# Machine Vision

Lecture Set – 03

Binary Image Processing

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# Binary Image Processing

- An image contains a continuum of intensity values before it is quantized to obtain a digital image
  - Commonly used quantization levels: 256 (8 bits)
  - Also used: 32, 64, 128, 512, and 4096 (12 bits, usually for medical images)
  - More quantization levels: better representation, more storage
  - Binary images: 2 gray level quantization (1 bit)
- Why binary images?
  - Memory and computing power are limited in early days
  - Algorithms are well understood
  - Require less memory and fast execution time
  - Object and background separation with mask
  - Easy to analyze

# Binary Image

- Binary images are particularly usefully for
  - Identifying objects with distinctive silhouettes
    - e. g. components on a conveyor in a manufacturing plant
  - Recognizing text and symbols
    - e.g. document processing or interpreting road signs
  - Determining the orientation of objects
- Disadvantages:
  - Need proper illumination control to obtain good contrast
  - Not possible to recover information using only two intensity levels for some applications

# Binary Image Processing

- Representation of binary image
  - An image with size of $m \times n$ pixels
  - 1 (white) for object pixel and 0 (black) for background pixel
- Topics on binary image processing
  - Formation of binary images
  - Geometric properties
  - Topological properties
  - Object recognition in binary images

# Image Segmentation

- Image segmentation
  - Partition an image into regions
  - Identify the subimage that represents objects
  - One of the most important problem in vision systems
  - It can be defined as a method to partition an image, $F[i, j]$, into subimages, called regions, $P_1, \ldots, P_k$, such that each subimage is an object candidate
- Region
  - A subset of an image

# Binary Image Segmentation

- Segmentation is grouping pixels into regions such that
  - $\cup^k_{i=1} P_i$ = Entire image ($\{P_i\}$ is an exhaustive partitioning)
  - $P_i \cap P_j = \emptyset$, $i \neq j$ ($\{P_i\}$ is an exclusive partitioning)
  - Each region $P_i$ satisfies a predicate; i.e., all points of the partition have *some* common property
  - Pixels belonging to adjacent regions, when taken jointly, do not satisfy the predicate
- The predicate can be "having uniform intensity", etc.
- A binary image is obtained using an appropriate segmentation of a gray image

# Thresholding & Segmentation

- If the intensity values of an object are in some some interval and that of background are outside that interval
  - Thresholding can be used to set one interval to 1 and the other interval to 0 to segment the object and background regions

- For binary vision, segmentation and thresholding are synonymous

# Thresholding for Binary Images

- Thresholding is a method to convert a gray scale image into a binary image so that objects of interest are separated from the background
- Sufficient contrast on objects and background is necessary to do the thresholding (why?)

# Thresholding for Binary Images

- If we use a threshold $T$ for the original image $F[i, j]$ to obtain a binary image $B[i, j] = F_T[i, j]$, then

$$F_T[i,j] = \begin{cases} 1 & \text{if } F[i,j] \le T \\ 0 & \text{otherwise} \end{cases}$$

- If the object intensity values are in the range $[T_1, T_2]$, then we can use the following equation for thresholding

$$F_T[i,j] = \begin{cases} 1 & \text{if } \mathrm{T}_1 \le F[i,j] \le T_2 \\ 0 & \text{otherwise} \end{cases}$$

# Thresholding for Binary Images

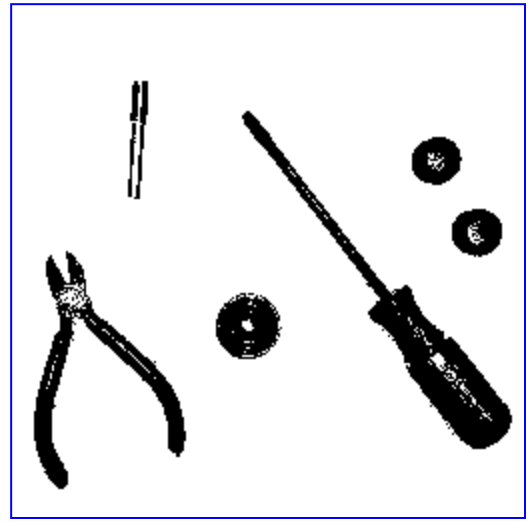
- A general thresholding scheme in which the intensity levels for an object may come from several disjoint intervals are represented as

$$F_T[i,j] = \begin{cases} 1 & \text{if } F[i,j] \in Z \\ 0 & \text{otherwise} \end{cases}$$

  where $Z$ is a set of intensity values for object components
- If there are two, or more threshold values, the threshold is usually selected on the basis of experience with the application domain
- Automatic thresholding of images is often the first step in the analysis of images in machine vision systems

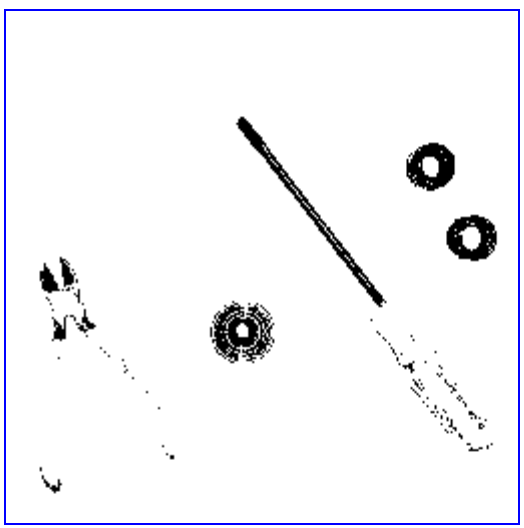
# Example

Original image



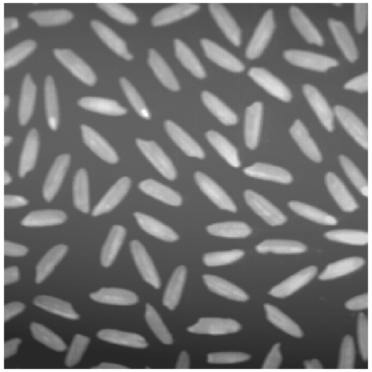
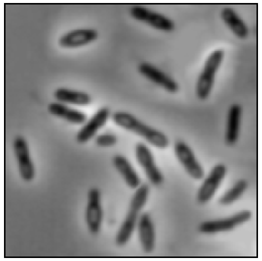
Threshold segmentation

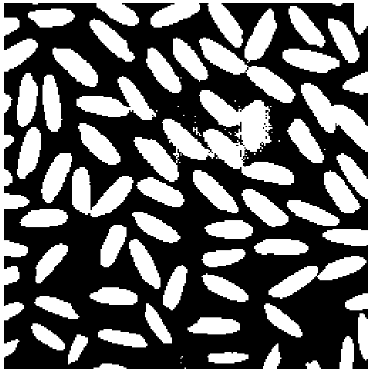
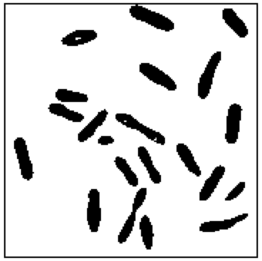
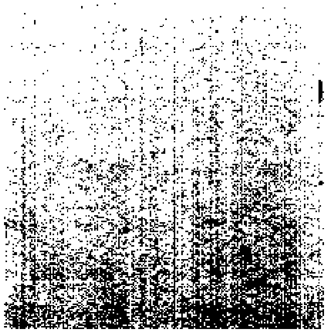
Threshold too low



Threshold too high

# Example – Applications

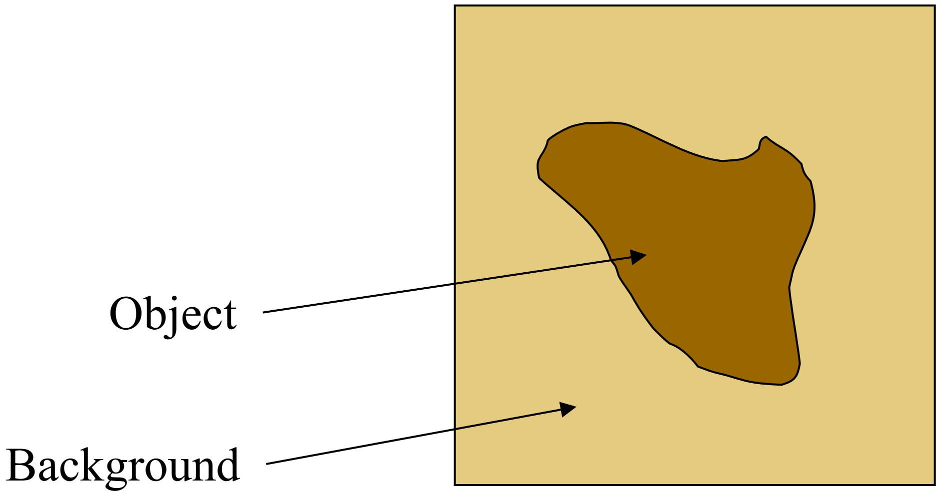



# 3/14/2023

# Geometric Properties

- The geometric properties of an object in a binary image include
  - Size
  - Position
  - Perimeter
  - Orientation




- They will be defined through moments of the object

# Moments of Objects

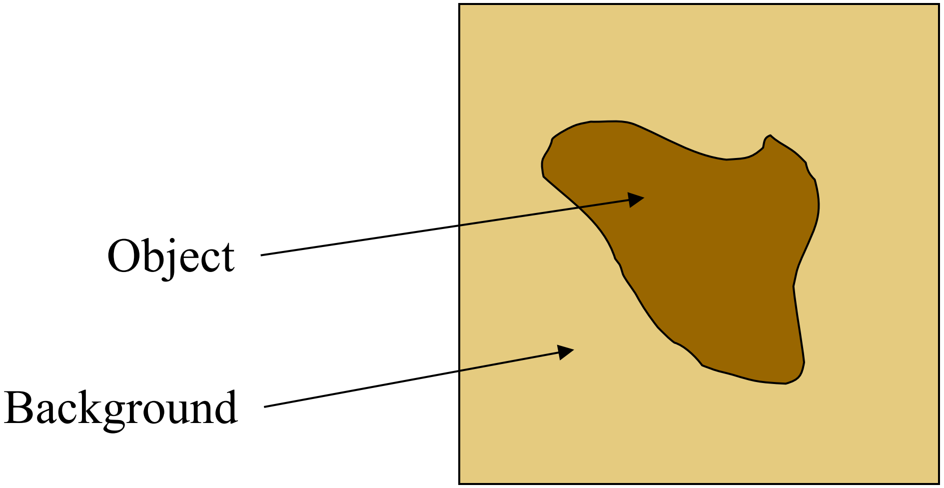
- Lots of useful information about a binary object can be gained from the moments of the object
- Define the binary image $B$ of an object to be

$$B(i,j) = \begin{cases} 1 & \text{for points on the object} \\ 0 & \text{else} \end{cases}$$

# Size of an Object

- The size of an object (area) is given by the $0^{th}$ *moment*

$$A = \sum_{i=1}^{n} \sum_{j=1}^{m} B(i, j)$$

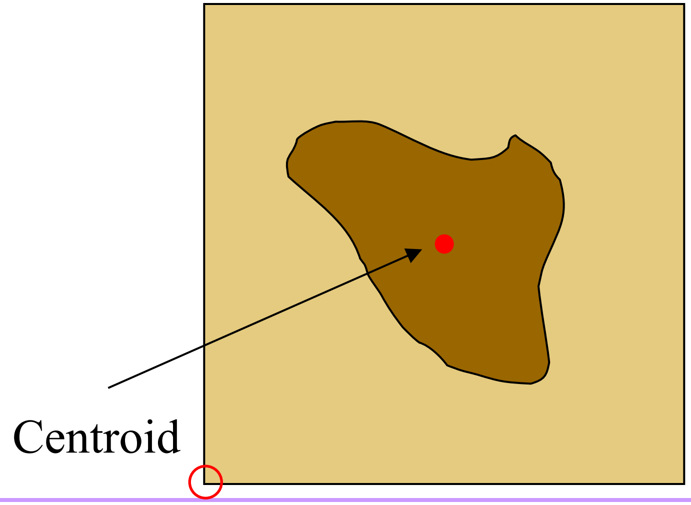


# Position of an Object

- The position of an object can be represented by
  - Enclosing rectangle – bounding box
  - Center of area – relatively insensitive to noise
- For a binary image, the center of area is the same as the center of mass (intensity value)
- The center of mass is given by the $1^{st}$ *moments*

$$\boxed{\bar{x} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} jB(i,j)}{\sum_{i=1}^{n}\sum_{j=1}^{m} B(i,j)}, \quad \bar{y} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} iB(i,j)}{\sum_{i=1}^{n}\sum_{j=1}^{m} B(i,j)}}$$

$$\int xf(x)dx = \bar{x}\int f(x)dx$$



# Orientation of an Object

- The orientation of an object is not necessary unique (such as circle)
- The orientation of an object is defined as the axis of minimum inertia
- This is the least 2nd moment, the orientation of which is

