**Minimization of Total Completion Time on a Single Processor Subject to Dynamic Arrivals**

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The considered problem is common to computer operating systems and operations management (manufacturing as well as service). Given $n$ jobs $N = \{1, 2, \ldots, n\}$ to process on a single processor. Each job $j \in N$ is characterized by positive processing length $p_j$ and non-negative arrival time $r_j$. Any job cannot start its processing before its arrival time. The processor processes at most one job at a time. The objective is to construct a feasible schedule that has the minimum sum of job completion times, or called total completion time. Let $C_j$ denote the completion time of job $j$ in a schedule. The objective function is $\sum_j C_j$, called the total completion time or the sum of completion times of the jobs. To determine an optimal schedule is unfortunately NP-hard.

Consider the following set of 6 jobs.

| job $j$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $r_j$ | 0 | 2 | 2 | 6 | 7 | 9 |
| $p_j$ | 6 | 2 | 5 | 2 | 8 | 2 |

- **Q1: Implement a full-enumeration tree to list all** $6!$ **schedules and their associated objective values.**
- **Q2: Construct the schedule obtained from the Shortest Remaining Processing Time First (SRPT) rule. Deploy a min-heap to facilitate your implementation of the SRPT rule.**
- **Q3: What is the difference between the optimal value of Q2 and the optimal value of Q1.**
- **Q4: By the way, what is the run time of the SRPT algorithm?**