25 Spring ECEN 610: Mixed-Signal Interfaces

Lab3: Analysis and Simulation of switched Gm-C Filters

Name: Yu-Hao Chen

UIN:435009528

Section:601

Professor: Sebastian Hoyos

TA: Sky Zhao

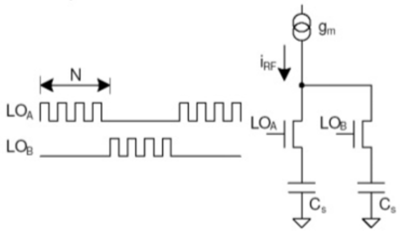
https://github.com/Yu-HaoChen/TAMU_ECEN610_Mixed_signal/tree/main

1. Consider the following circuit, where N=8 and the frequency of the clocks is 2.4 GHz and Cs=15.925 pF. The capacitors are charged in a cyclic fashion by the input current iRF. Similarly, the voltage stored in the capacitors Cs is read cyclically at the end of each consecutive N cycles. The output of the circuit is the concatenation of the cyclic readings of the voltages. Consider the following 2 cases:

   a) The capacitors are discharged after each read out operation, i.e. the charge of the capacitors is zero at the beginning of the integration of every N cycles.
   b) The capacitors are never discharged.

   In both cases find the filter transfer function H(f)=Vo(f)/iRF(f) where Vo(f) is the capacitor voltage. Please use a mathematical description of how the transfer function is found and then use Python to plot the transfer functions.

$$H(f) = \frac{Vo(f)}{iRF(f)}$$



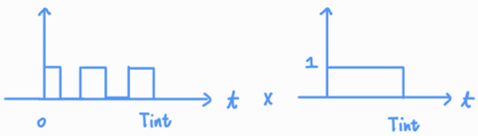
$$T_{LO} = \frac{1}{2.4G} \approx 0.4167\ ns$$

$$V_{cs}(t) = \frac{1}{C}\int_0^t iRF(\tau)\, d\tau \qquad 0 \le t \le T_{int}$$

$$0.4167\ n \times 8 = 3.33\ ns$$

$$w(t) \begin{cases} 1, & 0 \le t \le T_{int} \\ 0, & \text{other} \end{cases}$$

Fourier



$$W(f) = \int_{-\infty}^{\infty} w(t)\, e^{-j2\pi ft}\, dt$$

$$X(t) \times w(t)$$

$$\updownarrow$$

$$X(f) * W(f)$$

$$W(f) = \int_0^{T_{int}} 1 \times e^{-j2\pi ft}\, dt$$

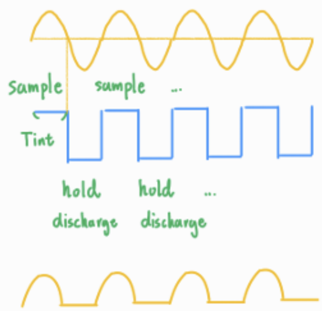
$$= \left.\frac{e^{-j2\pi ft}}{-j2\pi f}\right|_{t=0}^{t=T_{int}}$$

$$= \frac{1}{-j2\pi ft}\left(e^{-j2\pi f T_{int}} - 1\right)$$

$$= \frac{1 - e^{-j2\pi ft\, T_{int}}}{j2\pi f}$$

$$Vo = H(f) \times \frac{1}{C} \times iRF$$

(a)



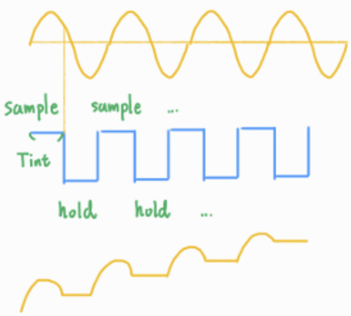
$$\lim_{f \to 0} \frac{1 - e^{-j2\pi f t\, Tint}}{j2\pi f} = Tint$$

$$Vc(t) = \frac{1}{C} \int_0^t i_{RF}(\tau)\, d\tau \qquad 0 \le t \le Tint$$

$$Ha(f) = \frac{1}{C} \left(\frac{1 - e^{-j2\pi f t\, Tint}}{j2\pi f} \right) \left(\frac{1 - e^{-j2\pi f t\, Tint}}{j2\pi f \times Tint} \right)$$

sample　　　　hold (V not change → f = 0)

