Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Yu Shuai

October 1, 2025

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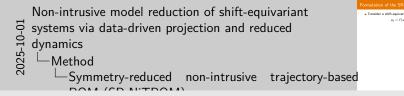
Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Method



• Consider a shift-equivariant FOM with traveling solutions:

$$u_t = f(u), \qquad u(x,t) = \hat{u}(x - c(t), t).$$
 (1)



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• Suppose we have a collection of training snapshots $\{u(t_m)\}_{m=0}^{N_t-1}$, $u(t) \in \mathbb{R}^N$.



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Method
Symmetry-reduced non-intrusive trajectory-based

DOMA (CD NITTONA)

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- We seek to find $\Phi, \Psi \in \mathbb{R}^{n \times r}$, such that $\Phi(\Psi^T \Phi)^{-1} \Psi^T$ is a projection.



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- We seek to find $\Phi, \Psi \in \mathbb{R}^{n \times r}$, such that $\Phi(\Psi^{\top}\Phi)^{-1}\Psi^{\top}$ is a projection.
- From this projection operator, we can encode the FOM state with a low-dim representation $a(t) \in \mathbb{R}^r$:

$$a = \Psi^{\top} u$$

$$\hat{u}_r = \Phi(\Psi^{\top} \Phi)^{-1} a. \tag{2}$$



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• The ROM dynamics is given in a symmetry-reduced form:

$$\dot{a}_i = A_{ij}a_j + B_{ijk}a_ja_k + \dot{c}M_{ij}a_j \tag{3a}$$

$$\dot{c} = -\frac{p_i a_i + Q_{ij} a_i a_j}{s_i a_i} \tag{3b}$$

$$M = \Psi^{\top} \partial_{\mathsf{x}} \Phi(\Psi^{\top} \Phi)^{-1}, \quad \mathsf{s} = \langle \partial_{\mathsf{x}} \Phi(\Psi^{\top} \Phi)^{-1}, \partial_{\mathsf{x}} \mathsf{u}_0 \rangle$$
 (3c)

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DOM (CD NITDOM)

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 $M = \Psi^{\top} \partial_x \Phi (\Psi^{\top} \Phi)^{-1}, \quad s = \langle \partial_x \Phi (\Psi^{\top} \Phi)^{-1}, \partial_x u_0 \rangle$

• The trajectory-based objective function:

$$J = \sum_{r=1}^{N_t-1} \|\hat{u}_r(t_m) - \hat{u}(t_m)\|^2 + \beta(c_r(t_m) - c(t_m))^2.$$
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• $\beta = \gamma \sum_m \|\hat{u}(t_m)\|^2 / \sum_m (c(t_0) - c(t_m))^2$: relative weights.

systems via data-driven projection and reduced

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Non-intrusive model reduction of shift-equivariant

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- γ : hyperparameter.



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• The trajectory-based objective function:

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- $\beta = \gamma \sum_m \|\hat{u}(t_m)\|^2 / \sum_m (c(t_0) c(t_m))^2$: relative weights.
- γ : hyperparameter.
- The unconstrained Lagrangian with multipliers:

$$L = \sum_{m=0}^{N_t - 1} \left(\|\hat{u}_r(t_m) - \hat{u}(t_m)\|^2 + \beta(c_r(t_m) - c(t_m)) + \int_{t_0}^{t_m} \lambda_m^\top (\dot{a} - Aa - B(a, a) - \dot{c}Ma) dt \right)$$
(5)

$$+\int_{t_0}^{t_m} \mu_m(\dot{c} + \frac{p_i a_i + Q_{ij} a_i a_j}{s_i a_i}) \mathrm{d}t \tag{6}$$

$$+\lambda_{m}(t_{0})(\mathsf{a}(t_{0})-\Psi^{\mathsf{T}}\hat{u}(t_{0}))\bigg), \quad \lambda_{m}\in\mathbb{R}^{r}, \mu_{m}\in\mathbb{R}.$$

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Symmetry-reduced non-intrusive trajectory-based

DOM (CD NITDOM)

The optimization problem of SR-NITROM

• The trajectory-based adjustment function: $J = \sum_{n \in \mathbb{N}} (a(c_n) - v(c_n))^2 + \beta(c_n(c_n) - c_n(c_n))^2. \quad (4)$ $J = \sum_{n \in \mathbb{N}} (a(c_n) - v(c_n))^2 + \beta(c_n(c_n) - c_n(c_n))^2.$ • The surrounding register with multipless: $1 + \sum_{n \in \mathbb{N}} \left(\frac{1}{n} (a(c_n) - c_n(c_n)) + \beta(c_n(c_n) - c_n(c_n)) + \int_0^\infty \lambda_n^2 (a - a - a - b_n(a))^2 + \beta(c_n(c_n) - c_n(c_n)) + \int_0^\infty \lambda_n^2 (a - a - a - b_n(a)) + \beta(c_n(c_n) - c_n(c_n)) + \int_0^\infty \lambda_n^2 (a - a - a - b_n(a)) + \beta(c_n(c_n) - c_n(c_n)) + \int_0^\infty \lambda_n^2 (a - a - a - b_n(a)) + \beta(c_n(c_n) - c_n(c_n)) + \beta(c_n(c_$

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Results



• FOM: Kuramoto-Sivashinsky equation

$$u_t = -uu_x - u_{xx} - \nu u_{xxx}, \quad x \in [0, 2\pi].$$
 (8)

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• $\nu = 4/87$ for traveling-wave patterns.



