DISCLAIMER: *If these strategies don't work it's not my fault! I tried my best, and not every single thing here will work for everyone. You can tell me if something is blatantly wrong, but don't say "Oh I tried checking and I got a 1, so this document deserves a 0/10".*

Note: *If you would like to suggest something, PM me at freeman66 on AoPS*

AIME Strategies

Dylan Yu

0 Table of Contents

1 Abstract

The purpose of this document is to explain some strategies you can employ during the AIME, and maybe it'll help you! If it doesn't, find your own strategy, and if you know any better strategies, let me know!

It should be noted that there has been a "revolution" in problem writing for the AIME, where a lot of the older techniques / types of problems don't show up as much, meaning the problems are a) harder and b) need more creativity. Thus, it is a good idea to expose yourself to all types of problems. Of course, this does not mean these new problems don't rely on the traditional techniques - those are still very important.

2 Preparation

"By failing to prepare, you are preparing to fail." -Benjamin Franklin

Preparing for the AIME is perhaps the greatest deciding factor on how well one does. Sure, some people might have a good testing day or get lucky and have seen one of the problems before. However, there is always a clear difference between the person who put in 1000 hours and the person who only put in 100. Preparing for the AIME is crucial, and to succeed and pass the AIME requires hard work to be put in. Not only must you spend a ton of time preparing, this time must also be used efficiently in order to be well spent.

2.1 Groups of People

Preparing for the AIME depends on what type of person you are and at what stage you are in regards to your math ability. There are typically three types of people: those who just barely qualified for the AIME, those who are aiming to qualify for the USA(J)MO for their first time but isn't sure they can qualify, and those who have already qualified for the USA(J)MO and know that they can for sure qualify. For the first group, don't take the AIME too seriously. If this is only your first time qualifying, then you should congratulate yourself for that success and try your best on the AIME, but don't be sad if you could only solve a couple of the problems. However, you should still practice. Try to be able to consistently solve the first 5 problems of any AIME - those are typically in the range of AMC10/12 as well. On the day of the test, relax and just have fun. The second group is probably the more common group to be in. For the second group, preparing for the AIME is critical. There are some people who skip the second group altogether and go straight to the olympiads as soon as they qualify for the AIME. You want to be those kinds of people. However, many more people find themselves qualifying for AIME year after year, barely missing the USA(J)MO cutoff. This group is probably one of the most stressful to be in, so you want to leave this group as quickly as possible. To do this, you need to start preparing for the AIME the first or second year you qualify. Most people fall into the trap of only trying to improve their AIME score. Do

not just do this. Instead, you must also make sure that you will score 135+ on the AMC10/12. Raising your AIME score by 1 or 2 points is incredibly tough, but making sure that you do not silly on the AMC10/12 can bring your index up by 12-18 points. In addition to this, you should spend a year increasing your AIME score to a consistent 10+ on mocks. Only after doing this will you have a chance at qualifying for the USA(J)MO. Lastly, if you are in the third group, do not worry too much. Do a couple AMC10/12's before the AMC10/12 so you don't get a terrible score, and do a couple AIME's before the real contest. Other than that, you should not be stressing out too much, instead focus your efforts on olympiad.

The rest of this section will be focusing on the preparation that the second group should do, as the first and third group do not need to focus too much on their preparation.

2.2 Over the summer and Fall (3-9 months before the AIME)

This time should be spent on improving your math knowledge. You have all the free time in summer and you shouldn't have too much homework in the fall, so make sure to use this time well. Attend camps, read books, attend local lectures, etc. You shouldn't be doing problem sets under time pressure at this point. Instead, try to focus more on theory and learn things that might even be more advanced than what is on the AIME. You should not be doing too many AIME problems instead you should be learning new theorems and their proofs.