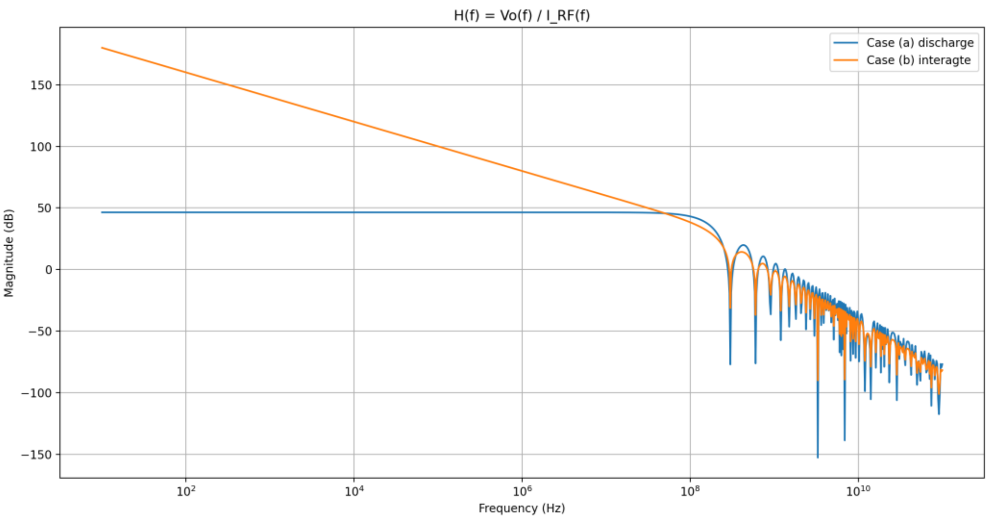
(b)

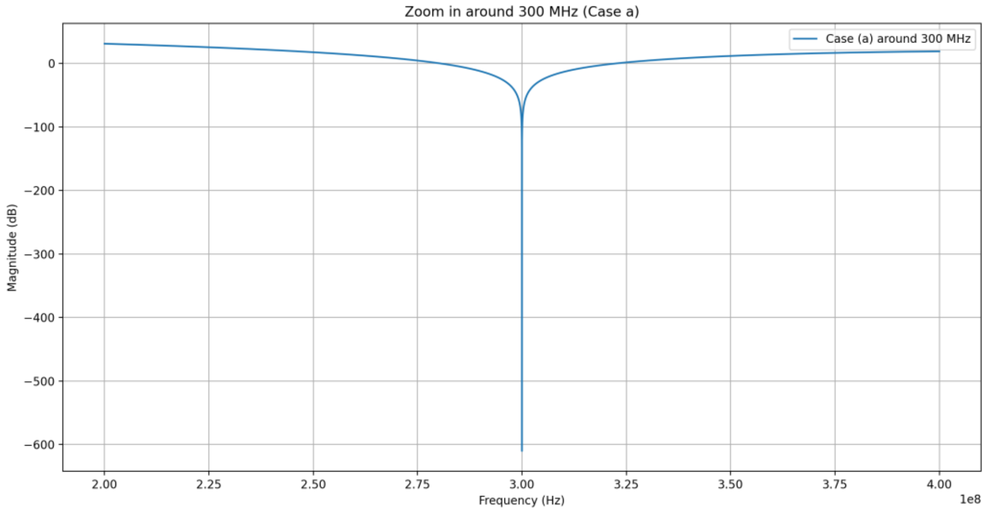


$$Vc(t) = \frac{1}{C} \int_0^t i_{RF}(\tau)\, d\tau \qquad t \ge 0$$

$$Hb(f) = \frac{1}{C} \left(\frac{1}{j2\pi f} \right) \left(\frac{1 - e^{-j2\pi f t\, lint}}{j2\pi f \times Tint} \right)$$

