Further Approximation Algorithms

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I. Exercise 1

Sketch the proofs of the following:

(a) The greedy algorithm for the k-center problem is a 2-approximation algorithm.

(b) There does not exist an α-approximation algorithm for the TSP (non-metric), for $\alpha > 1$, unless $P = NP$.

(c) Christofides algorithm is a $\frac{3}{2}$-approximation algorithm.

II. Exercise 2

Suppose we have $n = 3$ boolean variables, and $m = 3$ clauses where:

$C_1 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$,

$C_2 = \bar{x}_1 \vee x_2 \vee x_3$ and

$C_3 = x_1 \vee x_2 \vee \bar{x}_3$,

with weights $w_1 = 3$, $w_2 = 2$ and $w_3 = 1$. Suppose we want to maximize the value of $\sum_{j=1}^{m} w_j C_j$, where the variables C_j, take the value 1 when the clause is satisfied.

(i) Find an optimal solution by inspection and give the value of the x variables and the objective value.

(ii) Approximate the solution using randomized rounding, using the random $0 - 1$ sequence $001110100\ldots$ to generate random boolean variables. What is the approximate objective value

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and the value of the x variables.

(iii) The randomized algorithm used in (ii) is a $\frac{1}{2}$-approximation. Could we get a value in (ii) which is less than $\frac{1}{2}$ times the optimal value OPT?

III. Exercise 3

Prove that a vertex cover heuristic that adds both of an uncovered edge's endpoints to a cover yields a 2-approximation. (Hint: compare the vertices added by the algorithm with those in an optimal cover.)

IV. Exercise 4

Consider the complete graph $G = (V, E)$, where $V = \{1, 2, 3, 4, 5\}$ and $c_{ij} = \frac{i+j}{2}$. Approximate an optimal tour for the TSP on G using:

(a) Show that the triangle inequality holds.

(b) The nearest addition algorithm.

(c) The double tree algorithm.

(d) Christofides algorithm.

(e) For each one of the above α-approximation algorithms, give the value of α and verify that the approximate solutions found is within α of OPT.