DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -2 : AKTU Syllabus

Functions

- *Definition of Function*
- *Classification of functions*
- *Operations on functions*
- *Growth of function*

Boolean Algebra:

- Introduction
- Axioms and Theorems of Boolean Algebra
- Algebraic manipulation of Boolean Expressions
- Simplification of Boolean functions
- Karnaugh maps

DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -1 : Functions and Boolean Algebra

Lecture - 01

Today's Target

- *Definition of Function*
- *Classification of functions*
- PYQ
- DPP

$R : A \rightarrow B$

Function

$R \subseteq A \times B$

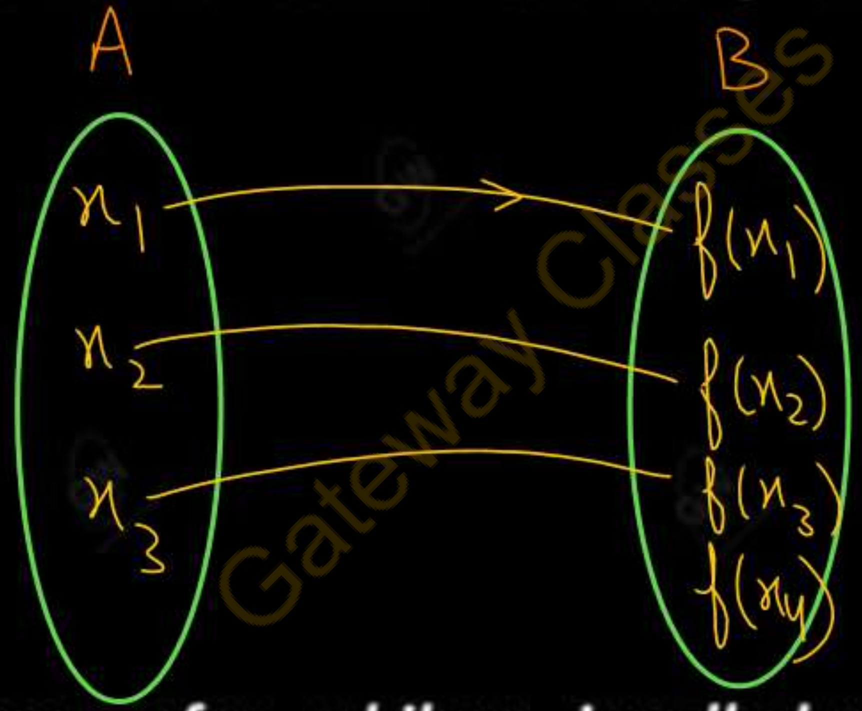
Let A and B are two non-empty sets

A function $\boxed{f : A \rightarrow B}$ is a relation / Mapping from set A to set B, such that each and every element of Set A is related to exactly one element of set B.

OR

Those relations in which each and every element of set A has unique image in Set B

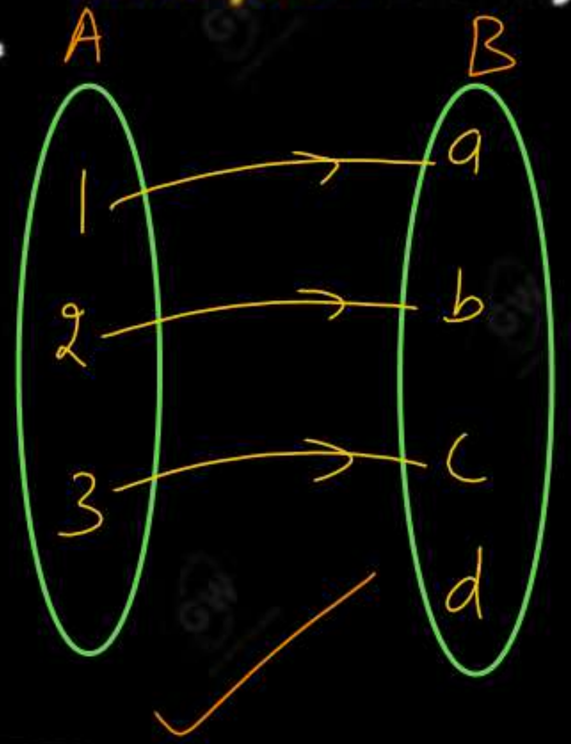
Conclusion : Every function is a relation but every relation is not a function .



Note: *$f(x_1)$ is called image of x_1 while x_1 is called pre-image of $f(x_1)$*

Example : Identify Function Mapping

(i)



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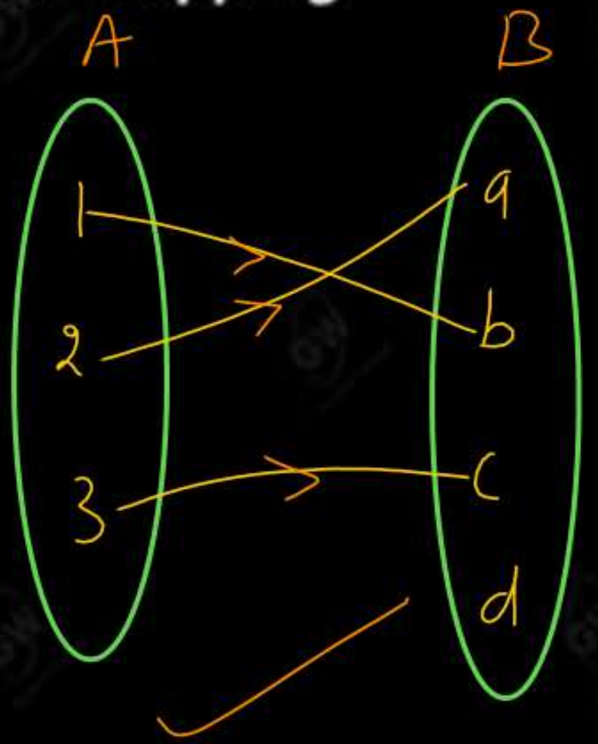

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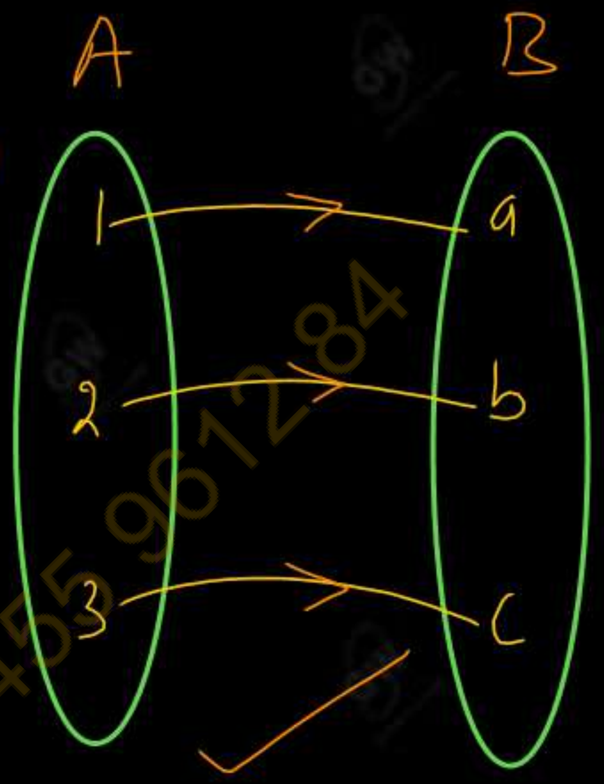

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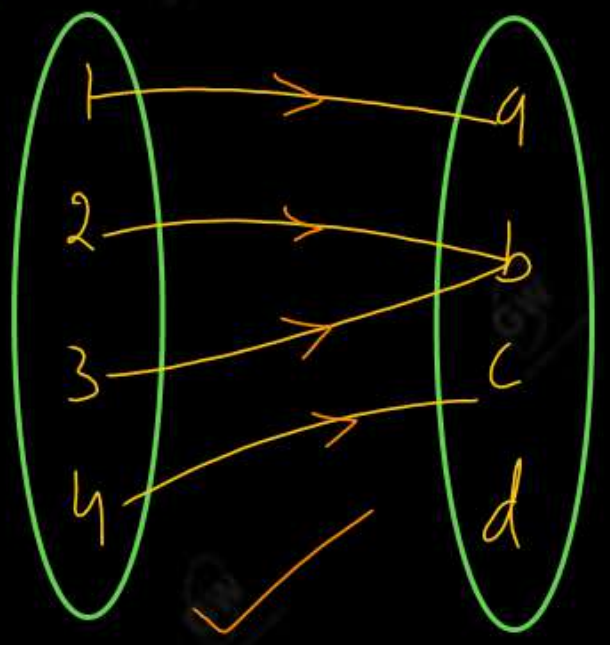

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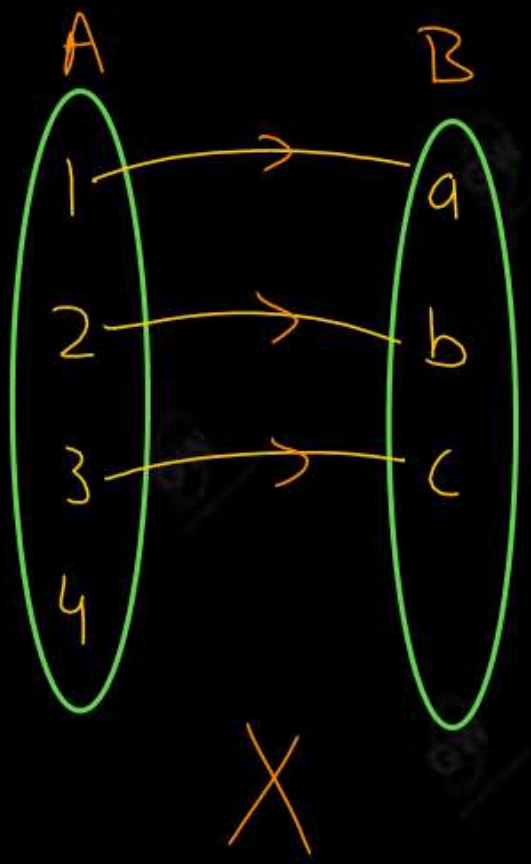

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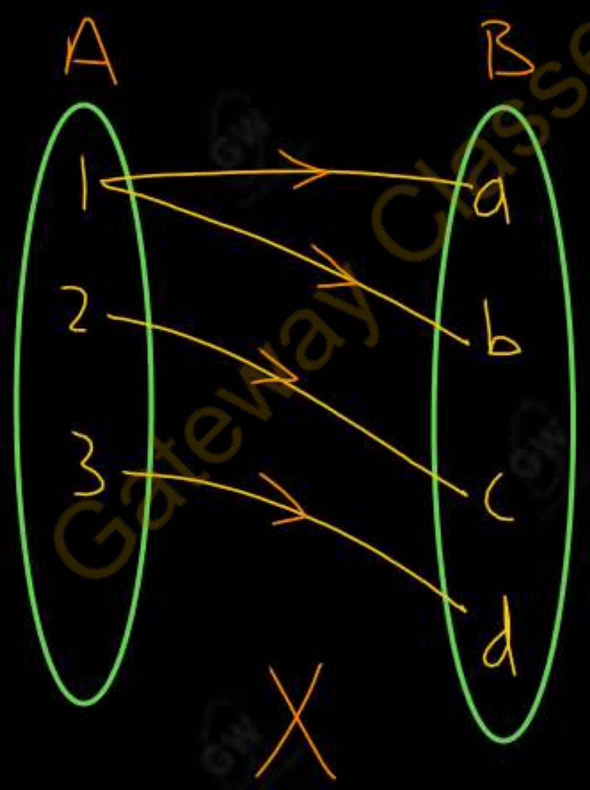

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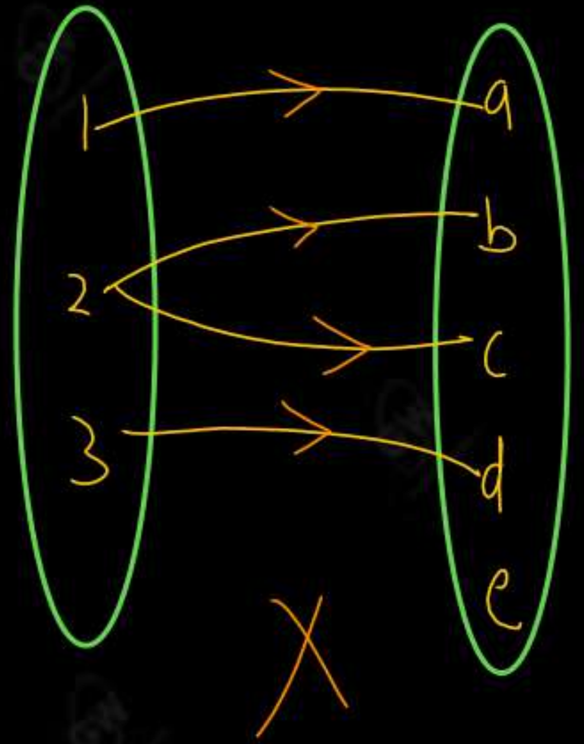

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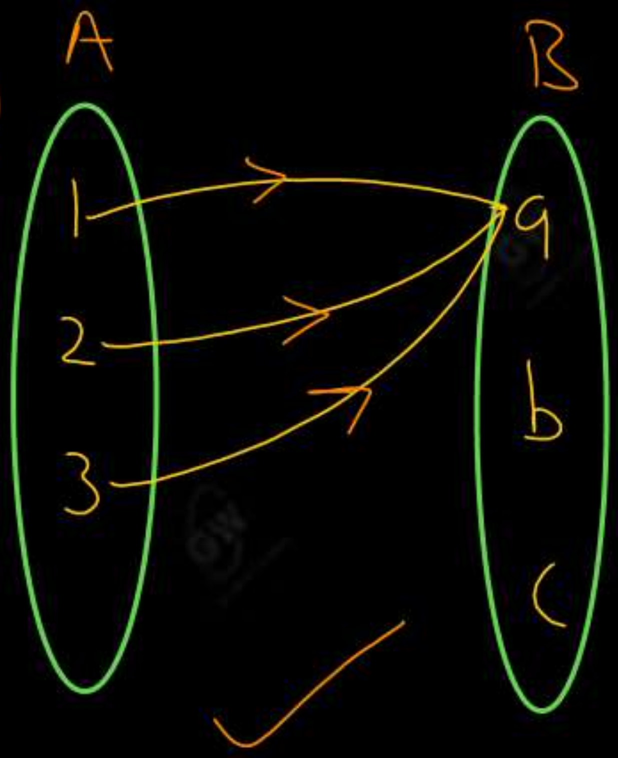
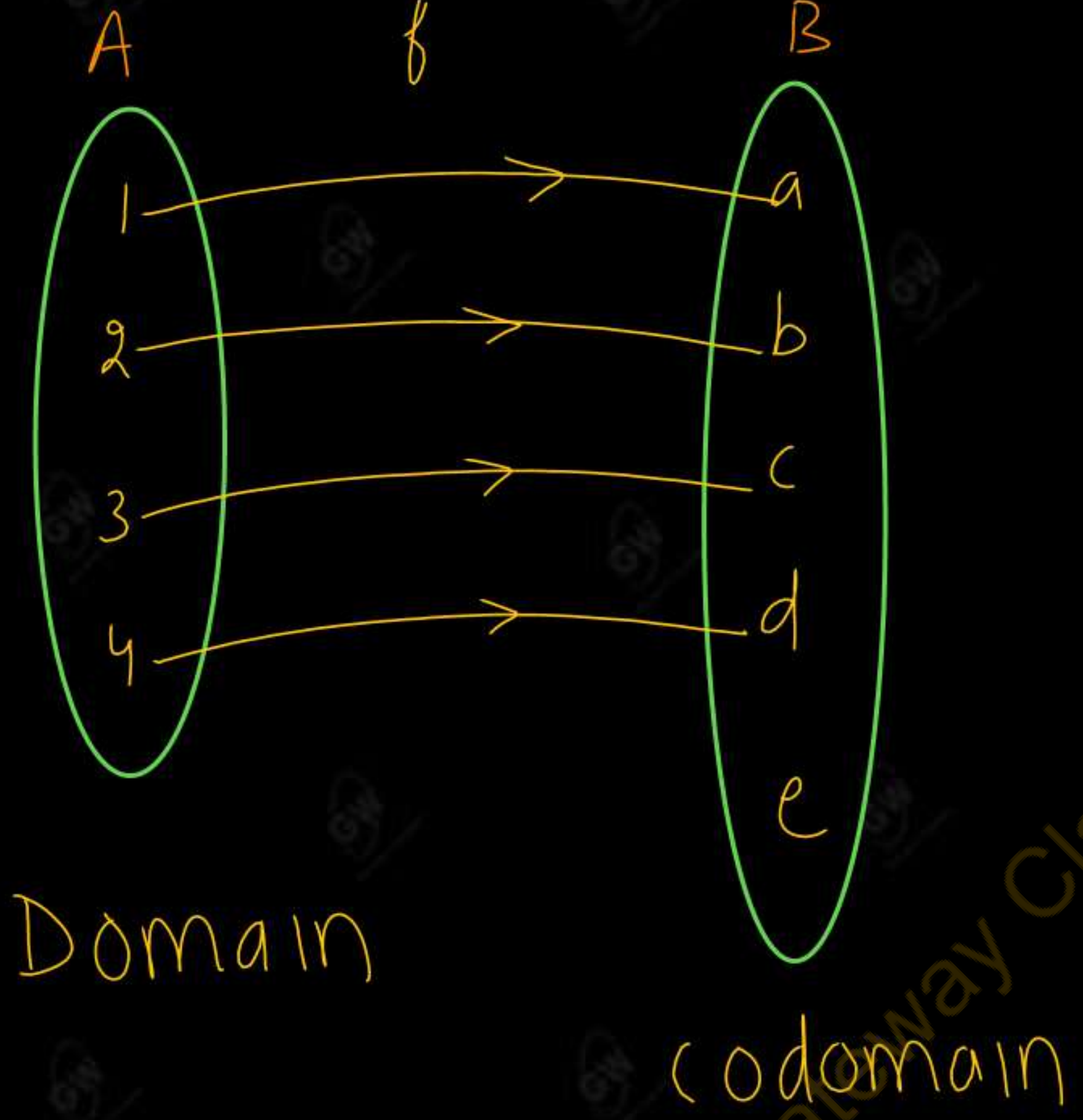


Domain, Co-domain and Range of function



$Dom(f) = \{1, 2, 3, 4\}$

$codomain = \{a, b, c, d, e\}$

$Range(f) = \{a, b, c, d\}$

$f : \underbrace{R}_{\text{Domain}} \longrightarrow \underbrace{R}_{\text{codomain}}$

Note:- *If* $n(A) = m$ *and* $n(B) = n$*, then total number of possible function from A to B*

number of functions $= (n)^m$ $\quad (5)^4 =$

Classifications of function (Mapping)

(1) ONE –ONE FUNCTION (or injective function)

(2) MANY-ONE FUNCTION

(3) ONTO FUNCTION (or surjective function)

(4) INTO FUNCTION

Imp (5) ONE-ONE ONTO FUNCTION (Bijective function)

(6) One-one into function

(7) Many-one onto function

(8) Many-one into function

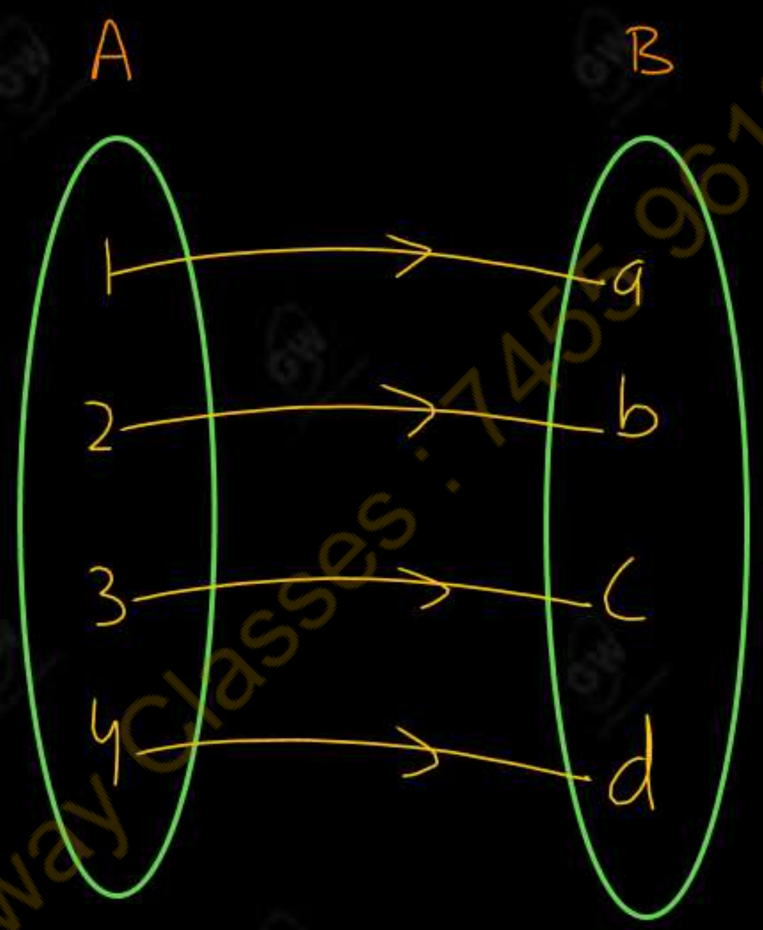
ONE – ONE FUNCTION OR One to one function OR Injective function

A function $f : A \to B$ is said to be one-one function if different elements of A have different images in B .

