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A novel model for the cumulative plastic strain of soft marine clay under long-term low cyclic loads

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ABSTRACT

Long-term cyclic loads with a stress level lower than the critical cyclic stress applied on soft soil can lead to soil deformation but not to failure. The Monismith model is known for its simplicity and capacity to describe the cumulative plastic strain of soil under cyclic loads. However, it is unsuccessfully applied in these cases because the plastic strain in this model would increase endlessly till to failure with increasing number of load cycles. To solve this problem, a novel empirical model with three parameters is proposed based on analogy analysis of the Hardin-Drnevich model and Monismith model. The proposed model is verified by experimental data from existing literature and is shown to have better capability and performance than the Monismith model in predicting the cumulative plastic strain of soft soil subjected to long-term low cyclic loads. The value of parameter b is recommended to be 0.5, and relationships of parameters a and c with the cyclic stress ratio are also proposed for soft clay. Applications of the proposed model are elaborated in detail, and in situ test results for settlement of soft subgrade are used to evaluate the performance of this model. The prediction results are consistent with the test results. The research results present a promising method for investigating the development of deformation and the settlement of soft foundation in the near-shore and off-shore areas caused by wave and/or traffic loads.

1. Introduction

Soft marine clay has a high void ratio, a high water content, low permeability, high compressibility and high sensitivity and is susceptible to outside disturbances. It is also widely distributed in the east coast area of China. In this marine environment, the foundation design of offshore installations (suction anchors, wind turbines, gravity platforms, etc.) or near-shore structures (seawalls, harbors, dockyards, lighthouse, etc.) is governed by the bearing capacity and the serviceability under cyclic loads (Andersen, 2009; Hu and Ding, 2010; Wichtmann et al., 2013). These cyclic loads are caused by waves, vehicle traffic and the operation of machinery, resulting in a reduction of strength (Rao and Panda, 1999; Moses and Rao, 2003; Li et al., 2011) and bearing capacity (Wichtmann et al., 2013; Tang et al., 2011), unexpected settlement (Ren et al., 2012; see Ng et al., 2013 for an example of settlement; Lei et al., 2016), and other geotechnical engineering problems (Mayoral et al., 2016).

A model for cumulative deformation is a theoretical basis for solving the problems of dynamic stability and settlement of subgrade under long-term cyclic loads. At present, the cumulative plastic deformation models under cyclic loading are mainly classified into two categories: theoretical models and empirical models. In the scientific literature, theoretical models mainly include the modified Cambridge model (Carter et al., 1982), nested yield surface model (Mroz, 1967; Prevost, 1977, 1978), bounding surface model (Dafalias, 1986a, 1986b; Hu and Liu, 2015), etc. These theoretical models are capable of calculating the plastic strain generated by each cyclic load, generally being high in accuracy but complex in calculation. However, to meet the accuracy requirements, most of these models need to adopt sufficiently small calculation steps to simulate the process of each cycle of loading and unloading. This often leads to model failure due to excessive calculation, especially for a large number of cyclic loads. Empirical models are usually established by using the results of laboratory tests and field tests, which avoids excessive calculation. As they meet the requirements of engineering practice, empirical models have been widely used in practical engineering despite being less accurate than theoretical models.

A number of empirical models have been developed for predicting cumulative plastic deformation in soil under cyclic loading. However, the most commonly used is the following power model (Monismith et al., 1975):

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$$\varepsilon_p = A \times N^b \tag{1}$$

where ε_p is the cumulative plastic strain(%); N is the number of cyclic load applications; and A and b are two parameters that depend on the soil type, soil properties and stress state. Afterward, many researchers improved the power model (1). Li and Selig (1996) took into account the influence of the cyclic deviator stress (σ_d) and soil statistic strength (σ_s) on coefficient A in (1) as follows:

$$A = a \times (\sigma_d/\sigma_s)^m \tag{2a}$$

$$\varepsilon_p = a \times (\sigma_d/\sigma_s)^m \times N^b \tag{2b}$$

where a and m are material parameters, m has a range of 1.0–4.2, and a has a range of 0.3–3.5 for 22 soils studied. Chai and Miura (2002) considered the effect of the initial deviator stress (σ_{id}) on the cumulative plastic strain and proposed the following equation:

$$\varepsilon_p = a \times \left(\frac{\sigma_d}{\sigma_s}\right)^m \left(1 + \frac{\sigma_{id}}{\sigma_s}\right)^n N^b \tag{3}$$

where n is a constant coefficient with a recommended value of 1.0. Various similar models have been developed (Parr, 1972; Moses et al., 2003; Abdelkrim et al., 2003; Huang et al., 2006; Shahin et al., 2011; Wang et al., 2013; Guo et al., 2013; Wang and Li, 2015; etc.) for different engineering backgrounds and practical applications. In general, these empirical models represent a relationship between the cumulative plastic strain and the number of repeated load applications. Some models also considered stresses, the strain rate, the soil type and soil properties.

The aim of this paper is to develop a novel empirical model for cumulative plastic deformation of soft marine clay subjected to cyclic loads characterized by a large number of repeated load applications and low stress. This will provide an opportunity to discuss the settlement and strength decrease of foundation soils in the near-shore and off-shore areas under wave or traffic loads and to provide a broad, even if not complete, overview of deformation characteristics and their controlling factors, as well as the relationship with time. In the following, we discuss the major influencing factors on cumulative deformation in terms of the stress state, soil type and soil properties in Section 2. Section 3 presents a cumulative plastic deformation model, and the proposed model will be verified and evaluated by test results in Section 4. Relationships between three model parameters, the physical soil state and the stress state, applications of the proposed model, and model limitations are discussed in Section 5. Summaries and main conclusions are drawn in the last part of the paper.

2. Influencing factors

A good prediction model for cumulative plastic deformation should take into account the major influencing factors. The list of the most frequently quoted factors includes the physical properties of soil (water content, Atterberg limits, specific gravity, particle size, specific surface area, etc.), stress state (dynamic stress, confining pressure, overconsolidation ratio, loading frequency, etc.), drainage conditions, and stress path (loading waveform, loading method, test control mode, the rotation of principal stress axis, etc.) (Li and Selig, 1996; Huang et al., 2006; Ren et al., 2012; Cai et al., 2013; Lei et al., 2015; Gu et al., 2016; Elia and Rouainia, 2016; Hicher, 2016). Among these factors, for a given soil, the most dominant influencing factor of cumulative plastic deformation is the cyclic deviator stress σ_d, which was recognized through laboratory test results conducted by many researchers such as Seed et al. (1955), Monismith et al. (1975), Li and Selig (1996), Huang et al. (2006), Shahin et al. (2011), etc. The second important factor is the confining pressure. These two factors mainly determine the development model of strain and the dissipation of excess pore water.

The existence of a critical cyclic stress (or threshold stress) has long been recognized by many researchers (e.g., Larew and Leonards, 1962; Mitchell and King, 1977; Sangrey et al., 1978; Lefebvre et al., 1989; Tang et al., 2003; Shahin et al., 2011), and the critical cyclic stress for several different soft soils is summarized in Table 1. The critical cyclic stress is defined as the stress below which the soil will not suffer failure regardless of the number of repeated load applications and above which soil deformation will continually increase up to failure. In other words, when the applied cyclic stress is lower than the critical stress, the cumulative plastic deformation of soil will not continually increase with an increase in the number of repeated load applications but tend to a stable limited value. Based on the critical cyclic stress level, Cai and Cao (1996) classified the development of the permanent deformation into three types, attenuation type, critical type, and destructive type, and suggested applying different models for the different deformation types.

In many practical cases, the cyclic loads applied on the mucky soft soil consist of long-term repeated applications and a low stress level, which is often lower than the critical stress of the soil. Therefore, the plastic accumulative deformation induced by these loads will gradually increase at the beginning, but the deformation rate gradually decreases. After a certain time, the cumulative plastic deformation reaches the maximum and tends to be stable, no longer increasing over time. However, Monismith's model, the most widely used empirical model, is not capable of predicting the deformation caused by the stress that is below the critical cyclic stress because the plastic strain it describes will increase endlessly with increasing number of cyclic loading applications.

