1. **(20%) For a directional (symmetrical) coupler, the coupling length typically determines the coupling ratio, and the amplitude will couple periodically back and forth between the waveguides. Please check the coupling length ($L_{50\%}$) for a conventional silicon 3 dB directional coupler with 50%-50% coupling at 1550 nm and cite a reference. You can choose the waveguide dimension / gap based on your citation.**

Ans: We found a paper in which they utilized silicon-on-insulator (SOI) with thicknesses of 220 nm and 3 μm to design a directional coupler (DC). They pointed out that the 3μm SOI DC, when used with larger waveguide dimensions, exhibits reduced propagation loss to shape variations. On the other hand, it offers the advantage of robust performance in broadband applications [1].

The schematic diagram of a conventional DC is depicted in Figure 1. The DC consists of two symmetric arms with a constant width of W. Each arm includes a straight waveguide with a length of L_C and two S-bends, which serve as the input and output ports, respectively. A cross-sectional view of the coupling region is shown in Figure 1b. Careful design of the rib waveguide height H, slab thickness h, gap size G, and W is required to achieve the desired coupling strength.

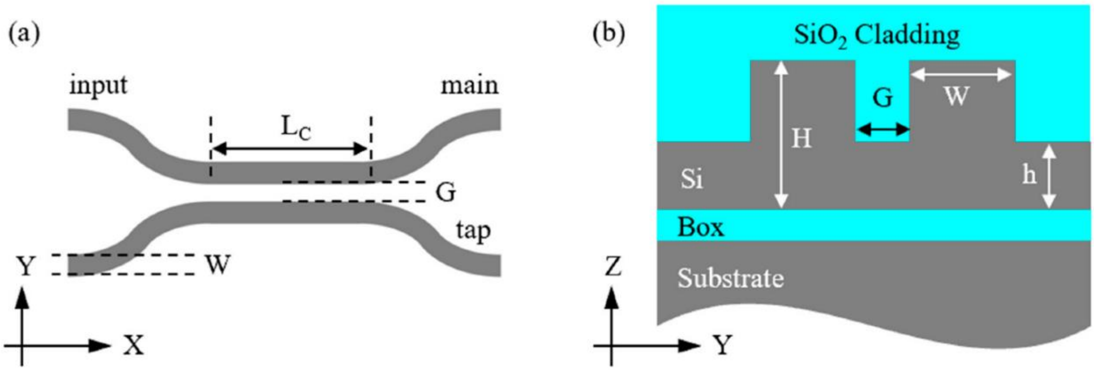

Figure 1. (a) Top view of a conventional DC. (b) Cross-sectional schematic of rib waveguides in the coupling region [1].

In a 3 μm SOI, the geometric parameters of the rib waveguide are represented by H = 3 μm, h = 1.8 μm, G = 2.2 μm, and W = 2.6 μm (as indicated by the blue line). In contrast, for a 220 nm SOI, the rib waveguide parameters are H = 220 nm, h = 70 nm, G = 150 nm, and W = 450 nm (as indicated by the red line). A finite difference eigenmode (FDE) solver was used to determine the effective indices of symmetric (even) and antisymmetric (odd) supermodes.

In Figure 2(a), the red line corresponds to the 220 nm SOI, while the blue line represents the 3 μm SOI. The solid line represents the symmetric (even) supermode of effective refractive index($n_{s,eff}$ 、 $n_{as,eff}$), and the dashed line represents the antisymmetric (odd) supermode. It can be observed that the 220 nm SOI exhibits significantly larger dispersion in its effective refractive index, with distinct dispersion characteristics between the even and odd modes. This leads to a substantial difference in effective indices between 1470 nm and 1630 nm, resulting in significant variations in the transmission coefficients T_{main} and T_{tap}.

Conversely, the 3 μm SOI has almost no dispersion in its effective refractive index, resulting in very small differences in coupling efficiency across the 1470-1630 nm range, as shown in Figure 2(b). For arm lengths L_C of 200 μm, 400 μm, and 600 μm, the coupling efficiency's sensitivity to wavelength is minimal within this wavelength range.

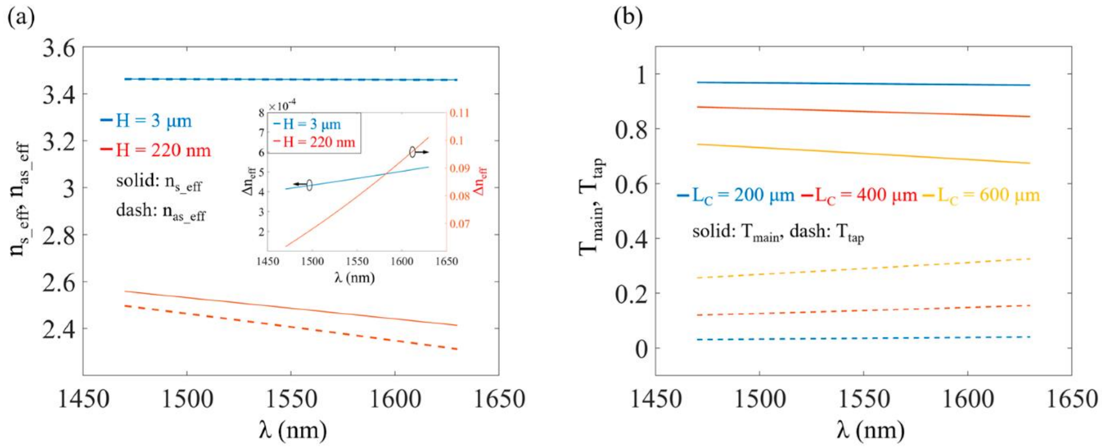

Figure 2. (a) Simulation results of the relationship between the n_{s_eff} (solid lines) and n_{as_eff} (dash lines) and λ in the coupling region of typical DC designs on the 3 μm SOI waveguide platform and the 220 nm SOI waveguide platform. The blue lines correspond to the structure with design parameters H = 3 μm, h = 1.8 μm, G = 2.2 μm and W = 2.6 μm, while the red lines represent the structure with H = 220 nm, h = 70 nm, G = 150 nm and W = 450 nm. The insert exhibits the relationship between Δn_{eff} and λ for designs based on both platforms. (**b**) Simulated T_{main} and T_{tap} as functions of λ for the design based on the 3 μm SOI waveguide platform when the L_C values are 200 μm, 400 μm, and 600 μm, respectively [1].

