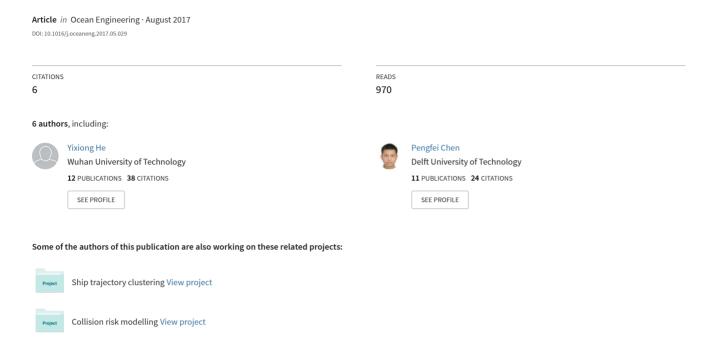
Quantitative analysis of COLREG rules and seamanship for autonomous collision avoidance at open sea



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Quantitative analysis of COLREG rules and seamanship for autonomous collision avoidance at open sea



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ABSTRACT

Ship collision avoidance is highly dependent upon seamanship and rules. When ship collision risk exists, proper collision avoidance actions must be taken according to the correct encounter situation and determined stage. All autonomous collision avoidance (ACA) operations in the future must comply with given rules and seamanship practices, which make the quantitative analysis of them prerequisites for ACA. This study presents a novel quantitative analysis system for the International Regulations for Preventing Collisions at Sea (COLREG) Rules and Seamanship. The proposed system consists of three parts: an encounter situation discrimination model based on the mutually relative bearing of the "target ship" (TS) and "own ship" (OS); a stage discrimination model representing the extent of collision risk per different domain models for every potential situation and a model to determine collision avoidance action per COLREG, seamanship, and ship maneuverability information was established accordingly. The collision avoidance plans appropriate for different situations and stages are generated based on the rules, seamanship, and rudder steering direction judgments. A simulation scenario was utilized to validate the effectiveness and feasibility of the system.

1. Introduction

Human error contributes to more than 80% of ship collision accidents, which pose a rather severe threat at sea. Research on ACA at open sea is important for mitigating the overall risk of collisions as well as easing the workload of ship officers. Intelligent navigation systems, integrated with ACA algorithms, have become invaluable tools on board. These systems are designed to support decision-making for navigators and isolate human errors during collision avoidance (Perera et al., 2011).

Ship officers perform anti-collision operations based on the Convention on the International Regulations for Preventing Collisions at Sea (COLREG) (U.S. Department of Homeland Security, U.S. Coast Guard Headquarters, 2010) rules and good seamanship. Naturally, ACA operations in the future must comply with them. Machines tasked with carrying out ACA, of course, cannot discriminate ship encounter situations or take proper anti-collision measures as human operators can. Therefore, quantitative analysis of rules and seamanship are the basic prerequisite for endowing machines with workable "knowledge" of COLREG.

COLREG categorizes encounters into three categories within which two ships are at risk of colliding with each other. The corresponding collision stages include risk of collision (CR), close-quarter situation (CS), and immediate danger (ID). COLREG only defines the basic concepts and principles for actions, however. In an effort to build more objective criteria for assessing encounter situations, Hilgert and Baldauf (1997) developed a common risk model with four risk levels based on COLREG. Montewka et al. (2011) presented another model that analyzes the risk of two common types of marine accidents. He et al. (2014) introduced a last steering point and collision risk index (CRI) model; Krata and Montewka (2015) proposed the concept of a "critical area" for a stand-on ship taking collision-evasive action when the give-way vessel does not fulfill her obligations.

Normally, collision avoidance (CA) actions are trigged after risk appears when two ships encounter each other and one ship enters the ship domain (SD) of the other one if they maintain their present speeds and courses. Therefore, SD is of major importance in regards to collision risk overall. SD was first presented in the early 1970s by Fujii and Tanaka (1971). Goodwin (1975) later established a

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Abbreviations: ACA, automatic collision avoidance; CA, collision avoidance; COLREG, International Regulations for Preventing Collisions at Sea 1972; CPA, closest point of approach; CR, risk of collision; CS, close-quarters situation; DCPA, distance to the closest point of approach; FTCS, first time-in-point of close-quarters situation; FTID, first time-in-point of immediate danger; ID, immediate danger; MMG, mathematical modeling group; OS, own ship; PCR, potential risk of collision; RPM, Revolutions Per Minute; SD, ship domain; TCPA, time to the closest point of approach; TCS, time to close-quarters situation; TID, time to immediate danger; TS, target ship

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Y. He et al. Ocean Engineering 140 (2017) 281–291

corresponding domain model in open sea. Over the past four decades, many researchers have presented various definitions of ship domains with different shapes and sizes, e.g., circle, elliptical, or polygonal (Weng et al., 2012; Pietrzykowski and Uriasz, 2009). Fuzzy SD has been developed as a technique based on historical statistics and navigational experience (Wang, 2010). Finding a universal SD which satisfies all situations remains a highly challenging endeavor. It is generally considered reasonable to apply different SD shapes to different situations.

The concepts and technologies available for ship ACA navigation have seen rapid progress alongside new advancements in artificial intelligence (Statheros et al., 2008). These concepts are closely linked with path planning and optimization operations. Kao et al. (2007), for example, proposed a fuzzy logic method that adds vessel CA capability to VTS/AIS systems for all potential collision ships. Perera et al. (2011) focused on a fuzzy-logic-based intelligent decision-making system for navigation safety and collision avoidance. There are major difficulties in adopting such approaches in regards to the adequate incorporation of COLREG and seamanship information, however. Unlike land-based navigation, the rules of ship collision avoidance operations are specific to each encounter due to the differences in situations and stages.

Real-world CA maneuvers are highly dependent on ship maneuverability. Curtis (1986) applied ship maneuverability in a study on very large crude carrier overtaking and heading-on TS. Montewka et al. (2010) introduced this idea into different encounter situations, and proposed a minimum distance to collision (MDTC) method to determine whether collision is avoidable. In another relevant study, Zhang et al. (2012) employed the Nomoto model to simulate anti-collision ship movement under COLREG. Ship maneuvers may differ considerably under various weather factors due to hull resistance and undesirable vessel motions (Perera et al., 2017). It is difficult to forecast accurate motorial conditions after the ship has been manipulated at sea.

