Matchings

Theorem:
The propose-and-reject algorithm produces a man-optimal matching.

Def: Man-optimal matching: a stable matching where each man is assigned to his best possible stable partner.

Lemma: In the propose-and-reject alg. (men propose) a woman never rejects a stable partner.

Proof of lemma:

M* stable matching produced by the propose-and-reject algorithm.

Suppose women reject stable partners. The first time that a woman rejects a stable partner is when Mary rejects David to be with John.

(A) Mary: John > David

- Let M be the stable matching that produces the stable pair ~~(Mary, David)~~ (David, Mary).
- By assumption, John was not earlier rejected by any of his stable partners, thus:
(since men propose in the same order as their rankings)

(B) John: prefers Mary to any other stable partner

- Suppose in M, John was assigned to Kate.
⇒ (John, Kate) is a stable pair:

(C) John: Mary > Kate

Since, M has pairs: (David, Mary)
(John, Kate)
} M is not stable by (A) & (C).

∴ → ←.

□