then G - G_.
```

Our work: Representing & generating correct music

Agda

Synquid

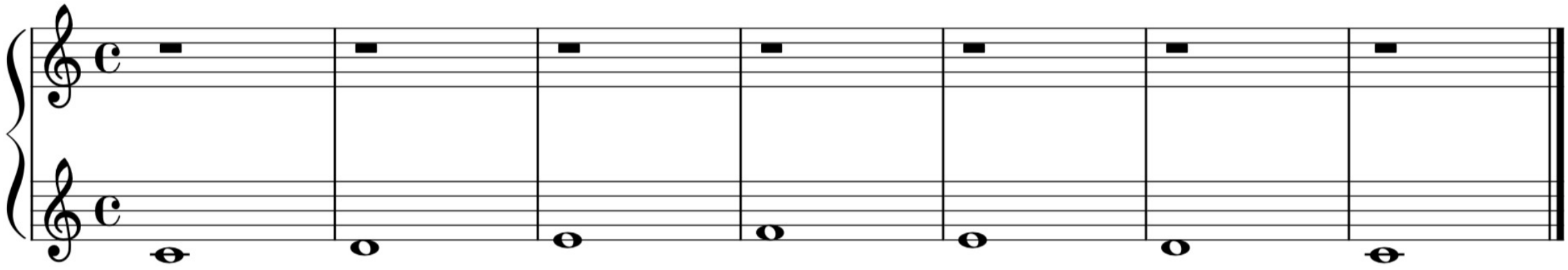
Musical correctness?

Introduction to Counterpoint

Counterpoint 101

- Technique for combining multiple melodies
- Has a strict set of rules (by J. J. Fux 1725)

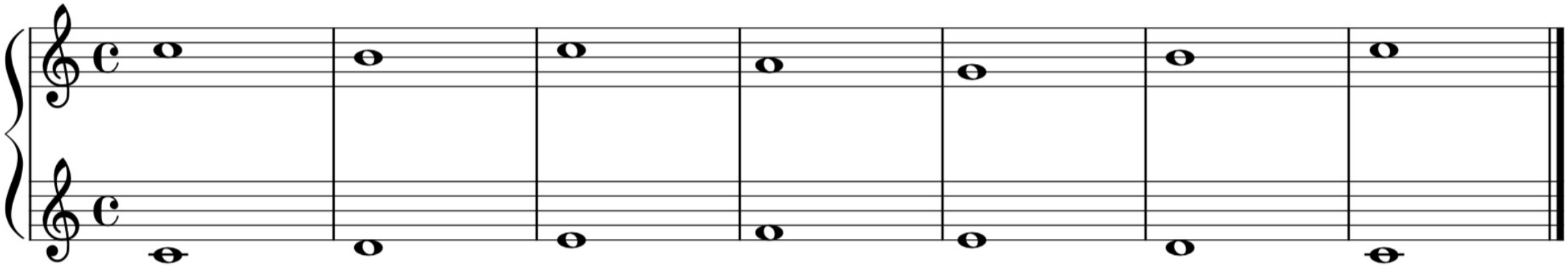
Example of counterpoint



Cantus firmus

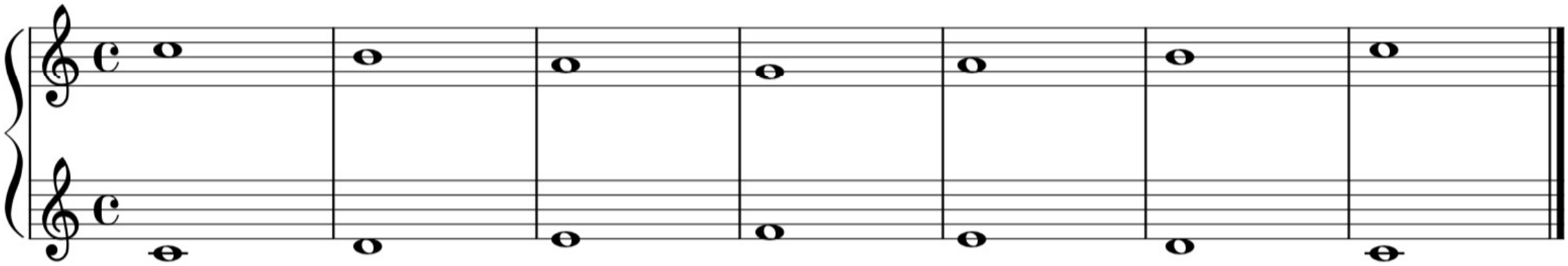
Example of counterpoint

Counterpoint

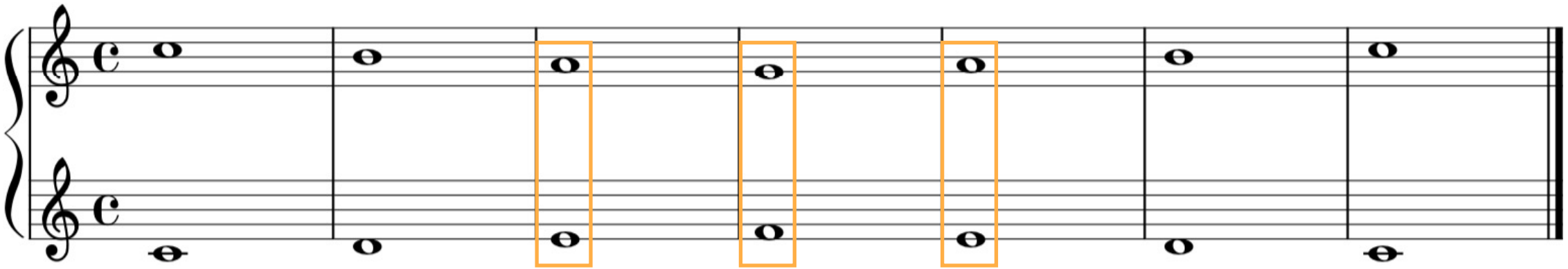


Cantus firmus

Rules of counterpoint



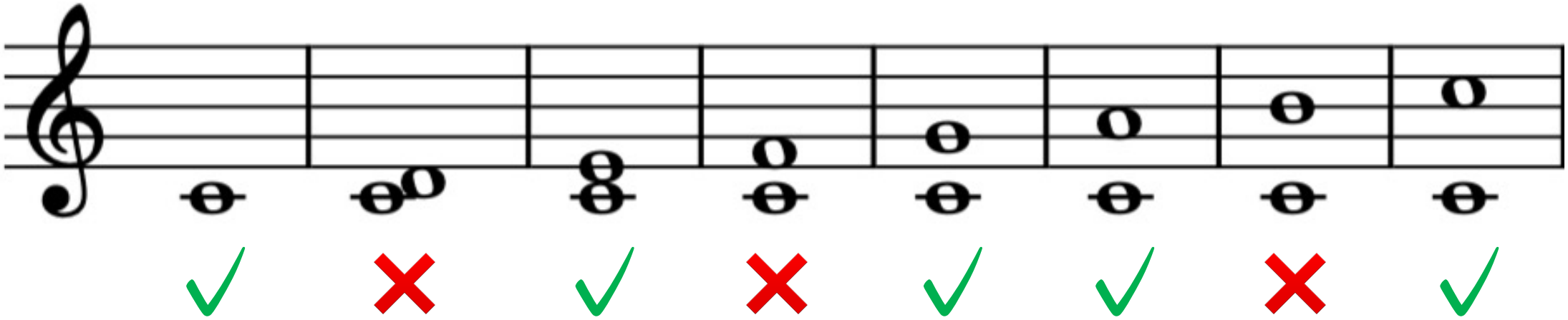
Rules of counterpoint



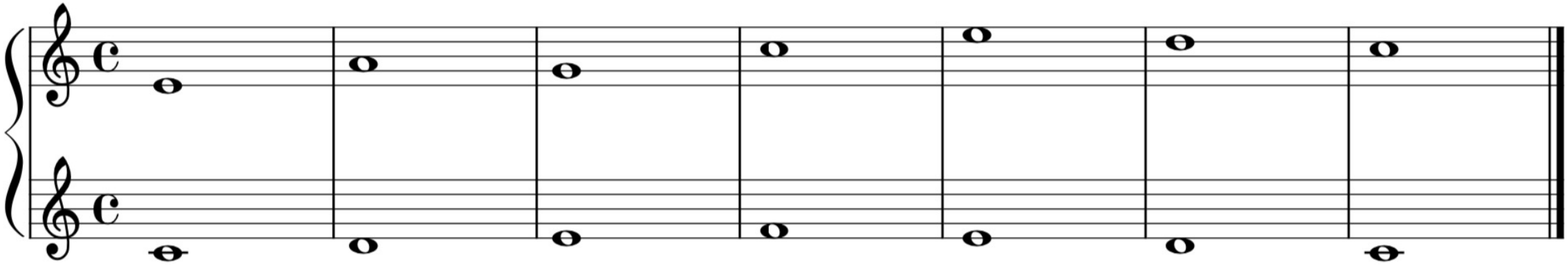
Dissonant

Rule #1

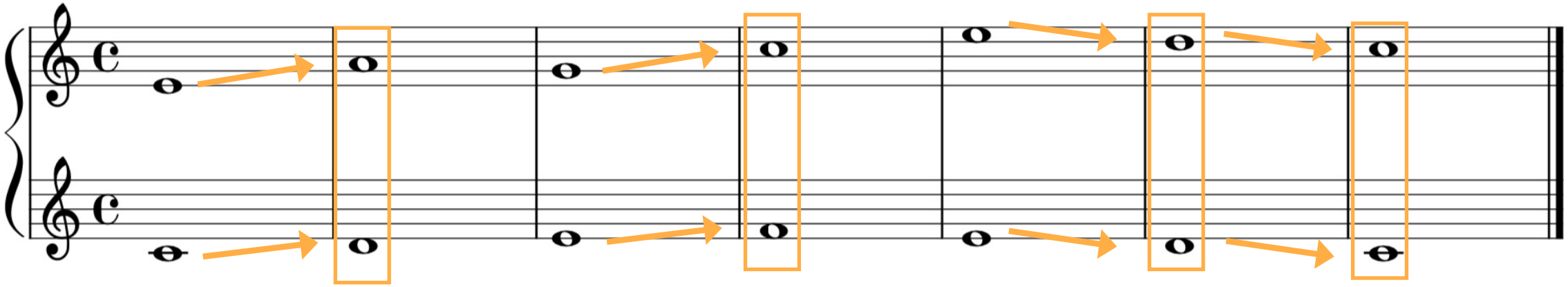
All intervals must be consonant.



Rules of counterpoint



Rules of counterpoint

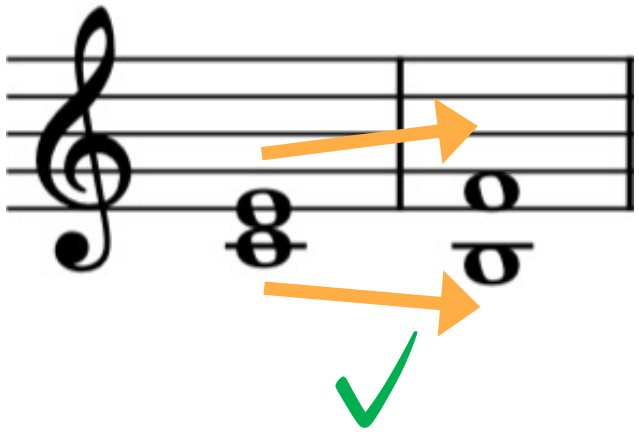


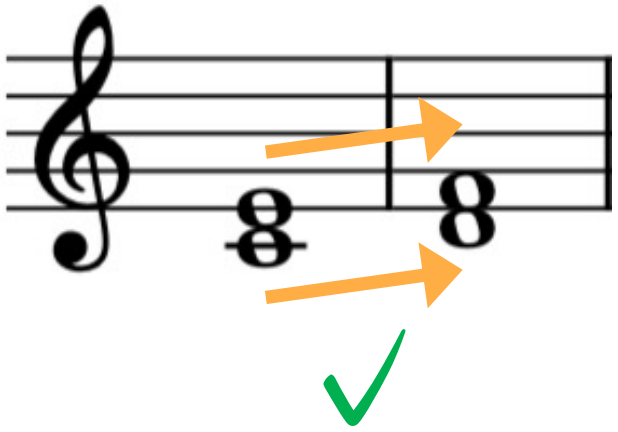
Direct fifth

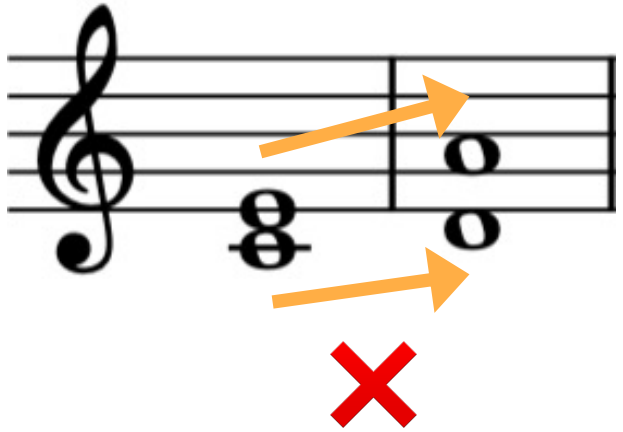
Direct octave

Rule #2

No direct fifth or octave is allowed.



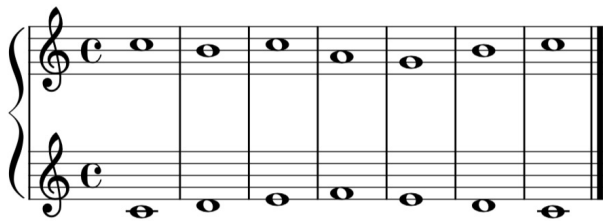


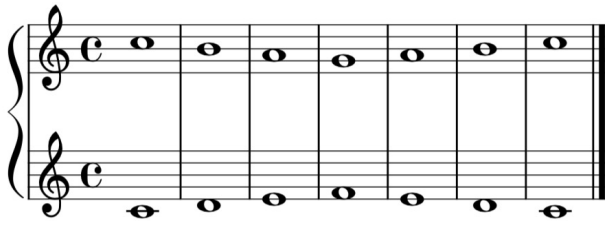


Rules of Counterpoint

1. All intervals must be consonant.

2. No direct fifth or octave is allowed.

$\vdash$  : CP

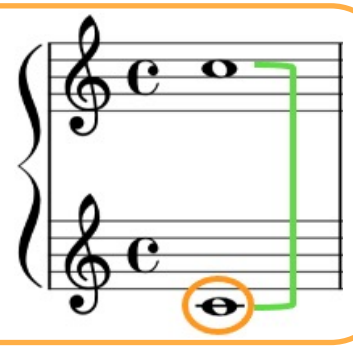
$\nvdash$  : CP

Representing Correct Counterpoint

Correct counterpoint as records