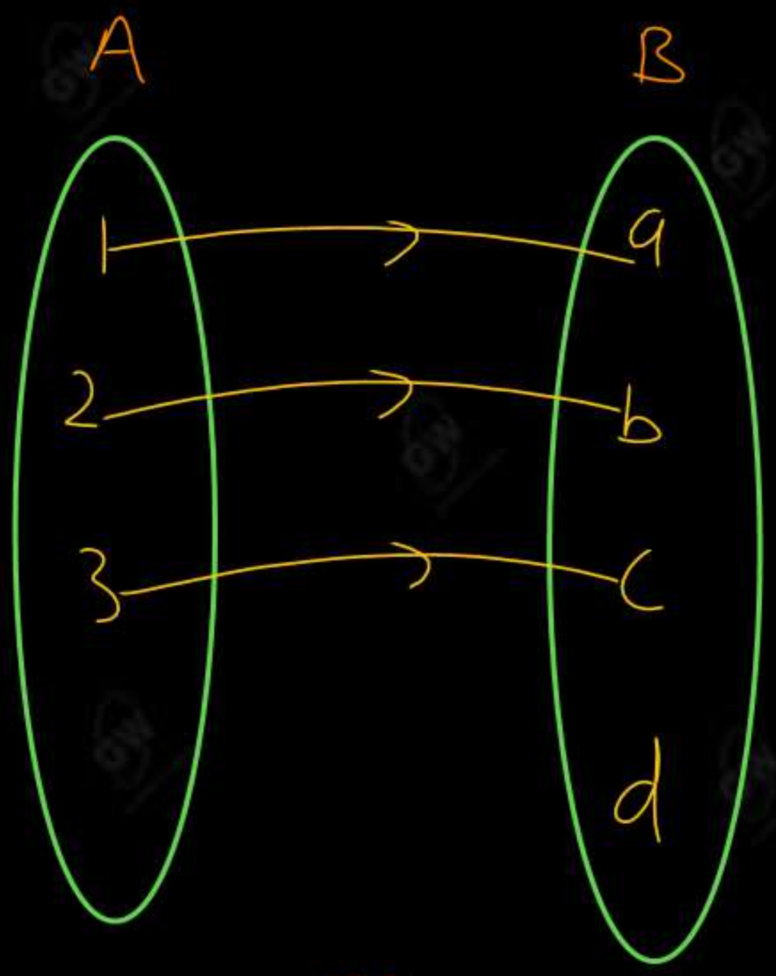
Thus, $f : A \to B$ is one-one if and only if

$$f(x_1) = f(x_2)$$

$$\Rightarrow \boxed{x_1 = x_2} \quad \forall\ x_1\ \ x_1\ \in A$$



(i)



(ii)

Note:- If $n(A) = m$ and $n(B) = n$, then

Total number of one-one function = ${}^nP_m = \frac{n!}{(n-m)!}$

Q. 1 :- *Show that the function* $f: N \to N$, *given by* $f(x) = 2x + 3$ *is one – one*

$$f(x) = 2x + 3$$

Let $x_1, x_2 \in N$

Put $f(x_1) = f(x_2)$

$$2x_1 + \cancel{3} = 2x_2 + \cancel{3}$$

$$\cancel{2}x_1 = \cancel{2}x_2$$

$$\boxed{x_1 = x_2}$$

Hence, $f(x)$ is a one-one function

Q.2:- *Check whether the function* $f(x) = x^2 - 1$ *is injective or not for* $f: R \to R$ AKTU : 2020-21

$$f(x) = x^2 - 1$$

Let $x_1, x_2 \in R$

Put $f(x_1) = f(x_2)$

$$x_1^2 - \cancel{1} = x_2^2 - \cancel{1}$$

$$x_1^2 = x_2^2$$

$$x_1^2 - x_2^2 = 0$$

$$(x_1 - x_2)(x_1 + x_2) = 0$$

$$x_1 = x_2 \text{ or } x_1 = -x_2$$

Hence, $f(x)$ is not injective

Q.3:- *Check whether the function* $f(x) = x^3 + 2$ *is one – one or not for* $f: R \to R$.

$f(x) = x^3 + 2$

Let $x_1, x_2 \in R$

Put $f(x_1) = f(x_2)$

$x_1^3 + \cancel{2} = x_2^3 + \cancel{2}$

$x_1^3 = x_2^3$

$\boxed{x_1 = x_2}$

Hence, $f(x)$ is one-one function

MANY – ONE FUNCTION

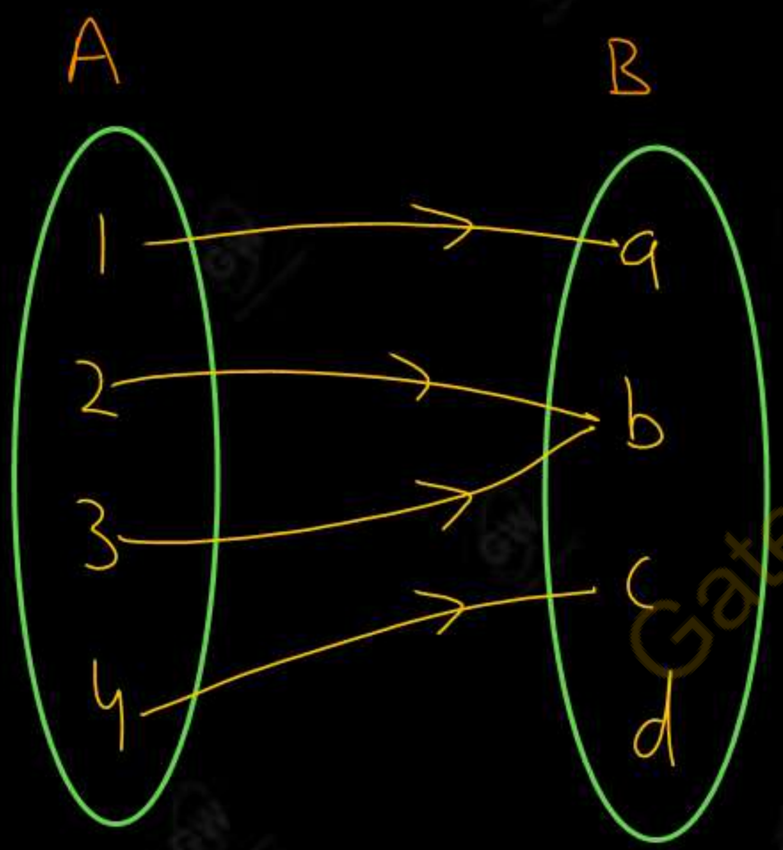
A function $f: A \to B$ is said to be a many – one function if two or more elements of set A have same image in B.

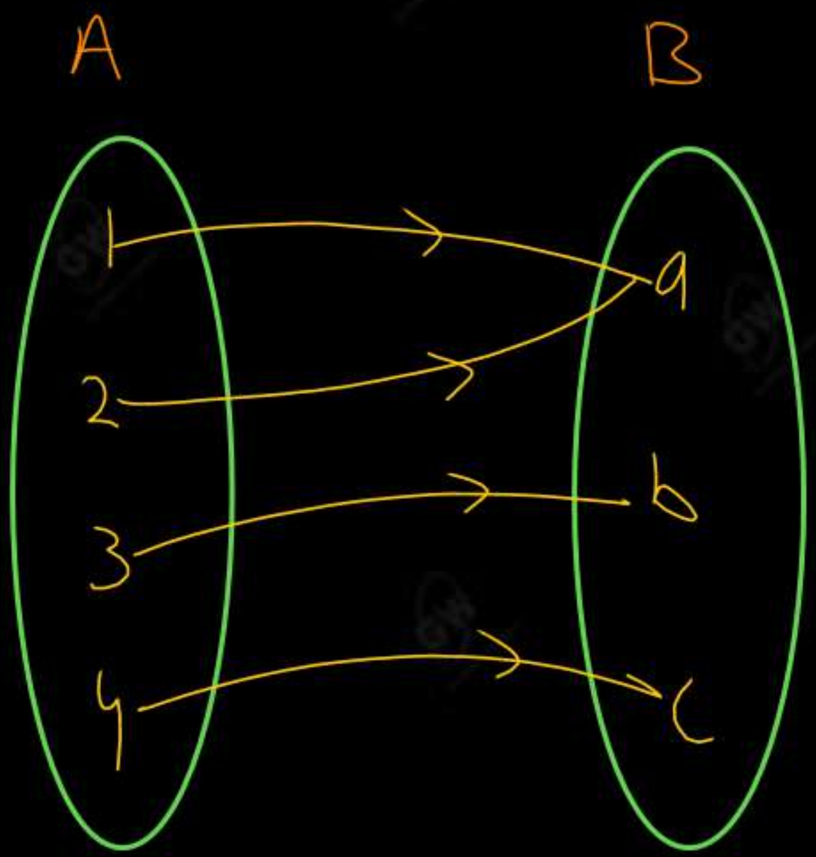
Thus, $f : A \to B$ is Many one if and only if

$$f(x_1) = f(x_2)$$

$$\Rightarrow x_1 \neq x_2 \quad \forall\ x_1,\ x_1\ \in A$$

In other words, $f : A \to B$ is Many one if it is not one – one function



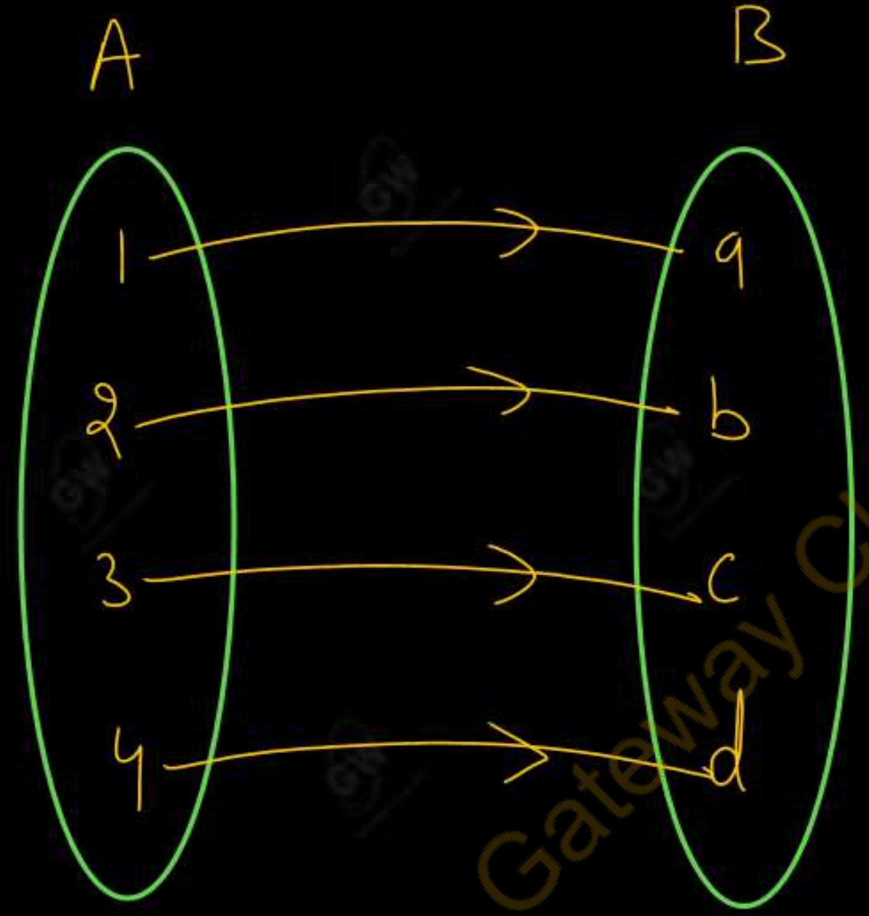


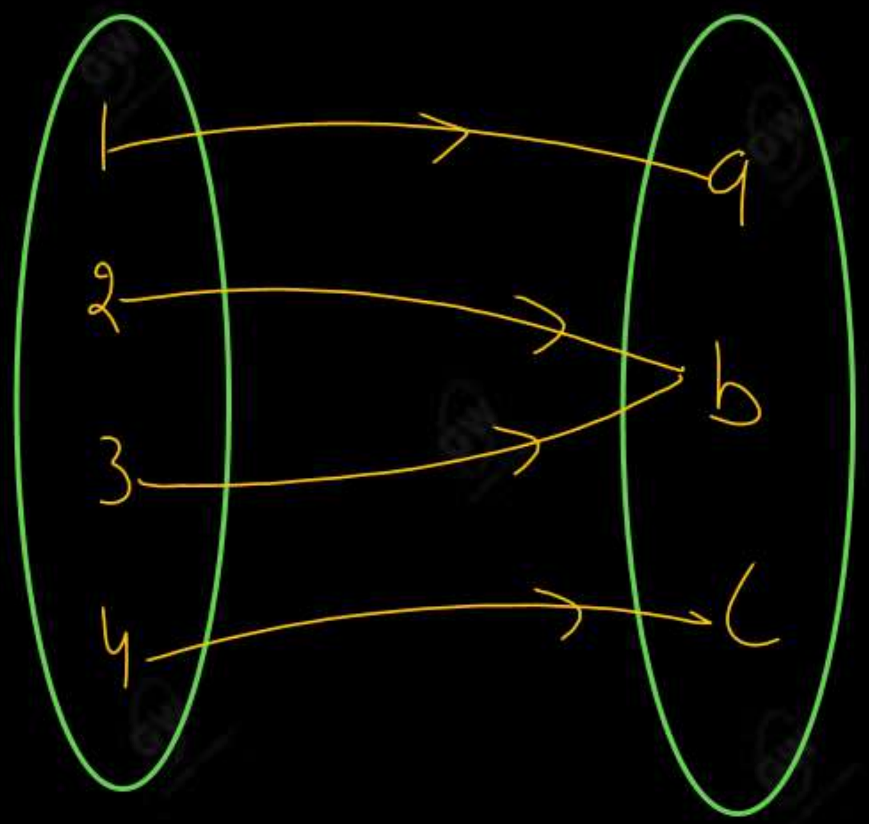
ONTO FUNCTION (or Surjective function)

A function $f: A \to B$ is said to be an ONTO function if every element of set B have pre-image in set A

i.e. $\boxed{\text{Range of function} = \text{codomain of function}}$

Example





Q.4:- Show that the function $f : R \to R^+$ defined by $f(x) = x^2 \ \forall \ x \in R$ is an Onto function

$f(x) = x^2$

$y = x^2$

$x^2 = y$

$x = \sqrt{y}$

$\Rightarrow y \in R^+$

Hence

$\boxed{\text{Range of function} = \text{codomain of function}}$

$\Rightarrow f(x)$ is an ONTO Function

Q.5:- Discuss the Surjectivity of the following functions

(i) $f : R \to R$ given by $f(x) = x^3 + 2$ for $x \in R$

(ii) $f : R \to R$ given by $f(x) = x^2 + 2$ for $x \in R$

(i) $f(x) = x^3 + 2$

$y = x^3 + 2$

$y - 2 = x^3$

$x^3 = y - 2$

$x = (y-2)^{1/3}$

$\Rightarrow y \in R$

$\boxed{\text{Range of } f = \text{codomain of } f}$

Hence

$f(x)$ is a surjective function

(ii) $f(x) = x^2 + 2$

$y = x^2 + 2$

$x^2 = y - 2$

$x = \sqrt{y-2}$

$\Rightarrow y \geq 2$

$\boxed{\text{Range of } f \neq \text{codomain of } f}$

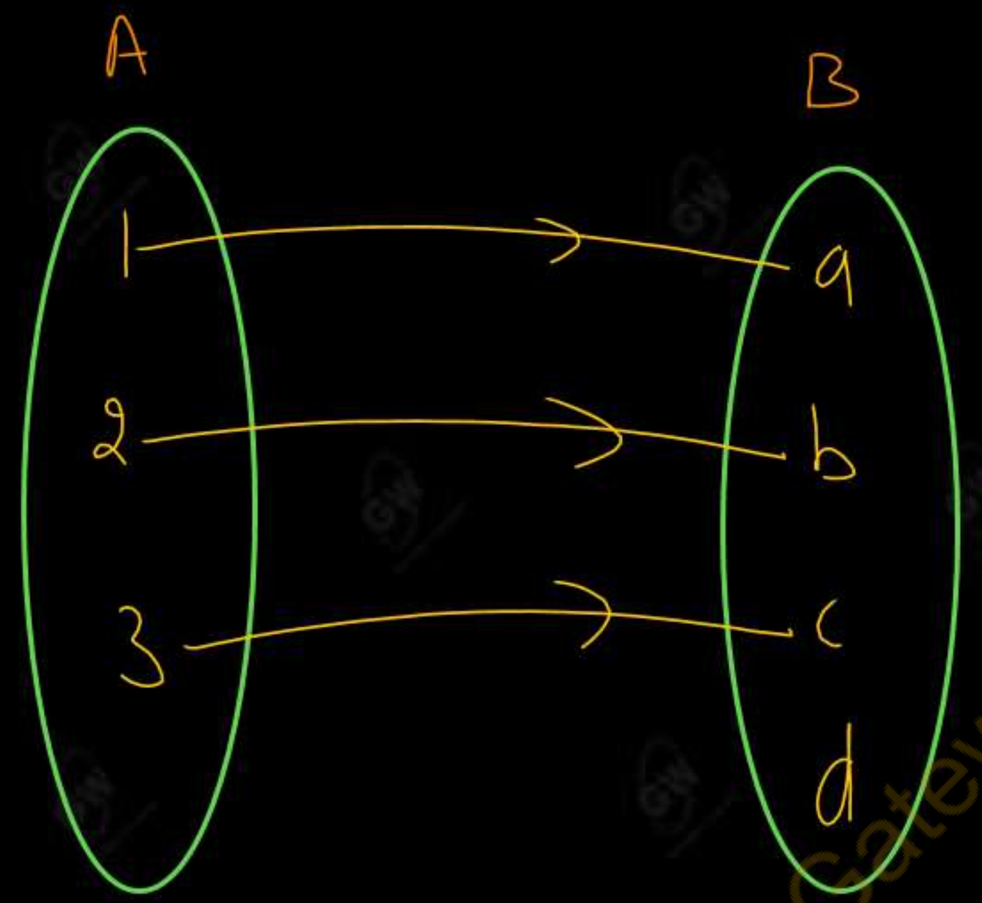
Hence, function is not surjective

INTO FUNCTION :

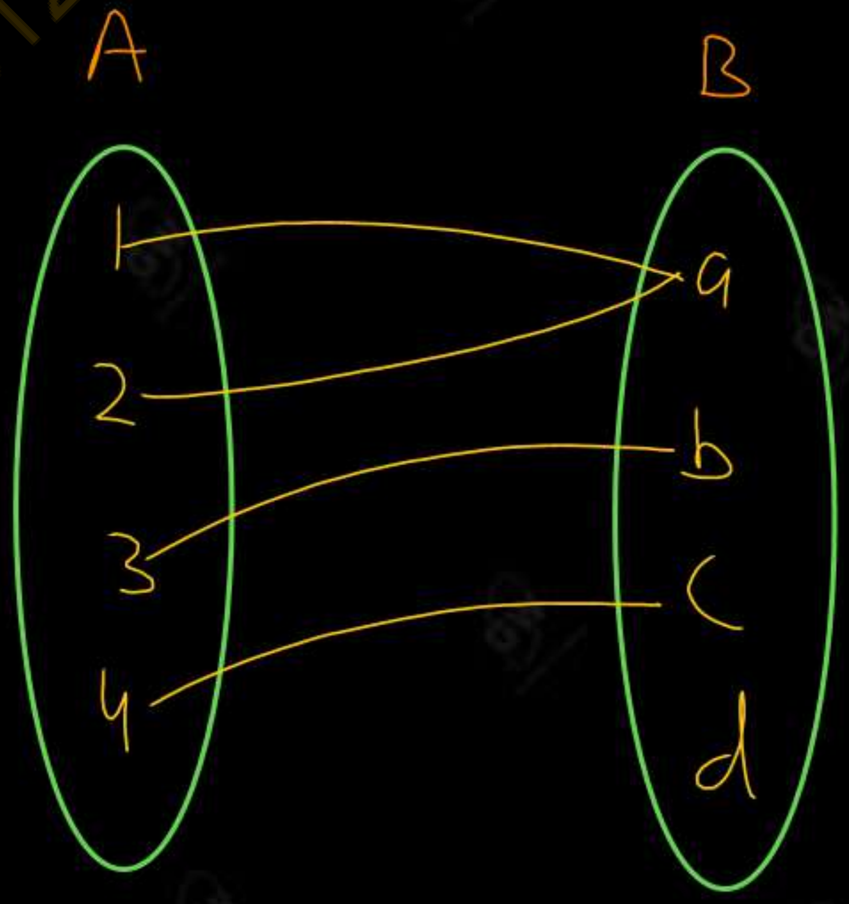
A function $f : A \to B$ is an Into function if there exist an element in B having no pre-image in A.