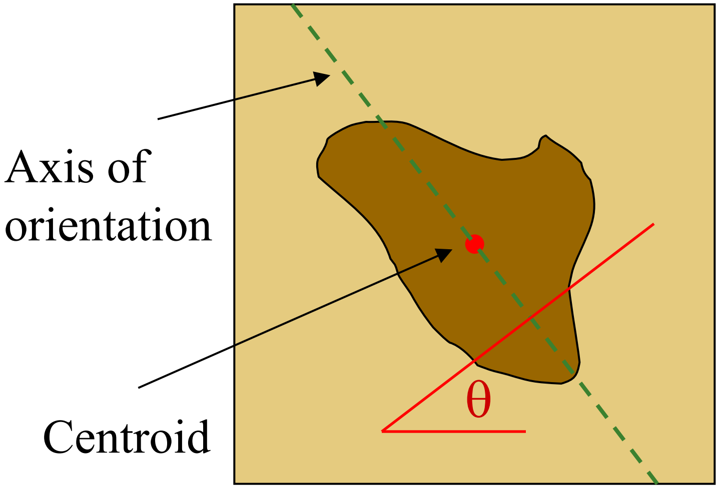
$$\theta = \frac{1}{2}\tan^{-1}\frac{M_{xy}}{M_{xx} - M_{yy}}$$

where the 2nd moments are

$$M_{xx} = \sum_x \sum_y (x - \bar{x})^2 B(x, y)$$

$$M_{xy} = \sum_x \sum_y 2(x - \bar{x})(y - \bar{y}) B(x, y)$$

$$M_{yy} = \sum_x \sum_y (y - \bar{y})^2 B(x, y)$$



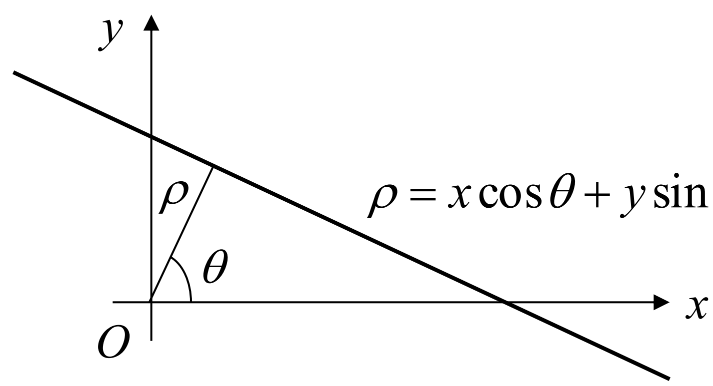
# Derivation of Orientation

- The axis of second moment
  - The line for which the sum of the squared distances between object points and the line is minimum
  - Compute the least-squares fit of a line to the object points
  - Let $r_{ij}$ be the distances of all object points from the line, then

$$\chi^2 = \sum_{i=1}^{n}\sum_{j=1}^{m} r_{ij}^2 B[i, j]$$

- Using polar coordinate system, let $\rho = x \cos \theta + y \sin \theta$, then for each objects to the line

$$r_{ij}^2 = (x_{ij}\cos\theta + y_{ij}\sin\theta - \rho)^2$$



# Derivation of Orientation

- Thus, $\chi^2 = \sum_{i=1}^{n} \sum_{j=1}^{m} (x_{ij} \cos\theta + y_{ij} \sin\theta - \rho)^2 B[i,j]$
- Take the derivative w.r.t. ρ, set to zero, and solve for ρ:*

$$\frac{\partial \chi^2}{\partial \rho} = -2 \sum_{i=1}^{n} \sum_{j=1}^{m} (x_{ij} \cos\theta + y_{ij} \sin\theta - \rho) B_{ij}$$

$$\Rightarrow \quad \rho = \bar{x} \cos\theta + \bar{y} \sin\theta$$

- Let $x' = x - \bar{x}$ and $y' = y - \bar{y}$, then $\chi^2 = a\cos^2\theta + b\sin\theta\cos\theta + c\sin^2\theta$ with*

$$a = \sum_{i=1}^{n} \sum_{j=1}^{m} (x_{ij}')^2 B[i,j], \; b = 2\sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij}' y_{ij}' B[i,j], \; c = \sum_{i=1}^{n} \sum_{j=1}^{m} (y_{ij}')^2 B[i,j]$$

- Thus, $\chi^2 = \frac{1}{2}(a+c) + \frac{1}{2}(a-c)\cos 2\theta + \frac{1}{2} b \sin 2\theta$
- Take the derivate w.r.t. θ, set to zero, and solve for θ:

$$\tan 2\theta = \frac{b}{a-c}$$

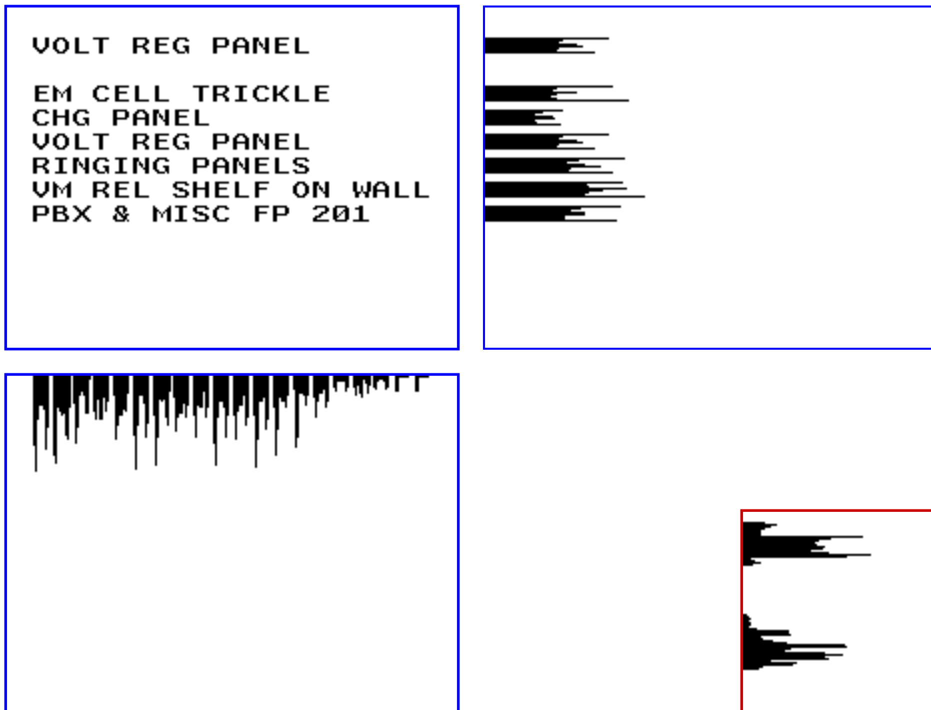
# Projections

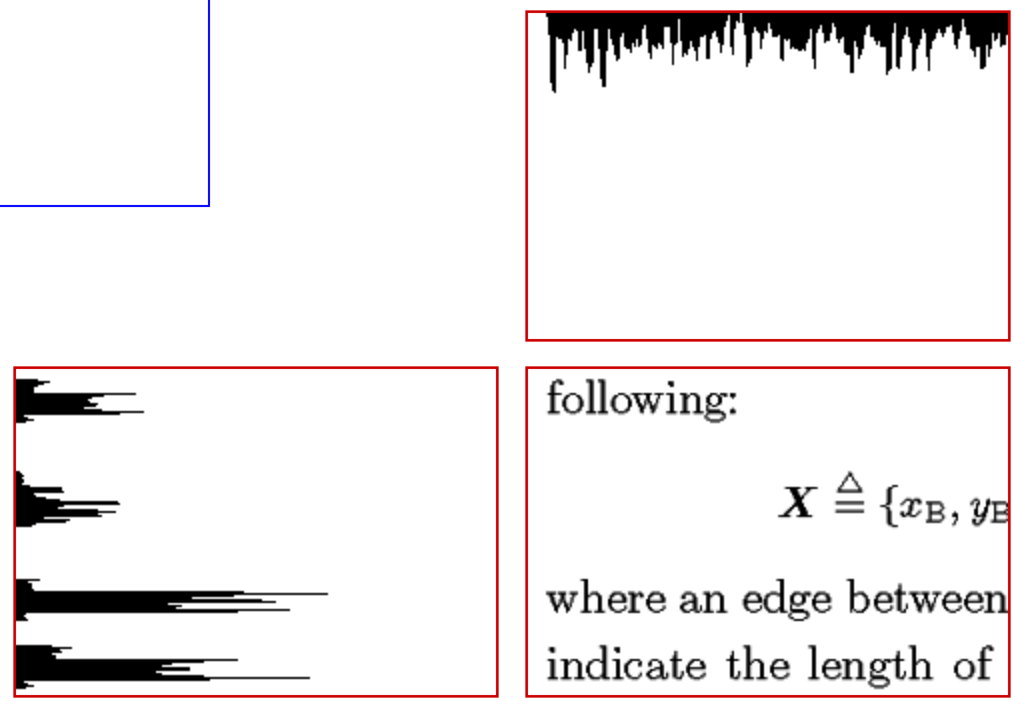
- Projection is a compact representation of binary images
  - Projections are not unique
  - More than one image may have the same projection
- Projection of a binary image onto a line
  - Partition the line into bins and finding the number of 1 pixels that are on lines perpendicular to each bin
- The projection $H[i]$ along the rows and the projection $V[j]$ along the columns of a binary image are given by

$$\boxed{H[i] = \sum_{j=1}^{m} I(i,j), \qquad V[j] = \sum_{i=1}^{n} I(i,j)}$$

# Projections

- Pictures of projections along both directions (Fig. 2.4, 2.5, 2.6)



# Projections and Position

- A general projection onto any line may be defined
- The first moments of an image equal to the first moments of its projection (Why?)
- Calculation of the position of an object requires only the first moment
- The position can be computed from the horizontal and vertical projections

$$A = \sum_{j=1}^{m} V[j] = \sum_{i=1}^{n} H[i], \quad \bar{x} = \frac{\sum_{j=1}^{m} jV[j]}{A}, \quad \bar{y} = \frac{\sum_{i=1}^{m} iH[j]}{A}$$

# Run-Length Encoding

- Used for image transmission
- Use numbers indicating the lengths of the runs of 1 pixels in the image
- Two common approaches:
  - The start position and lengths of runs of 1s for each row are used
  - Use only the length of runs, starting with the length of the 1 run

# Run-Length Encoding

- Binary image:

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |

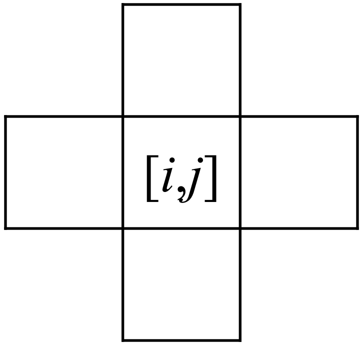
- Start and length of 1 runs:
  - (1,3) (7,2) (12,4) (17,2) (20,3)
  - (5,13) (19,4)
  - (1,3) (17,6)
- Length of 1 and 0 runs:
  - 3,3,2,3,4,1,2,1,3
  - 0,4,13,1,4
  - 3,13,6

