# Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Yu Shuai

October 29, 2025

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© Before we start: Photos from the West

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# On the driveway





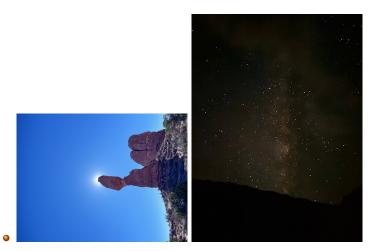
Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Before we start: Photos from the West

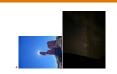
Photos



#### The Arches National Park



Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics Before we start: Photos from the West —Photos



### The Grand Canyon



Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Before we start: Photos from the West
Photos



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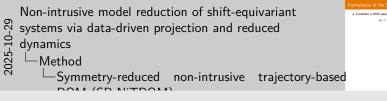
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  - Symmetry-reduced non-intrusive trajectory-based ROM (SR-NiTROM)
- Results

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics  $\begin{tabular}{ll} \label{eq:continuous} $\mathsf{Non-intrusive}$ model reduction of shift-equivariant systems via data-driven projection and reduced <math display="block"> \begin{tabular}{ll} \label{eq:continuous} \begin{tabular}{ll} \labe$ 



• Consider a shift-equivariant FOM with traveling solutions:

$$u_t = f(u), \qquad u(x,t) = \hat{u}(x - c(t), t).$$
 (1)



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Non-intrusive model reduction of shift-equivariant
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dynamics

Method
Symmetry-reduced non-intrusive trajectory-based

DOMA (CD NITTONA)

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- We seek to find  $\Phi, \Psi \in \mathbb{R}^{n \times r}$ , such that  $\Phi(\Psi^{\top}\Phi)^{-1}\Psi^{\top}$  is a projection.



Non-intrusive model reduction of shift-equivariant
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Method
Symmetry-reduced non-intrusive trajectory-based

DOM (CD NITDOM)

nulation of the SR-NiTROM

ift-equivariant FOM with traveling solutions:  $u_t = f(u), \quad u(x,t) = \dot{u}(x-c(t),t).$ 

 Suppose we have a collection of training snapshots {u(t<sub>rel</sub>)}<sub>n=1</sub><sup>N<sub>t</sub>−1</sup> u(t) ∈ ℝ<sup>N</sup>.
 We seek to find Φ, Ψ ∈ ℝ<sup>n×r</sup>, such that Φ(Ψ<sup>T</sup>Φ)<sup>-1</sup>Ψ<sup>T</sup> is a proj

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- From this projection operator, we can encode the FOM state with a low-dim representation  $a(t) \in \mathbb{R}^r$ :

$$a = \Psi^{\top} u$$

$$\hat{u}_r = \Phi(\Psi^{\top} \Phi)^{-1} a. \tag{2}$$



Non-intrusive model reduction of shift-equivariant
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dynamics

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Symmetry-reduced non-intrusive trajectory-based

DOM (CD NITDOM)

ulation of the SR-NiTROM

a shift-equivariant FOM with traveling solutions:

 $u_t = f(u),$   $u(x,t) = \bar{u}(x - c(t), t).$ we have a collection of training snapshots  $\{u(t_m)\}_{m=1}^{N_0}$ 

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 $(t) \in \mathbb{R}'$ :  $\mathbf{a} = \mathbf{\Phi}^{\top} \mathbf{u}$  $\hat{\mathbf{u}}_t = \mathbf{\Phi}(\mathbf{\Phi}^{\top} \mathbf{\Phi})^{-1} \mathbf{a}$ .