2.3 1-3 months before the AIME

The time to transition to this new style of preparation is announced by the beginning of winter break. Start mixing AIME problem sets along with your books and other materials. Try to do one timed AIME every week, and make sure that you understand how to solve all the problems. For each problem that you got wrong, do not just read the solution, and don't even just say that you understood it. Instead, you must understand the solution and be able to write the solution 3 days after reading the solution. This will test if you really understood the solution or if you just read it without thinking. In addition to these AIME problem sets, also mix in computational problems from other contests such as HMMT, PUMAC, CMIMC, ARML, and the like. During this time is the AMC10/12. Two weeks before the AMC10/12, start doing timed AMC10/12 problem sets. Make sure you get a good score on the AMC10/12. Getting a good score practically determines what you can do on the AIME, so you must make sure you get the score that you wished for - preferable 130+.

2.4 1-4 weeks before the AIME

By this point, you should start only doing problems. I recommend doing at least 1 timed AIME practice set once every three days, as well as reading their solutions and rewriting the solutions. This is where every single minute of your day should be spent on

doing problems in order to raise your AIME score by as much as possible. The number of practice AIME's that you do during this time period have the greatest effect on your AIME score. In addition to doing past AIME's, I also recommend mocks, which are typically harder than a normal AIME. You can find many high quality ones inside of the Aops Mock Contest Forum on Aops.

2.5 1 week before the AIME

You should try to squeeze in at least one more practice AIME in the last week. You should also start to plan your strategy, ex) What would I go for, a P13 Geo or a P10 Algebra? Planning ahead is crucial because choosing the right problems to do can either give you that one extra point or waste an hour of your time. You should also start to review the problems that you've gotten wrong before and try to resolve them. In addition, review concepts and theorems. If you realize that you have no idea on one of the concepts, then try to cram it in as quickly as possible. The last couple days shouldn't be spent much on math. Instead, relax, maybe review some more theorems, but by this point you should just rest to ensure you have the best possible AIME score.

2.6 Resources

Below are some common resources that I've used for my own AIME preparation. This is by no means an exhaustive list, so if you find a good resource that is not on the list, feel free to use it. Also, by Google searching and chasing threads you can find a lot of good handouts. Study by **individual handouts** (or books if you like those), **not by randomly asking people for resources.**

Volume 1: A lot of people ignore this book as they think it's "too easy" for AIME. However, it's great for teaching you the basics and is sort of like a fundamental

Volume 2: This will teach you much more advanced topics than Volume 1, and it is good for the middle and last 5 AIME problems

Introductory Aops Books: Great for building a foundation - will help you get through the first and middle 5.

[This Handout](): It has all the AIME problems sorted into categories by subject and then by difficulty.

Intermediate Algebra: Covers pretty much all the AIME algebra topics

Intermediate Counting and Probability: Covers pretty much all the AIME counting and probability topics

BOGTRO's AIME study guide: Very useful study guide for the AIME

Markan's AIME syllabus: Good for review

CMC series: These are a series of mock contests. Their mock AMC's are probably way too hard, but their AIME is fine.

2015 Mock AIME I by djmathman and Binomial-Theorem: Quite a nice, high quality mock AIME

djmathman's AIME practice set: [**Link here**](#)
Olympiad Number Theory by Justin Stevens: Probably a bit too advanced, but it covers many topics that would be on the AIME (Many Olympiad NT topics are useful on AIME, for ex.: LTE)
djmathman's 100 Geometry problems handout: Also probably a bit too advanced, but it contains some nice Geometry Problems
Cjquines' Geometry Handout: Most of the theorems on here aren't too useful but reviewing them can't hurt. [Link here](#)

3 Time Management

At the AMC we have 3 minutes per problem. At the AIME we have 12. That means the timing strategies need to be different. Indeed, the AMC is so fast-paced that it is reasonable to save time by not reading a problem twice. If you read it, you either solve it or skip it and go on. The student who is not trying to achieve a perfect score can decide in advance not to read those final, highly-difficult problems.

Strategy 1. When deciding between checking your work or doing another problem, and it's crunch time, I recommend using the following algorithm:

1. If less than 15 minutes left
   a. Check your work
2. If less than 30 minutes left
   a. Try the problem for 15 minutes
3. Anything else
   a. Split around ⅔ ratio: in other words, spend two-thirds of the time checking

Note: the ratio depends on preference, and if you get the gut feeling this isn't going to work out, abort and go check your other problems.