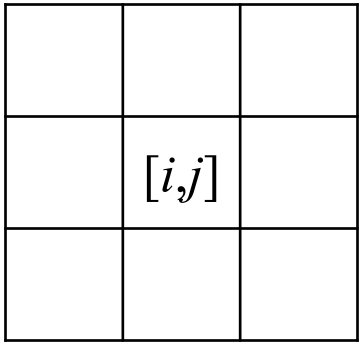
# 3/16/2023

- HW#1 due 3/23

# Neighbors

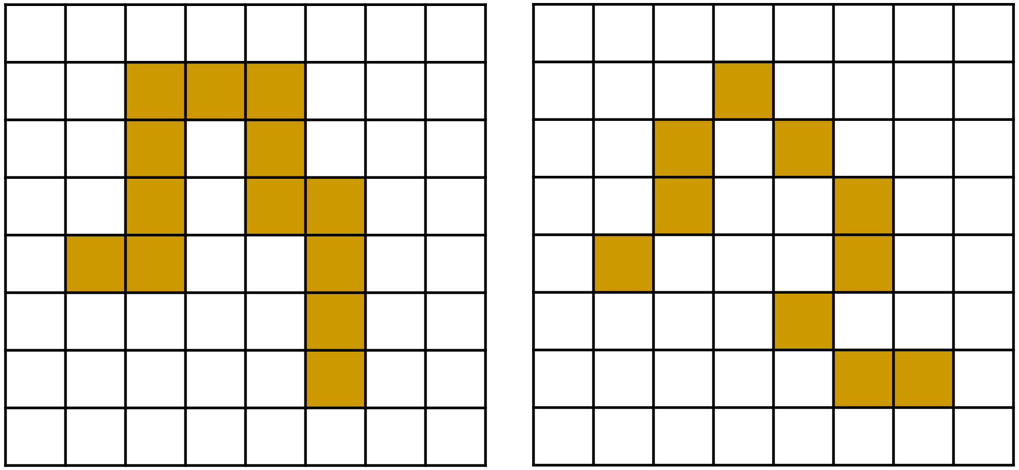
- In a digital image, a pixel has a common boundary with four pixels and shares a corner with four additional pixels
  - Two pixels are *4-neighbors* if they share a common boundary
  - Two pixels are *8-neighbors* if they share at least one corner
  - The pixel at location $[i, j]$ has 4-neighbors $[i+1, j]$, $[i-1, j]$, $[i, j+1]$, $[i, j-1]$
  - The 8-neighbors of the pixel include the 4-neighbors plus $[i+1, j+1]$, $[i+1, j-1]$, $[i-1, j+1]$, $[i+1, j-1]$
- A pixel is said to be *4-connected* to its 4-neighbors and *8-connected* to its 8-neighbors



# Path

- A *path* from the pixel at $[i_0, j_0]$ to the pixel at $[i_n, j_n]$ is a sequence of pixel indices $[i_0, j_0], [i_1, j_1], \ldots, [i_n, j_n]$ such that the pixel at $[i_k, j_k]$ is a neighbor of the pixel at $[i_{k+1}, j_{k+1}]$ for all $k$ with $0 \le k \le n-1$
- If the neighbor relation uses 4-connection, the path is a *4-path*; If the neighbor relation uses 8-connection, the path is a *8-path*



# Perimeter

- The length of the perimeter $P$ of a region is a global property
- A definition of the perimeter of a region without holes is the set of its interior border pixels
- Perimeter:

$$P_4 = \{(r,c) \in R \mid N_8(r,c) - R \neq \varnothing\}$$
$$P_8 = \{(r,c) \in R \mid N_4(r,c) - R \neq \varnothing\}$$

(Check with a simple image.)

- To compute length $|P|$ of perimeter $P$, the pixels in $P$ must be ordered in a sequence $P = \langle (r_0,c_0),\ldots,(r_{k-1},c_{k-1}) \rangle$
- Perimeter length:

$$|P| = |\{k \mid (r_{k+1},c_{k+1}) \in N_4(r_k,c_k)\}| + 2^{1/2}\,|\{k \mid (r_{k+1},c_{k+1}) \in (N_8(r_k,c_k) - N_4(r_k,c_k))\}|$$

# Circularity

- Circularity (or compactness) can be defined as the length of the perimeter squared divided by the area

$$C_1 = |P|^2 / A$$

- In this definition, it has the smallest value for digital octagons or diamonds depending on whether 4- or 8-neighbor used
- *Smaller is better!*

# Circularity

- Circularity can also defined as $C_2 = \mu_R/\sigma_R$ where $\mu_R$ and $\sigma_R$ are the mean and standard deviation of the distance from the centroid of the shape
  - Mean radial distance:

$$\mu_R = \frac{1}{K}\sum_{k=0}^{K-1} \| (r_k, c_k) - (\bar{r}, \bar{c}) \|$$

  - Standard deviation of radial distance:

$$\sigma_R = \left( \frac{1}{K}\sum_{k=0}^{K-1} [\| (r_k, c_k) - (\bar{r}, \bar{c}) \| - \mu_R]^2 \right)^{\frac{1}{2}}$$

  - *Larger is better!*

# Example

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 2 | 2 | 2 | 0 | 0 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 2 | 2 | 2 | 0 | 0 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 2 | 2 | 2 | 0 | 0 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| region number | region area | row of center | column of center | perimeter length | circularity 1 | circularity 2 | radius mean | radius variance |
|---|---|---|---|---|---|---|---|---|
| 1 | 44 | 6 | 11.5 | 21.2 | 10.2 | 15.4 | 3.33 | .05 |
| 2 | 48 | 9 | 1.5 | 28 | 16.3 | 2.5 | 3.80 | 2.28 |
| 3 | 9 | 13 | 7 | 8 | 7.1 | 5.8 | 1.2 | 0.04 |

# Connectivity

- A pixel $p \in S$ is said to be *connected* to $q \in S$ if there is a path from $p$ to $q$ consisting entirely of pixels of $S$
- Connectivity is an *equivalence relation*
- For any three pixel $p$, $q$ and $r$ in $S$, we have the following properties:
  - *Reflectivity*: pixel $p$ is connected to $p$
  - *Commutativity*: if $p$ is connected to $q$, then $q$ is connected to $p$
  - *Transitivity*: if $p$ is connected to $q$ and $q$ is connected to $r$, then $p$ is connected to $r$
- A set of pixels in which each pixel is connected to all other pixels is called a *connected component*

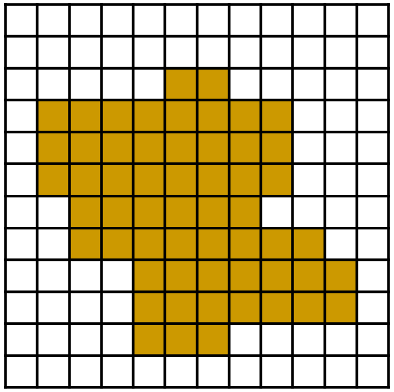
# Foreground, Background

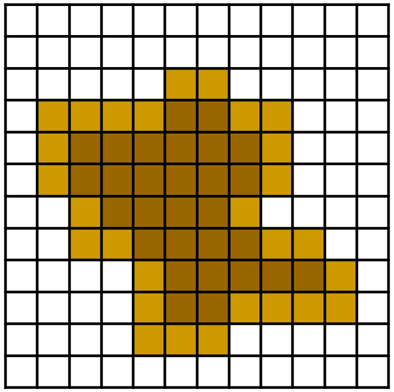
- Foreground
  - The set of all 1 pixels in an image is called the foreground and is denoted by $S$
- Background
  - The set of all connected components of $\hat{S}$ (the complement of $S$) *that have points on the border of an image* is called the background
- All other components of $\hat{S}$ are called holes
- Different connectedness should be used for object and background
  - If 8-connectedness is used for $S$, the 4-connectedness should be used for $\hat{S}$
  - (Why? Check page 43 in the textbook.)

| | | |
|---|---|---|
| | 1 | |
| 1 | | 1 |
| | 1 | |

# Boundary, Interior, Surrounds

- Boundary
  - The *boundary* of $S$ is the set of pixel of $S$ that have 4-neighbors in $\hat{S}$
  - The boundary is usually denoted by $S'$
- Interior
  - The *interior* is the set of pixels of $S$ that are not in its boundary
  - The interior of $S$ is $(S - S')$
- Surrounds
  - Region $T$ *surrounds* region $S$ (or $S$ is inside $T$), if any 4-path from any point of $S$ to the border of the picture must intersect $T$



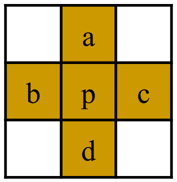


# Component Labeling

- Uniquely label each cluster of positive connected components
- Zero-elements are considered part of the background and remain zero
- Two algorithms:
  - Recursive algorithm (very inefficient, used only on parallel machines)
  - Sequential algorithm
- Recursive Algorithm:
  - Scan the image to find an unlabeled 1 pixel and assign it a new label L
  - Recursively assign a label L to all its 1 neighbors
  - Stop if there are no more unlabeled 1 pixels
  - Go to step 1

# Sequential Algorithm (Labeling)

- Labeling 4-connected components via a raster scan (row by row starting top left)
  - if **p** = 0 ignore
  - else if **a** and **b** not labeled, increment label and label **p**
  - else if only one of **a** or **b** labeled then copy label to **p**
  - else if **a** and **b** labeled
    - if **a** and **b** labeled the same then copy label to **p**
    - else copy either label to **p** and record the equivalence of the labels




| | | | |
|---|---|---|---|
| 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 |

→

| | | | |
|---|---|---|---|
| 1 | 1 | 0 | 2 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 3 | 0 |
| 0 | 0 | 0 | 0 |

# Counting Objects in an Image

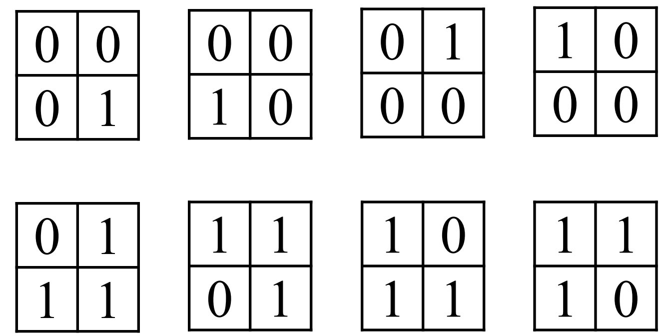
- For counting foreground objects
  - The *external corner patterns* (E) are 2×2 masks that have three 0's and one 1-pixel
  - The *internal corner patterns* (I) are 2×2 masks that have three 1's and one 0-pixel

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |

- Algorithm count_objects:
  - E = 0, I = 0
  - For L = 0 to row_number
    - For P = 0 to column_number
      - If external_match(L,P) then E = E + 1
      - If internal_match(L,P) then I = I + 1
  - Return (E-I)/4

# Counting Foreground Objects

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | |
| | 1 | 1 | 1 | | | | | | 1 | 1 | | | 1 | 1 | |
| | 1 | 1 | 1 | | | | | | | 1 | | | 1 | 1 | |
| | | | | | 1 | 1 | | | | 1 | 1 | | | 1 | |
| | | | | 1 | 1 | 1 | 1 | | | 1 | 1 | 1 | 1 | 1 | |
| | | | | | 1 | 1 | | | | | | | | | |
| | | | | | | | | | | | | | | | |




| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| e | | | e | | | | | e | | e | | e | | e | |
| | | | | | | | | e | i | | | | | | |
| e | | | e | e | | e | | | | i | e | e | i | | |
| | | | e | i | | i | e | | | | i | | i | | |
| | | | e | i | | i | e | | e | | | | | e | |
| | | | | e | | e | | | | | | | | | |
| | | | | | | | | | | | | | | | |

Number of e's : 21
Number of i's : 9
Number of objects = (21-9) / 4 = 3
(Why?)

