

GraphChallenge.org

Triangle Counting Performance

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Abstract—The rise of graph analytic systems has created a need for new ways to measure and compare the capabilities of graph processing systems. The MIT/Amazon/IEEE Graph Challenge has been developed to provide a well-defined community venue for stimulating research and highlighting innovations in graph analysis software, hardware, algorithms, and systems. GraphChallenge.org provides a wide range of pre-parsed graph data sets, graph generators, mathematically defined graph algorithms, example serial implementations in a variety of languages, and specific metrics for measuring performance. The triangle counting component of GraphChallenge.org tests the performance of graph processing systems to count all the triangles in a graph and exercises key graph operations found in many graph algorithms. In 2017, 2018, and 2019 many triangle counting submissions were received from a wide range of authors and organizations. This paper presents a performance analysis of the best performers of these submissions. These submissions show that their state-of-the-art triangle counting execution time, T_{tri} , is a strong function of the number of edges in the graph, N_e , which improved significantly from 2017 ($T_{\text{tri}} \approx (N_e/10^8)^{4/3}$) to 2018 ($T_{\text{tri}} \approx N_e/10^9$) and remained comparable from 2018 to 2019. Graph Challenge provides a clear picture of current graph analysis systems and underscores the need for new innovations to achieve high performance on very large graphs.

I. INTRODUCTION

The importance of graph analysis has dramatically increased and is critical to a wide variety of domains that include the analysis of genomics [1]–[6], brain mapping [7], computer networks [8]–[13], social media [14], [15], cybersecurity [16], [17], and sparse machine learning [18]–[24].

Many graph processing systems are currently under development. These systems are exploring innovations in algorithms [25]–[36], software architecture [37]–[46], databases [47], [48], software standards [49]–[55], and parallel computing hardware [56]–[65]. The rise of graph analysis systems has created a need for new ways to measure and compare the capabilities of these systems. The MIT/Amazon/IEEE Graph Challenge has been developed to provide a well-defined community venue for stimulating research and highlighting innovations in graph analysis software, hardware, algorithms, and systems. GraphChallenge.org provides a wide range of pre-parsed graph data sets, graph generators, mathematically

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defined graph algorithms, example serial implementations in a variety of languages, and specific metrics for measuring performance.

Scale is an important driver of the Graph Challenge and graphs with billions to trillions of edges are of keen interest. The Graph Challenge is designed to work on arbitrary graphs drawn from both real-world data sets and simulated data sets. Examples of real-world data sets include the Stanford Large Network Dataset Collection (see <http://snap.stanford.edu/data>), the AWS Public Data Sets (see aws.amazon.com/public-data-sets), and the Yahoo! Webscope Datasets (see webscope.sandbox.yahoo.com). These real-world data sets cover a wide range of applications and data sizes. While real-world data sets have many contextual benefits, synthetic data sets allow the largest possible graphs to be readily generated. Examples of synthetic data sets include Graph500, Block Two-level Erdos-Renyi graph model (BTER) [66], Kronecker Graphs [67]–[69], and Perfect Power Law graphs [70]–[72]. The focus of the Graph Challenge is on graph analytics. While parsing and formatting complex graph data are necessary in any graph analysis system, these data sets are made available to the community in a variety of pre-parsed formats to minimize the amount of parsing and formatting required by Graph Challenge participants. The public data are available in a variety of formats, such as linked list, tab separated, and labeled/unlabeled.

Graph Challenge 2017 received a large number of submissions that highlighted innovations in hardware, software, algorithms, systems, and visualization that allows the state-of-the-art in graph processing for 2017 to be estimated [73]. The goal of this paper is to analyze and synthesize the 2018 and 2019 submissions to provide an updated picture of the current state of the art of graph analysis systems. The organization of this paper is as follows. First, a recap of triangle counting is provided, along with a few standard algorithms. Next, an overview is presented of the Graph Challenge 2018 and 2019 submissions. The core of the paper is the section on the analysis of the 17 submission that all performed the triangle counting challenge. Based on this analysis, these results are synthesized to provide a picture of the current state of the art.

