

2019 AIME I #6

Tristan Shin

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In convex quadrilateral $KLMN$ side \overline{MN} is perpendicular to diagonal \overline{KM} , side \overline{KL} is perpendicular to diagonal \overline{LN} , $MN = 65$, and $KL = 28$. The line through L perpendicular to side \overline{KN} intersects diagonal \overline{KM} at O with $KO = 8$. Find MO .

Let $P = LO \cap KN$. From $\triangle KPL \sim \triangle KLN$ we have $\frac{KP}{KL} = \frac{KL}{KN}$. From $\triangle KPO \sim \triangle KMN$ we have $\frac{KP}{KO} = \frac{KM}{KN}$. So

$$KO(KO + MO) = KO \cdot KM = KP \cdot KN = KL^2$$

from which $MO = \frac{KL^2}{KO} - KO = \frac{28^2}{8} - 8 = \boxed{090}$ as desired. ■