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An analysis of domain-based ship collision risk parameters

Rafal Szlapczynski [a], Joanna Szlapczynska [b,*]

[a] Faculty of Ocean Engineering, Gdansk University of Technology, Poland
[b] Faculty of Navigation, Gdynia Maritime University, Poland

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ABSTRACT

According to a lot of contemporary research on ship collision avoidance the classic approach parameters – distance at closest point of approach (DCPA) and time to the closest point of approach (TCPA) – are not sufficient for estimating ship collision risk and for planning evasive manoeuvres. Consequently new measures are introduced, often utilizing the concept of a ship domain. Their drawback, up to this point, was the lack of analytic solutions that would make it possible to efficiently use ship domains in real-time systems where computational time is of essence. The current paper aims to change this, offering analytic formulas for domain-based collision risk parameters: degree of domain violation (DDV) and time to domain violation (TDV). Explicit derivations of formulas for DDV and TDV are presented here for any elliptic domain. For domains of other shapes elliptic approximations are discussed, so that the derived formulas could still be used. A comparison of TCPA/DCPA with domain-based parameters is presented, evidencing the superiority of the latter.

1. Introduction

Despite extensive research that has been done on ship safety domains, the classic approach parameters – distance at closest point of approach (DCPA) and time to the closest point of approach (TCPA) remain an industry standard in on board collision avoidance and decision support systems (Chin and Debnath, 2009), especially in Automatic Radar Plotting Aid (ARPA). There is certainly no lack of evidence that a ship domain is more than a theoretical concept. As of late, in (Hansen et al., 2013, Wang and Chin, 2016) domain models based on real AIS data from southern Danish waters and Singapore Port area respectively have been developed. Those domain models show significant similarities to classic ship domain shapes of the past, both theoretical and empirical (Fuji and Tanaka, 1971; Goodwin, 1975; Davis et al., 1982; Coldwell, 1983). A form of a ship domain has also been suggested by an AIS data-based analysis of distances between ships in convoys in ice conditions (Goerlandt et al., 2016). Despite all the abovementioned research the world of commercial system designers and manufacturers has still not warmed up to the idea of a ship domain. Even brand new systems of state-of-the art functionality choose to rely on combination of TCPA and DCPA (Pietrzykowski et al., 2012).

If ship domains are used in practice, it is mostly in marine traffic engineering, e.g. for determining the capacity of traffic lanes and assessing statistical collision risk (Rawson et al., 2014; Liu et al., 2015). Yet even when this purpose is considered, the domain-oriented researchers are in minority - the synonymous terms of a safe distance or safety radius dominate in this kind of projects (Mou et al., 2010; Wen et al., 2015; Zhang et al., 2015a; Li and Pang, 2013; Sang et al., 2015; Xiao et al., 2015; Goerlandt et al., 2012). The same is true for collision avoidance systems and safe path planning: the majority of researchers use concepts of a safe distance or diameter (Tam et al., 2009; Tam and Bucknall, 2013; Zhang et al., 2015b; Perera and Guedes Soares, 2015; Lee et al., 2015) and compare those values to DCPA when assessing collision risk. However, it must be noted that ship domains – and context-specific approach in general – are utilized in alerts-oriented collision risk decision support systems, whose authors (Baldauf et al., 2011; Bukhari et al., 2013; Simsir et al., 2014 and Goerlandt et al., 2015) realize the limitations of DPCA-TCPA based solutions and opt for more advanced alternatives.

When compared to ship domains, TCPA/DCPA major flaw is their ignorance of the target's bearing and aspect. But it is not its only limitation. Another related problem is that if we assume a large safe distance (which may be reasonable in some cases, e.g. in restricted visibility) then TCPA may be a long time value, even if the safe distance is soon to be violated. In the worst case scenario TCPA may be practically irrelevant – the close quarters situation may happen long before the closest point of approach is actually reached. Therefore in (Lenart, 2015) a new collision threat parameter – time to safe distance (meaning: time to violating the safe distance) was introduced. Lenart presented an extensive analysis of DCPA and TCPA and based on that – argued and evidenced the superiority of newly defined time to safe distance over TCPA.

* Corresponding author.
E-mail address: asiasz@am.gdynia.pl (J. Szlapczynska).

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The current paper addresses the abovementioned limitations of TCPA/DCPA by offering their domain-based equivalents. At the same time, the authors are aware of the fact that complexity of ship domains and related computations is a serious discouraging factor when their practical applications in commercial systems are considered. After all, the computational time is of essence in on-board decision support systems and in that matter it is hard to compete with simple analytical formulas for TCPA and DCPA. This problem however can be overcome if elliptic domains or elliptic approximations of other-shaped domains are used. As the paper shows, practically any domain can be approximated with reasonable accuracy by a decentralised ellipse and once this is done, all necessary formulas can be derived analytically. The rest of the paper is organized as follows. In Section 2 the issue of elliptic approximation of ship domain is discussed. An analysis of general domain-based parameters is provided in Section 3, where a degree of domain violation (DDV) is defined. In Section 4 a new parameter – time to domain violation (TDV) is introduced, which is a domain-based generalization of time to safe distance by (Lenart, 2015). Examples of applying the proposed parameters are given in Section 5, followed by paper's conclusions in Section 6.

2. Elliptic approximation of a ship domain

The major advantages of an ellipse as an approximating figure are its flexibility and computational simplicity. As for the former, a decentralised elliptic domain used here (Fig. 1) is described by four length-dependent parameters:

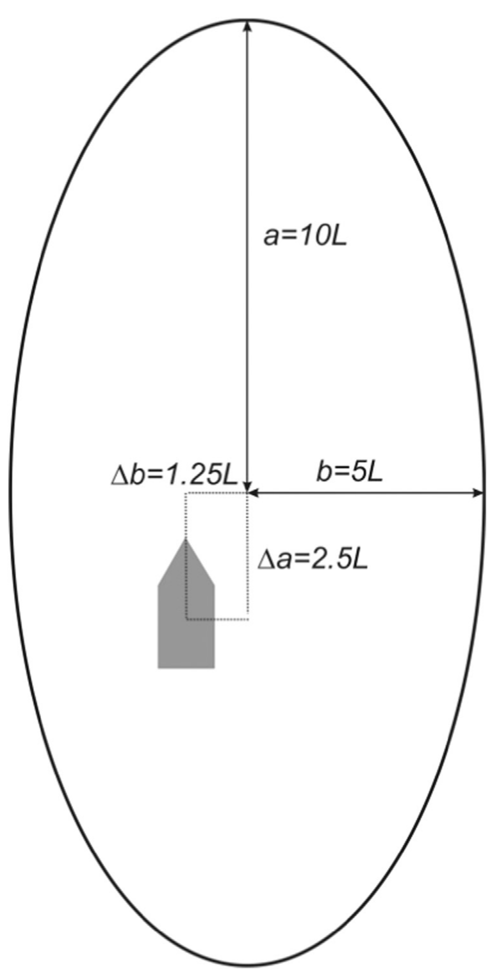

Fig. 1. A decentralised elliptic ship domain (L – ship's length).

a – semi-major axis,
b – semi-minor axis,
Δa – a ship's displacement from the ellipse's centre towards aft along the semi-major axis,
Δb – a ship's displacement from the ellipse's centre towards port along the semi-minor axis.

The first two variables make it possible to specify the domain's length and width, the other two – to vary the sizes of the fore, aft, starboard and port sectors. In practice this is enough to make a domain compliant with COLREGS (IMO, 1972, Cockcroft and Lameijer, 2011) by favouring passing astern and manoeuvres to starboard, while reflecting navigator's perception of collision risk. At the same time, an ellipse is the most complex geometric figure which still makes it possible to formulate all necessary equations as quadratic polynomials, which can be solved analytically. Finally, it is also worth noting that according to many researchers ship domains actually are ellipses (Fuji and Tanaka, 1971; Davis et al., 1982; Coldwell, 1983; Hansen et al., 2013; Liu et al., 2015).

In (Szlapczynski and Szlapczynska, 2015) the authors have shown how a ship domain can be approximated by a polygon and that 16 nodes is sufficient for a quality approximation with additional nodes carrying little to no significant information. Below the reverse process will be shown. Contemporary ship domain models are often based on empirical data, which means that the researchers either use polygonal shapes (Wang and Chin, 2016; Rawson *et al.*, 2014, Pietrzykowski et al. 2009) or at least these shapes come from a set of points around a central ship that initially form a polygon (Hansen et al., 2013).

The literature is rich in algorithms approximating given shapes (including polygons) with regular figures. In particular, various methods of bounding a set of points with an ellipse are discussed in detail in (van Loan, 2008; Gander *et al.*, 1994; Rosin, 1993). In (van Loan, 2008) it has been shown there that using a conic representation of an ellipse (pages 8–9) allows to reduce the approximation to a linear least squares problem with five unknowns (page 47). In conic representation a set of points (x,y) defines an ellipse if:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \tag{1}$$

where

$$B^2 - 4AC < 0 \tag{2}$$

and (to avoid degenerate matrix):

$$\frac{D^2}{4A} + \frac{E^2}{4C} - F > 0 \tag{3}$$

Without loss of generality it may be assumed that A = 1, which gives:

$$x^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \tag{4}$$

$$B^2 - 4C < 0 \tag{5}$$

$$\frac{D^2}{4} + \frac{E^2}{4C} - F > 0 \tag{6}$$

For such a defined ellipse and a given set of points $\{(x_1,y_1),\dots(x_n,y_n)\}$, the distance (in the least squares sense) from the ellipse to the set of points is:

$$dist = \sum_{i=0}^{n} (x_i^2 + x_i y_i B + y_i^2 C + x_i D + y_i E + F)^2 \tag{7}$$

The values of parameters B, C, D, E, F minimizing (7) can be found by various deterministic iterative algorithms (van Loan, 2008; Gander et al., 1994; Rosin, 1993). In (Szlapczynski and

Szlapczynska, 2015) is also shown, that having found the values of those five parameters, it is possible to do an analytic conversion from conic to a more user-friendly parametric representation. Although (4) is a general equation taking into account the tilt of the ellipse, in practice the ellipse will be parallel to the ship's velocity vector, so the following parametric formula can be assumed:

$$\left(\frac{x-x_0}{a}\right)^2+\left(\frac{y-y_0}{b}\right)^2=1, \tag{8}$$

where: (x_0, y_0) is the ellipse's centre and a and b are semi axes.

It must be noted here that it is enough to perform the approximation process only once for a specified domain. The process does not have to be repeated in real-time during encounter situations and thus will not affect the constant computational complexity of determining domain-based parameters analytically.

Since the approximation process is presented here only for illustrative purposes, a very generic algorithm has been used in the paper to approximate a given polygon with an ellipse. It works as follows.

1) Find the minimal and maximal X and Y coordinate's values of polygon's vertices (in the Cartesian system with the ship in the centre): $X_{min}, X_{max}, Y_{min}, Y_{max}$.
2) Set $a=\frac{Y_{max}-Y_{min}}{2}$, $b=\frac{X_{max}-X_{min}}{2}$, $\Delta a=Y_{max}-a$, $\Delta b=X_{max}-b$.
3) If needed, rescale the elliptic domain by multiplying its parameters a, b, Δa and Δb by a common factor, so that the ellipse minimizes Eq. (7).

The first step of the algorithm finds the coordinates of a rectangle enclosing the ellipse. The second one sets the ellipse's proportions and the third one updates its size if needed. The factor mentioned in Step 3) can be easily found iteratively, e.g. by means of a binary search algorithm. However, in many cases values set in Step 2) are already acceptable or even optimal.

To complement the analysis of ship domain elliptic approximation a discussion on its accuracy is presented in Appendix A. The results presented there suggest that many domains can be approximated by an off-centred ellipse with reasonable accuracy.

