2008 China TST #1

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Let ABC be a triangle with $AB > AC$. Its incircle touches side BC at point E. Point D is the second intersection of the incircle with segment AE. Point F (different from E) is taken on segment AE such that $CE = CF$. The ray CF meets BD at point G. Show that $CF = FG$.

Let the A-intouch chord meet BC at T. Since the quadrilateral formed by the three intouch points and D is harmonic, DT is tangent to the incircle. Then

$$\measuredangle EDT = \measuredangle TED = \measuredangle CEF = \measuredangle EFC$$

so $CF \parallel TD$. Then

$$-1 = (B, C; E, T) \stackrel{D}{=} (G, C; F, \infty_{CF})$$

and thus F is the midpoint of CG as desired. ■