Take-home 02 (due April 6th)

General Instructions for take-home exams: Please submit your answers as one pdf document, where you describe the logic and procedure you used to solve the problem and where you plot your results. Attach your python or matlab or R scripts and indicate which code solves which problem. Annotate your codes properly. We will evaluate your answers and codes in terms of correctness and clarity. Compress all your relevant files into one compressed file and submit to tengtianyuan@pku.edu.cn.

可以用中文写作业。文件/邮件命名格式请参照“编程语言—本科生/研究生—姓名—学号”。例：“python—研究生—张三—1901111111”。

1. (5 points) According to the Cha Sa-soon example of Lee & Wagenmakers' (2013) book (Section 5.5), the number of correct answers in Cha Sa-soon's 949 unsuccessful attempts had ranged from 15 to 25. She passed the exam on the 950 attempts by scoring the required minimum of 30 correct questions. The exam had 50 four-choice questions. Lee & Wagenmakers had made Bayesian inferences about the latent probability (θ) that Cha Sa-soon can correctly answer any one question, assuming that this probability did not change across the 950 exams. Please estimate θ using maximum likelihood estimates (MLE) and compare the MLE result with that of Bayesian inference (i.e. Fig. 5.9).

(1) Please estimate θ using maximum likelihood estimates (MLE) and compare the MLE result with that of Bayesian inference (i.e. Fig. 5.9).

(2) Imagine a grandpa who took the same exams. He passed the exam on his fifth attempt. The number of correct answers in his four unsuccessful attempts also ranged from 15 to 25. Assume the same prior distribution as in Section 5.5. Plot the posterior distribution about his latent probability θ. Is the posterior distribution of his latent probability different from Cha Sa-soon's? If so, explain the reason of such difference.

2. **(3 points)** In the two-country quiz problem (Section 6.4 of Lee & Wagenmakers 2013), suppose we know Person 1 is a Moldovan. Please run TwoCountryQuiz_jags.R and plot each person's posterior probability of being a Moldovan and each question's posterior probability of being a Moldovan history question.

3. **(7 points)** Below is a revised version of the two-country quiz problem. Suppose the probability for a Thai to correctly answer a Moldovan history question does not necessarily equal to the probability for a Moldovan to answer a Thai history question, which means we need β_1 and β_2 instead a single β. Other setting is the same as the original two-country quiz problem in Section 6.4 of Lee & Wagenmakers 2013.

(1) Please draw the graphical model for the revised problem (use the notations of Fig. 6.4).

(2) Please revise the model definition and RJAGS files (TwoCountryQuiz.txt & TwoCountryQuiz_jags.R) to implement your graphical model.

(3) Please run the revised codes until the MCMC chains have converged. Plot posterior distributions for β_1 and β_2.

(4) In the revised two-country quiz question, we are actually proposing a new model to explain the data. Please compare the DIC of this model ($\beta_1 \neq \beta_2$) with the DIC of the original model ($\beta_1 = \beta_2$).

(Hint: Beware that you may run into a problem known as model indeterminacy or label-switching. Please read Exercise 6.4.2 and its solution for details.)