Disturbance growth in a flat plate boundary layer with zero pressure gradient

– Introduction –

1. Background and purpose

Laminar boundary layers are subjected to a convective wave instability called Tollmien-Schlichting (TS) waves. TS-waves with a given frequency amplify within a range of Reynolds numbers, which can be assessed by linear theory (see Appendix A). When the wave reaches large enough amplitude, non-linear phenomena initiate the transition to turbulence. This is the dominating scenario in a low-disturbance environment, e.g. on an airfoil or a spacecraft under atmospheric conditions. In noisy environments, e.g. on turbine blades, the TS-wave instability competes with other transition mechanisms, which may lead to transition at lower Reynolds numbers (see Appendix A.). This lab will give some practical experience in how to measure the characteristics of the TS wave instability.

The experiments will be performed on a flat plate mounted in the BL wind tunnel in the Fluid Physics Laboratory at KTH – Engineering Mechanics. This tunnel has a test-section of 4.2 m and a crosssectional area of 0.5×0.75 m^2. The flow quality is very good, with exceptionally low level of background disturbances. The flat plate is mounted vertically in the test section and the stagnation line flow is controlled by means of a flap at the down-stream end of the plate. The pressure gradient on the flat plate is adjusted to near-zero pressure gradient conditions, so that the boundary layer profile matches the Blasius solution.

2. Experimental equipment

2.1 Introducing disturbances into the boundary layer

Different kinds of disturbances can be introduced into the boundary layer: we can disturb with regular wave trains at a fixed frequency, with a puff that immediately creates a turbulent spot or simply with continuous forcing. Continuous TS-waves are, here, generated by a loudspeaker, which is fed with a signal from a function generator, amplified by a hi-fi-system. The waves (by means of periodic blowing and suction) are introduced into the boundary layer through a slot in the plate, and they excite TS-waves with the same frequency.

2.2 Hot wire anemometry and traversing system

The streamwise velocity and its fluctuations are measured with a hot-wire. The time signal is logged on a computer where the signal is processed and its power spectrum computed. This gives the dominating wave frequency and the wave amplitude at this frequency. The wave can also be observed qualitatively by watching an oscilloscope connected to the hot wire probe.

The hot-wire is calibrated against a Prandtl tube placed in the free stream (see appendix B. for more information). Note that the anemometer output is a strongly non-linear function of the velocity, which means that the amplitude of velocity fluctuations observed in the oscilloscope will depend on the local mean velocity at different y-positions (i.e. different wall-normal positions). The hot-wire probe can be traversed normal to the plate as well as in the downstream direction. This makes it possible to measure the amplitude profile of the wave across the boundary layer as well as its downstream development.

Disturbances in a zero-pressure gradient boundary layer

– theoretical and experimental background –

1. Tollmien–Schlichting waves

For low environmental disturbances the transition scenario from laminar to turbulent flow on a flat plate boundary layer is rather well understood. This class of transition starts with the amplification of streamwise traveling two-dimensional waves, so called Tollmien–Schlichting (TS) waves. The first experimental observation of TS–waves was made in a low turbulence level wind tunnel by Schubauer & Skramstad (1947). They were able to determine the critical Reynolds number for different frequencies by using a vibrating ribbon to trigger the TS–waves and hot-wire anemometry to measure the fluctuating velocity signal. The concept of TS–waves is the basis for engineering calculation methods (so called e^N-methods) for transition prediction in simple boundary layer flows.

For viscous plane flows an ordinary differential equation can be obtained which describes the development of TS–waves. The normal velocity component v is assumed to have a wave character given by