The transmission of the DC can be expressed as:

$$T_{main} = \cos^2\left(\pi L_C \cdot \frac{\Delta n_{eff}}{\lambda}\right)$$
$$T_{main} = \sin^2\left(\pi L_C \cdot \frac{\Delta n_{eff}}{\lambda}\right) \quad \text{-(eq.1)}$$

where T_{main} and T_{tap} represent the transmission of the main and tap ports, respectively.

Since this paper did not include the calculation of Tmain and Ttap for the 220 nm SOI, I derived the Δ_{neff} required by eq.1 from Figure 2(a) and calculated T_{main} and T_{tap}, as illustrated in Figure 3(a-c). Figure 3(d-g) corresponds to the 3 μm SOI.

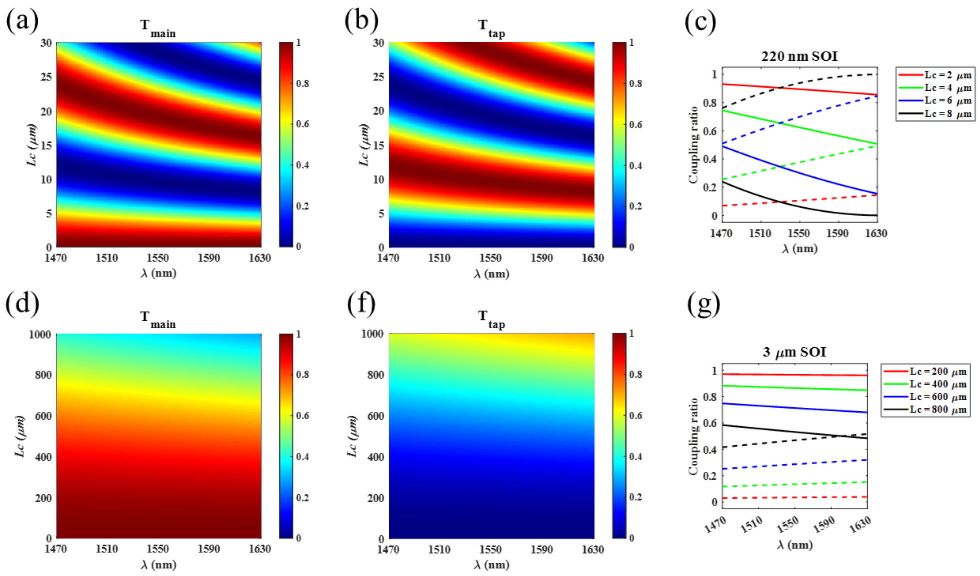

Figure 3 The coupling ratio for different arm lengths (Lc), where (a)-(c) correspond to 220 nm SOI (H = 220 nm, h = 70 nm, G = 150 nm and W = 450 nm), and (d)-(g) correspond to 3 μm SOI (H = 3 μm, h = 1.8 μm, G = 2.2 μm and W = 2.6 μm).

2. (30%) Please verify your answer above by the coupled mode theory $P(z)$ = $P0\cos^2(Kz)$ From chapter 10.4. If you do not have an effective index difference (Δn), you can use 0.05 as an example. Please compare the results.

Ans: We can get the Δ_{neff} values for 3 μm SOI and 220 nm SOI through Figure 1(a), which are 4.6e-4 and 0.08, respectively. Using Eq. 1, when we set the left-hand side of T_{main} and T_{tap} to 0.5, we can determine that the arm length L_c that makes cosine and sine equal to π/4 is denoted as $L_{50\%}$. Of course, we can also directly scan the values of T_{main} and T_{tap} for different L_c arm lengths through numerical calculations, as shown in Figure (4).

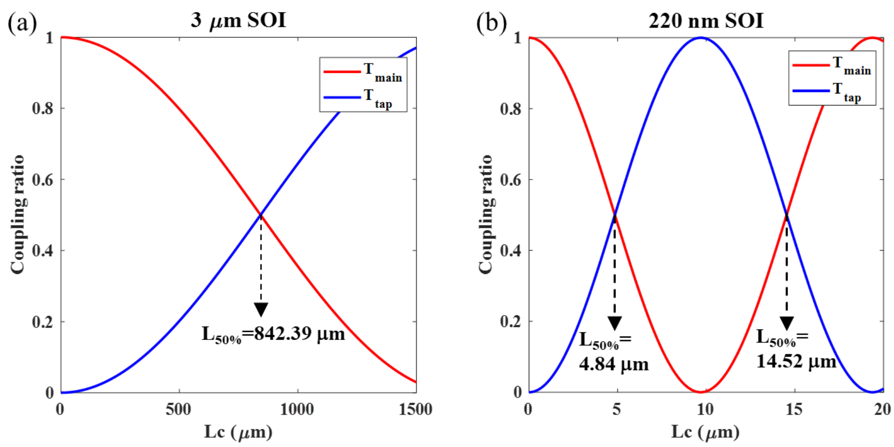

Figure 4 The coupling ratio of the 1550 nm wavelength DC at different arm lengths, where (a) corresponds to an arm length of 842 μm for the 3 μm SOI, denoted as $L_{50\%}$, and (b) shows an arm length of 4.84 μm for the 220 nm SOI, also denoted as $L_{50\%}$.

In addition to comparing $L_{50\%}$, we also compared the results of T_{main} and T_{tap} at L_c = 200, 400, and 600 μm, as shown in Figure 5. The left-hand side the results calculated using Equation 1, while the right-hand side shows the results obtained from the paper's simulations.

