Appendix

A Proofs

Proposition 1. *A correlation plan x is the team's strategy in a TME if and only if it is a solution of the following linear program:*

$$\begin{aligned}&\max_x v(\mathcal{I}_n(\phi))\\ &v(\mathcal{I}_n(\sigma_n)) - \sum_{I_n\in\mathcal{I}_n:\mathrm{seq}_n(I_n)=\sigma_n} v(I_n)\\ &\le \sum_{z\in Z,\mathrm{seq}_n(z)=\sigma_n} U_T(\mathrm{seq}_N(z))x(\mathrm{seq}_T(z)) \quad \forall\sigma_n\in\Sigma_n\\ &x\in\mathcal{X}.\end{aligned}$$

Proof. By Eqs.(3) and (4), we know that $\mathcal{R}_T$ is equivalent to $\mathcal{X}$, i.e., for each joint realization plan, there is an equivalent correlation plan and vice versa. Then the team's strategy in a TME can be computed by:

$$\max_{x\in\mathcal{X}}\min_{r_n\in\mathcal{R}}\sum_{z\in Z}U_T(\mathrm{seq}_N(z))x(\mathrm{seq}_T(z))r_n(\mathrm{seq}_n(z)).$$

After expanding the constraint $r_n\in\mathcal{R}_n$ by using the constraints for the realization plan r_n in Eqs.(1a)-(1c), we obtain the following inner minimization problem:

$$\begin{aligned}&\min_{r_n\in\mathcal{R}}\sum_{z\in Z}U_T(\mathrm{seq}_N(z))x(\mathrm{seq}_T(z))r_n(\mathrm{seq}_n(z))\\ &r_n(\phi)=1\\ &\sum_{a_n\in\psi_n(I_n)} r_n(\mathrm{seq}_n(I_n)a_n)=r_n(\mathrm{seq}_n(I_n))\ \forall I_n\in\mathcal{I}_n\\ &r_n(\sigma_n)\ge 0\quad\forall\sigma_n\in\Sigma_n.\end{aligned}$$

Introducing the free dual variable $v(\phi)$ (i.e., $v(\mathcal{I}_n(\phi))$) for the first constraint of r_n and $v(I_n)$ for the constraint of each $I_n\in\mathcal{I}_n$, similar to the variables in Program (2), we obtain the following dual linear program:

$$\begin{aligned}&\max v(\mathcal{I}_n(\phi))\\ &v(\mathcal{I}_n(\sigma_n)) - \sum_{I_n\in\mathcal{I}_n:\mathrm{seq}_n(I_n)=\sigma_n} v(I_n)\\ &\le \sum_{z\in Z,\mathrm{seq}_n(z)=\sigma_n} U_T(\mathrm{seq}_N(z))x(\mathrm{seq}_T(z))\ \forall\sigma_n\in\Sigma_n.\end{aligned}$$

Therefore, x is the team's strategy in a TME if and only if it is a solution of Program (5). □

Proposition 2. *$\mathcal{X}\subseteq\mathcal{Y}$ in any EFG.*

Proof. Given any σ_T, obviously, if $y(\sigma_T)=x(\sigma_T)=\prod_{i\in T}r_i(\sigma_T(i))$, then constraints in Eq.(6) will hold. Specifically, Constraint (6c) holds due to that: $r_{T\setminus\{i\}}(\sigma_{T\setminus\{i\}})\sum_{a_i\in\psi_i(I_i)}r_i(\mathrm{seq}_i(I_i)a_i) = r_{T\setminus\{i\}}(\sigma_{T\setminus\{i\}})r_i(\mathrm{seq}_i(I_i))$ by Eq.(1c). Therefore, $\mathcal{Y}$ includes $\mathcal{X}$. □

Proposition 3. *In any EFG satisfying the non-unique-path property, $\mathcal{X}$ is a strict subset of $\mathcal{Y}$, i.e., $\mathcal{X}\subset\mathcal{Y}$.*

Proof. By Proposition 2, we have $\mathcal{X}\subseteq\mathcal{Y}$. Now we show that $\mathcal{X}\neq\mathcal{Y}$. Suppose there is an information set I_i of team member i with σ_T and $\sigma'_T\in\Sigma_T(I_i)$ and $|\psi_i(I_i)|\ge 2$. By Eq.(6c), we have:

$$\sum_{a_i\in\psi_i(I_i)} y(\sigma_{T\setminus\{i\}},\mathrm{seq}_i(I_i)a_i)=y(\sigma_T),$$
$$\sum_{a_i\in\psi_i(I_i)} y(\sigma'_{T\setminus\{i\}},\mathrm{seq}_i(I_i)a_i)=y(\sigma'_T).$$