II. TRIANGLE COUNTING

The Graph Challenge consists of a pre-challenge and three challenges

- Pre-challenge: PageRank pipeline [74]

- Static graph challenge: subgraph isomorphism [75]
- Streaming graph challenge: stochastic block partition [76]
- Sparse deep neural network challenge [77]

The static graph challenge is further broken down into triangle counting and k-truss. Triangle counting is the focus of this paper.

Triangles are the most basic, trivial sub-graph. A triangle can be defined as a set of three mutually adjacent vertices in a graph. As shown in Figure 1, the graph G contains two triangles comprising nodes $\{a,b,c\}$ and $\{b,c,d\}$. The number of triangles in a graph is an important metric used in applications such as social network mining, link classification and recommendation, cyber security, functional biology, and spam detection [78].

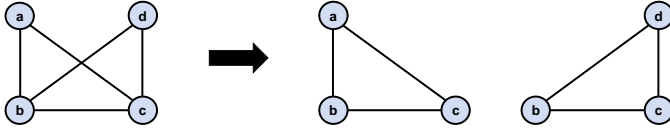


Fig. 1. The graph shown in this example contains two triangles consisting of nodes $\{a,b,c\}$ and $\{b,c,d\}$.

The number of triangles in a given graph G can be calculated in several ways. We highlight two algorithms based on linear algebra primitives. The first algorithm proposed by Wolf et al [79] uses an overloaded matrix multiplication approach on the adjacency and incidence matrices of the graph and is shown in Algorithm 1. The second approach proposed by Burkhardt et al [80] uses only the adjacency matrix of the given graph and is shown in Algorithm 2.

Algorithm 1: Array based implementation of triangle counting algorithm using the adjacency and incidence matrix of a graph [79].

Data: Adjacency matrix A and incidence matrix E
Result: Number of triangles in graph G
 initialization;
 $C = AE$
 $n_T = nnz(C)/3$

Multiplication is overloaded such that

$C(i, j) = \{i, x, y\}$ iff
 $A(i, x) = A(i, y) = 1$ & $E(x, j) = E(y, j) = 1$

Another algorithm for triangle counting based on a masked matrix multiplication approach has been proposed by Azad et al [81]. The serial version of this algorithm based on the MapReduce implementation by Cohen et al [82] is shown in Algorithm 3. Finally, a comparison of triangle counting algorithms can be found in [83].

III. COMMUNITY SUBMISSIONS

Graph Challenge has received a wide range of submissions across all its various challenges that have included hundreds of authors from over fifty organizations. In 2018, eighteen

Algorithm 2: Array based implementation of triangle counting algorithm using only the adjacency matrix of a graph [80].

Data: Adjacency matrix A

Result: Number of triangles in graph G
 initialization;

$C = A^2 \circ A$
 $n_T = \sum_{ij} (C)/6$

Here, \circ denotes element-wise multiplication

Algorithm 3: Serial version of triangle counting algorithm based on MapReduce version by Cohen et al [82] and [81].

Data: Adjacency matrix A

Result: Number of triangles in graph G
 initialization;

$(L, U) \leftarrow A$
 $B = LU$
 $C = A \circ B$
 $n_T = \sum_{ij} (C)/2$

Here, \circ denotes element-wise multiplication

submissions were selected for publication [84]–[101] and nine provided sufficient triangle counting performance data for analysis [84]–[92]. In 2019, twenty submissions were selected for publication [102]–[121] and eight provided sufficient triangle counting performance data for analysis [102]–[109].

Numerous submissions implemented the triangle counting challenge in a comparable manner, resulting in over 800 distinct measurements of triangle counting execution time, T_{tri} . The number of edges, N_e , in the graph describes the overall size of the graph. The rate of edges processed in triangle counting is given by

$$\text{Rate} = N_e / T_{tri}$$

Analyzing and combining all the performance data from the submissions can be done by fitting a piecewise model to each submission and then comparing the models. For each submission, T_{tri} vs N_e is plotted on a log-log scale from which a model can be fit to the data by estimating the parameters N_1 and β in the formula

$$T_{tri} = (N_e / N_1)^\beta$$

where N_1 is the number edges that can be processed in 1 second. The triangle counting execution time vs number of edges and corresponding model fits are given in Appendix A. The model fits illustrate the strong dependence of T_{tri} on N_e .

