2019 AIME I #14

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Find the least odd prime factor of $2019^8 + 1$.

Let p be an odd prime dividing $2019^8 + 1$. Since p divides $\Phi_{16}(2019)$, either $p \mid 16$ or $16 \mid p - 1$. Clearly the former is impossible, so $p \equiv 1 \pmod{16}$.

Now, the first two primes that are $1 \pmod{16}$ are 17 and 97. Check that

$$2019^8 + 1 \equiv 13^8 + 1 \equiv 4^8 + 1 \equiv 16^4 + 1 \equiv 2 \pmod{17}$$

while

$$\begin{aligned} 2019^8 + 1 &\equiv 79^8 + 1 \equiv 18^8 + 1 \equiv 324^4 + 1 \equiv 33^4 + 1 \\ &\equiv 1089^2 + 1 \equiv 22^2 + 1 \equiv 485 \equiv 0 \pmod{97} \end{aligned}$$

so the answer is $\boxed{097}$ as desired. ■