Suppose $a_i,a'_i\in\psi_i(I_i)$ and there is a solution with $y(\sigma_T)>0$ and $y(\sigma'_T)>0$, satisfying that $y(\sigma_{T\setminus\{i\}},\mathrm{seq}_i(I_i)a_i)=y(\sigma_T)$ and $y(\sigma'_{T\setminus\{i\}},\mathrm{seq}_i(I_i)a'_i)=y(\sigma'_T)$. Note that $\mathrm{seq}_i(I_i)=\sigma_T(i)=\sigma'_T(i)$. By Eq.(4), we have $r_i(\mathrm{seq}_i(I_i)a_i)=r_i(\mathrm{seq}_i(I_i))>0$ and $r_i((\mathrm{seq}_i(I_i)a'_i))=r_i(\mathrm{seq}_i(I_i))>0$, which will violate the definition of r_i because $r_i(\mathrm{seq}_i(I_i))\ge r_i(\mathrm{seq}_i(I_i)a_i)+r_i(\mathrm{seq}_i(I_i)a'_i)=2r_i(\mathrm{seq}_i(I_i))>0$. Then, $\mathcal{X}\neq\mathcal{Y}$ because R_T is equivalent to $\mathcal{X}$. Therefore, $\mathcal{X}\subset\mathcal{Y}$. □

Proposition 4. *In any EFG satisfying the unique-path property, $\mathcal{X}=\mathcal{Y}$.*

Proof. By Proposition 2, we have $\mathcal{X}\subseteq\mathcal{Y}$. Now we show that $\mathcal{Y}\subseteq\mathcal{X}$. For each information set $I_i\in\mathcal{I}_i$ of player $i\in T$ in EFG satisfying the unique-path property, if $|\Sigma_T(I_i)|=1$ with that $\sigma_T\in\Sigma_T(I_i)$, by Eq.(6c), we have:

$$\sum_{a_i\in\psi_i(I_i)} y(\sigma_{T\setminus\{i\}},\mathrm{seq}_i(I_i)a_i)=y(\sigma_T).$$

Then, similarly as in Eq.(4), we define the behavioral strategy (equivalent to a realization plan r_i) in I_i with $\sigma'_T=(\sigma_{T\setminus\{i\}},\mathrm{seq}_i(I_i)a_i)$:

$$\beta_i(I_i,a_i)=\begin{cases}\frac{y(\sigma'_T)}{y(\sigma_T)}=\frac{r_i(\mathrm{seq}_i(I_i)a_i)}{r_i(\mathrm{seq}_i(I_i))} & \text{if } y(\sigma_T)>0,\\ \frac{1}{|\psi_i(I_i)|} & \text{if } y(\sigma_T)=0.\end{cases}$$

If $|\psi_i(I_i)|=1$ with that $a_i\in\psi_i(I_i)$, by Eq.(6c), we have:

$$y(\sigma_{T\setminus\{i\}},\mathrm{seq}_i(I_i)a_i)=y(\sigma_T),\quad\forall\sigma_T\in\Sigma_T(I_i).$$

Then, similarly as in Eq.(4), we have a behavioral strategy (equivalent to a realization plan) in I_i such that $\beta_i(I_i,a_i)=1$. Then, in each information set I_i, β_i (equivalent to a realization plan r_i) is well-defined based on y. It means that for any $y\in\mathcal{Y}$, there is an equivalent $r_T\in\mathcal{R}_T$, i.e., there is an equivalent $x\in\mathcal{X}$. Thus, $\mathcal{Y}\subseteq\mathcal{X}$. Therefore, $\mathcal{X}=\mathcal{Y}$. □

Theorem 1. *In any EFG satisfying the unique-path property, a TME can be computed in polynomial time.*

Proof. By Proposition 4, $\mathcal{X}=\mathcal{Y}$. Then, we can use the polynomial-sized Constraint (6) to represent $\mathcal{X}$ in Program (5) of Proposition 1, which has a polynomial number of variables. Therefore, there is a polynomial-time algorithm for computing a TME in these EFGs. □