4 Problem Management

What problems should we do, and which should we abort? This will tell you what to do!

Strategy 2. When reading through the problems, I recommend using the following algorithm:

1. If solution is immediately found
   a. If solution is a long bash
      i. Do the bash now, put priority on checking (see Checking section)

      b. If solution is short and sweet
          i. (Obviously) Do it now, put checking priority towards end of list
  2. If solution is not immediately found but an idea is found
      a. Write down your idea next to the problem, skip
  3. If solution is not immediately found with no idea
      a. Skip
  4. If step 1 is no longer possible
      a. Proceed to step 2, then step 3, etc.

A few notes: *What if I don't immediately see the solution, but can probably do it?* Duh. do it. *Won't it take a lot of time to put the idea next to the problem?* Then just memorize it.

For the AIME it is not expensive, in relative terms of time, to read all the problems. You can read the problems and choose the most promising ones to start with, knowing that if there is time they can always come back to other problems.

Strategy 3. Streamlined solutions are the best, meaning you just go with the flow, and this reduces the amount of work you actually have to do. This only works if you've seen a problem like this before, because the more you try a certain idea or method the easier it is for you to do it. Another idea is to blindly apply a formula you've learned before. This might allow you to learn something very important regarding the problem.

5 Checking

I've noticed that the accuracy level of students who take the AIME for the first time drops significantly. It seems that they are so used to multiple choice questions that they rely on multiple choices as a confirmation that they are right. So when someone solves a problem, they compare their answer to the given choices and if the answer is on the list they assume that the answer must be correct. Their pattern is broken when there are no choices. So they arrive at an answer and since there is no way to check it against choices, they just submit it. Because of this lack of confirmation, checking their answer in other ways becomes more important.

Strategy 4. When checking the problems, I recommend using the following algorithm:
  1. If it was bash
      a. Check longest bash first

i. Methods of checking: check your work, redo the problem, or try to find another way to do the problem
2. If it was not bash
   a. Check longest solution first

As a general rule of thumb, check the longer solutions, because those are more prone to mistakes. That being said, if a #14 seems like a one-liner, you (probably) have an error in your thinking. Reread the problem!

6 Guessing

Guessing at the AMC is very profitable if you can exclude three choices out of the given five. Guessing for the AIME is a waste of time because the answers are integers between 000 and 999. So the probability of a random guess is one in a thousand. Actually, this is not quite right, because the problem writers are human and it is much easier to write a problem with an answer of 10 than one with an answer of 731. But the AIME designers are trying very hard to make answers that are randomly distributed. So the probability of a random guess is not one in a thousand, but it is very close. You can improve your chances by an intelligent guess. For example, you might notice that the answer must be divisible by 10. But guessing is still a waste of time. Thinking about a problem for two minutes in order to increase the probability of a correct guess to one in a 100 means that your expected gain is 1/200 points per minute. Which is usually much less than the gain for checking your answers. You can play the guessing game if you have exhausted your other options.

I remember someone collected (outdated data) that stated the most popular answer choice was 025. This is not likely to be correct, but feel free to put it!

Strategy 5. This is more like "guessing the formula", but whatever. If you forget part of a formula, try to think of what it basically looked like. Plug in values to make that formula match the data you put in (this works only if you know what goes into the formula and what will come out). Just assume the formula, and if you have time (which you probably won't) try to prove it.

Strategy 6. If you see a pattern, use **engineer's induction**! However, note that sometimes the pattern a) grows too big, and it's hard to find the first few terms, and b) the pattern sometimes isn't what it seems, so this can be solved by just trying more terms.

Strategy 7. (Engineer's Induction) If a pattern seems reasonable, assume it's true! Sometimes, even if it doesn't seem reasonable, still assume it's true if you are running low on time. Usually, the pattern is right.

Strategy 8. (Collinearity and Concurrency) Draw a few cases you know, and if it works out assume collinear / concurrent. If you running low on time, just assume it without proof, or just try the equilateral case (which usually doesn't work out because everything is nice in an equilateral triangle).

