MA427 – Exercise set 1

Academic year 2018-19

Exercise 1.1 Solve the following two LPs using Fourier-Motzkin elimination:

(a)

$$\begin{pmatrix} 2 & -3 & 3 \\ -1 & 1 & 0 \\ 1 & 0 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \leq \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 2 & -1 & 3 \\ -1 & 1 & 0 \\ 1 & 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \leq \begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix}$$

Exercise 1.2 Apply Farkas' lemma for the following system (that is, formulate the alternative system).

$$\begin{array}{rcr} 2x_1 - 3x_2 + 4x_3 & = & -1 \\ -x_1 - 2x_2 + 3x_3 & \geq & 0 \\ x_2, x_3 & \geq & 0 \end{array}$$

Exercise 1.3 Consider two polyhedra $P = \{x \in \mathbb{R}^n : Ax \leq b\}$ and $P' = \{x \in \mathbb{R}^n : A'x \leq b'\}$, and assume they are disjoint: $P \cap P' = \emptyset$. Using Farkas' lemma, prove that there exists a hyperplane strictly separating these two polyhedra: there is a vector $c \in \mathbb{R}^n$ such that $\max\{c^\top x : x \in P\} < \min\{c^\top x : x \in P'\}$.

Exercise 1.4 Consider the following LP:

$$\begin{array}{rrcl} \min & 3x_1 - 2x_2 - x_3 + x_4 & & \\ & x_1 + x_2 + x_3 - x_4 & \leq & 4 \\ & 2x_1 - x_2 \quad\quad + x_4 & \geq & 4 \\ & -x_1 + 2x_2 + 3x_3 + x_4 & = & 2 \\ & x_1 \geq 0,\ x_3 \geq 0,\ x_4 \leq 0 & & \end{array}$$

(a) Write the dual of the above problem.

(b) Using the complementary slackness theorem, determine which of the following points is an optimal solution. $(\frac{5}{3}, -\frac{2}{3}, 3, 0)$; $(0, -\frac{26}{3}, 8, -\frac{14}{3})$; $(\frac{13}{6}, 0, \frac{3}{2}, -\frac{1}{3})$.

Exercise 1.5 Formulate the dual of the system $\max\{c^\top x : Ax \leq b, x \geq 0\}$, and derive the strong duality theorem for this form using the form in the lecture notes.