3. An analysis of domain-based approach parameters

In (Szlapczynski, 2006) the author has defined a new domain-oriented collision risk measure – an approach factor f_{min}. For an encounter of a target, the approach factor f_{min} is the scale factor by which the target's domain has to be multiplied so that the own ship passed on the boundary of the f_{min}-scaled target's domain (assuming unchanged courses and speeds of both ships), as shown in Fig. 2. Consequently $f_{min} \geq 1$ represents safe passage and $f_{min} < 1$ means domain violations. For a given f_{min}, the degree of domain violation (DDV) can be defined as:

$$DDV=\max(1-f_{min},0) \tag{9}$$

DDV is a useful alternative for DCPA when ship domains are used, and in (Szlapczynski, 2006) it has been shown how using f_{min}/ DDV instead of DCPA favours COLREGS-compliant manoeuvres (IMO, 1972; Cockcroft and Lameijer, 2011) when planning evasive actions. However, true motion parameters were used there and developed f_{min} formulas were limited to Fuji domain (an ellipse with a ship in its centre). Below it will be shown how using relative parameters of a target makes it possible to derive all equations for a more general decentralised elliptic domain depicted in Fig. 1. All formulas have to be derived for f_{min} first and then DDV can be obtained by using (9).

A coordinate system with the own ship in its centre and the Y axis parallel to the own ship's velocity vector is given in Fig. 2, where a predicted domain violation is shown.

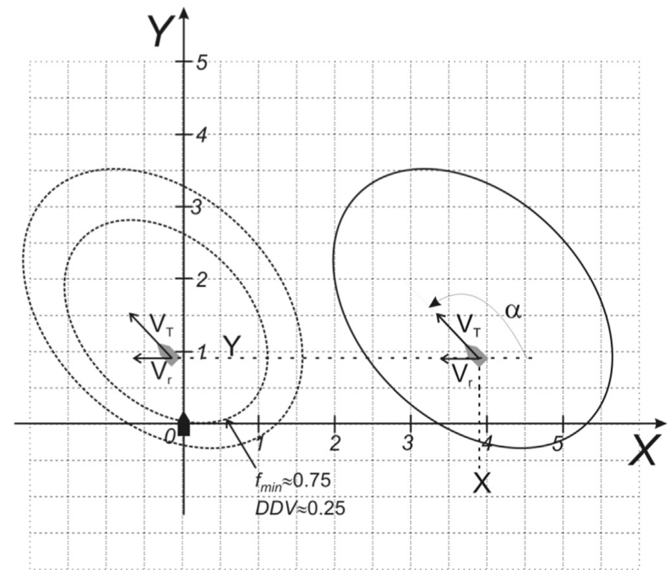

Fig. 2. A predicted violation of a target's domain presented in the own ship's relative coordinate system (V_T – true speed, V_r – relative speed).

Let us denote:

(X,Y) – relative position of a target,
(X_e, Y_e) – relative position of the centre of an ellipse being the target's domain,
(V_x,V_y) – components of the relative velocity of a target,
α – the rotation angle of the target's domain (being equal to course angle of the target), measured counter clockwise from X axis to the tip of a target's true speed vector (shown in Fig. 2).

The coordinates of the rotated ellipse's centre are:

$$X_e = X + h \tag{10}$$

$$Y_e = Y + k \tag{11}$$

where

$$h=\Delta a\cos\alpha+\Delta b\sin\alpha \tag{12}$$

$$k=\Delta a\sin\alpha-\Delta b\cos\alpha \tag{13}$$

For an f-scaled domain, that is one whose semi-axes dimensions are multiplied by f factor, Eqs. (10) and (11) have a more general form:

$$X_e = X + hf \tag{14}$$

$$Y_e = Y + kf \tag{15}$$

The elliptic domain moves with the relative speed of a target:

$$X_e(t) = X + hf + V_x t \tag{16}$$

$$Y_e(t) = Y + kf + V_y t \tag{17}$$

The parametric equation of a rotated ellipse with a centre in $(X_e(t), Y_e(t))$ as a function of time is:

$$\frac{(X_e(t)\cos\alpha + Y_e(t)\sin\alpha)^2}{a^2} + \frac{\left(X_e(t)\sin\alpha - Y_e(t)\cos\alpha\right)^2}{b^2} = 1 \tag{18}$$

Likewise, the parametric equation of the f-scaled ellipse (with the same centre) as a function of time is:

$$\frac{(X_e(t)\cos\alpha + Y_e(t)\sin\alpha)^2}{f(t)^2 a^2} + \frac{\left(X_e(t)\sin\alpha - Y_e(t)\cos\alpha\right)^2}{f(t)^2 b^2} = 1 \tag{19}$$

Solving (19) gives a formula for $f(t)$, as presented in Appendix B by (B-12) with (B-22), whose minimum over time t is the approach factor f_{min}. DDV is then obtained by substituting f_{min} to (9).

4. Determining time to domain violation

Time to domain violation (TDV) is the time remaining to entering the target's domain by the own ship. It can be determined similarly to approach factor f_{min}. A target's domain will be violated in the moment when the own ship will enter the ellipse, that is when (18) is fulfilled. After collecting like terms in (18) and using (B-1), (B-2) and (B-3) from Appendix B we get:

$$A_1X_e(t)^2 + B_1X_e(t)Y_e(t) + C_1Y_e(t)^2 - 1 = 0 \tag{20}$$

Substituting (12) for $X_e(t)$ and (13) for $Y_e(t)$ leads to

$$A_1\left(X_e^2 + 2X_eV_xt + V_x^2t^2\right) + B_1\left(X_eY_e + X_eV_yt + Y_eV_xt + V_xV_yt^2\right) + C_1\left(Y_e^2 + 2Y_eV_yt + +V_y^2t^2\right) - 1 = 0 \tag{21}$$

After collecting like terms and substituting

$$A_3 = A_1V_x^2 + B_1V_xV_y + C_1V_y^2 \tag{22}$$

$$B_3 = 2\left(A_1X_eV_x + C_1Y_eV_y\right) + B_1\left(X_eV_y + Y_eV_x\right) \tag{23}$$

$$C_3 = A_1X_e^2 + B_1X_eY_e + C_1Y_e^2 - 1 \tag{24}$$

we get

$$A_3t^2 + B_3t + C_3 = 0 \tag{25}$$

If $B_3^2 - 4A_3C_3 < 0$ then the domain will not be violated ($f_{min} > 1$, DDV$=0$) and (25) will have no solutions. If $B_3^2 - 4A_3C_3 = 0$ then the domain will be touched ($f_{min} = 1$, DDV$=0$) and (25) will have one solution. If $B_3^2 - 4A_3C_3 > 0$ then the domain will be violated ($f_{min} < 1$, DDV>0) and (25) will have two solutions, the first of them being TDV:

$$t_1 = \min\left(\frac{-B_3 - \sqrt{B_3^2 - 4A_3C_3}}{2A_3}, \frac{-B_3 + \sqrt{B_3^2 - 4A_3C_3}}{2A_3}\right) \tag{26}$$

$$t_2 = \max\left(\frac{-B_3 - \sqrt{B_3^2 - 4A_3C_3}}{2A_3}, \frac{-B_3 + \sqrt{B_3^2 - 4A_3C_3}}{2A_3}\right) \tag{27}$$

where $t_1 < t_2$,

t_1 – the time remaining to entering the target's domain - TDV,
t_2 – the time remaining to leaving the target's domain.