There is no universally accepted solution to the problems described above. Many researchers have attempted to integrate COLREG rules and artificial intelligent methods by building mathematical models into ACA decision-making systems. One such algorithm for autonomous collision avoidance navigation of marine vehicles was proposed in 2004 which integrates both fuzzy logic and COLREG guidelines (Sang-Min Lee et al., 2004). Benjamin et al. (2006) proposed a behavior-based control framework with multi-objective optimization for representing the navigation rules, within which some COLREG rules were introduced. Perera et al. (2009) and Perera and Soares, (2015) presented an overview of existing autonomous guidance and navigation systems with respect to the collision avoidance, as well as fuzzy-logic-based decisionmaking processes in accordance with COLREG rules; the same research team later presented a collision detection methodology and collision risk assessment in an integrated bridge system. Kuwata et al. (2014) presented an autonomous motion planning algorithm for unmanned surface vehicles in which obstacles (i.e., collision risks) are determined through three basic encounter situations. Goerlandt et al. (2015) also presented a maritime riskinformed collision alert system framework.

Research is yet underway on other critical parts of ACA, especially in regards to applying complete COLREG rules and good seamanship to ACA decision-making systems with appropriate consideration of ship maneuverability. Such ACA techniques may be applicable in integrated bridge systems. Integrated bridge systems are the primary focus of e-navigation research (Perera et al., 2015).

For every TS involved in the ACA decision-making system, the following questions must be answered in order to avoid a collision:

- In what situation and stage are the TS at present?
- What ACA actions should be taken?

This study presents a novel quantitative analysis system for ship

ACA at open sea. This system, which includes a series of adaptive judging models, can be flexibly applied into modern navigation systems to provide practicable on-scene tactics.

This paper is structured into six main sections including the introduction. Section 2 describes the situation and stage discrimination model. Section 3 details the computing model of the first time-in-point of CR (FTCR)/first time-in-point of close-quarters situation (FTCS)/first time-in-point of immediate danger (FTID), which are utilized to discriminate the various stages of an encounter. Section 4 introduces actions at different stages and situations based on COLREG rules and seamanship. Section 5 describes the algorithm embedded in the model discussed in Section 3, and the numerical simulation of a multi-ship encounter scenario. Section 6 concludes the paper.

2. Situation and stage discrimination model

The models described below were developed under the following assumption: Ships are sailing at open and calm sea under good visibility, and the dynamic data (position, course, and speed) of TS and OS are known.

2.1. Situation discrimination model

This model is used to discriminate different encounter situations between the TS and OS. The following coordinate system (Fig. 1) is adopted (He et al., 2014).

(1) Coordinate system

Coordinate system XOY is fixed to the earth, while system xoy is fixed to the OS. The meaning and relationships of true course (TC), true bearing (TB), and relative bearing (Q) are shown in Fig. 1 and Eq. (1) respectively:

$$TB = Q + TC \tag{1}$$

The position and speed of the OS in XOY can be converted from the calculated values in xoy by Eq. (2):

$$\begin{cases} [X, Y] = [x, y] \times A + [X_0, Y_0] \\ [\dot{X}, \dot{Y}] = [\dot{x}, \dot{y}] \times A \end{cases}$$
 (2)

where $A = \begin{bmatrix} cos\ (TC) & -sin\ (TC) \\ sin\ (TC) & cos\ (TC) \end{bmatrix}$ and $[X,\ Y]$, $[x,\ y]$ are the coordinates of the OS in system XOY and xoy, respectively. $[X_0,\ Y_0]$ are the coordinates of xoy's origin in XOY.

(2) Situation discrimination model

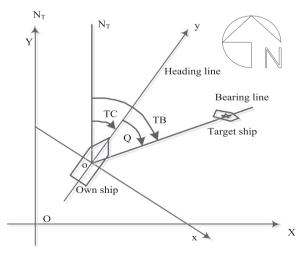


Fig. 1. Coordinate systems.

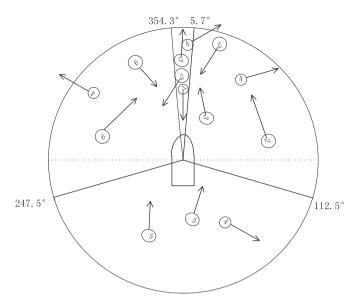


Fig. 2. Ships situations within one sector.

In previous situation discrimination models (Perera et al., 2011; Xu et al., 2014), situations were discriminated in terms of the value of the relative bearing of TS (Q) from OS, e.g., TSs with Q from 355°to 5°are in head-on situations. However, there are some discrepancies between these discrimination standards when strictly compared to COLREG. The relative bearing of OS from TS (Q_1) should be checked in addition to Q. As shown in Fig. 2, four TSs are within the interval of Q from 354.3°to 5.7°, but TS1 represents "head-on", TS2 "crossing", TS3 "overtaking", and TS4 another kind of TS in which the time to the closest point of approach (TCPA) is below 0, which means the TS has passed the closest point of approach (CPA).

Encounter situations with TCPA above zero can be divided to five categories according to the role of the OS in each situation, as described in Table 1.

Remarks:

- 1) PCR is "potential risk of collision", as introduced in Section 2.4.
- 2) Crossing and overtaking situations are explicitly defined in COLREG rules. Head-on situation is somewhat vaguely defined by Rule 14 as "...meeting on reciprocal or nearly reciprocal courses...", however. It is generally accepted that OS/TS should be within the half-compass point (11.25) of the TS/OS heading line on either side, so 5.7° is used in this study rather than 5°.
- 3) In an overtaking situation, the stand-on vessel should be before the abeam direction of the overtaking vessel according to Rule 13.

2.2. SD model for different situations

The SD model is utilized to determine if collision risk emerges. However, ship domain is subjected to the different situations described above. We use the following domain models to assess risk in different situations (Fig. 3).

