# 2017 ELMO #2

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15 Nov 2017

Let $ABC$ be a triangle with orthocenter $H$, and let $M$ be the midpoint of $\overline{BC}$. Suppose that $P$ and $Q$ are distinct points on the circle with diameter $\overline{AH}$, different from $A$, such that $M$ lies on line $PQ$. Prove that the orthocenter of $\triangle APQ$ lies on the circumcircle of $\triangle ABC$.

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Let $A'$ be the antipode of $A$ in $(ABC)$ and $H'$ be the orthocenter of $\triangle APQ$. Reflecting $A'$ over $M$ gives $H$, but reflecting $H$ over the midpoint of $PQ$ gives $H'$, so $A'$ is on the line through $H'$ parallel to $PQ$, hence $\angle A'H'A = 90^\circ$ and we're done. $\blacksquare$