• Consider a shift-equivariant FOM with traveling solutions:

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$$\mathbf{a} = \Psi^{\top} \mathbf{u}$$

$$\hat{\mathbf{u}}_r = \Phi(\Psi^{\top} \Phi)^{-1} \mathbf{a}.$$
(2)

• The ROM dynamics is given in a symmetry-reduced form:

$$\dot{a}_i = A_{ij}a_j + B_{ijk}a_ja_k + \dot{c}M_{ij}a_j \tag{3a}$$

$$\dot{c} = -\frac{p_i a_i + Q_{ij} a_i a_j}{s_i a_i} \tag{3b}$$

$$M = \Psi^{\top} \partial_{\mathsf{x}} \Phi(\Psi^{\top} \Phi)^{-1}, \quad \mathsf{s} = \langle \partial_{\mathsf{x}} \Phi(\Psi^{\top} \Phi)^{-1}, \partial_{\mathsf{x}} u_0 \rangle$$
 (3c)

Non-intrusive model reduction of shift-equivariant
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Method
Symmetry-reduced non-intrusive trajectory-based

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• The trajectory-based objective function:

$$J = \sum_{r=0}^{N_t-1} \|\hat{u}_r(t_m) - \hat{u}(t_m)\|^2 + \beta(c_r(t_m) - c(t_m))^2.$$
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Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Method

Symmetry-reduced non-intrusive trajectory-based

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•  $\beta = \gamma \sum_m \|\hat{u}(t_m)\|^2 / \sum_m (c(t_0) - c(t_m))^2$ : relative weights.

Non-intrusive model reduction of shift-equivariant
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Symmetry-reduced non-intrusive trajectory-based

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optimization problem of SR-NiTROM he trajectory-based objective function:  $J = \sum_{m=0}^{N_0-1} \| \tilde{u}_r(t_m) - \tilde{u}(t_m) \|^2 + \beta(c_r(t_m) - c(t_m))^2.$ 

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- $\beta = \gamma \sum_m \|\hat{u}(t_m)\|^2 / \sum_m (c(t_0) c(t_m))^2$ : relative weights.
- $\gamma$ : hyperparameter.



Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Method
Symmetry-reduced non-intrusive trajectory-based

DOMA (CD NITTONA)

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- $\beta = \gamma \sum_m \|\hat{u}(t_m)\|^2 / \sum_m (c(t_0) c(t_m))^2$ : relative weights.
- $\gamma$ : hyperparameter.
- The unconstrained Lagrangian with multipliers:

$$L = \sum_{m=0}^{N_t - 1} \left( \|\hat{u}_r(t_m) - \hat{u}(t_m)\|^2 + \beta(c_r(t_m) - c(t_m)) + \int_{t_0}^{t_m} \lambda_m^\top (\dot{a} - Aa - B(a, a) - \dot{c}Ma) dt \right)$$
(5)

$$+\int_{t_0}^{t_m} \mu_m(\dot{c} + \frac{p_i a_i + Q_{ij} a_i a_j}{s_i a_i}) \mathrm{d}t \tag{6}$$

$$+\lambda_{m}(t_{0})(a(t_{0})-\Psi^{\top}\hat{u}(t_{0}))$$
,  $\lambda_{m}\in\mathbb{R}^{r},\mu_{m}\in\mathbb{R}$ . (7)

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics -Method -Symmetry-reduced non-intrusive trajectory-based  $+\lambda_m(t_0)(a(t_0) - \Psi^\top \hat{u}(t_0))$ ,  $\lambda_m \in \mathbb{R}', \mu_m \in \mathbb{R}$ . (7

DOM (CD NITDOM)

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  - Reconstruction of multiple training KSE trajectories with dim-8 ROMs
  - Reconstruction of multiple testing KSE trajectories with dim-8 ROMs
  - Ongoing: Reconstruction of channel-flow solutions to the 3D linearized Navier-Stokes equations

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics  $\begin{array}{c} \text{Results} \end{array}$ 

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we start: Photos from the West

(3) Method

Results

Reconstruction of a single training KSE trajectory: SR-NiTROM of SR-Galerkin

Reconstruction of multiple training KSE trajectories with dim-8

ROMs

Reconstruction of multiple testing KSE trajectories with dim-8

Ongoing: Reconstruction of channel-flow solutions to the 3D

• FOM: Kuramoto-Sivashinsky equation

$$u_t = -uu_x - u_{xx} - \nu u_{xxxx}, \quad x \in [0, 2\pi].$$
 (8)

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Results

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•  $\nu = 4/87$  for traveling-wave patterns.