```
record CP : Set where
  constructor cp
  field
    bars       : List PitchInterval
    intervalOK : collectIntervalError bars ≡ []
    motionOK   : collectMotionError bars ≡ []
```

(c, oct) =



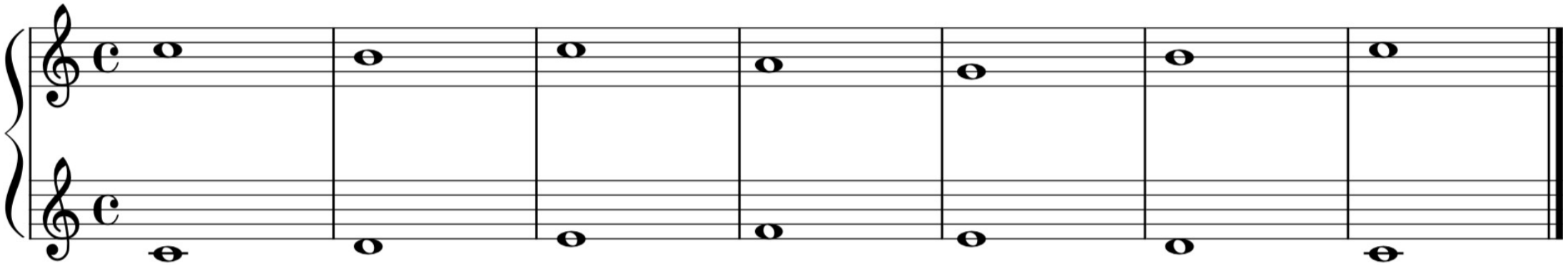
Interval rule as a function

