MA421 Advanced Algorithms   Lent 2019

Exercises 2

1. Let decision problem HAM-CYCLE be defined as

$$\text{HAM-CYCLE} = \{\langle G\rangle \mid \text{Graph } G \text{ has a Hamiltonian cycle}\},$$

   where a *Hamiltonian cycle* in an undirected graph is a simple cycle that contains each vertex of the graph. Show that if HAM-CYCLE is in $\mathcal{P}$, then the problem of listing the vertices of a Hamiltonian cycle of a graph in order is polynomial-time solvable.

2. Let ϕ be a boolean formula constructed from the boolean input variables $x_1, x_2, \ldots, x_k$, negations ($\neg$), ANDs ($\wedge$), ORs ($\vee$), and parentheses. The formula ϕ is a tautology if it evaluates to 1 for every assignment of 0s and 1s to the input variables. Define TAUTOLOGY as the language of boolean formulas that are tautologies. Show that TAUTOLOGY is in co-$\mathcal{NP}$.

3. Prove that $\mathcal{P} \subseteq$ co-$\mathcal{NP}$.

4. Prove that if $\mathcal{NP} \neq$ co-$\mathcal{NP}$, then $\mathcal{P} \neq \mathcal{NP}$.

5. Show that the relation $\leq_{\mathrm{P}}$ is a transitive relation on languages. That is, show that, if $L_1 \leq_{\mathrm{P}} L_2$ and $L_2 \leq_{\mathrm{P}} L_3$, then $L_1 \leq_{\mathrm{P}} L_3$.

6. Prove that $L \leq_{\mathrm{P}} L^c$ if and only if $L^c \leq_{\mathrm{P}} L$.

Solutions to these exercises are to be handed in at the lecture on **29 January 2019**.

