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Let ABC be a triangle with $\angle ABC$ obtuse. The *A-excircle* is a circle in the exterior of $\triangle ABC$ that is tangent to side BC of the triangle and tangent to the extensions of the other two sides. Let E, F be the feet of the altitudes from B and C to lines AC and AB, respectively. Can line EF be tangent to the A-excircle?

Since $\triangle AEF \sim \triangle ABC$ with ratio $\cos A$, we get that the ratio of A-exradii is also $\cos A$. But if EF is tangent to the A-excircle of $\triangle ABC$, then the A-excircles of $\triangle ABC$ and $\triangle AEF$ are the same, and thus this ratio is 1. So $\cos A = 1$, contradiction. ∎