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

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- Periodic BCs, N = 40 Fourier modes,  $\Delta t = 10^{-3}$ .



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- Optimization of the SR-NiTROM:

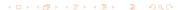


Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics Results -Reconstruction of a single training KSE trajectory:

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- Optimization of the SR-NiTROM:
  - Alternating training: 5 iterations for the tensors fixing the bases, then 5 iterations for the bases fixing the tensors. Switch 20 times.



Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Results

Reconstruction of a single training KSE trajectory:

 $\omega_{\theta} = -\omega_{\theta}$ ,  $\omega_{\theta} = -p\omega_{\text{cons}}$ ,  $\kappa \in [0, 2\pi]$ . (8) s = J/S for traceling even patterns. Periodic BCs, M = 0 Fourier modes,  $\Delta t = 10^{-3}$ . Semization of the SS-NTTROM. Alternative trainers: Furnishing for the scarse, there is bases, then S

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Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics Results

Periodic BCs, N = 40 Fourier modes,  $\Delta t = 10^{-1}$ 

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Reconstruction of a single training KSE trajectory: CD NUTDONA

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  - Initial conditions: POD bases (capturing >99.5% energy) + Galerkin-projected tensors. (imitating the training result of the re-projected SR-OpInf ROM)



Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics Results re-projected SR-OpInf ROM) Reconstruction of a single training KSE trajectory:

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  - If the training is not stable, then train on short trajectories first and then extend in time gradually ("curriculum learning").

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics 2025-Results Reconstruction of a single training KSE trajectory:

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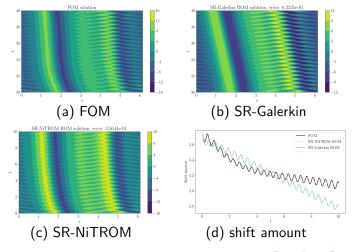
## Results: single transient trajectory from t = 30 to t = 40

• Relative weight: 10.0, 4-dim ROM

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Results
Reconstruction of a single training KSE trajectory:

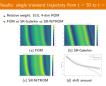
## Results: single transient trajectory from t = 30 to t = 40

- Relative weight: 10.0, 4-dim ROM
- FOM vs SR-Galerkin vs SR-NiTROM

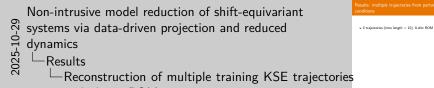


Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Results
Reconstruction of a single training KSE trajectory:



• 9 trajectories (time length = 10), 8-dim ROM



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- Initial conditions: post-transient solution snapshot + perturbations  $u(t = 80) + \{0, \sin(x), ..., \sin(4x), \cos(x), ..., \cos(4x)\}.$



Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Results

Reconstruction of multiple training KSE trajectories

s: multiple trajectories from perturbed initial ons

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- Initial conditions: post-transient solution snapshot + perturbations  $u(t = 80) + \{0, \sin(x), ..., \sin(4x), \cos(x), ..., \cos(4x)\}.$
- Two types of errors: The raw  $L_2$  error and the fitted  $L_2$  error.

$$\epsilon = \frac{\sum_{m=0}^{N_t - 1} \|u(t) - u_r(t)\|_2^2}{\sum_{m=0}^{N_t - 1} \|u(t)\|_2^2}$$
(9)

$$\epsilon_f = \frac{\sum_{m=0}^{N_t-1} \|\hat{u}(t) - \hat{u}_r(t)\|_2^2}{\sum_{m=0}^{N_t-1} \|\hat{u}(t)\|_2^2}$$
(10)

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics Results Reconstruction of multiple training KSE trajectories

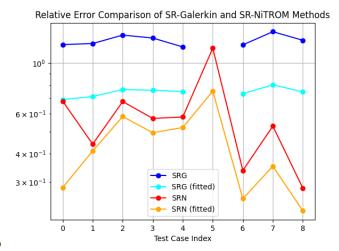


Figure: Comparison of relative prediction errors of training set.