The "phantom ship" concept, under which an imaginary ship is situated in the center of the SD, was first proposed by Davis et al. (1980). The shape and parameters of the three SDs depicted above are considered to be a combination of COLREG rules and good seamanship. These three kinds of SDs apply to different situations for the following reasons:

For ships in head-on situations, Rule 14 of COLREG states: "When two power-driven vessels are meeting on reciprocal or nearly reciprocal courses so as to involve risk of collision, each shall alter her course to starboard so that each shall pass on the port side of the other...". It is safer if the TS passes the portside rather than starboard side, because misunderstandings are less likely when ships pass portside to portside. A "dangerous head on situation" is created when ships pass "starboard side to starboard side" due to the susceptibility to misunderstandings, as most experienced navigators know.

For ships in a crossing situation, Rule 15 asks the vessel with the TS on her starboard side, if circumstances permit, to avoid crossing ahead of TS . In most cases, the OS should alter course to the starboard side and give more room starboard side than portside to perform collision-evasive action.

For ships in an overtaking situation, Rule 13 and 17 ask the overtaking ship to take CA actions while the overtaken ship should keep her initial course and speed at the beginning stage. The ship should thus keep more room ahead than astern.

The 19° angle was set by Davis et al. (1980); other SD parameters (Fig. 3) are only set for research purposes. In practice, they are determined by the ship captain who is fully aware of prevailing conditions.

2.3. Ship motion model

When no rudder is steered, the OS and TS sail at their present speed and course. Ship positions can be determined at any point in time by Formula (3) after the rudder is steered:

$$\begin{cases} \dot{TC} = r \\ [\dot{X}, \dot{Y}] = [\dot{x}, \dot{y}] \times A = [u, v] \times A \end{cases}$$
 (3)

where u and v denote the speed of the OS along the x-axis and y-axis, respectively. The meanings of other variables are same as those defined in Section 2.1.

Ship maneuverability should be taken into consideration when determining the stage CS and ID. Since pitch, roll, and heave during ACA are negligible, in this study, three dimensions of ship motion (surge, sway, and yaw) are modeled based on the mathematical modeling group (MMG) (He et al., 2014).

$$\begin{cases} (m' + m'_{y})\dot{v}' - (m' + m'_{x})u'r' = Y'_{H} + Y'_{P} + Y'_{R} \\ (m' + m'_{x})\dot{u}' - (m' + m'_{y})v'r' = X'_{H} + X'_{P} + X'_{R} \\ (I'_{ZZ} + i'_{ZZ})\dot{r}' = N'_{H} + N'_{R} \end{cases}$$
(4)

Table 1Limitations for different situations.

Encounter situation		Limitations	
First class: PCR exists and TCPA > 0	Head-on-①		$Q \in [0,5.7^{\circ}] \cup [354.3^{\circ},360^{\circ}), \ Q_{1} \in [0,5.7^{\circ}] \cup [354.3^{\circ},360^{\circ})$
	Overtaking	Given-way-③ Stand-on-⑤	$\begin{aligned} &Q_1 \in (112.5^{\circ}, 247.5^{\circ}), Q \in [0.90^{\circ}] \cup [270^{\circ}, 360^{\circ}) \\ &Q \in (112.5^{\circ}, 247.5^{\circ}), Q_1 \in [0.90^{\circ}] \cup [270^{\circ}, (360^{\circ}) \end{aligned}$
	Crossing	Given-way-@	$Q \in [000^{\circ}, 112.5^{\circ}]$ and there is no head-on or overtaking situation
		Stand-on-®	$Q \in [247.5^{\circ}, 360^{\circ})$ and there is no head-on or overtaking situation
Second class: No PCR or TCPA ≤ 0 (TS has p	passed the CPA)-@		

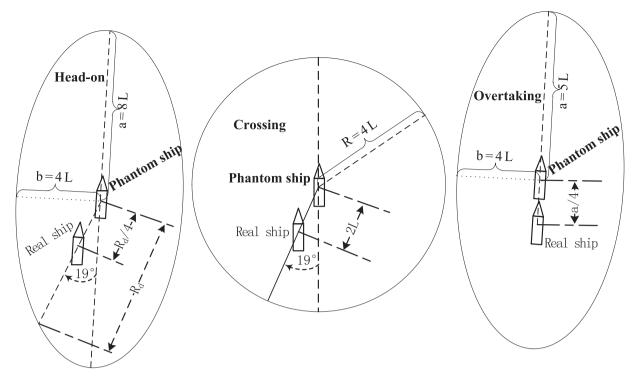


Fig. 3. SD for different situations.

Appendix A provides detailed definitions of the symbols and parameters used in this paper. A panama maximal size bulk carrier, the HUAYANG DREAM, was simulated. To verify the accuracy of the digital MMG model, the speed performance at different revolutions per minute (RPM) of propeller and turning circle were compared between the model and the real ship. Slight discrepancies were found, but the accuracy is acceptable overall (Appendix B).

2.4. Stage model

The stage model is defined in terms of the scale of collision risk. CR is frequently referred to in the COLREG rules. CS is applied in Rules 8 and 19, and ID is applied in Rules 2 and 10.

According to COLREG, in a dangerous encounter where two power-driven vessels are approaching each other and will collide if they maintain their speeds and courses, CR, CS, ID, and the final collision can be described consecutively (Zhong, 1999). The four stages (no CR, CR, CS, and ID) are shown in Fig. 4.

The stages of CR/CS/ID are introduced by different COLREG rules. It follows that the extent of danger gradually increases across the four stages, but COLREG does not directly define them. To determine the first time-in-point of CR/CS/ID, these stages must be clearly defined. This section proposes such definitions as-determined by COLREG, good seamanship, and ship maneuverability.

