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Genetic Algorithm Based Collaborative Optimization of a Tiltrotor Configuration

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The rotorcraft optimization problem is characterized by challenges stemming from computationally demanding analyses, complex multidisciplinary interactions, and a design space that may contain several local optima. These problems are compounded further by the additional complexity of the tiltrotor configuration. At the very outset, the aerodynamic and structural design of the proprotor blade is tightly coupled, exhibiting strong multidisciplinary interactions. The rotor and wing systems also exhibit interactions, ranging from moderate to strong. A rotor and wing design problem has been formulated that is sufficiently complex so as to represent industrial practice, and is easily scalable from moderate to large size. The details of the problem formulation have been presented in a companion paper. The current paper explores the solution of the optimal design problem. A collaborative optimization scheme was considered to handle the potentially large-scale problem and manage the multi-disciplinary design interactions. Separation of the rotor and wing into subsystems was shown to be a reasonable choice for decomposition, though approximations were necessary to ensure a manageable set of shared variables between the two sub-systems. Optimal search was based on a collaborative optimization approach that computed the sub-problem optimal design solution at each iteration, or function evaluation of the master problem. While this approach magnified the number of function evaluations, it allowed a global search of the design space when coupled to genetic algorithm optimal search at the master and sub-problem levels. Neural network function surrogates were used for the sub-problems; this provided a relatively quick run time for the sub-problem optimal searches. The strategy proposed in the paper allowed use of surrogate models in a manner that mitigated the influence of inaccuracy in their predictive capability. The strategy was shown to be relatively efficient in terms of the required number of exact function evaluations, particularly when compared to an all-in-one approach.

Nomenclature

d	=	individual hub load desirability
D	=	combined hub loads desirability
F	=	objective function (multiple objectives)
K	=	hub loads weighting factor
G	=	constraint vector
W_{rotor}	=	rotor system weight
W_{fuel}	=	fuel weight for cruise mission segment
W_{wing}	=	wing system weight
X	=	design variable vector
Y	=	design objective vector, system responses
V	=	hub loads vector

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c_{tip}	=	rotor tip chord
Ω_{cruise}	=	airplane mode cruise rotor speed
t/c	=	wing thickness ratio

I. Introduction

A problem formulation has been developed for the integrated design of the rotor and wing systems of a tiltrotor aircraft. The problem was shown to exhibit multidisciplinary interactions between the aerodynamic and structural design of the rotor as well as interactions between the rotor and wing systems. The primary objective was to minimize the combined weights of the rotor and wing, and the fuel load required for the cruise flight segment. Additional objectives were six vibratory hub loads, which are the primary source of fuselage vibrations. The proposed design problem attempted to represent key disciplines though inclusion of aerodynamic performance, loads, strength, and aeroelastic stability. Rotor and wing design interactions were primarily in the coupled flutter analysis (stability and natural frequencies) and the effect of wing thickness on wing strength and aerodynamic performance. Increased wing thickness increases profile drag, which increases the power required for airplane mode cruise. However, whirl flutter stability requirements may push the wing stiffness up, requiring a thicker wing. Details of this problem are more fully described in a companion paper[1].

The set of design variables included the numbers of plies in various parts of several cross sections for the rotor blade and the wing. An additional structural design variable for the rotor included the chord-wise location of the heel of the blade spar. Three design variables defined the blade twist verses span and two design variables defined the blade aerodynamic and structural thickness ñ span-wise locations of 18 percent and 12 percent thick airfoils. The blade planform was straight-tapered and defined by the tip chord design variable and the fixed root chord. Rotor speed for the airplane mode cruise configuration was also included as a design variable. The total number of design variables associated with the rotor was 18.

Wing design variables were essentially the same as the rotor blade structural design variables. Additional wing structural design variables included the chord-wise locations of the forward and aft spars, and the number and area of the stringers. Wing thickness was a key design variable for its effect on stiffness, stability, and strength, as well as its effect on power required to overcome the profile drag that varies with wing thickness. The total number of design variables associated with the wing was 33.

Problem size and function execution time made all-in-one execution unpractical. Therefore surrogates were considered. Neural networks have been used in rotorcraft optimization with a measure of success, though inaccuracies in function approximations may result in the search identifying an infeasible design. Decomposition was also considered. Collaborative optimization is a decomposition strategy for managing the efforts of separate teams while decomposing the problem to reduce the solution effort. This paper presents a solution strategy for the tiltrotor design problem using collaborative optimization employing genetic algorithms[2] in a global search strategy. The approach used neural network surrogate functions to reduce the total number of executions of the full analysis but does so in a way that tends to mitigate inaccuracy.

II. Collaborative Optimization

Collaborative optimization[3] is a bi-level strategy that allows a large-scale design problem to be decomposed in a manner that mimics design practice. That is, the decomposed design problems are parsed to their natural owners such as disciplinary experts, sub-system design groups, or integrated product teams. The experts maintain a certain level of autonomy in the formulation and execution of the sub-problems. Sub-problems handle local constraints; however they do not control the design objectives; these are handled by the master problem.