Proposition 5. *$\mathcal{X}\subset\mathcal{M}$ in any EFG.*

Proof. In an EFG with $|T| = 2$, given any σ_T, obviously, if $m(\sigma_T) = x(\sigma_T) = r_1(\sigma_T(1))r_2(\sigma_T(2))$, then constraints in Eq.(7) will hold. Specifically, Constraint (7b) holds due to that $(1-r_1(\sigma_T(1)))(1-r_2(\sigma_2)) \geq 0$. Therefore, $\mathcal{M}$ includes $\mathcal{X}$. However, let $m(\sigma_T) = \min\{r_1(\sigma_T(1)), r_2(\sigma_T(2))\}$ for any $\sigma_T \in \Sigma_T^{\bowtie}$, which satisfies constraints in Eq.(7), but $m(\sigma_T) \neq x(\sigma_T) = r_1(\sigma_T(1))r_2(\sigma_T(2))$ when $0 < r_1(\sigma_T(1)) < r_2(\sigma_T(2)) < 1$. That is, $\mathcal{X} \neq \mathcal{M}$. Therefore, $\mathcal{X} \subset \mathcal{M}$. Because the case with $r_1(\sigma_T(1)) < r_2(\sigma_T(2)) < 1$ can happen in any EFG, therefore, $\mathcal{X} \subset \mathcal{M}$ in any EFG with $|T| = 2$. This result also holds in any EFG with $|T| > 2$ by just using a variable $m'(\sigma_{T\setminus\{1\}}) \in [0, 1]$ to replace $r_2(\sigma_2)$. $\square$

Proposition 6. *In any EFG satisfying the non-unique-path property,* $\mathcal{X} \subset (\mathcal{M} \cap \mathcal{Y}) \subset \mathcal{M}$.

Proof. Suppose there is an information set I_1 with $a, b \in \psi_1(I_1)$ in a game with $|T| = 2$. Given $\sigma_T \in \Sigma_T(I_1)$, set

$$m(\sigma_T) = \min\{r_1(\sigma_T(1)), r_2(\sigma_T(2))\}$$
$$m(\sigma_T(1)a, \sigma_T(2)) = \min\{r_1(\sigma_T(1)a), r_2(\sigma_T(2))\}$$
$$m(\sigma_T(1)b, \sigma_T(2)) = \min\{r_1(\sigma_T(1)b), r_2(\sigma_T(2))\}.$$

As a consequence, $m(\sigma_T) \neq m(\sigma_T(1)a, \sigma_T(2)) + m(\sigma_T(1)b, \sigma_T(2))$ when $0 < r_2(\sigma_T(2)) < \min\{r_1(\sigma_T(1)), r_1(\sigma_T(1)a), r_1(\sigma_T(1))b\} < 1$. Therefore, $\mathcal{M} \nsubseteq \mathcal{Y}$, which means that $(\mathcal{M} \cap \mathcal{Y}) \subset \mathcal{M}$.

Now consider $m(\sigma_T) = m(\sigma_T(1)a, \sigma_T(2)) + m(\sigma_T(1)b, \sigma_T(2)) > 0$ and $m(\sigma_T(1)a, \sigma_T(2)) > m(\sigma_T(1)b, \sigma_T(2))$, which implies $r_1(\sigma_T(1)a) > r_1(\sigma_T(1)b)$ by Eq.(4). Consider another $\sigma'_T \in \Sigma_T(I_1)$ with $m(\sigma'_T) = m(\sigma'_T(1)a, \sigma'_T(2)) + m(\sigma'_T(1)b, \sigma'_T(2)) > 0$ and $m(\sigma'_T(1)a, \sigma'_T(2)) < m(\sigma'_T(1)b, \sigma'_T(2))$, which implies $r_1(\sigma_T(1)a) = r_1(\sigma'_T(1)a) < r_1(\sigma'_T(1)b) = r_1(\sigma_T(1)b)$ by Eq.(4). This contradiction implies that $\mathcal{X} \subset (\mathcal{M} \cap \mathcal{Y}) \subset \mathcal{M}$. This result also holds in EFGs with $|T| > 2$ by using a variable $m'(\sigma_{T\setminus\{1\}}) \in [0, 1]$ to replace $r_2(\sigma_T(2))$. $\square$