Thus, $f : A \to B$ is an Into function if it is not Onto function

Example



(i)



(ii)

ONE -ONE ONTO FUNCTION (Bijective function)

A function $f : A \rightarrow B$ is Bijective if it is

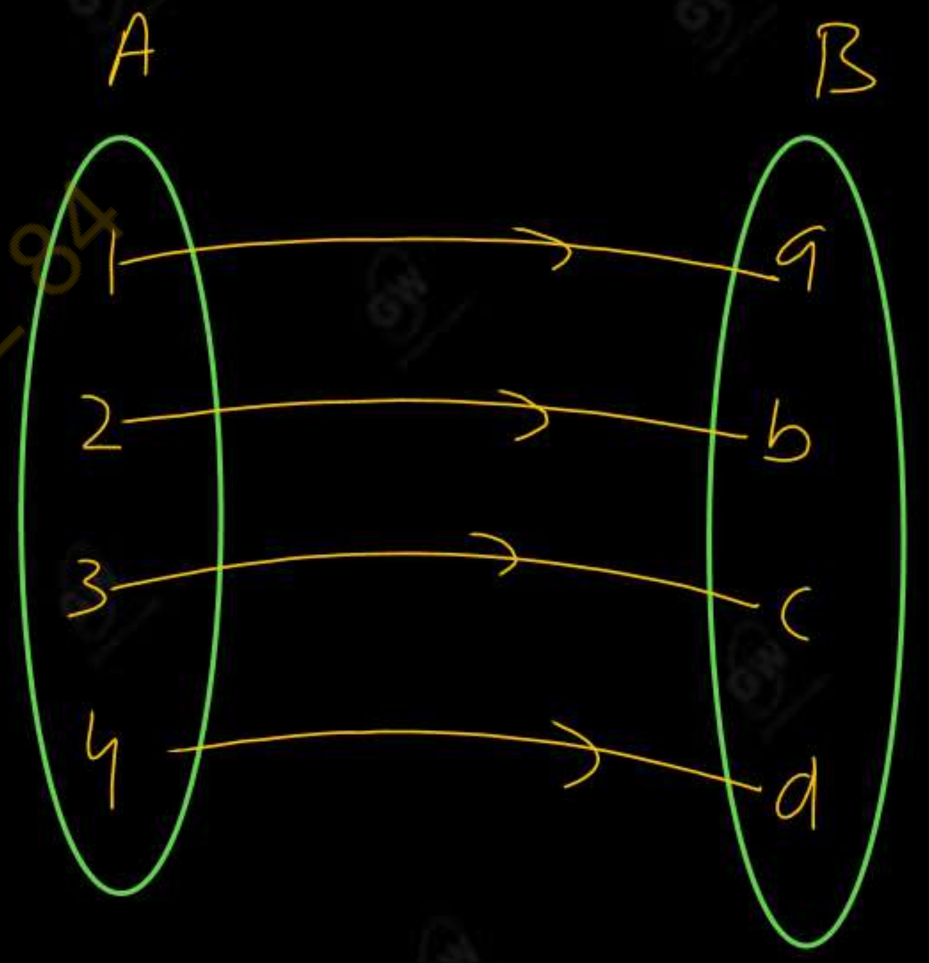
(i) One – One Function

$$f(x_1) = f(x_2)$$

$$\Rightarrow x_1 = x_2$$

(ii) Onto Function

For all $y \in B$, there exist $x \in A$ such that $f(x) = y$



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Q.6:- Check the Injectivity or Surjectivity of the following function

(i) $f: N \to N$ given by $f(x) = x^2$

(ii) $\boxed{f: Z \to Z \text{ given by } f(x) = x^2}$ HW

(i) $f(x) = x^2$

Let $x_1, x_2 \in N$

Put $f(x_1) = f(x_2)$

$x_1^2 = x_2^2$

$\boxed{x_1 = x_2}$

Hence

$f(x)$ is injective

Again

$f(x) = x^2$

$y = x^2$

$x^2 = y$

$x = \sqrt{y}$

$\boxed{\text{Range}(f) \neq \text{codomain}(f)}$

$f(x)$ is surjective

Q.7:- Show that the function $f : \underbrace{R-\{3\}} \rightarrow \underbrace{R-\{1\}}$ given by $f(x)=\frac{x-2}{x-3}$ is a bijection

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$$f(x) = \frac{x-2}{x-3}$$

Let $x_1, x_2 \in R-\{3\}$

Put $f(x_1) = f(x_2)$

$$\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

$$(x_1-2)(x_2-3) = (x_1-3)(x_2-2)$$

$$\cancel{x_1x_2} - 3x_1 - 2x_2 + \cancel{6} = \cancel{x_1x_2} - 2x_1 - 3x_2 + \cancel{6}$$

$$-3x_1 - 2x_2 = -2x_1 - 3x_2$$

$$+x_1 = +x_2$$

$$\boxed{x_1 = x_2}$$

$f(x)$ is injective

Again

$$f(x) = \frac{x-2}{x-3}$$

$$y = \frac{x-2}{x-3}$$

$$y(x-3) = x-2$$

$$xy - 3y = x - 2$$

$$xy - x = 3y - 2$$

$$x(y-1) = 3y-2$$

$$x = \frac{3y-2}{y-1}$$

$\Rightarrow$ Range $= R-\{1\}$

$\Rightarrow$ $\boxed{\text{Range}(f) = \text{codomain } f}$

Hence

$f(x)$ is surjective

since $f(x)$ is injective as well as surjective

$\Rightarrow$ $f(x)$ is bijective

Discrete Mathematics by Gulshan Sir

DPP- 1

Topic : *Classification of function*

Q.1. *Show that the function* $f : Z \to Z$ *defined by* $f(x) = x^2 + 1$ *for all* $x \in Z$ *is a many – one function.*

Q.2. *Show that* $f : R \to R$, *given by* $f(x)$ *an + b where a, b* $\in R, a \neq 0$ *is a bijection*

Q.3. *Show that* $f : R \to R$, *idefined as* $f(x) = x^3$ *is a bijection.*

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Thank You

DISCRETE STRUCTURES & THEORY OF LOGICS
(Discrete Mathematics)

UNIT - 2: Functions and Boolean Algebra

Lecture - 02

Today's Target

- *Inverse of a Function*
- PYQ
- DPP

ONE – ONE FUNCTION

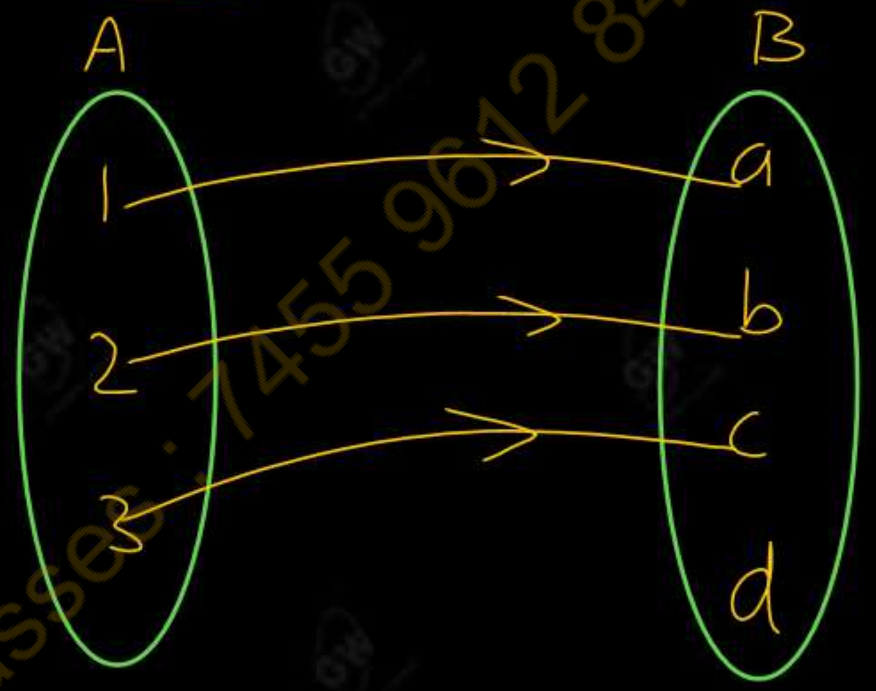
Each

A function $f : A \to B$ is said to be one-one function if different elements of A have different images in B .

Thus, $f : A \to B$ is one-one if and only if

$$f(x_1) = f(x_2)$$

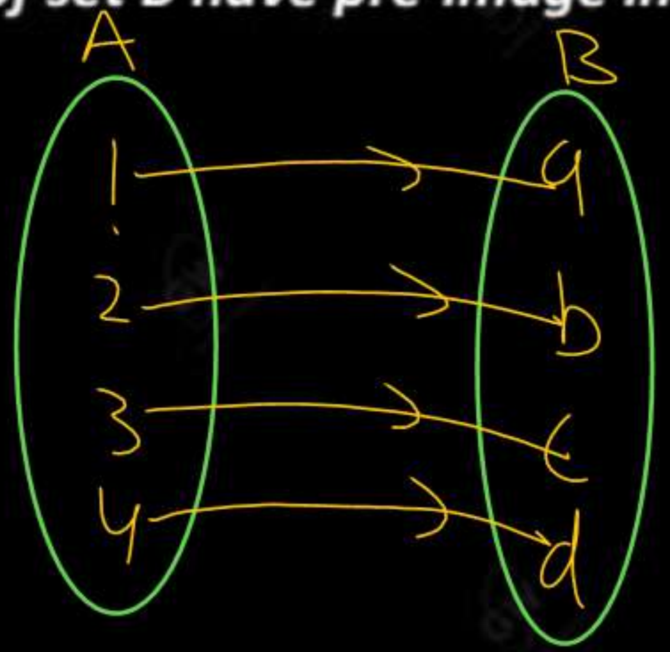
$$\Rightarrow x_1 = x_2 \quad \forall\ x_1\ \ x_1\ \in A$$



ONTO FUNCTION (or Surjective function)

A function $f\colon\ A \to B$ is said to be an ONTO function if every element of set B have pre-image in set A

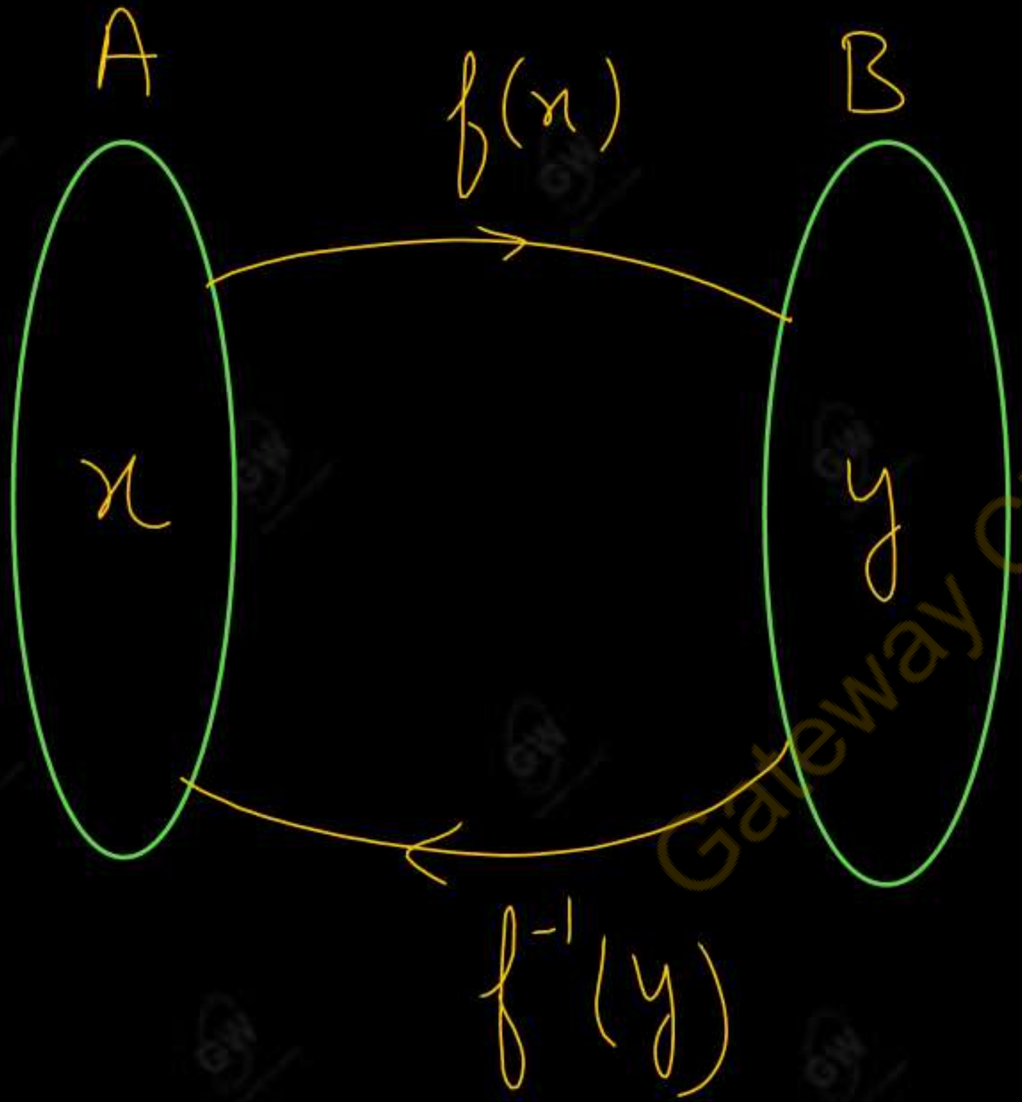
i.e. Range of function = codomain of function



Inverse of a Function

A function $\boxed{f : A \longrightarrow B}$ *is called invertible function if it is one one onto function and its inverse is given as*

