MA421 Advanced Algorithms Lent 2019

Exercises 7

1. Suppose A is a randomised algorithm for a decision problem. Furthermore, suppose that, for any input x to A, the following condition holds:

$$\Pr[A(x) \text{ is correct}] > \frac{3}{4}.$$

   In a lecture, we have intuitively argued that, through repetitions of Algorithm A, we can devise an algorithm for the same decision problem with error probability less than any $\delta > 0$. Using a rigorous analysis, give a sufficent number of repetitions, for a given error probability bound δ.

   [Hint: Using a tail inequality might help here.]

2. Suppose n is even. Consider the oblivious routing problem on the hypercube of order n and the transpose permutation on n-bit labels: writing i as the concatenation of two $(n/2)$-bit strings a_i and b_i, we want to route the packet at $a_i b_i$ to destination $b_i a_i$. Show that the bit-fixing strategy takes $\Omega(2^{n/2})$ steps on this permutation.

3. Consider the following variant of the bit-fixing strategy for oblivious routing. Each packet randomly orders the bit positions in the label of its source and then corrects the mismatched bits in that order. Show that there is a permutation for which, with high probability, this given algorithm uses $2^{\Omega(n)}$ steps to route. An inequality that might be helpful is

$$\left(\frac{n}{k}\right)^k \leq \binom{n}{k} \leq \left(\frac{en}{k}\right)^k.$$

   [Hint: It might help to concentrate on only a subset of all packets that are likely to go through the same node.]

4. Consider a collection of n random variables X_i drawn independently from the geometric distribution with mean 2: that is, X_i is the number of flips of an unbiased coin up to and including the first occurrence of heads. Let $X = \sum_{i=1}^{n} X_i$. Derive an upper bound on the probability that $X > (1+\delta)(2n)$ for any fixed δ, by reducing this to a question involving just the sum of independent Poisson (i.e., indicator) variables, allowing you to apply the Chernoff bound.

Solutions to these exercises are to be handed in at the lecture on **5 March 2019**.