$$v(x, y, t) = \mathrm{Re}\{\hat{v}(y)e^{i(\alpha x - \omega t)}\} ,$$

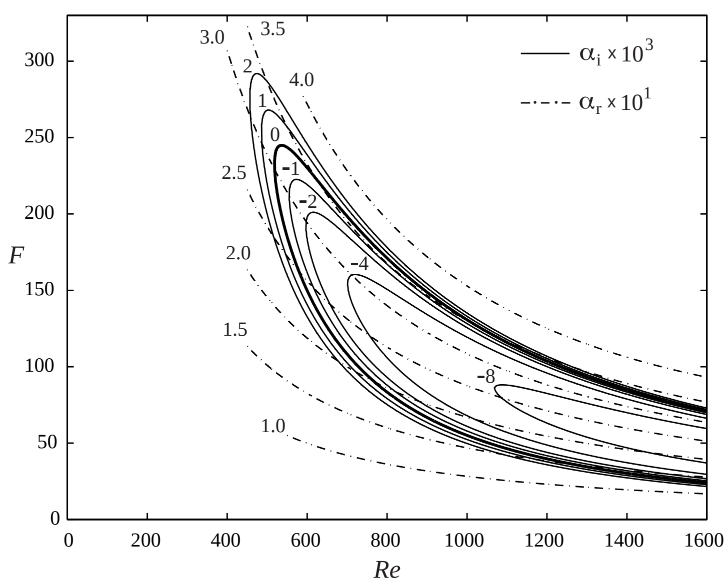

Figure 1: Spatial stability curves for two-dimensional waves in a Blasius boundary layer. Solid lines are for constant imaginary parts of the streamwise wave number (α_i) and dash-dotted for constant real parts (α_r). The bold solid line is the neutral stability curve. The displacement thickness (δ_1) is the characteristic length scale. Figure from Fransson 2003.

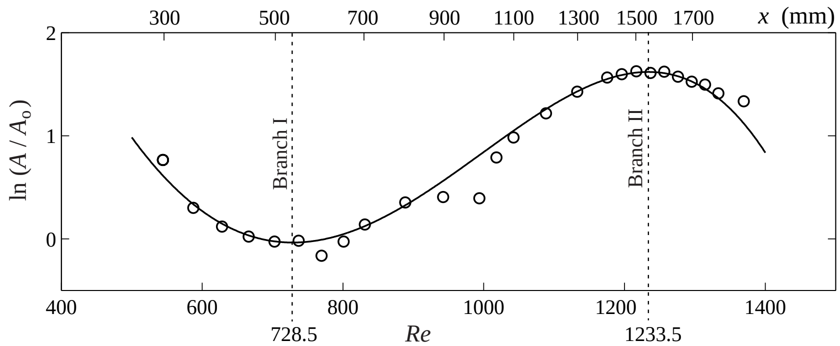

Figure 2: Amplitude evolution of the TS-wave at $F = 100$. Symbols are experimental results, and solid lines are the OS-solution of the Blasius profile. Figure from Fransson 2003.

where α is the complex streamwise wave number and where the real part gives the wavenumber in the streamwise x-direction and the negative imaginary part is the disturbance amplification, ω is the frequency of the wave, y is the wall normal coordinate and t is time. $\hat{v}(y)$ can be viewed as the complex amplitude function of the normal velocity component. Furthermore, the mean flow is assumed to be parallel $\overline{U} = U(y)\overline{e}_x$, *i.e.* the streamwise development of the boundary layer is not taken into account. Under these assumptions the Navier-Stokes equations may be reduced to the so called Orr-Sommerfeld (OS) equation:

$$\left[(-i\omega + i\alpha U)(\mathcal{D}^2 - \alpha^2) - i\alpha U'' - \frac{1}{Re}(\mathcal{D}^2 - \alpha^2)^2\right]\hat{v} = 0\ ,$$

where $\mathcal{D} = \partial/\partial y$, $Re = U_\infty \delta_1/\nu$ is the Reynolds number based on the free stream velocity U_∞, ν the kinematic viscosity and $\delta_1 = \int_0^\infty (1 - U/U_\infty)dy$ the displacement thickness. For the Blasius boundary layer $\delta_1 = 1.72\sqrt{\nu x/U_\infty}$.

A wave traveling in the streamwise direction may be amplified only in a certain region of the wave frequency-Reynolds number (F-Re) parameter plane, see figure 1. Here, $F = 2\pi f\nu\cdot 10^6/U_\infty^2$ is a non-dimensionalized frequency and $Re = 1.72\sqrt{Re_x}$ is the Reynolds number, where $Re_x = U_\infty x/\nu$ and x is the distance from the leading edge.

The downstream development of the TS–wave can be seen in figure 2. The wave is damped downstream of the TS–wave generation slot, until it enters the unstable region and begins to amplify, and finally decreases again. Neutral points can be determined from the downstream evolution of the TS–wave amplitude maximum. The minimum and maximum amplitude positions correspond to branch I and branch II, respectively (see figure 1).

Schubauer & Skramstad (1947) found in their experiment that the amplitude distribution of the disturbance has two maxima, the largest close to the wall and the second maximum at the boundary layer edge, see figure 3. In the same figure the corresponding phase distribution profiles are also plotted, and they clearly show the phase shift of π radians which can be shown to appear where $\partial\hat{v}/\partial y$ changes sign, i.e. at the wall-normal amplitude ($\hat{v}$) maxima.