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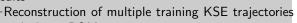
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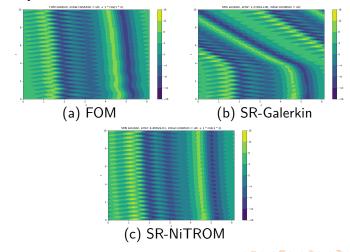
Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Results





• Trajectory 2:



Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

(a) FOM (b) SR-Galerkin

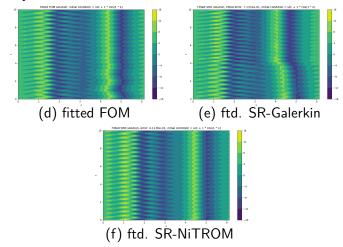
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• Trajectory 2:

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Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

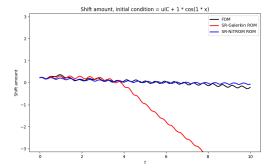
(d) fitted FOM (e) field SR-Gallerki

Results

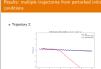
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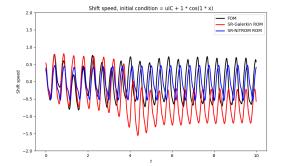
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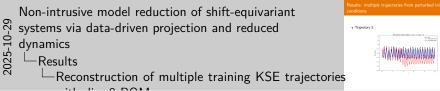


Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced Trajectory 2 dynamics Results -Reconstruction of multiple training KSE trajectories

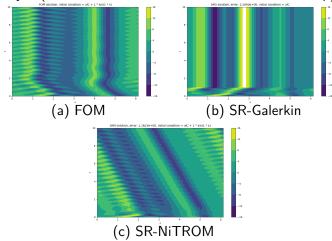


#### • Trajectory 2:





• Trajectory 6 (where the SR-Galerkin ROM solution blows up):



Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

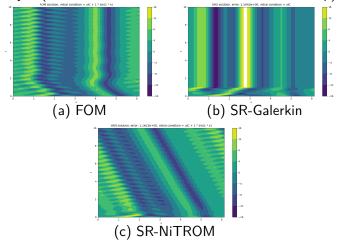


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• Trajectory 6 (where the SR-Galerkin ROM solution blows up):

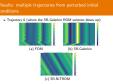


• When the solution blows up, we manually set the shifting speed ≡ 0.00

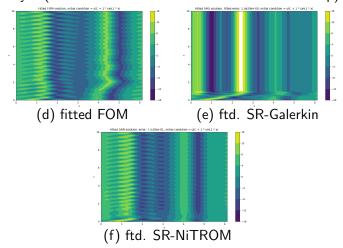
Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Results

-Reconstruction of multiple training KSE trajectories



• Trajectory 6 (where the SR-Galerkin ROM solution blows up):



Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Results

multiple trajectories from perturbed initial is tray 6 (where the SR-Caleria ROM solution bloos up)

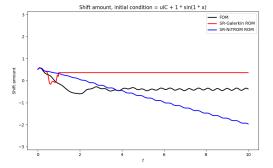
(d) fissel FOM

(e) fis. SR-Caleria

(f) fis. SR-Caleria

Results
Reconstruction of multiple training KSE trajectories

• Trajectory 6 (where the SR-Galerkin ROM solution blows up):





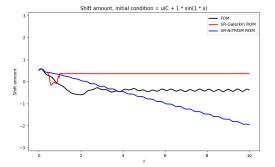
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Reconstruction of multiple training KSE trajectories



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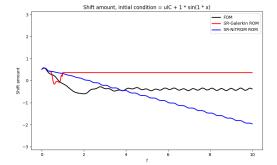


• For this specific trajectory, we find it difficult to train the SR-NiTROM model regardless of fine-tuning the hyperparameters.