(1) Risk of collision

COLREG does not contain clear descriptions of CR. As Cockcroft and Lameijer (2011) points out, "...CR has not been held to apply at long distance... Nobody could seriously contend that if two ships are six miles apart the COLREG are applicable to them". For a vessel at least 50 m in length, the visibility of the masthead light and stern light are at

least six and three miles, respectively (Rule 22). To this effect, CR potentially applies to head-on or crossing situations only when the distance between vessels is equal to or less than six miles, and three miles for an overtaking situation according to Rule 13. In a crossing situation, however, the TS approaching speed may be very slow. It is necessary to develop another limitation which requires that TCPA or the time to CS (TCS) be below a critical value. Hilgert Baldauf (1997) advised that CR exists at open sea only when $18 \, \text{min} > TCPA > 6 \, \text{min}$. This value is dependent on the ship's maneuverability, and the best way to define it is based on the ship captain's assessment. $TCS \leq 20 \, \text{min}$ is the requirement set in this study.

The meaning of PCR is based on the definition of SD. If both ships maintain their present speed and course, PCR exists when TS enters the SD of the OS regardless of present distance. At long range, PCR exists while CR appears when the distance reaches the threshold mentioned above. In other words, CR exists only when PCR exists.

All limitations for the existence of CR are listed in Table 2.

The first time-in-point of CR (FTCR) is the first time-in-point when a limitation (Table 2) is satisfied.

(2) CS and ID

Although no clear descriptions of CS and ID exist in COLREG, many researchers have presented their interpretations of these terms (Hilgert and Baldauf, 1997; Zhong, 1999). The following definitions are generally accepted.

CS: A situation in which safe passing distance cannot be ensured if only one ship fully maneuvers.

ID: A situation in which collision cannot be avoided if only one ship fully maneuvers.

There are two main methods of avoiding collision: Changing speed or changing course. Vessels sailing in open sea operate at sea speed and

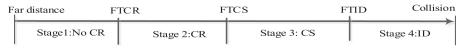


Fig. 4. Different stages of ship encounter.

Table 2 CR limitations.

Situations	Limitations
Head-on	PCR exists, $TCPA > 0$ and $D \le 6$ miles
Crossing	PCR exists, $TCPA > 0$, $TCS \le 20 \text{ min}$, $D \le 6 \text{ miles}$
Overtaking	PCR exists, $TCPA > 0$, $D \le 3$ miles

most modern ships are fitted with fixed pitch propellers. The engines are protected by special programs which restrain any sudden changes in engine rotation speed, so it is difficult (or even impossible) to rapidly reduce it. The large inertia of the ship also means that reducing propeller revolutions does not necessarily mean a useful reduction in speed. A few ships (often are smaller ships) are equipped with controllable pitch propellers that have screw pitch can be adjusted to change speed. These operations are typically carried out at port and channel, and are not the best choice during CA operation for big ships running at full speed in open sea; it is less convenient than steering the rudder. In short, CA via speed change is not a suitable option for ACA at open sea, so only course alterations are considered in this study. "Full ship maneuvers" can be restricted by steering the rudder to hard a-starboard or hard a-port.

3. Mathematic model of FTCS/FTID

Feasible CA routes are calculated and carried out in practice by the navigator. In ACA, this must be performed by a computer. Machines can judge the stage and generate ACA actions intelligently only when all situations and stages of encounter process are identifiable. In order to separate the entire encounter process into stages, the first time—inpoint of CR, CS, and ID should then be strictly defined, analyzed, and computed in a quantifiable manner. FTCS and FTID are defined as follows.

3.1. FTCS

The SD is the area where another ship cannot safely enter; TSs can maintain safe distance if passing outside the OS's SD. FTCS, also called "last steering point", must satisfy the following limitations: For two approaching ships (TS and OS), if the rudder is steered to hard a-starboard or hard a-port by the OS at the time-in-point of FTCS, the TS will touch the border of the OS's SD but not enter it.

To calculate FTCS, the physical process of the problem must be addressed. For two approaching ships (TS and OS), the encounter process with OS maintaining course and speed is shown in Fig. 5(1). The encounter process of OS steered hard a-starboard at FTCS (t_m^0) is shown in Fig. 5(2). Fig. 5(3) and 5(4) show the encounter process of an OS steered hard a-starboard before and after this time.

 t_m is the time when the rudder is steered. t_m^0 is the time-in-point defined as FTCS. The TS touches the domain border of the OS but does not enter it when the OS is steered hard a-starboard (a-port) at that time (Fig. 5(2)). FTCS is calculated through the following process.

OS and TS are approaching at speeds (v_0/v_I) and courses (TC_0/TC_I) before t_m , but the OS's rudder is steered to hard a-starboard (or aport) at t_m . t denotes the length of time that has passed. The denotations of Dis, D, and R_T are shown in Fig. 6.

If the rudder was not steered, the value of Dis is solely decided by t. If the OS rudder was steered, the value of Dis is determined by t and t_m . Thus, Dis is a dual function with two independent variables: t and t_m , as shown in Eq. (5):

$$Dis = D - R_T = f(t, t_m)$$
(5)

The physical meaning of Eq. (5) is that the value of *Dis* is decided by the time-in-point when the rudder was steered and the length of time passed since it was steered.

Dis varies across different time points after the rudder is steered. The minimum value of Dis is positive if the TS would eventually pass outside the SD of the OS. It is negative if the TS eventually enters the SD of the OS, and is zero if the TS touches the boundary of the OS's SD without entering it. t_m (the time point rudder steered) is the key to decide the minimum value of Dis. When rudder is steered at t_m^0 (FTCS), the minimum value of Dis will be zero.

The process to solve FTCS (t_m^0) is to find t_m^0 which satisfies Eq. (6):

$$\min_{t_m = t_m^0} (Dis) = \min(f(t, t_m^0)) = 0$$
(6)

The physical meaning of Eq. (6) is to find a time-in-point (t_m^0) where the TS will touch but not enter the OS SD if the OS rudder angle is steered to hard a-starboard (a-port) at t_m^0 . If the rudder is steered at a random time point, one of three processes will occur as shown in Fig. 5(2)- Fig. 5 (4).