Note: Strategy 8 applies to all areas: if you are running low on time, just look at your diagram and assume a bunch of stuff.

Strategy 9. Draw a VERY DETAILED diagram. This means make lengths reasonable - if the answer is an integer you could very easily guess the answer. This might lead you to solve a #15 before a #3 (most of the time, the AIME writers are smart and add in square roots and fractions, but it couldn't hurt to estimate the answer so you can double check your real answer).

7 Fakesolving

Fakesolving is the idea of getting the correct answer while not necessarily proving or justifying that your answer is correct. It can be used in many cases where the problem statement doesn't specify certain things, and is incredibly useful to get one or two extra points. Obviously, this strategy only works for computational competitions and not olympiads.

Strategy 10. If you have a couple degrees of freedom in a geometry problem, use that to your advantage. Assume the central shape is something like an equilateral triangle or that a point is on top of another point. Doing this can sometimes trivialize the problem.

Strategy 11. Using a compass and a ruler to draw a very accurate diagram, then use a ruler to measure out the length. This can sometimes give you an extra point.

Strategy 12. Using expected value on problems where they ask you to count something can be incredibly useful, especially when paired with symmetry. For example, look at 2017 AIME II Problem 12.

8 Strategies and Techniques for Specific Topics

I wanted to include this, but I wasn't sure where: *look out for the definition to fact problems*. What I mean by this is, if the problem states *"A point is 3 inches away from 3 other points…"*, you immediately know this point is the circumcenter of the other 3 points, and the circumradius has a length of 3 inches! So knowing the definition extremely well is important. For example, when you draw a good figure, and you see a point that looks like the orthocenter, it probably is. Knowing various properties of orthocenters can help prove that.

8.1 Algebra

Variables. Use common notation, like x for distance, t for speed, etc.

Equations. If you get a high-degree polynomial, and it is not symmetric abort. Similarly, a lot of square roots and cube roots could be bad. There is probably a simpler way then, or perhaps $x = 1$ or $x = -1$ will work. If you square an equation, make sure that you check your work, because there is the plus-minus deal to worry about. Basically, check *non-reversible* steps.

Factorizations. The more factorizations you know the better. This is also useful in Number Theory, when you check if a polynomial can be prime. Also, prime factorization is extremely important.

Substitutions. Use substitutions in two ways: a) to reduce a multi-variable equation, or b) to simplify something inside a root. For example, in
$$\sqrt{4n + 5},$$
If you make the substitution $n = t^2 - t - 1$,
$$\sqrt{4t^2 - 4t + 1} = 2t - 1,$$
Reducing the square root.

Sequences and Series. Know arithmetic, geometric, arithmetico-geometric (https://artofproblemsolving.com/wiki/index.php?title=Arithmetico-geometric_series) , telescoping, and periodic sequences / series. Also, telescoping series can sometimes come in the form of polynomials (e.g. 2016 AIME I Problem 11: https://artofproblemsolving.com/wiki/index.php/2016_AIME_I_Problems/Problem_11)

Rate Problems. Only equation necessary is $d = rt$. Sometimes, even relativity helps solve the problem (i.e. assume one object isn't moving).

Polynomials. Vieta's kills most. Transformations like $x^n P(z)$ where $z = \dfrac{1}{x}$ sometimes works, because it reverses all the coefficients.

Logarithms. Knowing all the rules destroys the problems.

Trigonometry. Knowing all the rules really helps. For the problems that require you to sum a bunch of them that form a pattern, use the sum to product / product to sum rules. Also, know that

$$\tan \frac{\pi}{4} = \frac{\tan \theta + \tan(\frac{\pi}{4} - \theta)}{1 - \tan \theta \cdot \tan(\frac{\pi}{4} - \theta)},$$

$$(\tan \theta + 1)(\tan(\frac{\pi}{4} - \theta) + 1) = 2.$$

Inequalities. Rarely shows up, but know that squares are nonnegative. Also, occasionally they will throw a troll out like 2016 AIME II Problem 15 (https://artofproblemsolving.com/wiki/index.php/2016_AIME_II_Probl ems/Problem_15), where the Cauchy-Schwarz Inequality is actually in the equality case. Sometimes, this helps with maximization / minimization problems, but just know: AM-GM, Cauchy-Schwarz, and Rearrangement and you'll (probably) be fine.

