ASE 2020-21 Sunday Notes

Lecture Notes by Dylan Yu*

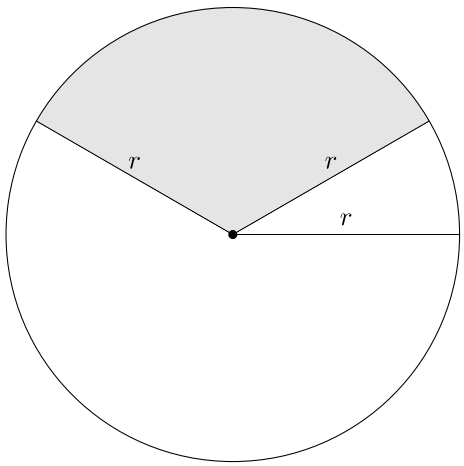
Last Updated September 20, 2020

Contents

§ 1 Geometry Formulas

*The ASE playlist can be found here.

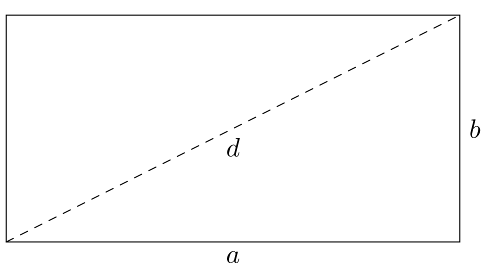
§ 1.1 Circles

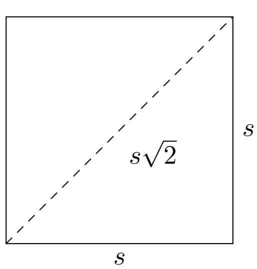


Everyone's seen a circle before. There are a few important properties about it.

1. The diameter of a circle with radius r is $d = 2r$.
2. The area of a circle with radius r is $A = \pi r^2$.
3. The circumference (perimeter) of a circle with radius r is $C = 2\pi r$.
4. The arc of a sector with degree θ has length $S = \frac{\pi r \theta}{180}$.
5. The area of a sector with degree θ is $A = \frac{\pi r^2 \theta}{180}$.

§ 1.2 Square & Rectangle





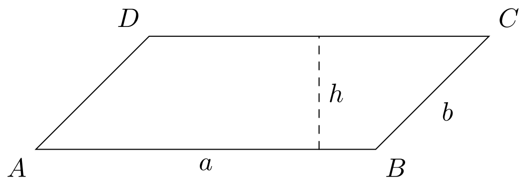
Squares and rectangles have very nice properties.

1. The angles in a rectangle are $90°$
2. The perimeter of a rectangle with length a and width b is $2(a + b)$.
3. The area of a rectangle with length a and width b is ab.
4. The diagonal of a rectangle has length $\sqrt{a^2 + b^2}$ (Pythagorean Theorem).

Squares are considered regular polygons.

1. Everything above applies to squares.
2. The side lengths are all the same.
3. The perimeter of a square with length s is $4s$.
4. The area of a square with length s is s^2.
5. The diagonal of a square has length $s\sqrt{2}$.

§ 1.3 Rhombus & Parallelogram



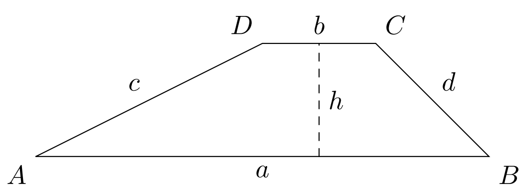
A rhombus is a parallelogram with equal side lengths. For parallelograms:

1. $\angle A = \angle C$, $\angle B = \angle D$.
2. $\angle A + \angle C = \angle B + \angle D = 180°$.
3. $2(AB^2 + BC^2) = AC^2 + BD^2$.
4. $[ABCD] = ah$.

For rhombi:

1. Everything above applies to rhombi.
2. $a = b$.

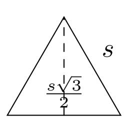
§ 1.4 Trapezoid



A trapezoid has one set of parallel sides.

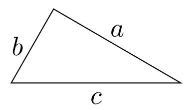
1. $\angle A + \angle D = 180°$.
2. $\angle B + \angle C = 180°$.
3. If $a > b$, $a = \sqrt{c^2 - h^2} + \sqrt{d^2 - h^2} + b$. Using this, if we know a, b, c, d we can solve for h.
4. $[ABCD] = \frac{1}{2}(a + b)h$.