```
data IntervalError : Set where
  dissonant : Interval → IntervalError

checkInterval : PitchInterval → Maybe IntervalError
checkInterval (p , i) with isConsonant i
checkInterval (p , i) | false = just (dissonant i)
checkInterval (p , i) | true  = nothing

collectIntervalError : List PitchInterval → List IntervalError
collectIntervalError = mapMaybe checkInterval
```

Motion rule as a function

```
data MotionError : Set where
  direct58 : PitchInterval → PitchInterval → MotionError

checkMotion : PitchInterval → PitchInterval → Maybe MotionError
checkMotion pi₁ pi₂ with isDirect pi₁ pi₂ | isPerfect (proj₂ pi₂)
checkMotion pi₁ pi₂ | true  | true  = just (direct58 pi₁ pi₂)
checkMotion pi₁ pi₂ | _     | _     = nothing

collectMotionError : List PitchInterval → List MotionError
collectMotionError = mapMaybe (uncurry checkMotion) ∘ pairs
```

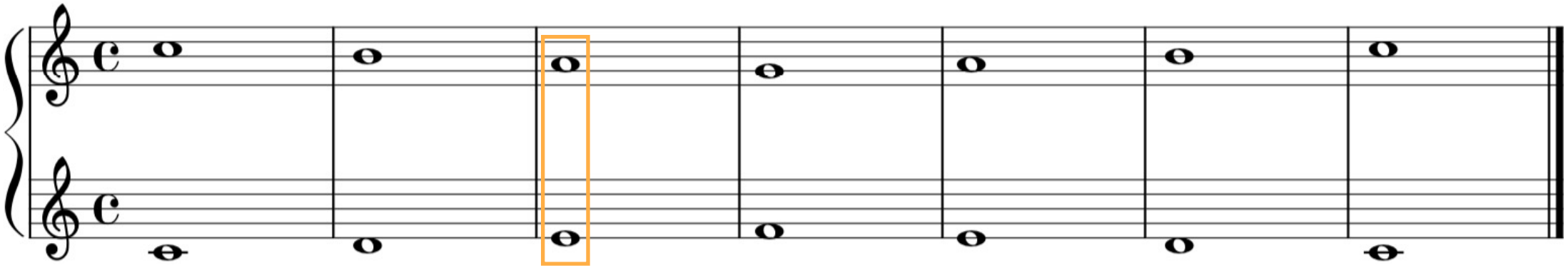
Type-checking correct counterpoint