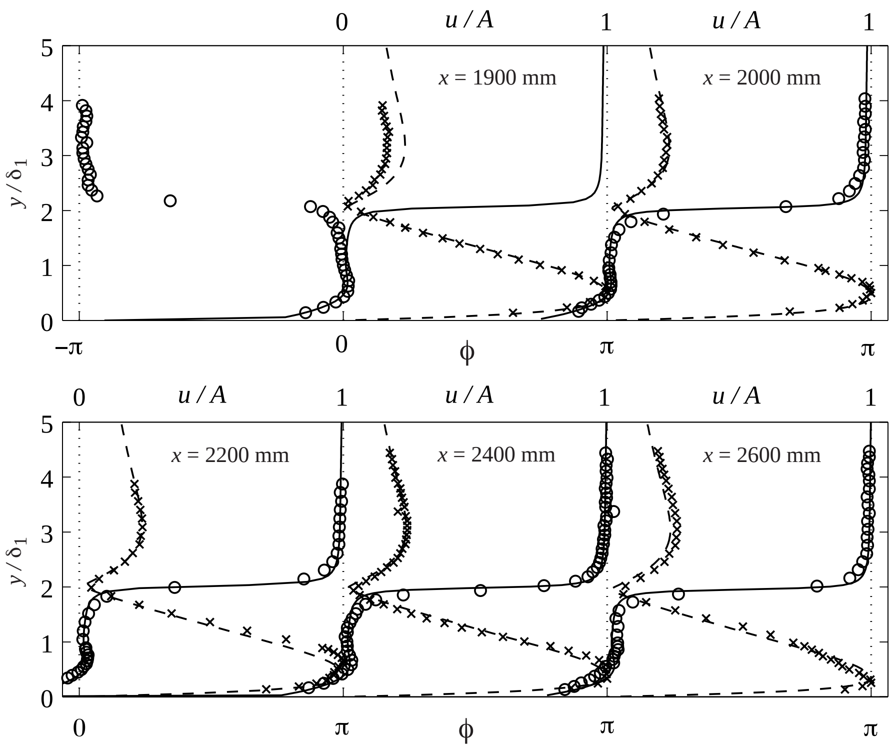

Figure 3: Amplitude- and phase distribution profiles for $F = 59$ at different x-positions. ($\times$)-symbols and dashed lines correspond to measured and theoretical amplitude profiles respectively. ($\circ$)-symbols and solid lines are the corresponding phase profiles. Figure from Fransson 2003.

2. Free stream turbulence and algebraic growth

It is well known that for the Blasius boundary layer free stream turbulence (FST) induces disturbances into the boundary layer which give rise to streamwise oriented structures of low and high speed fluid. These structures grow in amplitude and establish a spanwise size which is of the order of the boundary layer thickness far away from the leading edge. When the streaks reach a certain amplitude they break down to turbulence, probably through a secondary instability mechanism. This type of boundary layer disturbance was originally called the breathing mode, since the wall-normal disturbance profile resembles that which would result from a locally continuous thickening and thinning of the boundary layer edge. However, this mode is nowadays recognized as the Klebanoff mode which was proposed by, and can be viewed as one scenario of by–pass transition. It is a relatively rapid process by–passing the traditional TS-wave dominated transition scenario resulting in breakdown to turbulence at subcritical Reynolds numbers when compared with the predicted value by traditional theory.

In figure 4 streamwise disturbance and mean velocity wall normal distributions are plotted for different levels of Tu ($= u_{rms}/U_\infty$, in the free stream). In figure 4a) it is clear that the presence of higher FST intensity causes a higher disturbance level inside as well as outside the boundary layer, without affecting the mean velocity (cf. figure 4c). It is both the Reynolds number and the Tu-level that sets the state, i.e. whether the flow is in the sub-transitional, transitional, or in the post-transitional state. At least up to the transitional state one can expect a self similar disturbance profile through the boundary layer. Thereafter, the disturbance peak moves towards the wall and the disturbance level spreads out more in the entire boundary layer, this may be observed in figure 4b). An interesting observation is that the level of the disturbance peak inside the boundary layer increases linearly with Tu which is shown in figure 4d), where solid lines are curve fits to the data (see caption for more information).