§ 1.5 Triangle



For equilateral triangles:

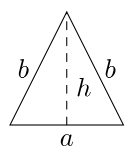
1. The angles are equal to $60°$ and the sides are equal.
2. The height of an equilateral triangle with side length s is $\frac{s\sqrt{3}}{2}$.
3. The area is $\frac{s^2\sqrt{3}}{4}$.



For right triangles:

1. The area of a right triangle with legs a and b is $\frac{ab}{2}$.

2. The hypotenuse is $c = \sqrt{a^2 + b^2}$.



For isosceles triangles:

1. The height from the vertex opposite the base bisects the base.

2. If $\triangle ABC$ is isosceles such that $AB = BC$, then the height from the vertex opposite the base bisects $\angle ABC$.

For all triangles:

1. If the side lengths are a, b, c, then $a + b > c, b + c > a, c + a > b$.

2. If the semiperimeter is $s = \frac{a+b+c}{2}$, then the area is $\sqrt{s(s-a)(s-b)(s-c)}$.

§ 1.6 Other Formulas

1. Area of a pentagon: if the side length is s, then $\frac{1}{4}\sqrt{5(5+2\sqrt{5})}s^2$.

2. Volume of a sphere: if the radius is r, then $\frac{4}{3}\pi r^3$.

3. Surface area of a sphere: if the radius is r, then $4\pi r^2$.

4. Volume of a cylinder: if the radius is r and height is h, then $\pi r^2 h$.

5. Surface area of a cylinder: if the radius is r and height is h, then $2\pi r(r+h)$.

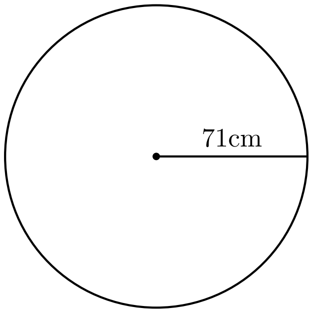
6. Volume of a cone: if the radius is r and height is h, then $\frac{1}{3}\pi r^2 h$

7. Surface area of a cone: if the radius is r and height is h, then $\pi r(r + \sqrt{r^2 + h^2})$.

8. Volume of a pyramid: if the area of the base is A and the height is h, then $\frac{1}{3}Ah$. For example, if the base is a square with side length s, then $A = s^2$.

§ 1.7 Novice Problems

1.



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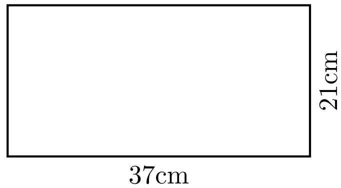

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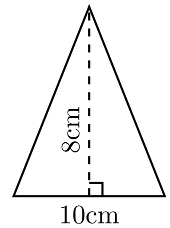

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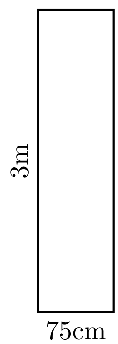

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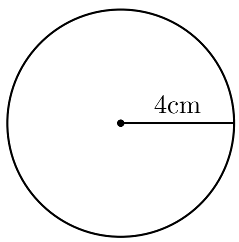

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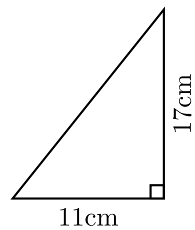

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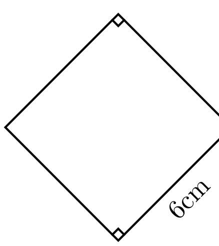

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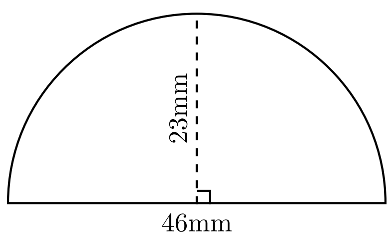

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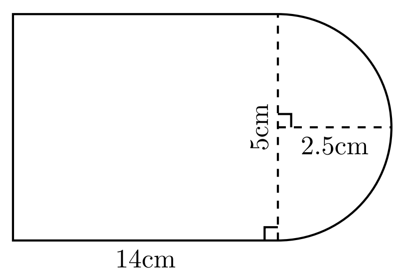

