Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Yu Shuai

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Formulation of the SR-NiTROM

- Consider a shift-equivariant FOM with traveling solutions:

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$$\dot{a}_i = A_{ij}a_j + B_{ijk}a_j a_k + \dot{c}M_{ij}a_j \tag{3a}$$

$$\dot{c} = -\frac{p_i a_i + Q_{ij}a_i a_j}{s_i a_i} \tag{3b}$$

$$M = \Psi^\top \partial_x \Phi(\Psi^\top \Phi)^{-1}, \quad s = \langle \partial_x \Phi(\Psi^\top \Phi)^{-1}, \partial_x u_0 \rangle \tag{3c}$$

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The optimization problem of SR-NiTROM

- The trajectory-based objective function:

$$J = \sum_{m=0}^{N_t-1} \|\hat{u}_r(t_m) - \hat{u}(t_m)\|^2 + \beta(c_r(t_m) - c(t_m))^2. \tag{4}$$

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- The unconstrained Lagrangian with multipliers:

$$L = \sum_{m=0}^{N_t-1} \Bigg(\left\|\hat{u}_r(t_m) - \hat{u}(t_m)\right\|^2 + \beta(c_r(t_m) - c(t_m))$$

$$+ \int_{t_0}^{t_m} \lambda_m^\top(\dot{a} - Aa - B(a,a) - \dot{c}Ma)\mathrm{d}t \tag{5}$$

$$+ \int_{t_0}^{t_m} \mu_m(\dot{c} + \frac{p_i a_i + Q_{ij} a_i a_j}{s_i a_i})\mathrm{d}t \tag{6}$$

$$+ \lambda_m(t_0)(a(t_0) - \Psi^\top \hat{u}(t_0)) \Bigg), \quad \lambda_m \in \mathbb{R}^r, \mu_m \in \mathbb{R}. \tag{7}$$

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Single trajectory: SR-NiTROM vs SR-Galerkin

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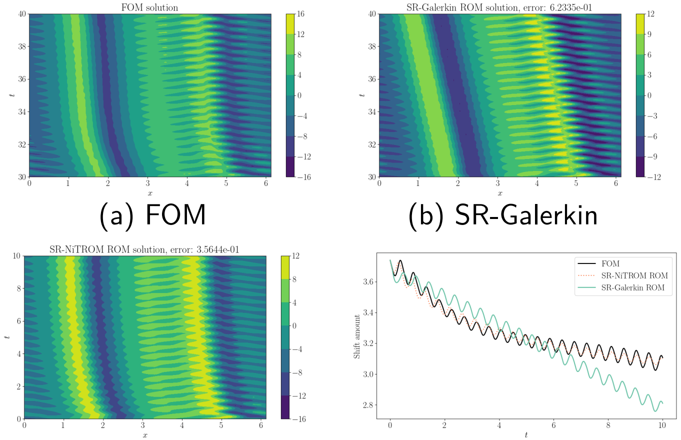
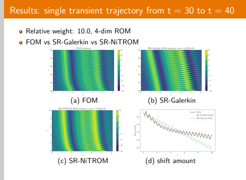
- FOM vs SR-Galerkin vs SR-NiTROM



(a) FOM (b) SR-Galerkin

(c) SR-NiTROM (d) shift amount



Results: multiple trajectories from perturbed initial conditions

- 9 trajectories, 7-dim ROM

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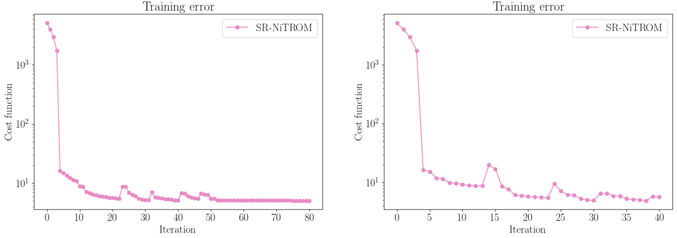

(a) Strategy A (b) Strategy B

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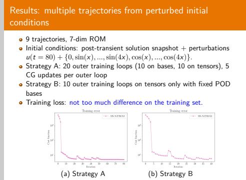
└Results

└Single trajectory: SR-NiTROM vs SR-Galerkin



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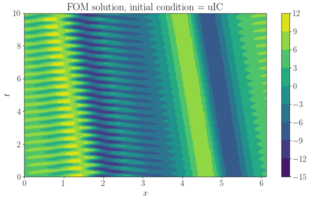
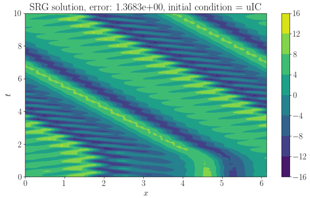
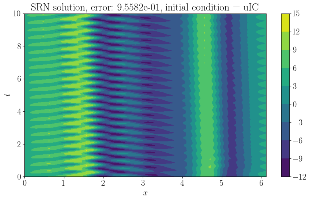


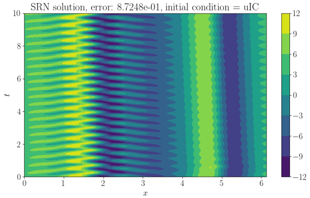
(a) FOM



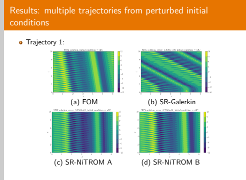
(b) SR-Galerkin



(c) SR-NiTROM A



(d) SR-NiTROM B



Results: multiple trajectories from perturbed initial conditions

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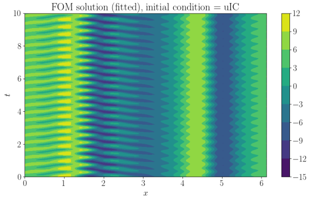

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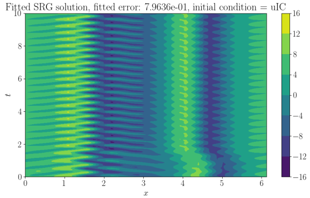

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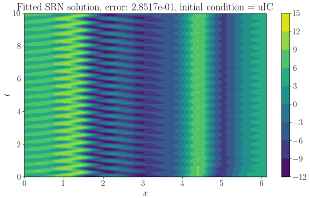

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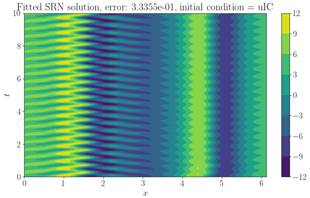

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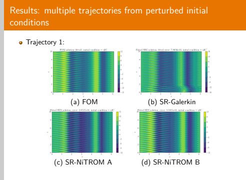
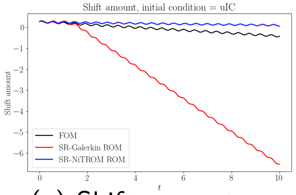
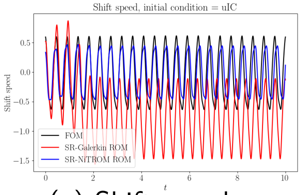


Results: multiple trajectories from perturbed initial conditions

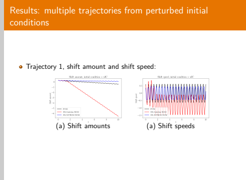
- Trajectory 1, shift amount and shift speed:



(a) Shift amounts



(a) Shift speeds



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