2019 AIME I #5

Tristan Shin

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A moving particle starts at the point $(4,4)$ and moves until it hits one of the coordinate axes for the first time. When the particle is at the point (a,b), it moves at random to one of the points $(a-1,b)$, $(a,b-1)$, or $(a-1,b-1)$, each with probability $\frac{1}{3}$, independently of its previous moves. The probability that it will hit the coordinate axes at $(0,0)$ is $\frac{m}{3^n}$, where m and n are positive integers, and m is not divisible by 3. Find $m+n$.

Call the steps L for moving left, D for moving down, and X for moving left and down in one move. Note that the last step must be X otherwise the second-to-last stop is on an axis already. Besides that, any combination of moves going from $(4,4)$ to $(1,1)$ suffices. So we can have any rearrangement of $LLLDDD$, $LLDDX$, $LDXX$, or XXX. Each of these happens with probability $\frac{1}{3^6}\binom{6}{3,3,0}$, $\frac{1}{3^6}\binom{5}{2,2,1}$, $\frac{1}{3^4}\binom{4}{1,1,2}$, and $\frac{1}{3^3}\binom{3}{0,0,3}$, respectively. So we add up and divide by 3 to get

$$\frac{1}{3}\left(\frac{20}{3^6}+\frac{30}{3^5}+\frac{12}{3^4}+\frac{1}{3^3}\right)=\frac{245}{3^7}$$

so the answer is $\boxed{252}$ as desired. ■