There are three possible cases here:

1. $t_1 < 0$ and $t_2 < 0$ represents past domain violations: a domain has already been entered and left,
2. $t_1 \leq 0$ and $t_2 \geq 0$ represents current violations: a domain has already been entered but not left,
3. $t_1 > 0$ and $t_2 > 0$ represents future violations: a domain will be entered in time t_1 and left in time t_2.

5. Examples of differences in using DCPA/TCPA and DDV/TDV

In this section examples of three kinds of comparison are presented. First, values of DCPA and degree of domain violation are compared for various encounter scenarios. Second, the sizes of minimum turns to starboard and port are determined for a safe distance and a decentralised elliptic domain. (In case of a safe distance the minimum manoeuvre means here passing at this safe distance and in case of a domain – passing at the domain's boundary.) Finally TCPA values are compared with TDV for a number of encounters. It must be noted here, that the section's purpose is not to argue the oversimplified nature of traditional approach parameters (hence statistical results are not provided). This has been done before by multiple researchers, who have shown that navigators naturally perceive risk as larger for some sectors around the ship. The section merely demonstrates how unacceptably far the inaptness of DCPA/TCPA combination can go in some typical encounter situations.

5.1. Domain model used in the examples

For all comparisons the parameterised elliptic shape shown in Fig. 1 has been used. The domain is similar in shape to classic domains proposed in (Goodwin, 1975; Davis et al., 1982; Coldwell, 1983): the starboard sector is here significantly larger than port sector and the fore sector larger than back sector. The domain used in this section has been updated so as to reflect more recent empirical observations (Hansen et al., 2013; Wang and Chin, 2016), where different dimensions have been suggested based on AIS data sets. However, the authors of (Hansen et al., 2013) mentioned that proposed domain is the smallest possible and in many cases navigators may opt for something larger. Also, in (Hansen et al., 2013) the authors did not explicitly take into account ship's speed, visibility conditions and manoeuvrability, all of which affect domain's size, as observed in (Wang and Chin, 2016; Zhu et al., 2001). Therefore, the domain's dimensions used in this work (shown in Fig. 1) are larger than those introduced in (Fuji and Tanaka, 1971) and in (Hansen et al., 2013) to cover the abovementioned effects:

$$a = 10L \tag{28}$$

$$b = 5L \tag{29}$$

where L is a ship's length. A target ship's length of about 370 m has been assumed for all encounters, which results in domain's semi-major axis a equalling 2 nautical miles – a value recommended in (IMO, 2007) as a threshold for warnings.

5.2. A comparison of DCPA and degree of domain violation

It is obvious that using a domain makes it possible to reflect the navigator's preferences and to make safe distance dependent on bearing. Below it will be shown to what extent the abovementioned holds and how inadequate DCPA can be, when compared with domain-based approach parameters: f_{min} and degree of domain violation (DDV). Domain violation has been checked for a total of 10 scenarios covering head-on, crossing and overtaking

Table 1
A comparison of DCPA and DDV – data.

Encounter number and type	Passing type	Coordinates of target's relative position		Coordinates of target's relative speed	
		X	Y	V_x	V_y
S1. head-on (180°)	Port to a target	12	1	−30	0
S2. head-on (180°)	Starboard to a target	12	−1	−30	0
S3. crossing (45°)	Astern of a target	1	−4	0	10
S4. crossing (45°)	Ahead of a target	−1	−4	0	10
S5. crossing (90°)	Astern of a target	7.42	−6	−15	15
S6. crossing (90°)	Aead of a target	4.58	−6	−15	15
S7. crossing (135°)	Astern of a target	10.24	−4	−20	10
S8. crossing (135°)	Ahead of a target	5.76	−4	−20	10
S9. overtaking (0°)	Port to a target	4	1	−10	0
S10. overtaking (0°)	Starboard to a target	4	−1	−10	0

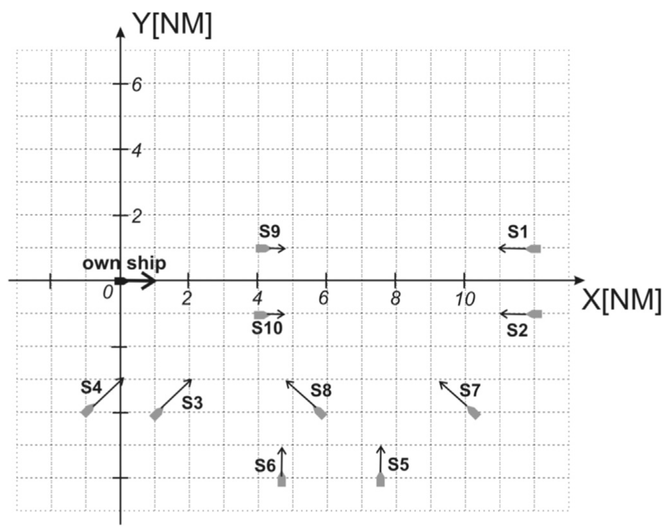

Fig. 3. Illustration of all scenarios from Tables 1 and 2.

encounters and passing ahead of a target, astern, port to a target, and starboard to a target. For all of the encounters DCPA is 1 NM, but domain-based parameters vary greatly. The data for the comparison are gathered in Table 1 and Fig. 3, the results – in Table 2.