intergrated　　　　hold



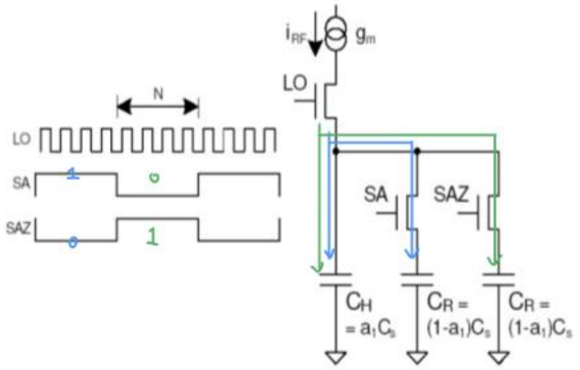


According to model as moving average, the filter has a sinc frequency response with null positions at k*f0/N= k*300M where k=1,2,3… (clear o discharge at every T*n period, in freq gain=0)

Case (a): Since the integrator resets (dumping the accumulated charge) at each interval, it only averages the input over that short time period. For very low-frequency signals (even DC), the output is limited to this brief average and then reset, resulting in a lower and flatter response.

Case (b): The integrator never resets, so the capacitor continuously accumulates charge. This continuous integration allows the output for very low-frequency signals (and DC) to build up over time, yielding a higher response at low frequencies compared to case (a).

2. Now consider the addition of a "history" capacitor CH=15.425 pF and a "rotating" capacitor CR=0.5 pF.
   a.) Explain the effect of adding the capacitor CH in the transfer function that was calculated in problem 1.
   b.) Find the new transfer function and plot it using Python.



$$a_1 = \frac{CH}{CH + CR}$$

- At switching time, C_H retains a_1 portion of its total charge and shares $(1-a_1)$ to the discharged C_R cap. At sampling time j, the system charge s_j is:

$$s_j = a_1 s_{j-1} + w_j$$

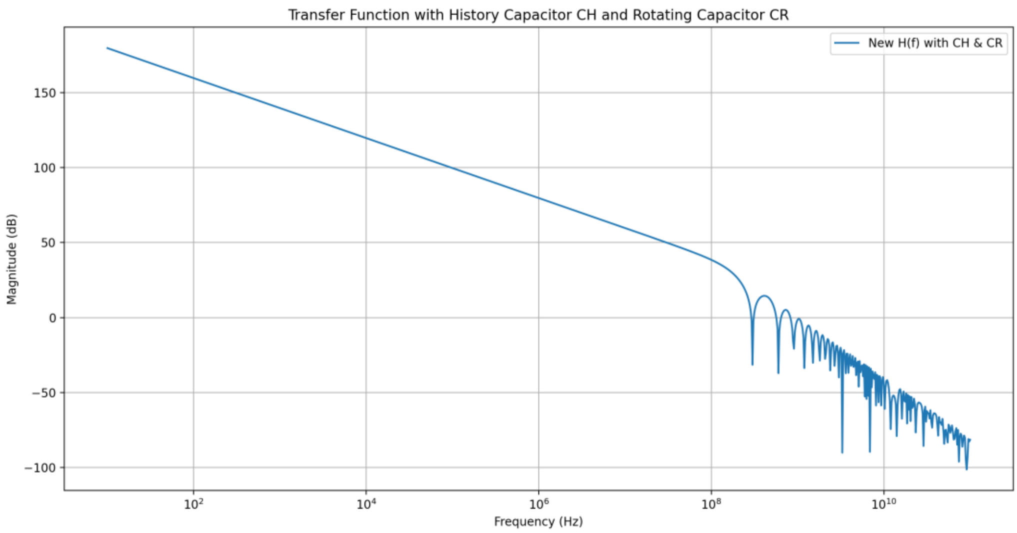
assume from Q1 (a) CR will be discharge

for charge $Cs = CH + CR$

$$CH \text{ (no discharge)} = \frac{1}{j2\pi f\, Cs}$$

$$CR \text{ (perodic discharge)} = \frac{1 - e^{-j2\pi f\, Tint}}{j2\pi f\, Cs}$$