IV. PERFORMANCE ANALYSIS

The normalized parameters N_1 and β , along with the largest values of N_e , are shown in Tables I and I for each submission in 2018 and 2019, respectively. Submissions with larger N_e ,

TABLE I
2018 Triangle counting time model fit coefficients for $T_{\text{tri}} = (N_e/N_1)^\beta$
for large values of N_e .

Ref	Submission	max N_e	N_1	β
[84]	Bisson-Nvidia-2018	1.8×10^9	3×10^8	1
[85]	Hu-GWU-2018	3.4×10^{10}	3×10^8	1
[86]	Yasar-GaTech-2018	3.3×10^9	1×10^8	1
[87]	Fox-LLNL-2018	5.2×10^8	2×10^8	4/3
[88]	Mailthody-UIUC-2018	1.0×10^9	1×10^7	4/3
[89]	Zhang-CMU-2018	3.4×10^{10}	8×10^6	1
[90]	Davis-TAMU-2018	1.8×10^9	5×10^7	4/3
[91]	Donato-UMassB-2018	6.2×10^9	5×10^7	4/3
[92]	Kuo-CUHK-2018	1.1×10^7	1×10^5	1

TABLE II
2019 Triangle counting time model fit coefficients for $T_{\text{tri}} = (N_e/N_1)^\beta$
for large values of N_e .

Ref	Submission	max N_e	N_1	β
[102]	Pandey-Stevens-2019	5.2×10^8	5×10^8	4/3
[103]	Pearce-LLNL-2019	1.1×10^{12}	5×10^5	1/2
[104]	Acer-Sandia-2019	3.6×10^9	6×10^7	3/2
[105]	Yasar-GaTech-2019	1.8×10^9	3×10^8	1
[106]	Hoang-UTexas-2019	3.7×10^{10}	5×10^8	2/3
[107]	Wang-UCDavis-2019	3.2×10^7	2×10^7	3/2
[108]	Gui-HuazhongU-2019	3.2×10^7	6×10^7	3/2
[109]	Pearson-UIUC-2019	1.8×10^9	6×10^7	4/3

larger N_1 , and smaller β perform best. The current state-of-the-art can be seen by plotting all the model fits T_{tri} together (see Figures 2 and 3). Combined, these suggest that state-of-the-art performance model of the 2018 and 2019 is

$$T_{\text{tri}} \approx N_e/10^9$$

which is a significant improvement over the 2017 state-of-the-art performance model of

$$T_{\text{tri}} \approx (N_e/10^8)^{4/3}$$

Given the enormous diversity in processors, algorithms, and software, this relatively consistent picture of the state-of-the-art suggests that the current limitations are set by common elements across these benchmarks, such as memory bandwidth.

V. CONCLUSION

The rapid increase in the use of graphs has inspired new ways to measure and compare the attributes of graph analytic systems. The MIT/Amazon/IEEE Graph Challenge was created to stimulate research in graph analysis software, hardware, algorithms, and systems. The GraphChallenge.org website makes available to the world many pre-processed graph data sets, graph generators, graph algorithms, prototype serial implementations in a several languages, and defined metrics for assessing performance. The triangle counting component of GraphChallenge.org tests the performance of graph processing systems to count all the triangles in a graph and exercises key graph operations found in many graph algorithms. In 2017, 2018, and 2019 many triangle counting submissions were received from a wide range of authors and organizations. These submissions show that their state-of-the-art triangle counting

