

$$H \equiv p^2 + V$$

$$\langle x^n \frac{dV}{dx} \rangle = 2nE \langle x^{n-1} \rangle - 2n \langle V x^{n-1} \rangle$$

$$\langle \frac{dV}{dx} e^{nx} \rangle = 2nE \langle e^{nx} \rangle - 2n \langle V e^{nx} \rangle$$

$$\langle \frac{dV}{dx} e^{mx} x^n \rangle = 2E \langle m e^{mx} x^n + n e^{mx} x^{n-1} \rangle - 2 \langle V (m e^{mx} x^n + n e^{mx} x^{n-1}) \rangle$$

$$n \langle \frac{dV}{dx} x^m p^{n-1} \rangle = 2Em \langle x^{m-1} p^{n-1} \rangle - 2m \langle V x^{m-1} p^{n-1} \rangle$$

$$E \langle x^m p^n \rangle = \langle x^m p^{n+2} \rangle + \langle V x^m p^n \rangle$$

$$n \langle \frac{dV}{dx} e^{im\theta} p^{n-1} \rangle = 2imE \langle e^{im\theta} p^{n-1} \rangle - 2im \langle V e^{im\theta} p^{n-1} \rangle$$

$$E \langle e^{im\theta} p^n \rangle = \langle e^{im\theta} p^{n+2} \rangle + \langle V e^{im\theta} p^n \rangle$$

Trigonometric potential

$$V = e^{i\theta} + e^{-i\theta}$$

$$(n + 2m - 1) \langle e^{im\theta} p^n \rangle = 2(m - 1)E \langle e^{i(m-1)\theta} p^n \rangle + (n - 2m + 3) \langle e^{i(m-2)\theta} p^n \rangle$$

$$(2m - 1) \langle e^{im\theta} \rangle = 2(m - 1)E \langle e^{i(m-1)\theta} \rangle + (3 - 2m) \langle e^{i(m-2)\theta} \rangle$$

$$E \langle e^{im\theta} p^n \rangle = \langle e^{im\theta} p^{n+2} \rangle + \langle e^{i(m+1)\theta} p^n \rangle + \langle e^{i(m-1)\theta} p^n \rangle$$

Coulomb potentia

$$V = -\frac{1}{r} + \frac{1}{r^2}$$

$$2(m + 1)Ef(m, n) = 2(m - n)f(m - 2, n) + (n - 2m - 1)f(m - 1, n)$$

$$Ef(m, n - 2) = f(m, n) - f(m - 1, n - 2) + f(m - 2, n - 2)$$

Toda

$$V = e^x + e^{-x}$$