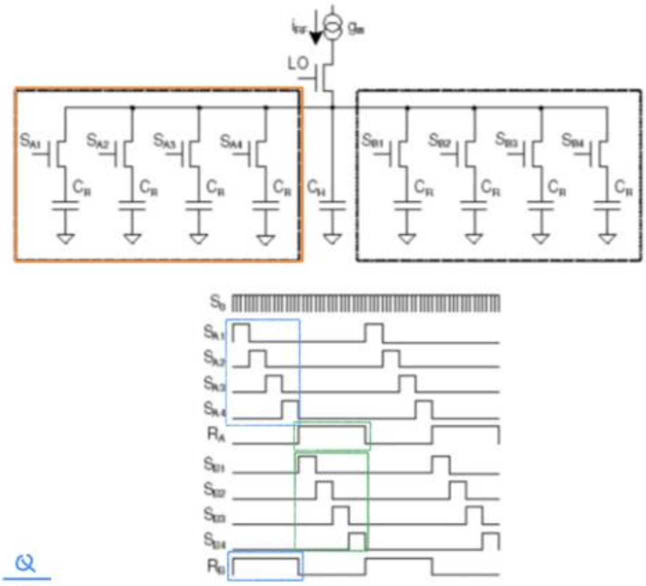
$$H(f) = \frac{1}{Cs} \underbrace{\left[a_1 \frac{1}{j2\pi f} + (1-a_1)\frac{1 - e^{-j2\pi f\, Tint}}{j2\pi f} \right]}_{\text{sample}} \times \underbrace{\left(\frac{1 - e^{-j2\pi f\, Tint}}{j2\pi f \times Tint} \right)}_{\text{hold}}$$



3. Consider the following circuit. This is just an extension of the previous circuit

where the cyclic operation is extended to 8 capacitors. As in the previous circuit, every capacitor also stores N=8 cycles of the input switched current. The output voltage is defined as the voltage resulting from the physical connection of the bank of 4 capacitors enclosed by the rectangle in the figure. This read out operation is also made in a cyclic fashion between the 2 bank of capacitors. Assume ideal transistors and an ideal transconductance gm. Find a mathematical expression for the transfer function and plot in Python for the following 2 situations.

H(f)



$v = \frac{Q}{C}$

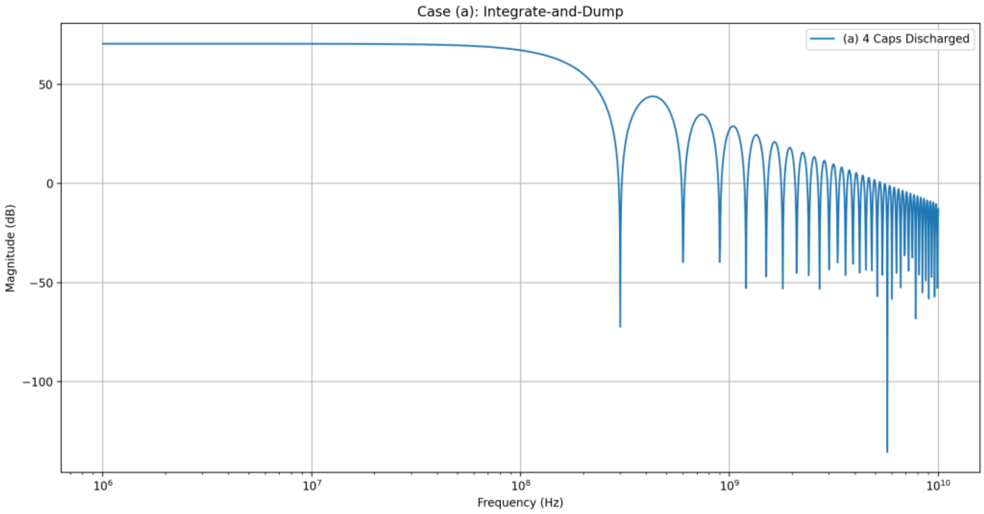
(a) $H(f) = \frac{1}{CR} \underbrace{\left(\frac{1 - e^{-j2\pi f T_{int}}}{j2\pi f} \right)}_{\text{sample}} \underbrace{\left(\frac{1 - e^{-j2\pi f T_{int}}}{j2\pi f \, T_{int}} \right)}_{\text{hold}}$

