1. The continuous-time signal $x_c(t) = cos(400\pi t)$ is sampled with a sampling period T to obtain the discrete-time signal $x(n) = cos(\frac{\pi n}{3})$. (a) Determine a choice for T consistent with this information. (b) Is your choice for T unique? If so, explain why. If not, specify another choice of T consistent with the given information.

(a)

$$x_c(t) = \cos(400\pi t) \xrightarrow{T_s} x[n] = \cos(\underbrace{\tfrac{\pi}{3}}_{w} n) = \cos(2\pi \cdot f_0 \cdot T_s \cdot n)$$

$2\pi f_0 t \quad f_0 = 200$ Hz

$$f_d = f_0 \cdot T_s$$

$$w_d = 2\pi f_d = \frac{\pi}{3}$$

$$\underline{T_s = \frac{1}{1200} \text{ sec.}}\ \#$$

(b) $w' = w + 2\pi k$

$$\frac{\pi}{3} = 2\pi \cdot \frac{f_0}{f_s} + 2\pi k = 2\pi \left(\frac{f_0 + f_s k}{f_s} \right)$$

$$\frac{1}{6} = \frac{200 + f_s k}{f_s} \qquad f_s = 1200 + 6k f_s \qquad (1-6k) f_s = 1200$$

$f_s = 1200 + 600k$ different T for same w

when

$f_s = 1800 \qquad w = \frac{\pi}{3} + 2\pi = \frac{\pi}{3} \qquad X(e^{j(w+2\pi k)}) = X(e^{jw})$

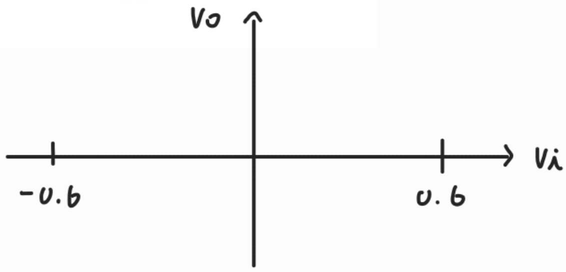
DTFT freq

2. Let $x[n]$ be a discrete-time sequence obtained by sampling a continuous-time signal $x_c(t)$ at $T = 50\mu s$. Suppose $X[k]$ is 4096-point DFT of $x[n]$. What is the frequency spacing of adjacent DFT samples?

$f_s = 20k \qquad N = 4096$

$$\frac{20k}{4096} = 4.88 \text{ Hz}$$

3. Calculate the required resolution of an ideal ADC if the full-scale range is 1.2 V and the desired quantization noise power must be below 5 μV^2.



$$P_q = \frac{\Delta^2}{12} \le 5\mu V^2$$

$$\Delta = \frac{2 \times V_{ref}}{2^n} = \frac{1.2}{2^n}$$

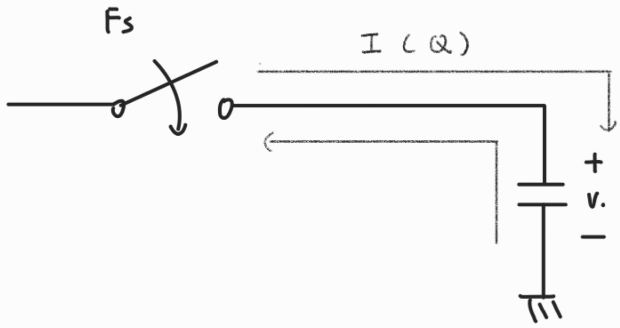
$$\left(\frac{1.2}{2^n}\right)^2 \le 60\mu \quad , \quad \frac{1.44}{2^{2N}} \le 60\mu \qquad 2^{2N} \ge 24000$$

$$2N = \log_2 24000 \approx 14.5$$

8 bits ADC #

4. Prove that the equivalent resistance of a switched-capacitor circuit is given by $R_{EQ} = \frac{1}{F_S C_S}$, where F_S is the rate of charge transfer. Now assuming a switching frequency of 2.5KHz and a desired equivalent resistance of 1000Ω, calculate the required capacitance in the SC circuit that could be used to emulate the resistor.

$$1000 = \frac{1}{2.5k \times C_s} \qquad C_s = 400nF$$



$$Q = C_s V$$

$$I = Q \cdot F_s = C_s \cdot V \cdot F_s \ / \text{sec.}$$

$$R_{EQ} = \frac{V}{I} = \frac{1}{C_s \cdot F_s}$$

5. A Power Spectral Density (PSD) is used to characterize the output of an ADC. If the sampling rate F_s is 10GHz, the number of sample point measurements M is 2^{14}, find a number of cycles C and an input frequency F_{in} that produces a PSD with a white noise floor (quantization errors is free of harmonics). The signal tone has to fall exactly on one of the PSD frequency bins.

$f_s = 10G \qquad M = 2^{14}$

$$\Delta f = \frac{10G}{M} = 610.35k$$

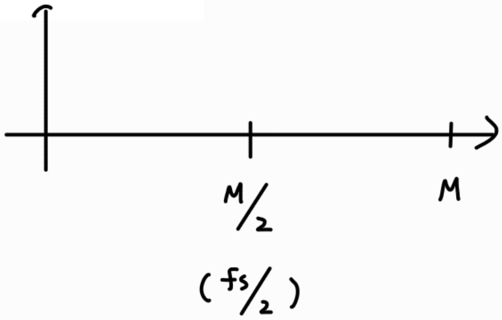
$$f_{in} = \frac{C}{M} \times f_s = \Delta f \times C \quad (\text{falls on bin})$$

ex.

$C = 1000$ cycle $\qquad f_{in} = 610.35\ MHz$

6. Prove that the noise floor $NF_{0_{dB}FS}$ of a normalized PSD plot is given by the expression $NF_{0_{dB}FS} = -SNR_{dB} - 10log_{10}(M/2)$. The normalization is such that the input tone full scale power is at 0dB and the normalized noise floor is $NF_{0_{dB}FS}$.

$P_{signal} = 1$



$$SNR_{dB} = 10\log_{10}\frac{P_{signal}}{P_{noise}}$$

$$0\,dBFS \longrightarrow P_{signal} = 1$$

$$P_{bin} = \frac{P_{total}}{M/2} \longrightarrow P_{total} = P_{bin} \times \frac{M}{2}$$

$$P_{noise} = P_{signal} \cdot 10^{\frac{-SNR_{dB}}{10}}$$

$$NF_{0\,dB\,FS} = 10\log_{10}\frac{P_{noise}}{M/2}$$

$$= 10\log_{10}\frac{P_{signal} \cdot 10^{\frac{-SNR_{dB}}{10}}}{\frac{M}{2}}$$

$$= 10\log_{10} 10^{\frac{-SNR_{dB}}{10}} - 10\log_{10}\frac{M}{2}$$