

## 2019 AIME I #4

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A soccer team has 22 available players. A fixed set of 11 players starts the game, while the other 11 are available as substitutes. During the game, the coach may make as many as 3 substitutions, where any one of the 11 players in the game is replaced by one of the substitutes. No player removed from the game may reenter the game, although a substitute entering the game may be replaced later. No two substitutions can happen at the same time. The players involved and the order of substitutions matter. Let  $n$  be the number of ways the coach can make substitutions during the game (including the possibility of making no substitutions). Find the remainder when  $n$  is divided by 1000.

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Casework on the number of substitutions made.

- 0 substitutions. Clearly only 1 way for this to happen.
- 1 substitution. There are 11 ways to choose the player to sub out, then 11 ways to choose the player to sub in for 121 ways.
- 2 substitutions. There are 121 ways for the first substitution, then  $11 \cdot 10$  ways for the second substitution as before. This gives  $310 \pmod{1000}$  ways.
- 3 substitutions. There are  $310 \pmod{1000}$  ways for the first two substitutions, then  $11 \cdot 9$  ways for the third substitution. This gives  $690 \pmod{1000}$  ways.

Adding up, we get  $1 + 121 + 310 + 690 \equiv \boxed{122} \pmod{1000}$  ways in total. ■