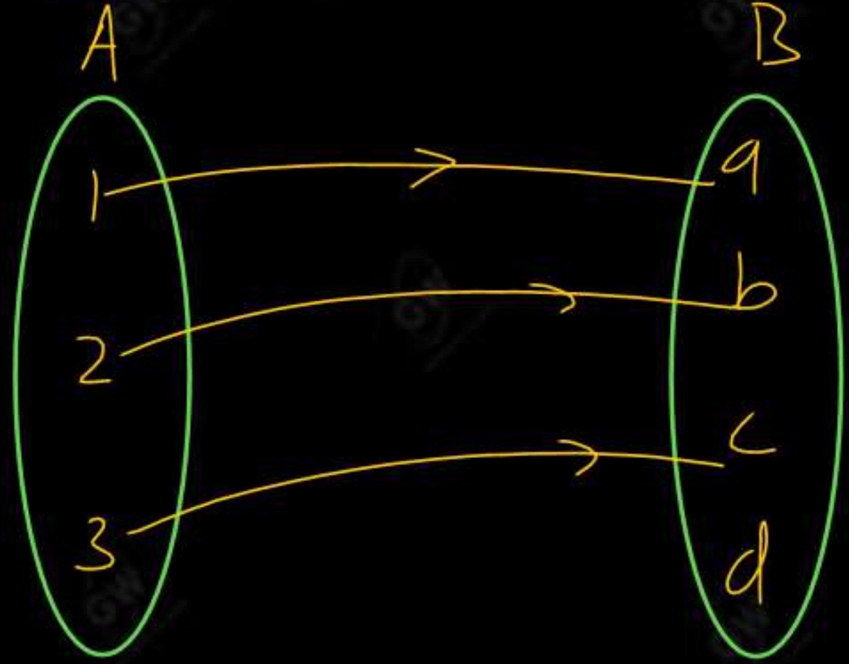
$$\boxed{f^{-1} : B \rightarrow A}$$



If $f(x) = y$ then

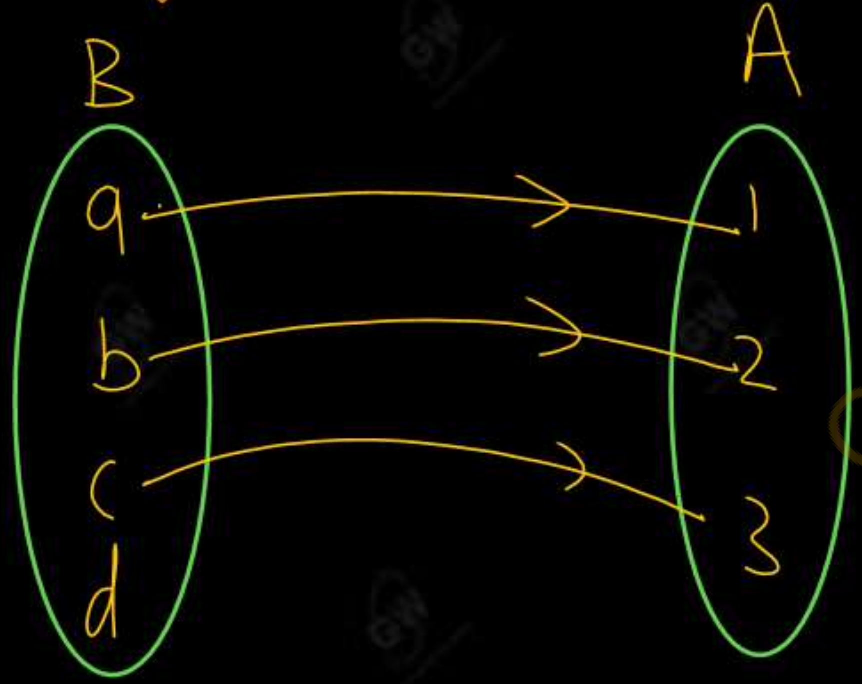
$$f^{-1}(y) = x$$

Example $f: A \longrightarrow B$



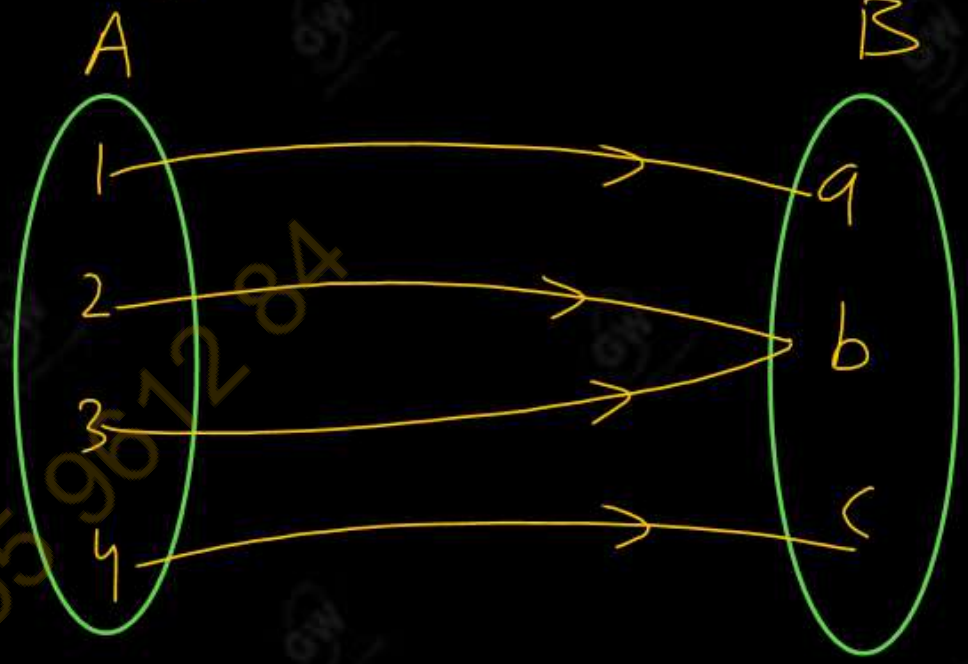
one-one
into

$f^{-1}: B \longrightarrow A$ ✗



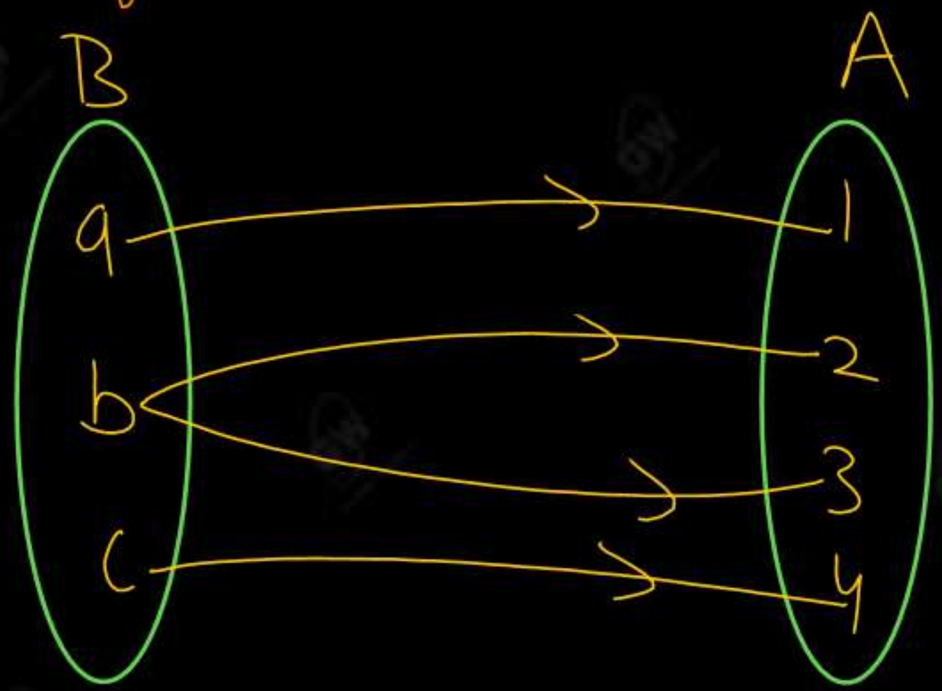
It is not a function

$f: A \longrightarrow B$



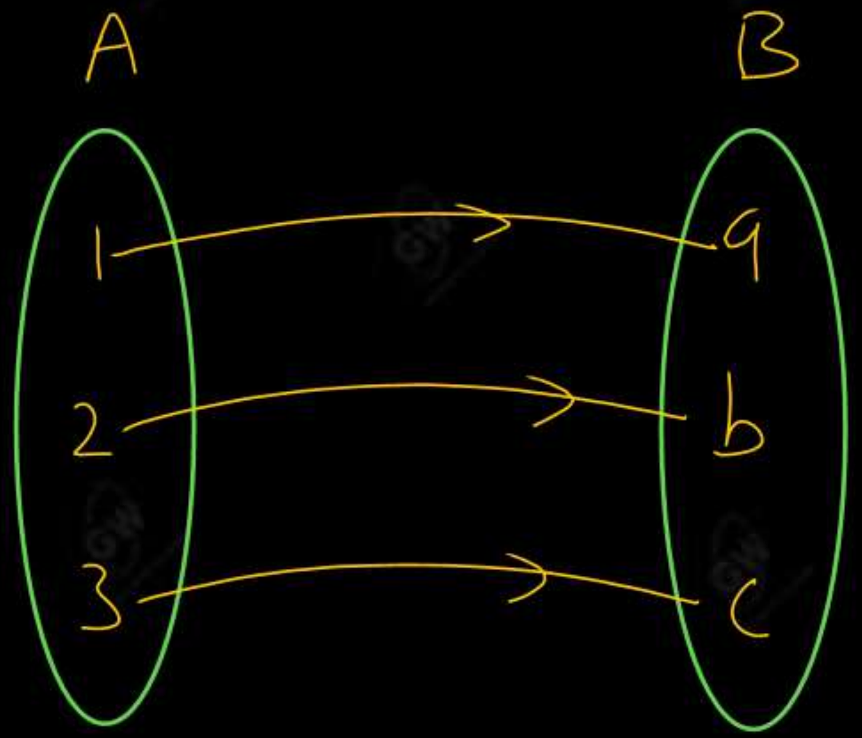
Many-one
onto

$f^{-1}: B \longrightarrow A$ ✗



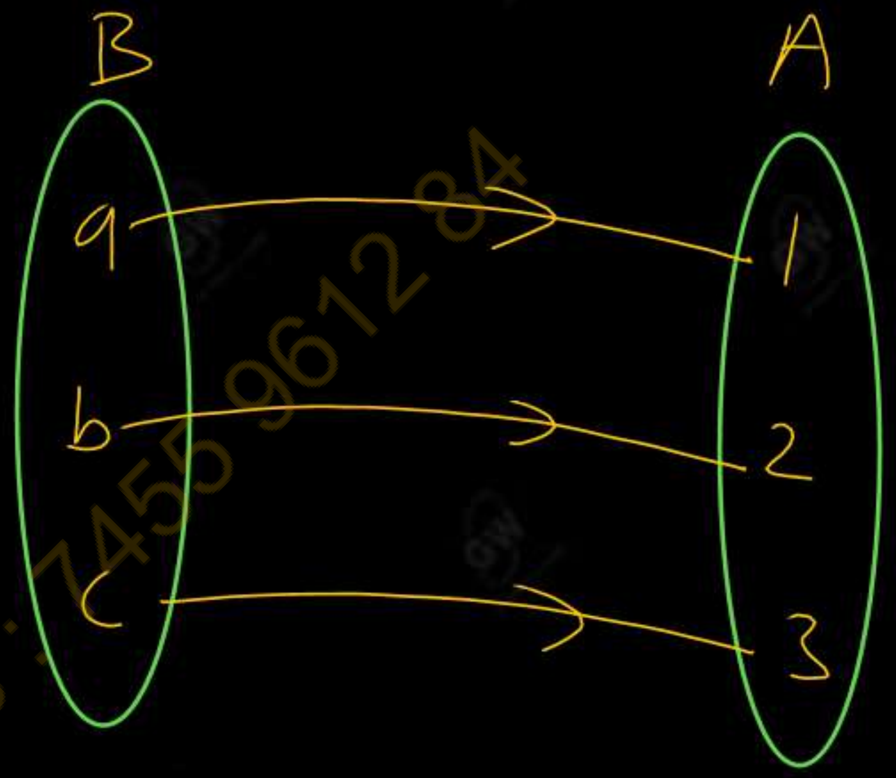
It is not a function

$f : A \longrightarrow B$



one-one onto

$f^{-1} : B \longrightarrow A$



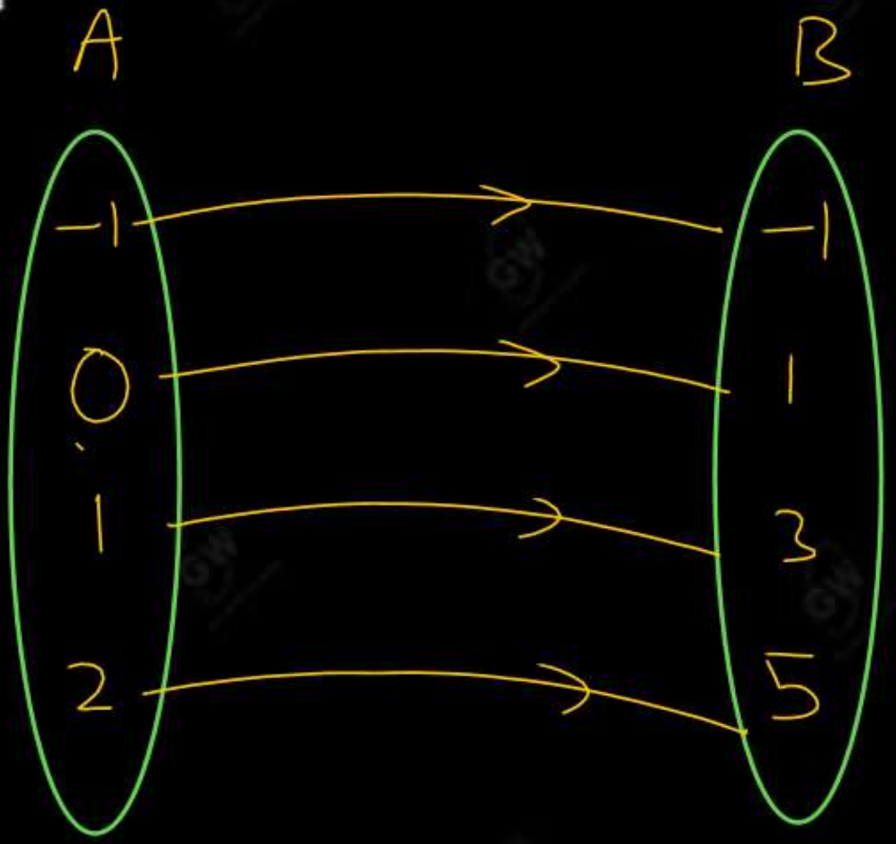
one-one onto

Conclusion : (i) *A function $f : A \longrightarrow B$ is invertible if it is one one onto function*

(ii) If f : $A \to B$ is ONE-ONE ONTO, then $f^{-1} : B \to A$ is also ONE-ONE ONTO

(iii) If f : $A \to B$ is ONE-ONE ONTO function, then $f^{-1} : B \to A$ is a unique function

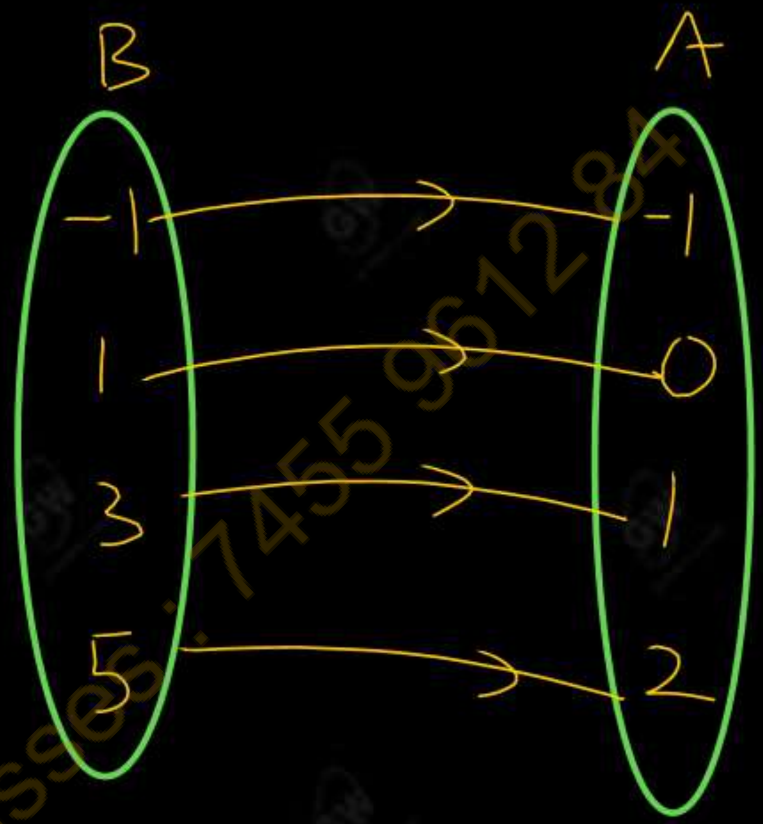
Example: $f(x) = 2x + 1$



$$y = 2x + 1$$

$$2x = y - 1$$

$$x = \frac{y-1}{2}$$



$$\boxed{f^{-1}(y) = \frac{y-1}{2}}$$

Steps to find INVERSE

(i) *A Put $f(x) = y$, where $y \in B$ and $x \in A$.*

(ii) Solve *$f(x) = y$ to obtain x in terms of y.*

(iii) Replace x by $f^{-1}(y)$ *to obtain the required inverse of f*

Q.1:- Let $f : R \to R$ is defined by $f(x) = 3x - 7$. Show that f is invertible and hence find the inverse of f.

$f(x) = 3x - 7$

Let $x_1, x_2 \in R$

Put $f(x_1) = f(x_2)$

$3x_1 - \cancel{7} = 3x_2 - \cancel{7}$

$\cancel{3}x_1 = \cancel{3}x_2$

$\boxed{x_1 = x_2}$

Hence

$f(x)$ is one-one function

Again

$f(x) = 3x - 7$

Put $f(x) = y$

$y = 3x - 7$

$3x = y + 7$

$\boxed{x = \frac{y+7}{3}}$ — (1)

$\Rightarrow y \in R$

For all $y \in R$, $x \in R$

$\Rightarrow \boxed{\text{Range} = \text{Codomain}}$

Hence $f(x)$ is an onto function

$\Rightarrow f(x)$ is one-one onto

Replace x by $f^{-1}(y)$ in (1)

$\boxed{f^{-1}(y) = \frac{y+7}{3}}$

Q.2:- Show that $f : R - \{0\} \to R - \{0\}$ given by $f(x) = \frac{1}{x}$, is invertible and find f^{-1}.

$$f(x) = \frac{1}{x}$$

Let $x_1, x_2 \in R - \{0\}$

Put $f(x_1) = f(x_2)$

$$\frac{1}{x_1} = \frac{1}{x_2}$$

$$\boxed{x_1 = x_2}$$

$\Rightarrow f(x)$ is one-one

Again

$$f(x) = \frac{1}{x}$$

$$y = \frac{1}{x}$$

$$x = \frac{1}{y} \quad \text{---} (1)$$

$y \in R - \{0\}$

For all $y \in R - \{0\}$, $x \in R - \{0\}$

$$\boxed{\text{Range } f = \text{codomain } f}$$

$\Rightarrow f(x)$ is an onto function

$\Rightarrow f(x)$ is one-one onto

Replace x by $f^{-1}(y)$ in (1)

$$\boxed{f^{-1}(y) = \frac{1}{y}}$$

Q.3:- Is the function $f : N \to N$ defined by $f(x) = x^2,\ x \in N$ is invertible

$f(x) = x^2$

Let $x_1, x_2 \in N$

Put $f(x_1) = f(x_2)$

$x_1^2 = x_2^2$

$\Rightarrow \boxed{x_1 = x_2}$

$\Rightarrow f(x)$ is one-one

$f(x) = x^2$

Put $f(x) = y$

$y = x^2$

$x^2 = y$

$x = \sqrt{y}$

For all $y \in N$

$x \notin N$

$\Rightarrow \boxed{\text{Range } f \neq \text{codomain } f}$

$\Rightarrow f(x)$ is not onto

Hence

$f(x)$ is not invertible

Q.4:- *Let a function is defined as $f : R - \{3\} \rightarrow R - \{1\}$, $f(x) = (x-2)/(x-3)$, then show that f is a bijective function and also compute the inverse of f. Where R is a set of real numbers.*

$$f(x) = \frac{x-2}{x-3}$$

Let $x_1, x_2 \in R - \{3\}$

Put $f(x_1) = f(x_2)$

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

$$\cancel{x_1 x_2} - 3x_1 - 2x_2 + \cancel{6} = \cancel{x_1 x_2} - 3x_2 - 2x_1 + \cancel{6}$$

$$-3x_1 - 2x_2 = -3x_2 - 2x_1$$

$$-3x_1 + 2x_1 = -3x_2 + 2x_2$$

$$+x_1 = +x_2$$

$$\boxed{x_1 = x_2}$$

$\Rightarrow f(x)$ is one-one

Again

$$f(x) = \frac{x-2}{x-3}$$

$$y = \frac{x-2}{x-3}$$

$$xy - 3y = x - 2$$

$$xy - x = 3y - 2$$

$$x(y-1) = 3y - 2$$

$$x = \frac{3y-2}{y-1} \quad \text{— (1)}$$

$y \in R - \{1\}$

For all $y \in R - \{1\}$

$x \in R - \{3\}$

$\boxed{\text{Range of function} = \text{Codomain of function}}$

$\Rightarrow f(x)$ is onto

$\Rightarrow f(x)$ is one-one onto function

Replace x by $f^{-1}(y)$ in (1)

$$\boxed{f^{-1}(y) = \frac{3y-2}{y-1}}$$

Q.5:- Find the inverse of function $f(x) = \frac{2x}{x-1}$

$$f(x) = \frac{2x}{x-1}$$

Put $f(x) = y$

$$y = \frac{2x}{x-1}$$

$$y(x-1) = 2x$$

$$xy - y = 2x$$

$$xy - 2x = y$$

$$x(y-2) = y$$

$$x = \frac{y}{y-2}$$

Replace x by $f^{-1}(y)$

$$\boxed{f^{-1}(y) = \frac{y}{y-2}}$$

$$f : R - \{1\} \to R - \{2\}$$

DPP- 2

Topic : *Inverse of a function*

Q.1. Let $f : R \to R$ is defined by $f(x) = ax + b$ where $a, b \in R$ and $a \neq 0$. Show that f is invertible and find the inverse of f.

Q.2. Show that $f : R - \{0\} \to R - \{0\}$, given by $f(x) = \frac{3}{x}$ is invertible and find f^{-1}.

Q.3. Let $A = R - \{3\}$ and $B = R - \{1\}$ Consider the function $f: A \to B$, defined by

$f(x) = \frac{x-2}{x-3}$ Show that f is invertible and find f^{-1}.