(single CR)

$V_{out} = \frac{Q_1 + Q_2 + Q_3 + Q_4}{CR_1 + CR_2 + CR_3 + CR_4}$

assume CR are all same $\longrightarrow \frac{Q}{CR}$ same with one CR

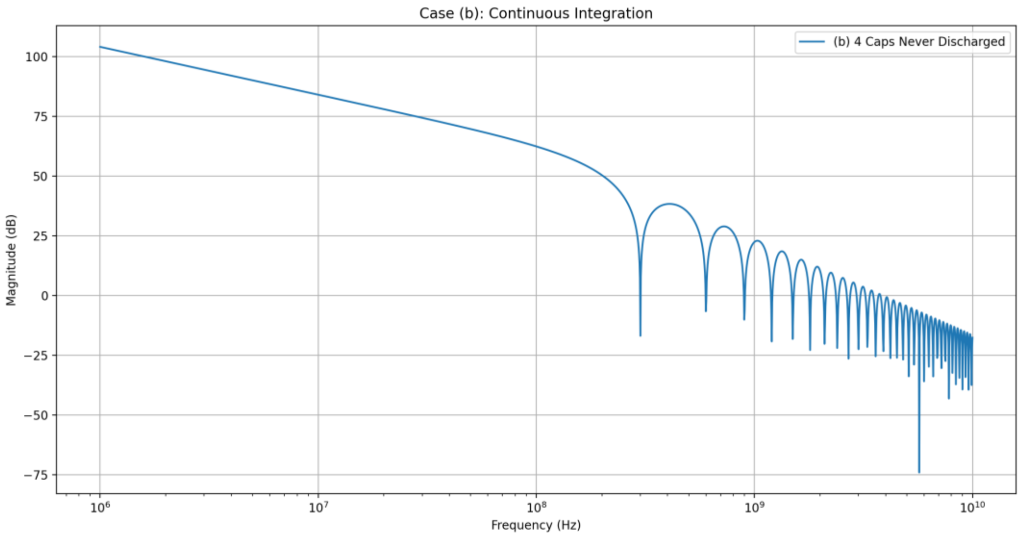
a) The 4 capacitors are discharged after their connection and read out operation, i.e. the charge of the capacitors is zero at the beginning of the integration of every N cycles.



b) The capacitors are never discharged.

(b) $H(f) = \frac{1}{CR} \left(\frac{1}{j2\pi f} \right) \left(\frac{1 - e^{-j2\pi f \, T_{int}}}{j2\pi f \, T_{int}} \right)$

(single CR)



c) The capacitors are discharged but they have different sizes, i.e. CR1, CR2, CR3, CR4.

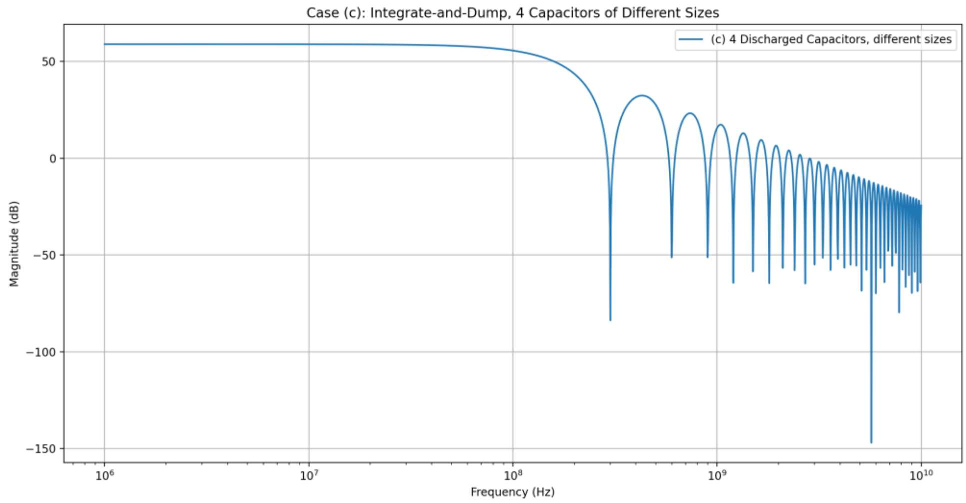
$$(c) \quad V_{ci}(f) = \frac{1}{CR}\left(\frac{1 - e^{-j2\pi f T_{int}}}{j2\pi f}\right)\left(\frac{1 - e^{-j2\pi f T_{int}}}{j2\pi f\ T_{int}}\right)$$