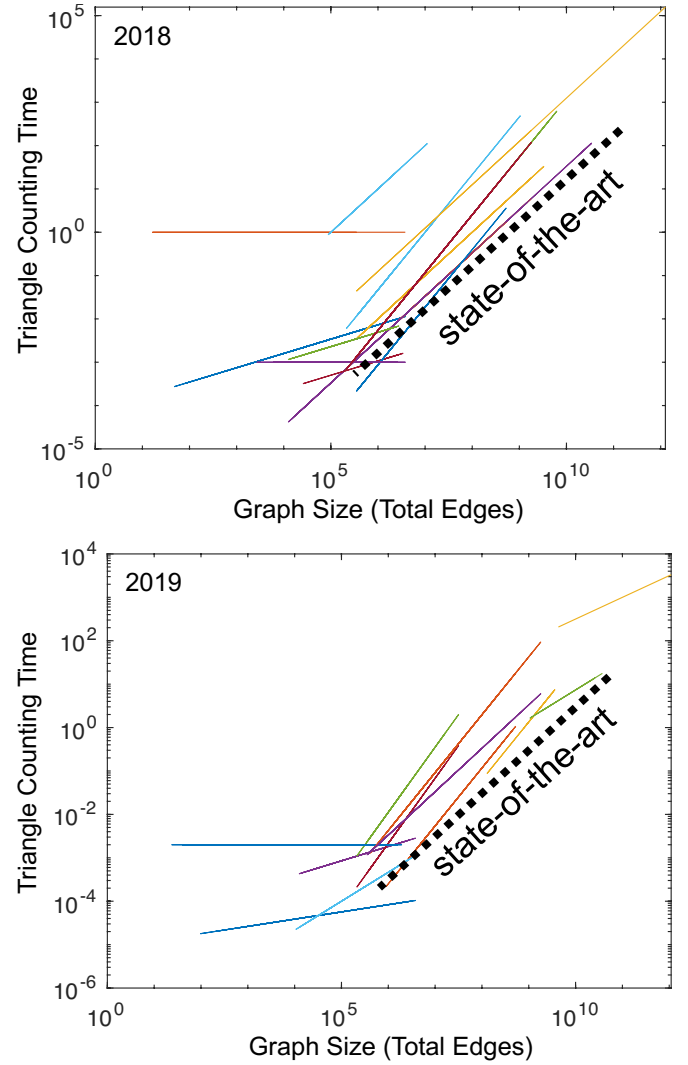


Fig. 2. Model fits of triangle execution time vs number edges for selected Graph Challenge 2018 (top) and 2019 (bottom) triangle counting submissions. State-of-the-art is denoted by the black dashed line.

execution time, T_{tri} , is a strong function of the number of edges in the graph, N_e , which improved significantly from 2017 ($T_{\text{tri}} \approx (N_e/10^8)^{4/3}$) to 2018 ($T_{\text{tri}} \approx N_e/10^9$) and remained comparable from 2018 to 2019. Graph Challenge provides a clear picture of current graph analysis systems and underscores the need for new innovations to achieve high performance on very large graphs.

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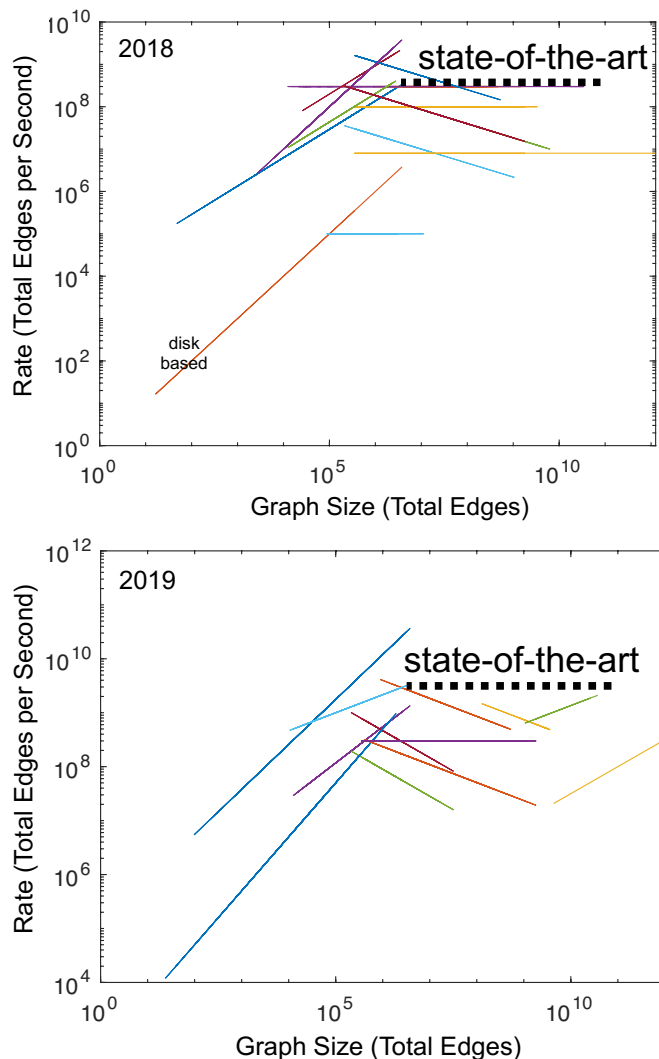