As can be observed, various values of DDV have been registered for the same DCPA, which is reasonable, since navigators perceive collision risk as much smaller when acting according to COLREGS, that is when passing astern of a target (crossing) or port-to port (head-on). Using a safe distance and relying on DCPA is often misleading. In case of navigating within Traffic Separation Schemes (TSS) a certain distance may be safe when two ships are using a lane and their courses are parallel, but it may also mean close quarters when a ship is crossing a lane ahead of a target. TDV is significantly smaller than TCPA for cases of serious domain violation. However, it might be slightly larger than TCPA for minor domain violations, especially if crossing astern of a target. The cases of perpendicular crossing astern and ahead of a target (from Table 2) are discussed in detail below. In Fig. 4a perpendicular crossing astern of a target (in the centre of each picture) is shown, with the own true and relative speeds denoted by parallel and diagonal arrows respectively. The ships are entering each other's radar range in Fig. 4a), followed by the own ship approaching target's domain in Fig. 4b). Own ship's relative positions at TCPA and TDV are shown in Fig. 4c) and d) respectively, with the latter being 75 s later. DCPA for this encounter is 1 nautical mile and DDV is about 0.04, which means only a slight domain violation, probably tolerable by most navigators.

Analogically, in Fig. 5 subsequent phases of a perpendicular crossing ahead of a target are shown. DCPA is again 1 nautical mile, however DDV is about 0.52, meaning a serious domain violation. Such a value of DDV reflects COLREGS (as opposed to DCPA): ships should avoid crossing ahead of a target on starboard and if they do cross ahead, a much larger distance should be kept than in case of crossing astern. Own ship's relative positions at TDV and TCPA are shown in Fig. 5c) and d) respectively, with TDV preceding TCPA by nearly 5 min. Actually, as depicted by Fig. 5d), at TCPA the own ship is about to leave target's domain and the encounter is nearly over. Thus, navigators relying on a DCPA / TCPA could have doubts whether to take a collision avoidance action at all and if so – when to take it. They could either assess the situation as safe (DCPA value) or start a manoeuvre much too late (TCPA) with both decisions resulting in a major domain violation at best and a collision at worst. Contrary to this, a DDV/TDV combination gives a good assessment of the collision risk, not only suggesting an avoidance action (DDV) but also an early one (TDV).

Table 2
A comparison of collision risk assessment by DCPA and DDV – results.

Encounter type and number	Passing type	DCPA [NM]	f_{min} [/]	DDV [/]	TCPA [min.]	TDV [min.]
S1. head-on (180°)	Port to a target	1.0	1.333	0	24.0	N/A[a]
S2. head-on (180°)	Starboard to a target	1.0	0.8	0.2	24.0	20.35
S3. crossing (45°)	Astern of a target	1.0	0.952	0.048	24.0	26.53
S4. crossing (45°)	Ahead of a target	1.0	0.474	0.526	24.0	14.0
S5. crossing (90°)	Astern of a target	1.0	0.956	0.044	26.833	28.083
S6. crossing (90°)	Ahead of a target	1.0	0.476	0.524	21.166	16.45
S7. crossing (135°)	Astern of a target	1.0	1.345	0	29.383	N/A[a]
S8. crossing (135°)	Ahead of a target	1.0	0.652	0.348	18.616	12.45
S9. overtaking (0°)	Port to a target	1.0	1.333	0	24.0	N/A[a]
S10. overtaking (0°)	Starboard to a target	1.0	0.8	0.2	24.0	19.066

[a] N/A - there is no domain violation (DDV = 0), thus TDV is not applicable.

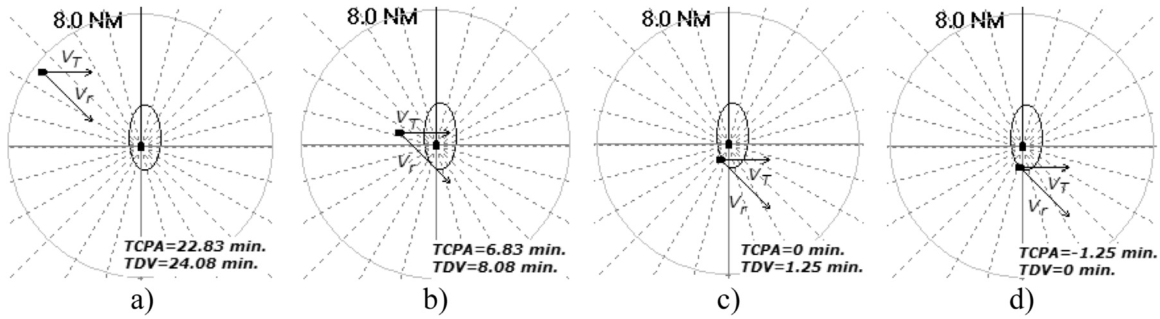

Fig. 4. Perpendicular crossing astern of a target: a) ships entering radar range, b) the own ship approaching target's domain, c) own ship's relative position at TCPA, d) own ship's relative position at TDV.

5.3. Turns to starboard and port necessary for keeping a safe distance or avoiding domain violation

As has been mentioned in the previous sub-section, according to COLREGS turns to starboard should be made to avoid collision, rather than turns to port, for crossing and head-on encounters. Using a domain with a starboard sector larger than port sector and fore sector larger than astern sector (Fig. 1) will usually favour correct decisions over incorrect ones. The data for the comparison are gathered in Table 3 and Fig. 6 (scenarios 1–4) and the results – necessary turns made to avoid violating a safe distance or a domain – in Table 4. The same simplified model of ship's dynamics has been applied for all scenarios to make the results easier to compare, though in practice the safe manoeuvre would depend on particular ship's manoeuvrability as well as speed, approach angle and environmental conditions (Zhang et al., 2012; Krata and Montewka, 2015). For all five scenarios DCPA equals 0 and DDV equals 1, which means that ships would crash if neither of them manoeuvred. Overtaking is not taken into account in Table 4 as COLREGS do not favour turning to port or starboard when overtaking. However data for this scenario is given in Table 3 because it is used further on for next experiment.

When ship domain is applied, then in three out of four cases it is easier to avoid domain violation by a COLREGS-compliant manoeuvre of turning to starboard (smaller turns to starboard than to port in the third and fourth column of Table 4). An exception is crossing encounter with a 45° difference between ships' courses. Yet, even in this case, the manoeuvre to starboard (65.5°) is not as large as the one determined for keeping a safe distance. The latter requires the same or nearly the same turns in either direction for three cases and a practically unrealistic turn of over 75° in the exceptionally hard fourth one (crossing encounter with a 45° course difference).

