Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Yu Shuai

October 1, 2025

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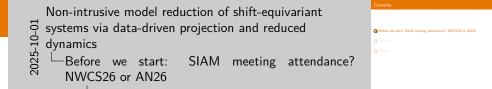
- Before we start: SIAM meeting attendance? NWCS26 or AN26
- 2 Method
- Results

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Before we start: SIAM meeting attendance? NWCS26 or AN26 Results

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  - Symmetry-reduced non-intrusive trajectory-based ROM (SR-NiTROM)
- Results

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Training as our SMM among absoluted SMMCSM as SMM

Medical

Medical

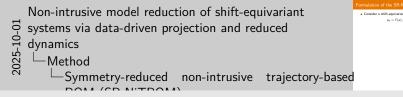
Symmetry-reduced non-introdes trajectory-based ROM (SR-AT TOM)

Comments

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$$u_t = f(u), \qquad u(x,t) = \hat{u}(x - c(t), t).$$
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- We seek to find  $\Phi, \Psi \in \mathbb{R}^{n \times r}$ , such that  $\Phi(\Psi^T \Phi)^{-1} \Psi^T$  is a projection.



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- From this projection operator, we can encode the FOM state with a low-dim representation  $a(t) \in \mathbb{R}^r$ :

$$a = \Psi^{\top} u$$

$$\hat{u}_r = \Phi(\Psi^{\top} \Phi)^{-1} a. \tag{2}$$



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(2)

• The ROM dynamics is given in a symmetry-reduced form:

$$\dot{a}_i = A_{ij}a_j + B_{ijk}a_ja_k + \dot{c}M_{ij}a_j \tag{3a}$$

$$\dot{c} = -\frac{p_i a_i + Q_{ij} a_i a_j}{s_i a_i} \tag{3b}$$

$$M = \Psi^{\top} \partial_{\mathbf{x}} \Phi(\Psi^{\top} \Phi)^{-1}, \quad \mathbf{s} = \langle \partial_{\mathbf{x}} \Phi(\Psi^{\top} \Phi)^{-1}, \partial_{\mathbf{x}} u_0 \rangle$$
 (3c)

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DOM (CD NITDOM)

Formulation of the  $SR \times RTMOM$ .

Consider a shift-position FOM with resulting solutions  $u_1 = \ell(s)$ , u(s, t) = u(s - e(t), t). (1)

Suppose we have a continction of braining supposition  $(u(s))_{t=1}^{\infty} u(s) \in \mathbb{R}^n$ ,  $u(s) \in \mathbb{R}^n$ . So the first  $(u(s))_{t=1}^{\infty} u(s) \in \mathbb{R}^n$ . So the first  $(u(s))_{t=1}^{\infty} u(s) \in \mathbb{R}^n$ . So the first  $(u(s))_{t=1}^{\infty} u(s) \in \mathbb{R}^n$ . (2)

This ROM dynamics is given in a symmetry-reduced form:  $u_1 = u(s)_{t=1}^{\infty} u(s)_{$ 

 $M = \Psi^{\top} \partial_x \Phi (\Psi^{\top} \Phi)^{-1}, \quad s = \langle \partial_x \Phi (\Psi^{\top} \Phi)^{-1}, \partial_x u_0 \rangle$ 

• The trajectory-based objective function:

$$J = \sum_{r=0}^{N_t-1} \|\hat{u}_r(t_m) - \hat{u}(t_m)\|^2 + \beta(c_r(t_m) - c(t_m))^2.$$
 (4)

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 $J = \sum_{m_r-1} ||\hat{u}_r(t_m) - \hat{u}(t_m)||^2 + \beta(c_r(t_m) - c(t_m))^2. \quad (4)$ 

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•  $\beta = \gamma \sum_m \|\hat{u}(t_m)\|^2 / \sum_m (c(t_0) - c(t_m))^2$ : relative weights.

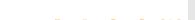
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- $\gamma$ : hyperparameter.



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 (4)

- $\beta = \gamma \sum_m \|\hat{u}(t_m)\|^2 / \sum_m (c(t_0) c(t_m))^2$ : relative weights.
- $\gamma$ : hyperparameter.
- The unconstrained Lagrangian with multipliers:

$$L = \sum_{m=0}^{N_t - 1} \left( \|\hat{u}_r(t_m) - \hat{u}(t_m)\|^2 + \beta(c_r(t_m) - c(t_m)) + \int_{t_0}^{t_m} \lambda_m^{\top} (\dot{a} - Aa - B(a, a) - \dot{c}Ma) dt \right)$$
(5)

$$+ \int_{t_0}^{t_m} \mu_m(\dot{c} + \frac{p_i a_i + Q_{ij} a_i a_j}{s_i a_i}) dt$$
 (6)

$$+\lambda_{m}(t_{0})(a(t_{0})-\Psi^{\top}\hat{u}(t_{0}))$$
,  $\lambda_{m}\in\mathbb{R}^{r},\mu_{m}\in\mathbb{R}$ . (7)

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The optimization problem of SR-NTROM

