BABU BANARASI DAS
NORTHERN INDIA INSTITUTE OF TECHNOLOGY, LUCKNOW
B. TECH.2nd YEAR–MODAL PAPER
ACADEMIC SESSION 2024-25(ODD SEMESTER)

Discrete Structure and Theory of Logic (BCS303)

Time: 3 hrs **Max Marks:70**

NOTE:Attempt all sections

SECTION A

Q01.Attempt ALL questions in brief: [07x02=14]

	Question	Marks	CO	BTL
1A	Define multiset and proper subset.	2	1	2
1B	Write the axioms of Boolean algebra.	2	2	1
1C	What are tautologies and contradictions in propositional logic?	2	3	1
1D	Define Ring in group theory?	2	4	1
1E	Write pigeonhole principle.	2	5	1
1F	"Every cyclic group is an abelian group" prove it.	2	4	2
1G	For any positive integer D_{36}, draw the Hasse diagram of (D_{36},'/').	2	2	3

SECTION B

Q02.Attempt any THREE of the following: [03x07 =21]

	Question	Marks	CO	BTL
2A	If A and B are two subsets of universal set, then prove the following: a. (A-B)=(B-A) iff A= B b. (A-B)=A iff A∩B= ø	07	1	2
2B	Show that the mapping f:R→R be defined by f(x)=ax+b, where a, b, x €R, a≠0 is invertible. Define its inverse.	07	2	3
2C	Write rules of inference in predicate logic.	07	3	1
2D	If the permutation of the elements of {1,2,3,4,5} are given by a=(1 2 3)(45), b=(1)(2)(3)(4 5), c=(152 4)(3). Find the value of x, if ax=b.	07	4	3
2E	Prove that for any connected planar graph v – e + r = 2, where v,e and r is the number of vertices, edges and regions of the graph respectively.	07	5	3

SECTION C

Q03. Attempt Any One of the Following: [01x07 =07]

	Question	Marks	CO	BTL
3A	Show that R={(a,b) l a ≡ b (mod m)} is an equivalence relation on Z. Show that if $x_1=y_1$ and $x_2=y_2$ then $(x_1+ x_2) \equiv (y_1+ y_2)$.	07	1	3
3B	In a distributive lattice, if an element has a complement then this complement is unique.	07	1	2

Q04.Attempt Any One of theFollowing: [01x07 =07]

	Question	Marks	CO	BTL
4A	Find the Sum-of-Product and Product-of-Sum expansion of the Boolean function F(x,y,z)= (x+y)z'	07	2	3
4B	Prove that the composition of functions satisfies the associative property.	07	2	2

Q05.Attempt Any One of the Following: [01x07 =07]

	Question	Marks	CO	BTL
5A	Give the symbolic form of the following statements. a. Some men are genius. b. For every x, there exists a y such that $x^2+y^2 \geq 100$. c. Given any positive integer, there is a greater positive integer. d. Everyone who likes fun will enjoy each of these plays.	07	3	1

5B	a. There are two restaurants next to each other one has signboard that says “Good food is not cheap” and other has sign board that says “Cheap food is not good”. Are the sign boards saying the same thing? b. Show that $\exists x Q(x)$ is a valid conclusion from the premises: $\forall(x)(P(x) \rightarrow Q(x))$ and $\exists x P(x)$	07	3	2

Q06.Attempt Any One of the Following: [01x07=07]

	Question	Marks	CO	BTL
6A	Show that G={(1,2, 4, 5,7,8),x_9} is cyclic. How many generators are there? What are they?	07	4	3
6B	State and prove Lagrange’s theorem for group. Is the converse true?	07	4	2

Q07.Attempt Any One of the Following: [01x07 =07]

	Question	Marks	CO	BTL
7A	Prove that for a simple graph with n vertices and k components can not have more Than (n-k)(n-k+1)/2 edges.	07	5	3
7B	Solve the following: a. How many 4 digit numbers can be formed by using the digits 2, 4, 6, 8 when repetition of digits is allowed? b. Find the minimum number of students in a class to show that five of them are born in same month. c. A connected plane graph has 10 vertices each of degree 3. Into how many regions, does a representation of this planar graph split the plane?	07	5	3

Answer of Model Paper

Discrete Structure and Theory of Logic (BCS303)

Time : 3 Hrs

Max. Mark: 70

SECTION - A

1A. Multiset :- Multisets are sets where an element can occur as a member more than once. For example,

$$A = \{a, a, a, b, b, c\}, \quad B = \{a, a, b, c, c, a\}$$

are multisets.