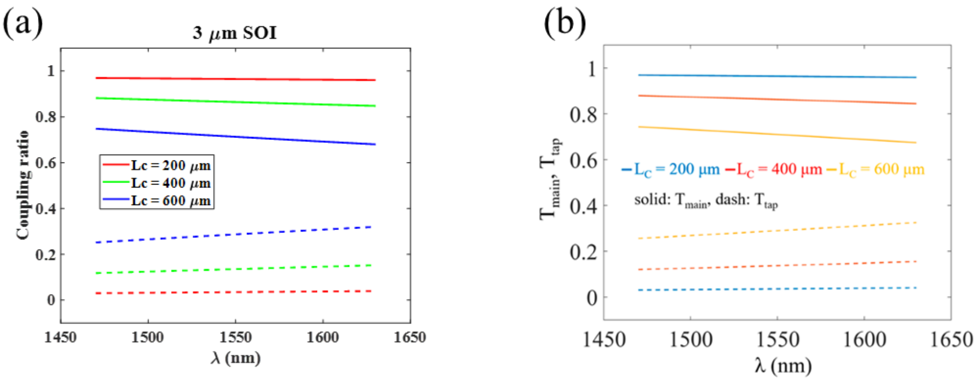

Figure 5(a) represents the results calculated using Equation 1. (b) shows the results simulated in the paper [1].

3. (50%) Please use BPM method to simulate the coupling length ($L_{50\%}$). How does this vary with different gap sizes?
   Hints: 1. you may use gap size between 0.4~2 μm for comparison. 2. The exemplary MATLAB for BPM is in Chapter 9-6.

Ans:

We build a waveguide with the same geometry as the directional coupler in the paper (W = 450 nm, G = 150 nm, the waveguide is Si, and the cladding is SiO2). We introduced a Gaussian-distributed light source into Waveguide 1 to represent the initial electric field intensity distribution within the waveguide, as shown in Figure 6(a). Additionally, we placed absorbing boundaries in the outer region of the waveguide, as depicted in Figure 6(b).

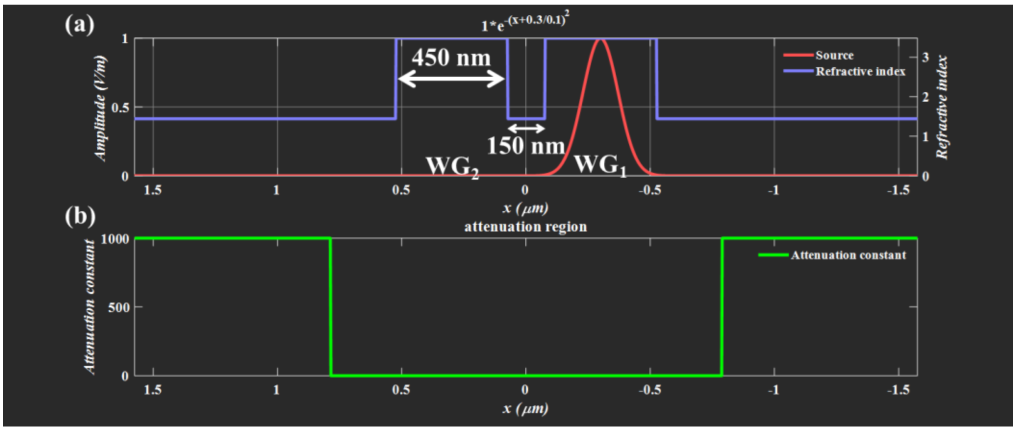

Figure 6 (a) The refractive index distribution of the waveguide in the directional coupler (W = 450 nm, G = 150 nm, the waveguide is Si, and the cladding is SiO2) and the initial electric field intensity distribution. (b) The absorbing boundaries for propagation.

The optical intensity distribution obtained after the initial electric field intensity propagates through the beam propagation method (BPM) is shown in Figure 7.

