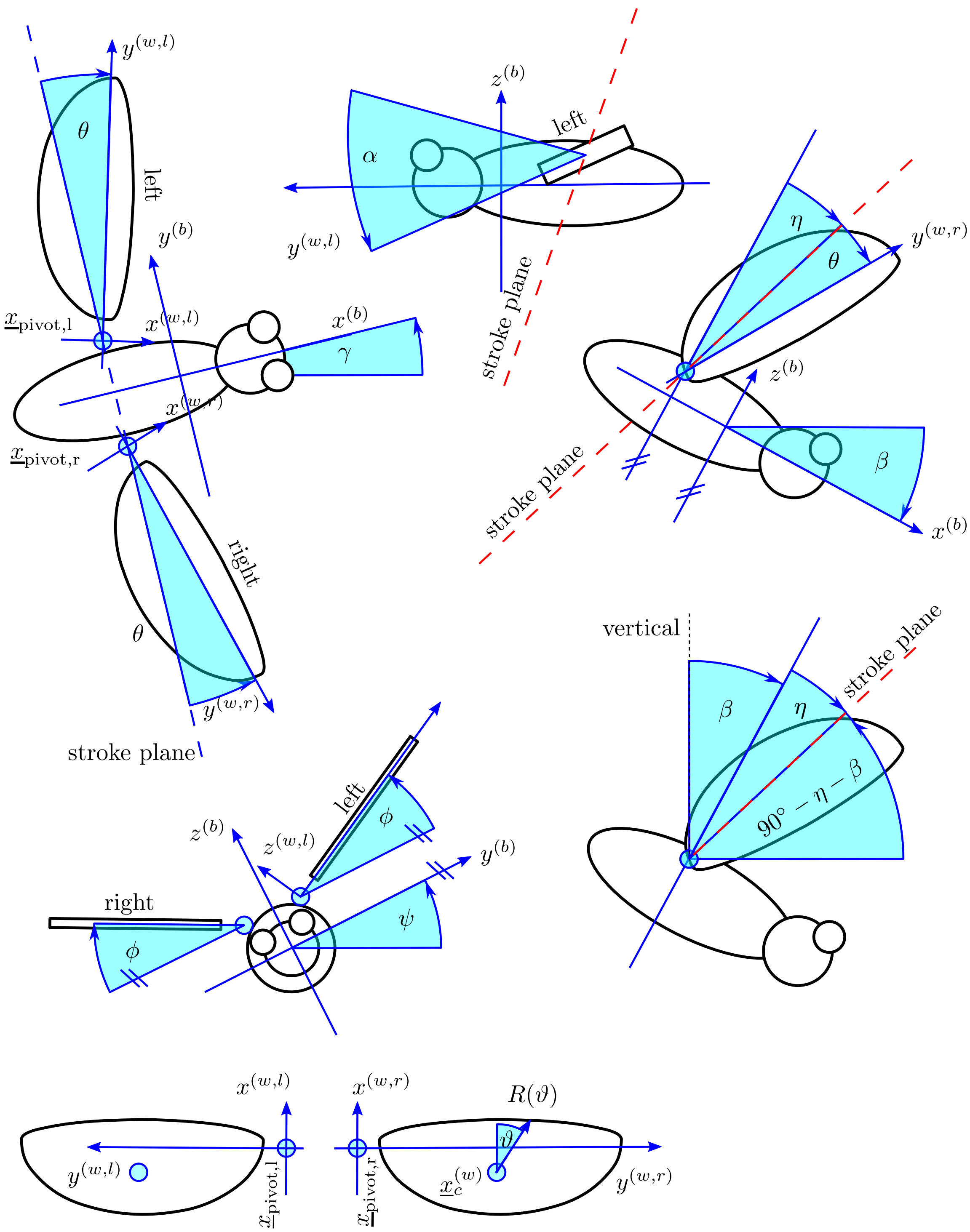
1. Insect Cheat Sheet



Angles:

Symbol	Name
α	wing feathering (Note this is NOT angle of attack)
θ	wing deviation
ϕ	wing flapping
η	stroke plane angle, relative to body
β	body pitch
γ	body yaw
ψ	body roll