```
cp-correct : CP
cp-correct = cp bars-correct refl refl
```

All Done

Type-checking wrong counterpoint

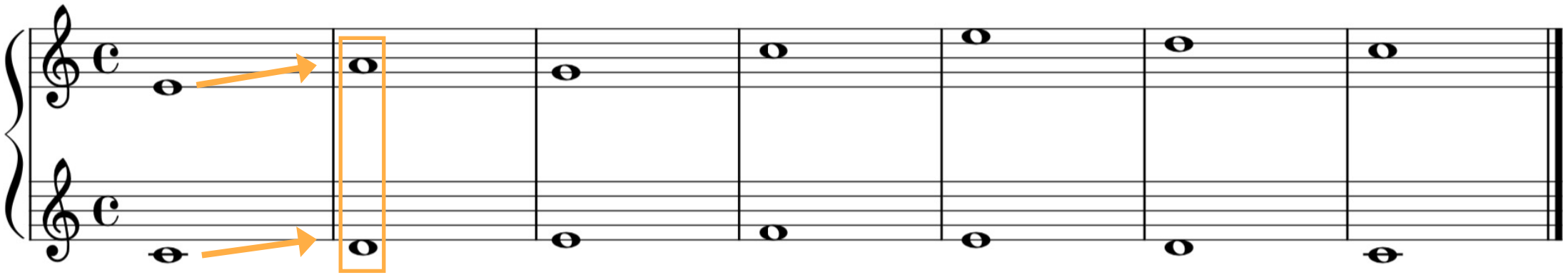


```
cp-dissonant : CP
cp-dissonant = cp bars-dissonant refl refl
```

Error

```
(dissonant per4) :: … != [] of type (List IntervalError)
```

Type-checking wrong counterpoint

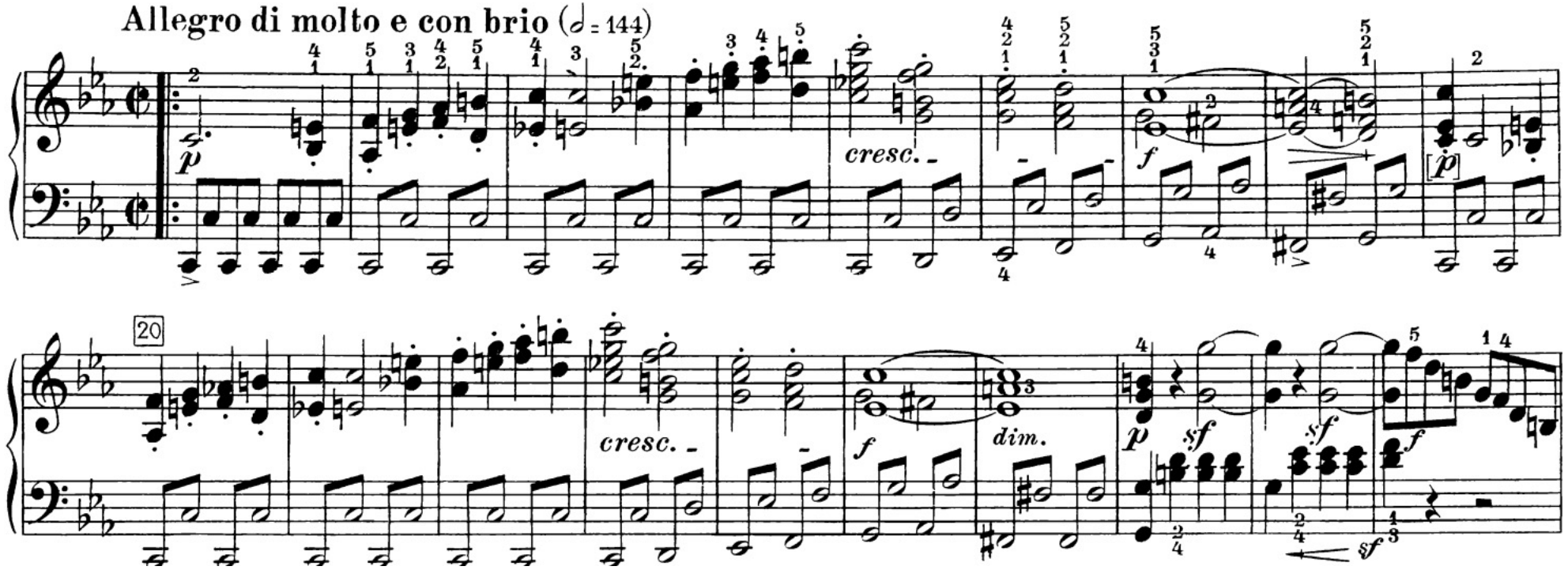


```
cp-direct58 : CP
cp-direct58 = cp bars-diect58 refl refl
```

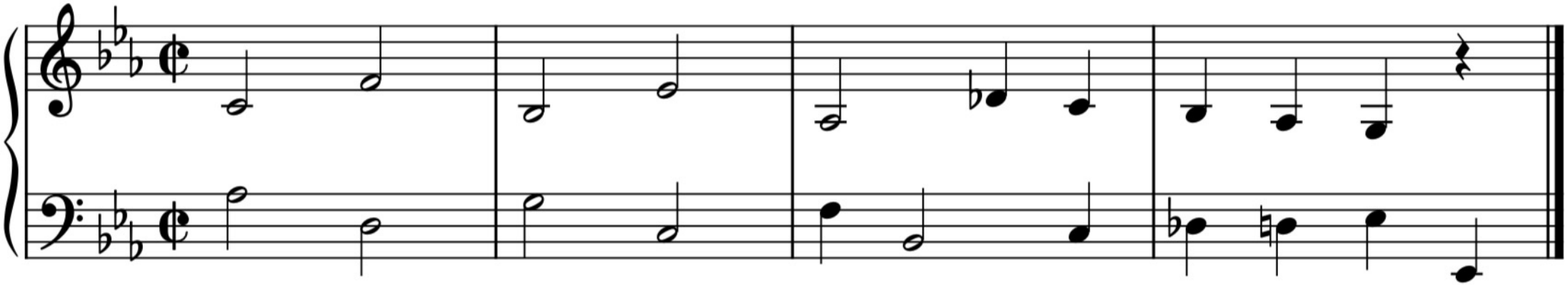
Error