Proper Subset :- Any subset A is said to be proper subset of another set B if A is subset of B, but there is at least one element of B which does not belong to A i.e., if $A \subseteq B$ but $A \neq B$. It is written as $A \subset B$. For example, if

$$A = \{1, 5\}, \quad B = \{1, 5, 6\}, \quad C = \{1, 6, 5\}$$

Then A and B are both subsets of C, but A is a proper subset of C, whereas B is not a proper subset of C since $B = C$.

1B. Axioms of Boolean Algebra:

If $a, b, c \in B$, then

1. Commutative Laws:

   a) $a + b = b + a$ b) $a \cdot b = b \cdot a$

2. Distributive Law:

   a) $a + (b \cdot c) = (a + b)(a + c)$ b) $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$

3. Identity Laws:

   a) $a + 0 = a$ b) $a \cdot 1 = a$

4. Complement Laws:

   a) $a + a' = 1$ b) $a \cdot a' = 0$

1C. Tautologies :- A compound proposition that is always true for all possible truth values of its variables is called a tautology. For example-

$p \vee \sim p$ where p is a proposition.

Contradictions :- A compound proposition that is always false for all possible value of its variable is called a contradictions. For example-

$p \wedge \sim p$.

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
T	F	T	F
F	T	T	F

1D. Ring :- An algebraic structure $(R, +, \cdot)$ where R is a non-empty set with two binary operations + (addition) and (multiplication) defined on R is called a ring if the following conditions are satisfied.

1. $(R,+)$ is an abelian group. i.e., $(R,+)$ must follow -
   a) Closure rule
   b) Associative property
   c) Identity element
   d) Inverse element
   e) Commutative property
2. $(R,\cdot)$ is semigroup, i.e., $(R,\cdot)$ must follow -
   a) closure rule
   b) Associative property
3. The operation is distributive over the operation +. i.e.,
   Left distributive law : $a\cdot(b+c) = a\cdot b + a\cdot c$, $\forall a,b,c \in R$
   Right distribution law : $(b+c)\cdot a = b\cdot a + c\cdot a$, $\forall a,b,c \in R$

1E. Pigeonhole principle:

If n pigeons are assigned to m pigeonholes than atleast one pigeonhole contains two or more pigeons $(m < n)$.

1F. Let G be a cyclic group and let a be a generator of G so that

$$G = \langle a \rangle = \{a^n : n \in \mathbb{Z}\}$$

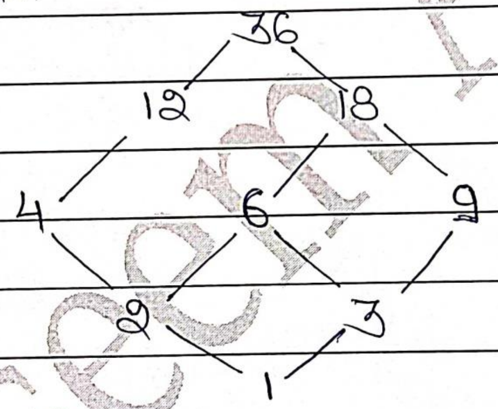
If g_1 and g_2 are two elements of G, there exist integers r and s such that $g_1 = a^r$ and $g_2 = a^s$, Then

$$g_1 g_2 = a^r a^s = a^{r+s} = a^{s+r} = a^s a^r = g_2 g_1$$

So, G is abelian.

1G. D_{36} = Divisor of 36 = $\{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

Hasse Diagram -



SECTION B

2A. a. Let $A = B$

Assume $x \in A - B$

$\Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in B$ and $x \notin A$ $[\because A = B]$

$\Rightarrow x \in B - A$

$\therefore A - B \subseteq B - A$ —— (I)

Conversely if $x \in B - A$

$\Rightarrow x \in B$ and $x \notin A$

$\Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in A - B$

$\therefore B - A \subseteq A - B$ —— (II)

from (I) & (II), $A = B \Rightarrow A - B = B - A$

Now let $A-B = B-A$

Let $x \in A-B$

$\therefore \quad x \in B-A$

Now $\quad x \in A-B \Rightarrow x \in A$ and $x \notin B$ —— (III)

and $\quad x \in B-A \Rightarrow x \in B$ and $x \notin A$ —— (IV)

equation (III) & (IV) can hold true when $A=B$.

b Let $A-B = A$

To show $A \cap B = \phi$

Let $A \cap B \neq \phi$ and let $x \in A \cap B$ and $x \notin \phi$

$\Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in (A-B)$ and $x \notin B \qquad [\because A-B = A]$

$\Rightarrow x \in A$ and $x \notin B$ and $x \in B$

$\Rightarrow x \in \phi$

which is a contradiction.

