2009 Putnam A1

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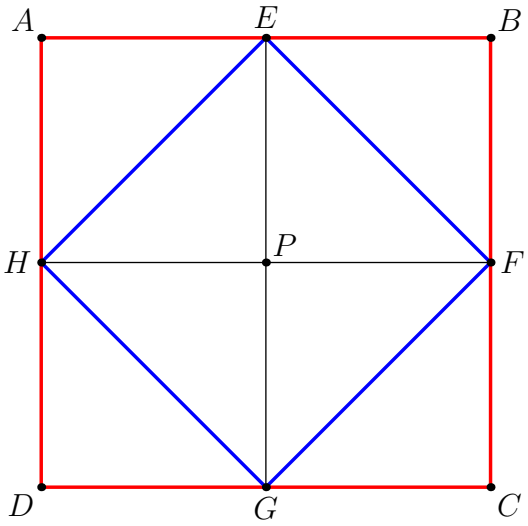
Let f be a real values function on the plane such that for every square $ABCD$ in the plane, $f(A) + f(B) + f(C) + f(D) = 0$. Does it follow that $f(P) = 0$ for all points P in the plane?

Yes. Let P be an arbitrary point. Let $ABCD$ be a square centered at P, and let E, F, G, H be the midpoints of AB, BC, CD, DA, respectively.



Then

$$\begin{aligned} f(P) &= f(P) + \frac{1}{4}[f(A) + f(B) + f(C) + f(D)] + \frac{1}{2}[f(E) + f(F) + f(G) + f(H)] \\ &= \frac{1}{4}[f(A) + f(E) + f(P) + f(H)] + \frac{1}{4}[f(E) + f(B) + f(F) + f(P)] \\ &+ \frac{1}{4}[f(P) + f(F) + f(C) + f(G)] + \frac{1}{4}[f(H) + f(P) + f(G) + f(D)] \\ &= 0 \end{aligned}$$

as desired. ■