# Example

- Component labeling

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |

=

4-connected

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 2 | 2 | 2 | 2 | 0 | 0 |
| 0 | 0 | 2 | 2 | 2 | 2 | 0 | 0 |
| 0 | 0 | 2 | 2 | 2 | 2 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 3 | 3 | 3 | 3 |
| 0 | 1 | 0 | 0 | 3 | 3 | 3 | 3 |
| 0 | 0 | 0 | 3 | 3 | 0 | 0 | 0 |

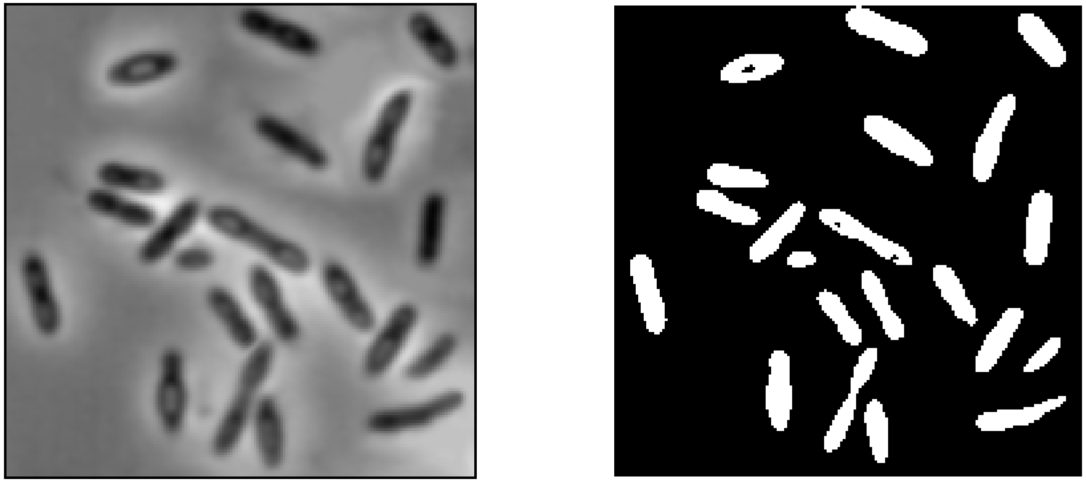
=

8-connected

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 2 | 2 | 2 | 2 |
| 0 | 1 | 0 | 0 | 2 | 2 | 2 | 2 |
| 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 |

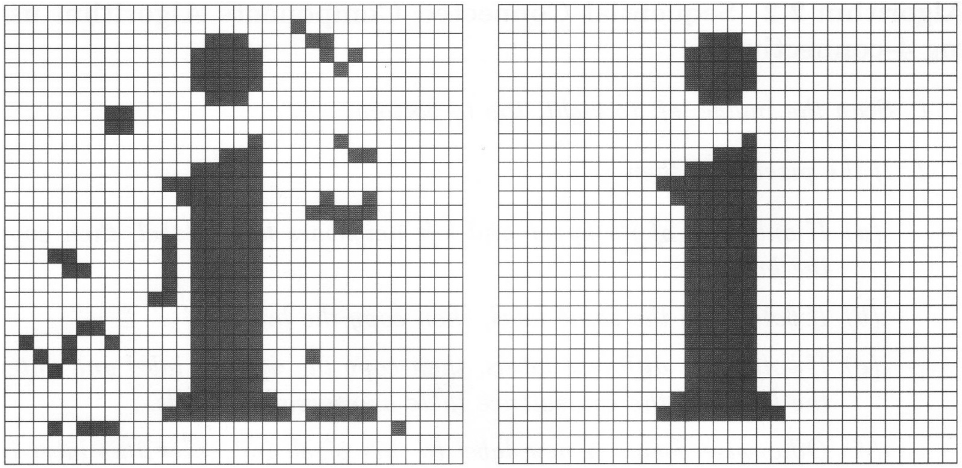
# Example

- Object counting

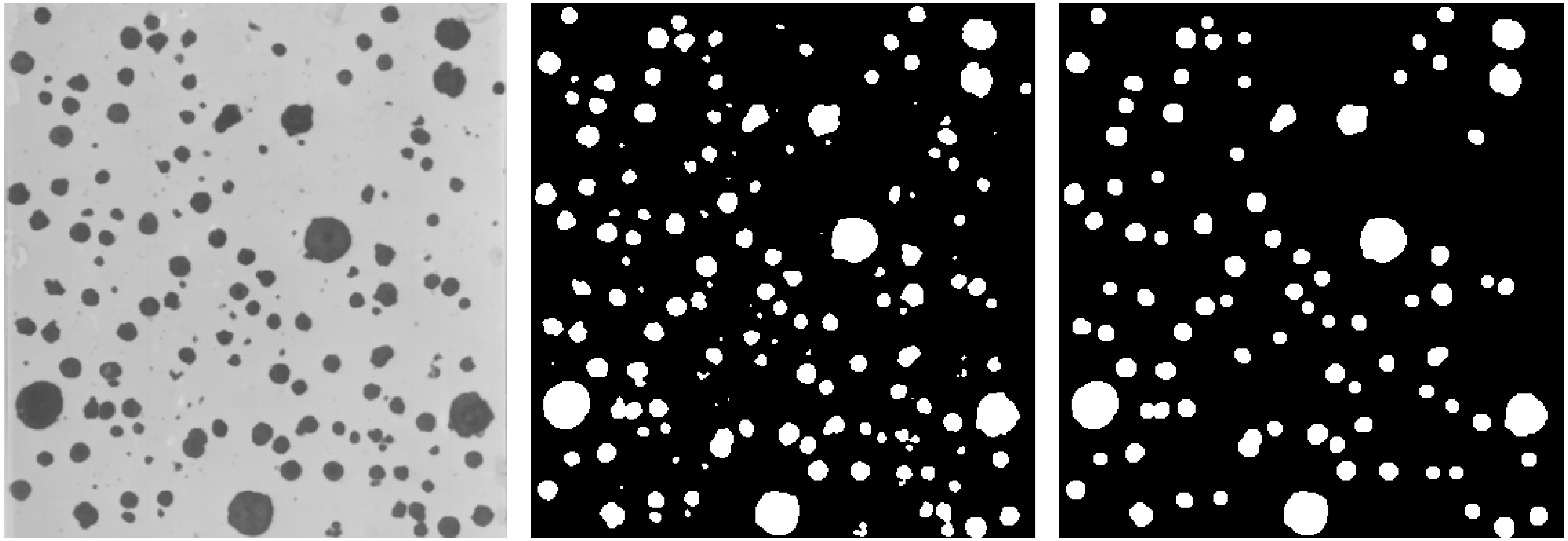


# Size Filtering

- In binary images, usually the noise regions are small
- If the size of object is greater than $T_0$ pixels, a size filter can be used to remove noise after component labeling
- All components below $T_0$ in size are removed by changing the corresponding pixels to 0

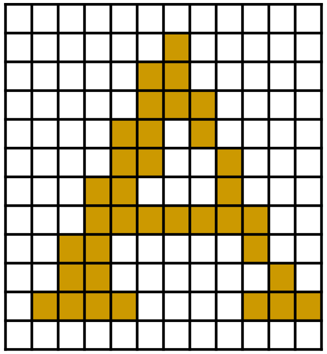


# Example

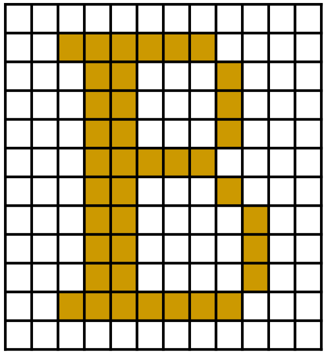


# Euler Number

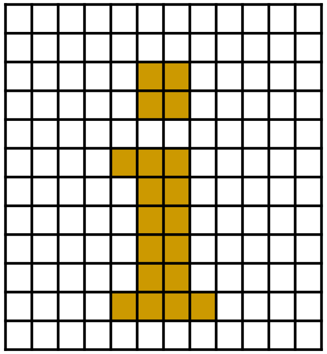
- Genus or Euler number can be used as a feature of an object
- Genus is defined as the number of components minus the number of holes: $E = C - H$
- Genus provides a simple topological feature that is *invariant to translation, rotation and scaling*



$E = 0$



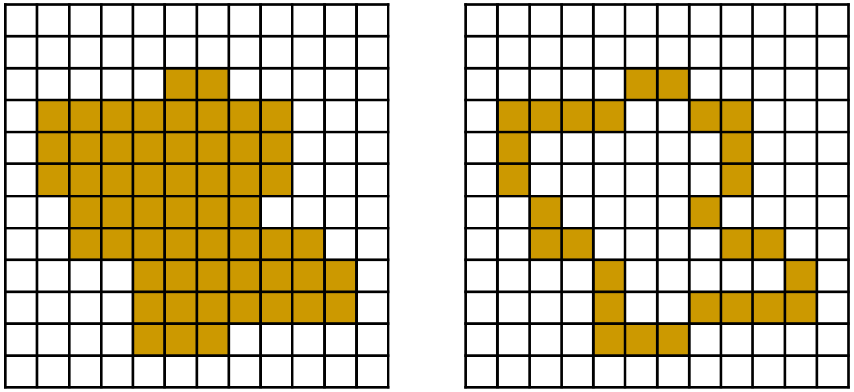
$E = -1$



$E = 2$

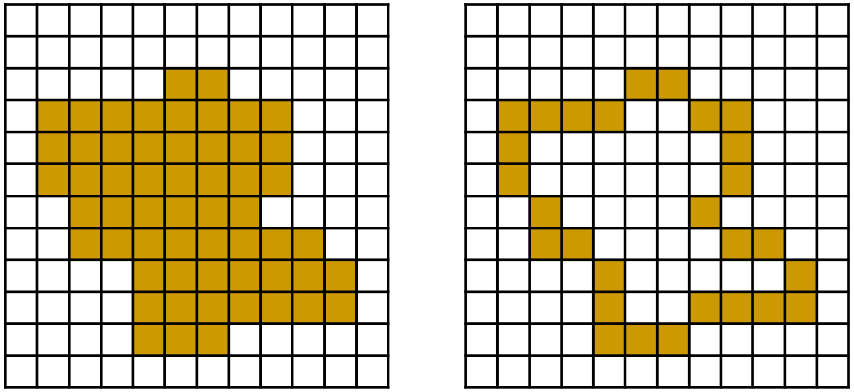
# Region Boundary

- The boundary of a connected component $S$ is the set of pixels of $S$ that are adjacent to $\bar{S}$
- In most application, we want to track pixels on the boundary in a particular order
- The boundary-following algorithm select a starting pixel $s \in S$ and track the boundary until it comes back to the starting pixel



# Region Boundary

- Boundary-Following Algorithm
  - Find a starting pixel $s \in S$ for the region via raster scan
  - Let the current pixel in boundary tracking be denoted by $c$, set $c = s$ and let the 4-neighbor to the west of $s$ be $b \in \hat{S}$
  - Let the eight 8-neighbors of $c$ starting with $b$ in clockwise order be $n_1, n_2, \ldots, n_8$. Find $n_i$, for the first $i$ that is in $S$
  - Set $c = n_i$ and $b = n_{i-1}$
  - Repeat above



# 3/21/2023

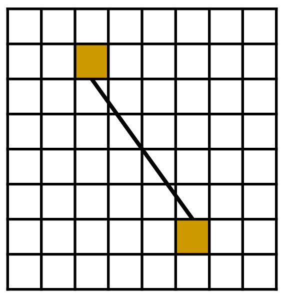
- No class next Tuesday (3/28)

# Distance Measure

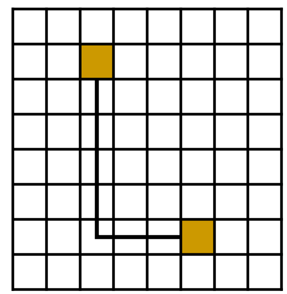
- To find the distance between two pixels or two components of an image
- For all pixels $p$, $q$ and $r$, any distance metric must satisfy all of the following properties:
  - $d(p,q) \geq 0$ and $d(p,q) = 0$ if and only if $p = q$
  - $d(p,q) = d(q,p)$
  - $d(p,r) \leq d(p,q) + d(q,r)$
- Common distance functions:
  - Euclidean: $d_{\text{Euclidean}}([i_1, j_1],[i_2, j_2]) = \sqrt{(i_1 - i_2)^2 + (j_1 - j_2)^2}$
  - City-block: $d_{\text{city}}([i_1, j_1],[i_2, j_2]) = |i_1 - i_2| + |j_1 - j_2|$
  - Chessboard: $d_{\text{chess}}([i_1, j_1],[i_2, j_2]) = \max(|i_1 - i_2|, |j_1 - j_2|)$

# Different Distance Measures

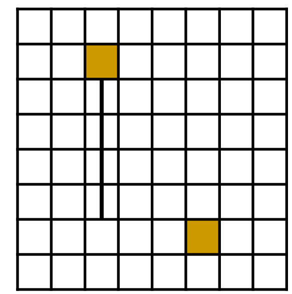
Euclidean distance



City-block distance



Chessboard distance



Euclidean distance:

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | 3 | | | |
| | $\sqrt{8}$ | $\sqrt{5}$ | 2 | $\sqrt{5}$ | $\sqrt{8}$ | |
| | $\sqrt{5}$ | $\sqrt{2}$ | 1 | $\sqrt{2}$ | $\sqrt{5}$ | |
| 3 | 2 | 1 | 0 | 1 | 2 | 3 |
| | $\sqrt{5}$ | $\sqrt{2}$ | 1 | $\sqrt{2}$ | $\sqrt{5}$ | |
| | $\sqrt{8}$ | $\sqrt{5}$ | 2 | $\sqrt{5}$ | $\sqrt{8}$ | |
| | | | 3 | | | |

City-block distance:

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | 3 | | | |
| | | 3 | 2 | 3 | | |
| | 3 | 2 | 1 | 2 | 3 | |
| 3 | 2 | 1 | 0 | 1 | 2 | 3 |
| | 3 | 2 | 1 | 2 | 3 | |
| | | 3 | 2 | 3 | | |
| | | | 3 | | | |