```
(direct58 (c,maj3) (d,per5)) :: … != [] of type (List MotionError)
```

Beethoven's Pathetique Sonata

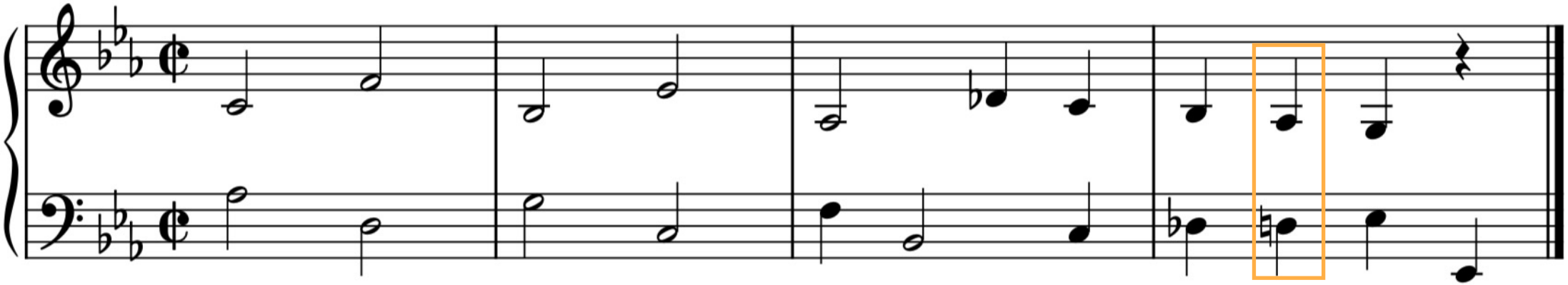


Type-checking Beethoven's Pathetique Sonata



```
cp-pathetique : CP
cp-pathetique = cp bars-pathetique refl refl
```

Type-checking Beethoven's Pathetique Sonata

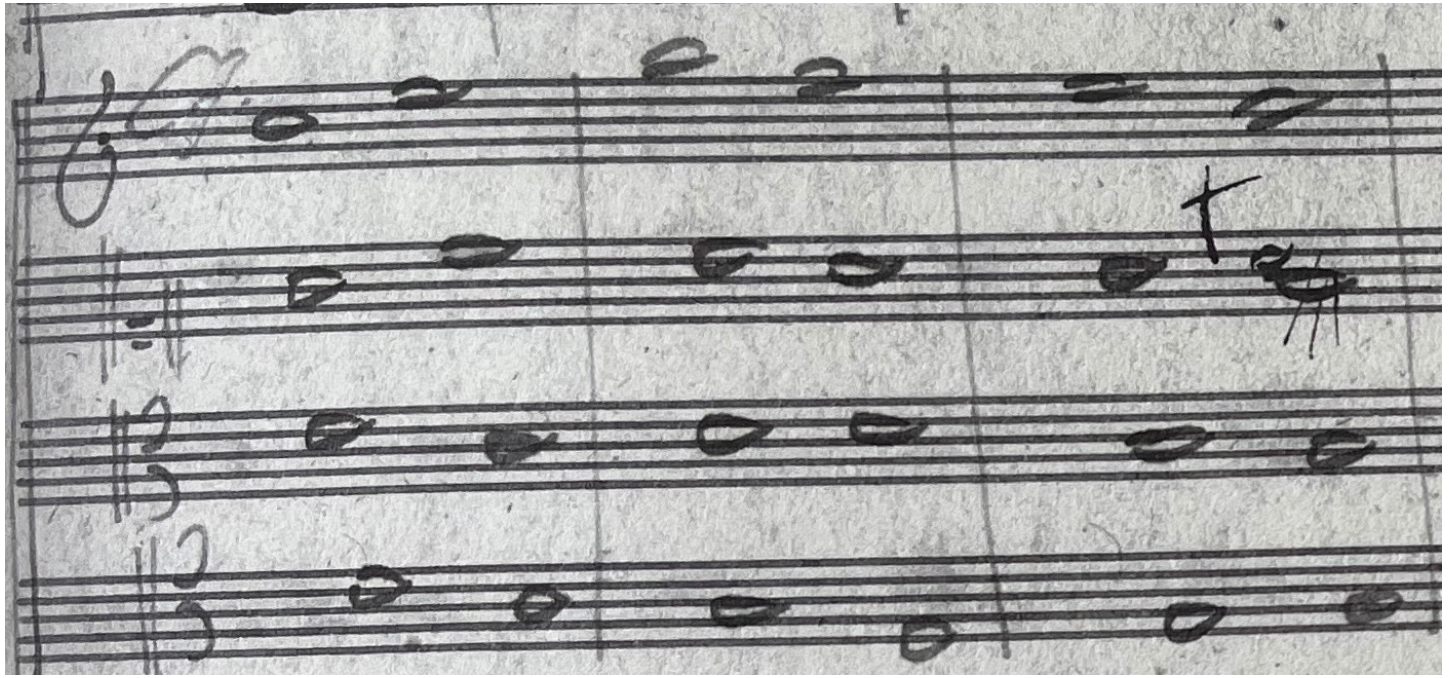


```
cp-pathetique : CP
cp-pathetique = cp bars-pathetique refl refl
```

Error

```
(dissonant aug4) :: … != [] of type (List IntervalError)
```

Beethoven’s exercises with Haydn



Generating Correct Counterpoint

Correct counterpoint as records

```
type CP =
  { List PitchInterval |
    intervalOK _v && motionOK _v }
```

Interval rule as a measure

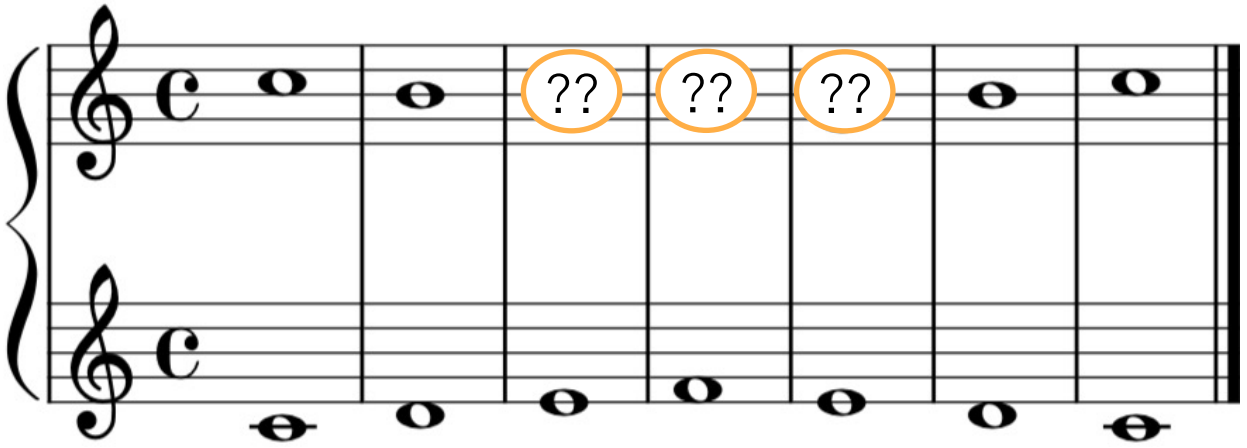
```
measure intervalOK :: List PitchInterval -> Bool where
  Nil -> True
  Cons pi pis -> isConsonant (proj2 pi) &&
                 intervalOK pis
```

Motion rule as a measure