Fig. 3. Model fits of triangle execution rate vs. number edges for selected Graph Challenge 2018 (top) and 2019 (bottom) triangle counting submissions. State-of-the-art is denoted by the black dashed line.

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APPENDIX A

2018 TRIANGLE COUNTING SUBMISSIONS

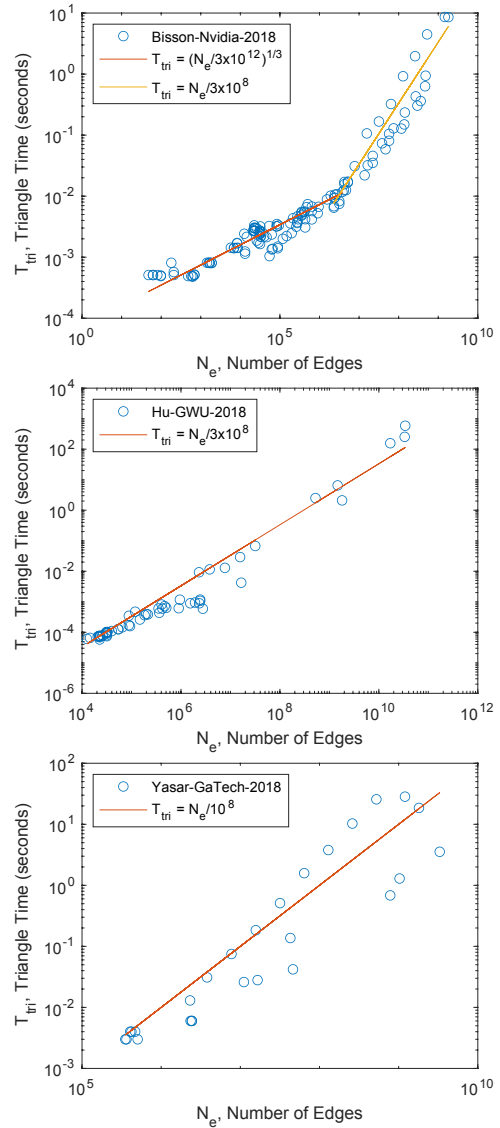


Fig. 4. Graph Challenge 2018 Champions. Triangle counting execution time vs number of edges and corresponding model fits for Bisson-Nvidia-2018 [84], Hu-GWU-2018 [85], and Yasar-GaTech-2018 [86].