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  - Single trajectory: SR-NiTROM vs SR-Galerkin
  - Multiple trajectories: SR-NiTROM vs SR-Galerkin

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics  $\begin{array}{c} \text{Results} \end{array}$ 



• FOM: Kuramoto-Sivashinsky equation

$$u_t = -uu_x - u_{xx} - \nu u_{xxx}, \quad x \in [0, 2\pi].$$
 (8)

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Results
Single trajectory: SR-NiTROM vs SR-Galerkin

Numerical details  $a \ {\sf FOM}. \ {\sf Kunmoto-Stovalinely reparts}$   $a_\theta=-m_\theta-m_\theta-\nu_{\rm const.} \quad x\in [0,2\pi]. \eqno(8)$ 

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 $\bullet$   $\nu=4/87$  for traveling-wave patterns.



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Numerical details s \in \mathsf{FOM}. \text{ Numerical details} u_r = -u_{rr} - u_{rr} - u_{rr} - u_{rr} \times \in [0, 2r]. \tag{9} s = 4/87 \text{ for transling wave patterns.}
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Numerical details  \begin{aligned} & \bullet \text{ FOM. Karamoto Sivabinely squatton} \\ & u_{R} = -u_{H} - u_{n} - v_{min}, \quad x \in [0, 2\pi], \\ & v = +(N! \text{ for transfergment patterns.} \\ & \cdot \text{Periodic BCs. } N = 00 \text{ Further modes.} \Delta t = 10^{-1}. \end{aligned}
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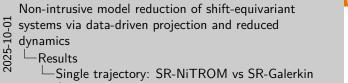
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Non-intrusive model reduction of shift-equivariant . FOM: Kuramoto-Sivashinsky equatio systems via data-driven projection and reduced dynamics Results . 20 outer loops, 5 CG updates per outer loops -Single trajectory: SR-NiTROM vs SR-Galerkin

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  - 20 outer loops, 5 CG updates per outer loops.
  - Initial conditions: POD bases (capturing >99.5% energy) + Galerkin-projected tensors. (imitating the training result of the re-projected SR-OpInf ROM)



Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Results
Single trajectory: SR-NiTROM vs SR-Galerkin

Numerical details  $= FOM. \ \, \text{Karameto-Sirabinishy squation}$   $= x - m_x - m_x - m_{\text{max}} - x \in [0, 2\pi].$  (8)  $= x - (1)\% \ \, \text{for transfig even patterns.}$   $= Finish \ \, \text{GeV} = 10^{-1} \ \, \text{GeV} = 10$ 

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## Results: single transient trajectory from t = 30 to t = 40

• Relative weight: 10.0, 4-dim ROM

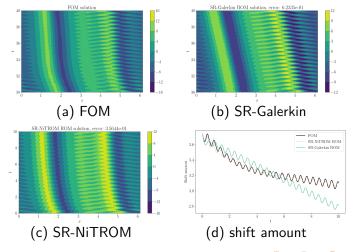
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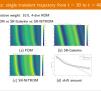
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Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Results
Single trajectory: SR-NiTROM vs SR-Galerkin



• 9 trajectories, 7-dim ROM

Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

Results
Multiple trajectories: SR-NiTROM vs SR-Galerkin

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Results

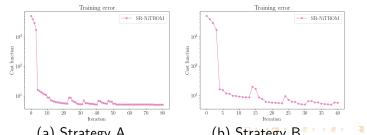
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- Strategy A: 20 outer training loops (10 on bases, 10 on tensors), 5 CG updates per outer loop
- Strategy B: 10 outer training loops on tensors only with fixed POD bases
- Training loss: not too much difference on the training set.

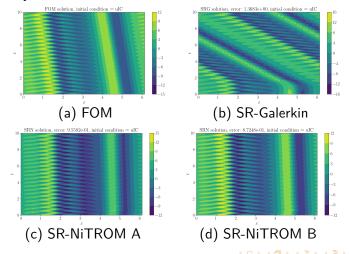


Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics Results

-Multiple trajectories: SR-NiTROM vs SR-Galerkin



• Trajectory 1:

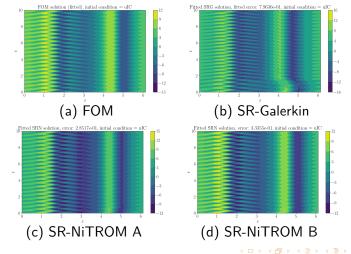


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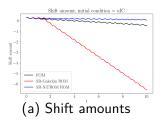


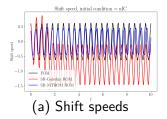
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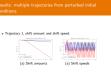


• Trajectory 1, shift amount and shift speed:





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 For the reconstruction of a single training trajectory including transient, SR-NiTROM outperforms SR-Galerkin ROM (and SR-OpInf of course). Non-intrusive model reduction of shift-equivariant systems via data-driven projection and reduced dynamics

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Conclusions:

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- To-dos: test our SR-NiTROM on unseen trajectories. Compute the obliqueness of projection.

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