Table 3
Data for a comparison of necessary turns / a comparison of TCPA and TDV.

Encounter type and number	Coordinates of target's relative position		Coordinates of target's relative speed	
	X	Y	V_x	V_y
S1. head-on (180°)	12	0	−30	0
S2. crossing (45°)	0	−4	0	10
S3. crossing (90°)	6	−6	−15	15
S4. crossing (135°)	8	−4	−20	10
S5. overtaking (0°)	4	0	−10	0

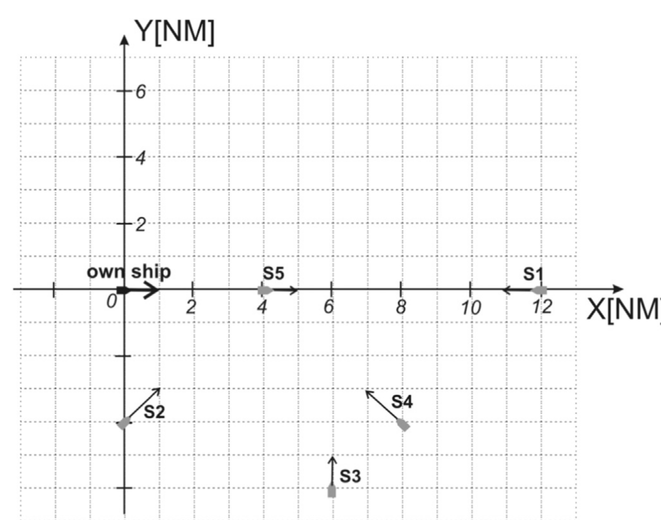

Fig. 6. Illustration of all scenarios from Table 3.

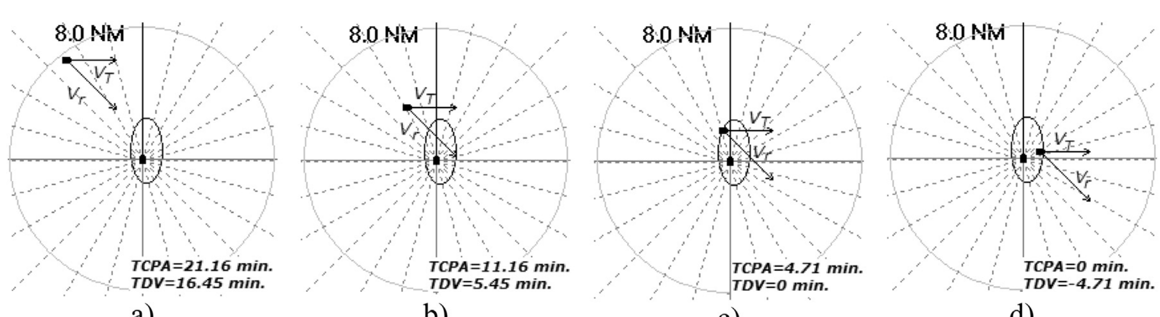

Fig. 5. Perpendicular crossing ahead of a target: a) ships entering radar range, b) the own ship approaching target's domain, c) own ship's relative position at TDV, d) own ship's relative position at TCPA.

Table 4
A comparison of turns necessary to avoid violating safe distance or a domain – results.

Encounter type and number	Turns necessary to avoid violating safe distance of 1 NM		Turns necessary to avoid violating elliptic domain	
	To starboard [°]	To port [°]	To starboard [°]	To port [°]
S1. head-on (180°)	19.33	19.33	10.25	17.39
S2. crossing (45°)	75.55	33.23	65.51	31.12
S3. crossing (90°)	28.1	28.1	17.12	47.38
S4. crossing (135°)	32.4	34.58	15.21	46.45

Table 5
A comparison of TCPA and TDV for an elliptic domain from Fig. 1.

Encounter type and number	DCPA [NM]	DDV [/]	TCPA	TDV
S1. head-on (180°)	0	1	24 min	19 min 8 s
S2. crossing (45°)	0	1	24 min	17 min 42 s
S3. crossing (90°)	0	1	24 min	21 min 2 s
S4. crossing (135°)	0	1	24 min	19 min 16 s
S5. overtaking (0°)	0	1	24 min	15 min 23 s

5.4. A comparison of TCPA and TDV for encounters with DCPA equal to zero

In the vast majority of cases a ship's domain will be violated before TCPA is reached, resulting in the TDV being significantly smaller than TCPA. For a fixed relative speed the difference between TCPA and TDV is proportional to the distance that a ship covers within target's domain before reaching closest point of approach (CPA), so obviously, the larger the domain and its violation, the larger this difference is. As evidenced by Table 5, those differences can be substantial for a domain from Fig. 1 and ships with DCPA=0 (data given in Fig. 6 and Table 3).

For the tested scenarios, the largest difference between TCPA and TDV (nearly 9 min) has been registered for an overtaking encounter and the relative speed of 10 knots. The reason for this is that the quotient of the astern sector's diameter and relative speed is larger than respective quotients for other domain sectors. It is worth noting, that for a head-on encounter with the relative speed of 30 knots, the difference between TCPA and TDV is still nearly 5 min (due to a large fore sector of a domain). The difference between TCPA and TDV is only 3 min for 90° crossing encounter (because of smaller domain's side sectors), and about 5–6 min for two other crossing encounters. In general TCPA can be so much larger than TDV, that it might contribute to the false sense of safety and consequently, to a delay in taking the necessary evasive action. TCPA also fails to distinguish between encounter types, giving the same values for all of them, despite the fact that domain violations will happen in much different times (nearly 6 min difference between TDV for perpendicular crossing and overtaking).

