# 2017 TARML I4

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Wanda and Lucas are playing a game with squares for each integer from 0 to 35, inclusive. They both start at square 0. Every minute, a fair standard coin is flipped. Assume that Wanda is currently at square $a$ and Lucas is at square $b$. If it turns up heads, Wanda moves from square $a$ to square $a+1$ and if it turns up tails, Lucas moves from square $b$ to square $b+1$. There is an additional condition that if in a move, Wanda moves from a square that is not a multiple of five to a square that is a multiple of five, Lucas instantaneously moves back to square 0. The game ends the second after either Wanda reaches square 35 or Lucas reaches square 5. Compute the probability that Lucas reaches square 5.

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Define a "round" to be a sequence of consecutive minutes $m+1, m+2, \ldots, m+k$ such that at minutes $m$ and $m+k$, Lucas had to reset to square 0 or Lucas advances to square 5 while at minutes $m+1, m+2, \ldots, m+k-1$, Lucas did not reset to square 0 and did not advance to square 5. (Set Lucas to reset to square 0 at minute 0 so that every minute in the game is in a round). Note that $k \leq 9$, otherwise by minute $m+10$, Wanda or Lucas must have either reached a multiple of 5. We say that Wanda "wins" the round if Lucas resets to square 0 and "loses" the round if Lucas advances to square 5. Note that by symmetry, there is a $\frac{1}{2}$ chance of Wanda winning a given round and a $\frac{1}{2}$ chance of Wanda losing a given round.

Thus, the probability that the game ends with Wanda reaching square 35 is the probability that she wins 7 consecutive rounds, which is $\frac{1}{128}$. By complementary counting, the probability that the game ends with Lucas reaching square 5 is $1 - \frac{1}{128} = \boxed{\dfrac{127}{128}}$. ■