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DISCRETE STRUCTURES & THEORY OF LOGICS
(Discrete Mathematics)

UNIT - 2 : Functions and Boolean Algebra

Lecture - 03

Today's Target

- Composition of Functions
- PYQ
- DPP

Composition of Functions

Let A, B and C be three non-empty set and let

$f : A \to B$ and $g : B \to C$

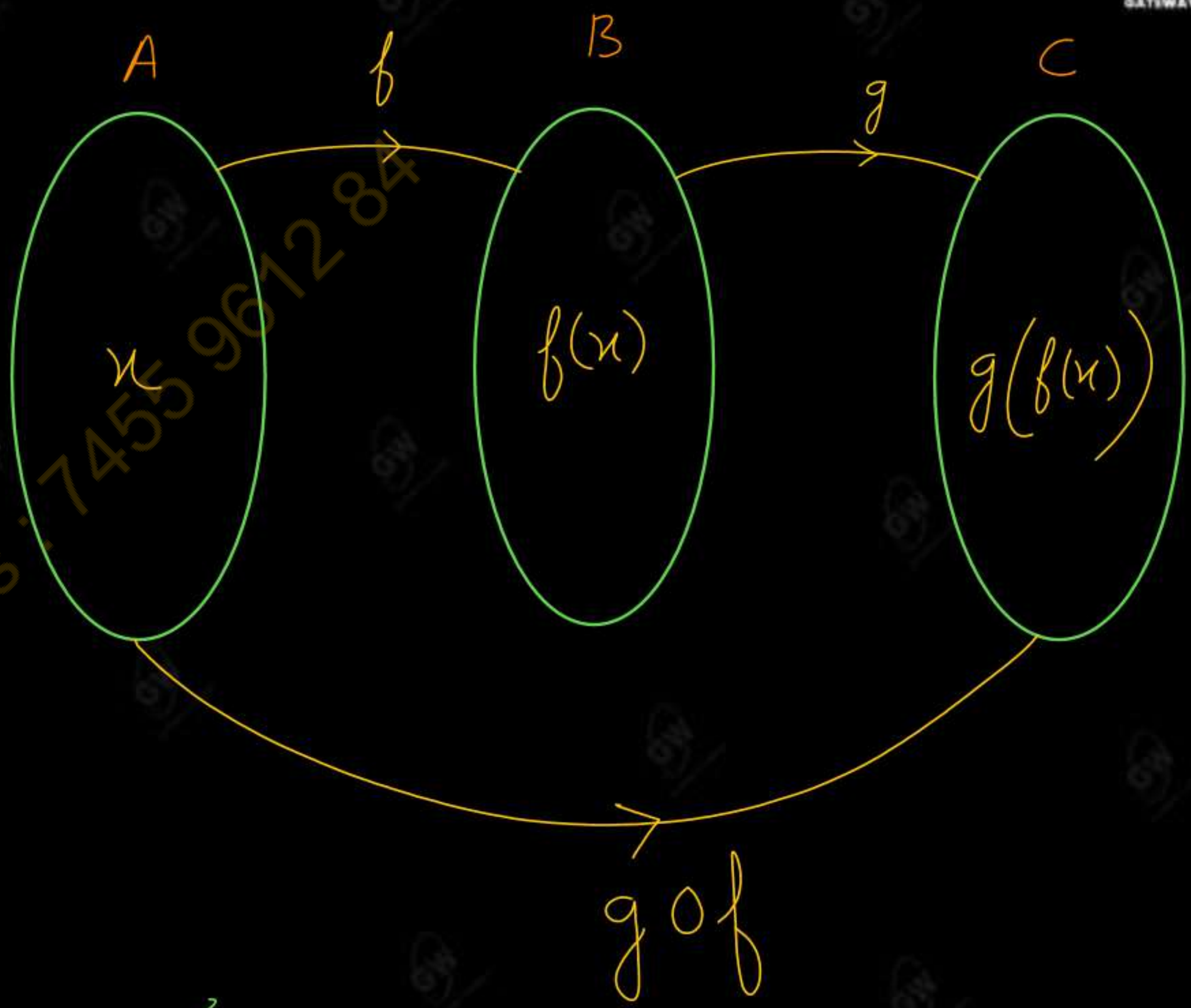
be two functions

Then the composition of f and g is a function denoted by

$gof : A \to C$

And is defined as

$\boxed{gof\,(x) = g\{f(x)\}}$ $\forall x \in A$



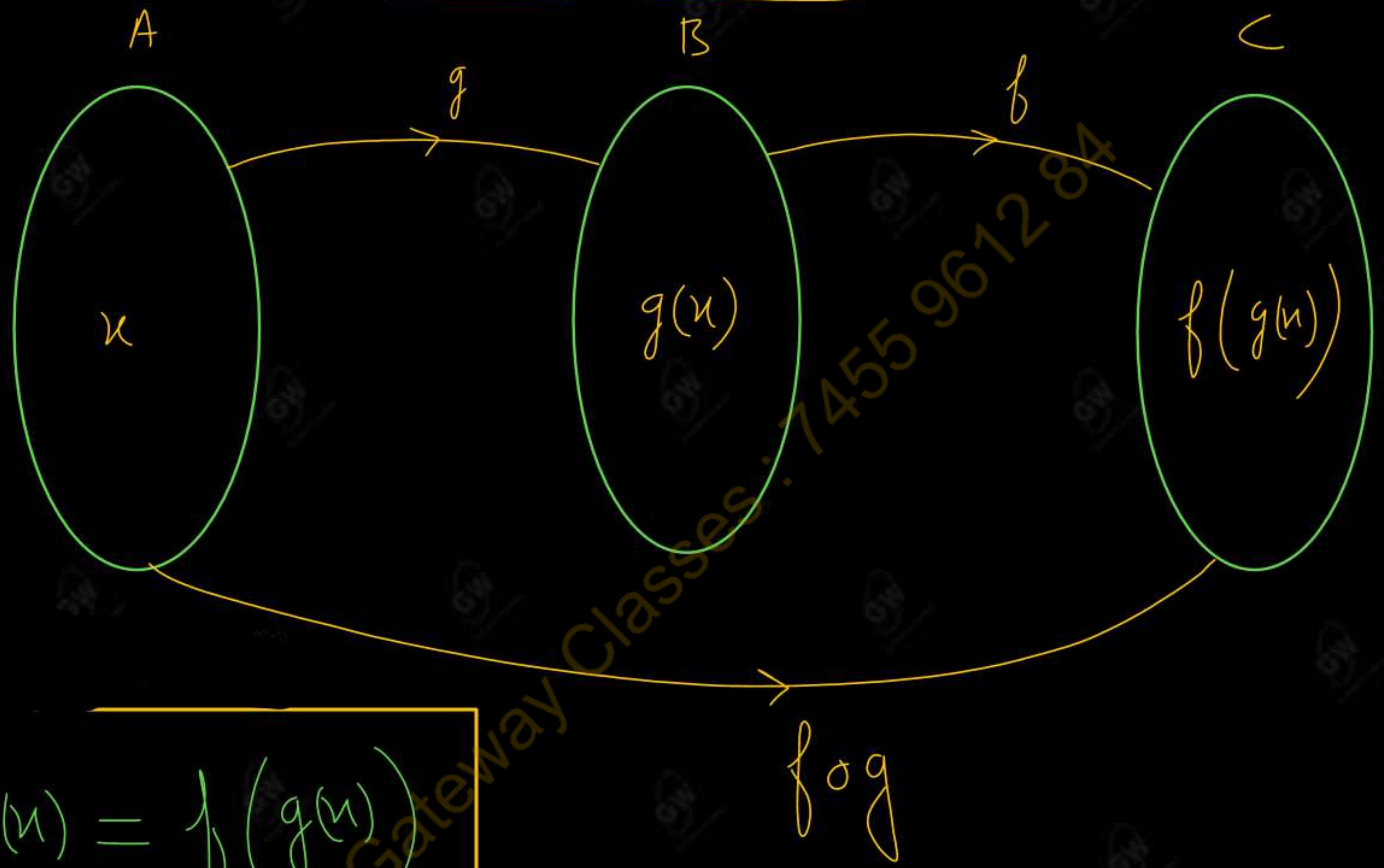
Note:- gof exist if and only if Range of $f \subseteq$ domain of g

Note:- $f \circ g$ exists if and only if, $\underbrace{\text{Range of } g} \subseteq \underbrace{\text{domain of } f}$

$g : A \longrightarrow B$

$f : B \longrightarrow C$

$f \circ g : A \longrightarrow C$



$$\boxed{(f \circ g)(x) = f(g(x))}$$

Q.1:- *Let* $f = \{1,3,4\} \to \{1,2,5\}$ *and* $g = \{1,2,5\} \to \{1,3\}$ *be defined as* $f = \{(1,2)(3,5)(4,1)\}$ *and* $g = \{(1,3)(2,3)(5,1)\}$, *Find gof and fog.*

Given

$f = \{1,3,4\} \longrightarrow \{1,2,5\}$

$g = \{1,2,5\} \longrightarrow \{1,3\}$

$f = \{(1,2)(3,5)(4,1)\}$

$g = \{(1,3)(2,3)(5,1)\}$

(i) $\text{Rang}(f) = \{1,2,5\}$

$\text{Dom}(g) = \{1,2,5\}$

$\Rightarrow \text{Rang}(f) \subseteq \text{Dom}(g)$

$\Rightarrow$ gof exists

$\text{dom}(gof) = \text{dom}(f)$

$\text{dom}(gof) = \{1,3,4\}$

$(gof)(1) = g(f(1))$

$= g(2) = 3$

$(gof)(3) = g(f(3))$

$= g(5)$

$= 1$

$(gof)(4) = g(f(4))$

$= g(1)$

$= 3$

$$\boxed{gof = \{(1,3),(3,1)(4,3)\}}$$

(ii) $rang(g) = \{1,3\}$

$dom(f) = \{1,3,4\}$

$\Rightarrow rang(g) \subseteq dom(f)$

$\Rightarrow$ fog exists

$dom(fog) = dom(g)$

$dom(fog) = \{1,2,5\}$

$(fog)(1) = f(g(1)) = f(3) = 5$

$(fog)(2) = f(g(2)) = f(3) = 5$

$(fog)(5) = f(g(5)) = f(1) = 2$

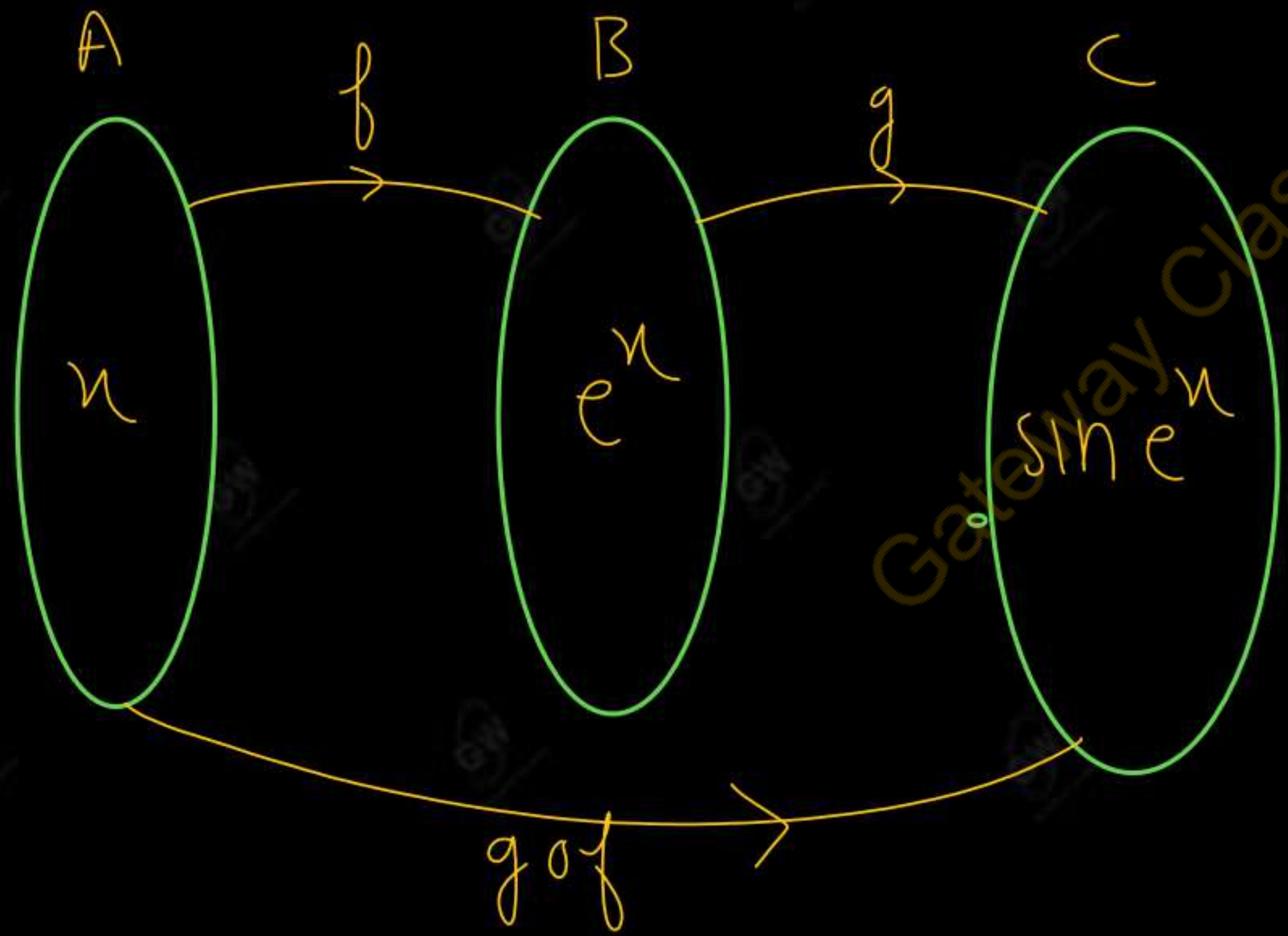
$$\boxed{fog = \{(1,5)(2,5)(5,2)\}}$$

Q.2:- Find the composition mapping gof if $f : R \rightarrow R$ is given by $f(x) = e^x$ and $g : R \rightarrow R$ is given by $g(x) = \sin x$. **(AKTU-2022-23)**

Given

$f : R \rightarrow R$, $f(x) = e^x$

$g : R \rightarrow R$, $g(x) = \sin x$



$$gof(x) = g(f(x))$$

$$= g(e^x)$$

$$\boxed{gof(x) = \sin e^x}$$

Q.3:- Let $A = \{1, 2, 3\}$

$B = \{a, b\}$

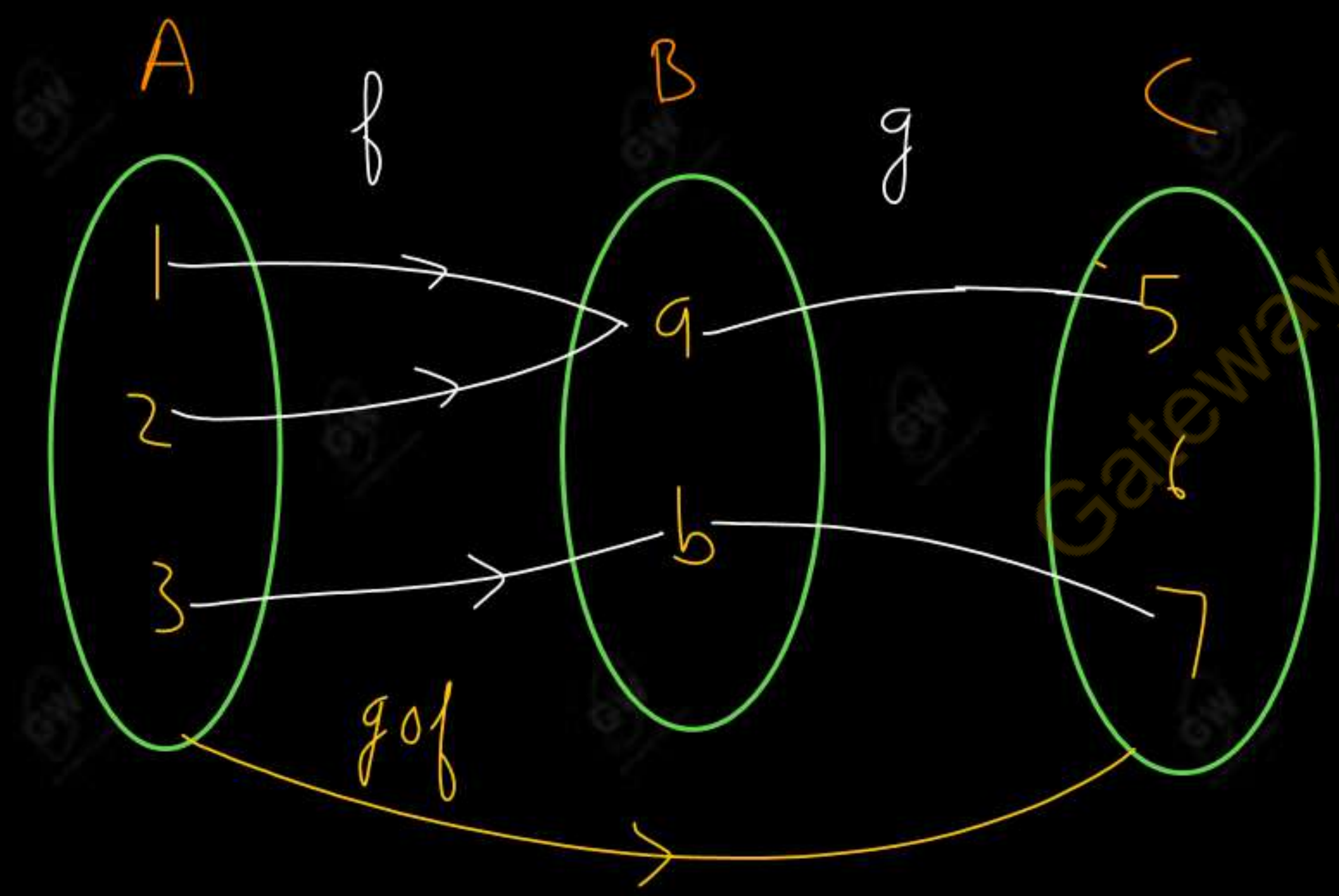
$C = \{5, 6, 7\}$

Define $f : A \rightarrow B$ where $f = \{(1, a)\ (2, a)(3, b)\}$

and $g : B \rightarrow C$ where $g = \{(a, 5)\ (b, 7)\}$

Find the composition of f and g.

I^{st} Method



$$\boxed{g \circ f = \{(1, 5)(2, 5)(3, 7)\}}$$

2^{nd} Method

$$dom(g \circ f) = dom(f)$$
$$= \{1, 2, 3\}$$

$$(g \circ f)(1) = g(f(1)) = g(a) = 5$$
$$(g \circ f)(2) = g(f(2)) = g(a) = 5$$
$$(g \circ f)(3) = g(f(3)) = g(b) = 7$$

$$\boxed{g \circ f = \{(1, 5)(2, 5)(3, 7)\}}$$

Q.4:- *Let $f : R \to R$ and $g : R \to R$ where R be the Set of real numbers.*

Find (i) gof (ii) fog (iii) fof (iv) gog. *Where $f(x) = x^2$ and $g(x) = x + 4$*

Given

$f : R \to R$, $f(x) = x^2$

$g : R \to R$, $g(x) = x+4$

(i) $gof = (gof)(x)$
$= g(f(x))$
$= g(x^2)$
$= x^2+4$

(ii) $fog = (fog)(x)$
$= f(g(x))$
$= f(x+4)$
$= (x+4)^2$

(iii) $fof = (fof)(x)$
$= f(f(x))$
$= f(x^2)$

$fof = (x^2)^2 = x^4$

(iv) $gog = gog(x)$
$= g(g(x))$
$= g(x+4)$
$= x+4+4$
$= x+8$

Q.5:- If $f : R \rightarrow R$ and $g : R \rightarrow R$ $h : R \rightarrow R$ defined by

$$f(x) = 3x^2 + 2, \quad g(x) = 7x - 5 \text{ and } h(x) = \frac{1}{x}.$$

Find the following composition functions.