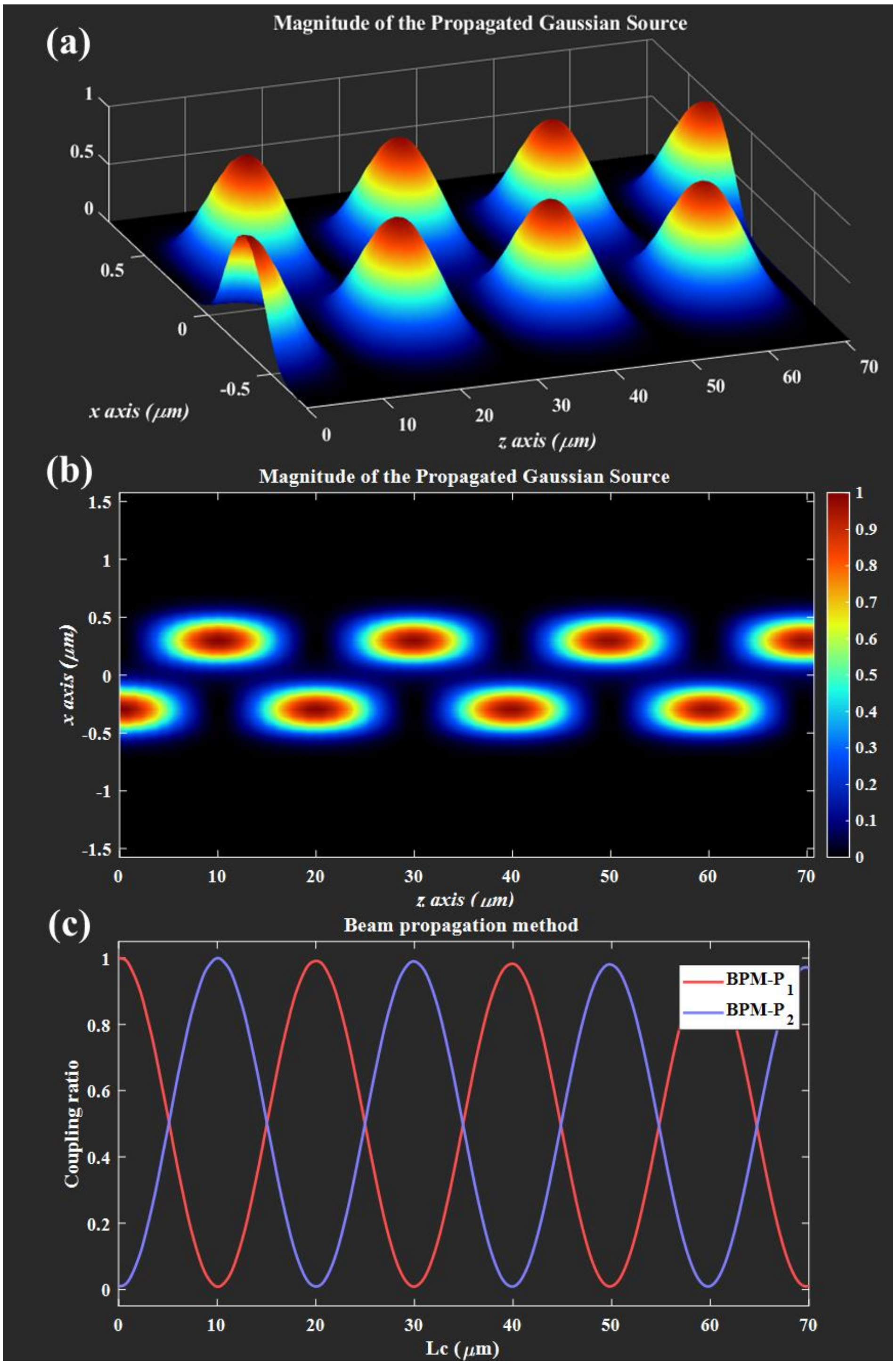

Figure 7 The results of the transfer back and forth between waveguides through BPM. (a) represents the side view, (b) the top view, and (c) illustrates the intensity distribution proportion within the waveguide.

We can get the BPM's $L_{50\%}$ and compare the coupling results of its propagation with those calculated by the Coupled Mode Theory (CMT), as shown in Figure 8. The results obtained from BPM and CMT are quite close, with BPM calculating a Δ_{neff} of

0.0775.

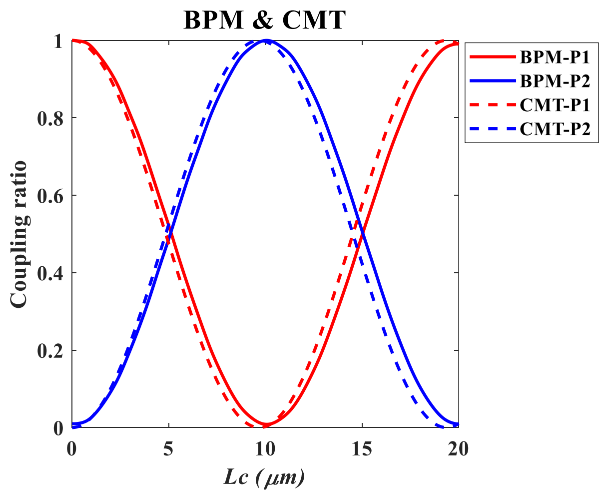

Figure 8 The results of the transfer back and forth between waveguides after BPM propagation (Solid line), the results Δ_{neff} in the paper is 0.08, based on using CMT calculated (dash line).

We chose a waveguide width of W = 450 nm, and the separation between the two symmetric waveguides is denoted as Gap. By scanning gap in the range of 100 to 250 nm, it can be observed that as gap increases, the Δ_{neff} of Supermodes decreases, leading to a reduced frequency of transfer back and forth between the waveguides, as depicted in Figures 9 and 10. The $L_{50\%}$ for Gap = 100 nm, 125 nm, 150 nm, 175 nm, 200 nm, and 250 nm are 2.4 μm, 3.05 μm, 5.00 μm, 8.15 μm, 17.15 μm, and 33.75 μm, respectively. Using Eq. 1, when we set the left-hand side of T_{main} and T_{tap} to 0.5, and we know the $L_{50\%}$ length, so we can determine that the Δ_{neff} that makes cosine and sine equal to π/4, and the Δ_{neff} for different gap sizes is shown in Figure 11.

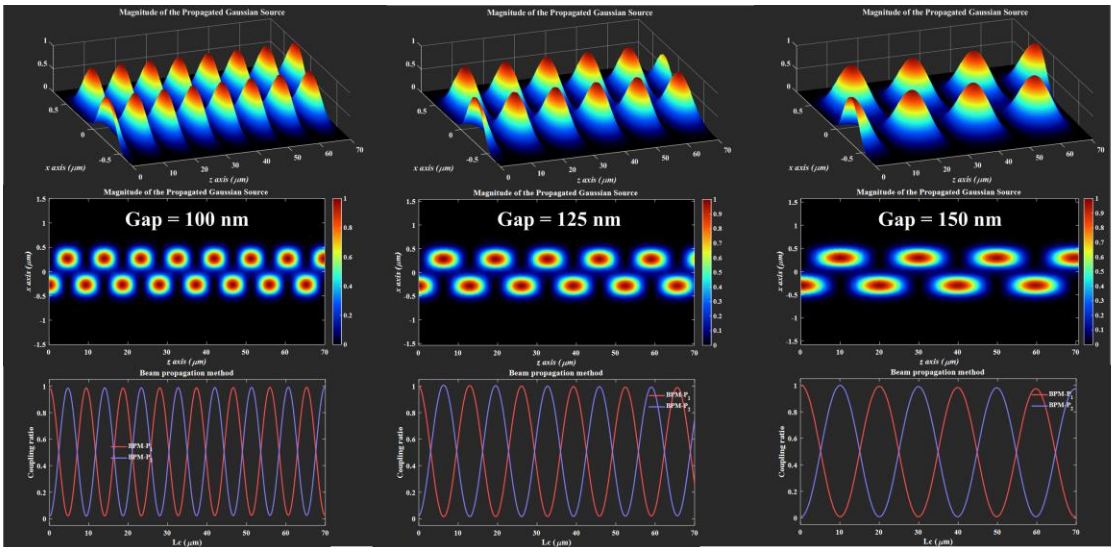

Figure 9 The results of the transfer back and forth between waveguides through BPM, the Gap = 100 nm, 125 nm, 150 nm, respectively.