$A \cap B = \phi$

Now conversely, let $A \cap B = \phi$

To show $A-B = A$

Let $x \in A-B$

$\Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in A \qquad [as\ A \cap B = \phi]$

$\Rightarrow A-B \subseteq A$ —— (I)

conversely, let $x \in A$

$\Rightarrow x \in A$ and $x \in B \qquad [as\ A \cap B = \phi]$

$\Rightarrow x \in A-B$

$\therefore A \subseteq A-B$ —— (II)

from (I) & (II), $A-B = A$.

2B. For, if $x_1, x_2 \in R$, then

$$f(x_1) = f(x_2) \Rightarrow ax_1 + b = ax_2 + b$$
$$\Rightarrow ax_1 = ax_2$$
$$\Rightarrow x_1 = x_2$$

This proves f is one-to-one.

Again, if $y \in R$

$$y = f(x) \Rightarrow y = ax + b$$
$$\Rightarrow x = (y-b)/a$$

Thus for $x \in R$, there exists $(y-b)/a \in R$ such that

$$f\left(\frac{1}{a}(y-b)\right) = a\left(\frac{1}{a}(y-b)\right) + b = y - b + b = y$$

Hence f is one-to-one and onto therefore f^{-1} exists and it is defined by

$$f^{-1}(y) = \frac{1}{a}(y-b)$$

2C. Inference Rule –

1. Universal instantiation:

$$\frac{\forall x\, P(x)}{\therefore P(c)}$$

c is some element of the universe

2. Existential instantiation :

$$\frac{\exists x\, P(x)}{\therefore P(c)}$$

c is some element for which P(c) is true.

3. Universal generalisation :

$$\frac{P(x)}{\forall x\, P(x)}$$

x should not be free in any of the given premises.

4. Existential generalisation:

$$\frac{P(c)}{\therefore \exists x\, P(x)}$$

c is some element of the universe.

5. Universal Modus Ponens:

$$\forall x, (P(x) \longrightarrow Q(x))$$

$$\frac{P(a)}{\therefore Q(a)}$$

a is some element of universe.

6. Universal Modus Tollens:

$$\forall x\, (P(x) \longrightarrow Q(x))$$

$$\frac{\sim Q(a)}{\therefore \sim P(a)}$$

a is some element of universe.

7. Universe Hypothetical Syllogism:

$$\forall x\, (P(x) \rightarrow Q(x))$$

$$\frac{Q(a) \rightarrow R(a)}{\therefore\ P(a) \rightarrow R(a)}$$

a is some element of the universe.

2.a $\quad ax = b$

$$\Rightarrow (1\,2\,3)(4\,5)\, x = (1)(2)(3)(4\,5)$$

$$\Rightarrow \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix} x = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix} x = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix}$$

$$x = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix}$$

$$x = \begin{pmatrix} 2 & 3 & 1 & 5 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 5 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 4 & 5 \end{pmatrix}$$

2E **Euler's Formula:-**

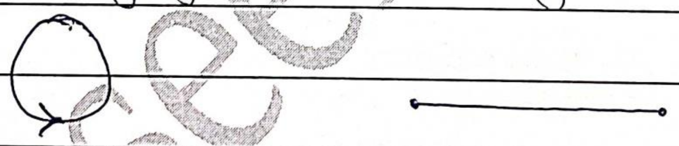
If a connected planar graph G has n vertices, e edges and r region, then $n-e+r=2$.

Proof: We prove Euler's formula by induction on e, number of edges of G.

Basis of Induction. If $e=0$, then G must have just one vertex, i.e., $n=1$ and one infinite region i.e., $r=1$. Then

$$n-e+r = 1-0+1 = 2$$

If $e=1$, then the number of vertices of G is either 1 or 2, the first possibility of occurring when the edge is a loop. These two possibility give rise to two regions and one region respectively.



