2019 HMMT Guts #19

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Complex numbers a, b, c form an equilateral triangle with side length 18 in the complex plane. If $|a+b+c| = 36$, find $|bc+ca+ab|$.

It is well-known that a, b, c form an equilateral triangle if and only if $a^2+b^2+c^2 = bc+ca+ab$. Thus

$$|bc+ca+ab| = \left|\frac{1}{3}\left(a^2+b^2+c^2+2bc+2ca+2ab\right)\right| = \frac{1}{3}|a+b+c|^2 = \boxed{432}$$

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