6. Conclusions and future work

While the literature is rich in works on ship domain models, publications on applying these domains in decision support systems up to this point there have been relatively scarce. Only a few researchers have been investigating this (Baldauf et al., 2011; Bukhari et al., 2013; Simsir et al., 2014 and Goerlandt et al., 2015). In particular there have been no analytic solutions which, for a given encounter situation, would address the three fundamental questions: will the ship's domain be violated, when and to what extent. The unfortunate consequence of this fact is the over-reliance of system designers and collision risk researchers on the old DCPA and TCPA approach parameters, which though often inadequate, are at least easy to determine. The paper aims to change the current state of things by presenting a solution to the domain violation problem. Two domain-based approach parameters: a degree of domain violation (DDV) and time to domain violation (TDV) are introduced and formulas for both of them are derived. The derivations assume elliptic domain model, though it is possible to use them for other shapes if a simple approximation is performed first. As it turns out, for an elliptic domain and relative motion parameters of a target, it is possible to reduce the problem to a sequence of quadratic polynomial equations, which are solvable analytically. In practice, analytical solution transfers to a constant computational time and thus it will not affect the computational complexity of an algorithm applied in a real-time system.

Possible systems where DDV and TDV parameters could be applied include collision avoidance systems (CAS) as well as systems dedicated to navigation in special environmental conditions. As for the former, (IMO, 2007) states that CAS functionality should be a part of Integrated Navigational Systems (INS) and should include collision avoidance alerts. According to this, "common criteria should be used for raising target related alerts, e.g., CPA/TCPA" and the resolution further recommends limits for collision warnings: DCPA of 2 nautical miles and TCPA of 12 min. Based on these IMO recommendations, some researchers propose their own four or five-level risk categorizations (Chin and Debnath, 2009; Baldauf et al., 2011 and Goerlandt et al., 2015). A similar alert-oriented system can be designed, based on DDV/TDV, with:

- cautions generated for passing in the proximity of a target domain,
- warnings generated for predicted minor domain violations and TDV lesser than a specified time,
- alarms raised for major domain violations and short TDV that is for situations demanding imminent action.

Precise threshold values for such a system, as well as other details are a subject of an ongoing work of the authors. As for navigation in special environment, it includes operating ships in ice conditions (Goerlandt et al., 2016), where a convoy of vessels follows an ice-breaker and keeping safe distances between particular ships is necessary. TDV would then be a natural replacement of TCPA, as for given motion parameters it would give time to violating those safe distances.

Hopefully the presented study will encourage both researchers and systems designers to complement traditional approach measures of TCPA/DCPA and related safety parameters (safe distance, radius or diameter) with their domain-based alternatives. The latter have long offered a better intuitive assessment of collision risk and now can do so without the past overhead of time-costing numerical computations.

Appendix A. – approximating ship domains with off-centred ellipses (Supplement to Section 2)

Altogether a total of five domains have been selected for the simulation: two crisp domains by Pietrzykowski and Uriasz (2009), two fuzzy ones by the same authors (with safety factor γ set to 0.5 and 1) and one empirically calibrated domain by Wang and Chin (2016), all presented, together with their possible elliptic approximations, in Fig. A1.

According to (Zhu et al., 2001) it is not the own ship's but the

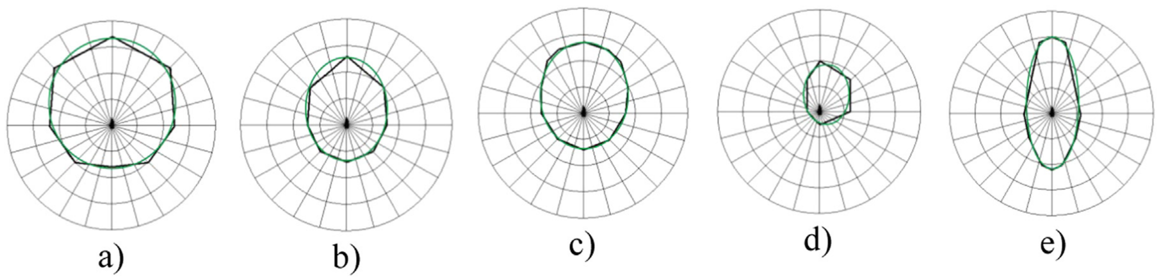

Fig. A1. Domain shapes (and their elliptic approximations) by a) Pietrzykowski crisp (mean size) b) Pietrzykowski crisp (minimal size) c) Pietrzykowski fuzzy (γ=0.5) d) Pietrzykowski fuzzy (γ=1.0) e) Wang and Chin.

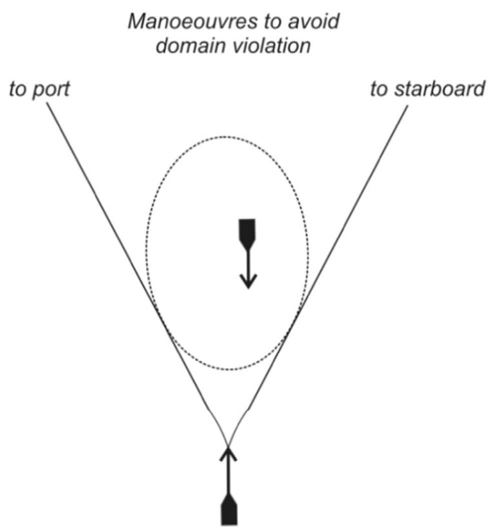

Fig. A2. Manoeuvres to port and starboard made to avoid violating target's domain.

Table A1
Errors of determining course alteration manoeuvres made to avoid violating target's domain.

Domain name	Manoeuvre	Errors of determining safe manoeuvres when using elliptic approximation			
		Diff. left [°]	Percentage diff. [%]	Diff. right [°]	Percentage diff. [%]
Pietrzykowski crisp (mean size)	Head-on	0.24	1.49	0.24	1.49
	Crossing 45°	0.35	1.47	0.98	2.42
	Crossing 90°	0.87	4.45	0.64	5.13
	Crossing 135°	0.19	1.46	0.11	1.09
	Overtaking	0.97	6.48	0.97	6.48
Pietrzykowski crisp (minimal size)	Head-on	0.04	0.23	0.04	0.23
	Crossing 45°	0.47	1.98	0.3	0.76
	Crossing 90°	0.92	4.41	0.65	5.04
	Crossing 135°	0.37	3.02	0.52	5.72
	Overtaking	0.73	3.96	0.73	3.96
Pietrzykowski fuzzy (γ=0.5)	Head-on	0.12	0.67	0.09	0.52
	Crossing 45°	0.04	0.19	0.08	0.20
	Crossing 90°	0.05	0.26	0.1	0.86
	Crossing 135°	0.12	0.98	0.35	3.89
	Overtaking	0.37	2.49	0.59	3.95
Pietrzykowski fuzzy (γ=1.0)	Head-on	0.43	1.95	0.41	1.94
	Crossing 45°	0.1	0.37	0.33	0.68
	Crossing 90°	0.04	0.17	0.29	1.81
	Crossing 135°	0.04	0.25	0.22	1.69
	Overtaking	0.24	1.72	0.18	1.21
Wang and Chin empirical domain	Head-on	0.91	5.21	0.91	5.21
	Crossing 45°	0.09	0.44	0.23	0.77
	Crossing 90°	0.31	1.91	0.13	1.32
	Crossing 135°	0.19	1.46	0.11	1.09
	Overtaking	0.34	2.97	0.34	2.97