(a) $(fogoh)(x)$ **(b)** $(hogof)(x)$ **(c)** $(gog)(x)$ **(d)** $(goh)(x)$

Given

$f : R \rightarrow R, \quad f(x) = 3x^2 + 2$

$g : R \rightarrow R, \quad g(x) = 7x - 5$

$h : R \rightarrow R, \quad h(x) = \frac{1}{x}$

(i) $(fogoh)(x) = f(g(h(x)))$

$= f\left(g\left(\frac{1}{x}\right)\right) = f\left(7 \times \frac{1}{x} - 5\right)$

$= 3\left(\frac{7}{x} - 5\right)^2 + 2$

(ii) $(hogof)(x) = h(g(f(x)))$

$= h(g(3x^2+2))$

$= h(7(3x^2+2)-5)$

$= h(21x^2+14-5)$

$= h(21x^2+9)$

$= \dfrac{1}{21x^2+9}$

(iii) $gog(x) = g(g(x)) = g(7x-5)$

$= 7(7x-5)-5$

$= 49x-40$

(iv) $goh(x) = g(h(x)) = g\left(\frac{1}{x}\right)$

$= \dfrac{7}{x} - 5$

Q.6:- Let $X = \{a, b, c\}$. Define $f : X \to X$ such that

$$f = \{(a, b), (b, a), (c, c)\}$$

Find (i) f^2 (ii) f^3 (iii) f^4 (UPTU-2014)

$f^2 = f \circ f$

Given

$X = \{a, b, c\}$

$f = \{(a, a)(b, a)(c, c)\}$

(i) $f^2(a) = (f \circ f)(a)$

$= f(f(a))$

$= f(a) = a$

$f^2(b) = (f \circ f)(b)$

$= f(f(b))$

$= f(a)$

$= b$

$f^2(c) = (f \circ f)(c)$

$= f(f(c))$

$= f(c)$

$f^2(c) = c$

$f^2 = \{(a, a)(b, b)(c, c)\}$

(ii) $f^3(a) = (f^2 \circ f)(a)$

$= f^2(f(a))$

$= f^2(a)$

$= a$

$$f^3(b) = (f^2 \circ f)(b)$$
$$= f^2(f(b))$$
$$= f^2(a)$$
$$= a$$

$$f^3(c) = (f^2 \circ f)(c)$$
$$= f^2(f(c))$$
$$= f^2(c)$$
$$= c$$

$$\boxed{f^3 = \{(a,a), (b,a) (c,c)\}}$$

(iii) $f^4(a) = (f^3 \circ f)(a)$
$$= f^3(f(a))$$
$$= f^3(a)$$
$$= a$$

$$f^4(b) = (f^3 \circ f)(b)$$
$$= f^3(f(b))$$
$$= f^3(a)$$
$$= a$$

$$f^4(c) = (f^3 \circ f)(c)$$
$$= f^3(f(c))$$
$$= f^3(c)$$
$$= c$$

$$\boxed{f^4 = \{(a,a) (b,a) (c,c)\}}$$

Theorem:- if $f : A \to B$ and $g : B \to C$ be one – one and onto function. Proof that

$$gof : A \to C \text{ is one } - \text{ one onto}$$

And

$$(gof)^{-1} = f^{-1} \, o \, g^{-1}$$

Given

$f : A \to B$ is one-one onto

$g : B \to C$ is also one-one onto

Proof: since f and g are one one onto function

$\therefore f^{-1}$ and g^{-1} both are exist

If $y = f(x) \Rightarrow f^{-1}(y) = x$ — (1)

If $z = g(y) \Rightarrow g^{-1}(z) = y$ — (2)

For one-one

Let $x_1, x_2 \in A$

$$(gof)(x_1) = (gof)(x_2)$$

$$g(f(x_1)) = g(f(x_2))$$

$$g(f(x_1)) = g(f(x_2))$$

since g is one-one

$$f(x_1) = f(x_2)$$

since f is one-one

$$x_1 = x_2$$

$\Rightarrow g \circ f$ is one-one function

For onto

Let $z \in C$

- since g is an onto function, then there exist an element $y \in B$ such that $g(y) = z$

- since f is an onto function, then there exist an element $x \in A$, such that $f(x) = y$

$\Rightarrow z = g(y) = g(f(x))$

$z = gof(x)$

Hence, gof is an onto function

$\Rightarrow gof$ is an one-one onto function

$\Rightarrow (gof)^{-1}$ exist

$$\boxed{(gof)^{-1}(z) = x} \quad \text{—— (3)}$$

<u>Now</u>

$(f^{-1} o g^{-1})(z) = f^{-1}(g^{-1}(z))$

$= f^{-1}(y)$ {From (2)}

$= x$ {From (1)}

$(f^{-1} o g^{-1})(z) = x$ —— (4)

From (3) and (4)

$(gof)^{-1}(z) = (f^{-1} o g^{-1})(z)$

$$\boxed{(gof)^{-1} = f^{-1} o g^{-1}}$$

DPP- 3

Topic : Composition of Functions

Q.1 Let $X = \{1,2,3\}, Y = \{q,p\}$ and $Z = \{a,b\}$, let $f: X \to Y$ be $f = \{(1,p),(2,p),(3,q)\}$ $g: Y \to Z$ be $g = \{(p,b),(q,b)\}$. Find gof and show it pictorially. (UPTU-2013)

$gof = \{(1,b)(2,b)(3,b)\}$

Q.2 If $f: R \to R$ and $g: R \to R$ defined by

$$f(x) = x^3 - 4x, g(x) = 1/(x^2+1) \quad h(x) = x^4.$$

Find the following composition functions.

(a) $(fogoh)(x)$ (b) $(hogof)(x)$ (c) $(gog)(x)$ (d) $(goh)(x)$

Ans (a) $(x^8+1)^{-3} - 4(x^8+1)^{-1}$

(b) $[(x^3-4x)^2+1]^{-4}$

(c) $(x^2+1)^2[1+(x^2+1)^2]$ (d) $(x^8+1)^{-1}$

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Q.3 If $f: Z \to Z,\ g: Z \to Z,\ h: Z \to Z$ defined by $f(n) = n^2, g(n) = n + 1$ and $h(n) = n - 1$. Find

(a) $gofoh$ (b) $fogoh$ (c) $hofog$.

Ans (a) n^2-2n+2 (b) n^2 (c) n^2+2n

Q.4 Given $f = \{(a,b),(b,a),(c,b)\}$ a function from $X = \{a,b,c\}$ to X:

(a) Write fof and $fofof$ as sets of ordered pairs. Ans $fof = \{(a,a)(b,b)(c,a)\}$

(b) Define $f^n = fofo\ \ldots\ldots\ of$ to be the $n-$fold composition of f with itself. Find f^9.

$fofof = \{(a,b)(b,a)(c,b)\}$

(b) $f^9 = f$

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DISCRETE STRUCTURES & THEORY OF LOGICS
(Discrete Mathematics)

UNIT -2 : Functions and Boolean Algebra

Lecture - 04

Today's Target

- Definitions of Boolean Algebra
- DPP

Boolean Algebra

Boolean Algebra is an algebraic structure which is based on the principle of logics.

Algebraic Structure for Boolean Algebra

(i) $(B, +, \cdot, ', 0, 1)$

(ii) $(B, \vee, \wedge, ', 0, 1)$

(iii) $(B, \cup, \cap, ', 0, 1)$

First Definition:

A non-empty set B with two binary operations $+$ and $\cdot$, a unary operation $'$, and two distinct elements 0 and 1 is called a Boolean algebra, denoted by $(B, +, \cdot, ', 0, 1)$ if and only if the following properties are satisfied

1. **Closure law :** $\forall\, a, b \in B$

(i) $a + b \in B$

(ii) $a \,.\, b \in B$

2. **Commutative law :** $\forall\, a, b \in B$

(i) $a + b = b + a$

(ii) $a \,.\, b = b \,.\, a$

3. **Distributive law :** $\forall\, a, b, c \in B$

(i) $a \,.\, (b + c) = a \,.\, b + a \,.\, c$

$(b + c) \,.\, a = b \,.\, a + c \,.\, a$

(ii) $a + (b \,.\, c) = (a + b) \,.\, (a + c)$

$(b \,.\, c) + a = (b + a) \,.\, (c + a)$

4. Identity law : $\forall a$ [redacted] ϵB

$$(i)\ a + 0 = 0 + a = a \qquad (ii)\ a\,.\,1 = 1\,.\,a = a$$

Additive Identity   *Multiplicative Identity*

5. Complement law : $\forall a \epsilon B$ *there exist* $a' \epsilon B$ such that

$$(i)\ a + a' = a' + a = 1 \qquad (ii)\ a\,.\,a' = a'.\,a = 0$$

Q1. Show that the structure $(B, +, \cdot, ')$ is a Boolean algebra where $B = \{0, 1\}$ and $+$ and $\cdot$ are two binary operation and ' ' a unary operation on B, defined by the following table

+	0	1
0	0	1
1	1	1

(i)

.	0	1
0	0	0
1	0	1

(ii)

a	a'
0	1
1	0

(iii)

Sol

① closure law : $B = \{0, 1\}$

(i) $0 + 1 = 1 \in B$ Hence closure satisfied under $+$ and $\cdot$

(ii) $0 \cdot 1 = 0 \in B$

② **Commutative law** : $\forall\ 0, 1 \in B$

$$0+1=1 \qquad 0.1=0$$

$$1+0=1 \qquad 1.0=0$$

$$\Rightarrow 0+1=1+0 \qquad 0.1=1.0$$

Hence commutative is satisfied under '+' and '•'

③ **Distributive law** : $\forall\ a, b, c \in B$

(i) $a.(b+c) = a.b + a.c$

(ii) $a+(b.c) = (a+b).(a+c)$

Let $a=1,\ b=0,\ c=1$

(i) $a.(b+c) = 1.(0+1)$
$= 1.1$
$= 1$

$a.b + a.c = 1.0 + 1.1$
$= 0+1$
$= 1$

$\Rightarrow a.(b+c) = a.b + a.c$

(ii) $a+(b.c) = 1+(0.1)$
$= 1+0$
$= 1$

$$(a+b)\cdot(a+c) = (1+0)\cdot(1+1)$$
$$= 1\cdot 1$$
$$= 1$$

$\Rightarrow a+(b\cdot c) = (a+b)\cdot(a+c)$

Hence Distributive law is satisfied

④ Identity law

$$0+0 = 0$$
$$1+0 = 1$$
$$0\cdot 1 = 0$$
$$1\cdot 1 = 1$$

Hence, 0 is additive identity

1 is multiplicative identity

⑤ complement law : For $0, 1 \in B$

$$1' = 0 \in B$$
$$0' = 1 \in B$$

$\Rightarrow$ complement law is satisfied

Hence $(B, +, \cdot, ', 0, 1)$ is a Boolean Algebra

Second Definition:

A complemented distributive lattice is called a Boolean algebra.

OR

A non-empty set B with two binary operations $\vee$ and $\wedge$, a unary operation $'$, and two distinct elements o and 1 is called a Boolean algebra, denoted by $(B, \vee, \wedge, ', 0, 1)$ if and only if the following properties are satisfied

1. Closure law : $\forall\, a, b \in B$

(i) $a \vee b \in B$ (ii) $a \wedge b \in B$

2. Commutative law : $\forall\, a, b \in B$

(i) $a \vee b = b \vee a$ (ii) $a \wedge b = b \wedge a$

3. Distributive law : $\forall\, a, b, c \in B$

(i) $a \vee (b \wedge C) = (a \vee b) \wedge (a \vee C)$

(ii) $a \wedge (b \vee C) = (a \wedge b) \vee (a \wedge C)$

4. Identity law : There exist two elements o and 1 in B such that

(i) $a \vee o = a$

(ii) $a \wedge 1 = a$

$\vee 1$

5. Complement law : $\forall\, a \in B$ there exist an element $a' \in B$ such that

(i) $a \vee a' = 1$

(ii) $a \wedge a' = o$

Q. 2: *Let B be the set of all +ve divisors of 30 and the operations $\vee$ and $\wedge$ on B are defined as follows*

$$\underbrace{a \vee b}_{} = \underbrace{LCM(a,b)}_{}$$

$$\underbrace{a \wedge b}_{} = \underbrace{HCF(a,b)}_{}$$

Prove that $(B, \vee, \wedge, ')$ is a Boolean algebra

Solution

$B = \{1, 2, 3, 5, 6, 10, 15, 30\}$

least element (1)

greatest element (30)

composition table for $\vee$

$\vee$	1	2	3	5	6	10	15	30
1	1	2	3	5	6	10	15	30
2	2	2	6	10	6	10	30	30
3	3	6	3	15	6	30	15	30
5	5	10	15	5	30	10	15	30
6	6	6	6	30	6	30	30	30
10	10	10	30	10	30	10	30	30
15	15	30	15	15	30	30	15	30
30	30	30	30	30	30	30	30	30

Composition table for $\wedge$

$\wedge$	1	2	3	5	6	10	15	30
1	1	1	1	1	1	1	1	1
2	1	2	1	1	2	2	1	2
3	1	1	3	1	3	1	3	3
5	1	1	1	5	1	5	5	5
6	1	2	3	1	6	2	3	6
10	1	2	1	5	2	10	5	10
15	1	1	3	5	3	5	15	15
30	1	2	3	5	6	10	15	30

Composition table for complement

a	a'
1	30
2	15
3	10
5	6
6	5
10	3
15	2
30	1

least element $= 1$

Greatest element $= 30$

$a \vee a' = 30$

$a \wedge a' = 1$

① closure law :

since all the element of $\vee$ and $\wedge$ table lie inside the set B

Hence closure law is satisfied under $\vee$ and $\wedge$

$\Rightarrow \forall\, a, b \in B$

$a \vee b \in B$ and $a \wedge b \in B$

② commutative law

since corresponding row and column are identical in $\vee$ and $\wedge$ table

$\Rightarrow$ commutative law is satisfied under $\vee$ and $\wedge$

$\Rightarrow \forall\, a, b \in B$

(i) $a \vee b = b \vee a$

(ii) $a \wedge b = b \wedge a$

③ Distributive law

(i) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

(ii) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

Let $a = 2, \; b = 3, \; c = 6$

(i) $a \vee (b \wedge c) = 2 \vee (3 \wedge 6)$

$= 2 \vee 3$

$= 6$

$(a \vee b) \wedge (a \vee c) = (2 \vee 3) \wedge (2 \vee 6)$

$= 6 \wedge 6$

$= 6$

$\Rightarrow a \vee (b \wedge c) = (a \vee) \wedge (a \vee c)$

Similarly

$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

Hence Distributive law is satisfied under $\vee$ and $\wedge$

(4) Identity law : $\forall\, a \in B$

From $\vee$ table

$a \vee 1 = a$

From $\wedge$ table

$a \wedge 30 = a$

Hence

1 is the identity element for $\vee$

30 is " " $\wedge$

(5) Complement law

$\forall\, a \in B$ there exist

an element $a' \in B$

such that

$a \vee a' = 30$

$a \wedge a' = 1$

Hence $(B, \vee, \wedge, ', 0, 1)$

Third Definition

A non-empty set B with two binary operations ∪ and ∩, a unary operation ′ , and two distinct elements o and 1 is called a Boolean algebra, denoted by $(B, \cup, \cap, ', 0, 1)$ if and only if the following properties are satisfied

1. Closure law : $\forall A_1, A_2 \in B$

$(i)\ A_1 \cup A_2 \in B$ $\qquad$ $(ii)\ A_1 \cap A_2 \in B$

2. Commutative law : $\forall A_1, A_2 \in B$

$(i)\ A_1 \cup A_2 = A_2 \cup A_1$ $\qquad$ $(ii)\ A_1 \cap A_2 = A_2 \cap A_1$

3. Distributive law : $\forall\ A_1, A_2, A_3 \in B$

$$(i)\ A_1 \cup (A_2 \cap A_3) = (A_1 \cup A_2) \cap (A_1 \cup A_3)$$

$$(ii)\ A_1 \cap (A_2 \cup A_3) = (A_1 \cap A_2) \cup (A_1 \cap A_3)$$

4. Identity law : $\forall\ A \in B$ *there exist two element* ϕ *and* $\cup$ *in* B such that

$(i)\ A \cup \phi = A$

$(ii)\ A \cap \cup = A$

5. Complement law : $\forall\ A \in B$ *there exist an element* $A' \in B$ such that

$(i)\ A \cup A' = \cup \qquad (ii)\ A \cap A' = \phi$

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(Discrete Mathematics)

UNIT -2 : Functions and Boolean Algebra

Lecture - 05

Today's Target

- Theorems of Boolean Algebra
- PYQ
- DPP

Boolean Algebra

$\boxed{(B, +, \bullet, ', 0, 1)}$ is a Boolean algebra if and only if the following properties are satisfied

1. Closure Laws: $\forall\ a,\ b \in B$

(i) $a + b \in B$ (ii) $a \,.\, b \in B$

2. Commutative Laws: $\forall\ a,\ b \in B$

(i) $a + b = b + a$ (ii) $a \,.\, b = b \,.\, a$

3. Associative Laws: $\forall\ a,\ b\ c \in B$

(i) $a + (b + c) = (a + b) + c$ (ii) $a\ (b\ c) = (a\ b)\ c$

4. Distributive Laws: $\forall\ a,\ b,\ c \in B$

(i) $a \,.\, (b + c) = a \,.\, b + a \,.\, c$

$(b + c) \,.\, a = b \,.\, a + c \,.\, a$

(ii) $a + (b \,.\, c) = (a + b) \,.\, (a + c)$

$(b \,.\, c) + a = (b + a) \,.\, (c + a)$

5. **Identity Law:** $\forall\ a \in B$

(i) $a + 0 = 0 + a = a$

Additive Identity

(ii) $a . 1 = 1 . a = a$

Multiplicative Identity

6. **Complement Law:** $\forall\ a \in B$ there exist $a' \in B$ such that

(i) $a + a' = a' + a = 1$

(ii) $a . a' = a' . a = 0$

Theorems of Boolean Algebra

Theorems 1 - (Idempotent Law) : For every element of a Boolean algebra B

(i) $a + a = a$ (ii) $a \cdot a = a$

(i) To Prove: $a + a = a$

Proof: RHS

$a = a + 0$ by identity law

$= a + a \cdot a'$ by complement law

$= (a+a) \cdot (a+a')$ by distributive law

$= (a+a) \cdot 1$ by complement law

$= a + a$ by identity law

$=$ RHS

Hence proved

(ii) To Prove : $a \cdot a = a$

Proof: LHS

$a \cdot a = a \cdot a + 0$ by identity law

$= a \cdot a + a \cdot a'$ by complement law

$= a \cdot (a + a')$ by distributive law

$= a \cdot 1$ by complement law

$= a$ by identity law

$=$ RHS

Hence proved

Theorems 2 – (Bounded laws) : For every element of a Boolean algebra B

(i) $a + 1 = 1$ | (ii) $a \cdot 0 = 0$

(i) To Prove: $a+1=1$

LHS:

$a+1 = a+(a+a')$ by complement law

$= (a+a)+a'$ by associative law

$= a+a'$ by idempotent law

$= 1$ by complement

$=$ RHS

Hence proved

To Prove: $a \cdot 0 = 0$

Proof: LHS

$a \cdot 0 = a \cdot (a \cdot a')$ by complement law

$= (a \cdot a) \cdot a'$ by Associative law

$= a \cdot a'$ by idempotent law

$= 0$ by complement law

$=$ RHS

Hence proved

Theorems 3 – (Absorption Laws) : For any two element a and b of a Boolean algebra B,

(i) $a + a \cdot b = a$

(ii) $a \cdot (a + b) = a$

(i) To Prove: $a + a.b = a$

Proof: LHS

$a + a.b = a.1 + a.b$ By identity law

$= a.(1+b)$ By distributive law

$= a.1$ By Boundedness law

$= a$ By identity law

$=$ RHS

Hence proved

(ii) To Prove: $a.(a+b) = a$

Proof: LHS

$a.(a+b) = (a+0).(a+b)$ by identity law

$= a+(0.b)$ by distributive law

$= a+0$ by boundedness la

$= a$ by identity law

$=$ RHS

Hence proved

Theorems 4 : *In a Boolean algebra B, the identity elements are complementary to each other*

i.e for $0, 1 \in B$ we have

(i) $0' = 1$ (ii) $1' = 0$

(i) To Prove : $0' = 1$

Proof :

LHS $= 0'$

$= 0' + 0$ by identity law

$= 1$ by complement law

$=$ RHS

Hence proved

(ii) To Prove : $1' = 0$

Proof :

LHS $= 1'$

$= 1' \cdot 1$ by identity law

$= 0$ by complement law

$=$ RHS

Hence proved

Theorems 5 : The complement of each element of Boolean algebra B is unique

OR

For each $a \in B$, a' is unique

Proof: Let $a \in B$ be any element

Suppose x and y are two complements of a

By complement law

$a+x = x+a = 1$ and $a.x = x.a = 0$ — ①

$a+y = y+a = 1$ and $a.y = y.a = 0$ — ②

Now

$x = x.1$ By identity law

$x = x.(a+y)$ From ②

$= x.a + x.y$ By Distributive law

$= 0 + x.y$ From ①

$= a.y + x.y$ From ②

$= (a+x).y$ By distributive law

$= 1.y$ From ①

$\boxed{x = y}$ By identity law

Hence, complement of each element is unique

Theorems 6 : The identity elements in a Boolean algebra are unique

OR

The elements 0 and 1 are unique in Boolean algebra.

