

2018 CGMO #5

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28 Mar 2019

Let $\omega \in \mathbb{C}$ such that $|\omega| = 1$. Find the maximum possible value of the magnitude of $z = (\omega + 2)^3 (\omega - 3)^2$.

Note that

$$3|\omega + 2|^2 + 2|\omega - 3|^2 = 3(\omega + 2)\left(\frac{1}{\omega} + 2\right) + 2(\omega - 3)\left(\frac{1}{\omega} - 3\right) = 35$$

so by AM-GM,

$$\sqrt[5]{|\omega + 2|^6 |\omega - 3|^4} \leq \frac{3|\omega + 2|^2 + 2|\omega - 3|^2}{5} = 7$$

and thus

$$|\omega + 2|^3 |\omega - 3|^2 \leq 49\sqrt{7}.$$

Equality holds when $\omega = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$. ■