3. Cumulative plastic deformation model

Fig. 1 (a) shows the skeleton curve of stress-strain relationship of soils subjected to cyclic loading, which was described as a hyperbolic equation by Hardin and Drnevich (1972), i.e. the well-known classical Hardin-Drnevich model (Eq. (4)).

$$\sigma_d = \frac{\varepsilon_d}{A + B\varepsilon_d} \tag{4}$$

where A and B are model parameters.

Fig. 1 (b) shows the typical curve of cumulative deformation of soft soils under long-term low cyclic stress. It can be seen from Fig. 1 that the two curves look very similar. It was therefore concluded that a function similar to that of Equation (4) might be applicable to the cumulative plastic strain of soft soils under cyclic load conditions. Combining the Monismith model (Eq. (1)) and Hardin-Drnevich model, a new model is proposed as Equation (5) for the cumulative plastic deformation caused by cyclic stress with long-term application and a low stress level, especially for the stress lower than the critical stress of soil.

$$\varepsilon_p = \frac{N^b}{a + cN^b} \tag{5}$$

where a, b and c are parameters that depend on the stress path, stress state and physical properties of soil.

The impact of parameters a, b and c on the cumulative plastic strain is shown in Fig. 2 (b), (a) and (c) respectively. Fig. 2(a) shows that for given values of parameters a and c, the slope of the curve increases with increasing exponent parameter b, while the initial and final strain remains the same regardless of b. Fig. 2(b) shows that the slope will remain stable and the curve will parallelly move toward the left or right if parameter b is given a certain value. This observation suggests that the exponent parameter b characterizes the rate of accumulative plastic strain and does not affect the final amount of deformation of soil. Following this observation, it might be reasonable to conclude that parameter b depends on the physical properties of the soil itself, such as the water content, void ratio, particle size and particle specific surface area. Monismith et al. (1975) suggested that the rate of cumulative plastic strain is unrelated to the dynamic stress level, namely, dynamic stress does not affect exponent parameter b. In addition, Li and Selig

Table 1
Critical cyclic stress for different soft soils.

References	Soil type	Water content (%)	Liquid limit (%)	Plastic limit (%)	Specific gravity	Frequency (Hz)	Cyclic number	Confining pressure (kPa)	Critical cyclic stress (kPa)	Critical cyclic strain (%)
Mitchell and King (1977)	Marine clay	33–38	34–41	18–20	2.75	0.25	10000	50	50	1.0
Cai and Cao (1996)	Chengdu clay	25.6	/	/	1.52 (dry density)	10	100000	0	55–60	5.7
Moses et al. (2003)	Clay	80–85	88	28	/	0.17 0.05	9000	100 100	46 38	6.2
Zhang et al. (2009)	Wuhan soft clay	57.7	69.1	32.2	1.65 (density)	4	3000	50 100 150	30–40 40–60 60–90	6.0 4.3 4.2
Liao et al. (2009)	Red caly	23.8 28.3 32.5 36.4	34.3	20.8	1.57 (dry density)	5	20000	/	59 44 38 29	4.2 5.1 7.3 12.3
Shahin et al. (2011)	Soft clay	/	53	26	2.65	1	150000	300	88–99	4.2
Wang et al. (2013)	Wenzhou soft clay	56–59	64	32	2.75	1	50000	100	47	2.0

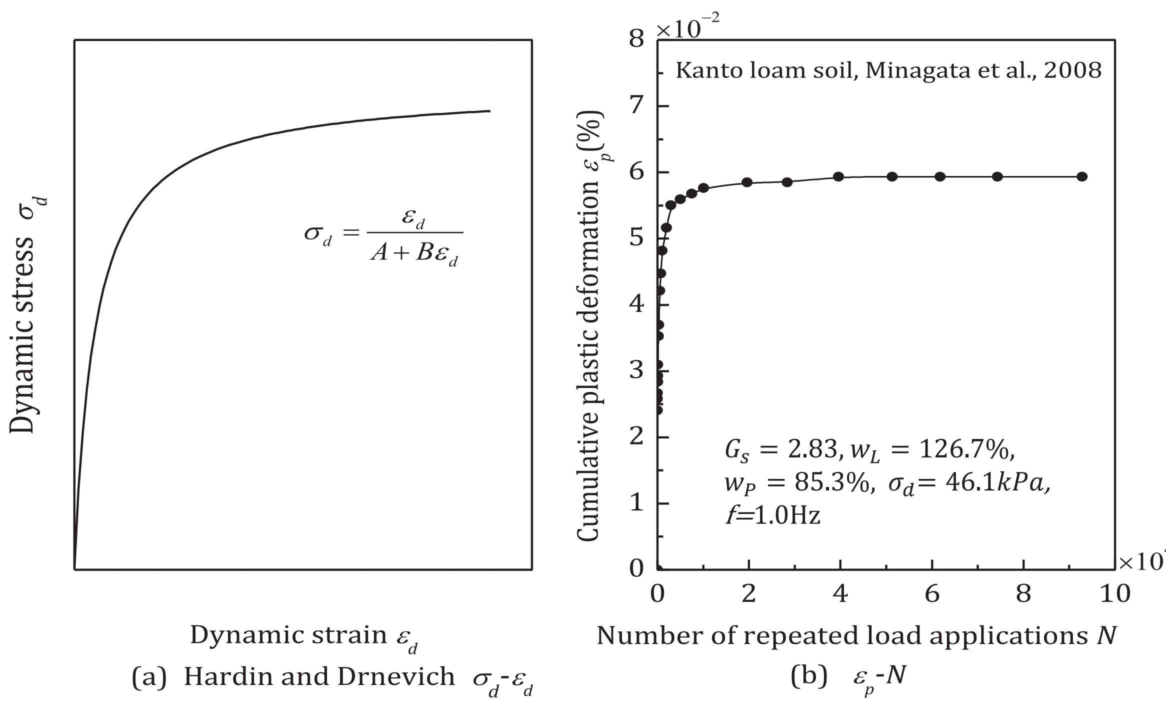

Fig. 1. Sketch diagram of the Hardin-Drnevich model and typical cumulative deformation curve of soft soils under long-term low cyclic stress.

(1996) obtained similar conclusions and suggested that for a given soil, parameter b should be a constant.

To examine whether parameter b in the proposed model also has a similar feature, we complied experimental data from previous research to discuss the relationships between parameter b and the dynamic deviator stress σ_d and loading stress ratio η, which is expressed as $\eta = \sigma_d/(2p)$, where $p = (\sigma_1 + \sigma_2 + \sigma_3)/3$. It can be seen from Fig. 3(a) and (b) that parameter b seems to have no clear relationship with σ_d and η, which indicates that the exponent parameter b only depends on the soil type and soil properties and is unrelated to the subjected loads. It should be a constant for a given soil. This observation is consistent with previous studies (see the references of Monismith et al., 1975; Li and Selig, 1996). Furthermore, most values of parameter b are within a narrow range of 0.18–0.82, with a mean value of approximately 0.5.

Fig. 2(b) shows the influence of parameter a on the cumulative plastic strain. For given parameters b and c, the strain curve moves toward the right with parameter a increasing, namely, the time or number of repeated load applications required for the final deformation to become stable increases with an increase in parameter a; meanwhile, the final deformation and strain rate remain constant. In addition, after the first cyclic load application ($N = 1$), the caused strain $1/(a+c)$ will increase with parameter a decreasing for a given parameter c. Therefore, parameter a characterizes the degree of difficulty of the soil beginning to deform and the initial strain during the first cyclic load application.