Chessboard distance:

| | | | | | | |
|---|---|---|---|---|---|---|
| 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 3 | 2 | 2 | 2 | 2 | 2 | 3 |
| 3 | 2 | 1 | 1 | 1 | 2 | 3 |
| 3 | 2 | 1 | 0 | 1 | 2 | 3 |
| 3 | 2 | 1 | 1 | 1 | 2 | 3 |
| 3 | 2 | 2 | 2 | 2 | 2 | 3 |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 |

# Distance Transform

- In some application, the *minimum distance* between *a pixel* of an object component and the *background* is needed
- Distance transform is to compute the distance to the background region $\hat{S}$, for all pixels in $S$
  - $f^0[i,j] = f[i,j]$
  - $f^m[i,j] = f^0[i,j] + \min(f^{m-1}[u,v])$

where $(u,v)$ is in the 4-neighbor of $(i,j)$

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | |
| | | | | | | | | | | | |
| | | | | | 1 | 1 | | | | | |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | | |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | | |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | | | | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | |
| | | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| | | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| | | | | 1 | 1 | 1 | | | | | |
| | | | | | | | | | | | |

0th pass

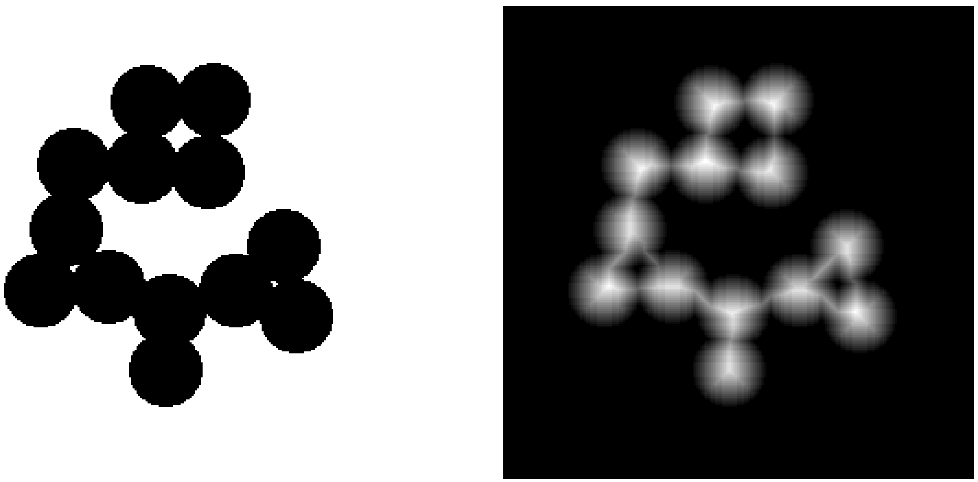
| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | |
| | | | | | | | | | | | |
| | | | | | 1 | 1 | | | | | |
| | 1 | 1 | 1 | 1 | 2 | 2 | 1 | 1 | | | |
| | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | | | |
| | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | | | |
| | | 1 | 2 | 2 | 2 | 2 | 1 | | | | |
| | | 1 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | | |
| | | | | 1 | 2 | 2 | 2 | 2 | 2 | 1 | |
| | | | | 1 | 2 | 2 | 1 | 1 | 1 | 1 | |
| | | | | 1 | 1 | 1 | | | | | |
| | | | | | | | | | | | |

1st pass

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | |
| | | | | | | | | | | | |
| | | | | | 1 | 1 | | | | | |
| | 1 | 1 | 1 | 1 | 2 | 2 | 1 | 1 | | | |
| | 1 | 2 | 2 | 2 | 3 | 3 | 2 | 1 | | | |
| | 1 | 2 | 3 | 3 | 3 | 3 | 2 | 1 | | | |
| | | 1 | 2 | 3 | 3 | 2 | 1 | | | | |
| | | 1 | 1 | 2 | 3 | 3 | 2 | 1 | 1 | | |
| | | | | 1 | 2 | 3 | 2 | 2 | 2 | 1 | |
| | | | | 1 | 2 | 2 | 1 | 1 | 1 | 1 | |
| | | | | 1 | 1 | 1 | | | | | |
| | | | | | | | | | | | |

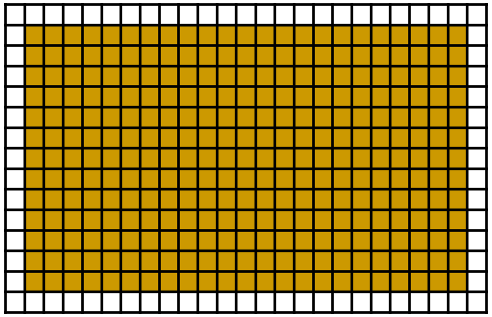
2nd pass

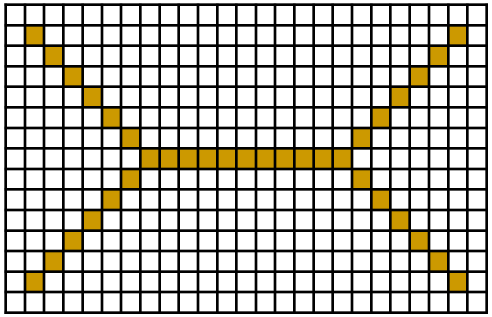
# Example



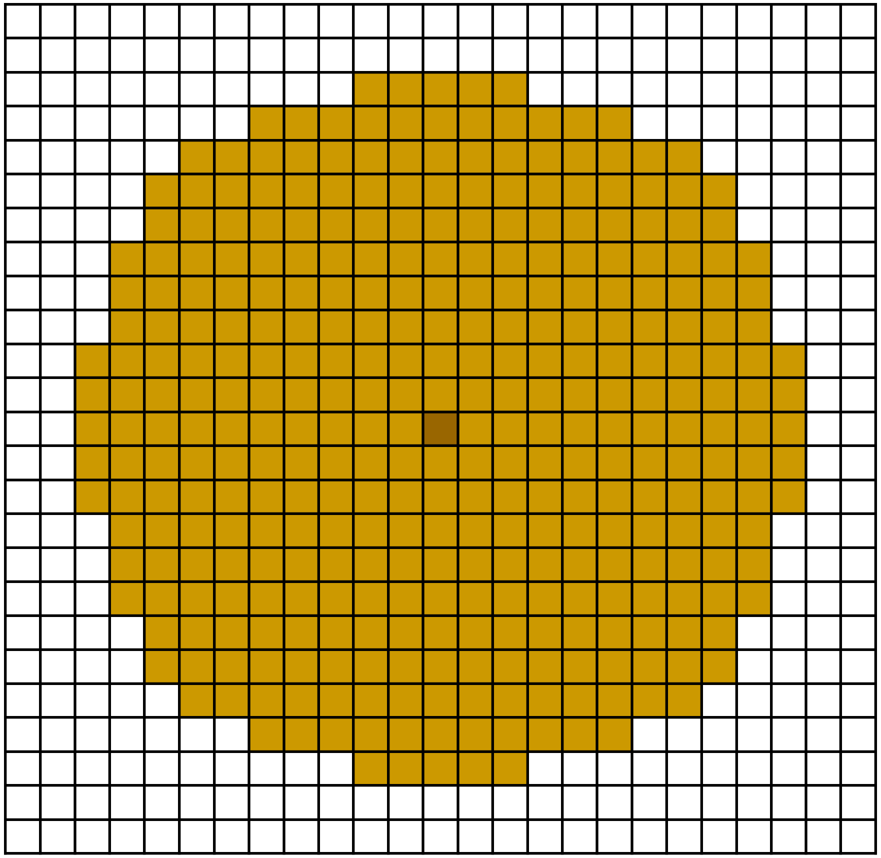
# Medial Axis

- The distance $d([i,j], \hat{S})$ from the pixel $[i,j]$ in $S$ to $\hat{S}$ is locally maximum if $d([i,j], \hat{S}) \geq d([u,v], \hat{S})$ for all pixels $[u,v]$ in the neighborhood of $[i,j]$
- The set of pixels in $S$ with distances from $\hat{S}$ that are locally maximum is called the skeleton, symmetric axis, or medial axis of $S$, and denoted by $S^*$