Roots of Unity. This also helps with the *summing a bunch of trig values in a pattern* problems. This usually works for equations of the form $\omega^n - 1$.

Functional Equations. On the AIME, it is likely the only functional equation problems that will show up are the ones where you simply plug in values after another in a pattern. These problems are of the following form:

$$a(f(x)) + b(f(\frac{k}{x})) = g(x),$$

Where a, b, k are constants, and $f(x), g(x)$ are functions. $g(x)$ is given. The solution to this type of problem is:

$$f(x) = \frac{a(g(x)) - b(g(\frac{k}{x}))}{a^2 - b^2}.$$

In addition try to guess the function, which can trivialize the problem sometimes.

Newton Sums. Know them. They're useful sometimes and are not too bad to memorize.

Complex Numbers. This happens more often on geometry problems, but know how to handle them and their basic properties, such as $z\bar{z} = |z|^2$ and other basics.

Lagrange Interpolation. Given a set of points, it can help find the smallest degree polynomial that passes through these points. A precursor to this is if you have a group of points right next to each other, by continuously taking the difference you can find the very next term. So for those questions that have $$f(i) = x_i$$ for $$i = 1,2,3,4,\ldots,10$$, you can find $$f(11)$$.

The *uvw* Method. By writing things in terms of $$u = x+y+z, v = xy+yz+zx, w = xyz,$$ you can solve many problems. For example, in the following set of equations,

$$x+y+z = 10,$$
$$x^2+y^2+z^2 = 10,$$
$$x^3+y^3+z^3,$$

You can derive both $$v$$ and $$w$$ ($$u$$ is given). You can also use the aforementioned Newton's Sums.

8.2 Geometry

Auxiliary Lines. This could be a parallel or perpendicular line, or just connecting two previously unconnected points.

Power of a Point. Know it and know how to use it

Similarity and Congruency. Self-explanatory. This is extremely useful when you have to find the area of two objects that are similar, and instead of finding the area you find two corresponding side lengths and just square it. Works with volumes too, but instead cube it.

Area / Perimeter / Surface Area / Volume Formulas. Memorize them. A weird one is the donut, and that may be useful. You can use calculus to derive them, but that's hard during the AIME. It is important to know a lot for triangles and quadrilaterals, especially cyclic quadrilaterals.

Triangles. Know a bunch of properties of them, including Incenter-Excenter, various concurrencies, and collinear points. In my opinion, most late AIME geometry problems (i.e. #11 - #15) are just JMO problems but with nice numbers. Trig Ceva's, Law of Sines, Law of Cosines, and other trig related formulas are great to use on the AIME - they come in handy a lot. Also, know the Kimberling Centers: https://faculty.evansville.edu/ck6/encyclopedia/ETC.html

Quadrilaterals. Know cyclic and tangential quadrilateral properties. Those are especially useful.

Bashing. Geometry has various bashing techniques: coordinate bashing, complex bashing, and barycentric bashing. The last one is hard, so just know mass points (a kindergarten version of barycentric coordinates). A note on coordinate bashing: there are many techniques for this, including

Shoelace, Pick's Theorem, distance from a point to a line, etc. There is also trig bashing, which involves using angles and trig functions to solve the problem (duh).

Coord Bashing. Use it when given a lot of intersections, perpendicular and parallel lines, or ratios. Also useful when you need to find the area of a figure. Shoelace is your best friend, along with choosing the right origin.

Trig Bashing. Law of sines is incredibly useful and can relate angles to lengths. Use it when you can find a lot of angles and their sine and cosine values.

Complex Bashing. Incredibly useful when you have 60, 90 degree angles. Often used for infinite path walking types of problems.

Note: As said before, doing JMO problems allows you to see the harder AIME questions in a new light and apply some quick techniques to finish off the problem. So do them.

