2019 HMMT T1

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Let $ABCD$ be a parallelogram. Points X and Y lie on segments AB and AD respectively, and AC intersects XY at point Z. Prove that

$$\frac{AB}{AX} + \frac{AD}{AY} = \frac{AC}{AZ}.$$

Consider an affine transformation which sends $A \to (0,0), B \to (1,0), C \to (1,1), D \to (0,1)$. This works because ratios along lines are preserved. Let $X = \left(\frac{1}{m}, 0\right)$ and $Y = \left(\frac{1}{n}, 0\right)$, then confirm that $Z = \left(\frac{1}{m+n}, \frac{1}{m+n}\right)$. The conclusion follows. ■