MA427 – Exercise set 9

Academic year 2018-19

This exercise set is part of the summative assessment. Please submit the solutions by 16:00 on Monday March 18 in COL.4.01 to Sarah Massey.

Exercise 9.1 Consider the following convex minimization problem.

$$\begin{aligned} &\min \quad -x_1 + x_2 \\ &e^{x_1 - x_2} - 1 \le 0 \\ &-x_1 + x_2 \le 0 \end{aligned}$$

(a) Determine the feasible region and the optimal solution. Are Slater's conditions satisfied?

(c) Derive the Lagrangian dual of the problem.

(b) Write the KKT conditions for the above problem. Do they have any solutions? Does strong duality hold?

Exercise 9.2 Consider the following convex minimization problem.

$$\begin{aligned} \min \quad & -\ln x \\ & |x - 2| \quad \le 1 \end{aligned}$$

(a) Write explicitly the Lagrangian dual of the above problem.

(b) Does the dual admit an optimum?

(c) Is there a solution (x, λ) to the KKT conditions? What is the primal optimal solution?

Exercise 9.3 Least-squares solution of linear equations: Consider the problem of finding a solution of minimum norm to a system of linear equations. That is, given and $m \times n$ matrix A and $b \in \mathbb{R}^m$

$$\begin{aligned} \min \quad & \|x\|^2 \\ & Ax = b. \end{aligned}$$

In this exercise you will prove that the problem can be solved in closed form.

(a) Compute the Lagrangian dual function of the above problem.

(b) Assuming (without loss of generality) that the matrix A has rank m, compute the optimal dual solution, and from that derive the optimal primal solution.

(Recall that $\|x\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2} = \sqrt{x^\top x}$. Recall also that, if A has rank m, then the $m \times m$ matrix $AA^\top$ is invertible.)

Exercise 9.4 Consider the following convex minimization problem.

$$\begin{array}{lrl} \min & x_1^2 + x_2^2 & \\ & (x_1-1)^2 + (x_2-1)^2 & \leq 1 \\ & (x_1-1)^2 + (x_2+1)^2 & \leq 1 \end{array}$$

(a) Determine the feasible region and the optimal solution x^*.

(b) Write the KKT conditions. Do they have any solutions?

(c) Determine the Lagrangian dual problem. Does strong duality hold?