(i) To Prove identity element 0 is unique for +

Suppose 0 and 0_1 are two identity elements of Boolean algebra B for +

If 0 is the identity element

$$0_1 + 0 = 0_1 \quad \text{—— (1)}$$

If 0_1 is the identity element

$$0 + 0_1 = 0 \quad \text{—— (2)}$$

By commutative law

$$0_1 + 0 = 0 + 0_1$$

$\Rightarrow 0 = 0_1$ (From (1) and (2))

Hence identity element 0 is unique

Similarly, we can prove identity element 1 is unique for $\cdot$

Theorems 7 : Involution Law

For each element a of Boolean algebra B

$$\boxed{(a')' = a}$$

Theorems 8 – (De Morgan's Law) : For any two elements a and b of Boolean algebra B DPP-1

(i) $(a+b)' = a' . b'$ (ii) $(a . b)' = a' + b'$ $\forall a, b \in B$

To Prove : $(a+b)' = a'.b'$

Proof : To show the complement of $(a+b)$ is $a'.b'$, it is sufficient to prove

$(a+b) + a'.b' = 1$ —— ①

$(a+b).(a'.b') = 0$ —— ②

LHS of ①

$(a+b)+a'.b' = (a+b+a').(a+b+b')$

By Distributive law

$= (b+a+a').(a+b+b')$

By commutative law

$= (b+1).(a+1)$

By complement law

$= 1.1$ By boundedness la

$= 1$ By identity law

$= RHS$

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LHS of (2)

$(a+b).(a'.b')$

$= a.(a'.b') + b.(a'.b')$

By distributive law

$= a.(a'.b') + b.(b'.a')$

By commutative law

$= (a.a').b' + (b.b').a'$

By Associative law

$= 0.b' + 0.a'$

By complement law

$= 0 + 0$ By Boundedness law

$= 0$ By idempotent law

$=$ RHS

$\Rightarrow (a+b)' = a'.b'$

Hence proved

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DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -2 : Functions and Boolean Algebra

Lecture - 06

Today's Target

- Truth table
- Boolean Function
- Minterm
- Maxterm
- Disjunction Normal Form (D N Form)
  OR
  Sum of Products Form (SOP Form)

Basic operation (Logic operation)

- The basic operations used in Boolean algebra are

  (i) Logical Addition

  (ii) Logical Multiplication

  (iii) Complementation

0	False	OFF	LOW	OPEN	NO
1	True	ON	HIGH	CLOSED	YES

- Each variable in Boolean algebra has either 0 or 1 (False or True)

- Here 1 or 0 do not represent actual numbers

Logical Addition : Logical Addition is denoted by + or by OR

Truth Table for logical addition

Total input combination $= 2^n$

$0 + 0 = 0$ otherwise 1

INPUT		OUTPUT
A	B	$C = A + B$
1	1	1
1	0	1
0	1	1
0	0	0

Logical Multiplication : Logical Multiplication is denoted by • or by AND

Truth Table for logical Multiplication

$1.1 = 1$ otherwise 0

INPUT		OUTPUT
A	B	$C = A.B$
1	1	1
1	0	0
0	1	0
0	0	0

Logical Complementation : Logical Complementation is denoted by ‾ or by ' $(\bar{A}, A')$

Truth Table for logical Complementation

INPUT	OUTPUT
A	$B = A'$
1	0
0	1

Boolean Function (or Boolean Polynomial)

An expression obtained by the application of binary operation $(+ \text{ or } \vee)$and $(\bullet \text{ or } \wedge)$ and unary operation (‘) on finite number of elements of Boolean algebra $(B, +, \bullet, ')$ or $(B, \vee, \wedge, ')$ is called a Boolean function or Boolean Polynomial.

A variable x is called a Boolean variable if it assumes values either 0 or 1.

Note:- (i) $\wedge$ or $\bullet$ $\longrightarrow$ Meet or Conjunction

(ii) $\vee$ or $+$ $\longrightarrow$ Join or Disjunction

Examples:

1. Function of one Variables

$$f(x) = x + x'$$

OR

$$f(x) = x \vee x'$$

2. Function of two variables x and y

$$f(x\,,\;y) = x\;.\;y' + x'\,.\,y$$

OR

$$f(x\,,\;y) = (x \wedge y') \vee (x' \wedge y)$$

3. Function of three variables x, y and z

$$f(x\,,\;y,\;\;z) = x\;.\;y\,.\,z + x'\,.\,y\,.\,z + x\;.\;y'\,.\,z'$$

OR

$$f(x\,,\;y,\;\;z) = (x \wedge y \wedge z) \vee (x' \wedge y \wedge z) \vee (x \wedge y' \wedge z')$$

Minterm (OR Minimal Boolean Function)

A minterm of n-variables is a $\boxed{\text{product}}$ of n literals in which each variable appears exactly once in either true or complemented form, but not both.

For Examples:

1. The list of all minterms of two variables x and y are $(2^2 = 4)$

$$xy,\ x'y,\ xy',\ x'y'$$

3. The list of all minterms of three variables x , y and z are $(2^3 = 8)$

$$xyz,\ xyz',\ xy'z,\ x'yz,\ x'y'z',\ x'y'z,\ x'yz',\ xy'z'$$

Note:

(1) *n-variables can be combined to form 2^n minterm*

(2) *The output value of a minterm is 1*

Maxterm (or Maximal Boolean Function)

A maxterm of n variables is a sum of n literals in which each variable appears exactly once in either true or complimented form, but not both.

For Examples:

1. All The maxterm of two variables x and y are $(2^2 = 4)$

$$x+y,\ x'+y,\ x+y',\ x'+y'$$

2. All The maxterm of three variables x , y and z are $(2^3 = 8)$

$$(x+y+z),\ (x+y+z'),\ (x+y'+z),\ (x'+y+z),\ (x'+y'+z'),\ (x'+y'+z),$$
$$(x'+y+z'),\ (x+y'+z')$$

Note:

(1) *The number of maxterm in n-variables are* 2^n

(2) *The output value of a maxterm is 0*

Note:

For Minterm:- 1 → Use same variable

0 → Use Complement of variable

For Maxterm:- 0 → Use same variable

1 → Use Complement

Truth Table:

1. For two variable x , y

x	y	Minterms(m)	Maxterms(M)
1	1	$m_1 = x \cdot y$	$M_1 = x' + y'$
1	0	$m_2 = x \cdot y'$	$M_2 = x' + y$
0	1	$m_3 = x' \cdot y$	$M_3 = x + y'$
0	0	$m_4 = x' \cdot y'$	$M_4 = x + y$

3. For three variable

x	y	z	Minterm(m)	Maxterm(M)
1	1	1	$m_1 = x.y.z$	$M_1 = x' + y' + z'$
1	1	0	$m_2 = x.y.z'$	$M_2 = x' + y' + z$
1	0	1	$m_3 = x.y'.z$	$M_3 = x' + y + z'$
1	0	0	$m_4 = x.y'.z'$	$M_4 = x' + y + z$
0	1	1	$m_5 = x'.y.z$	$M_5 = x + y' + z'$
0	1	0	$m_6 = x'.y.z'$	$M_6 = x + y' + z$
0	0	1	$m_7 = x'.y'.z$	$M_7 = x + y + z'$
0	0	0	$m_8 = x'.y'.z$	$M_8 = x + y + z$

Boolean Function

Disjunction Normal Form

Conjunction Normal Form

Next Lecture

Disjunction Normal Form (D N Form)

OR

Sum of Product Form (SOP Form) / sum of minterm

OR

Canonical Form

A Boolean function which can be written as the sum of Minterms, is called disjunctive normal form or D N Form

Note:- As Minterm is product of variables, so D N form is also called sum of minterm or sum of product form (SOP)

Step to find D N form

STEP-1: Make a truth table for given Boolean function

$\wedge$ or $\bullet$	Conjunction Meet Product	$T \quad T \Rightarrow T$ otherwise F $1 \quad 1 \Rightarrow 1$ otherwise 0
$\vee$ or $+$	Disjunction Join Addition	$F \quad F \Rightarrow F$ otherwise T $0 \quad 0 \Rightarrow 0$ otherwise 1
$-$ or $'$	Complement Prime Negation	$T \longrightarrow F \qquad F \longrightarrow T$ $1 \longrightarrow 0 \qquad 0 \longrightarrow 1$

STEP-2: See the entries 1 in the last column of truth table and write corresponding minterms.

STEP-3: By adding the all minterms, we will get required D N form or SOP form

Q. 1. *Express the Boolean function in to D N form* $f(x, y) = x + x'.y$

$$f(x,y) = x + x'.y$$

x	y	x'	$x'.y$	$x+x'.y$	Minterm
1	1	0	0	1	$m_1 = xy$
1	0	0	0	1	$m_2 = xy'$
0	1	1	1	1	$m_3 = x'y$
0	0	1	0	0	

✓ ✓

Required DN Form

$$\boxed{f(x,y) = xy + xy' + x'y}$$

2nd Method

$$f(x,y) = x + x'.y$$

$$f(x,y) = x.1 + x'.y$$

$$f(x,y) = x(y+y') + x'.y$$

$$\boxed{f(x,y) = xy + xy' + x'y}$$

Q. 2: Express the Boolean polynomial $f(x, y, z) = (x + y) . (x + z) + y + z'$ in to disjunctive normal form.

(AKTU-2020)

x	y	z	$(x+y)$	$(x+z)$	$(x+y).(x+z)$	z'	$y+z'$	$(x+y).(x+z)+y+z'$	Minterm
1	1	1	1	1	1	0	1	1	$m_1 = xyz$
1	1	0	1	1	1	1	1	1	$m_2 = xyz'$
1	0	1	1	1	1	0	0	1	$m_3 = xy'z$
1	0	0	1	1	1	1	1	1	$m_4 = xy'z'$
0	1	1	1	1	1	0	1	1	$m_5 = x'yz$
0	1	0	1	0	0	1	1	1	$m_6 = x'yz'$
0	0	1	0	1	0	0	0	0	
0	0	0	0	0	0	1	1	1	$m_8 = x'y'z'$

Required DN Form

$$\boxed{f(x,y,z) = xyz + xyz' + xy'z + xy'z' + x'yz + x'yz' + x'y'z'}$$

2nd Method

$$f(x,y,z) = (x+y).(x+z) + y + z'$$

$$= x + yz + y + z'$$

$$= x.1.1 + 1.yz + 1.y.1 + 1.1.z'$$

$$= x(y+y')(z+z') + (x+x')yz + (x+x')y(z+z') + (x+x')(y+y')z'$$

$$= (xy+xy')(z+z') + xyz + x'yz + (xy+x'y)(z+z') + (x+x')(yz+y'z')$$

$$= xyz + xyz' + xy'z + xy'z' + \cancel{xyz} + x'yz + \cancel{xyz} + \cancel{xyz'} + \cancel{x'yz} + x'yz' + \cancel{xyz}$$
$$+ \cancel{xy'z'} + \cancel{x'yz} + x'y'z'$$

Q. 3. Let $E(x, y, z) = [(x+y)' + (x' . z)]'$ be a Boolean expression. Write $E(x, y, z)$ in disjunctive normal form.

x	y	z	$x+y$	$(x+y)'$	x'	$x'.z$	$(x+y)'+(x'.z)$	$[(x+y)'+(x'.z)]'$	Minterms
1	1	1	1	0	0	0	0	1	$m_1 = xyz$
1	1	0	1	0	0	0	0	1	$m_2 = xyz'$
1	0	1	1	0	0	0	0	1	$m_3 = xy'z$
1	0	0	1	0	0	0	0	1	$m_4 = xy'z'$
0	1	1	1	0	1	1	1	0	
0	1	0	1	0	1	0	0	1	$m_5 = x'yz'$
0	0	1	0	1	1	1	1	0	
0	0	0	0	1	1	0	1	0	

Required DN Form

$$\boxed{E(x,y,z) = xyz + xyz' + xy'z + xy'z' + x'yz'}$$

2nd Method

$$E(x,y,z) = \left[(x+y)' + (x'.z)\right]'$$

$$= ((x+y)')'.(x'.z)'$$

$$= (x+y).(x+z')$$

$$= x + yz'$$

$$E(x,y,z) = x + yz'$$

$$= x(y+y')(z+z') + (x+x')yz'$$

$$= (xy+xy')(z+z') + xyz' + x'yz'$$

$$= xyz + xyz' + xy'z + xy'z' + \cancel{xyz'} + x'yz'$$

$$\boxed{E(x,y,z) = xyz + xyz' + xy'z + xy'z' + x'yz'}$$

DPP-6

Topic : Disjunction Normal Form (D N Form)

Q.1. *Represent each of the following Boolean function in disjunctive normal form.*

(i) $f(x_1, x_2) = (x_1 + x_2)(x'_1 + x'_2)$

(ii) $f(x, y) = x + xy$

(iii) $f(x_1, x_2, x_3) = x_1 x'_2 + x_3$

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Thank You

DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -2 : Functions and Boolean Algebra

Lecture - 07

Today's Target

- *Conjunctive Normal Form (C N Form)*

  OR

  Product of Sum Form (POS Form)

- *Conversion of disjunctive normal form(DNF)of a boolean function to its conjunctive normal form and vice*
- *AKTU PYQs*
- *DPPs*

Boolean Function

Disjunction Normal Form

Sum of minterm

Conjunction Normal Form

Product of Maxterm

Conjunctive Normal Form (C N Form)

OR

Product of Sum Form (POS Form)

OR

Canonical Form

A Boolean function which can be written as the product of Maxterm is called conjunctive normal form or C N Form

Note:- As Maxterm is the Sum of variables, so C N form is also called product of sum of form.

Step to find C N form

STEP-1: Make a truth table for given Boolean function

$\wedge$ or $\bullet$	Conjunction Meet Product	$T \quad T \;\Rightarrow\; T$ otherwise F $1 \quad 1 \;\Rightarrow\; 1$ otherwise 0
$\vee$ or $+$	Disjunction Join Addition	$F \quad F \;\Rightarrow\; F$ otherwise T $0 \quad 0 \;\Rightarrow\; 0$ otherwise 1
$-$ or $'$	Complement Prime Negation	$T \longrightarrow F \qquad F \longrightarrow T$ $1 \longrightarrow 0 \qquad 0 \longrightarrow 1$

STEP-2: See the entries 0 in the last column of truth table and write corresponding maxterms.

STEP-3: By product of all maxterm, we will get required C N form or POS form

Note:

For Minterm:- 1 → *Use same variable*

0 → *Use Complement of variable*

For Maxterm:- 0 → *Use same variable*

1 → *Use Complement*

Q. 1:- Express the Boolean function f(a,b,c) = a b + a' c as a product of maxterm.

$f(a,b,c) = ab + a'c$

a	b	c	ab	a'	a'c	f(a,b,c) = ab + a'c	Maxterm (M)
1	1	1	1	0	0	1	
1	1	0	1	0	0	1	
1	0	1	0	0	0	0	$M_1 = a' + b + c'$
1	0	0	0	0	0	0	$M_2 = a' + b + c$
0	1	1	0	1	1	1	
0	1	0	0	1	0	0	$M_3 = a + b' + c$
0	0	1	0	1	1	1	
0	0	0	0	1	0	0	$M_4 = a + b + c$

Required CN Form

$$\boxed{f(a,b,c) = (a'+b+c')(a'+b+c)(a+b'+c)(a+b+c)}$$

2nd Method

$f(a,b,c) = (ab + a'c)$

$= (ab+a')(ab+c)$

$= (a'+ab).(c+ab)$

$= (a'+a)(a'+b)(c+a)(c+b)$

$= 1\,(a'+b)(a+c)(b+c)$

$= (a'+b)(a+c)(b+c)$

① $(a+b)' = a'.b'$

② $(ab)' = a'+b'$

De Morgan's law

$\boxed{a+b.c = (a+b).(a+c)}$ Distributive law

$$f(a,b,c) = (a'+b+0)(a+0+c)(0+b+c)$$

$$= (a'+b+cc')(a+bb'+c)(aa'+b+c)$$

Using Distributive law

$$= (a'+b+c)(a'+b+c')(a+b+c)(a+b'+c)\overset{X}{(a+b+c)}\overset{X}{(a'+b+c)}$$

$$\boxed{f(a,b,c) = (a'+b+c)(a'+b+c')(a+b+c)(a+b'+c)}$$

Q. 2:- Let $f(x, y, z) = [(x + y)' + .(x'.z)]'$ be a Boolean expression. Write $f(x, y, z)$ in conjunctive normal form **(AKTU-2016)**

$$f(x,y,z) = [\underbrace{(x+y)' + (x'.z)}]'$$

x	y	z	$x+y$	$(x+y)'$	x'	$x'.z$	$(x+y)'+(x'.z)$	$f(x,y,z)=[(x+y)'+(x'.z)]'$
1	1	1	1	0	0	0	0	1
1	1	0	1	0	0	0	0	1
1	0	1	1	0	0	0	0	1
1	0	0	1	0	0	0	0	1
0	1	1	1	0	1	1	1	0 ✓
0	1	0	1	0	1	0	0	1
0	0	1	0	1	1	1	1	0 ✓
0	0	0	0	1	1	0	1	0 ✓

$M_1 = (x + y' + z')$

$M_2 = (x + y + z')$

$M_3 = (x + y + z)$

Required CNF Form

$$\boxed{\begin{aligned} f(x,y,z) = &(x+y'+z') \\ &(x+y+z') \\ &(x+y+z) \end{aligned}}$$

2nd Method

$$f(x,y,z) = [(x+y)' + (x'.z)]'$$

$$= (x+y) . (x'.z)'$$

$$= (x+y) . (x+z')$$

$$= (x+y+zz')(x+yy'+z')$$

$$= (x+y+z)(x+y+z')(x+y+z')(x+y'+z')$$

$$\boxed{f(x,y,z) = (x+y+z)(x+y+z')(x+y'+z')}$$

Conversion of disjunctive normal form(DNF) of a boolean function to its conjunctive normal form and vice – versa

DN $\longrightarrow$ CN OR CN $\longrightarrow$ DN

Q.3 Convert the boolean function $f(x, y) = xy' + x'y + x'y'$ to its conjunctive normal form

$f(x, y) = xy' + x'y + x'y'$

complete DN Form for 2 variables x and y

$f(x, y) = xy + xy' + x'y + x'y'$

$f'(x, y) = xy$

$f''(x, y) = (xy)'$

$\boxed{f''(x, y) = x' + y'}$

Required CN Form

Q.4 *Convert the boolean function* $f(x,y,z) = (x'+y+z')(x'+y+z)(x+y'+z)$ *to its disjunctive normal form.*

$$f(x,y,z) = (x'+y+z')(x'+y+z)(x+y'+z)$$

complete CN Form For three variables x, y and z

$$f(x,y,z) = (x+y+z)(x+y+z')(x+y'+z)(x'+y+z)(x'+y'+z')$$
$$(x'+y'+z)(x'+y+z')(x+y'+z')$$

$$f'(x,y,z) = (x+y+z)(x+y+z')(x'+y'+z')(x'+y'+z)(x+y'+z')$$

$$f''(x,y,z) = [\underbrace{(x+y+z)(x+y+z')(x'+y'+z')(x'+y'+z)}(x+y'+z')]'$$

$$f''(x,y,z) = [(x+y+z)(x+y+z')(x'+y'+z')(x'+y'+z)]' + (x+y'+z')'$$

$$f''(x,y,z) = [(x+y+z)(x+y+z')(x'+y'+z')]' + (x'+y'+z)' + x'yz$$

$$f''(x,y,z) = [(x+y+z)(x+y+z')]' + (x'+y'+z')' + xyz' + x'yz$$

$$f''(x,y,z) = (x+y+z)' + (x+y+z')' + xyz + xyz' + x'yz$$

$$\boxed{f''(x,y,z) = x'y'z' + x'y'z + xyz + xyz' + x'yz}$$

Required DN Form

DPP- 7

Topic : *Conjunctive Normal Form (C N Form)*

Q.1. *Express the Boolean function in to C N form* $f(x\,,\ y) = x + x'.y$

Q.2. *Find the product of sum expansion of each of the following*

(i) $f(x\,,y,z) = (x+z)\,y$

(i) $f(x\,,y,z) = xy'$

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DISCRETE STRUCTURES & THEORY OF LOGICS
(Discrete Mathematics)

UNIT - 2 : Functions and Boolean Algebra

Lecture - 08

Today's Target

- *K – Map for SOP form*
- PYQ
- DPP

Minimization of Boolean Function

1. By Karnaugh Map OR By K-Map
2. By Boolean algebra

K – Map :

The K – Map is a pictorial representation of truth table of the Boolean functions and used to minimize the Boolean functions

Note :

K – Map Matrix contain $\boxed{2^n \text{ cell / square}}$, where n is the total number of variables

K – Map

For SOP form

For POS form

Next Lecture

K – Map for SOP form

1. For two variables x and y :

- No. of cells in K – Map $= 2^2 = 4$
- Each cell contain a minterm

$x \backslash y$	y'	y
x'	$x'y'$ 0	$x'y$ 1
x	xy' 2	xy 3

$x \backslash y$	0	1
0	00 0	01 1
1	10 2	11 3

2. For three variables x , y and z

- No. of cells in K – Map $= 2^3 = 8$
- Each cell represent a minterm

$x \backslash yz$	$y'z'$	$y'z$	yz	yz'
x'	$x'y'z'$ 0	$x'y'z$ 1	$x'yz$ 3	$x'yz'$ 2
x	$xy'z'$ 4	$xy'z$ 5	xyz 7	xyz' 6