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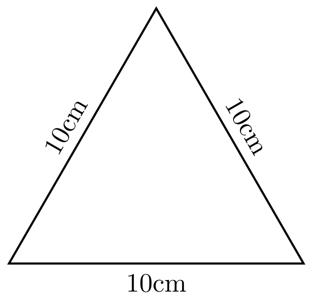

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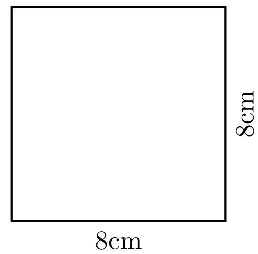

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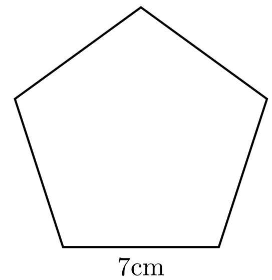


Answer: ____________ Answer: ____________ Answer: ____________

13. What is the volume and surface area of a sphere with diameter 6?

14. What is the area of a triangle with sides $13, 14$, and 15?

15. What is the volume and surface area of a cylinder with radius 3 and height 6?

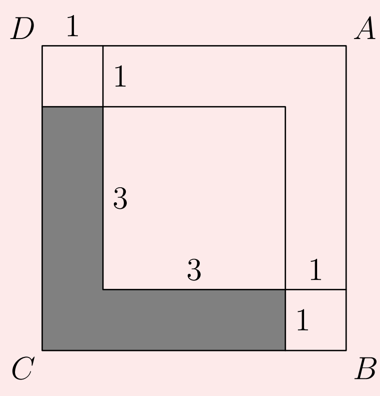
16. What is the volume and surface area of a cone with radius 3 and height 6?

17. What is the area of a square pyramid with a square base with side length of 3 and height of 6?

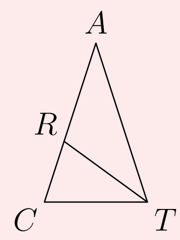
§ 1.8 Advanced Problems

These problems require more skills than the formulas above. This is just to practice for AMC 8.

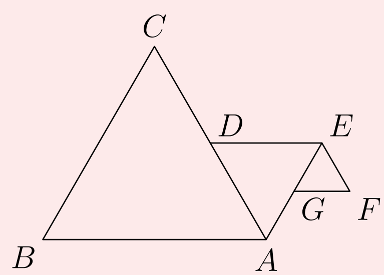
Problem 1 (AMC 8 2000/6). Figure $ABCD$ is a square. Inside this square three smaller squares are drawn with the side lengths as labeled. The area of the shaded L-shaped region is



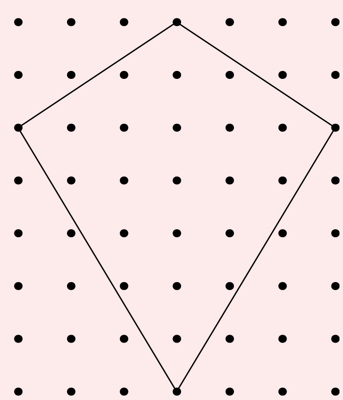
Problem 2 (AMC 8 2000/13). In triangle CAT, we have $\angle ACT = \angle ATC$ and $\angle CAT = 36^\circ$. If $\overline{TR}$ bisects $\angle ATC$, then $\angle CRT =$



Problem 3 (AMC 8 2000/15). Triangles ABC, ADE, and EFG are all equilateral. Points D and G are midpoints of $\overline{AC}$ and $\overline{AE}$, respectively. If $AB = 4$, what is the perimeter of figure $ABCDEFG$?



3 Part Question To promote her school's annual Kite Olympics, Genevieve makes a small kite and a large kite for a bulletin board display. The kites look like the one in the diagram below. For her small kite Genevieve draws the kite on a one-inch grid. For the large kite she triples both the height and width of the entire grid.

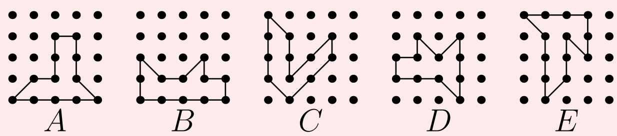


Problem 4 (AMC 8 2001/7). What is the number of square inches in the area of the small kite?