In figure 5a) the energy distribution versus the downstream distance for the three different Tu-levels are shown. It is seen that the disturbance (u_{rms}/U_∞) reaches levels around 14% for Tu=4.0% before it starts to decrease, which is connected to the transitional nature of the boundary layer. What is observed in figure 5a) is the algebraic growth followed by transition. That the peak becomes smaller with decreasing Tu is probably connected to the relation between turbulent scales in the free stream (that are different for all three cases), the disturbance level, the streak spacing, and the boundary layer thickness. Note, also that the energy seems to asymptote to a constant level around $E = 0.007$ independent of the Tu-level after reaching the maximum value. The intermittency function for the three different cases in a) are shown in figure 5b). The maximum (in figure 5a) is closely related to the point of γ=0.5, i.e. the point where the flow alternatively consists of laminar portions and turbulent spots which explains the high u_{rms} value. In a) it is seen that the higher the Tu the smaller the Re_x for which the maximum occurs, and figure 5b) shows that the relative extent of the transitional zone is larger for Tu=4.0% than the other two.

References

- Fransson J. H. M. 2003 Flow control of boundary layers and wakes. PhD thesis, KTH, Stockholm, TRITA-MEK Tech. Rep. 2003:18.

- Schubauer, G.B. & Skramstad, H.K. 1947 Laminar boundary layer oscillations and transition on a flat plate. *J. Res. Nat. Bur. Stand.*, **38**, 251-292.

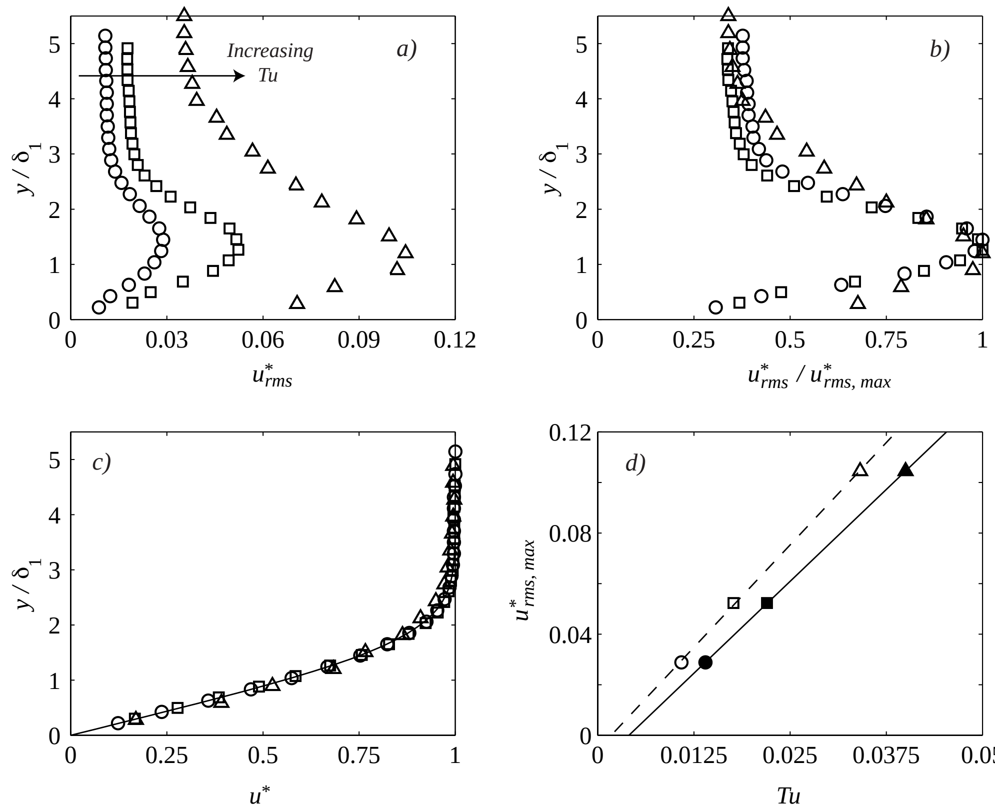

Figure 4: Wall normal perturbation- and mean velocity profiles for different Tu-levels at $Re = 544$ (based on δ_1) or $Re_x = 10^5$, $u^* = u/U_\infty$ and $u^*_{rms} = u_{rms}/U_\infty$. a) and c) show the perturbation- and the mean velocity profile for different Tu ($= u_{rms}/U_\infty$, in the free stream). b) correspond to the data in a) but normalized to unity and d) shows the normalization value versus the local (open symbols) and the leading edge (filled symbols) Tu-value, respectively. (◯) $Tu = 1.4\%$, (□) $Tu = 2.2\%$, and (△) $Tu = 4.0\%$. Figure from Fransson 2003.

