2019 Putnam B1

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(filler)

The answer is $\boxed{5n+1}$. When $n = 0$ this is clearly true.

An easy descent argument implies that the solutions to $x^2+y^2 = 2^n$ are $(\pm 2^{\frac{n}{2}}, 0)$ and $(0, \pm 2^{\frac{n}{2}})$ if n is even, or $(\pm 2^{\frac{n-1}{2}}, \pm 2^{\frac{n-1}{2}})$ is n is odd.

Now, we count the number of squares in P_n that use an element from $P_n \setminus P_{n-1}$. Check that the convex hull of P_n is the square formed by the vertices in $P_n \setminus P_{n-1}$. So such a square must two edges along the sides of the large square. This only produces two types of squares: the big square formed by $P_n \setminus P_{n-1}$; and the four smaller squares which use one vertex in $P_n \setminus P_{n-1}$, two vertices in $P_{n-1} \setminus P_{n-2}$, and the vertex $(0, 0)$. So there are 5 such squares.

This immediately implies by induction that the total count is $5n + 1$ as desired. ■