The later the rudder is steered, the smaller the minimum distance from the TS to the SD of the OS boundary. It is a positive value (Fig. 5(4)) if the rudder is steered at any time point before t_m^0 , but negative after t_m^0 (Fig. 5(3)). $\exists t_1 > 0, t_2 > 0, t_m \in [t_m^0 - t_1, t_m^0 + t_2],$ min (f(t,t_m)) descends monotonously near t_m^0 . In addition, there is only one $t_m^0 \in [0, TCPA]$ that satisfies Eq. (6).

This problem can be solved by bisection method. Two time-inpoints when the rudder is steered to hard a-starboard (a-port), must be identified and included in the range of 0 to TCPA. One ensures the TS passes outside the SD of the OS, and the other one ensures the TS enters the OS's SD.

If the OS steers to hard a-starboard (a-port) immediately when the TS is far away and PCR exists, the TS will pass outside the SD of the OS (Fig. 5(4)), so $\min(f(t, 0)) > 0$. On the contrary, if the OS steers to hard a-starboard (a-port) at the clearest point of approach (CPA), the TS is already inside the OS's SD due to the existence of PCR, so $\min(f(t, TCPA)) < 0$.

The value of Dis should be calculated at different time-in-points and situations. The function of $Dis = f(t,t_m)$ for a crossing situation is:

$$\begin{cases} Dis^{t} = \left[(X_{0}^{t} - X_{R}^{t})^{2} + (Y_{0}^{t} - Y_{R}^{t})^{2} \right]^{0.5} - R \\ X_{0}^{t} = x_{0}^{t} + 0.25R \sin(TC_{0}^{t} + 19) \\ Y_{0}^{t} = y_{0}^{t} + 0.25R\sin(TC_{0}^{t} + 19) \end{cases}$$

$$(7-1)$$

For a head-on situation:

$$\begin{cases} Dis^{t} = [(X_{0}^{t} - X_{R}^{t})^{2} + (Y_{0}^{t} - Y_{R}^{t})^{2}]^{0.5} - R_{T}^{t} \\ X_{0}^{t} = x_{0}^{t} + 0.25R_{d}sin (TC_{0}^{t} + 19) \\ Y_{0}^{t} = y_{0}^{t} + 0.25R_{d}cos (TC_{0}^{t} + 19) \end{cases}$$

$$(7-2)$$

And for an overtaking situation:

$$\begin{cases} Dis^{t} = [(X_{0}^{t} - X_{R}^{t})^{2} + (Y_{0}^{t} - Y_{R}^{t})^{2}]^{0.5} - R_{T}^{t} \\ X_{0}^{t} = x_{0}^{t} + 0.25R_{d} \sin(TC_{0}^{t}) \\ Y_{0}^{t} = y_{0}^{t} + 0.25R_{d} \cos(TC_{0}^{t}) \end{cases}$$

$$(7-3)$$

Where (x_0^t, y_0^t) is the position of the OS at time t, which is calculated through Eqs. (9)–(12). The angle 19° in the above equations is the angle between the headline of the real ship and the line connecting the real ship to the phantom ship (Fig. 3).

 R_I^t is the distance from the domain center to the boundary along the direction of the TS at time t. It equals the SD radius in the crossing situation. For other situations, it is defined by the parameters of an elliptical SD and Q^t ; Q^t is the relative bearing of the TS from the domain center at time t (Fig. 3).

a and b denote the lengths of the long and short semi-axes of an elliptical SD. R_d , as shown in Fig. 3, indicates the distance of the phantom ship to the SD boundary of the OS along the direction of the

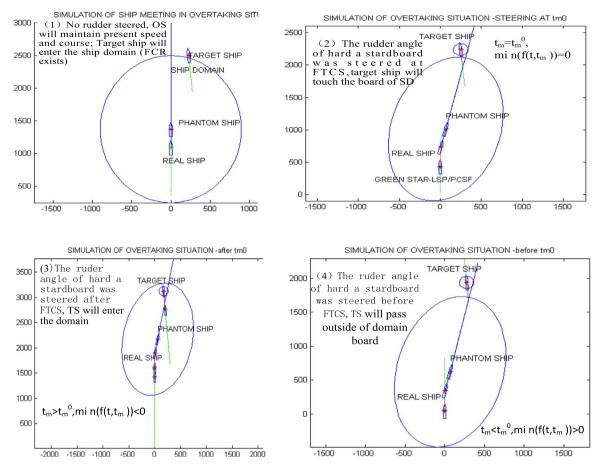


Fig. 5. Rudder is steered at different times (overtaking).

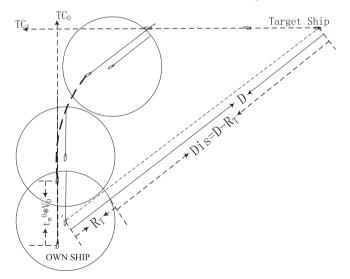


Fig. 6. Dis (crossing situation).

real ship, (x_0, y_0) , (X_0, Y_0) , (X_R, Y_R) denote the position of the real ship, phantom ship, and TS, respectively. Superscript t indicates the values of variables are determined at time t. For example, Dis' denotes the value of Dis at time t. U and V denote the speed of the OS along the X-axis and Y-axis, respectively.

Eq. (8) is obtained as shown in Figs. 3 and 6 after simple mathematical calculation:

$$R_T^t = ab/((a\sin(Q^t))^2 + (b*\cos(Q^t))^2)^{0.5}$$
(8)

The position of the OS at time t can be obtained as $t \le t_m$, where the motion of the OS is uniform in a straight line because no rudder is

steered:

$$(x_0^t, y_0^t) = (x_0^0, y_0^0) + (\sin(TC_0), \cos(TC_0))v_0t$$
(9)