To make parameter a much clearer, the frequency of cyclic load f and application time t are introduced to replace the number of cyclic load applications N by the relation $N = tf$. Thus, Equation (6) could be rewritten as:

$$\varepsilon_p = \frac{(tf)^b}{a + c(tf)^b} \tag{6}$$

For a given parameter $b = 1$, both sides of Equation (6) are divided by

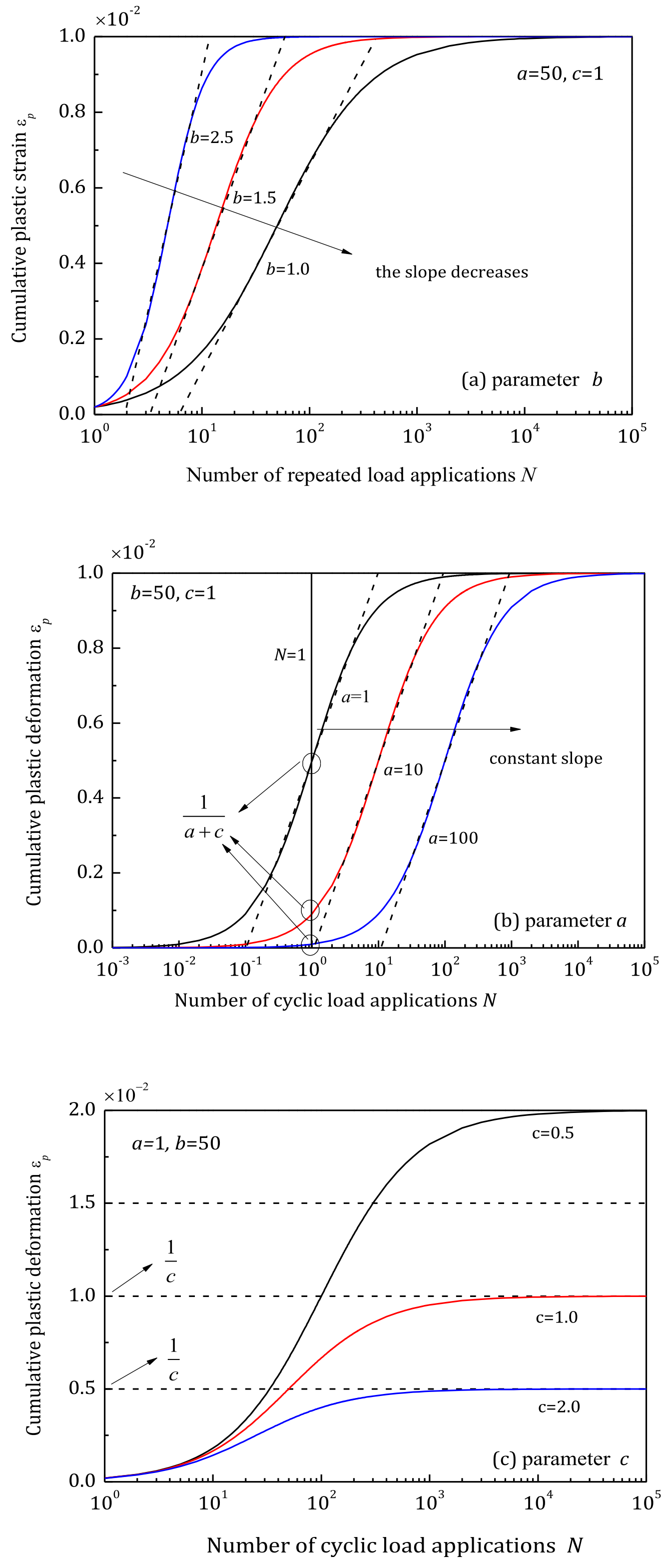

Fig. 2. Impact of model parameters on the cumulative plastic strain: theoretical analysis.

the time t, then yielding:

$$\dot{\varepsilon}_p = \frac{\varepsilon_p}{t} = \frac{f}{a + ctf} \tag{7}$$

When time $t \to 0$, the strain rate becomes:

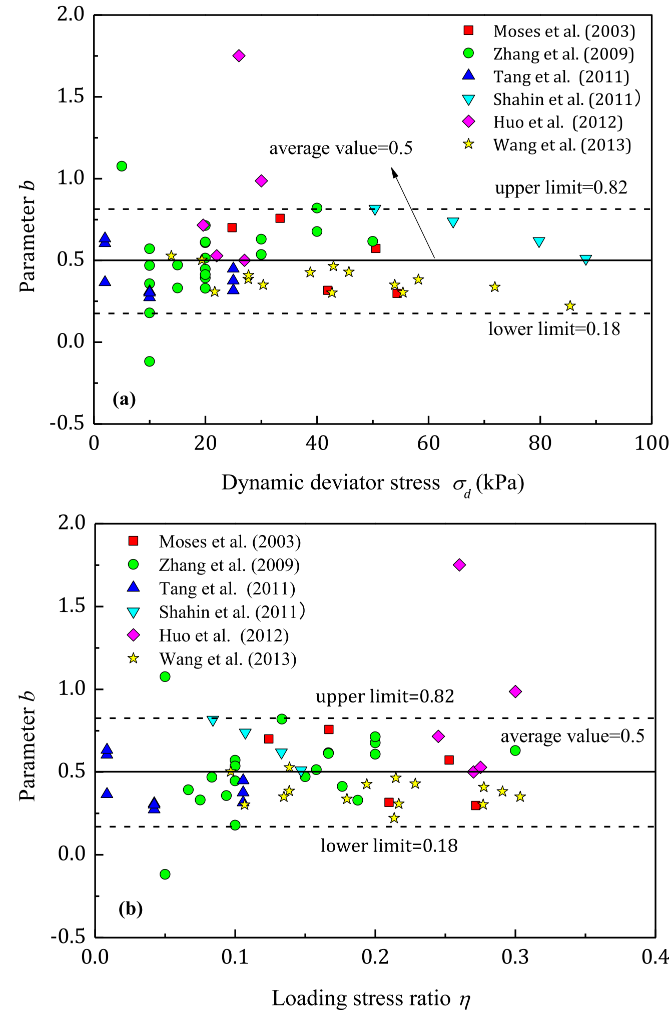

Fig. 3. Effect of the stress state on parameter b: (a) relationship between parameter b and dynamic deviator stress σ_d; (b) relationship between parameter b and loading stress ratio η.

$$\dot{\varepsilon}_p = \frac{f}{a} \tag{8}$$

Equation (8) further indicates that parameter a controls the initial strain of soil, that is, the degree of difficulty of the soil beginning to deform. In addition, since the first cyclic loading application will cause more or less deformation, the caused strain $1/(a+c)$ is always greater than zero, namely, $a+c>0$. To explore the physical meaning of parameter c, the bounds of Equation (6) are discussed. When the number of cyclic load applications N approaches infinity, namely, $N \to \infty$, the final cumulative plastic strain is:

$$\varepsilon_{pf} = \frac{1}{c} \tag{9}$$

Equation (9) indicates that the final strain is only dependent on parameter c, regardless of the parameters a and b, which could also be seen from Fig. 2(c). For given parameters a and b, the final strain increases with parameter c decreasing; meanwhile, both the initial strain (stain at $N=1$) and the time to reach the final strain are the same for different c. Thus, the factors that affect the final deformation, e.g., physical properties of soils and cyclic deviator stress, will also control parameter c. Since this paper's focus is on the case in which the applied stress is less than the critical cyclic stress, the caused final strain should not exceed the critical strain of soils ε_{pc}, that is, $c > 1/\varepsilon_{pc}$. Many researchers have investigated the critical strain of different types of soft soil under various cyclic stress and suggested a range of $1.0 \le \varepsilon_{pc} \le 12(\%)$, as shown in Table 1. Consequently, the range of values for parameter c can be estimated as $c > 0.08$, and parameter $a > -c > -0.08$.

In summary, we proposed a new model with three parameters *a, b* and *c*, which denote the initial strain (strain at $N = 1$), strain rate and final cumulative strain, respectively. Unfortunately, although the physical meaning of every parameter is clear, there is no appropriate experimental method to measure the parameters directly, which weakens the application of the proposed model in practice. Therefore, it is important to build links between the parameters and other easily measurable parameters, such as the liquid limit, undrained static strength, confining pressure and cyclic deviator stress. These links will be thoroughly analyzed in the following section.

4. Verification and validation

To verify and validate the proposed model, some available laboratory test results from the existing literature are compared with their corresponding predicted results.

A series of experimental data of cumulative plastic deformation under cyclic loads from the existing literature are compiled to examine the performance of the proposed model (Eq. (5)) and Monismith model (Eq. (1)). The data involved four different soft clay-type soils and were obtained via cyclic triaxial tests, whose applied cyclic loads had a large number of cyclic load applications and a lower stress than critical stress level. The Least Square Method was used to obtain the parameters in Equations (1) and (5) by fitting the data set of cumulative plastic strain ε_p and number of cyclic load applications *N*. Using the fitted parameters, the cumulative plastic strain was computed via the proposed model and Monismith model, and we made a comparison with the test results shown in Fig. 4.

