2015 AIME I #1

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The expressions $A = 1\times2+3\times4+5\times6+\cdots+37\times38+39$ and $B = 1+2\times3+4\times5+\cdots+36\times37+38\times39$ are obtained by writing multiplication and addition operators in an alternating pattern between successive integers. Find the positive difference between integers A and B.

Write out $B-A$ as

$$\begin{array}{rccccccccccc}
 & 1 & + & 2\times3 & + & 4\times5 & + & \cdots & + & 38\times39 & & \\
- & & & 1\times2 & + & 3\times4 & + & \cdots & + & 37\times38 & + & 39 \\
\hline
 & 1 & + & 2(3-1) & + & 4(5-3) & + & \cdots & + & 38(39-37) & - & 39 \\
= & 1 & + & 2\times2 & + & 4\times2 & + & \cdots & + & 38\times2 & - & 39
\end{array}$$

so that

$$B-A = 1+2(2+4+\cdots+38)-39 = 4\cdot\frac{19\cdot20}{2}-38 = \boxed{722}.$$

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