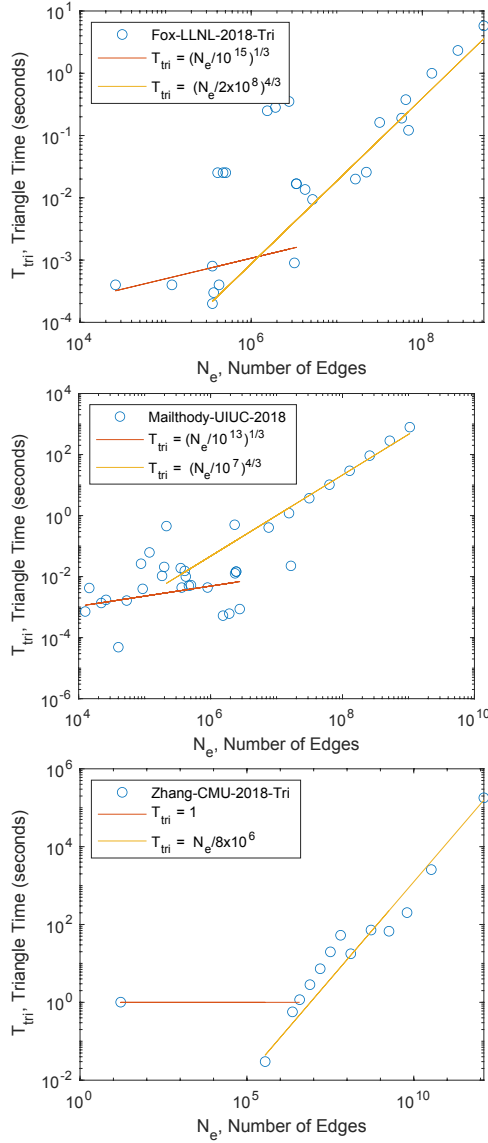


Fig. 5. Graph Challenge 2018 Finalists. Triangle counting execution time vs number of edges and corresponding model fits for Fox-LLNL-2018 [87], Mailthody-UIUC-2018 [88], and Zhang-CMU-2018 [89].

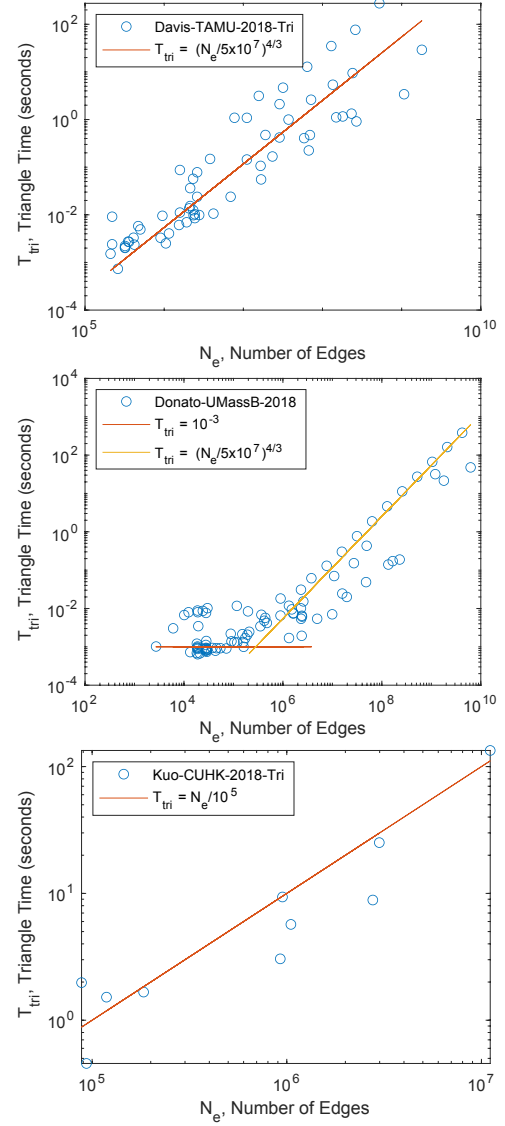


Fig. 6. Graph Challenge 2018 Innovation Award and Honorable Mentions. Triangle counting execution time vs number of edges and corresponding model fits for Davis-TAMU-2018 [90], Donato-UMassB-2018 [91], and Kuo-CUHK-2018 [92].