$x \backslash yz$	00	01	11	10
0	000 0	001 1	011 3	010 2
1	100 4	101 5	111 7	110 6

3. For Four variables x , y, z and w

- No. of cells in K – Map = $2^4 = 16$
- Each cell represent a Minterm

$xy \backslash zw$	$z'w'$	$z'w$	zw	zw'
$x'y'$	$x'y'z'w'$ 0	$x'y'z'w$ 1	$x'y'zw$ 3	$x'y'zw'$ 2
$x'y$	$x'yz'w'$ 4	$x'yz'w$ 5	$x'yzw$ 7	$x'yzw'$ 6
xy	$xyz'w'$ 12	$xyz'w$ 13	$xyzw$ 15	$xyzw'$ 14
xy'	$xy'z'w'$ 8	$xy'z'w$ 9	$xy'zw$ 11	$xy'zw'$ 10

$xy \backslash zw$	00	01	11	10
00	0000 0	0001 1	0011 3	0010 2
01	0100 4	0101 5	0111 7	0110 6
11	1100 12	1101 13	1111 15	1110 14
10	1000 8	1001 9	1011 11	1010 10

Working Rule

1. Construct the K- Map

2. Put 1 in the cell corresponding to each Minterm given in the Boolean function

3. Making Groups of 1

   (i) Group of 1 contain 1, 2, 4, 8 …….. Cells

   (ii) Diagonal groups are not allowed

   (iii) Overlapping allowed in groups

   (iv) Group should be as large as possible

   (v) No 1 should be as left without grouping

4. Write the Minimised term for all the groups keeping in mind

   (i) Write only unchanged variable for each group

   (ii) Check rows and column values for each group

5. Add all the minimised term to get required result

For 2 Variables

Q. 1. Using the K – Map to find minimal form of the Boolean functions $f(X,\ Y) = XY + X'Y + X'Y'$

$= \sum(0,1,3)$

K-map for two variables

X \ Y	Y'	Y
X'	1	1
X		1

Hence, the required minimal form

$f(X,Y) = X' + Y$

Q. 2. *Using the K – Map to find minimal form of the Boolean functions* $f(x, y) = x\,y + x'y'$

$= \Sigma(0,3)$

K-map for two variables

$x \backslash y$	y'	y
x'	1	
x		1

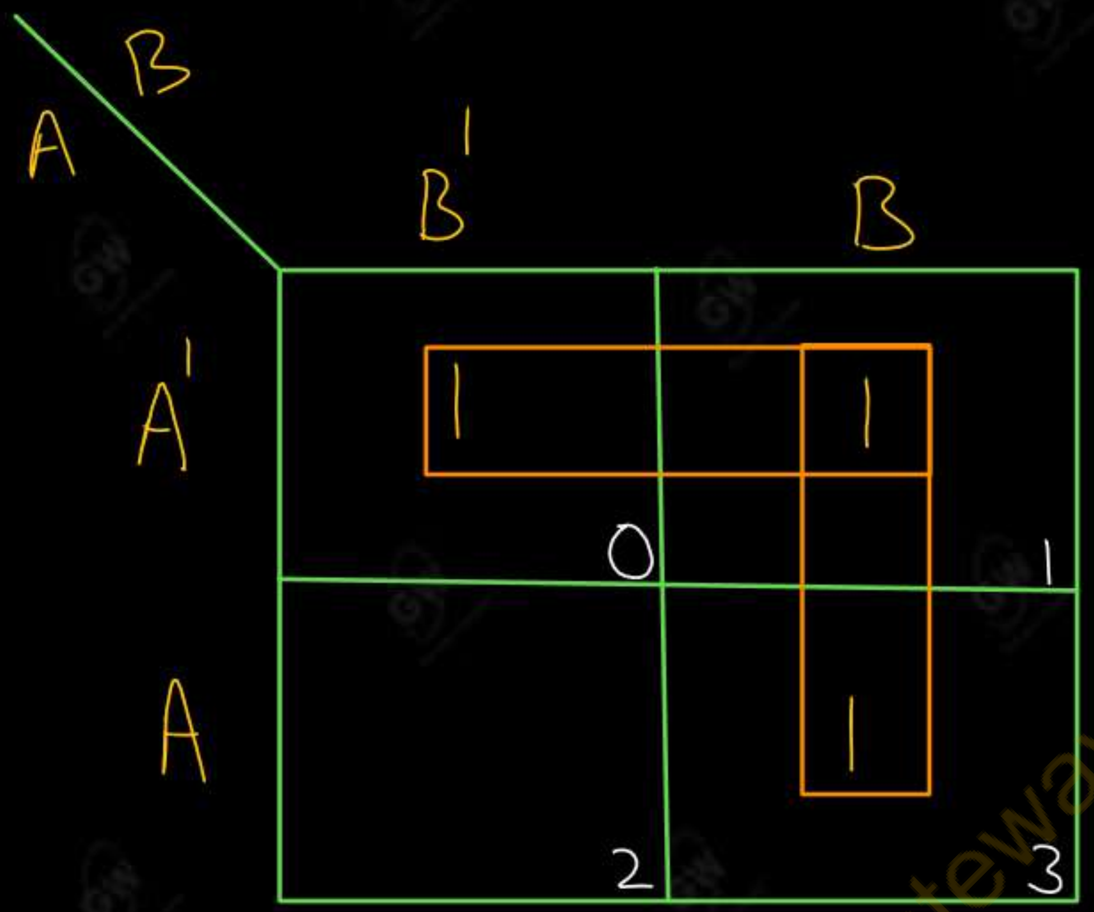
Required Minimal form

$f(x,y) = x'y' + xy$

Q. 3. Using the Karnaugh Map to find minimal form of the Boolean functions $f(A,\ B) = \Sigma(0,\ 1, 3)$

$= A'B' + A'B + AB$

K-map for 2 variables



Required Minimal form

$f(A, B) = A' + B$

Q. 4. Find using the K – Map a minimal form of the Boolean functions

K-map for 3 variables

$$f(x,\ y,\ z) = x\,y\,z + x\,y\,z' + x\,y'\,z + x'\,y\,z + x'\,y'\,z$$

$x \backslash yz$	$y'z'$	$y'z$	yz	yz'
x'		1	1	
x		1	1	1

Required Minimal form

$$f(x,y,z) = z + xy$$

Q. 5. Find using the K – Map a minimal form of the Boolean functions

$$f(A, B, C) = ABC + ABC' + A'BC' + A'B'C$$

K-map for 3 variables

$$= \sum(1, 2, 6, 7)$$

A \ BC	$B'C'$ 00	$B'C$ 01	BC 11	BC' 10
A' 0		1		1
A 1			1	1

Required minimal form

$$f(A, B, C) = AB + BC' + A'B'C$$

Q. 6. Find using the K – Map a minimal form of the Boolean functions

$$F(A,\ B,\ C) = \Sigma\,(0,\ 1,\ 2,\ 4,\ 5,\ 6)$$

K-map for 3-variables

A \ BC	B'C'	B'C	BC	BC'
A'	1 (0)	1 (1)	(3)	1 (2)
A	1 (4)	1 (5)	(7)	1 (6)

Required minimal form

$$F(A,B,C) = B' + C'$$

For Four Variables

Q. 7:- Use K – Map to find a minimal form $= \sum_m (2, 7, 8, 10, 12)$

$$f(A\ B, C, D) = A'B\,C\,D + A\,B'C\,D' + A'B'C\,D' + AB\,C'D' + A\,B'C'D'$$

AB \ CD	C'D'	C'D	CD	CD'
A'B'				1
A'B			1	
AB	1			
AB'	1			1

Required minimal form

$$f(A, B, C, D) = AC'D' + B'CD' + A'BCD$$

Q. 8:- Simplify the Boolean function $F(A, B, C, D) = \Sigma(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11)$ V. IMP

AB \ CD	C'D'	C'D	CD	CD'
A'B'	1 0	1 1	1 3	1 2
A'B	1 4	1 5	1 7	1 6
AB	12	13	15	14
AB'	1 8	1 9	1 11	10

Simplified form

$$F(A, B, C, D) = A' + B'C' + B'D$$

Q. 9:- Solve the following Boolean function using K – Map

$$f(A, B, C, D) = \Sigma_m(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$$

AB \ CD	$C'D'$	$C'D$	CD	CD'
$A'B'$	1 (0)	(1)	(3)	1 (2)
$A'B$	1 (4)	1 (5)	1 (7)	1 (6)
AB	(12)	1 (13)	1 (15)	(14)
AB'	1 (8)	(9)	(11)	1 (10)

Required minimal form

$$f(A, B, C, D) = BD + A'B + B'D'$$

Q. 10:- Solve the following Boolean functions using K – Map.

$$F(A,\ B,\ C,\ D) = \Sigma_m(m_0, m_1, m_2, m_4, m_5, m_6, m_8, m_9, m_{12}, m_{13}, m_{14})$$

AB \ CD	C'D'	C'D	CD	CD'
A'B'	1 0	1 1	3	1 2
A'B	1 4	1 5	7	1 6
AB	1 12	1 13	15	1 14
AB'	1 8	1 9	11	10

Required function

$$F(A, B, C, D) = C' + A'D' + BD'$$

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DPP- 8

Topic : K – Map for SOP form

Q.1. Using the K – Map to find minimal form of the Boolean functions $f(x, y) = x'y + xy$

Q. 2. Using the Karnaugh Map to find minimal form of the Boolean functions $f(A, B) = \Sigma(0, 2)$

Q.3. Find using the K – Map a minimal form of the Boolean functions $f(X, Y, Z) = XYZ + XYZ' + X'YZ$

Q. 4. Find using the K – Map a minimal form of the Boolean functions $f(A, B, C) = \Sigma_m (0, 2\ 3\ 4, 7)$

Q. 5. Use K – Map to find a minimal form $f(x, y, z, w) = x\,y\,z\,w + x\,y\,z\,w' + x\,y'z\,w' + x'y'z\,w + x'y'z\,w'$

Q.6 Solve $E(x, y, z, t) = \Sigma\,(0, 2, 6, 8, 10, 12, 14, 15)$ using K – Map.

Q.7. Solve the following Boolean functions using K – Map $F(A, B, C, D) = \Sigma\,(0, 2, 5, 7, 8, 10, 13, 15)$

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DISCRETE STRUCTURES & THEORY OF LOGICS
(Discrete Mathematics)

UNIT - 2 : Functions and Boolean Algebra

Lecture - 09

Today's Target

- *K – Map for POS form*
- *Simplification of Boolean function by Boolean algebra*
- PYQ
- DPP

K – Map

For SOP form

OR

(Sum of Minterm)

OR

(DN Form)

For POS form

OR

(Product of Maxterm)

OR

(CN Form)

K – Map for POS form

1. For two variables x and y :

- No. of cells in K – Map = 4
- Each cell contain a Maxterm

$x \backslash y$	y	y'
x	$x+y$ (0)	$x+y'$ (1)
x'	$x'+y$ (2)	$x'+y'$ (3)

$x \backslash y$	0	1
0	00 (0)	01 (1)
1	10 (2)	11 (3)

2. For three variables x , y and z

No. of cells in $K-Map = 2^3 = 8$

Each cell represent a Maxterm

$x \backslash yz$	$y+z$	$y+z'$	$y'+z'$	$y'+z$
x	0	1	3	2
x'	4	5	7	6

$x \backslash yz$	00	01	11	10
0	0	1	3	2
1	4	5	7	6

3. For Four variables x, y, z and w

No. of cells in K – Map $= 2^4 = 16$

Each cell represent a Maxterm

xy \ zw	z+w	z+w'	z'+w'	z'+w
x+y	0	1	3	2
x+y'	4	5	7	6
x'+y'	12	13	15	14
x'+y	8	9	11	10

xy \ zw	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Working Rule

1. Construct Karnaugh Map

2. Put 0 in the cell corresponding to each *Maxterm* given in the Boolean function

3. Making Groups of 0

    (i) Group of 0 contain 1, 2, 4, 8 .... Cells

    (ii) Diagonal groups are not allowed

    (iii) Overlapping allowed in groups

    (iv) Group should be as large as possible

    (v) No 0 should be as left without grouping

4. Write the Minimised term for all the groups keeping in mind

    (i) Write only unchanged variable for each group

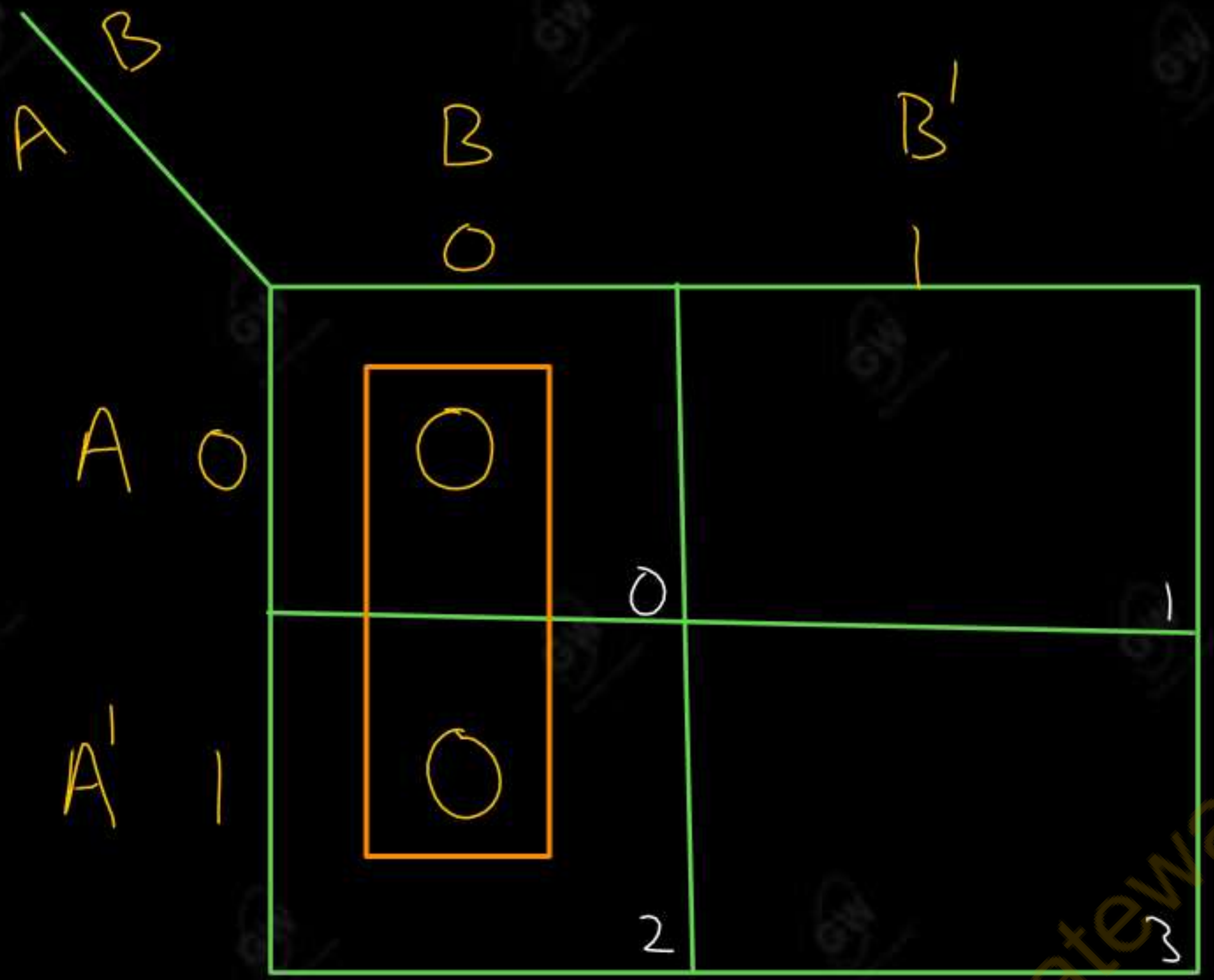
    (ii) Check rows and column values for each group

5. Add all the minimised term to get required result

For 2 Variables

Q. 1:- Minimise the Boolean expression $f(A, B) = \prod M(0, 2)$

$$= (A+B)(A'+B)$$



Required minimal form

$$f(A, B) = B$$

For 3 Variables

Q. 2:- *Draw the K – map and simplify the Boolean expression* $F(A, B, C) = \prod M(0, 1, 4, 5)$

$$= (A + B + C)(A + B + C')(A' + B + C)(A' + B + C')$$

A \ BC	$B+C$ 00	$B+C'$ 01	$B'+C'$ 11	$B'+C$ 10
A 0	0 (0)	0 (1)	(3)	(2)
A' 1	0 (4)	0 (5)	(7)	(6)

Required simplified form

$$F(A, B, C) = B$$

For 4 Variables

Q. 3:- Draw the K – map and simplify the Boolean function

$$F(A, B, C, D) = \prod M(1, 2, 5, 6, 8, 9, 15)$$

AB \ CD	$C+D$ 00	$C+D'$ 01	$C'+D'$ 11	$C'+D$ 10
00 $A+B$	0	$\boxed{0}$ 1	3	$\boxed{0}$ 2
01 $A+B'$	4	$\boxed{0}$ 5	7	$\boxed{0}$ 6
11 $A'+B'$	12	13	$\boxed{0}$ 15	14
10 $A'+B$	$\boxed{0}$ 8	$\boxed{0}$ 9	11	10

Simplified Boolean function

$$F(A, B, C, D) = (A + C + D')$$
$$(A + C' + D)(A' + B + C)$$
$$(A' + B' + C' + D')$$

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Q. 4:- Simplify using K – map to obtain a minimum POS expression :

$$F(A, B, C, D) = (A' + B' + C + D)(A + B' + C + D)(A + B + C' + D')(A' + B + C' + D')(A + B + C' + D)$$

AB \ CD	$C+D$ 00	$C+D'$ 01	$C'+D'$ 11	$C'+D$ 10
00 $A+B$			0	0
01 $A+B'$	0			
11 $A'+B'$	0			
10 $A'+B$			0	

Simplified Boolean function

$$F(A, B, C, D) = (B' + C + D)$$
$$(A + B + C')(B + C' + D')$$

Simplification of Boolean function by Boolean algebra

Q.5 Simplify the Boolean expressions

(a) $C(B + C)(A + B + C)$

(b) $XY + X'Z + YZ$

(C) $A + B(A + B) + A(A' + B)$

(a) $C(B+C)(A+B+C)$

$= (CB+CC)(A+B+C)$

$= (CB+C)(A+B+C) \quad \{\because CC=C\}$

$= C(A+B+C) \quad \{\because a + ac = a\}$

$= CA + CB + CC$

$= CA + CB + C$

$= CA + C$

$= C$

$\{a + ac = a\}$

(b) $XY + X'Z + YZ$

$= XY + X'Z + YZ(X+X')$

$\{X+X'=1\}$

$= \underline{XY} + X'Z + \underline{XYZ} + X'YZ$

$= XY + XYZ + X'Z + X'ZY$

$= XY(1+Z) + X'Z(1+Y)$

$= XY + X'Z \quad \{\because a+1=1\}$

(c) $A + B(A+B) + A(A'+B)$

$= A + AB + BB + AA' + AB$

$= A + AB + AB + B + 0 \quad \left\{\begin{array}{l}\because a \cdot a' = 0 \\ a \cdot a = a\end{array}\right.$

$= A + AB + B \quad \{\because a+a=a\}$

$= A + B \quad \{a + ac = a\}$

DPP- 9

Topic : K – Map for POS form

Q.1. Draw the K – map and simplify the Boolean function $F(A, B, C, D) = \prod M(1, 2, 4, 6, 8, 9)$

Q.2. Draw the K – map and simplify the Boolean function $F(A, B, C, D) = \prod M(3, 4, 6, 7, 11, 12, 13, 14, 15)$

Q. 3. Simplify the following using Boolean algebra

(a) $X = (A + B + AB)(A + C)$

(b) $A = XY + (XZ)' + XY'Z\,(XY + Z)$

(C) $A = XY + XYZ + X'Y + XY'Z$

(d) $A = X\,[Y + Z\,(XY + XZ)']$

(e) $A = XY + X'YZ' + YZ$

(f) $A = (XY' + Z)(X + Y')Z$

(g) $A = XY' + Z + (X' + Y)\,Z'$

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