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Numerical details s \text{ FOM. Karantos-Sixohindy equation} u_s = -u_s - u_s - u_{source}, \quad s \in [0, 2\pi]. \tag{B} s = 4/67 \text{ for two diag wave patients.}
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Numerical details a_1 \text{FOM. Nuraneto-Sivashindy equation} a_2 = -aa_2 - aa_{21} - aa_{22} \text{,} \qquad x \in [0, 2\pi]. \tag{(} a_1 = 0.037 \text{ for traveling-same patients.} \bullet \text{Periods BCs. } N = 00 \text{ Finative modes, } \Delta t = 10^{-3}.
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Single trajectory: SR-NiTROM vs SR-Galerkin



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 - 20 outer loops, 5 CG updates per outer loops.
 - Initial conditions: POD bases (capturing >99.5% energy) + Galerkin-projected tensors. (imitating the training result of the re-projected SR-OpInf ROM)



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Single trajectory: SR-NiTROM vs SR-Galerkin

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Results: single transient trajectory from t = 30 to t = 40

• Relative weight: 10.0, 4-dim ROM

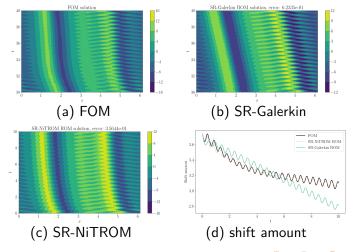
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Single trajectory: SR-NiTROM vs SR-Galerkin

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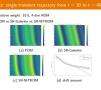
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- FOM vs SR-Galerkin vs SR-NiTROM



Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Results
Single trajectory: SR-NiTROM vs SR-Galerkin



• 9 trajectories, 7-dim ROM

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics Results —Single trajectory: SR-NiTROM vs SR-Galerkin

- 9 trajectories, 7-dim ROM
- Initial conditions: post-transient solution snapshot + perturbations $u(t = 80) + \{0, \sin(x), ..., \sin(4x), \cos(x), ..., \cos(4x)\}.$

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- Strategy B: 10 outer training loops on tensors only with fixed POD bases



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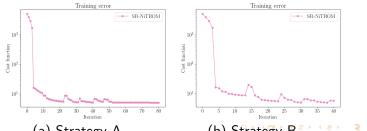
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- Training loss: not too much difference on the training set.



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Single trajectory: SR-NiTROM vs SR-Galerkin

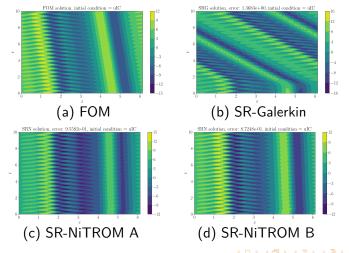
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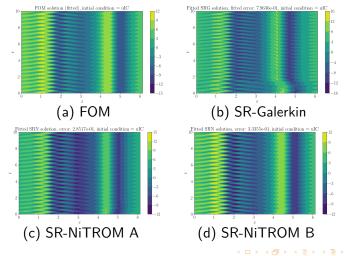


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Single trajectory: SR-NiTROM vs SR-Galerkin

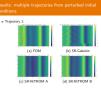


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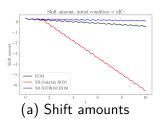


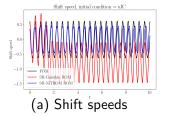
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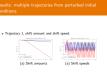
Results
Single trajectory: SR-NiTROM vs SR-Galerkin



• Trajectory 1, shift amount and shift speed:







 For the reconstruction of a single training trajectory including transient, SR-NiTROM outperforms SR-Galerkin ROM (and SR-OpInf of course). Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics
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Conclusions:

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Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics 2025-Results Single trajectory: SR-NiTROM vs SR-Galerkin

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 - However, SR-NiTROM with only trained tensors and POD bases attains the least reconstruction error of the raw snapshots.

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 - Why: our loss function = error in aligned snapshots + error in shift amounts, not error in raw snapshots.

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Non-intrusive model reduction of shift-equivar

- Why: our loss function = error in aligned snapshots + error in shift amounts, not error in raw snapshots.
- Trade-off: we may want to switch to error in raw snapshots, but then the optimizer doesn't know if the error comes from mismatch of aligned profiles or shift amounts.

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- For the reconstruction of multiple transient trajectories, we find that:
 - SR-NiTROM gives better approximation of template-aligned snapshots and shift amounts than the SR-Galerkin ROM.
 - It's better to optimize both the bases and the tensors to minimize our loss function.
 - However, SR-NiTROM with only trained tensors and POD bases attains the least reconstruction error of the raw snapshots.
 - Why: our loss function = error in aligned snapshots + error in shift amounts, not error in raw snapshots.
 - Trade-off: we may want to switch to error in raw snapshots, but then the optimizer doesn't know if the error comes from mismatch of aligned profiles or shift amounts.
 - This new loss function is reasonable since small shift mismatch can lead to large error in raw snapshots.

2025-

dynamics

Results

Non-intrusive model reduction of shift-equivariant

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