2011 Putnam B1

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Let h and k be positive integers. Prove that for every $\epsilon > 0$, there are positive integers m and n such that

$$\epsilon < |h\sqrt{m} - k\sqrt{n}| < 2\epsilon.$$

Take $m = ak^2, n = bh^2$ so that $\frac{\epsilon}{hk} < |\sqrt{a} - \sqrt{b}| < \frac{2\epsilon}{hk}$. This is possible by letting $a = \left\lfloor \left(\sqrt{b} + \frac{\epsilon}{hk}\right)^2 \right\rfloor + 1$ for b large. ■