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics Results Reconstruction of multiple training KSE trajectories



• Trajectory 6 (where the SR-Galerkin ROM solution blows up):



- For this specific trajectory, we find it difficult to train the SR-NiTROM model regardless of fine-tuning the hyperparameters.
- Our next step will be trying a different loss function raw  $L_2$  error instead of (fitted  $L_2$  error + shifting amount error).

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

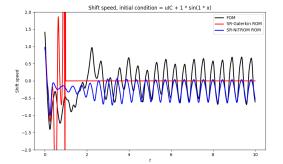
Results
Reconstruction of multiple training KSE trajectories.

Reads: multiple trajectories from perturbed initial conditions

• Trajectory 6 (where the SR-Calaria ROM solution Book up)

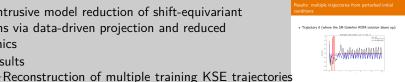
For this specific trajectory, we find it difficult to train the SR-NiTROM model regardless of fine-turing the hyperparam Our next step will be trying a different loss function — raw I instead of (fitted  $L_2$  error + shifting amount error).

• Trajectory 6 (where the SR-Galerkin ROM solution blows up):



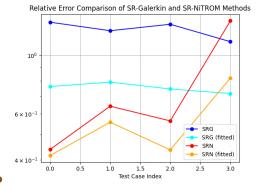


Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics Results



## Results: mean reconstruction errors of testing trajectories with 8-dim ROMs

• 4 testing trajectories. Initial conditions are:  $u(t = 80) + \{0.6\cos(x) + 0.8\sin(3x), \cos(5x), 0.7\cos(2x) + 0.7\sin(5x), 2\sin(x)\cos(4x)\}.$ 



• Overall, the SR-NiTROM ROM retains its outperformance over the SR-Galerkin ROM except on the 4th testing trajectory.

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

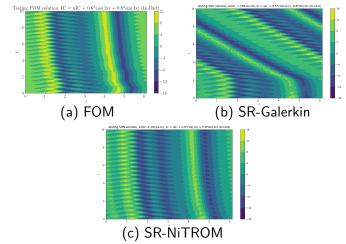
Results
Reconstruction of multiple testing KSE trajectories

• Count for the first mean reconstruction over the first graph (and first graph).

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• Count for the first mean reconstruction over the first graph (and first graph).

• Testing trajectory 1:



Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

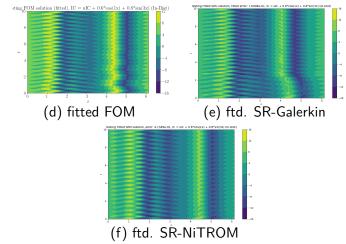
Results

Operatory 1:

(a) FOM
(b) SR-Galerko
(c) SR-MTROM

Results
Reconstruction of multiple testing KSE trajectories

• Testing trajectory 1:



Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Telling trajectory 1:

(d) fitted FOM

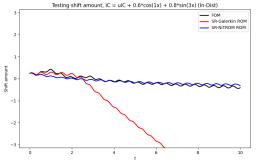
(ii) field SR-Callerian

(i) field SR-Callerian

Results

Reconstruction of multiple testing KSE trajectories

• Testing trajectory 1:

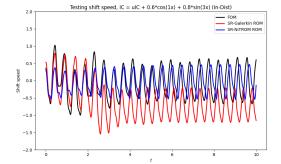




Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced · Testing trajectory 1 dynamics Results -Reconstruction of multiple testing KSE trajectories



• Testing trajectory 1:

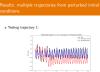




Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Results
Reconstruction of multiple testing KSE trajectories

... .. .. .. ...



