2012 Putnam A2

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11 Aug 2019

Let $*$ be a commutative and associative binary operation on a set S. Assume that for every x and y in S, there exists z in S such that $x * z = y$. (This z may depend on x and y.) Show that if a, b, c are in S and $a * c = b * c$, then $a = b$.

Let $d, e \in S$ such that $a = acd$ and $b = ae$. Then

$$a = acd = bcd = aecd = ae = b$$

as desired. ■