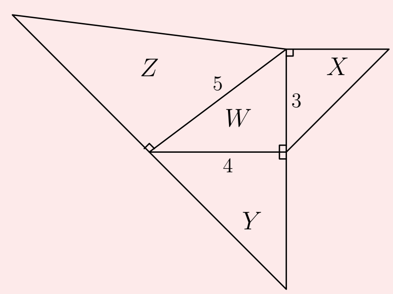
Problem 5 (AMC 8 2001/8). Genevieve puts bracing on her large kite in the form of a cross connecting opposite corners of the kite. How many inches of bracing material does she need?

Problem 6 (AMC 8 2001/9). The large kite is covered with gold foil. The foil is cut from a rectangular piece that just covers the entire grid. How many square inches of waste material are cut off from the four corners?

Problem 7 (AMC 8 2002/15). Which of the following polygons has the largest area?

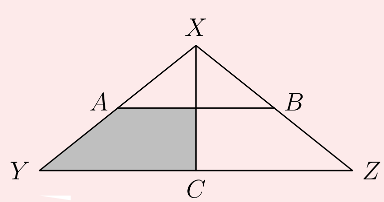


Problem 8 (AMC 8 2002/16). Right isosceles triangles are constructed on the sides of a 3-4-5 right triangle, as shown. A capital letter represents the area of each triangle. Which one of the following is true?

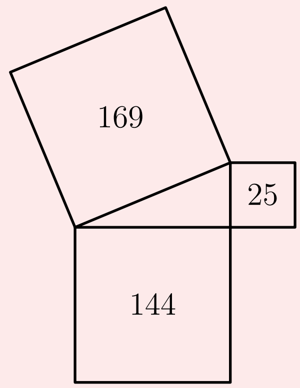


(A) $X + Z = W + Y$ (B) $W + X = Z$ (C) $3X + 4Y = 5Z$ (D) $X + W = \frac{1}{2}(Y + Z)$ (E) $X + Y = Z$

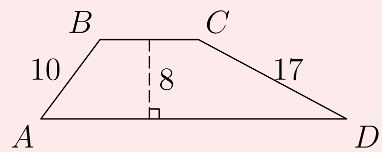
Problem 9 (AMC 8 2002/20). The area of triangle XYZ is 8 square inches. Points A and B are midpoints of congruent segments $\overline{XY}$ and $\overline{XZ}$. Altitude $\overline{XC}$ bisects $\overline{YZ}$. The area (in square inches) of the shaded region is



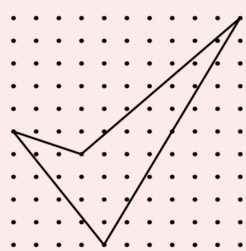
Problem 10 (AMC 8 2003/6). Given the areas of the three squares in the figure, what is the area of the interior triangle?



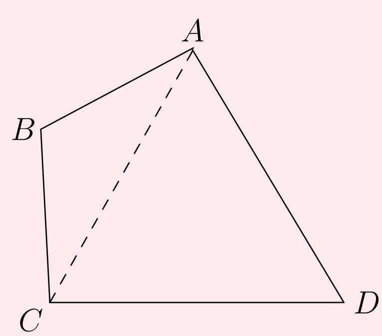
Problem 11 (AMC 8 2003/21). The area of trapezoid $ABCD$ is 164 cm^2. The altitude is 8 cm, AB is 10 cm, and CD is 17 cm. What is BC, in centimeters?



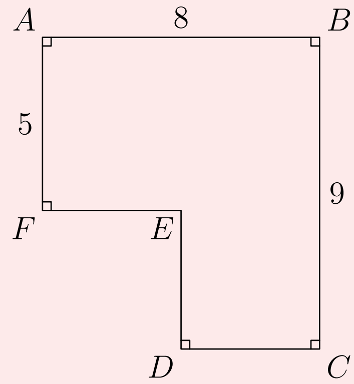
Problem 12 (AMC 8 2004/14). What is the area enclosed by the geoboard quadrilateral below?



Problem 13. In quadrilateral $ABCD$, sides $\overline{AB}$ and $\overline{BC}$ both have length 10, sides $\overline{CD}$ and $\overline{DA}$ both have length 17, and the measure of angle ADC is 60°. What is the length of diagonal $\overline{AC}$?



Problem 14. The area of polygon $ABCDEF$ is 52 with $AB = 8$, $BC = 9$ and $FA = 5$. What is $DE+EF$?



Problem 15. What is the perimeter of trapezoid $ABCD$?