$e=1, n=1$ $\qquad\qquad$ $e=1, n=2$

In the case of loop, $n-e+r = 1-1+2 = 2$ and

In the case of non-loop, $n-e+r = 2-1+1 = 2$.

Hence the result is true.

Induction hypothesis: Now we suppose that the result is true for any connected plane graph G with $e-1$ edges.

Induction step: We add one new edge K to G to form a connected supergraph of G which is denoted by $G+K$. There are following three possibilities.

i> K is a loop, in which case a new region bounded by the loop is created by but the number of vertices remain unchanged.

ii> K joins two distinct vertices of G, in which case one of the region of G is split into two, so that number of regions is increased by 1, but the number of vertices remains unchanged.

iii> K is incident with only one vertex of G on which case another vertex must be added, increasing the number of vertices by one, but leaving the number of regions unchanged.

If let n', e' and r' denote the number of vertices, edges and regions in G and n, e and r denote the same in $G+K$. Then

In case(i), $n-e+r = n'-(e'+1)+(r'+1) = n'-e'+r'$.

In case(ii), $n-e+r = n'-(e'+1)+(r'+1) = n'-e'+r'$

In case(iii), $n-e+r = (n'+1)-(e'+1)+r' = n'-e'+r'$

But by our induction hypothesis, $n'-e'+r'=2$. Thus in each case $n-e+r=2$.

Hence by mathematical induction the formula is true for all plane graph.

SECTION-C

3A. $R = \{(a,b) \mid a \equiv b \pmod m\}$

For an equivalence relation it has to be reflexive, symmetric and transitive.

Reflexive: For reflexive $\forall a \in \mathbb{Z}$ we have $(a,a) \in R$ i.e.,

$$a \equiv a \pmod m$$

$\Rightarrow a-a$ is divisible by m i.e., 0 is divisible by m

Therefore aRa, $\forall a \in \mathbb{Z}$, it is reflexive.

Symmetric : Let $(a,b) \in \mathbb{Z}$ and we have

$(a,b) \in R$ i.e. $a \equiv b \pmod{m}$

$\Rightarrow$ $a-b$ is divisible by m

$\Rightarrow$ $a-b = km$, $k \in \mathbb{Z}$

$\Rightarrow$ $b-a = (-k)m$

$\Rightarrow$ $b-a = dm$, $d \in \mathbb{Z}$

$\Rightarrow$ $b-a$ is also divisible by m

$\Rightarrow$ $b \equiv a \pmod{m} \Rightarrow (b,a) \in R$

It is symmetric.

Transitive : Let $(a,b) \in R$ and $(b,c) \in R$ then

$(a,b) \in R \Rightarrow$ $a-b$ is divisible by m

$\Rightarrow a-b = p \cdot m$, $p \in \mathbb{Z}$ —— (I)

$(b,c) \in R \Rightarrow$ $b-c$ is divisible by m

$\Rightarrow b-c = q \cdot m$, $q \in \mathbb{Z}$ —— (II)

From (I) & (II)

$a-b+b-c = (p+q)m$

$a-c = r \cdot m$, $r = (p+q) \in \mathbb{Z}$

$a-c$ is divisible by m

$a \equiv c \pmod{m}$

$(a,c) \in R$

It is transitive.

Since R is reflexive, symmetric and transitive,

Hence R is an equivalence class.

3B. Suppose that an element a has two complements b and c. Then

$a \vee b = 1$, $a \wedge b = 0$

$a \vee c = 1$, $a \wedge c = 0$

we have $b = b \wedge 1$

$= b \wedge (a \vee c)$ $\quad$ $(\because a \vee c = 1)$

$= (b \wedge a) \vee (b \wedge c)$ $\quad$ by distributive law

$= (a \wedge b) \vee (b \wedge c)$ $\quad$ by commutative law

$= 0 \vee (b \wedge c)$ $\quad$ $(\because a \wedge b = 0)$

$= (a \wedge c) \vee (b \wedge c)$

$= (a \vee b) \wedge c$

$= 1 \wedge c$

$= c$

4A. $F(x,y,z) = (x+y)\bar{z}$

x	y	z	$x+y$	z'	$(x+y)z'$
1	1	1	1	0	0
1	1	0	1	1	1
1	0	1	1	0	0
1	0	0	1	1	1
0	1	1	1	0	0
0	1	0	1	1	1
0	0	1	0	0	0
0	0	0	0	1	0

