2019 HMMT Guts #4

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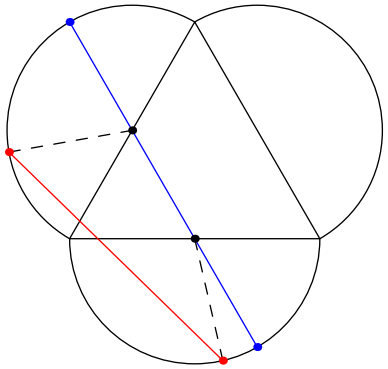
Tessa has a figure created by adding a semicircle of radius 1 on each side of an equilateral triangle with side length 2, with semicircles oriented outwards. She then marks two points on the boundary of the figure. What is the greatest possible distance between the two points?

Clearly the two points must be on different semicircles. Let the points be P and Q, and let the centers of the semicircles that they are on by O_P and O_Q, respectively. Then

$$PQ \leq PO_P + O_PO_Q + O_QP = 1 + 1 + 1 = \boxed{3}$$

with equality when P, O_P, O_Q, Q collinear.



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