

2017 ISL G2

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Let R and S be different points on a circle Ω such that RS is not a diameter. Let ℓ be the tangent line to Ω at R . Point T is such that S is the midpoint of the line segment RT . Point J is chosen on the shorter arc RS of Ω so that the circumcircle Γ of triangle JST intersects ℓ at two distinct points. Let A be the common point of Γ and ℓ that is closer to R . Line AJ meets Ω again at K . Prove that the line KT is tangent to Γ .

We use directed angles.

Note that

$$\angle KRT = \angle KRS = \angle KJS = \angle XJS = \angle XTA = \angle RTA,$$

so $RK \parallel AT$.

Thus, if A' is the point such that $RA'TA$ is a parallelogram, then $A' \in RK$. Furthermore, the center of $RA'TA$ is S , so A , S , and A' are collinear.

Now, note that

$$\angle A'TS = \angle A'TR = \angle ART = \angle ARS = \angle RKS = \angle A'KS,$$

so $SKA'T$ is cyclic.

Now, note that

$$\angle KTS = \angle KA'S = \angle RA'A = \angle TAA' = \angle TAS,$$

so KT is tangent to Γ . ■