<u>A Short List</u>
1. Projective Geometry
2. Radical Axis
3. Symmedians

Transformations. This includes noticing symmetry, rotations, reflections, translations, etc. An important one is the cut-and-paste method, where you cut up an object and move it around - this preserves area but also simplifies the object.

3D Figures. These problems are hated because you have a 2D space to draw a 3D figure, making it hard to visualize. However, by simply knowing a list of formulas (e.g. for tetrahedrons, look through this: https://en.wikipedia.org/wiki/Tetrahedron), you can bash through the problem without even drawing the figure! Of course, a picture is still better for weird cases. In fact, taking the right cross section usually trivializes these problems.

Pythagorean + De Gua's theorems. Memorize them.

Path over 3-D objects. Learn how to unfold 3-d objects so you can find the shortest path

Heron's Shortest Path Problem Technique. Try to understand this technique and know how to use reflections to get the shortest path.

Ptolemy's Theorem. Appears quite often, knowing the formula is usually enough

Stewart's Theorem. Just memorize the formula to find cevian lengths without having to go through a trig bash

Menelaus/Ceva. Useful for finding ratios of sides and is also useful on olympiads. Use it when you are given a lot of ratios.

Ratio Lemma. Good when you are given ratios of the sine of angles. Can sometimes devolve into a Law of Sines bash, and if you find two it can devolve into length bash.

Pitot's Theorem. Gives you lengths and is sometimes useful.

Moving Points. If SOMEHOW a #15 has this, search it up on AoPS and you will get results for this.

8.3 Combinatorics

Combinatorial Arguments. This includes committee forming and block walking.

Simplification. Try doing a simpler problem by removing a constraint, then adding in the constraint later. For example, if you have something in a circle, do it in a line, then transform it back into a circle.

Combinations and Permutations. Know them. They are the foundation of every AIME combinatorics problem.

Symmetry. (*cough cough one of this year's events with states on AMC.*) If something works the same way as another thing but it's reflected, just multiply by 2 and move on. Pretty self-explanatory, and there are so many examples I can't list them all here.

Stars and Bars. Know it.

The Classic 3. (Casework, Constructive, Complementary Counting) These are basically the ways to count, so this is important.

PIE. Principle of Inclusion Exclusion is very important, especially for case work.

Binary. It's possible to transfer some problems to binary notation, where as long as something is yes or no, on or off, etc. (i.e. two options), you can write it as 1s and 0s.

Invariants. If something doesn't change, take full advantage! This is where JMO techniques help.

Bijection. Be able to see them and create them between problems to make the problem easier to solve.

Events with States. If you are ever doing path walking, or considering things that return to its original position, use this. Take advantage of symmetry to simplify the equations.

Recursion. Know how to form and solve recursions, and when to use recursion and when to use casework.

Expected Value. Linearity of Expectation is super powerful - use it.

Catalan. Know when the catalan numbers appear in which problems and know the formula: $C_n = \dfrac{1}{n+1}\dbinom{2n}{n}$. Also be able to form bijections between problems involving catalan.

Fibonacci. Know when fibonacci numbers appear (ex. Stair walking problem, rabbits problem, …)

Roots of Unity Filter. Read it on BOGTRO's AIME List, it helps for solving the summation of combinations.

Generating Functions. Again, read on BOGTRO. Generating functions can even be used on imaginary numbers.

Geometric Probability. Using this, you can transfer the problem to a geometry problem, you can simply just solve for the ratio of areas or volumes.

Combinatorial Identities. In the 2020 AIME 1, Vandermonde's showed up, so you should know that. Also: Pascal's Identity, the summation of a row in Pascal's triangle, etc. A quick Google search should give you plenty of results. Something important is Generalized Vandermonde's:
https://en.wikipedia.org/wiki/Vandermonde%27s_identity#Generalized_Vandermonde's_identity

Chromatic Polynomials. For some reason, these are super useful. They are good for coloring questions regarding cycles, especially the ones where you can't color two adjacent the same:
https://en.wikipedia.org/wiki/Chromatic_polynomial
(Ex: 2016 AIME II Problem 12 is a one line with chromatic polynomials, but also read the other solutions:
https://artofproblemsolving.com/wiki/index.php/2016_AIME_II_Problems/Problem_12)

Hook Length. From Mathcounts to IMO, these are super useful. It might be a good idea to actually understand how it works, but the formula itself requires no insight: https://en.wikipedia.org/wiki/Hook_length_formula
(Note: It wasn't allowed on the IMO because it trivialized the problem, but on AIME it's fair game.)