Taking Fig. 4(a) as an example, the comparisons of the predicted and experimental cumulative plastic strain for remolded kaolinite clay are shown. The test results were obtained by Shahin et al. (2011). The soil physical states (specific gravity $G_s = 2.65$, liquid limit $w_L = 53\%$, plastic limit $w_P = 26\%$), stress state (effective confining pressure $\sigma_3 = 2.65$, cyclic stress ratio $CSR = 0.63$, loading frequency $f = 1.0$ Hz) were taken into account in the tests, and the applied dynamic deviator stress was lower than the critical stress level. Specific details for the test procedures and other information of the soil and stress are available in

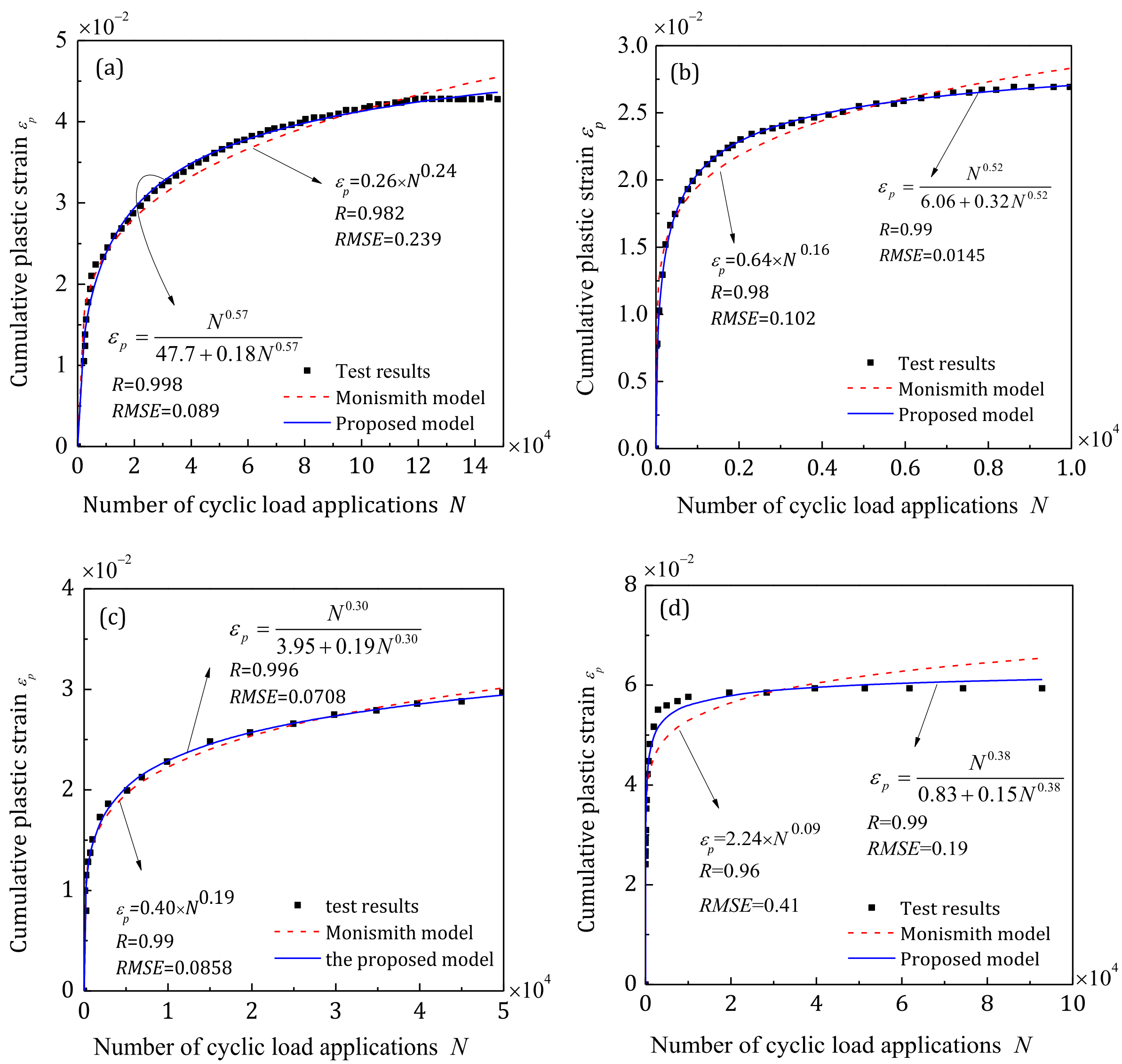

Fig. 4. Comparison of predicted and experimental cumulative plastic strain. Data source: (a) Shahin et al. (2011), remolded kaolinite clay, $G_s = 2.65$, $w_L = 53\%$, $w_P = 26\%$, $\sigma_3 = 300kPa$, $CSR = 0.63$, $f = 1.0$ Hz; (b) Huang et al. (2006), Shanghai marine clay, $G_s = 2.75$, $w_L = 44.8\%$, $w_P = 24\%$ (soil properties refer to Chen and Xu, (2011)), $\sigma_3 = 100kPa$, $f = 0.5$ Hz; (c) Wang et al. (2013), Wenzhou soft clay, $G_s = 2.75$, $w_L = 64\%$, $w_P = 32\%$, $\sigma_3 = 100kPa$, $CSR = 0.66$, $f = 1.0$ Hz; (d) Minagata et al. (2008), Kanto loam soil, $G_s = 2.83$, $w_L = 126.7\%$, $w_P = 85.3\%$, $\sigma_{max} = 101.3kPa$, $f = 1.0$ Hz.

the literature of Shahin et al. (2011). The predicted results of the Monismith model and the proposed model in this paper are shown by the red dotted line and blue solid line, respectively. As illustrated, the proposed model successfully reproduced the experimental data and produced a much lower root mean square error than the classical Monismith model, especially at the larger number of cyclic load applications. The same conclusion can also be drawn from the other three figures. The results indicate that the proposed model performs better in describing the cumulative plastic strain caused by long-term low cyclic stress and is more suitable than the Monismith model for describing the final deformation of soft clay when the applied cyclic deviator stress is below the critical stress level of soil.

Wang et al. (2013) carried out a series of triaxial tests with a large number of load applications on soft marine clay under various stress levels and different confining pressures of 50 kPa, 100 kPa, and 200 kPa. For the test procedures and physical properties of the soils, refer to this literature. These test results are used to further validate the applicability of the proposed model. We assume that the exponent parameter b in Equation (5) is a constant of 0.5 according to Fig. 3, and the other two parameters a and c are fitted by the Least Square Method using the test results for the confining pressures of 100 kPa and 200 kPa. The fitted results are shown in Fig. 5. It can be seen that both parameters a and c have a strong correlation with the cyclic stress ratio CSR, and their relationship expressions are also obtained for this soil.