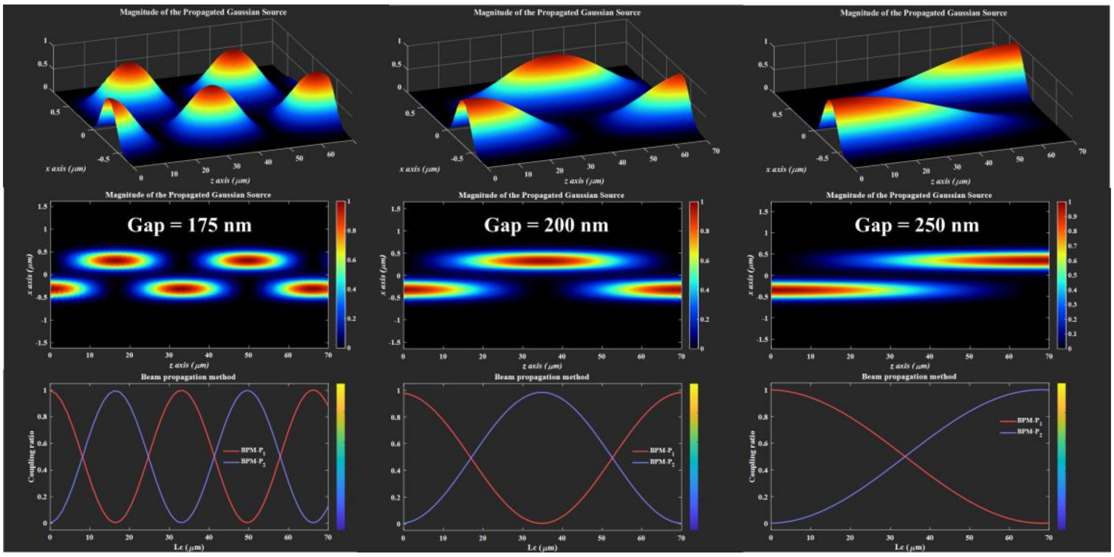

Figure 10 The results of the transfer back and forth between waveguides through BPM, the Gap = 175 nm, 200 nm, 250 nm, respectively.

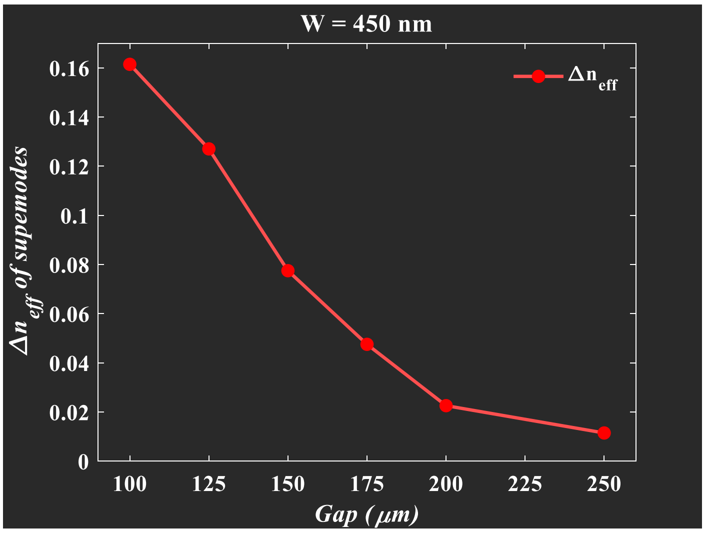

Figure 11 The Δ_{neff} for different gap sizes

Reference:

[1] Lv, D.; Wu, L.; Liu, C.; Li, A.; Wang, R.; Wu, A. Broadband and Low-Loss Silicon Photonic Directional Coupler for Signal Power Tapping on the 3 μm SOI Waveguide Platform. Photonics 2023, 10, 776. https://doi.org/10.3390/photonics10070776

Appendix:

Optical coupler and optical switch

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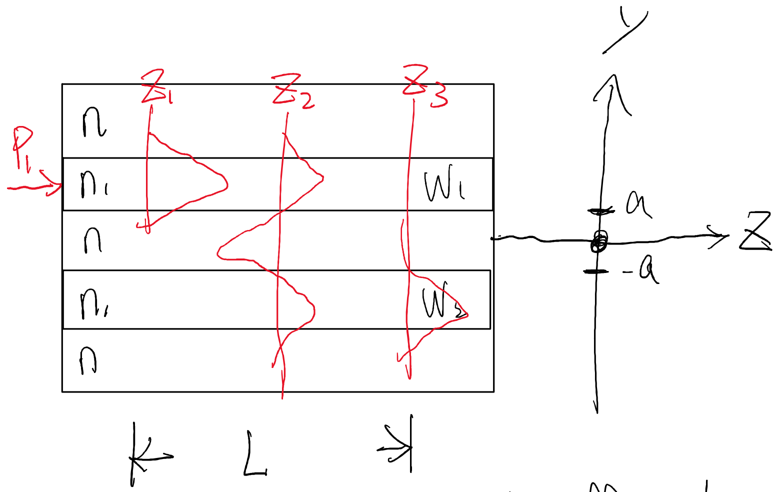


Assuming that the coupling only affects the amplitude of the waveguides and not their distribustion and propagation constants.