#### Numerical details

• Consider the 3D Navier-Stokes equations (3DLNS) linearized about the base flow  $\boldsymbol{U} = (U(y), 0, 0)$  for fluctuations (u, v, w, p) of flow field in a channel domain (periodic in streamwise/spanwise direction, no-slip in normal direction):

$$u_t + Uu_x + vU_y = -p_x + \frac{1}{Re}\nabla^2 u \tag{11a}$$

$$v_t + Uv_x = -p_y + \frac{1}{Re} \nabla^2 v \tag{11b}$$

$$w_t + Uw_x = -p_z + \frac{1}{Re} \nabla^2 w \tag{11c}$$

$$u(y = \pm 1) = v(y = \pm 1) = w(y = \pm 1) = 0$$
 (11d)

- Why we want to study this full-order model?
  - The solutions to 3DLNS has large transient growth before the final decay (e.g., the Orr mechanism, the lift-up mechanism, streamwise elongated structures).
  - In 3D Navier-Stokes system, the transient will trigger nonlinear effects before decay, resulting in transition to turbulence.

Non-intrusive model reduction of shift-equivariant Consider the 3D Navier-Stokes equations (3DLNS) linearized about the base flow  $\mathbf{U} = (U(y), 0, 0)$  for fluctuations (u, v, w, p) of flow field in a channel domain (periodic in streamwise/spanwise direction systems via data-driven projection and reduced dynamics -Results The solutions to 3DLNS has large transient growth before the fina Ongoing: Reconstruction of channel-flow solutions

#### Numerical details

• Change of variables: denote the normal vorticity  $\eta = u_z - w_x$  and eliminate p (substitute  $\nabla^2 p$  into (11b)), we can rewrite the 3DLNS using  $q = (v, \eta)$  as state variables.

$$q_{t} = Lq$$

$$L = \begin{bmatrix} \nabla^{2} & I \end{bmatrix}^{-1} \begin{bmatrix} -U\partial_{x}\nabla^{2} + U_{yy}\partial_{x} + \nabla^{4}/\text{Re} & \\ -U_{y}\partial_{z} & -U\partial_{x} + \nabla^{2}/\text{Re} \end{bmatrix}$$
(12a)
$$(12b)$$

$$v(y = \pm 1) = v_y(y = \pm 1) = \eta(y = \pm 1) = 0.$$
 (12c)

- Spatial discretization:
  - Fourier-based differentiation in x and z direction.
  - Chebyshev collocation method in y direction.
  - Enforcing the clamped boundary conditions.
- Time stepping: fourth-order Runge-Kutta exponential time differencing scheme (ETDRK4, Cox et al. 2002)
  - Equal to the exponential time stepping  $q(t + \Delta t) = e^{\Delta t L} q(t)$  without external forcing.

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics 2025-

Results Ongoing: Reconstruction of channel-flow solutions

Change of variables: denote the normal vorticity  $n = \mu_r - w_r$  and

#### Numerical details

• Re = 3000. Initial conditions (Henningson et al. 1993):

$$\psi = (1 - y^2)^2 (x/2) z e^{-(x/2)^2 - (z/2)^2}$$
(13)

$$(u, v, w) = (0, \psi_z, -\psi_v)$$
 (14)

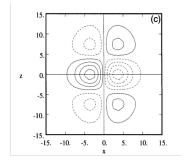
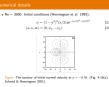


Figure: The contour of initial normal velocity at y=-0.56. (Fig. 4.16(c), Schmid & Henningson 2001).

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Results
Ongoing: Reconstruction of channel-flow solutions



#### Results: simulation of 3DLNS compared with previous studies

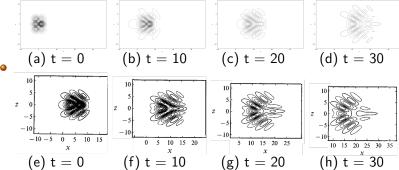


Figure: The contour of normal velocity at y = -0.56 and t = 0, 10, 20, 30compared to Fig. 2 in Henningson et al. (1993).

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics Results -Ongoing: Reconstruction of channel-flow solutions