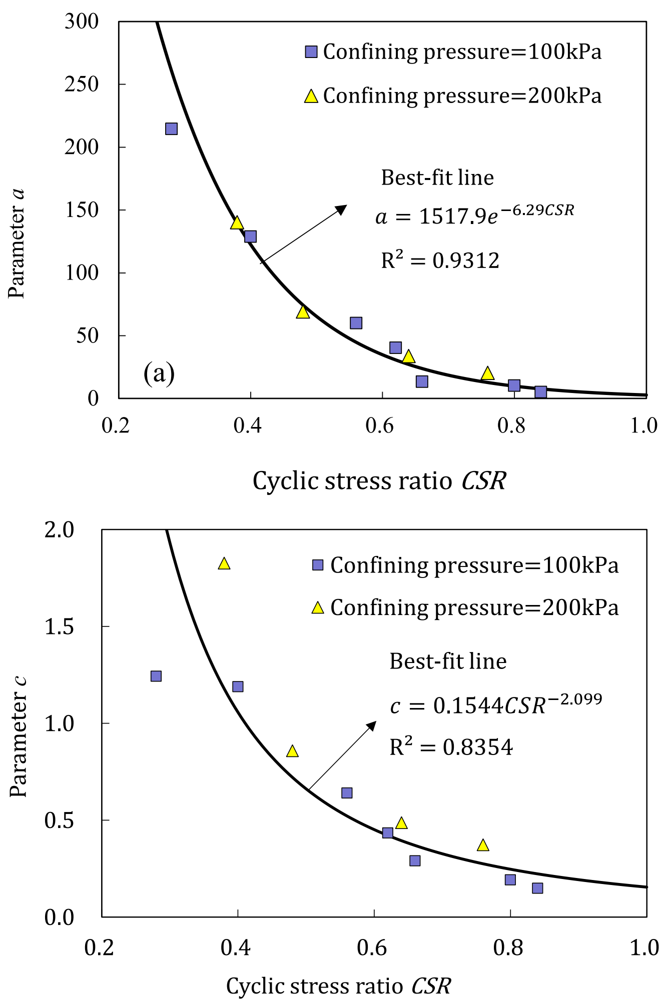

Fig. 5. Fitted parameters in Equation (5): (*a*) parameter a versus the cyclic stress ratio *CSR*; (*b*) parameter c versus the cyclic stress ratio *CSR*. Parameter b is assumed to be 0.5. The data source refers to Wang et al. (2013). Note that the *CSR* in this paper is defined as the ratio of the cyclic deviator stress σ_d to static strength τ_u, which is different from the definition in the reference of Wang et al. (2013).

Then, using best-fit parameters, the cumulative plastic strain of soils for different cyclic stress ratios is predicted with the proposed Equation (5). Comparisons are made between the predicted values and measured values for the confining pressure of 50 kPa, which was not used to fit the parameters. Fig. 6(a) and (b) show the comparisons of the predicted and experimental cumulative plastic strains. It can be seen that the predicted data agree very well with the measured experimental data. Although several relatively large errors (the maximum error is 31%) exist at the low number of cyclic load applications, most of the predicted values fall within the 10% error area. This indicates that the proposed model has a good prediction capability for the cumulative plastic strain of soft soil, especially for that under long-term (large number of cyclic load applications) cyclic loads with a low stress level.

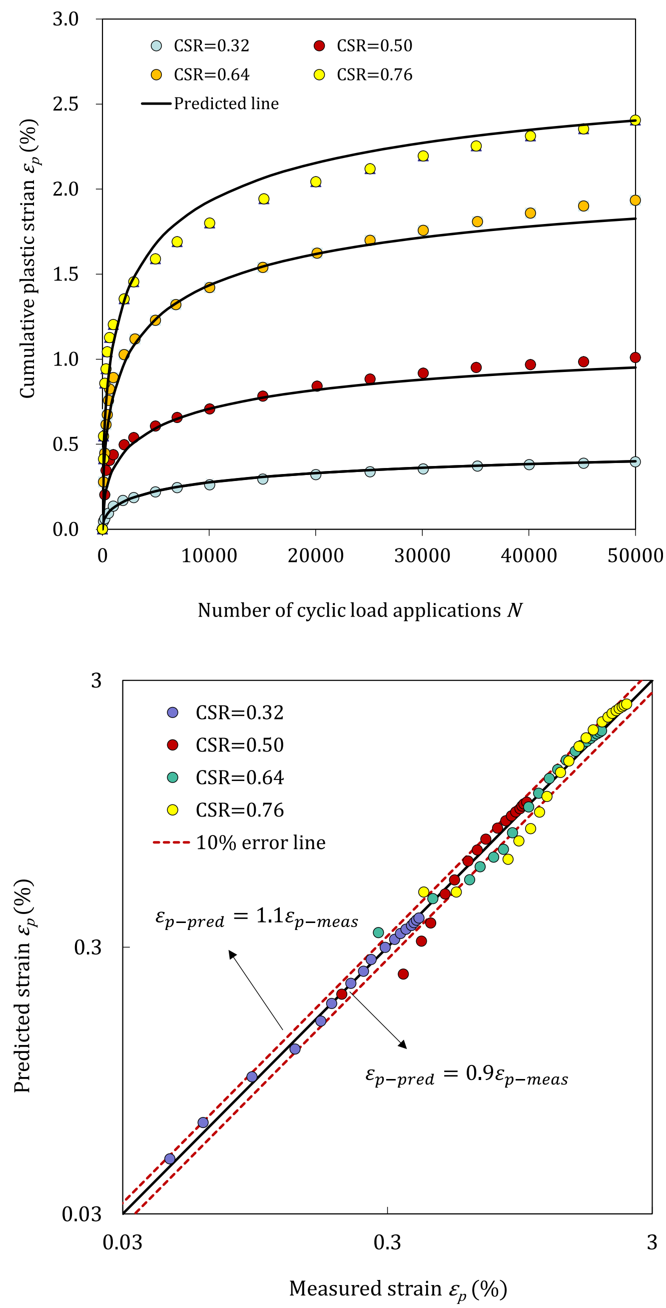

Fig. 6. Comparison of predicted and experimental plastic strain. Data source refers to Wang et al. (2013).

5. Discussions and applications

Since no simple test exists to directly determine the parameters *a*, *b* and *c*, it is essential to make an estimation based on the soil state and stress state. For these purpose, the relationships of these parameters with soil properties and the stress state are discussed in this part.

5.1. Model parameters

Following the above observations, since the exponent parameter *b* only depends on the physical properties of soil regardless of the stress state, we try to determine the exponent parameter *b* by using easily measurable parameters of the physical properties of soils. The average value of parameter *b* was calculated under different stress conditions for a given soil, and its relationship with the water content ratio w_R is shown in Fig. 7. The water content ratio is defined as the ratio of the natural water content *w* to the liquid limit *LL*, namely, $w_R = w/LL$. For a given soil, its water content ratio is a constant. It can be seen from Fig. 4 that a linear relationship exists between parameter *b* and the water content ratio w_R.

$$b = 0.584w_R - 0.007 \tag{10}$$

The correlation coefficient is equal to 0.87 ($R = 0.87$) and the root mean square deviation is 0.08 ($RMSE = 0.08$) for Equation (10), which suggests that this correlation is very significant. It is worth noting that this relationship is built upon fine-grained soils; whether it is applicable for coarse-grained soils needs further study. In addition, for soft marine clay generally with high water content ratio, parameter *b* is recommended to be a constant value of 0.5 as shown in the shadow area of Fig. 7, which is also highlighted by Fig. 3.

According to previous discussions, parameter *c* characterizes the final plastic strain of soils, mainly depending on the type of soils, physical properties and subjected stress. Fig. 8 shows the influence of dynamic deviator stress σ_d, confining stress σ_c, dry density and water content on parameter *c*. As seen in Fig. 8, all of these factors have a remarkable effect on parameter *c*. Thus, a good formula to determine parameter *c* should cover these main factors and must maintain sufficient simplicity at the same time. Many researchers (Monismith et al., 1975; Li and Selig, 1996; Seed et al., 1955; etc.) suggested that the cyclic deviator stress σ_d is the most dominant influencing factor on the plastic strain of soil, namely, on parameter *c*. This indicates that σ_d is an essential element to determine parameter *c*. In addition, the undrained static strength τ_u is introduced to represent the influence of the confining pressure, soil type, and physical properties of soils, which can be easily determined by undrained triaxial tests. The cyclic stress ratio (*CSR*), defined as the ratio of cyclic shear stress $\tau_d = \sigma_d/2$ to static strength τ_u, is introduced to determine parameter *c*.

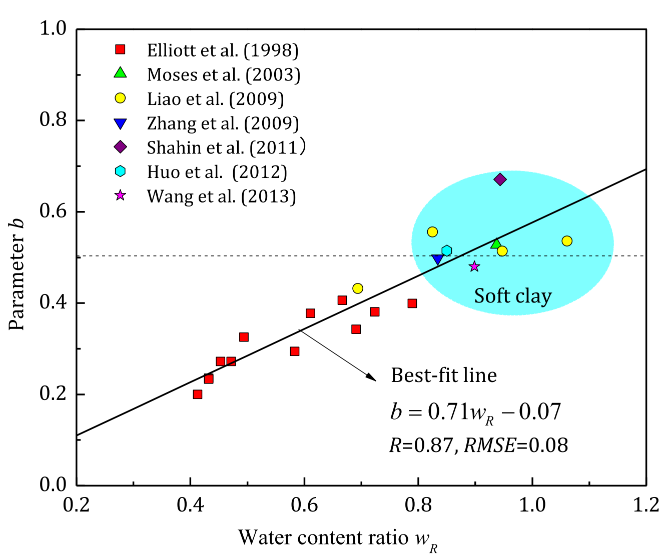

Fig. 7. Relationship between parameter *b* and the water content ratio w_R.

$$CSR = (\sigma_d/2)/\tau_{cu} = \sigma_d/q_{cu} \tag{11}$$

where σ_d is the dynamic deviatoric stress imposed in the cyclic triaxial tests, and $q_{cu} = 2\tau_{cu}$ is the deviatoric stress at failure obtained from an undrained monotonic triaxial test.

