MA427 – Exercise set 7

Academic year 2018-19

Exercise 7.1 Given a graph $G = (V, E)$, a *vertex cover* of G is a subset of the nodes $C \subseteq V$ such that every edge $e \in E$ contains at least one element in C. The *minimum cardinality vertex cover problem* is the problem of finding a vertex cover of G with the smallest possible number of nodes.

(a) Formulate the minimum cardinality vertex covering problem as a pure integer program with binary variables.

(b) Show that, if G is bipartite, then formulation is ideal.

(c) Write the dual of the LP relaxation of the formulation given in (a). What is its meaning?

Exercise 7.2 A company needs to plan its production for then next n time-periods. We assume that each item of product is "indivisible" (such as boat, a computer, etc.). The production cost per item in period $t = 1, \dots, n$ is c_t, while the demand at period t is d_t. The company may decide to store products from one period to the next; the unit cost of storing an item from period t to period $t + 1$ is h_t. Assume that there is no initial stock.
Write an integer programming problem to determine the minimum cost production plan. Show that the formulation is ideal.

Exercise 7.3 Let $P = \{x \in \mathbb{R}^n : Ax \leq b, x \geq 0\}$, where $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$, and let $X = \{x \in P : x_j \in \mathbb{Z}\ \forall j \in I\}$, where $I \subseteq \{1, \dots, n\}$. Show that, if $\bar{x}$ is a vertex of P such that $\bar{x} \notin X$, then there exists a valid inequality $\alpha^\top x \leq \beta$ for X that cuts off $\bar{x}$ (i.e. $\alpha^\top \bar{x} > \beta$). (Hint: use the definition of vertex.)

Exercise 7.4 Solve the following integer programming problem using Gomory fractional cuts.

$$\begin{array}{lrrcl}
\max & 2x_1 & +x_2 & & \\
 & -x_1 & +x_2 & \leq & 0 \\
 & 6x_1 & +2x_2 & \leq & 21 \\
 & x_1, & x_2 & \geq & 0 \text{ integer.}
\end{array}$$

For every cut added, express it in terms of the original variables x_1, x_2 and represent it graphically.

Exercise 7.5 Solve the previous exercise using branch and bound (at every iteration, use the dual simplex to re-optimize).