OS7201 Electromagnetic Analysis Using FDTD Method

Term Project Due: 01/13/2023 at 11:59 a.m.

1. (100%) The right-angle-face slanted surface relief silicon (Si) grating shown in Fig. 1 was designed to operate at $\lambda_0 = 1550$ nm. The grating parameters are $\Lambda = 630.8$ nm, $F = 0.7746$, $d = 691$ nm, $n_{\text{Si}} = 3.476$, and $\phi = 54.746°$ (due to the anisotropic etching of Si). Assume the grating is structurally invariant along the z axis and is infinitely periodic in the $\pm x$ direction. The substrate is assumed to extend to infinity in the $+y$ direction (pointing downward). Use PMLs at the top and bottom boundaries of the problem space and apply periodic boundary conditions to the left and right boundaries. Simulate the transmittance and reflectance spectra from this grating for $\lambda_0 = [1500,\ 1600]$ nm using your FDTD code for a TE wave at normal incidence.

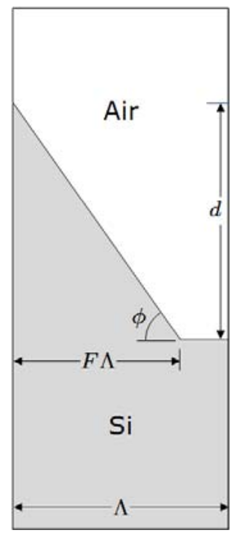

Fig. 1. Schematic of the Si grating considered in this term project.

Major steps for your code development are provided below:

1. *Define the problem*

   Source parameters, grating parameters, grid parameters

2. *Compute grid resolution*

   Compute initial grid resolution, snap grid to critical dimensions, determine the size of the problem space and compute the number of cells in the x and y directions

3. *Build the device on the 2× grid*

   Initialize materials to free-space, compute position indices, add sawtooth, add infinite substrate

4. *Compute source*

   Courant stability condition, specify $j_{y,\text{src}}$, compute source parameters: $\tau = 0.5/f_{\max}$, $t_0 = 6\tau$, $A = \sqrt{\varepsilon_{r,\text{src}}/\mu_{r,\text{src}}}$, $\delta t = n_{\text{src}}\Delta y/(2c_0) + \Delta t/2$, compute number of iterations, compute source functions ($E_{z,\text{src}}$ and $H_{x,\text{src}}$)

5. *Initialize Fourier transforms*

   Compute kernels, initialize steady-state fields,

```
Eref = zeros(Nx,NFREQ);
Etrn = zeros(Nx,NFREQ);
SRC = zeros(1,NFREQ);
```

   specify positions of record planes $j_{y,\text{ref}}$ and $j_{y,\text{trn}}$. Note that the refractive indices in record planes are given by

$$n_{\text{ref}} = \sqrt{\varepsilon_{zz}\Big|^{1,j_{y,\text{ref}}} \mu_{xx}\Big|^{1,j_{y,\text{ref}}}},$$

$$n_{\text{trn}} = \sqrt{\varepsilon_{zz}\Big|^{1,j_{y,\text{trn}}} \mu_{xx}\Big|^{1,j_{y,\text{trn}}}},$$

   where $j_{y,\text{ref}}$ and $j_{y,\text{trn}}$ are both one cell away from their corresponding PML region.

6. *Compute the PML*

Compute the 2× grid `Nx2` and `Ny2` and σ'_y. Notice that $\sigma'_x = 0$.

7. *Compute Update Coefficients for* H_x, H_y, D_z, and E_z.

```
sigHx = sigx(1:2:Nx2, 2:2:Ny2);
sigHy = sigy(1:2:Nx2, 2:2:Ny2);
mHx0  = (1/dt) + sigHy/(2*epsilon_0);
mHx1 = …
mHx2 = …
mHx3 = …
```

8. *Initialize FDTD data arrays*

The data arrays include `Hx`, `Hy`, `Dz`, `Ez`, `CEx`, `CEy`, `CHz`, `ICEx`, `ICEy`, and `IDz`

9. *Main FDTD loop (by following the TF/SF block diagram for the* E_z *mode given in Lecture 15)*

- Compute the x component of the curl of $\boldsymbol{\mathcal{E}}$

$$C_x^{\mathcal{E}}\Big|_t^{i,j} = \begin{cases} \dfrac{\tilde{\mathcal{E}}_z\big|_t^{i,j+1} - \tilde{\mathcal{E}}_z\big|_t^{i,j}}{\Delta y}, & j < N_y \\[2ex] \dfrac{\tilde{\mathcal{E}}_z\big|_t^{i,1} - \tilde{\mathcal{E}}_z\big|_t^{i,N_y}}{\Delta y}, & j = N_y \end{cases}$$

- Compute the y component of the curl of $\boldsymbol{\mathcal{E}}$

$$C_y^{\mathcal{E}}\Big|_t^{i,j} = \begin{cases} -\dfrac{\tilde{\mathcal{E}}_z\big|_t^{i+1,j} - \tilde{\mathcal{E}}_z\big|_t^{i,j}}{\Delta x}, & j < N_x \\[2ex] -\dfrac{\tilde{\mathcal{E}}_z\big|_t^{1,j} - \tilde{\mathcal{E}}_z\big|_t^{N_x,j}}{\Delta x}, & i = N_x \end{cases}$$