The two waveguides of wave function can expressed as

$$E_1(y,z) = a_1(z)\, e_1(y,z) = a_1(z)\, u_1(y)\, e^{-j\beta_1 z}$$

$$E_2(y,z) = a_2(z)\, e_2(y,z) = a_2(z)\, u_2(y)\, e^{-j\beta_2 z}$$

The waveguide 2 can be regarded as a scatter with Δn in waveguide 1, allowing the E_2 field to induce an additional P field.

$$\nabla^2 E - \frac{\varepsilon}{c^2}\frac{\partial^2 E}{\partial t^2} = \boxed{\mu_0 \frac{\partial^2 P}{\partial t^2}}$$

(wave equation in anisotropic medium)

$$P = \chi E = (\varepsilon_2 - \varepsilon)E_2 = (n_2^2 - n^2)E_2$$

The induced P can be regared as a new source $S_1 = \mu_0 \frac{\partial^2 P}{\partial t^2}$ at waveguide 1

$$n_2 = \frac{c_0}{c_2}$$

$$c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$c_2 = \frac{1}{\sqrt{\mu_0 \varepsilon_2}}$$

$$S_1 = \mu_0 \omega^2 P = \mu_0 \omega^2 (n_2^2 - n^2) E_2$$

$$= (k_2^2 - k^2) E_2$$

$= (k_2 - k)E_2$ $\qquad \omega\mu_0\varepsilon_0\varepsilon_2$

The New Helmholz equation :

$$\nabla^2 E_1 + k_1^2 E_1 = (k_2^2 - k)E_2$$

$$\nabla^2 E_2 + k_2^2 E_2 = (k_1^2 - k)E_1$$

$$E_1(y,z) = a_1(z)\, e_1(y,z) = a_1(z)\, u_1(y)\, e^{-j\beta_1 z}$$

$$E_2(y,z) = a_2(z)\, e_2(y,z) = a_2(z)\, u_2(y)\, e^{-j\beta_2 z}$$

$$\nabla^2 E_1 = \frac{\partial^2}{\partial z^2}\left[a_1(z) e_1(y,z)\right] = \frac{\partial}{\partial z}\left[a_1' e_1 + a_1 e_1'\right]$$

$$= a_1'' e_1 + a_1' e_1' + a_1' e_1' + a_1 e_1''$$

$$= a_1'' e_1 + 2a_1' e_1' + a_1 e_1''$$

$$e_1 \frac{\partial^2}{\partial z^2} a_1 + 2\frac{\partial}{\partial z} a_1 \frac{\partial}{\partial z} e_1 + \underbrace{a_1 \frac{\partial^2}{\partial z^2} e_1}_{\cancel{-k_1^2 a_1 e_1}} + \cancel{k_1^2 a_1 e_1} = (k_2^2 - k^2)E_2$$

$$e_1 \frac{\partial^2 a_1}{\partial z^2} + 2\frac{\partial a_1}{\partial z}\frac{\partial e_1}{\partial z} = (k_2^2 - k^2)\, a_2 e_2$$

$$e_2 \frac{\partial^2 a_2}{\partial z^2} + 2\frac{\partial a_2}{\partial z}\frac{\partial e_2}{\partial z} = (k_1^2 - k^2)\, a_1 e_1$$

If propagated amplitude slowly vary

$\frac{\partial^2 a}{\partial z^2} \ll \frac{\partial a}{\partial z}\frac{\partial e}{\partial z}$, then

$a_2 \quad \beta_2 \quad u_2(y) \quad \beta_2 \qquad k_1^2 \quad a_1\, u_1(y) \quad \beta_1$

$$2\frac{\partial a_1}{\partial z}(-j\beta_1)\, u_1(y)\, e^{-j\beta_1 z} = (k_2^2 - k^2)\, a_2 u_2(y)\, e^{-j\beta_2 z}$$

$$\Rightarrow \frac{\partial a_1}{\partial z} e^{-j\beta_1 z} = -\frac{1}{2j\beta_1}\cdot (k_2^2 - k^2)\, a_2 u_2(y)\, e^{-j\beta_2 z}$$

$$\frac{\partial a_2}{\partial z} e^{-j\beta_2 z} = -\frac{1}{2j\beta_2}\cdot (k_1^2 - k^2)\, a_1 u_1(y)\, e^{-j\beta_1 z}$$

$n_1 k_0 = k_1 \int u_1^* (y)\, dy \qquad \beta_0 \qquad \times \int u_1^*(y)\, dy$

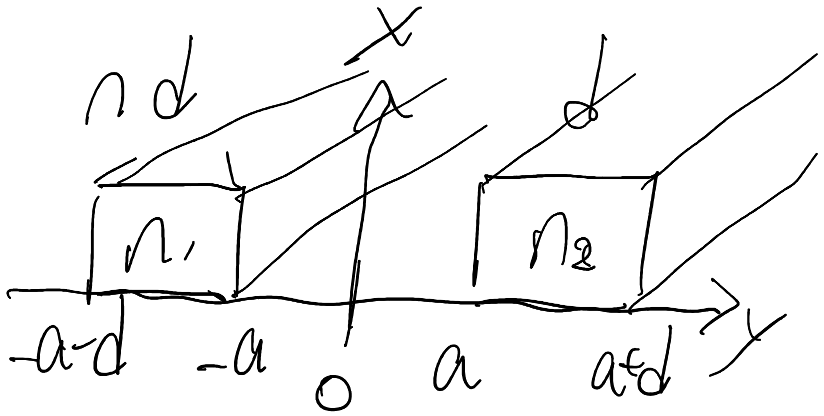
$\partial z \quad -j\beta_2$

$n_1 k_0 = k_1$

$\frac{\partial a_1}{\partial z} e^{-j\beta_1 z} = \frac{-j}{2}\frac{k_0^2}{\beta_1}(n_2^2-n^2) a_2 e^{-j\beta_2 z} u_2(y) \quad \times \int u_1^*(y)\,dy$

$\frac{\partial a_2}{\partial z} e^{-j\beta_2 z} = \frac{-j}{2}\frac{k_0^2}{\beta_2}(n_1^2-n^2) a_1 e^{-j\beta_1 z} u_1(y) \quad \times \int u_2^*(y)\,dy$

$$\frac{\partial a_1}{\partial z} e^{-j\beta_1 z}\int_a^{a+d} u_1^*(y)\,dy = -j\frac{1}{2}\frac{k_0^2}{\beta_1}(n_2^2-n^2) a_2 e^{-j\beta_2 z}\int_a^{a+d} u_1^*(y)u_2(y)\,dy$$



assume $C_{21} \equiv \frac{1}{2}\frac{k_0^2}{\beta_1}(n_2^2-n^2)\int_a^{a+d} u_1^*(y)u_2(y)\,dy$

