Assignment 3

Numerical Methods, 2025 Spring

Due on May 8

Note: You should explain how you obtain your solution in your submission. If you use MATLAB or any other software to compute your results, you should provide your code or describe your solving process. This is a good practice for you to explain things in a logical, organized, and concise way!

1. (20%) Use this ordinary difference table:

x	$f(x)$	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
0.12	0.79168	-0.01834	-0.01129	0.00134	0.00038
0.24	0.77334	-0.02963	-0.00995	0.00172	0.00028
0.36	0.74371	-0.03958	-0.00823	0.00200	
0.48	0.70413	-0.04781	-0.00623		
0.60	0.65632	-0.05404			
0.72	0.60228				

   (a) Estimate $f(0.231)$ from the Newton-Gregory polynomial of degree-2 with $x_0 = 0.12$.

   (b) Add one term to part (a) to get $f(0.231)$ from the third-degree polynomial.

   (c) Estimate the errors of both parts (a) and (b).

   (d) Is it better to start with $x_0 = 0.24$ or with $x_0 = 0.36$ when getting $f(0.42)$ from quadratic? Justify your answer.

2. (20%) Fit the function below with a natural cubic spline that matches to $f(x)$ at five evenly spaced points in [-1, 1] and forces the slopes at the ends to be zero. Plot the spline curve together with $f(x)$.

$$f(x) = \begin{cases} 0, & -1 < x < -0.5 \\ 1 - |2x|, & -0.5 < x < 0.5 \\ 0, & 0.5 < x < 1 \end{cases}$$

3. (15%)Compute the connected Bezier curve from this set of points:

	Point#									
	0	1	2	3	4	5	6	7	8	9
x	10	50	75	90	105	150	180	190	160	130
y	10	15	60	100	140	200	140	120	100	80

   (a) Draw a graph determined by the ten points.

   (b) Why is the graph smoothly connected at points 3 and 6?

   (c) Rewrite the Bezier equations so that the parameter u is defined on [0, 1] for points 0 to 3, on [1, 2] for points 3 to 6, and on [2, 3] for points 6 to 9.

4. (10%) Repeat Problem 3 for a B-spline curve.

5. (20%) The equation of a plane is $z = ax + by + c$. We can fit experimental data to such a plane using the least-squares technique. Here are some data for $z = f(x, y)$:

x	0.40	1.2	3.4	4.1	5.7	7.2	9.3
y	0.70	2.1	4.0	4.9	6.3	8.1	8.9
z	0.031	0.933	3.058	3.349	4.870	5.757	8.921

   (a) Develop the normal equations to fit the (x, y) data to a plane.

   (b) Use these equations to fit $z = ax + by + c$.

   (c) What is the sum of the squares of the deviations of the points from the plane?

6. (25%) Find Padé approximations for these functions, with numerators and denominators each of the third degree:

$$\cos^2(x), \sin(x^4 - x), xe^x.$$

7. (15%) (Bonus) Let $f(x) = xe^{-x}, x_0 = 1, x_1 = 2, x_2 = 3$

   (a) Construct the Hermite cubic interpolation polynomial for f at the specified interpolating points.

   (b) Approximate $f(1.5)$ using the polynomial from part (a).