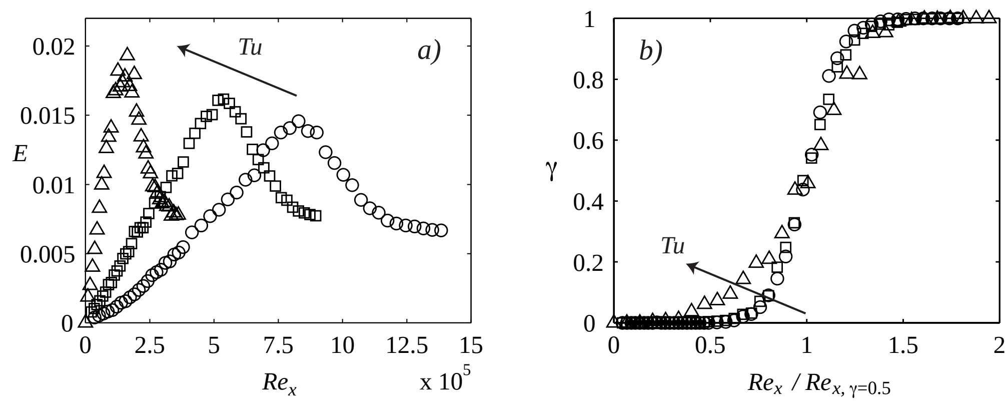

Figure 5: a) Energy growth ($E = u_{rms}^2/U_\infty^2$) as function of Re_x for three different Tu-levels. Measurements are made at y/δ_1=1.4. b) The corresponding intermittency distribution of the data in a) versus Re_x normalized with ditto for $\gamma = 0.5$. (◯) $Tu = 1.4\%$, (□) $Tu = 2.2\%$, and (△) $Tu = 4.0\%$. Figure from Fransson 2003.

Brief introduction to CTA hot-wire anemometry

Constant–Temperature Anemometry (CTA) for hot-wires is based on convective heat loss from a heated resistance sensor. In CTA the hot-wire sensor is held at a constant temperature. If the fluid properties remain constant the instantaneous heat loss is a measure of the instantaneous velocity. A hot-wire is simply a thin wire, usually made of platinum and in the range 0.5–5 μm in diameter, mounted between two prongs. The length to diameter ratio of the sensor should be at least 200 in order to have large heat loss to the fluid as compared to the heat conduction to the prongs. With CTA the sensor temperature is constant by means of a highly amplified feed-back loop of a servo system. The anemometer essentially consists of a Wheatstone bridge and a servo amplifier. The sensor resistance, typically around 10 Ω, forms one of four legs of the bridge. By setting the resistance of one of the other legs to a chosen value, the sensor resistance at operating conditions will be given. The difference between the sensor resistance at operating conditions and the cold resistance (at ambient temperature) determines the temperature difference between the sensor and the fluid. This overheat is typically 150 degrees in air flows, which make the system insensitive to small changes in air temperature. The electrical current needed to keep the sensor at constant temperature, and thereby the bridge in balance, is measure indirectly by measuring the voltage between the bridge ”top” and ground. This voltage increases with increasing velocity, and rapid changes in velocity can be measured. The frequency response of the system is usually of the order of 10-100 kHz.

The relation between output voltage E and velocity U is given by King's Law

$$E^2 = A + BU^n ,$$

where E is the anemometer output voltage at the velocity U, and A, B and n are constants to be determined by calibrating the wire against a set of known velocities. The value of n is typically close to 0.5 and A should be close to the square of the output voltage at zero velocity, $A \approx E_0^2$.