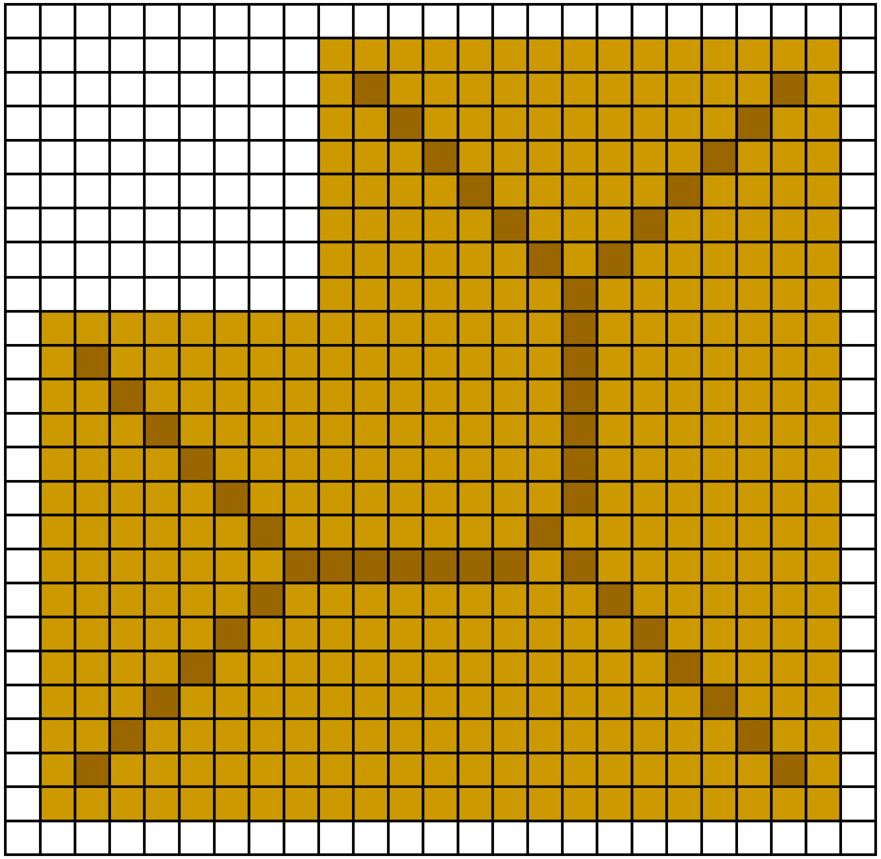




# Example



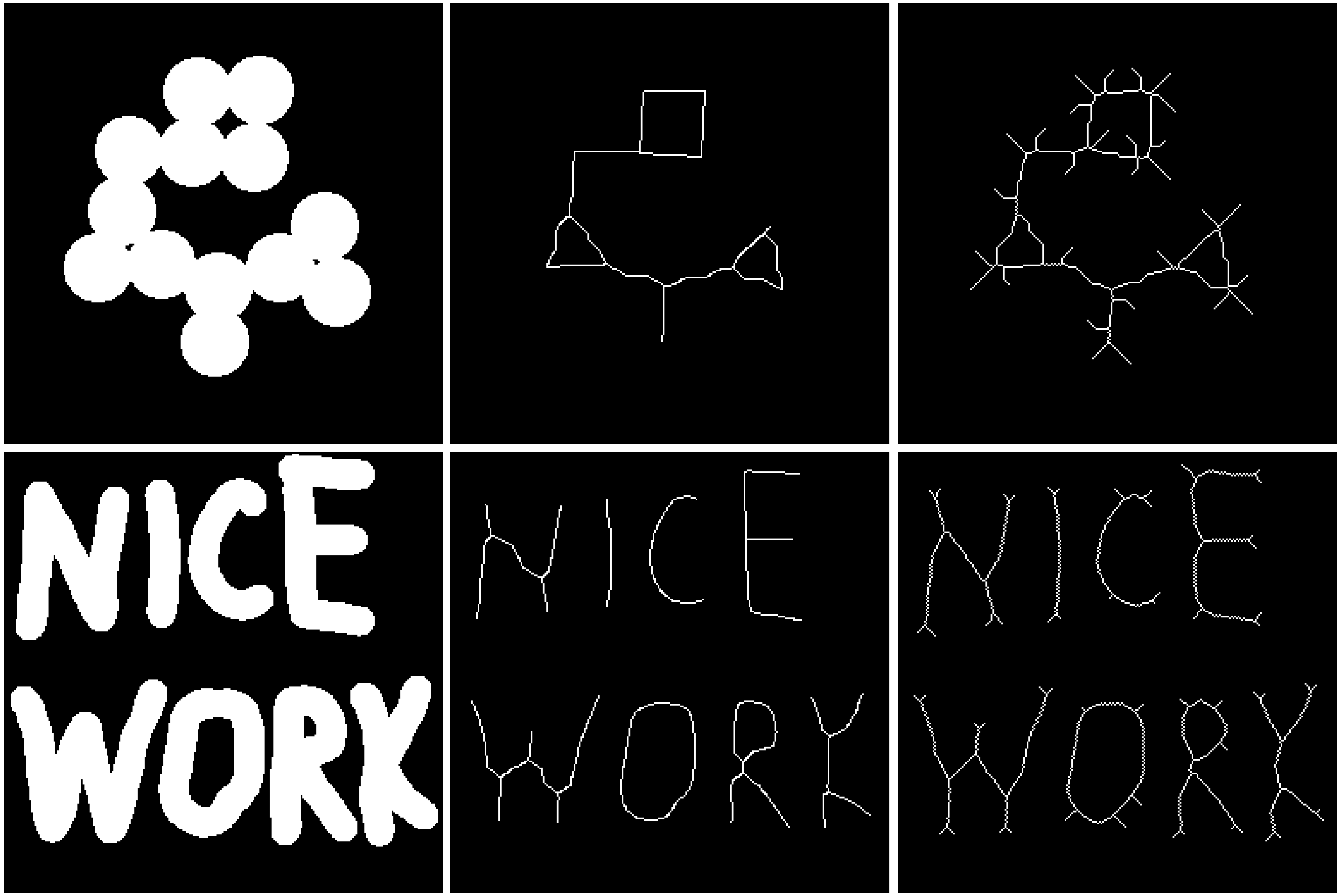
# Example



# Medial Axis

- The original set $S$ can be reconstructed from $S^*$ and the distance of each pixel of $S^*$ from $\hat{S}$
- $S^*$ is a compact representation of $S$
- $S^*$ is used to represent the shape of a region
- By deleting pixels of $S^*$ whose distances from $\hat{S}$ are small, we can create a simplified version of $S^*$
- Two representations – boundary and medial axis:
  - For arbitrary objects, a boundary is a more compact representation of a region
  - To find whether a given pixel is in the region or not, medial axis is a better representation

# Example



# Thinning

- Thinning:
  - Binary image *regions* are reduced to *lines* that approximate their center lines, also called skeleton or core-line
  - To reduce the image components to their essential information so that further analysis and recognition are facilitated
- Thinning requirement:
  - *Connected image regions* must thin to *connected line structures*
  - The thinned result should be minimally 8-connected
  - *Approximate end-line locations should be maintained*
  - *The thinning results should approximate the medial lines*
  - Extraneous spurs (short branches) caused by thinning should be minimized

# Thinning

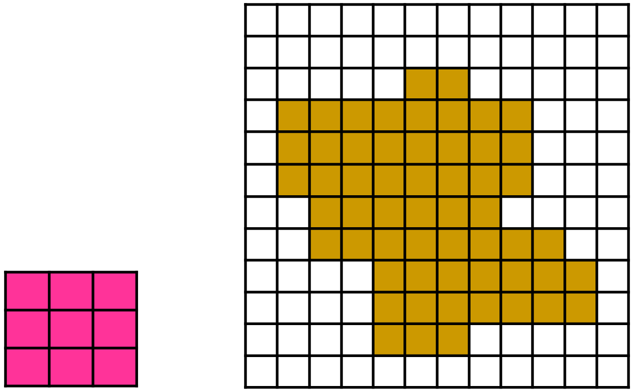
- Examine each pixel in the image within the context of its neighborhood region of at least 3×3 pixels and to "peel" the region boundaries, one pixel at a time, until the regions have been reduced to thin lines
- The process is performed iteratively

# 3/23/2023

- No class next Tuesday (3/28)
- Homework #2 will be given later, at 11:40.

# Expanding and Shrinking

- *Expanding*
  - A component is allowed to change such that some background pixels are converted to 1
  - Change a pixel from 0 to 1 if <u>any</u> neighbors of the pixel are 1
- *Shrinking*
  - Object pixels are systematically deleted or converted to 0
  - Change a pixel from 1 to 0 if <u>any</u> neighbors of the pixel are 0



# Expanding and Shrinking

- Let $S^{(k)}$ : $S$ expanded $k$ times and $S^{(-k)}$ : $S$ shrunk $k$ times, then
  - $(S^m)^{-n} \neq (S^{-n})^m \neq S^{(m-n)}$
  - $S \subset (S^k)^{-k}$
  - $S \supset (S^{-k})^k$
- Expanding and shrinking can be used to determine isolated components and clusters (Fig. 2.23 and 2.24)
  - Expanding followed by shrinking can be used for *filling undesired holes*
  - Shrinking followed by expanding can be used for *removing isolated noise pixels*

# Binary Image Morphology

- The word *morphology* refers to form and structure
- In computer vision, it can be used to refer to the shape of a region
- The operations of mathematical morphology were originally defined as *set operations*
- *Morphological operators* can:
  - Thin,
  - Thicken,
  - Find boundaries,
  - Find skeletons (medical axis),
  - Convex hull,
  - And more

# Structuring Elements

- The operations of binary morphology input a binary image B and a structuring element S
- The structuring element S is usually another smaller binary image
  - It represents a shape
  - It can be any size and have arbitrary structure that can be represented by a binary image
  - Examples:

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

| | | | | |
|---|---|---|---|---|
| | 1 | 1 | 1 | |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| | 1 | 1 | 1 | |

| | | | | |
|---|---|---|---|---|
| | 1 | 1 | 1 | |
| 1 | | | | 1 |
| 1 | | | | 1 |
| 1 | | | | 1 |
| | 1 | 1 | 1 | |

| |
|---|
| 1 |
| 1 |
| 1 |
| 1 |
| 1 |

# Point Sets and Notation

- Binary objects are considered as *point sets*
- For point sets $A$ and $B$ denote the:
  - Translation of $A$ by $x$ as $A_x = \{a_i + x \mid a_i \in A\}$
  - Reflection of $B$ as $B^r = \{-b_i \mid b_i \in B\}$
  - Complement of $A$ as $A^c = \{a_i \mid a_i \notin A\}$
  - Difference of $A$ and $B$ as $A - B = \{c_i \mid (c_i \in A) \text{ XOR } (c_i \in B)\}$

# Basic Operations

- The basic operations of binary morphology are *dilation*, *erosion*, *closing*, and *opening*
  - Dilation enlarges a region
  - Erosion makes a region smaller
  - A closing operation can close up internal holes in a region and eliminate bays along the boundary
  - An opening operation can get rid of small portions of the region that just out from the boundary into the background region

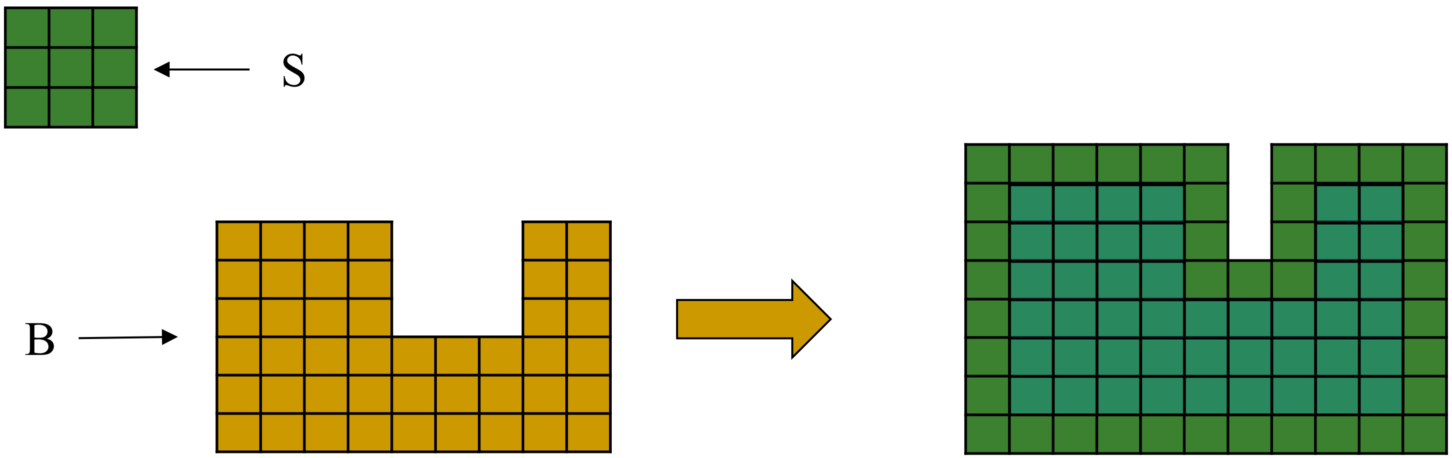
# Some Applications

- Binary morphology can be used to extract primitive features of an object that can be used to recognize the object
- A shape matching system can use morphological feature detection to rapidly detect primitives that are used in object recognition

# Dilation

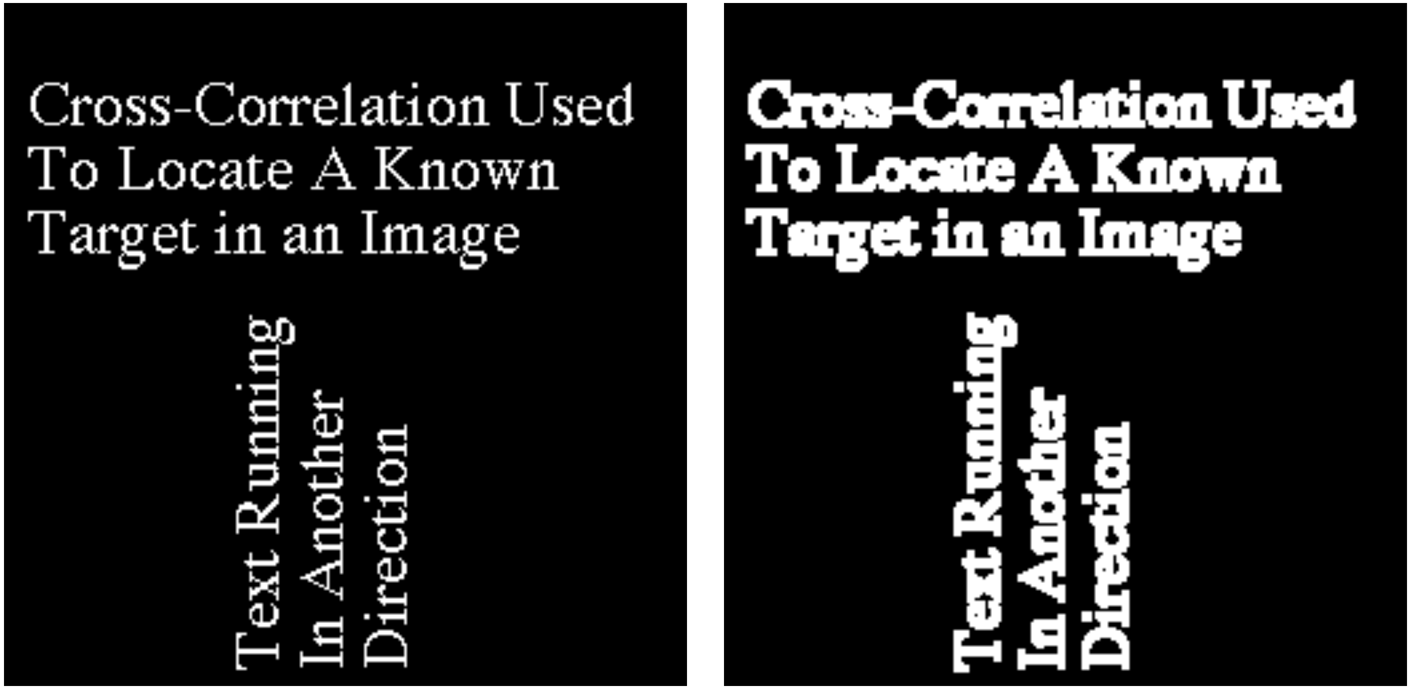
- Dilation of B by structuring element S:

$$B \oplus S = \{x \mid (S^r{}_x \cap B) \neq \varnothing\} = \cup_{b \in B} S_b$$