Sum-of-Product:

$$F(x,y,z) = xy\bar{z} + x\bar{y}\bar{z} + \bar{x}y\bar{z}$$

Product-of-sum:

$$F(x,y,z) = (x+y+z)(x+\bar{y}+z)(\bar{x}+y+z)(\bar{x}+\bar{y}+z)(\bar{x}+\bar{y}+\bar{z}).$$

4B. Associative law of Function Composition :

Let $f: A \to B$, $g: B \to C$ and $h: C \to D$

then $gof: A \to C$ and $hog: B \to D$

Hence $ho(gof): A \to D$ and $(hog)of: A \to D$

So $dom[ho(gof)] = dom[(hog)of]$

Let $x \in A$, $y \in B$, $z \in C$ such that $f(x) = y$ and $g(y) = z$

Then $[(hog)of](x) = (hog)[f(x)] = (hog)(y) = h[g(y)] = h(z)$ —— ①

Also $[ho(gof)](x) = [ho(gof)(x)] = h[g(f(x))] = h[g(y)] = h(z)$ —— ⑪

from ① & ⑪ we get

$$[(hog)of](x) = [ho(gof)](x) \quad \forall x \text{ in } A$$

$$(hog)of = ho(gof)$$

Hence composition of function satisfy the associative property.

5A. i) Let $M(x)$: x is a man

$G(x)$: x is genious

Hence, the given statement can be written symbolically,

$(\exists x)(M(x) \wedge G(x))$

ii) $(\forall x)(\exists y)(x^2 + y^2 \geq 100)$

iii) Let x and y be two variables belong to $\mathbb{Z}$. The given statement can be written as :

For all x, there exists a y such that y is greater than x.

Let $G(x, y)$: x is greater than y

The given statement is.

$(\forall x)(\exists y)\, G(y, x)$

iv) Let $L(x)$: x likes fun.

$P(x)$: x is play

$E(x, y)$: x will enjoy y.

The given statement in symbolic form is

$$(\forall x)(\forall y)[(L(x) \wedge P(y) \Rightarrow E(x,y))]$$

5B. a) Let p: Food good

q: Food is cheap.

Then the statement "Good food is not cheap' is written as

$$p \to \sim q$$

and the statement 'Cheap food is not good" is written as

$$q \to \sim p$$

The truth table for the statements are given below.

p	q	$\sim p$	$\sim q$	$p \to \sim q$	$q \to \sim p$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

Since two columns of of truth table are same i.e. $p \to \sim q$ and $q \to \sim p$, so they are saying the same thing.

b)

1.	$\exists x\, P(x)$	Premise (Given)
2.	$P(x)$	Existential Instantiation on 1
3.	$\forall x\,(P(x) \to Q(x))$	Premise (Given)
4.	$P(y) \to Q(y)$	Universal Instantiation on 3
5.	$Q(y)$	Modus Ponens on 2 & 4
6.	$\exists x\, Q(x)$	Existential Generalization on 5

Hence $\exists x\, Q(x)$ is a valid conclusion.

6A. Composition table for $\times_9$ is

$\times_9$	1	2	4	5	7	8
1	1	2	4	5	7	8
2	2	4	8	1	5	7
4	4	8	7	2	1	5
5	5	1	2	7	8	4
7	7	5	1	8	4	2
8	8	7	5	4	2	1

From the table 1 is identity element of group G.

$$2^1 = 2 \equiv 2 \bmod 9$$
$$2^2 = 4 \equiv 4 \bmod 9$$
$$2^3 = 8 \equiv 8 \bmod 9$$
$$2^4 = 16 \equiv 7 \bmod 9$$
$$2^5 = 32 \equiv 5 \bmod 9$$
$$2^6 = 64 \equiv 1 \bmod 9$$

Therefore, 2 is generator of G. Hence G is cyclic.

Similarly, 5 is also generator of G. (5 is inverse of 2)

Hence there are two generators 2 and 5.

6B. Lagrange's Theorem :-

The order of each sub-group of a finite group G is a divisor of the order of the group G.

Proof :- Let H be any sub-group of order m of a finite group G of order n. We consider the left coset decomposition of G relative to H.

We first show that each coset aH consists of m different elements.

Let $H = \{h_1, h_2, h_3, \ldots, h_m\}$.