Instructions for the LAB REPORT

Disturbance growth in a flat plate boundary layer with zero pressure gradient

Maximum number of students per report is 2. Below items should be included:

0. Title page with your name/names and personal number/numbers.
1. Introduction of the experimental lab
   - 1.i. Summary of the lab (from your notes and Appendix A of LabPM.)
   - 1.ii. Experimental setup (from you own notes and the introduction from the lab assistant)
   - 1.iii. Experimental measurement technique (from Appendix B of LabPM and your notes)
2. Results

   In the first step, determine the virtual origin of the experiment using the two mean velocity profiles. Calulate the boundary layer displacement thickness δ_1 (see instructions for Figure 2 below) and compare it to the theoretical Blasius value, solving for the virtual origin x_0:

   $$\delta_1 = 1.7208\sqrt{\frac{(x-x_0)\nu}{U_\infty}} \quad . \tag{1}$$

   Perform the calculation separately for both mean velocity profiles and average the two results. Report the values and use it to correct the experimental data. *Hint: the virtual origin should be on the order of -120 mm.*

   Figures to be included in the report:

   - **Figure 1**: Plot the wave growth rate as N=ln(A/A_0), where A_0 is the amplitude at branch I, as a function of Re based on $\delta = (x \times \nu/U_\infty)^{1/2}$, x being the downstream distance from the leading edge. Linear stability results are provided in `LinTheory_F160_2023.mat`. Compare the predictions from linear theory with experimental data in the same plot.
   - **Figure 2**: Scaled mean velocity profiles at branch I and branch II locations (measured from **figure 2**)
     The wall-normal coordinate y should be scaled with the displacement thickness δ_1, while the mean velocity should be scaled with the local free stream velocity (or equivalently the boundary layer edge velocity U_e).
     The experimental profiles are taken at discrete positions above the wall with a known relative displacement, but with an unknown absolute position. This

means that the position of the wall, `Ywall`, has to be determined in order to get the absolute positions correct. Once `Ywall` is known the wall position can be subtracted from the y-vector of the profile and in that way re-defining the position of the wall to be at $y = 0$. The Matlab program

$$\texttt{[Ywall, ny] = TS\_LAB\_JF\_P1(Y, Uy)} \tag{2}$$

takes as argument two vectors, the mean streamwise velocity distribution in the wall-normal direction (`Uy`) and its corresponding wall-normal coordinates (`Y`). The program returns estimates of the wall-position `Ywall` and the acceleration parameter `ny` using the Falkner-Skan similarity equation in an iterative least-squares fit sense to the data. Note the dimensions of the arguments: `Y` [mm] and `Uy` [m/s]. *Hint: The function requires the data to go from the wall to the free stream. If necessary, flip the input vectors.*
In order to plot the data

$$\frac{u(y/\delta_1)}{U_e} \tag{3}$$

δ_1 and U_e have to be determined. The latter can be taken as the mean of the last points of the measured velocity in the free stream (check so that the points chosen to calculate the mean has roughly reached a constant value). The displacement thickness is defined as

$$\delta_1 = \int_0^\infty \left(1 - \frac{u(y)}{U_e}\right) \mathrm{d}y\,, \tag{4}$$

which may be integrated using the `TRAPZ` command in Matlab (type "help `TRAPZ`" in the command window and read/learn about `Z = TRAPZ(X,Y)`). **Note:** you should add the no-slip boundary condition at the wall before calculating the above integral. The ∞ symbol simply indicates a point *infinitely* far away from the plate, in practice it corresponds to the points in the free stream outside the boundary layer. You should repeat above procedure for each one of the measured profiles. Compare experimental data with theoretical results (Blasius).

– **Figure 3(a)**: Measured wall-normal amplitude and phase distribution at branch I. Normalize the amplitude with the maximum value and plot it against $\eta = y/\delta$. Compare amplitude distributions between experiments and linear stability theory. **Note:** The reference length in the linear stability results is δ, it needs to be converted to δ_1.

– **Figure 3(b)**: Measured wall-normal amplitude and phase distribution at branch II. Normalize the amplitude with the maximum value and plot it against $\eta = y/\delta$. Compare amplitude distributions between experiments and linear stability theory.

– **Figure 4**: Plot the phase distribution along the streamwise direction. Does the wavelength of the TS waves estimated during the lab match the wavelength that we can compute from the experimental phase distribution? Remembering that $\partial\Phi/\partial x$ corresponds to real part of the streamwise wavenumber (α_r), calculate the phase speed c/U_∞ using a linear fit through the point (shifted of a full period if needed). Compare the experimentally measured value with the one obtained with linear stability theory and the once manually calculated during the lab.

3. Conclusions

   – Write something about how well the experimental data from the lab agree with linear stability theory? Try to come up with potential reasons for the deviations.

   – What determines the instability of the TS-wave? How does the external pressure gradient affect the instability?