Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Yu Shuai

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Formulation of the SR-NiTROM

- Consider a shift-equivariant FOM with traveling solutions:

$$u_t = f(u), \qquad u(x,t) = \hat{u}(x - c(t), t). \tag{1}$$

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- From this projection operator, we can encode the FOM state with a low-dim representation $a(t) \in \mathbb{R}^r$:

$$\begin{aligned} a &= \Psi^\top u \\ \hat{u}_r &= \Phi(\Psi^\top \Phi)^{-1} a. \end{aligned} \tag{2}$$

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- The ROM dynamics is given in a symmetry-reduced form:

$$\dot{a}_i = A_{ij} a_j + B_{ijk} a_j a_k + \dot{c} M_{ij} a_j \tag{3a}$$

$$\dot{c} = -\frac{p_i a_i + Q_{ij} a_i a_j}{s_i a_i} \tag{3b}$$

$$M = \Psi^\top \partial_x \Phi(\Psi^\top \Phi)^{-1}, \quad s = \langle \partial_x \Phi(\Psi^\top \Phi)^{-1}, \partial_x u_0 \rangle \tag{3c}$$

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The optimization problem of SR-NiTROM

- The trajectory-based objective function:

$$J = \sum_{m=0}^{N_t-1} \|\hat{u}_r(t_m) - \hat{u}(t_m)\|^2 + \beta(c_r(t_m) - c(t_m))^2. \qquad (4)$$

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$$L = \sum_{m=0}^{N_t-1} \Bigg(\|\hat{u}_r(t_m) - \hat{u}(t_m)\|^2 + \beta(c_r(t_m) - c(t_m))$$

$$+ \int_{t_0}^{t_m} \lambda_m^\top (\dot{a} - Aa - B(a, a) - \dot{c}Ma)\mathrm{d}t \tag{5}$$

$$+ \int_{t_0}^{t_m} \mu_m (\dot{c} + \frac{p_i a_i + Q_{ij} a_i a_j}{s_i a_i})\mathrm{d}t \tag{6}$$

$$+ \lambda_m(t_0)(a(t_0) - \Psi^\top \hat{u}(t_0)) \Bigg), \quad \lambda_m \in \mathbb{R}^r, \mu_m \in \mathbb{R}. \tag{7}$$

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- Multiple trajectories: SR-NiTROM vs SR-Galerkin

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$$u_t = -uu_x - u_{xx} - \nu u_{xxxx}, \quad x \in [0, 2\pi]. \tag{8}$$

Numerical details

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- Relative weight: 10.0, 4-dim ROM

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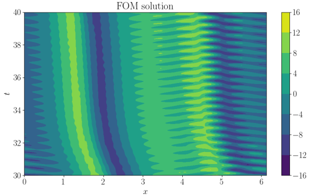
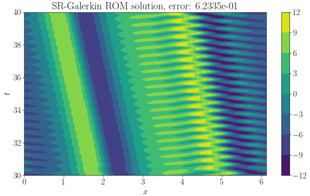
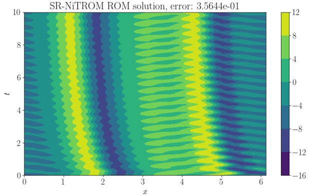
- FOM vs SR-Galerkin vs SR-NiTROM



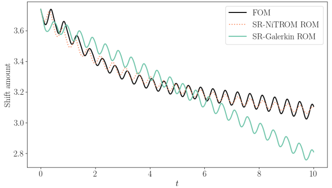
(a) FOM



(b) SR-Galerkin



(c) SR-NiTROM



(d) shift amount

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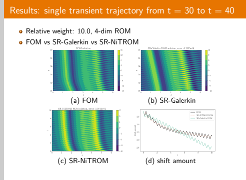

Results: multiple trajectories from perturbed initial conditions

- 9 trajectories, 7-dim ROM

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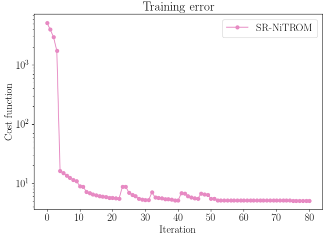
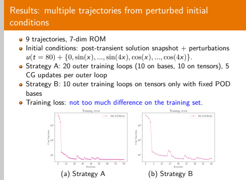
- Strategy B: 10 outer training loops on tensors only with fixed POD bases
- Training loss: not too much difference on the training set.