 $t > t_m, \Delta t$ denotes the calculating time step, $k \geq 0$ and $k \in N$:

$$t - t_m = k * \Delta t \tag{10}$$

If the rudder is steered to hard a-starboard (a-port) at t_m , the position/course of the OS at t can be ascertained according to the previous position/course and speed/rotation speed of the heading angle as Eq. (11) indicates:

$$(x_0^t, y_0^t, TC_0^t) = (x_0^{t_m + k * \Delta t}, y_0^{t_m + k * \Delta t}, TC_0^{t_m + k * \Delta t}) = (x_0^{(k)}, y_0^{(k)}, TC_0^{(k)})$$

$$= (x_0^{(k-1)}, y_0^{(k-1)}, TC_0^{(k-1)}) + (U^{(k-1)}, V^{(k-1)}, r^{(k-1)}) \times \Delta t$$

$$(11)$$

According to Eq. (3):

$$(U^{(k-1)}, V^{(k-1)}) = (u^{(k-1)}, v^{(k-1)}) \times A$$
(12)

The MMG model is used to simulate the motion of the OS after the rudder is steered. Therefore, $(u^{k-1}, v^{k-1}, r^{k-1})$ can be solved using Eq. (4) based on the Runge-Kutta method when the initial motion conditions are known. Initial motion conditions are determined when the rudder of the OS is steered to hard a-starboard (a-port) $(t = t_m)$. In practice, the hard a-port (a-starboard) angle is 35°. Other initial values for computing $(u^{k-1}, v^{k-1}, r^{k-1})$ include the longitudinal speed, lateral speed, and rotational angle speed of the OS. The position and TC of the OS at t_m can be expressed as follows: $(u^{(0)}, v^{(0)}, r^{(0)}) = (0, v_0, 0), (x_0^{(0)}, y_0^{(0)}, TC_0^{(0)}) = (x_0^{t_m}, y_0^{t_m}, TC_0)$.

Details of the computing procedure are provided in Section 5.2. The TCS is the length of time to t_m^0 :

$$TCS = t_m^0 - t^{(0)} = t_m^0 (13)$$

3.2. FTID

Provided that collision between two ships occurs when the distance between their centers of gravity are less than half the sum of the two ships' lengths, FTID satisfies following limitations:

- The minimum distance between centers of gravity is equal to half the sum of the two ships' lengths when the OS is steered hard astarboard (a-port) at FTID.
- 2) The FTID model is similar to the FTCS model. A circular SD should be used to substitute the elliptical/circular SD described in Section 2.2, the radius of which is half of the sum of two ships' lengths with its center located at the OS weight center.

4. ACA actions at different stages and situations

If there is no risk (lack of PCR), there is no anti-collision action required. Otherwise, action is necessary per the situation and stage at hand.

4.1. OS in a head-on situation or given-way ship in a crossing/overtaking situation

When an OS is in a head-on situation or a given-way ship is in a crossing/overtaking situation, ACA actions include the following.

- Stage 1: At long range, before CR exists, the OS is free to take any available action.
- Stage 2: When CR first appears, the OS is required to take early and substantial action to obtain a safe passing distance according to COLREG Part B.
- Stage 3: After FTCS, CS exists and ID begins to develop. The OS must alter course to avoid ID or collision, and depart from COLREG according to Rule 2 if necessary.

In the first and third stages, because course altering is the only available anti-collision action, it is very important to determine the rudder steering direction appropriately. The model for choosing the rudder direction is presented in Section 4.3. "Close-quarter" and "immediate danger" stages are included in the third stage for these ships in these situations.

4.2. OS as stand-on ship

For stand-on vessels, according to Hilgert and Baldauf (1997), "... there may be four stages relating to the permitted or required action for each vessel" under Rule 17.

- Stage 1: At long range, before CR exists, the OS is free to take any available action.
- Stage 2: When CR first appears, the OS must maintain her course and speed.
- Stage 3: When the given-way vessel is explicitly not taking appropriate action in compliance with known regulations, the OS is permitted to take any available action to avoid collision.
- Stage 4: When collision cannot be avoided by the given-way vessel alone, the OS must take whatever action necessary to avoid collision.

Obviously, the first time-in-points in Stages 1, 2, and 4 are same as the first time-in-points of the stages described in Section 2.4. The first time-in-point of Stage 3 is different from FTCS, however – it is earlier. Due to the various interpretations of the third stage by different captains, it is difficult to determine the exact time at which the stage applies. This time-in-point can be set in a computer program by the captain on-scene in the future via ACA. In this study, this time-in-point is set at the midpoint of FTCS and FTID.

4.3. Appropriate rudder steering direction

When the OS is free to or should take CA measures, speed changing is not optional (Section 2.4) so the rudder steering direction should be selected to alter the course. In coordinate system xoy, when the TS is approaching from the first quadrant ($Q \in (0, \pi/2)$), the most appropriate rudder direction can be determined easily by comparing the following angles.

- 1) TB00: The true bearing of the OS observed from the TS, which is the angle from TS N_T to the bearing line of the OS (Fig. 7).
- 2) RVC: The relative velocity course of the TS, which is the angle from TS N_T to the relative velocity line of the TS (Fig. 7).
- 3) C0: The course of the OS.

Of course, no PCR exists if the RVC deviates from the OS heading. RVC should point toward the head of the OS when PCR exists. When TS approaches from the first quadrant, five PCR conditions may exist and are separated by RVC (Fig. 7):

Condition 1: RVC \in (C0 -0.5π ,C0), the TS will pass the bow of the OS if the rudder is not steered. Altering course to port side will increase the distance to the closest point of approach (DCPA), but DCPA will decrease if course is altered to the starboard side (Fig. 8). In other words, altering course to port side is more efficient when the TS comes from the first quadrant and has RVC belonging to Condition 1.

Condition 2: RVC = $C0 - 0.5\pi$, in which altering course is equally efficient toward either port or starboard side.

Condition 3: RVC \in (TB00, $C0 - 0.5\pi$), in which altering course toward starboard side is more efficient.

Condition 4: RVC = TB00, in which the TS will collide with the OS if the rudder is not steered. This is equal to altering course toward port or starboard side.

Condition 5: RVC<TB00, in which altering course to port side is more efficient.

Fig. 9 shows the more appropriate direction of altered OS courses when the TS approaches from the first quadrant.

The above discussion focuses on the TS in one quadrant, but situations in which the TS originates in the other three quadrants of coordinate system xoy can be interpreted similarly. The results for TSs originating from all quadrants are shown in Table 3.

