Forests' Hereditary Property:

$E = \{edges\}$
$\Phi = \{forests\}$

if $I \in \Phi$ is a forest, then $\forall I' \subseteq I$ will also be a forest, i.e. $I' \in \Phi$.

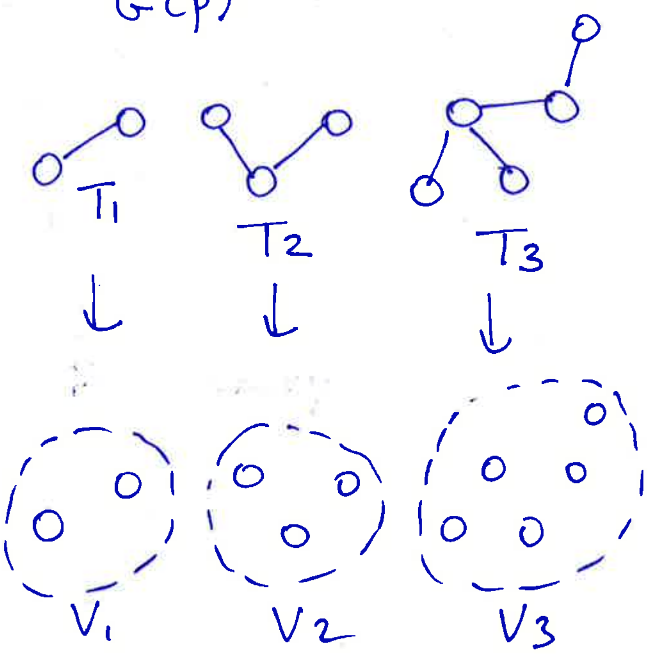
Forests' Growth Property:

If $G(p), G(p+1) \in \Phi$ are forests (subsets of edges with no cycles), then $\exists\ e \in G(p+1)/G(p)$ s.t. $\{e\} \cup G(p) \in \Phi$.

Proof: $G(p)$ forest of p edges: trees $T_1, T_2, \dots, T_k$
$|T_1| + |T_2| + \dots + |T_k| = p$.
$G(p+1)$ forest of $p+1$ edges.

- We want to find an $e \in G(p+1)$ that does not create a cycle with edges in $G(p)$.
- For an edge not to create a cycle with $G(p)$, it must ~~have~~ not have both its end nodes in the same T_i, $i = 1, \dots, k$.

$G(p)$



$G(p) = T_1 \cup T_2 \cup T_3$
$|T_1| = 1$
$|T_2| = 2$
$|T_3| = 4$
$p = |T_1| + |T_2| + |T_3| = 7$

- How many edges can we put in the same V_i without forming a cycle? $|T_i|$
- What is the maximum # of edges in $G(p+1)$ with both ends in the same V_i, $i = 1, \dots, k$? $|T_1| + |T_2| + \dots |T_k| = p$

Since $G(p+1)$ has $p+1$ edges there must be at least one edge e that does not have both its ends in the same V_i.

Thus $\{e\} \cup G(p)$ a forest. $\square$