Reynolds number:

$$
\begin{aligned}
Re &= \frac{\bar{U}_{\mathrm{tip}} R}{\nu} = \frac{2\Phi f R \cdot c_m}{\nu} \\
c_m &= A/R
\end{aligned}
$$

Rotation matrices

For further reference, we use the following notation:

$$
R_x(\xi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\xi & \sin\xi \\ 0 & -\sin\xi & \cos\xi \end{pmatrix} \qquad R_y(\xi) = \begin{pmatrix} \cos\xi & 0 & -\sin\xi \\ 0 & 1 & 0 \\ \sin\xi & 0 & \cos\xi \end{pmatrix}
$$

$$
R_z(\xi) = \begin{pmatrix} \cos\xi & \sin\xi & 0 \\ -\sin\xi & \cos\xi & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

Body system

We define the body system: the x-axis points forward and the z-axis points upward. Therefore, in a right handed coordinate system, we have the y-axis pointing to the left. Therefore, as illustrated in the figure as well, note that positive rotation results in:

- around x-axis (angle: ψ) rolls to the right side
- around y-axis (angle: β) pitches nose down
- around z-axis (angle: γ) yaws to the right

To get from global to body frame, we apply

$$\begin{aligned}
\underline{x}^{(b)} &= M_{\text{body}}\left(\psi,\beta,\gamma\right)\left(\underline{x}^{(g)}-\underline{x}^{(g)}_{\text{cntr}}\right)\\
&= R_x\left(\psi\right)R_y\left(\beta\right)R_z\left(\gamma\right)\underline{X}^{(g)}\\
&= \begin{pmatrix} 1 & 0 & 0\\ 0 & \cos\psi & \sin\psi\\ 0 & -\sin\psi & \cos\psi \end{pmatrix}\begin{pmatrix} \cos\beta & 0 & -\sin\beta\\ 0 & 1 & 0\\ \sin\beta & 0 & \cos\beta \end{pmatrix}\begin{pmatrix} \cos\gamma & \sin\gamma & 0\\ -\sin\gamma & \cos\gamma & 0\\ 0 & 0 & 1 \end{pmatrix}\underline{X}^{(g)}\\
&= \begin{pmatrix} \cos(\beta)\cos(\gamma) & \cos(\beta)\sin(\gamma) & -\sin(\beta)\\ \sin(\psi)\cos(\gamma)\sin(\beta)-\cos(\psi)\sin(\gamma) & \cos(\psi)\cos(\gamma)+\sin(\psi)\sin(\beta)\sin(\gamma) & \sin(\psi)\cos(\beta)\\ \sin(\psi)\sin(\gamma)+\cos(\psi)\cos(\gamma)\sin(\beta) & \cos(\psi)\sin(\beta)\sin(\gamma)-\sin(\psi)\cos(\gamma) & \cos(\psi)\cos(\beta) \end{pmatrix}\underline{X}^{(g)}
\end{aligned}$$

and the other way:

$$\underline{x}^{(g)} = M^{-1}_{body}\underline{x}^{(b)} + \underline{x}^{(g)}_{cntr}$$

body angular velocity:

$$\begin{aligned}
\underline{\Omega}^{(g)}_b &= R_z^{-1}\left(\gamma\right)\left[\begin{pmatrix}0\\0\\\dot\gamma\end{pmatrix}+R_y^{-1}\left(\beta\right)\left[\begin{pmatrix}0\\\dot\beta\\0\end{pmatrix}+R_x^{-1}\left(\psi\right)\begin{pmatrix}\dot\psi\\0\\0\end{pmatrix}\right]\right] = \begin{pmatrix}\dot\psi\cos(\beta)\cos(\gamma)-\dot\beta\sin(\gamma)\\ \dot\beta\cos(\gamma)+\dot\psi\cos(\beta)\sin(\gamma)\\ \dot\gamma-\dot\psi\sin(\beta)\end{pmatrix}\\
\underline{\Omega}^{(b)}_b &= M_{\text{body}}\,\Omega^{(g)}_b = \begin{pmatrix}\dot\psi-\dot\gamma\sin(\beta)\\ \dot\beta\cos(\psi)+\dot\gamma\cos(\beta)\sin(\psi)\\ \dot\gamma\cos(\beta)\cos(\psi)-\dot\beta\sin(\psi)\end{pmatrix}
\end{aligned}$$