Symmetric Groups. Apparently these are useful for those questions where the function maps to itself, and you have to find how many ways this can be done, but I'm not sure:
https://en.wikipedia.org/wiki/Symmetric_group

(A note on those problems: sometimes, just doing case work on the amount of pairs (x,x) is better)

8.4 Number Theory

Divisibility. This involves taking the prime factorization, or just knowing the rules you learn from doing Number Sense problems (basically fast paced problems like "does 17 divide 51?").

Simon's Favorite Factoring Trick. Useful in many scenarios, one of which is finding the number of solutions to $\frac{1}{x} + \frac{1}{y} = \frac{1}{n}$.

Difference of Squares. If you need to find out if a polynomial has integer roots, look at the discriminant, and do difference of squares.

Modular Arithmetic. So many techniques here, I'm not going to list them all - but you have to know them. Know how to do inverses and fractions in modular arithmetic - this might require Legendre's. Also know which mods are good for what, for example 7 and 9 are good for cubes while 8 is good for squares, etc.

Bounding. To show there are no more solutions, just place something between two consecutive integers and say that because of this, there are no more integer solutions.

Fibonacci. This relates to geometry too, especially for pentagons and decagons. Know Binet's theorem, and various properties of Fibonacci numbers (like the summation of all the odd Fibonacci numbers up to a certain point).

Euclidean Algorithm and Fermat's Little Theorem. The phi function is super important. Know these two.

Diophantine Equations. Know Bezout's and know how to find solutions to these equations.

Chicken Mcnuggets. Just memorize it and know when to apply it

Lifting the Exponent. See BOGTRO - it's good for finding how many powers of one prime divide a number.

Cyclotomic Polynomials. These are kind of advanced, but it could still help for the integer polynomial type questions.

Zsigmondy's Theorem. Useful for finding if $p | a^n - b^n$. I'm not sure exactly how it works either, but a quick Google search should give you some good results.

Quadratic Residues. This is actually pretty useful for the ones with weird constraints. Even just trying small residues is a good idea (e.g. 2016

AIME I Problem 12:
https://artofproblemsolving.com/wiki/index.php/2016_AIME_I_Problems/Problem_12 (this is not a quadratic residues problem, but it is a "try small residues" problem.)

9 Final Notes / Strategies

Be calm. When in doubt, bash. Go with your gut. Eat a good breakfast. There are two strategies, explained in this video:
https://www.youtube.com/watch?v=x01gndt4qOM
Good luck, have fun, and eat your vegetables! Also, drink a lot of water / boba.

Jeffrey's Advice: Be organized. I recommend having at least 30 sheets of scratch paper and at least 5 pencils, along with your favorite snacks, water, and any necessary medication (inhaler, cough drops, …). Give each problem its own sheet of paper and write neatly so you can easily check your work. For more difficult problems (particularly those in the last 5), try using 2 or 3 sheets of paper each. Even for the earlier ones, where you might only need ⅓ a sheet of paper per problem, still use an entire sheet, front and back, for that one problem. It makes checking your work and finding your ideas for problems so much easier. If you like, you might also want to leave little notes around your work so that you can easily come back to what you were thinking about for that problem.

10 Sources

1. https://blog.tanyakhovanova.com/2012/02/approaching-the-aime-strategically/
2. BOGTRO's AIME List
3. http://markan.net/aime/syllabus.pdf
4. https://www.youtube.com/watch?v=x01gndt4qOM
5. http://blog.tanyakhovanova.com/2010/01/how-to-boost-your-guessing-accuracy-during-tests/