The relationships of parameters *a* and *c* with *CSR* for different soft soil are plotted and shown in Figs. 9 and 10, respectively. In addition, the parameter values in these relationships are recommended and listed in Table 2. These values are determined based on back-calculated results with a given constant value of parameter $b = 0.5$. Fig. 9 shows that parameter *c* has a good power relation with *CSR* ($R^2 > 0.8611$) and can be expressed as the following equation.

$$c = C_1(CSR)^{C_2} \tag{12}$$

C_1 and C_2 are parameters. Equation (12) shows that when the cyclic stress ratio tends to zero ($CSR \to 0$), parameter *c* tends to infinity ($c \to \infty$), and then, the final deformation tends to zero ($1/c \to 0$). This indicates that the proposed Equation (12) is reasonable since it meets the physical meaning of parameter *c*. It can be seen from Table 2, for soft soils, the parameter C_1 ranges from 0.0293 to 0.1156 with an average value of 0.0645, while the parameter C_2 ranging from −2.2073 to −4.3040 with an average value of −2.9211.

Fig. 10 shows that the model parameter *a* has a strong correlation with CSR ($R^2 > 0.9433$), and can be expressed as an exponential function of CSR,

$$a = A_1 e^{A_2 \ CSR} \tag{13}$$

The values of parameter A_1 and A_2 are presented in Table 2. For soft soils, the parameter A_1 ranges from 457 to 1539 with an average value of 705, while the parameter A_2 ranging from −4.43 to −8.48 with an average value of −6.42. To facilitate the use of Equations (12) and (13) in practice, for soft soils, the average values for the parameters C_1, C_2 in Equation (12) and A_1, A_2 in Equation (13) are recommended when lack of experimental data.

5.2. Applications

To elaborate the application of the proposed model in calculating settlement, four steps can be followed.

5.2.1. Determine the cyclic stress ratio (CSR)

According to the definition of *CSR*, it can be determined by the cyclic deviatoric stress σ_d and static strength τ_u. These two stress states can be calculated using theoretical constitutive models combined with a finite element numerical simulation. For example, Dong et al. (2010) proposed a method to determine σ_d and τ_u as follows (see more details in Dong et al., 2010).

$$\sigma_d = \sqrt{3J_2} = \sqrt{\frac{1}{2}\left[\left(\sigma_{xd} - \sigma_{yd}\right)^2 + \left(\sigma_{xd} - \sigma_{zd}\right) + \left(\sigma_{zd} - \sigma_{yd}\right) + 6\tau_{xyd}^2\right]}$$
$$\tau_{cu} = c_{cu}\frac{\cos\varphi_{cu}}{1 - \sin\varphi_{cu}} + \sigma_z\frac{1 + k_0}{2}\frac{\sin\varphi_{cu}}{1 - \varphi_{cu}} \tag{14}$$

In addition, σ_d and τ_u can also be directly determined by field tests.

5.2.2. Determine model parameters

Model parameters can be determined by dynamic triaxial tests under the stress state determined in the first step. For soft marine clay or CH soils, in the absence of test results, parameter *b* is recommended to remain constant at 0.5, and the parameters *a* and *c* can be estimated with proposed Equations (13) and (12), respectively.

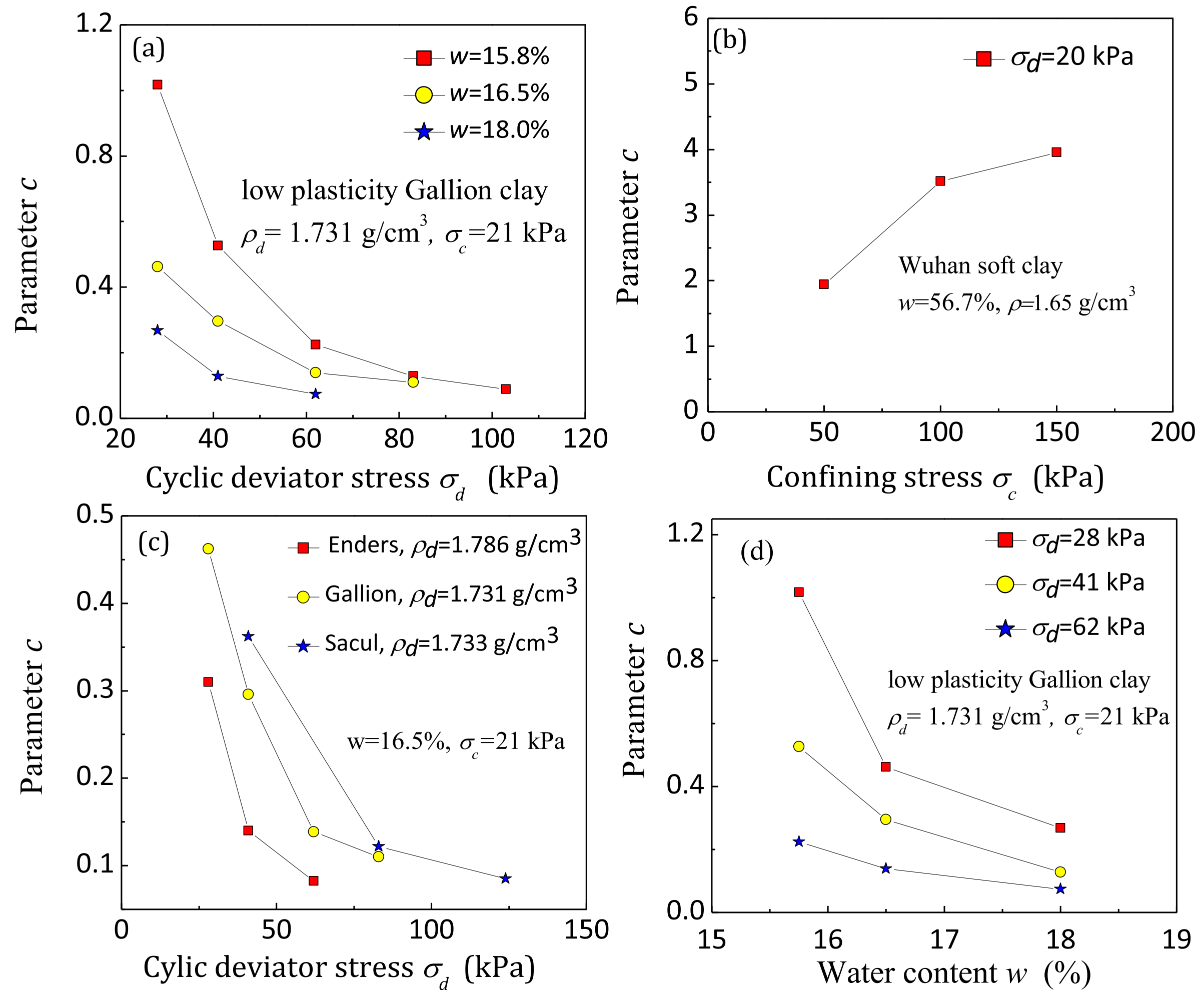

Fig. 8. Influence of the cyclic deviator stress σ_d, confining stress σ_c, dry density ρ_d and water content w on parameter c. (Data source: (a), (c) and (d) refer to Elliott et al. (1998) (b) refers to Zhang et al. (2009)).

5.2.3. Determine the cumulative plastic strain

The cumulative plastic strain for each subdivided layer of soil can be easily determined by the proposed model (5) with known model parameters.