```
measure motionOK :: List PitchInterval -> Bool where
  Nil -> True
  Cons pi pis ->
    if pis = Nil then True
    else {- pi and (head pis) do not form direct motion -} ||
         not (is58 (interval (head pis)))
```

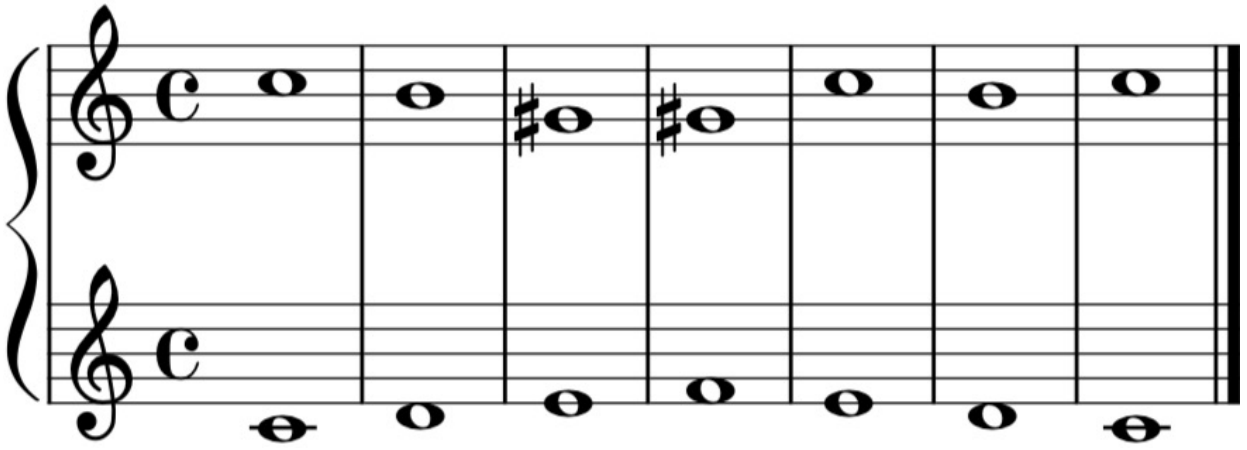
Generating correct counterpoint

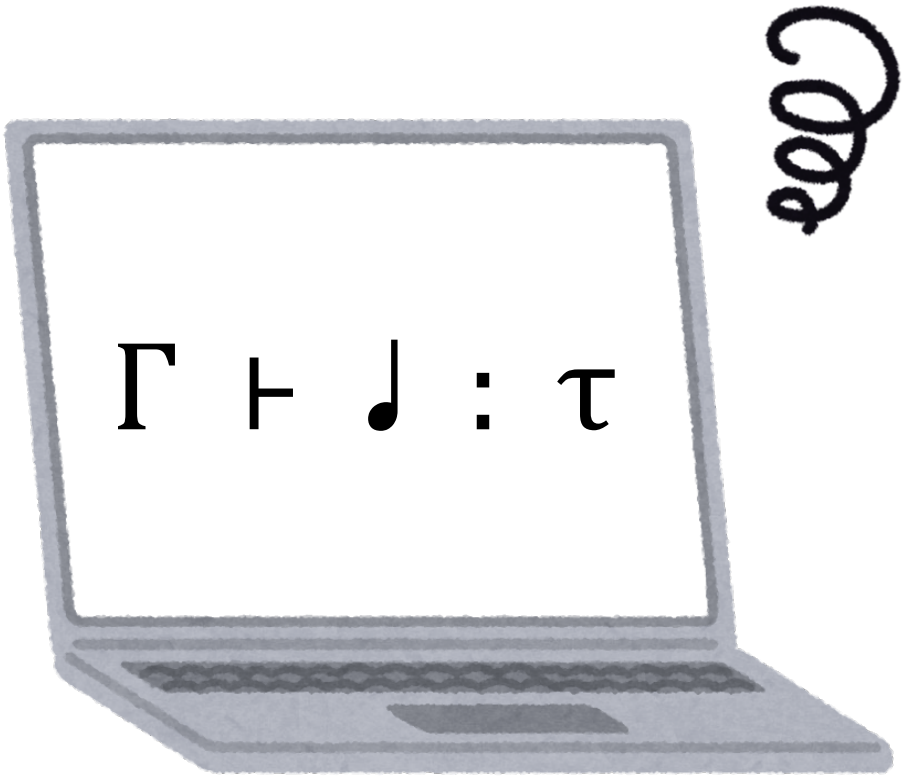
```
cp :: CP
cp =
  Cons (C, Oct)
   (Cons (D, Maj6)
    (Cons (E, ??)
     (Cons (F, ??)
      (Cons (E, ??)
       (Cons (D, Maj6)
        (Cons (C, Oct)
         Nil))))))
```



Generating correct counterpoint