- Example: dilating A with a 3×3 structuring element B centered at the origin

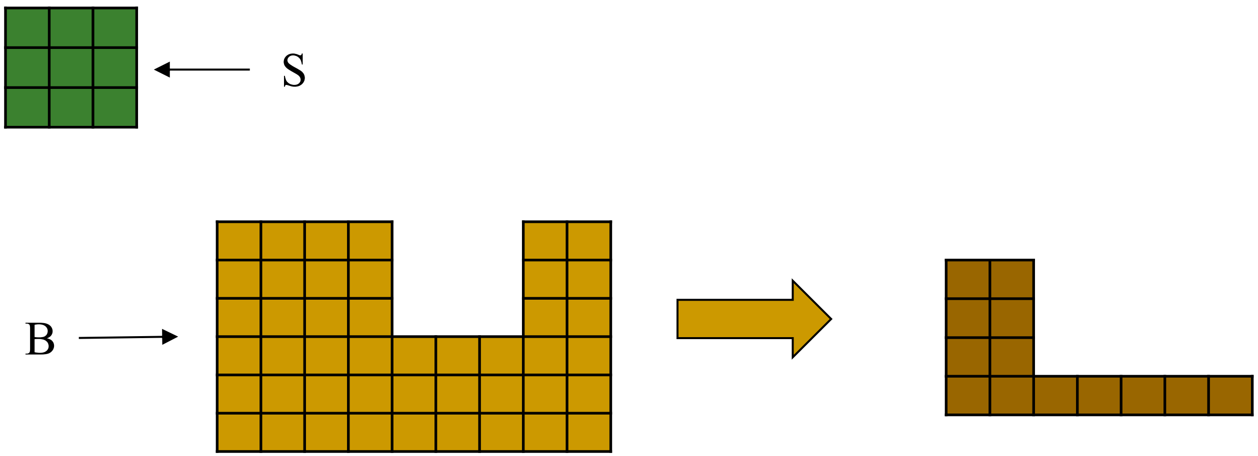
# Example


Cross-Correlation Used
To Locate A Known
Target in an Image
Text Running
In Another
Direction
Cross-Correlation Used
To Locate A Known
Target in an Image
Text Running
In Another
Direction

# Erosion

- Erosion of B by structuring element S:

$$B \ominus S = \{x \mid S_x \subseteq B\} = \{b \mid b + s \in B, \forall s \in S\}$$




- Example: eroding A with a 3×3 structuring element B centered at the origin

# Example

# Combining Dilation and Erosion

- Combining dilation and erosion for
  - *Opening*
  - *Closing*
  - Thickening
  - Thinning
  - Skeleton

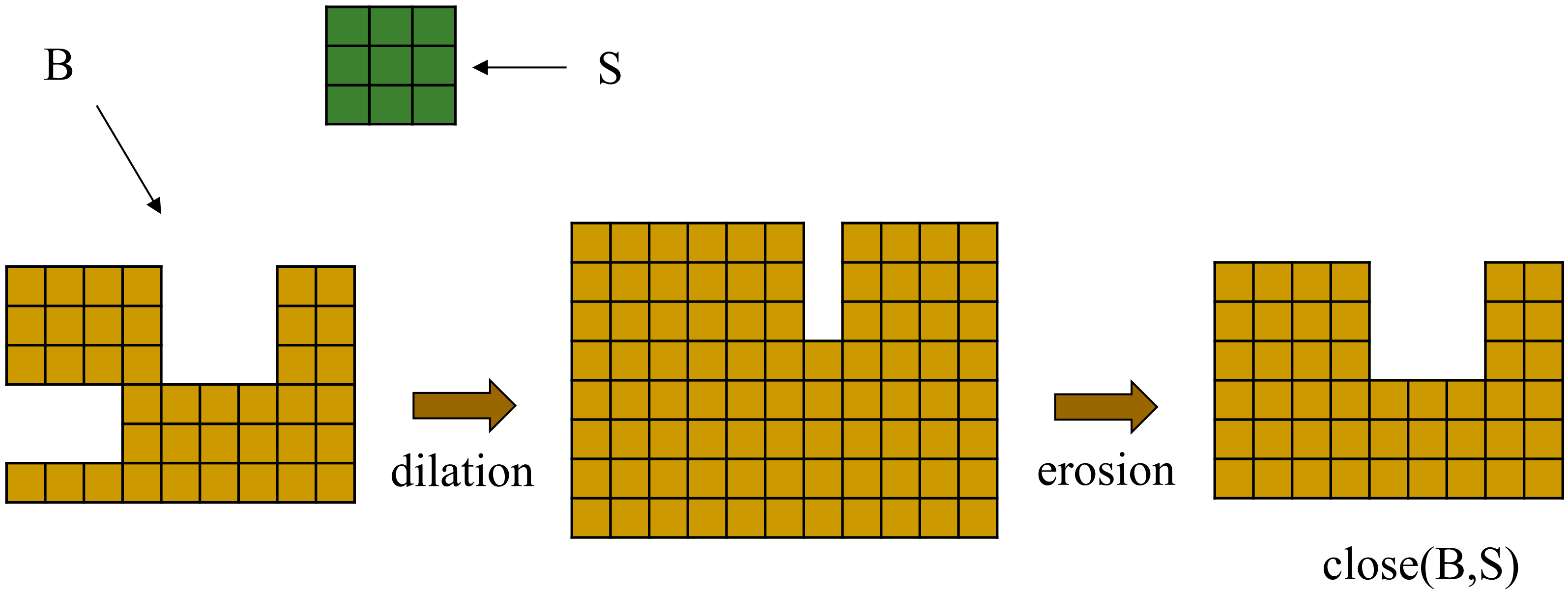
# Intuitive Interpretation

- Dilation expands an object
- Erosion contracts an object
- Opening
  - Smooths contours
  - Enlarges narrow gaps
  - Eliminates thin protrusions
- Closing
  - Fills narrow gaps, holes and small breaks

# Closing

- Closing: Like "smoothing from the outside"

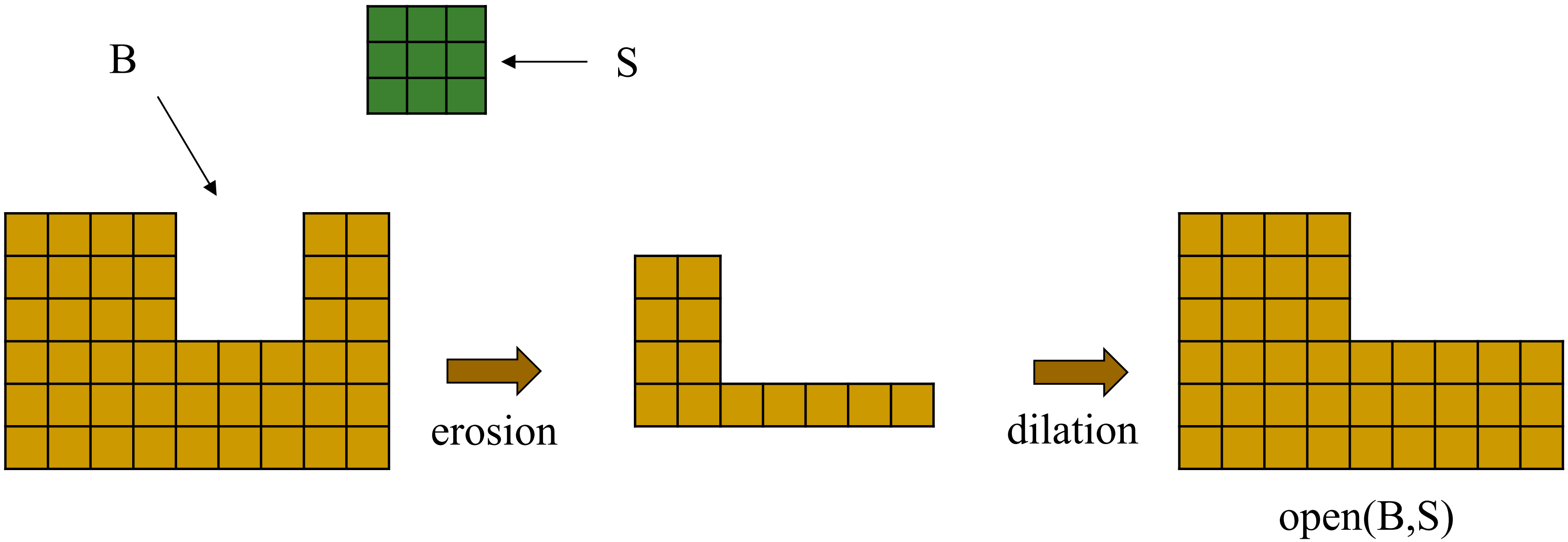
$$B \bullet S = (B \oplus S) \ominus S$$



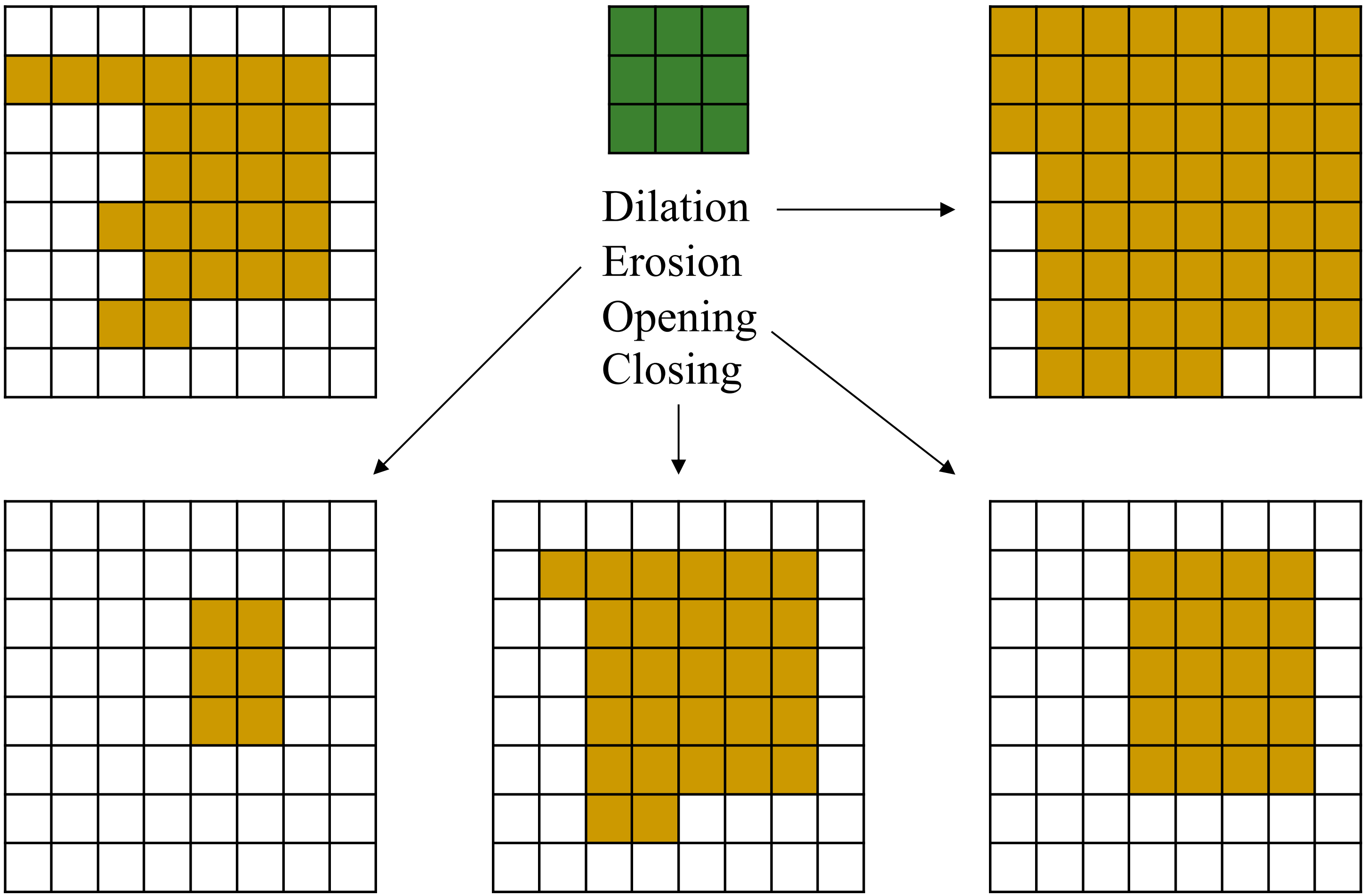
# Opening

- Opening: Like "smoothing from the inside"

