

2012 HMMT G8

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Hexagon $ABCDEF$ has a circumscribed circle and an inscribed circle. If $AB = 9$, $BC = 6$, $CD = 2$, and $EF = 4$, find $\{DE, FA\}$.

By Brianchon, AD, BE, CF concurrent. By the famous China TST,

$$AB \cdot CD \cdot EF = BC \cdot DE \cdot FA.$$

By Pitot,

$$AB + CD + EF = BC + DE + FA.$$

Combining these two gives

$$DE + FA = 9$$

$$DE \cdot FA = 12$$

which by Vieta implies $\{DE, FA\} = \left\{ \frac{9 + \sqrt{33}}{2}, \frac{9 - \sqrt{33}}{2} \right\}$.

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