Numerical Methods, 2025 Spring

Assignment 1
Due on Mar 27

Note: In your submission, you should explain how you arrived at your solution. If you use MATLAB or any other software to compute your results, include your code and describe your solving process. This will help you practice presenting explanations in a logical, organized, and concise manner.

1. (30%) The function $f(x) = x^2 + \sin(x) - \frac{e^x}{4} - 1$ has zeros for two values near $x = 0$. Please compute both roots, starting with $[-2, 0]$ and $[0, 2]$, to attain an accuracy of 10^{-5} using the following methods: (a) the bisection method (b) the secant method, and (c) Newton's method.

2. (25%) Use Newton's method on the polynomial $P(x) = (x-2)^3(x-4)^2$ with $x_0 = 3$. Does it converge? To which root? Is convergence quadratic?

3. (30%) Below are three different g(x) functions. All are rearrangements of the same f(x). What is f(x)?
   (a) $(4 + 2x^3)/x^2 - 2x$
   (b) $\sqrt{4/x}$
   (c) $(16 + x^3)/(5x^2)$
   Which of them converge? What x-value is obtained? Are there starting values for which one or more diverge? Which diverge?

4. (30%) Solve the following system of nonlinear equations using Newton's method or fixed-point method.

$$x - 3y - z^2 = -3$$
$$2x^3 + y - 5z^2 = -2$$
$$4x^2 + y + z = 7$$

   Hints: There are six solutions to this system. Two of the real solutions are near (1, 1, 1) and (1.3, 0.9, -1.2). Your score for this question will be based on the number of solutions you find.