$C_{12} \equiv \frac{1}{2}\frac{k_0^2}{\beta_2}(n_1^2-n^2)\int_{-a-d}^{-a} u_2^*(y)u_1(y)\,dy$

$\Delta\beta = \beta_1 - \beta_2$

$\boxed{\text{一次}}$

$\frac{da_1}{dz} = -jC_{21} e^{j\Delta\beta z} a_2(z)$ (Coupled Mode Equations)

$\frac{da_2}{dz} = -jC_{12} e^{-j\Delta\beta z} a_1(z)$

$\boxed{\text{二次}}$ $\frac{\partial}{\partial z}$

$$\frac{d^2 a_1}{dz^2} - j\Delta\beta\frac{da_1}{dz} + C_{12}C_{21}a_1 = 0$$

$$\frac{d^2 a_2}{dz^2} + j\Delta\beta\frac{da_2}{dz} + C_{21}C_{12}a_2 = 0$$

$j\Delta z$

guess: $a_1(z) = a_0 e^{j\alpha z}$

$$-\alpha^2 + \Delta\beta\,\alpha + C_{21}C_{12} = 0$$

solve: $\alpha = \frac{\Delta\beta}{2} \pm \sqrt{\left(\frac{\Delta\beta}{2}\right)^2 + C_{21}C_{12}} = \frac{\Delta\beta}{2} \pm \gamma$

assume $\gamma^2 = \left(\frac{\Delta\beta}{2}\right)^2 + C_{21}C_{12}$

$$a_1(z) = a_1(0)\, e^{j\left(\frac{\Delta\beta}{2} \pm \gamma\right)z}$$

$$= a_1(0)\, e^{j\frac{\Delta\beta}{2}z}\left[\cos\gamma z - j\frac{\Delta\beta}{2\gamma}\sin\gamma z\right]$$

同理

$$a_2(z) = a_2(0)\,\frac{C_{12}}{j\gamma}\, e^{-j\frac{\Delta\beta}{2}z}\,\sin\gamma z$$

$$I_1 = |a_1|^2$$

$$I_1(z) = I_1(0)\left[\cos^2(\gamma z) + \left(\frac{\Delta\beta}{2\gamma}\right)^2\sin^2(\gamma z)\right]$$

$$I_2(z) = I_2(0)\,\frac{|C_{12}|^2}{\gamma^2}\,\sin^2(\gamma z)$$

$$\Delta\beta = \beta_1 - \beta_2$$

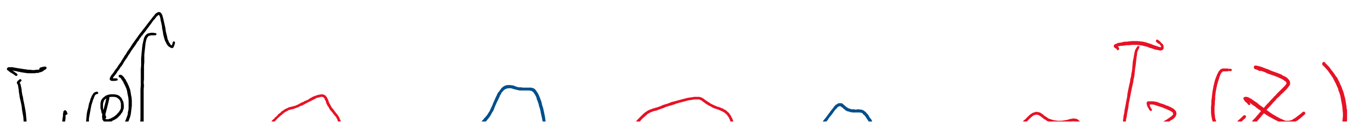
when $WG_1 = WG_2$, $\beta_1 = \beta_2$

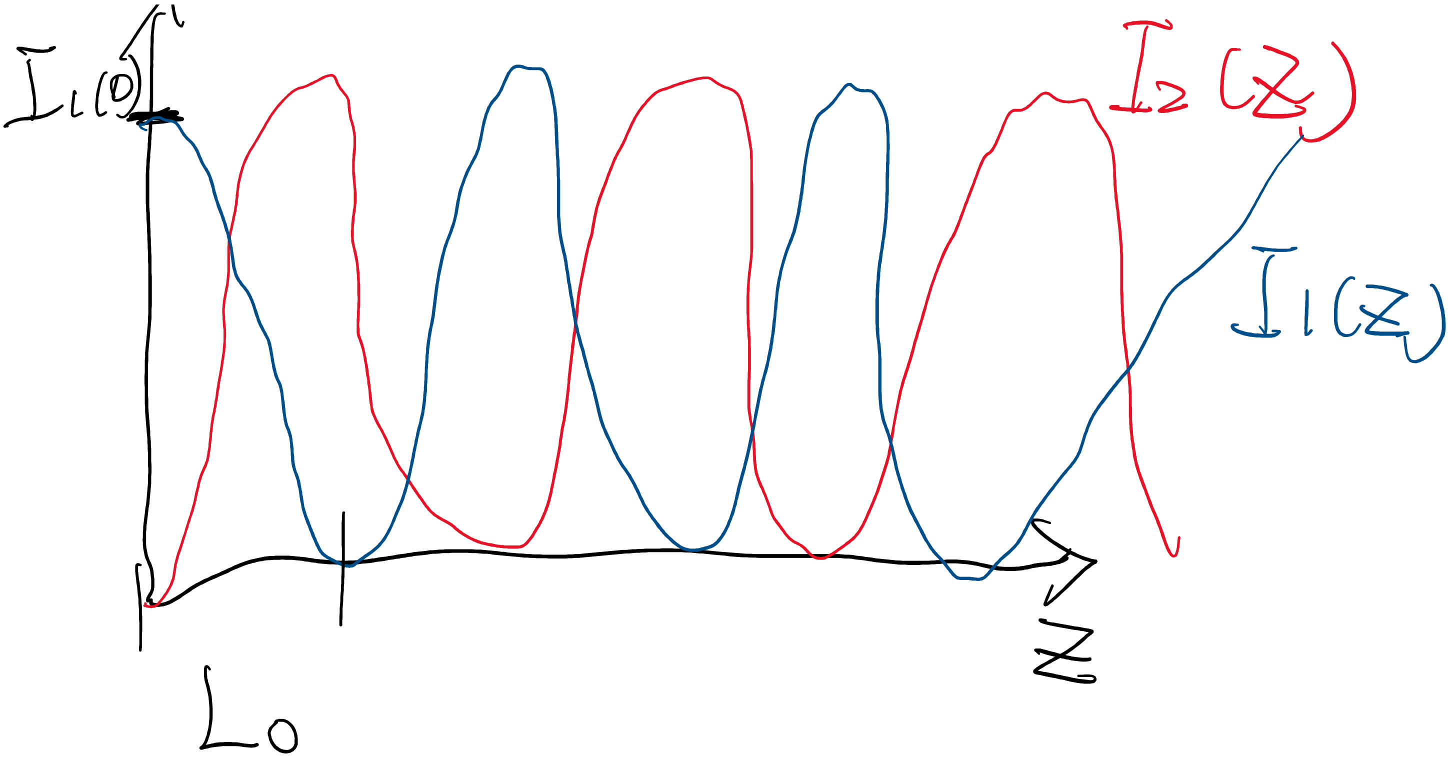
$\because \Delta\beta = 0$, $\gamma = 0 + C = C$ $\qquad C_{12} = C_{21}$