5.2.4. Determine the cumulative deformation

The cumulative deformation or final settlement can be determined by summing up the deformations of all subdivided layers using the following equation:

$$s = \sum_{i=1}^{n} \varepsilon_p^i h_i \tag{15}$$

where h_i is the thickness of each subdivided layer.

In situ tests were performed on a soft subgrade at the Transportation Technology Center (TTC) in Pueblo, CO to investigate soft subgrade performance under repeated heavy axle loading. For detailed test program and subgrade description, refer to Li and Selig (1996). The subgrade settlements are predicted by the proposed model (5), and the prediction results are shown in Table 3. The subgrade was filled with soft soil of Vicksburg Buckshot clay (PI = 40–45, LL = 60–70), and the soft subgrade consisted of 5 sublayers, each 0.3 m thick. The deviator stress at the center of each subdivided layer was determined by the GEOTRACK model (Li, 1994; Li and Selig, 1996) and is shown in Table 3. The total number of repeated load applications N is 770,000 (Li, 1994). Since the filled subgrade soil is soft clay (CH), the model parameter b is assumed to be 0.5, and parameters a and c are estimated by Equations (13) and (12) with the recommended average values for C_1, C_2, A_1 and A_2, respectively. The cumulative plastic strain ε_p^i of each subdivided layer was calculated with Equation (5), and the final settlement of subgrade was determined with Equation (15).

The results of subgrade settlement predicted by this paper and by Li and Selig (1996) are also plotted in Fig. 11 with the test results. The range of results predicted by both this paper and Li and Selig (1996) are consistent with the range of test results. In addition, the average settlement predicted by this paper is much closer to the average settlement obtained via test compared to that predicted by Li and Selig (1996).

Notably, the recommended values for parameter b and for parameters a and c, estimated by Equations (13) and (12), are proposed based on back-calculation results for soft clay (CH), which may be not applicable for other soil classifications such as ML, MH and CL.

5.3. Limitations

(1) Although the physical meaning of parameters in the proposed model is clear, there is no appropriate experimental method to measure these parameters directly, which weakens the application of the proposed model in practice. (2) The method using the cyclic stress ratio (*CSR*) to estimate parameters a and c misses some other important factors such as the stress history (overconsolidation ratio), static deviator stress and frequency of applied cyclic loads. (3) The proposed model performs well in predicting the cumulative plastic strain and final settlement for a given soft deposit; however, for different soft soils, it still lacks enough

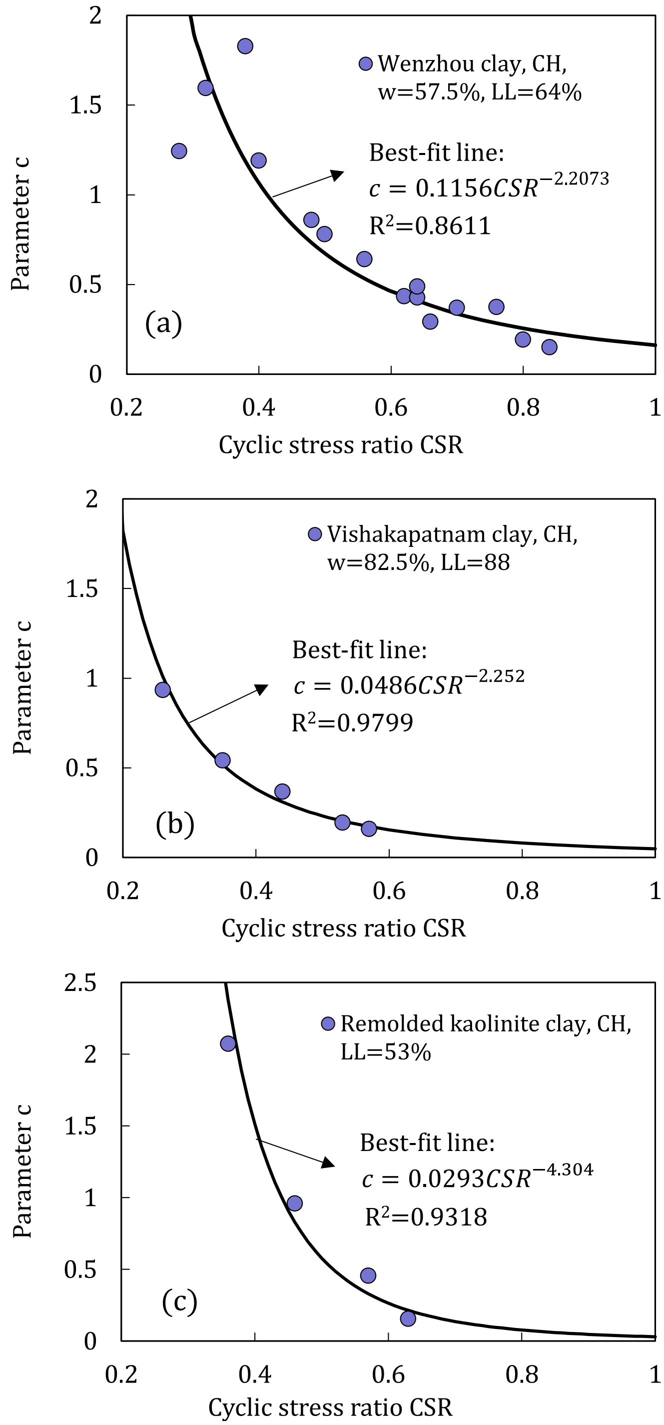

Fig. 9. Relationship between parameter *c* and the cyclic stress ratio *CSR* for different soft clays (Data source: (a): Wenzhou clay refers to Wang et al. (2013); (b): Vishakapatnam clay refers to Moses et al. (2003) (c): Remolded Kaolinite refers to Shahin et al. (2011); all of the data points in this figure are best fit by Equation (5) based on experimental data.).

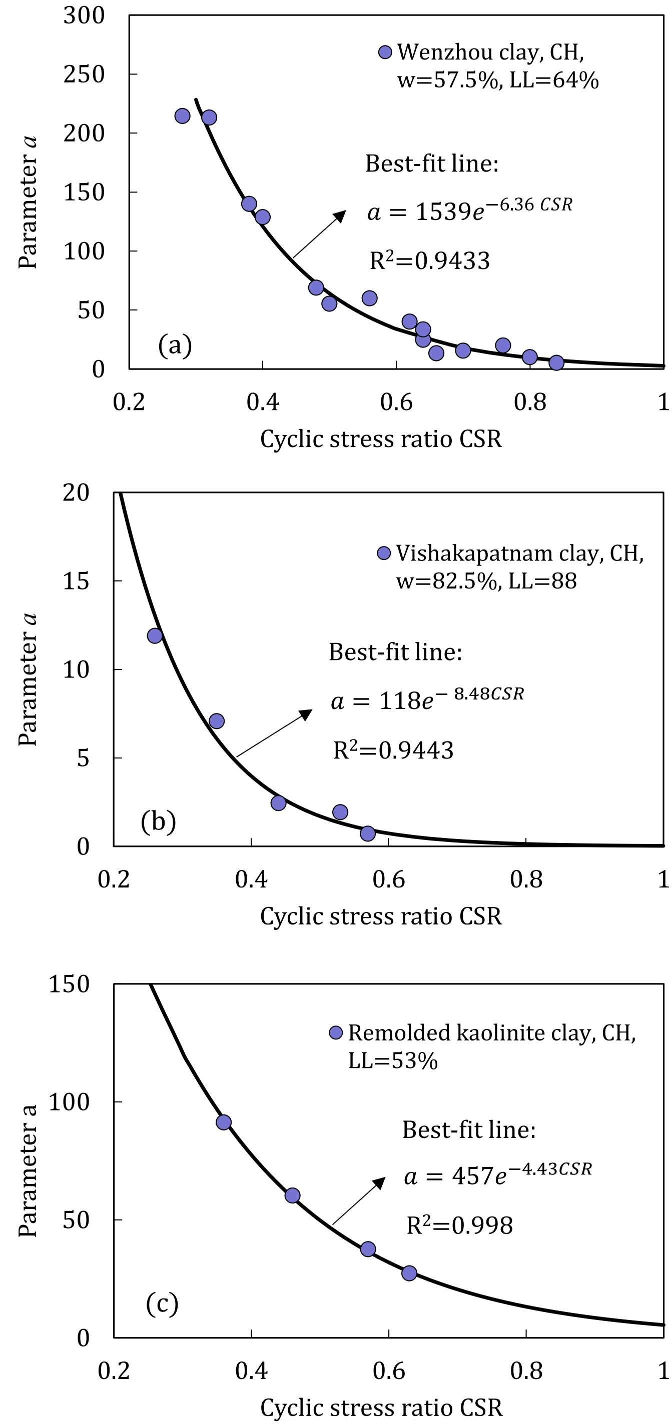

Fig. 10. Relationship between parameter *a* and the cyclic stress ratio *CSR* for different soft clays (Data source: (a): Wenzhou clay refers to Wang et al. (2013); (b): Vishakapatnam clay refers to Moses et al. (2003) (c): Remolded Kaolinite refers to Shahin et al. (2011); all of the data points in this figure are best fit by Equation (5) based on experimental data.).

information to determine model parameters with universal applicability. It needs further experimental research.

