2019 Putnam A3

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(filler)

The answer is $\boxed{2019^{-\frac{1}{2019}}}$ achieved by $b_k = 2019^{\frac{k}{2019}}$. This polynomial is

$$\sum_{k=0}^{2019} 2019^{\frac{k}{2019}} x^k = \frac{2019^{\frac{2020}{2019}} x^{2020} - 1}{2019^{\frac{1}{2019}} x - 1}$$

all of whose roots have magnitude $2019^{-\frac{1}{2019}}$. Now, by AM-GM we have

$$\mu \geq \left(\prod_{k=1}^{2019} |z_k|\right)^{\frac{1}{2019}} = \left|\prod_{k=1}^{2019} z_k\right|^{\frac{1}{2019}} = \left|-\frac{b_0}{b_{2019}}\right|^{\frac{1}{2019}} \geq \left(\frac{1}{2019}\right)^{\frac{1}{2019}}$$

as desired. ■