```
cp :: CP
cp =
  Cons (C, Oct)
   (Cons (D, Maj6)
    (Cons (E, Maj3)
     (Cons (F, Min3)
      (Cons (E, Min6)
       (Cons (D, Maj6)
        (Cons (C, Oct)
         Nil))))))
```

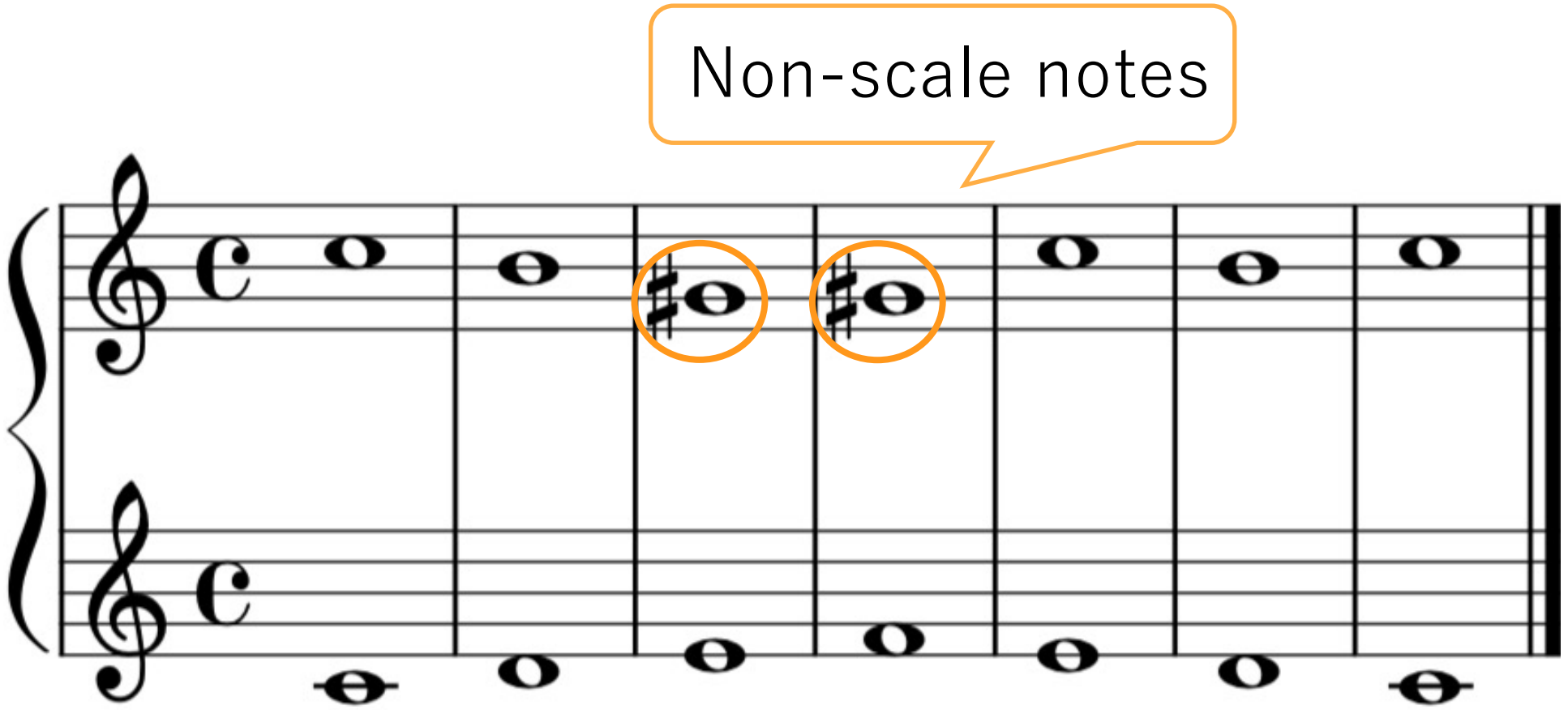




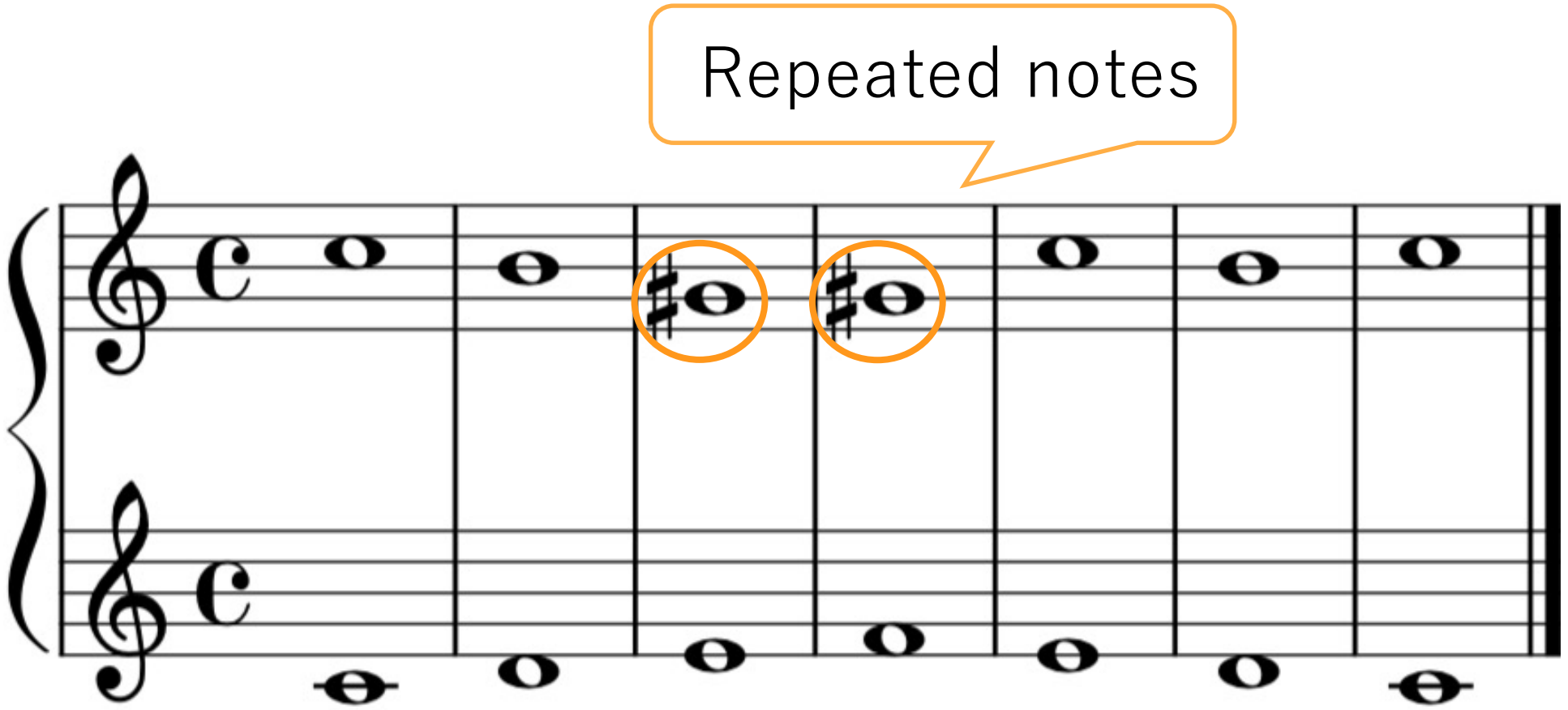
Well-typed music
may sound wrong.

Towards Musical Soundness

Problems with Synquid's composition



Problems with Synquid's composition



Hard and soft rules of Counterpoint

1. All intervals must be consonant. (hard)
2. No direct fifth or octave is allowed. (hard)
3. Non-scale notes are not preferred. (soft)
4. Repeated notes are not preferred. (soft)

Limitation of standard refinement types