6. Summary and conclusions

In many practical cases, the cyclic loads applied on mucky soft soil are of long-term repeated applications and a low stress level even lower than the critical stress of soil, which can lead to deformation but not failure regardless of the number of repeated applications. These cases are very common for soft soils subjected to wave or traffic loads in the near-shore and off-shore areas of China. However, there is still no suitable model to describe deformation development over time. In response to this situation, a novel empirical model was proposed to predict the cumulative plastic deformation of soft marine clay subjected to long-term low cyclic loads. This model was based on analogy analysis of the Hardin-Drnevich model and Monismith model with three parameters. Comparisons were made between prediction results and available experimental results in the literature that had not been used to fit the parameters. The

Table 2
Model parameters.

Soil type	Parameter b	Parameter c = A * CSR^B			Parameter a = M * e^{N*CSR}			Source
		C_1	C_2	R^2	A_1	A_2	R^2	
Wenzhou clay	0.5	0.1156	−2.2073	0.8611	1539	−6.36	0.9433	Wang et al., 2013
Vishakapatnam clay	0.5	0.0486	−2.2520	0.9799	118	−8.48	0.9443	Moses et al. (2003)
Remolded Kaolinite	0.5	0.0293	−4.3040	0.9318	457	−4.43	0.9980	Shahin et al. (2011)
Average value	0.5	0.0645	−2.9211	0.9243	705	−6.42	0.9619	

Table 3
Prediction results of soft subgrade under long-term repeated loads.

Soil condition	Subdivided layer number	σ_d (kPa)	q_{cu} (kPa)	*CSR*	ε_p^i (%) calculated by equation (5)	Settlement (mm)	
						Predicted by Eq. (15)	Predicted by Li and Selig (1996)
w = 28.4%	1	66	193	0.34	0.67	5.46	9.4
	2	59		0.31	0.48		
	3	51		0.26	0.31		
	4	44		0.23	0.20		
	5	40		0.21	0.16		
w = 32.4%	1	59	90	0.66	4.59	42.30	51
	2	55		0.61	3.71		
	3	48		0.53	2.47		
	4	44		0.49	1.90		
	5	40		0.44	1.43		
w = 35.4%	1	47	48	0.98	15.55	153.28	142
	2	44		0.92	12.72		
	3	40		0.83	9.52		
	4	37		0.77	7.50		
	5	34		0.71	5.80		
The calculated average settlement for the three soil conditions						62.17	67.47

Note：Each subdivided layer is 0.3 m thick. The test results of minimum settlement for lower water content (w = 28.4%) is about 4.9 mm, and maximum settlement for higher water content (w = 35.4%) is about 164 mm. The average settlement for the three conditions is about 60 mm.

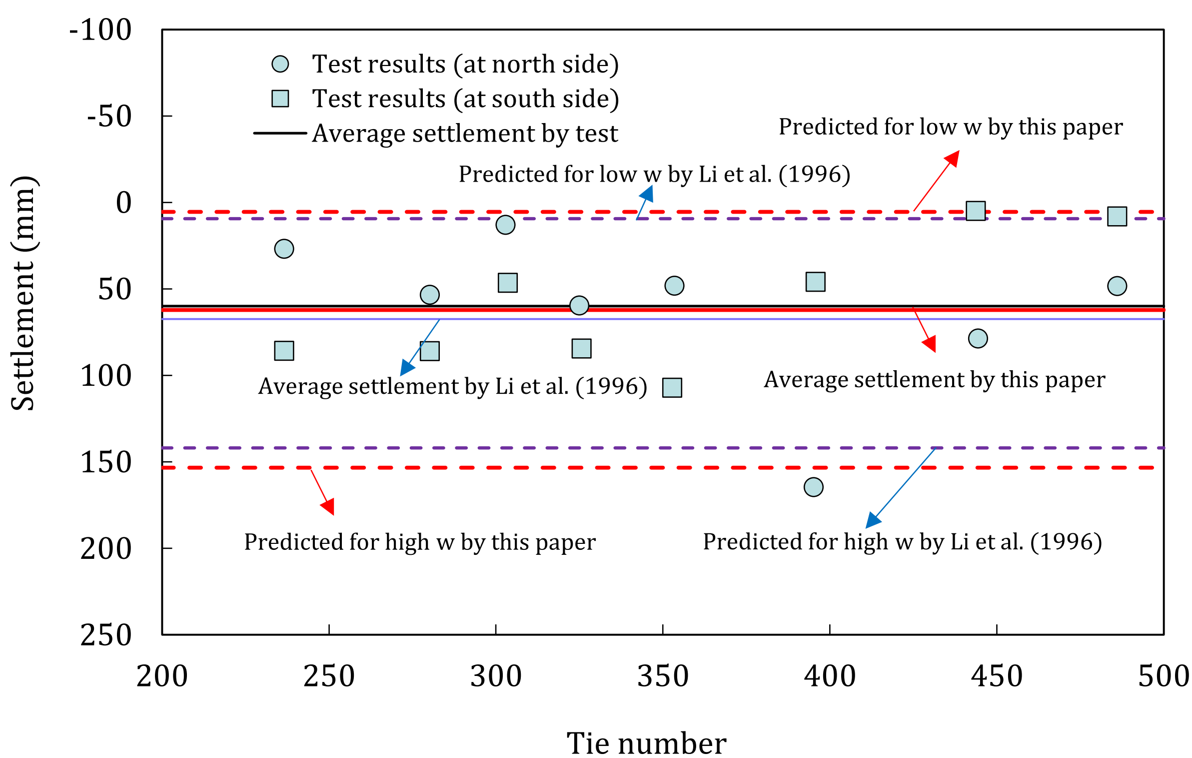

Fig. 11. Comparison of in situ test results with the results predicted by this paper and Li and Selig (1996).

comparison showed that the new model presents better capability and performance than the Monismith model in predicting the cumulative plastic strain of soft soil, especially for that under a long-term (large number of cyclic load applications) low stress level.

Relationships between the three parameters for the prediction model and readily measurable soil parameters, as well as the stress state, in the absence of test results, are recommended for soft soils (CH). These relationships were obtained by statistical regression analysis of available test results in the literature. The exponent parameter *b* in this prediction model was proved to only depend on the physical properties of soil regardless of the stress state and is recommended to be 0.5 for soft soils. The dynamic deviator stress is the most important stress factor for the cumulative plastic strain and is taken into account by the other two parameters *a* and *c* in the prediction model. Meanwhile, the effect of the soil physical state (moisture content, dry density, porosity, etc.) captured by the soil static strength is also reflected by these two parameters. Both of them can be expressed as a function of the cyclic stress ratio (*CSR*). Applications of the proposed model were elaborated in detail, and in situ test results for settlement of soft subgrade were used to evaluate the performance of this model. The prediction results are consistent with the test results.

The model presented in this paper mainly considered the influence of the dynamic deviator stress, the number of cyclic load applications, the soil physical state, and other factors probably indirectly captured by the static strength through the cyclic stress ratio. However, the frequency of the applied cyclic load and static deviator stress, which probably has an important impact on the cumulative plastic strain, is not considered in this paper. In addition, although the proposed model has been verified and validated with test results, it still needs further study to improve the model parameters for expansive applicability.

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