MA427 – Exercise set 6

Academic year 2018-19

Exercise 6.1

(a) Show that if both $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{m\times m}$ are T.U. matrices, then the following $(n+m)\times(n+m)$ matrix is also T.U.:

$$C = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}.$$

(b) Is it true that if A and B are both T.U. matrices, then the juxtaposition $(A|B)$ is always T.U.?

Exercise 6.2 Let A be an $m\times n$ totally unimodular matrix, and $b, b' \in \mathbb{Z}^m$ be integer vectors such that all components of $b+b'$ are even numbers. Assume that $Ax = b$, $x \geq 0$ and $Ax = b'$, $x \geq 0$ are both feasible. Show that there exists an *integer* vector $z \in \mathbb{Z}^n$, $z \geq 0$, such that $Az = (b+b')/2$.

Exercise 6.3 Let G be a bipartite graph with bipartition V_1, V_2 and $|V_1| = |V_2|$. Assume that, for a given positive integer k, every node of G has exactly k neighbours. Show that G has a perfect matching (hint: show that the ideal formulation of perfect matchings has a feasible solution).

Exercise 6.4 A company produces a K products at I plants, and then ships these products to J market zones. For $k = 1,\ldots,K$, $i = 1,\ldots,O$ and $j = 1,\ldots,J$, the following data are given.

v_{ik} = cost of producing one unit of product k at plant i;
c_{ijk} = cost of transporting one unit of product k from plant i to market zone j;
f_{ik} = fixed cost associated with producing product k at plant i;
Q_i = resource capacity at plant i;
q_{ik} = amount resource capacity used to produce product k at plant i;
d_{jk} = demand for product k at market zone j.

(a) Formulate as a mixed-integer linear programming problem the problem of minimizing the total cost of production and transportation that the company is facing.

(b) Can you think of two different formulations for the problem? Which is the strongest?

(c) Modify your formulation to introduce the further restriction that no plant may produce more than $k_{\max}$ products.

(d) Modify your formulation to introduce the further restriction that any product can be produced in at most $i_{\max}$ plants.