(a) Strategy A

(b) Strategy B



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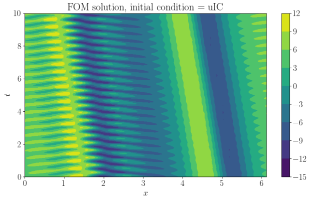

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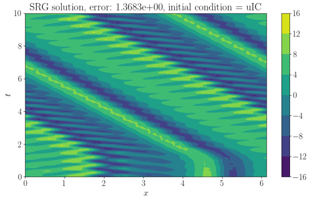
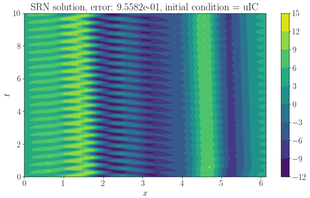
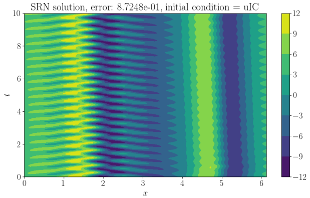


(b) SR-Galerkin



(c) SR-NiTROM A



(d) SR-NiTROM B

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└Results

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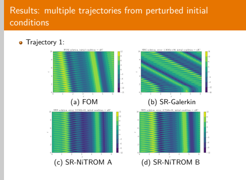
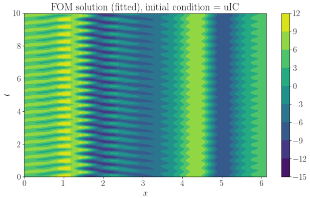
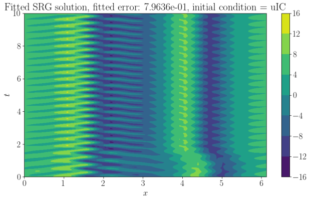


Results: multiple trajectories from perturbed initial conditions

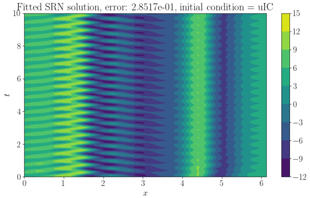
- Trajectory 1:



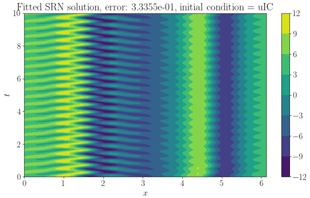
(a) FOM



(b) SR-Galerkin



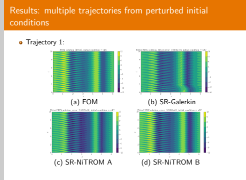
(c) SR-NiTROM A



(d) SR-NiTROM B

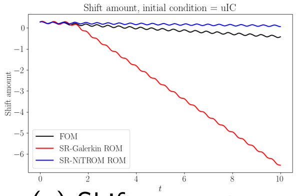
2025-10-01

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics
└Results
└Multiple trajectories: SR-NiTROM vs SR-Galerkin

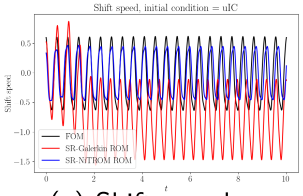


Results: multiple trajectories from perturbed initial conditions

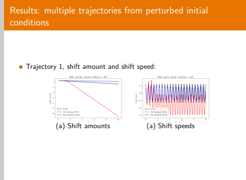
- Trajectory 1, shift amount and shift speed:



(a) Shift amounts



(a) Shift speeds



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