```
type CP = { List PitchInterval |

            allConsonant _v && noDirect58 _v &&

            allScaleNotes _v && noRepeat _v }
```

Representing hard and soft rules

```
type CP = List PitchInterval < r1, r2 >
```

Representing hard and soft rules

```
type CP = List PitchInterval < r1, r2 >
```

Rules on individual intervals

Representing hard and soft rules

```
type CP = List PitchInterval < r1, r2 >
```

Rules on adjacent intervals

Representing hard and soft rules

```
type CP = List PitchInterval < r1, r2 >

where

  r1 = (isConsonant, ∞) ∧ (isScaleNote, 80)

  r2 = (notDirect58, ∞) ∧ (notRepeated, 60)
```

Representing hard and soft rules

```
type CP = List PitchInterval < r1, r2 >

where

  r1 = (isConsonant, ∞) ∧ (isScaleNote, 80)

  r2 = (notDirect58, ∞) ∧ (notRepeated, 60)
```

Hard

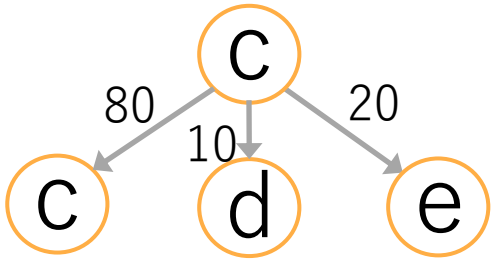
Representing hard and soft rules

```
type CP = List PitchInterval < r1, r2 >

where

  r1 = (isConsonant, ∞) ∧ (isScaleNote, 80)

  r2 = (notDirect58, ∞) ∧ (notRepeated, 60)
```

Soft

Implications of weighted types

- Type checking and synthesis as Max-SAT problem



- Soundness as gradable property

Theorem. Lowly weighted music does not sound too wrong.

Ideas for implementation

- Extend Synquid

  ☺ Has reusable components
  ☹ May not be suited for music

- Use Turnstile+ and Rosette [Chang et al. ‘20, Torlak et al. ‘13]

  ☺ Easy to customize
  ☹ May not combine well

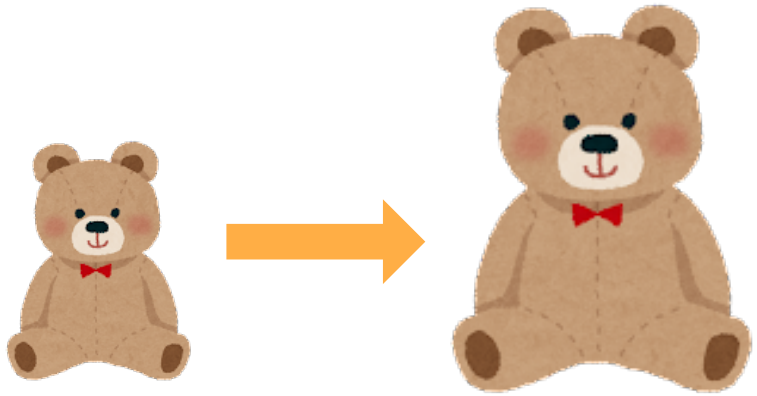
Ideas for soundness

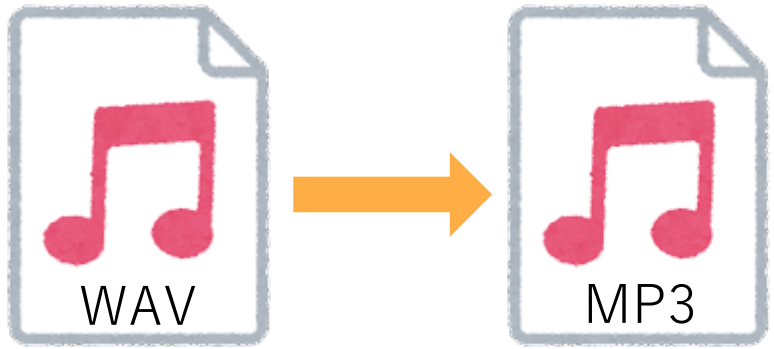
$$\frac{\begin{array}{c} e : PitchInterval \\ l : List\ PitchInterval < r_1, r_2 >;\ w \\ cost(r_1\ e) = w_1 \\ cost\big(r_2\ e\ (head\ l)\big) = w_2 \end{array}}{(e :: l\ ) : List\ PitchInterval < r_1, r_2 >;\ w + w_1 + w_2}$$

Discussion

How would weighted refinement types be useful for programming?

Possible application: Approximate computing





Precision types (Sampson+ ’11, Boston+ ‘14)

```
@Approx(0.8) int square(@Approx(0.9) int x) {
  @Approx(0.8) int xSquared = x * x;
  return xSquared;
}
```

Precision types (Sampson+ ’11, Boston+ ‘14)

```
@Approx(0.8) int square(@Approx(0.9) int x) {
  @Approx(0.8) int xSquared = x * x;
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Precision types (Sampson+ ’11, Boston+ ‘14)

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@Approx(0.8) int square(@Approx(0.9) int x) {
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```

Precision types (Sampson+ ’11, Boston+ ‘14)

```
@Approx(0.8) int square(@Approx(0.9) int x) {
  @Approx(0.8) int xSquared = x * x;
  return xSquared;
}
```

Takeaway

Type theorists can get inspiration from music!

Code & Google Form:
https://github.com/YouyouCong/counterpoint/types22/

Thanks to JST for funding this project.