Then $ah_1, ah_2, ah_3, \ldots, ah_m$ are the m members of aH, all distinct.

For, we have

$$ah_i = ah_j \Rightarrow h_i = h_j, \text{ by cancellation law in } G.$$

Thus every left coset of H in G has m distinct elements.

Since G is a finite group, the number of distinct left cosets will also be finite.

Let it be k. Then the union of these k-left cosets of H in G is equal to G.

$$G = a_1H \cup a_2H \cup a_3H \cup \ldots \cup a_kH.$$

$$\therefore\ O(G) = O(a_1H) + O(a_2H) + O(a_3H) + \cdots + O(a_kH)$$

(since two distinct left cosets are mutually disjoint)

$$n = m + m + m + \ldots + m \text{ (k-times)}$$

$$n = mk \Rightarrow k = \frac{n}{m}$$

$$\therefore\ k = \frac{O(G)}{O(H)}$$

Thus order of each subgroup of a finite group G is a divisor of the order of the group.

7A Let the number of vertices in each of the k-components of a graph G be $n_1, n_2, n_3, \ldots, n_k$ then we get

$$n_1 + n_2 + n_3 + \cdots + n_k = n \quad \text{where } n_i \geq 1 \ (i = 1, 2, \ldots k)$$

Now $\displaystyle\sum_{i=1}^{k} (n_i - 1) = \sum_{i=1}^{k} n_i - \sum_{i=1}^{k} 1 = n - k$

$$\therefore \left(\sum_{i=1}^{k} (n_i - 1)\right)^2 = n^2 + k^2 - 2nk$$

or $\displaystyle\sum_{i=1}^{k} (n_i - 1)^2 + 2 \sum_{i=1}^{k} \sum_{\substack{j=1 \\ i \neq j}} (n_i - 1)(n_j - 1) = n^2 + k^2 - 2nk$

or $\sum_{i=1}^{k} (n_i - 1)^2 + 2\,(\text{Non-negative terms}) = n^2 + k^2 - 2nk$

$(\because n_i - 1 \geq 0,\ n_j - 1 \geq 0)$

or $\sum_{i=1}^{k} (n_i - 1)^2 \leq n^2 + k^2 - 2nk$

or $\sum_{i=1}^{k} n_i^2 + \sum_{i=1}^{k} 1 - 2\sum_{i=1}^{k} n_i \leq n^2 + k^2 - 2nk$

or $\sum_{i=1}^{k} n_i^2 + k - 2n \leq n^2 + k^2 - 2nk$

or $\sum_{i=1}^{k} n_i^2 - n \leq n^2 + k^2 - 2nk - k + 2n$

$= n(n-k+1) - k(n-k+1)$

$= (n-k)(n-k+1)$ ——— (1)

We know that the maximum number of edges in the ith component of G $= {}^{n_i}C_2 = \frac{n_i(n_i - 1)}{2}$

Therefore, the maximum number of edges in G is

$$\frac{1}{2}\sum_{i=1}^{k} n_i(n_i - 1) = \frac{1}{2}\left[\sum_{i=1}^{k} n_i^2 - \sum_{i=1}^{k} n_i\right]$$

$$= \frac{1}{2}\left(\sum_{i=1}^{k} n_i^2 - n\right)$$

$$\leq \frac{1}{2}(n-k)(n-k+1) \text{ by eqn } (1)$$

7B. a) When repetition is allowed :

The thousands place can be filled by 4 ways.

The hundreds place can be filled by 4 ways

The tens place can be filled by 4 ways.

The unit place can be filled by 4 ways.

$\therefore$ Total number of 4 digit number $= 4 \times 4 \times 4 \times 4$

$= 256$.

b). Five of them are born in same month, so $n = ?$, $m = 12$.

$$5 = \left\lfloor \frac{n-1}{m} \right\rfloor + 1$$

$$4 = \frac{n-1}{12}$$

$$\therefore\ 48 = n-1$$

$$n = 49$$

$\therefore$ 49 students are there to show that at 5 of them are born in same month.

c). Here $n=10$ and degree of each vertex is 3.

$$\sum \deg(v) = 3 \times 10 = 30$$

but $\sum \deg(v) = 2e \Rightarrow 30 = 2e$

$$\Rightarrow e = 15$$

by Euler's formula, we have

$$n - e + r = 2$$

$$10 - 15 + r = 2$$

$$\Rightarrow r = 7.$$