$$V_{out} = \frac{Q_1 + Q_2 + Q_3 + Q_4}{CR_1 + CR_2 + CR_3 + CR_4} = \frac{CR_1 V_{c1} + CR_2 V_{c2} + CR_3 V_{c3} + CR_4 V_{c4}}{CR_1 + CR_2 + CR_3 + CR_4}$$

(caverage)

$$H(f) = \frac{4}{\sum_{i=1}^{4} CR_i}\left[\frac{1 - e^{-j2\pi f T_{int}}}{j2\pi f}\right] \times \left(\frac{1 - e^{-j2\pi f T_{int}}}{j2\pi f \times T_{int}}\right)$$

$C_1 = 2p \quad C_2 = 3p \quad C_3 = 4p \quad C_4 = 6p$



■ Appendix

Q1

```python
import numpy as np
import matplotlib.pyplot as plt

# 參數設定
C = 15.925e-12       # 電容值 (F)
f_LO = 2.4e9         # LO 頻率 (Hz)
N = 8
T_int = N / f_LO     # 積分時間 (s)

# 頻率掃描範圍：從 10Hz ~ 100 GHz
f = np.logspace(np.log10(10), np.log10(1e11), 1000)

w = 2 * np.pi * f

# (a) 每次讀出後放電
# H_a(f) = (1/C)*[(1 - e^{-j w T_int}) / (j w)] * [(1 - e^{-j w T_int}) / (j w T_int)]
Hw_a_num = (1 - np.exp(-1j * w * T_int))   # 分子 1 - e^{-j w T_int}
Hw_a_int = Hw_a_num / (1j * w)             # 有限積分
Hw_a_hold = Hw_a_num / (1j * w * T_int)    # ZOH
H_a = (1/C) * Hw_a_int * Hw_a_hold

# (b) 從不放電
# H_b(f) = (1/C)*[1/(j w)] * [(1 - e^{-j w T_int})/(j w T_int)]
Hw_b_int = 1 / (1j * w)                    # 連續積分 1/(j w)
Hw_b_hold = Hw_a_num / (1j * w * T_int)    # ZOH (同上)
H_b = (1/C) * Hw_b_int * Hw_b_hold

# 轉成絕對值(dB)
H_a_mag_dB = 20 * np.log10(np.abs(H_a))
H_b_mag_dB = 20 * np.log10(np.abs(H_b))
```

Q1_check

lab3-1_check.py > ...

```python
import numpy as np
import matplotlib.pyplot as plt

C = 15.925e-12
f_LO = 2.4e9
N = 8
T_int = N / f_LO

# 針對 200 MHz ~ 400 MHz 之間做高解析度掃描 (線性分佈)
f_lin = np.linspace(2.0e8, 4.0e8, 20001)  # 2萬個點
w_lin = 2 * np.pi * f_lin

Hw_a_num = (1 - np.exp(-1j * w_lin * T_int))
Hw_a_int = Hw_a_num / (1j * w_lin)
Hw_a_hold = Hw_a_num / (1j * w_lin * T_int)
H_a = (1/C) * Hw_a_int * Hw_a_hold

H_a_mag_dB = 20 * np.log10(np.abs(H_a))
```

