(程式檔裡有註解解釋 code)

(1) It's finished in code 1.py

之間隨便找一點當根，誤差 $< 10^{-5}$

Result:

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Bisection
First solution between 0.9119110107421875, 0.9119186401367188
Second solution between -1.4318161010742188, -1.4318084716796875
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Secant
First solution near 0.9119167281094818
Second soltion near -1.4318067119384044
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Newton
First solution near 0.9119180886916561
Second solution near -1.4318086009771325
```

(2) (Some code is finished in 2.py)

Choose starting point $x_0 = 3$

Use Newton's method $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$p(x) = (x-2)^3(x-4)^2$

$p'(x) = 3(x-2)^2(x-4)^2 + 2(x-2)^3(x-4)$

$$\frac{p(x)}{p'(x)} = \frac{(x-2)(x-4)}{3(x-4)+2(x-2)}$$

$x_0 = 3$, $x_1 = 3 - 1 = 2$

$x_2 = 2 - 0 = 2 \Rightarrow$ converge to root 2

For simple root, Newton's method will converge quadratic

However, p has multiple root at $x=2$, so it's linearly convergent

If we modify the Newton's method to be $x_{n+1} = x_n - k\frac{f(x_n)}{f(x_n)}$

Where k is the number of root at that point, it will become quadratically convergent.

(3)

(a) $x-\left(\frac{(4+2x^3)}{x^2}-2x\right)=0$

$\Rightarrow x-\frac{4}{x^2}=0 \Rightarrow x^3-4=0$

(b) $x=\sqrt{\frac{4}{x}} \Rightarrow x^2-\frac{4}{x}=0$

$\Rightarrow x^3-4=0$

(c) $x=\frac{(16+x^3)}{5x^2} \Rightarrow x^3-4=0$

Found $f(x)=x^3-4$

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g1 diverge
g2 converge
Solution = 1.5873982537565454
g3 diverge
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(a)

$g'(x)=\frac{-8}{x^3}$

$|g'(R)|=\left|\frac{-8}{4}\right|=2>1$

Therefore, (a) will diverge no matter the starting point

(b) $g(x)=\frac{2}{\sqrt{x}}$

$g'(x)=-\frac{1}{\sqrt[3]{x^2}}$, $|g'(x)|<1$ for $1<x<2 \Rightarrow |g'(R)|<1$

Therefore, exist an interval that will make x_{n+1} converge

Chouse $x_0=2$, and $x_{n+1}=g(x_n)$

The result converge at $x=1.58739 \fallingdotseq 2^{\frac{2}{3}}$ (See the code 3.py)

If $x_0>0$ and iterate enough of times, (b) will always converge to $x=1.58739$

(c)

$g(x)=\frac{16}{5x^2}+\frac{x}{5}$

$g'(x)=\frac{-32}{5}\frac{1}{x^3}+\frac{1}{5}$

$|g'(R)|=\left|\frac{-32}{5}\times\frac{1}{4}+\frac{1}{5}\right|>1 \Rightarrow$ (C) will diverge no matter the starting value x_0

(4)

$$f(x,y,z) = \begin{bmatrix} x-3y-z^2+3 \\ 2x^3+y-5z^2+2 \\ 4x^2+y+z-7 \end{bmatrix}$$

$$J(x,y,z) = \begin{bmatrix} 1, & -3, & -2z \\ 6x^2, & 1, & -10z \\ 8x, & 1, & 1 \end{bmatrix}$$

Using Newton's method. $X_{n+1} = X_n - J^{-1}(X_n) \cdot f(X_n)$

Solve S_x, $J(X_n) \cdot S_x = -f(X_n)$ for each iteration
And Update X_{n+1}, $X_{n+1} = X_n + S_x$

See 4.py for the result

Found the six roots

$X_0 = (1,1,1) \longrightarrow X = [1.1114, 0.98821, 1.07088]$

$X_0 = (1.3, 0.9, -1.2) \longrightarrow X = [1.35375, 0.92543, -1.25597]$

$X_0 = (-1.1838, 0.7337, 0.0841) \rightarrow X = [32.88463, -4434.08662, 115.49088]$

$X_0 = (50, 0, 0) \longrightarrow X = [31.15140, -3768.15713, -106.48297]$

$X_0 = (-1+i, 0, 0) \longrightarrow X = [-1.25060+0.48994i, 0.66505-0.0134i, 0.08859+0.50359i]$

$X_0 = (-1-i, 0, 0) \longrightarrow X = [-1.25060-0.48994i, 0.665053+0.0134i, 0.08859-0.50359i]$