

2019 HMMT Guts #18

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2019 points are chosen independently and uniformly at random on the interval $[0, 1]$. Tairitsu picks 1000 of them randomly and colors them black, leaving the remaining ones white. Hikari then computes the sum of the positions of the leftmost white point and the rightmost black point. What is the probability that this sum is at most 1?

Note that the probability that the leftmost white point is x is $1019(1-x)^{1018}$ and the probability that the rightmost black point is y is $1000y^{999}$. Thus the probability that the sum is at most 1 is

$$\int_0^1 \int_0^{1-x} 1019(1-x)^{1018} \cdot 1000y^{999} dy dx = \int_0^1 1019(1-x)^{1018} (1-x)^{1000} dx = \boxed{\frac{1019}{2019}}$$

as desired. ■