APPENDIX B

2019 TRIANGLE COUNTING SUBMISSIONS

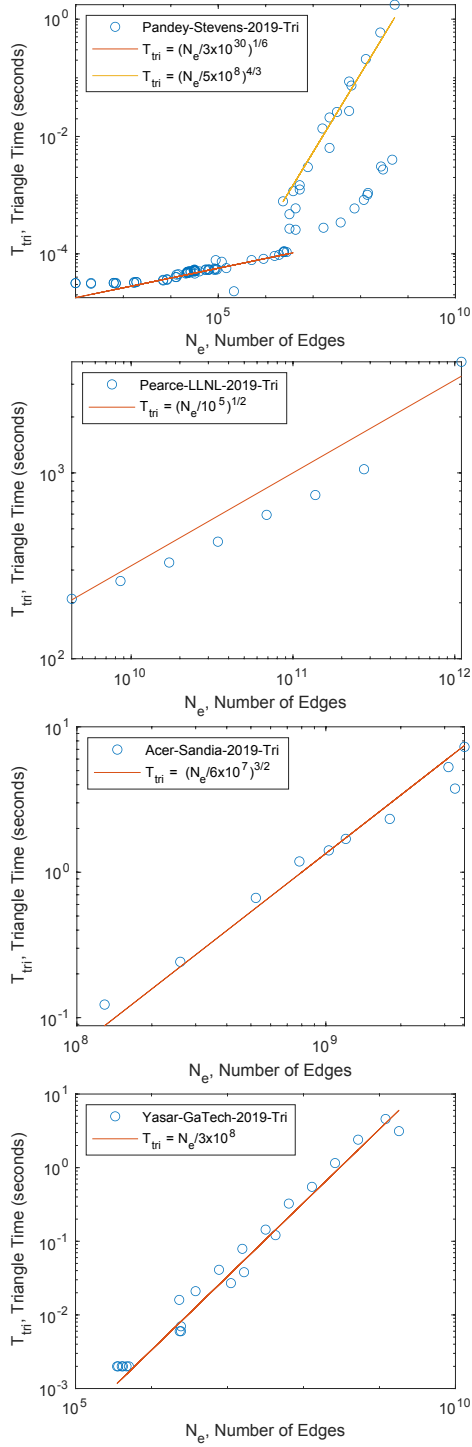


Fig. 7. Graph Challenge 2019 Champions and Innovation Awards. Triangle counting execution time vs number of edges and corresponding model fits for Pandey-Stevens-2019 [102], Pearce-LLNL-2019 [103], Acer-Sandia-2019 [104], and Yasar-GaTech-2019 [105].

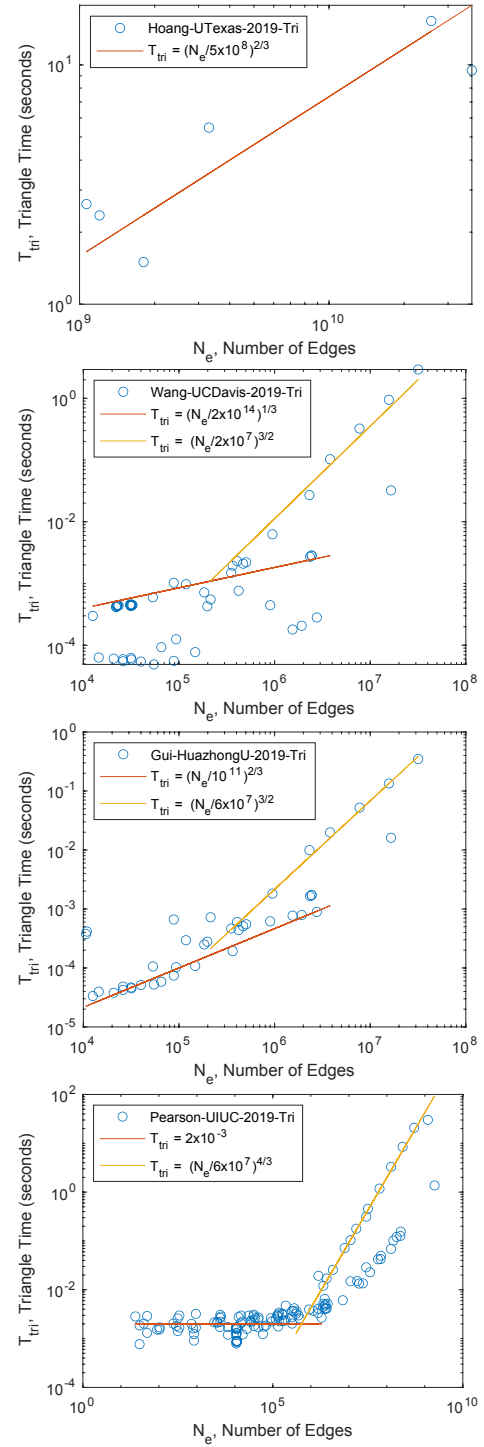


Fig. 8. Graph Challenge 2019 Student Innovation Award, Finalist, and Honorable Mentions. Triangle counting execution time vs number of edges and corresponding model fits for Hoang-UTexas-2019 [106], Wang-UCDavis-2019 [107], Gui-HuazhongU-2019 [108], and Pearson-UIUC-2019 [109].