2008 China TST Quiz 6 #2

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30 Apr 2019

Prove that for all $n \geq 2$, there exists a polynomial $f(x) = x^n + a_1 x^{n-1} + \ldots + a_n$ such that:

(1) $a_1, a_2, \ldots, a_n$ are all non-zero;

(2) $f(x)$ cannot be factored into the product of two non-constant polynomials with integer coefficients;

(3) For any integer x, $|f(x)|$ is not prime.

Consider the polynomial $f(x) = x^n + 210(x^{n-1} + \ldots + x^2) + 105x + 5$. This is constructed so that:

- f only outputs even numbers
- f is irreducible by Eisenstein on $p = 5$
- $f - 2$ is irreducible by Eisenstein on $p = 3$
- $f + 2$ is irreducible by Eisenstein on $p = 7$

Thus $|f(x)|$ is an even integer but not 2 and is thus composite. ■