$\qquad \gamma = \sqrt{C_{12}C_{21}} = C$

$\therefore I_1(z) = I_1(0)\cos^2(Cz)$

$$I_2(z) = I_2(0)\sin^2(Cz)$$



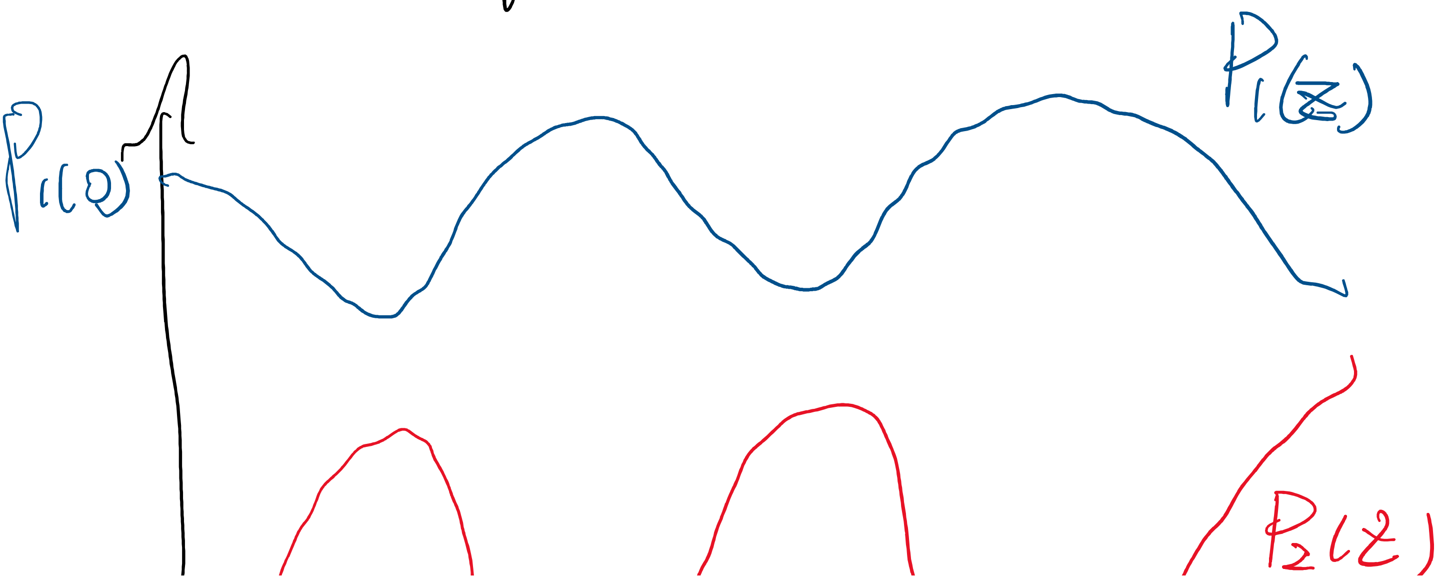


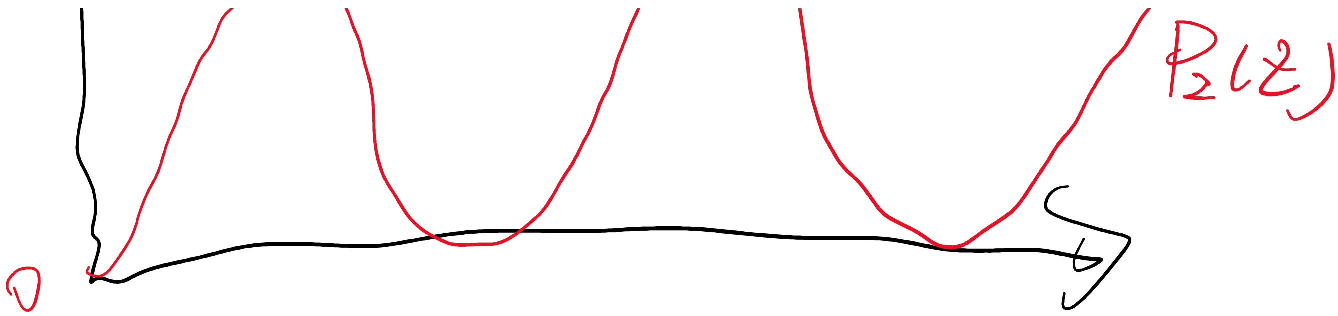
$$L_0 = \frac{\pi}{2C}$$

$$I_1(z) = I_1(0)\cos^2\left(\frac{\pi}{2L_0} z\right)$$

$$I_2(z) = I_2(0)\sin^2\left(\frac{\pi}{2L_0} z\right)$$

When $\Delta\beta \neq 0$




$P_2(z)$
0