- Correct curl $C_x^{\mathcal{E}}$ for TF/SF

$$C_x^{\mathcal{E}}\Big|_t^{i,j_{\text{src}}-1} = \frac{\tilde{\mathcal{E}}_z\big|_t^{i,j_{\text{src}}} - \tilde{\mathcal{E}}_z\big|_t^{i,j_{\text{src}}-1}}{\Delta y} - \frac{1}{\Delta y}\tilde{\mathcal{E}}_z^{\text{src}}\Big|_t^{i,j_{\text{src}}}$$

  Since $C_x^{\mathcal{E}}$ has been computed previously, we just need to incorporate the second term on the right-hand side into all values of `i` in $C_x^{\mathcal{E}}$. The code for this expression is

```
CEx(:, ny_src-1) = CEx(:, ny_src-1) – Ez_src(t_step)/dy;
```

- Update $\boldsymbol{\mathcal{H}}$ field
  - ✓ Update integration terms

```
ICEx = ICEx + CEx;
ICEy = ICEy + CEy;
```

  - ✓ Update $\mathcal{H}_x$ and $\mathcal{H}_y$

```
Hx = mHx1.*Hx + mHx2.*CEx + mHx3.*ICEx;
Hy = mHy1.*Hy + mHy2.*CEy + mHx3.*ICEy;
```

- Compute the z component of the curl of $\boldsymbol{\mathcal{H}}$

$$C_z^{\mathcal{H}}\Big|_{t+\Delta t/2}^{i,j} = \begin{cases} \dfrac{\tilde{\mathcal{H}}_y\Big|_{t+\Delta t/2}^{i,j} - \tilde{\mathcal{H}}_y\Big|_{t+\Delta t/2}^{i-1,j}}{\Delta x} - \dfrac{\tilde{\mathcal{H}}_x\Big|_{t+\Delta t/2}^{i,j} - \tilde{\mathcal{H}}_x\Big|_{t+\Delta t/2}^{i,j-1}}{\Delta y}, & i > 1 \text{ and } j > 1 \\ \dfrac{\tilde{\mathcal{H}}_y\Big|_{t+\Delta t/2}^{1,j} - \tilde{\mathcal{H}}_y\Big|_{t+\Delta t/2}^{N_x,j}}{\Delta x} - \dfrac{\tilde{\mathcal{H}}_x\Big|_{t+\Delta t/2}^{1,j} - \tilde{\mathcal{H}}_x\Big|_{t+\Delta t/2}^{1,j-1}}{\Delta y}, & i = 1 \text{ and } j > 1 \\ \dfrac{\tilde{\mathcal{H}}_y\Big|_{t+\Delta t/2}^{i,1} - \tilde{\mathcal{H}}_y\Big|_{t+\Delta t/2}^{i-1,1}}{\Delta x} - \dfrac{\tilde{\mathcal{H}}_x\Big|_{t+\Delta t/2}^{i,1} - \tilde{\mathcal{H}}_x\Big|_{t+\Delta t/2}^{i,N_y}}{\Delta y}, & i > 1 \text{ and } j = 1 \\ \dfrac{\tilde{\mathcal{H}}_y\Big|_{t+\Delta t/2}^{1,1} - \tilde{\mathcal{H}}_y\Big|_{t+\Delta t/2}^{N_x,1}}{\Delta x} - \dfrac{\tilde{\mathcal{H}}_x\Big|_{t+\Delta t/2}^{1,1} - \tilde{\mathcal{H}}_x\Big|_{t+\Delta t/2}^{i,N_y}}{\Delta y}, & i = 1 \text{ and } j = 1 \end{cases}$$

- Correct $C_z^{\mathcal{H}}$ for TF/SF

$$C_z^{\mathcal{H}}\Big|_{t+\Delta t/2}^{i,j_{\text{src}}} = \frac{\tilde{\mathcal{H}}_y\Big|_{t+\Delta t/2}^{i,j_{\text{src}}} - \tilde{\mathcal{H}}_y\Big|_{t+\Delta t/2}^{i-1,j_{\text{src}}}}{\Delta x} - \frac{\tilde{\mathcal{H}}_x\Big|_{t+\Delta t/2}^{i,j_{\text{src}}} - \tilde{\mathcal{H}}_x\Big|_{t+\Delta t/2}^{i,j_{\text{src}}-1}}{\Delta y} + \frac{1}{\Delta y}\mathcal{H}_x^{\text{src}}\Big|_{t+\Delta t/2}^{i,j_{\text{src}-1}}$$

  Since $C_z^{\mathcal{H}}$ has been computed previously, the code for correcting $C_z^{\mathcal{H}}$ is given below:

```
CHz(:,ny_src) = CHz(:,ny_src) + Hx_src(t_step)/dy;
```

- Update the integration term for $\mathcal{D}_z$

```
IDz = IDz + Dz;
```

- Update $\mathcal{D}_z$

```
Dz = mDz1.*Dz + mDz2.*CHz + mDz4.*IDz;
```

- Update $\mathcal{E}_z$

```
Ez = mEz1.*Dz;
```

- Update the Fourier transforms at each frequency component

```
for nfreq = 1 : N_freq
     Eref(:,nfreq) = … ;
     Etrn(:,nfreq) = … ;
     SRC(nfreq) = …;
end
```

- Draw the field superpositioned on the materials

Use `draw2d` provided on empossible.net to draw $\mathcal{E}_z$ along with the grating structure. Update the visualization every 10 or 15 iterations.

10. Produce professional-looking figures