$$B \circ S = (B \ominus S) \oplus S = \cup \{S_x \mid S_x \subseteq B\}$$



# More Example



# Idempotency

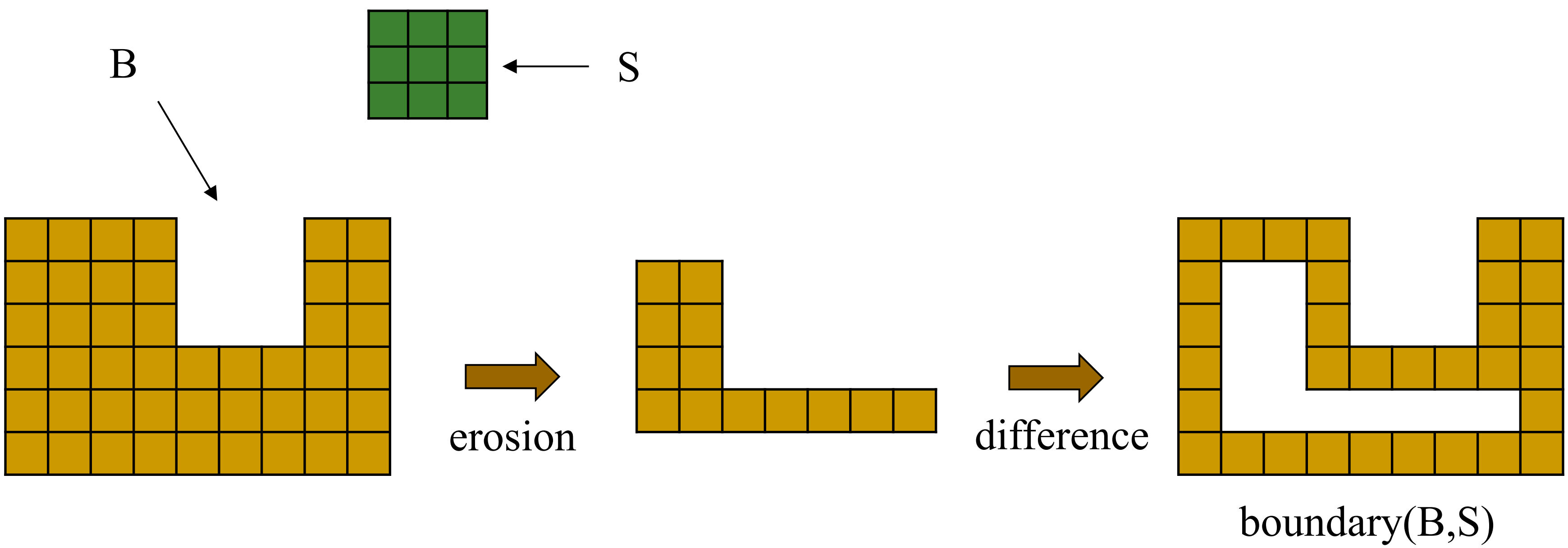
- Applying opening or closing more than once has no further effect
  - open(open(A,B),B) = open(A,B)
  - close(close(A,B),B) = close(A,B)

# Additional Structuring Operations

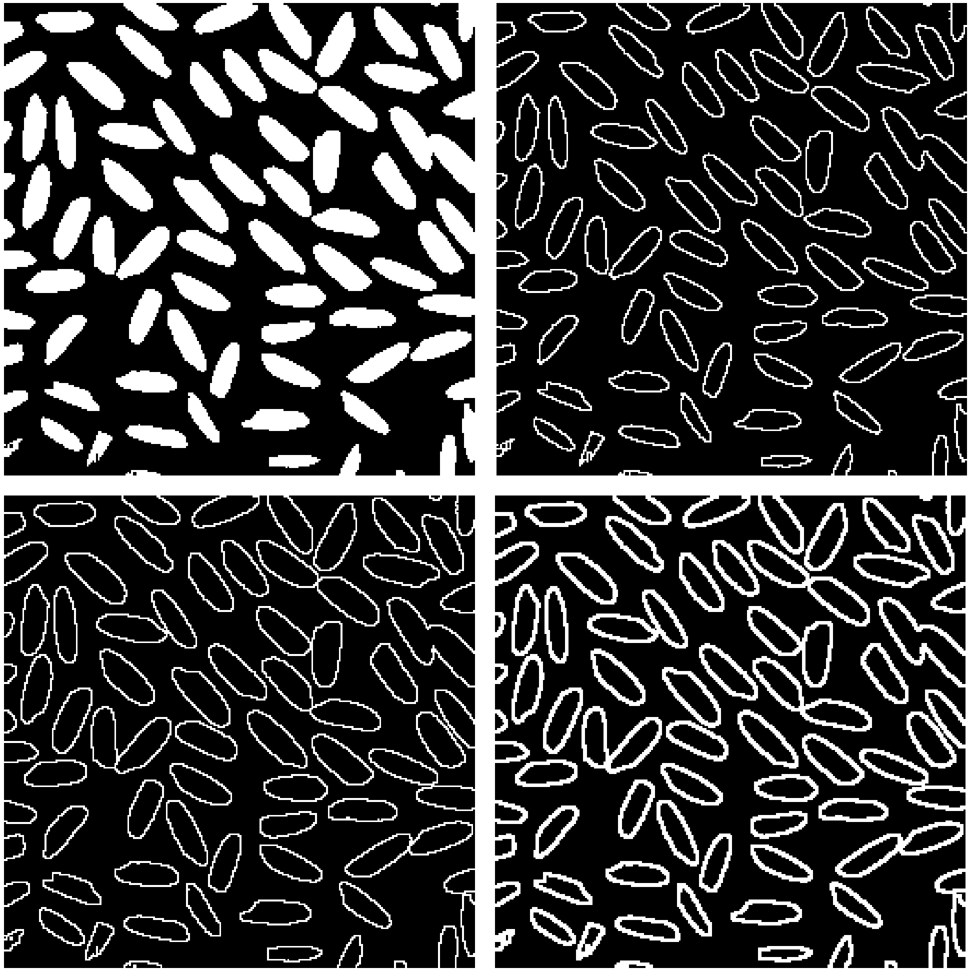
- Find boundary of an object
- Region filling
- Skeleton

# Find Boundary

$$\text{boundary}(B,S) = B - B \ominus S$$

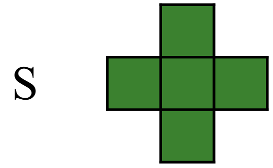


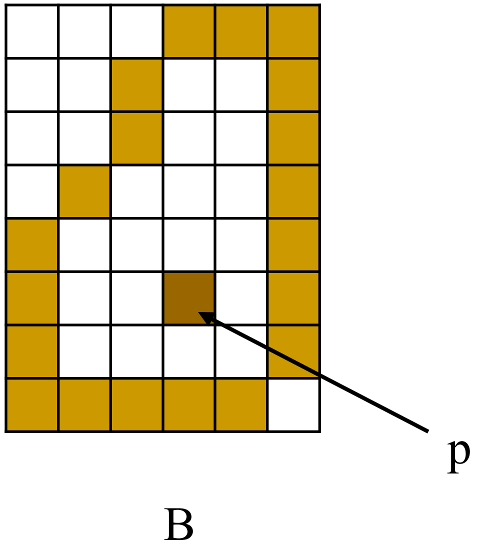
# Example



# Region Filling

- Problem:
  - Fill 8-connected boundary A with 1's given a point inside the boundary p




- Use structuring element S, and
  - Iterative dilations
  - Intersection
  - Complement

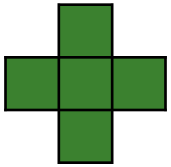
# Region Filling

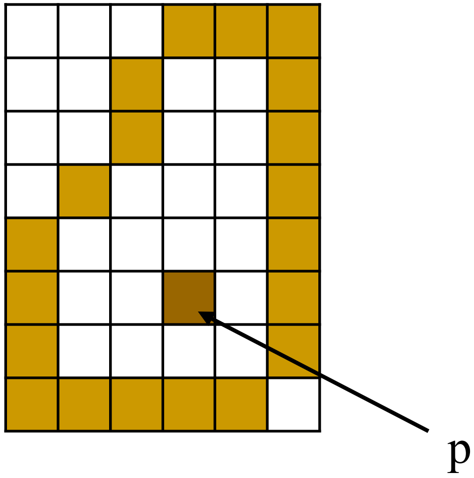
- Let $C_0 = p$
- Calculate

$$C_k = (C_{k-1} \oplus S) \cap B^c, \text{ for } k = 1,2, \ldots$$

- Stop when $C_k = C_{k-1}$
- $C_k$ is the interior of B

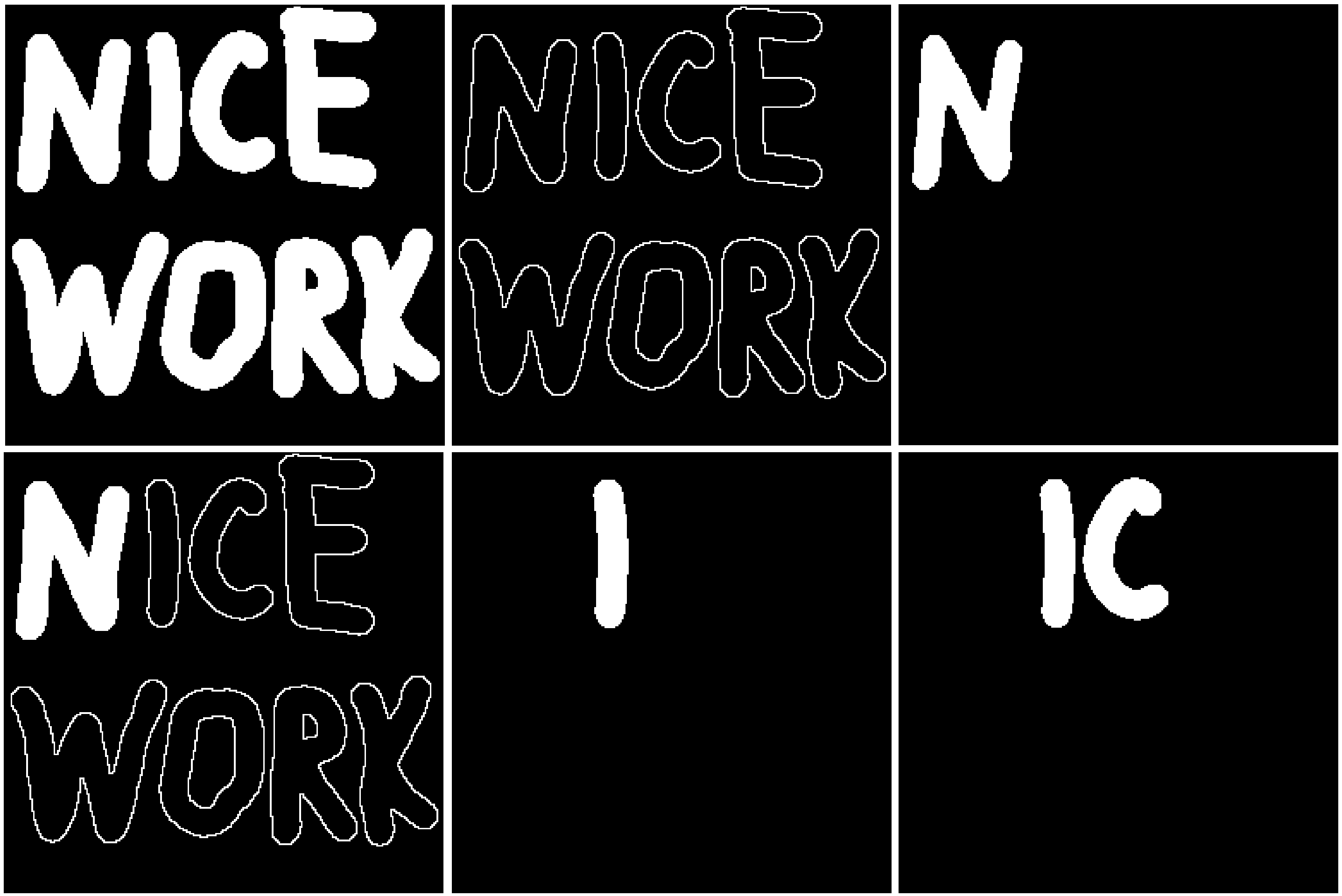
S






B

# Example



# Reading

- Chapter 2 of Jain's book