body angular acceleration

$$\frac{d}{dt}\underline{\Omega}^{(g)}_b = \begin{pmatrix} \cos(\beta)\cos(\gamma)\ddot\psi-\sin(\gamma)\ddot\beta-\dot\beta\dot\gamma\cos(\gamma)-\dot\beta\dot\psi\cos(\gamma)\sin(\beta)-\dot\gamma\dot\psi\cos(\beta)\sin(\gamma)\\ \cos(\gamma)\ddot\beta+\cos(\beta)\sin(\gamma)\ddot\psi-\dot\beta\dot\gamma\sin(\gamma)+\dot\gamma\dot\psi\cos(\beta)\cos(\gamma)-\dot\beta\dot\psi\sin(\beta)\sin(\gamma)\\ \ddot\gamma-\sin(\beta)\ddot\psi-\dot\beta\dot\psi\cos(\beta) \end{pmatrix}$$

body velocity field

$$\underline{u}^{(g)}_b = \underline{u}^{(g)}_{\text{cntr}} + M^{-1}_{\text{body}}\left(\underline{\Omega}^{(b)}_b\times\underline{x}^{(b)}\right)$$

Stroke plane

we use the anatomical stoke plane angle η, thus the global (with respect to vertical) is $\eta_0 = \eta+\beta$ or the complement angle $\eta_1 = 90^\circ - \eta - \beta$. In hovering, where $\beta \neq 0$, and with a perfectly horizontal stroke plane we have $\eta_1 = 180^\circ$ and thus $\eta = -\left(\beta + 90^\circ\right)$

Wing System

We define the wing coordinate system: the y-axis is the span and points root-tip. The x-axis is the chord and points forward. Then, by consequence, z-axis points down for right wing and up for left wing.

Internally, flusi always uses mathematically positive rotation for ALL angles. We can change the sign of the deviation angle, as given in the SISC paper, in the wing kinematics INI file.

To get to the wing system we have

$$\begin{aligned}
\underline{x}^{(w)} &= M_{\text{wing}} M_{\text{stroke}} \left(\underline{x}^{(b)} - \underline{x}^{(b)}_{\text{pivot}} \right) \\
&= M_{\text{wing}} M_{\text{stroke}} M_{body} \left(\underline{x}^{(g)} - \underline{x}^{(g)}_{pivot} \right) \\
&\overset{\text{left}}{=} \left(R_y(\alpha) R_z(+\theta) R_x(\phi) R_y(\eta) R_x(\psi) R_y(\beta) R_z(\gamma) \right) \underline{X}^{(g)} \\
&\overset{\text{right}}{=} \left(R_y(-\alpha) R_z(+\theta) R_x(-\phi) R_x(\pi) R_y(\eta) R_x(\psi) R_y(\beta) R_z(\gamma) \right) \underline{X}^{(g)}
\end{aligned}$$

thus

$$\begin{aligned}
M_{\text{stroke,l}} &= R_y(\eta) & M_{\text{stroke,r}} &= R_x(\pi) R_y(\eta) \\
M_{\text{wing,l}} &= R_y(\alpha) R_z(\pm\theta) R_x(\phi) & M_{\text{wing,r}} &= R_y(-\alpha) R_z(\pm\theta) R_x(-\phi)
\end{aligned}$$