target's domain that is taken into account, when collision avoidance manoeuvre is planned by navigators. The same policy has been applied here. The accuracy of an elliptic approximation of each of the five chosen domains has been tested for five scenarios involving manoeuvres to avoid violating target's domain. Those scenarios were: a head-on encounter, an overtaking encounter and three crossing encounters (the differences between two ships courses for the crossing encounters being 45, 90 and 135 degrees). For each domain and each scenario manoeuvres to avoid target's domain have been determined for both the original domain and its approximation. An example of such manoeuvres to starboard and port for elliptic approximation is shown in Fig. A2. Errors of determining those manoeuvres (differences between manoeuvres made for original domains and their approximations) are given in Table A1.

The average difference between course alterations determined for the original domain and its approximation is 0.36° (about 2%). The largest errors have been registered for the first of the test domains (Fig. A1a) due to the polygon having 8 vertices only, but even here errors do not exceed 1°. In general, the elliptic approximation described above can be considered sufficiently accurate to emulate other user-specified shapes in decision support systems, especially if more refined approximation algorithms are used.

Appendix B. –derivation of formula for DDV (Supplement to Section 3)

By substituting in (19)

$$A_1 = \frac{\cos^2\alpha}{a^2} + \frac{\sin^2\alpha}{b^2} \tag{B-1}$$

$$B_1 = 2\sin\alpha\cos\alpha\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \tag{B-2}$$

$$C_1 = \frac{\sin^2\alpha}{a^2} + \frac{\cos^2\alpha}{b^2} \tag{B-3}$$

and collecting like terms we get

$$A_1X_e(t)^2 + B_1X_e(t)Y_e(t) + C_1Y_e(t)^2 = f(t)^2 \tag{B-4}$$

$X_e(t)$ and $Y_e(t)$ are linear functions of scale factor *f*, which in turn is a function of time *t*. After substituting (16) and (17) to (B-4) and collecting like terms we get

$$A_2 f(t)^2 + (B_{21} + B_{22}t)f(t) + C_{21} + C_{22}t + C_{23}t^2 = 0 \quad \text{(B-5)}$$

where

$$A_2 = A_1h^2 + B_1hk + C_1k^2 - 1 \quad \text{(B-6)}$$

$$B_{21} = (2A_1X + B_1Y)h + (2C_1Y + B_1X)k \quad \text{(B-7)}$$

$$B_{22} = (2A_1V_x + B_1V_y)h + (2C_1V_y + B_1V_x)k \quad \text{(B-8)}$$

$$C_{21} = A_1X^2 + B_1XY + C_1Y^2 \quad \text{(B-9)}$$

$$C_{22} = 2(A_1XV_x + C_1YV_y) + B_1(XV_y + YV_x) \quad \text{(B-10)}$$

$$C_{23} = A_1V_x^2 + B_1V_xV_y + C_1V_y^2 \quad \text{(B-11)}$$

Solving quadratic Eq. (B-5) gives formulas for *f(t)*

$$f_{1,2}(t) = \frac{-(B_{21} + B_{22}t) \mp \sqrt{D_1t^2 + E_1t + F_1}}{2A_2} \quad \text{(B-12)}$$

where

$$D_1 = B_{22}^2 - 4A_2C_{23} \quad \text{(B-13)}$$

$$E_1 = 2B_{21}B_{22} - 4A_2C_{22} \quad \text{(B-14)}$$

$$F_1 = B_{21}^2 - 4A_2C_{21} \quad \text{(B-15)}$$

$f(t) = f_{min}$, when $\frac{f(t)}{dt} = 0$. The formula for $\frac{f(t)}{dt}$ is

$$f_{1,2}'(t) = -\frac{B_{22}}{2A_2} \mp \frac{2D_1t + E_1}{4A_2\sqrt{D_1t^2 + E_1t + F_1}} \quad \text{(B-16)}$$

$f_{1,2}'(t) = 0$ when

$$\frac{B_{22}}{2A_2} = \mp \frac{2D_1t + E_1}{4A_2\sqrt{D_1t^2 + E_1t + F_1}} \quad \text{(B-17)}$$

After raising both sides of (B-17) to the square and rearranging we get

$$D_2t^2 + E_2t + F_2 = 0 \quad \text{(B-18)}$$

where

$$D_2 = D_1(B_{22}^2 - D_1) \quad \text{(B-19)}$$

$$E_2 = E_1(B_{22}^2 - D_1) \quad \text{(B-20)}$$

$$F_2 = F_1B_{22}^2 - E_1^2 \quad \text{(B-21)}$$

After collecting like terms and solving the resulting quadratic equation, we get formulas for t_{min1} and t_{min2}, which minimize (B-12).

$$t_{min1,2} = \frac{-E_2 \mp \sqrt{E_2^2 - 4D_2F_2}}{2D_2} \quad \text{(B-22)}$$

Substituting $t_{min1,2}$ from (B-22) as t to (B-12) gives up to four potential values of approach factor *f*. Since *f* is by definition a non-negative value, the potential negative solutions can be eliminated, as can be those, which are not real numbers. In case of more than one non-negative solution, f_{min} is always the smallest value of *f* obtained from (B-12) and t_{min} is the value that has been used in (B-12) to get f_{min}. Contrary to f_{min}, t_{min} can be negative. In such cases it signals that the own ship is moving away from the target's domain and it represents the time that has elapsed from reaching f_{min}. Once f_{min} is calculated DDV can be obtained by substituting it to (9).

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