Q2

lab3-2.py > ...

```python
import numpy as np
import matplotlib.pyplot as plt

CH = 15.425e-12
CR = 0.5e-12
CS = CH + CR   # 15.925 pF
a1 = CH / CS   # 約 0.9686

f_LO = 2.4e9      # LO freq
N = 8
T_int = N / f_LO  # 积分區間
f = np.logspace(1, 11, 1000)  # 10 Hz ~ 100 GHz
w = 2*np.pi*f

# Part 1: 連續積分(不放電) × a1
#         有限窗積分(會放電) × (1-a1)
Hw_part = a1 * (1/(1j*w)) + (1-a1)*( (1 - np.exp(-1j*w*T_int)) / (1j*w) )

# Part 2: 再乘上 ZOH
Hw_zoh = (1 - np.exp(-1j*w*T_int)) / (1j*w*T_int)

# 合併
H_new = (1/CS)*Hw_part * Hw_zoh

# Magnitude in dB
H_new_mag_dB = 20*np.log10(np.abs(H_new))
```

Q3-a

lab3-3-a.py > ...

```python
import numpy as np
import matplotlib.pyplot as plt

# --- 參數設定 ---
C_R  = 1e-12          # 單顆電容值 (假設四顆相同)
f_LO = 2.4e9          # LO 頻率
N    = 8
T_int = N / f_LO      # 積分時間

# --- 頻率軸 ---
f_min, f_max = 1e6, 1e10   # 1 MHz ~ 10 GHz
num_points   = 2000
f = np.logspace(np.log10(f_min), np.log10(f_max), num_points)
w = 2 * np.pi * f

# --- (a) Integrate & Dump 傳遞函式 ---
Hw_num   = (1 - np.exp(-1j * w * T_int))        # 分子 1 - e^{-j w T_int}
Hw_int   = Hw_num / (1j * w)                    # 有限窗積分
Hw_hold  = Hw_num / (1j * w * T_int)            # 零階保持(ZOH)
H_a      = (1 / C_R) * Hw_int * Hw_hold         # 乘上 1/C

# --- 轉成 dB ---
H_a_mag_dB = 20*np.log10(np.abs(H_a))
```

Q3-b

lab3-3-b.py > ...

```python
import numpy as np
import matplotlib.pyplot as plt

# --- 參數設定 ---
C_R  = 1e-12
f_LO = 2.4e9
N    = 8
T_int = N / f_LO

f_min, f_max = 1e6, 1e10
f = np.logspace(np.log10(f_min), np.log10(f_max), 2000)
w = 2 * np.pi * f

# (b) Continuous Integration
Hw_b_int  = 1 / (1j * w)                     # 1/(j w)
Hw_b_hold = (1 - np.exp(-1j * w * T_int)) / (1j * w * T_int)
H_b       = (1 / C_R) * Hw_b_int * Hw_b_hold

H_b_mag_dB = 20*np.log10(np.abs(H_b))
```

Q3-c

lab3-3-c.py > ...

```python
import numpy as np
import matplotlib.pyplot as plt

# --- 1. 參數 ---
C_R_list = [2e-12, 3e-12, 4e-12, 6e-12]  # 不同大小的放電電容
C_sum    = sum(C_R_list)

f_LO = 2.4e9
N    = 8
T_int = N / f_LO

# --- 2. 頻率軸 ---
f_min, f_max = 1e6, 1e10
num_points   = 2000
f = np.logspace(np.log10(f_min), np.log10(f_max), num_points)
w = 2 * np.pi * f

# --- 3. F(f) = [ (1 - e^{-jwT_int}) / (j w ) ] * [ (1 - e^{-jwT_int}) / ( j w T_int ) ]
Hw_num  = (1 - np.exp(-1j * w * T_int))
Ff      = (Hw_num / (1j * w)) * (Hw_num / (1j * w * T_int))

# --- 4. H_c(f) = [ 4 * F(f ) ] / sum_i CRi
# (同時間窗積分, 最後並聯讀出)
H_c = (len(C_R_list) * Ff) / C_sum

# --- 轉 dB ---
H_c_mag_dB = 20 * np.log10(np.abs(H_c))
```