Remark: If the angle in the table exceeds 2π , it should be subtracted by 2π . For example: $1.5\pi + C0$ equals $C0 - 0.5\pi$ when it is more than 2π .

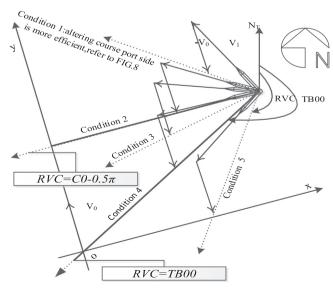


Fig. 7. Five RVC conditions of TS in first quadrant.

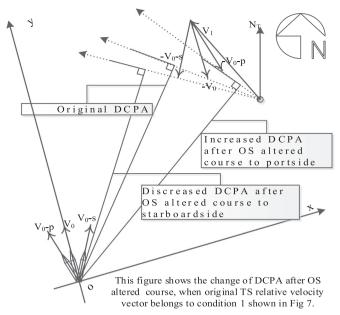


Fig. 8. Comparison of DCPAs of condition 1 when OS altered course.

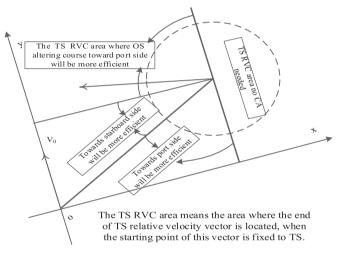


Fig. 9. Most efficient course altering direction when TS comes from the first quadrant.

4.4. ACA action choices

All the potential ACA-related actions under COLREG, seamanship, and the other factors described above are listed in Table 4, where "time to immediate danger" is denoted as TID.

5. Algorithm design and simulation

In order to separate the stages as necessary, an algorithm for FTCS and FTID models was built as discussed below. A scenario including one OS and seven TSs was simulated to validate the feasibility and effectiveness of the model.

5.1. Prototype OS ship

A fully loaded Panama maximum bulk carrier on which the author once served as Captain was simulated for the purposes of this study. The ship is described in detail in Appendix B. The Runge-Kutta method was designed and implemented in MATLAB*.

5.2. Algorithm for FTCS and FTID

A bisection method was applied to compute FTCS and FTID with the initial conditions introduced in Section 3:

- 1) As described in Section 3.1, two necessary initial values of time-inpoint rudder steering should be chosen: At present $(t_m^{-1} = 0)$ and at CPA $(t_m^{-2} = \text{TCPA})$. $t_m^1 = 0$ ensures that the TS will pass outside the OS's SD, implying $\min_{t_m = t_m^1}(Dis) = \min(f(t,t_m^1)) > 0$. $t_m^2 = TCPA$ ensures that the TS will enter the OS's SD, meaning $\min_{t_m = t_m^2}(Dis) = \min(f(t,t_m^2)) < 0$. The initial t_m^0 equals $(t_m^1 + t_m^2)/2$.
- 2) If any $Dis^{(k)} \le 0$, the TS has entered the OS's SD. In this case, t_m^0 is substituted with t_m^2 , and computing returns to the first step.
- 3) If all Dis values are larger than zero, the TS will not enter the OS's SD. At this point, the minimum value of Dis must be checked. If it is larger than a pre-defined positive value, t_m⁰ is substituted with t_m¹ and computing returns to the first step. Otherwise, computing ends and t_m⁰ is the value to be solved (FTCS or FTID).

Fig. 10 depicts the above procedure in flow chart.

5.3. Simulations

Simulations were run to verify the mathematical models and algorithms we propose and to answer the two questions posed in the

Table 3Most efficient direction of rudder to be steered at all quadrants.

Quadrants $(Q \in)$	Towards port side (RVC)	To starboard side (RVC)	Port equal to starboard (RVC =)
$[0, 0.5\pi)$	$>1.5\pi + C0$ or $< TB00$	\in (TB00, 1.5 π + C0)	TB00, $1.5 * \pi + CO(Q \in [0, \pi))$ or $0.5 * \pi + CO(Q \in [\pi, 2\pi))$
$[1.5\pi, 2\pi)$	$\in (0.5\pi + C0, TB00)$	$>TB00 \text{ or } < 0.5\pi + C0$	
$[\pi, 1.5\pi)$	\in (TB00, 0.5 π + C0)	$>0.5\pi + C0 \text{ or } < TB00$	
$[0.5\pi, \pi)$	$>TB00 \text{ or } < 1.5\pi + C0$	\in (1. 5 π + C0, TB00)	

Table 4 ACA actions against TS.

Situation			Stage1	Stage2	Stage 3 TID \geq 0 Stage 4 TID $<$ 0
PCR exists and TCPA > 0	Head-on Crossing situation Overtaking situation	Given-way Stand-on Stand-on	Free for any CA actions, more efficient direction of altering course should be chosen	Starboard (Rule 14) Altering course to starboard side, as Rules 8, 15, 16 No CA action needed before midpoint of time range (Rules 2, 15, and 17). Altering course to starboard side for crossing and more efficient actions for overtaking (Rules 2 and 17) after midpoint	Emergency situation is formed; more efficient direction of course should be chosen according to Rule 2.
		No CA actions needed	More efficient actions		

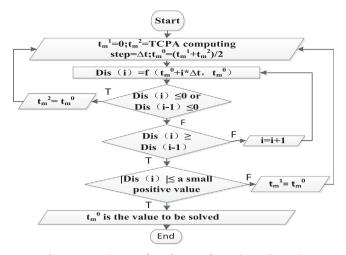


Fig. 10. Computing procedure of FTCS and FTID (He et al., 2014).

Table 5 Simulation results.

Ship		Coordinates	Course (°)	Speed	L	Result		
		(X, Y) (N/ M)		(m/s)		Direction of rudder should be steered by OS	TCS (seconds)	Stage
os		(0,0)	000	6	225			
TS	1	(-0.1,7)	180	6	180	port	952	1
	2	(2,2)	270	6.5	180	starboard	289	2
	3	(0.5, -0.5)	350	7	180	starboard	176	2
	4	(-2,2)	090	6	180	starboard	405	2
	5	(0.3,1.1)	000	5	180	port	1042	2
	6	(0.4,3)	182	6	180	starboard	291	2
	7	(-0.3,4)	178	6	180	starboard	463	2

Introduction. The results are presented in Table 5.