The angular velocity was defined as

$$\begin{aligned}
\underline{\Omega}^{(b)}_{wr} &= M^{-1}_{\text{stroke,r}} \left(R_x^{-1}(\phi) \left[\left(\begin{array}{c} \dot{\phi} \\ 0 \\ 0 \end{array} \right) + R_z^{-1}(\theta) \left(\left(\begin{array}{c} 0 \\ 0 \\ \dot{\theta} \end{array} \right) + R_y^{-1}(\alpha) \left(\begin{array}{c} 0 \\ \dot{\alpha} \\ 0 \end{array} \right) \right) \right] \right) \\
\underline{\Omega}^{(b)}_{wl} &= M^{-1}_{\text{stroke,l}} \left(R_x^{-1}(\phi) \left[\left(\begin{array}{c} -\dot{\phi} \\ 0 \\ 0 \end{array} \right) + R_z^{-1}(\theta) \left(\left(\begin{array}{c} 0 \\ 0 \\ \dot{\theta} \end{array} \right) + R_y^{-1}(\alpha) \left(\begin{array}{c} 0 \\ -\dot{\alpha} \\ 0 \end{array} \right) \right) \right] \right) \\
\underline{\Omega}^{(w)}_{w} &= M_{\text{wing}} \, \underline{\Omega}^{(b)}_{w}
\end{aligned}$$

which simplifies to

$$\underline{\Omega}^{(b)}_{wl} = \left(\begin{array}{c} \dot{\phi}\cos(\eta) - \dot{\alpha}\sin(\theta)\cos(\eta) + \dot{\theta}\cos(\phi)\sin(\eta) + \dot{\alpha}\cos(\theta)\sin(\phi)\sin(\eta) \\ \dot{\alpha}\cos(\phi)\cos(\theta) - \dot{\theta}\sin(\phi) \\ \dot{\theta}\cos(\phi)\cos(\eta) - \dot{\phi}\sin(\eta) + \dot{\alpha}\sin(\theta)\sin(\eta) + \dot{\alpha}\cos(\theta)\sin(\phi)\cos(\eta) \end{array} \right)$$

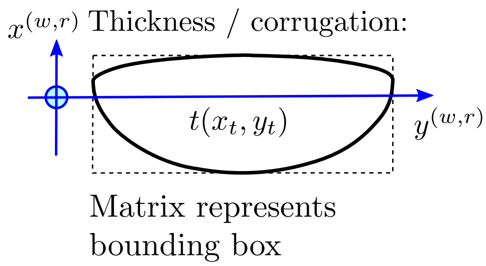
$$\underline{\Omega}^{(w)}_{wl} = \left(\begin{array}{c} \dot{\phi}\cos(\alpha)\cos(\theta) - \dot{\theta}\sin(\alpha) \\ \dot{\alpha} - \dot{\phi}\sin(\theta) \\ \dot{\theta}\cos(\alpha) + \dot{\phi}\sin(\alpha)\cos(\theta) \end{array} \right)$$

wing solid body velocity

$$\begin{aligned}
\underline{u}^{(w)}_{w} &= \underline{\Omega}^{(w)}_{w} \times \underline{x}^{(w)} \\
\underline{u}^{(g)}_{w} &= M^{-1}_{\text{body}} \, M^{-1}_{\text{wing}} \, M^{-1}_{\text{stroke}} \, \underline{u}^{(w)}_{w}. \qquad (1)
\end{aligned}$$

Wing corrugation

If the wing is corrugated (that means any deviation from a flat plate, $z = z(x^{(w)}, y^{(w)})$) or has variable thickness $t(x^{(w)}, y^{(w)})$, a matrix is read from the ini-file for the wing (this is a new feature!). This matrix is interpreted as $x^{(w)}$, $y^{(w)}$ values, with its bounds inside the wing shape bounding box, which is the dashed line below in the figure below



The ini files parser reads from top to bottom and from right to left, e.g.

```
wing_thickness_profile=(/0.1 0.5

0.2 0.6

0.3 0.7/);
```

is indeed read into a matrix

$$t = \begin{pmatrix} 0.1 & 0.5 \\ 0.2 & 0.6 \\ 0.3 & 0.7 \end{pmatrix}$$

Note the bounding box does not include the part at the wing root, where no wing exists (in the gap), since we'd just set zeros anyways, which is not incredible useful.

The matrix is interpreted as $t(ix, iy)$ data, so lines (1st index) are x-direction and columns (2nd index) are y direction.