

2015 ISL G2

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Triangle ABC has circumcircle Ω and circumcenter O . A circle Γ with center A intersects the segment BC at points D and E , such that B, D, E , and C are all different and lie on line BC in this order. Let F and G be the points of intersection of Γ and Ω , such that A, F, B, C , and G lie on Ω in this order. Let K be the second point of intersection of the circumcircle of triangle BDF and the segment AB . Let L be the second point of intersection of the circumcircle of triangle CGE and the segment CA .

Suppose that the lines FK and GL are different and intersect at the point X . Prove that X lies on the line AO .

Observe that

$$\begin{aligned}\angle AFK &= \angle AFG + \angle GFD + \angle DFK \\ &= \angle ABG + \angle GED + \angle DBK \\ &= \angle ABG + \angle GEC + \angle CBA \\ &= \angle CBG + \angle GEC \\ &= \angle CAG + \angle GLC \\ &= \angle LAG + \angle GLA \\ &= \angle LGA,\end{aligned}$$

so lines FK and GL are symmetric in AO (since AF and AG are symmetric in AO), so they intersect on AO . ■