Table 5 shows the coordinates, courses, speeds, and lengths of seven TSs sailing around one OS. To avoid these different TSs, the ACA decision-making system must analyze them individually. The results of every encounter situation with each TS are listed in the third column, including the optimal rudder steering direction, time to close-quarter

situation, and stage of every encounter.

The initial ACA action is normally taken by giving the most dangerous TS top priority; preventing collision with the most dangerous TS is the basic premise of any ACA plan. A new collision risk index system was developed recently based on comparison between TCS and the time from FTCR to FTCS (He et al., 2014). The most dangerous TS can be distinguished by this system, and the course-altering direction of initial ACA actions can be determined per the results of Table 5.

6. Conclusions

A novel quantitative analysis framework was presented in this paper for ship ACA at sea. The contributions of this paper can be summarized as follows:

- We presented a situation identifying model and defined four distinct stages in the entire encounter process for ACA, where appropriate CA actions must be taken according to different situations and stages.
- 2) We built quantitative computing models for FTCS and FTID.
- We established a model for determining the most efficient (altered) course direction for the OS.
- We generalized initial ACA actions in different stages and situations.

We focused on quantitative analysis work regarding COLREG rules and seamanship in open sea under good weather. Integrating COLREG and seamanship into a computer system is a common problem in ACA studies; the results presented here can be utilized as a foundation for follow-up research on ACA systems.

Final ACA plans can be started from the initial ACA actions defined in this study and designed to effectively eliminate collision risk with any given TS. Further studies on path planning are yet necessary to accomplish this.

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Appendix A. List of variables

See Table A1.

Table A1 Variables.

Variable	Definition
TC	True Course of OS, angle from True North (N_T) to OS heading line
TB, TB00	True Bearing of TS/OS, angle from True North to bearing line of OS/TS
$Q_{i}Q_{1}$	Relative bearing of TS/OS, angle from OS/TS heading line to bearing line of TS/OS
[X, Y], [x, y]	Coordinates in coordinate system XOY/xoy
$[X_0, Y_0]$	Coordinates of xoy origin in coordinate system XOY
m', m'_x, m'_y	OS mass, added mass along x-axis, added mass along y-axis, respectively
$I_{zz}^{\prime},i_{zz}^{\prime}$	OS moment of inertia, added moment of inertia according to z-axis
$X'_{H}, X'_{P}, X'_{R}, Y'_{H}, Y'_{P}, Y'_{R}$	External force along x-axis, y-axis of ship's hull, propeller, and rudder, respectively
$N_H^\prime,N_R^\prime,r^\prime$	Yaw moment around z-axis of hull and rudder, yaw (continued on next page)

Table A1 (continued)

Variable	Definition	
$(x_f, y_f), (b_1 b_2)$	angular velocity, respectively Position of ship at time <i>t,</i> zero, respectively	
u, v t_m, t_m^0	OS speed along x axis and y axis, respectively. Time when rudder is steered, FTCS respectively.	
(v_0, TC_0) (v_1, TC_1) Dis, D, R_T	Speed, true course of OS, TS, respectively Distance from TS to the domain's board of OS along the direction of TS to "phantom ship", TS to "phantom ship", domain center to the boundary along the direction of TS, respectively	
Superscript t	Variables at present moment since time t	
Subscript 0, R	Variables about OS/TS	
R, a, b	Radius of circular SD, length of long and short axis of elliptical SD, respectively	
R_d	Distance of "phantom ship" to the boundary of SD along the direction of the real ship	
$(x_0, y_0), (X_0, Y_0), (X_R, Y_R)$	Position of OS, "phantom ship", TS	
u_{dT}, u_{tT}, u_{T}	SCRI, TCRI, CRI values	
t_0	Time since which CR first applies to FTCS	

Appendix B. Simulated vessel and experiment results

The simulated ship is the M/V HUAYANG DREAM, which is a Panama maximum size bulk carrier. Table B1 shows information relevant to the digital model.

One author served as Captain of this ship from January 15, 2014, to July 28, 2014.

A comparison of the turning cycles is shown in Figs. A1 and A2.

A comparison of speed performance between the digital model and real ship is shown in Fig. A3.

There were slight differences between the real ship and MMG model which could be adjusted via system identification; in this work, however, MMG was only used to simulate the turning process after the ship's rudder is steered, so the model's accuracy is acceptable.

Table B1 ship's particular.

Name	Huayang dream	Displacement	90,000*10 ³ (kg)
Draft	14.5 (m)	Breadth	32.5 (m)
LOA	225 (m)	Density of water	$1000 \text{ (kg/m}^3\text{)}$
$C_{\rm b}$	0.8715	RPM	90 (r/min)
Acreage of rudder	56.88 (m ²)	Propeller advance	4.738 (m)

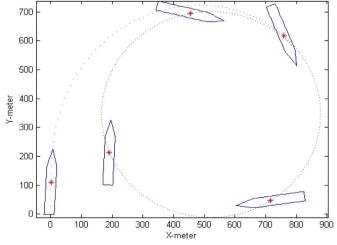


Fig. A1. Digital MMG model turning cycles.

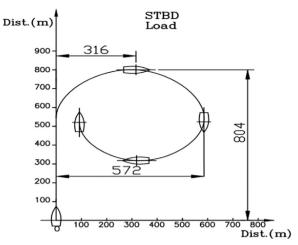


Fig. A2. Real ship model turning cycle.

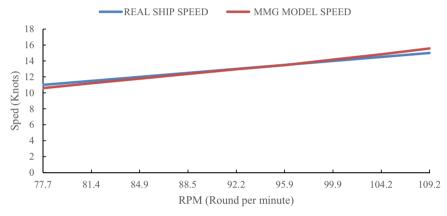


Fig. A3. Comparison of speed performance of MMG model and real ship.

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