Sub-problems contain the associated local constraints and design variables. The sub-problem objective is to minimize the difference between certain sub-system design variables and system responses, and the corresponding system-level targets from the master problem. This discrepancy is usually formulated as the sum of the square of the differences. The integrity of the sub-problems is maintained by the local constraints associated with each sub-problem. Design variable sets include the specific design variables that are associated with each sub-problem as well as shared variables. It is these shared variables that are included in the sub-problem objective function, along with certain calculated responses that must match their targets from the master problem. Design variables that are used to compute responses in two or more sub-problems are included in the set of shared variables.

The master problem minimizes the system objective subject to coordination constraints. These constraints are formulated identically as the sub-problem objectives, though their function is to ensure that the targets found by the master problem allow a feasible solution to the sub-problems. The master problem manipulates shared variables to

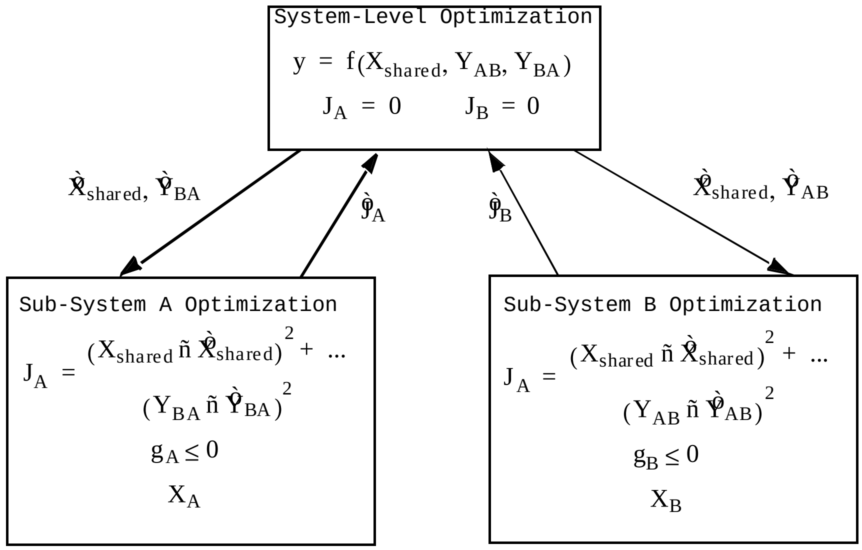

Figure 1. Collaborative Optimization Schematic

find targets for the sub-problems. In addition to shared design variables, the set of targets that are passed to the sub-problems can include system responses, or values that are functions of design variables.

Collaborative optimization is outlined in Figure 1, which shows two sub-problems attempting to match shared variables and system responses to the target values from the master problem using local variables subject to local constraints. The sub-problems return information about the ability of the sub-problems to match the targets within the bounds of the local sub-problem constraints. The size of the coordination constraint in the master problem represents the ability of the sub-problem to find a feasible design for the given set of master problem targets.

Data interchange generally includes sensitivities of the sub-problem with respect to problem parameters, which in this case are the targets, so that an approximation to the discrepancy constraint can be used by the master problem. This will be further explored below.

III. Tiltrotor Decomposition Strategy

The current problem is easily scaled from moderate to large size by considering additional blade and wing cross sections, or by converting cross section details that are fixed parameters into design variables. It was assumed that, while the general formulation represented a fairly complete design problem, actual practice would employ many more design variables as well as additional constraints. The conventional wisdom when considering collaborative optimization is that the line of separation should be drawn where the interactions are not too strong. It is also desirable that the size of each sub-problem should be significantly smaller than the original problem.

Shared variables are selected from the design variable set by those that are required in more than one sub-system to compute any system responses. Ref. 1 presented the complete set of design variables for the proposed tiltrotor problem. Selection of shared variables will be expounded next. They are the blade chord at the tip c_{tip}, airplane mode cruise rotor speed Ω_{cruise}, and wing thickness ratio t/c. All other design variables will be noted simply as part of the design variable vector X.

Table 1 summarizes the association of the grouped rotor and wing design variables for the calculation of objectives (ΣW and Hub Loads) and constraints (G). Note that the rotor, wing, and fuel weights require all design variables but one (the angle of the cross plies in the wing X_{25}). All other design variables either contribute directly to structural weight or directly to the cruise performance that defines fuel weight. Cruise performance requires one wing design variable, wing thickness t/c, which contributes to the aerodynamic drag from the wing. The secondary objectives, six vibratory hub loads, require only those design variables associated with the rotor.

Constraints are divided into those associated with the rotor and those associated with the wing; each requiring only their respective design variables for evaluation. A third group has those associated with the coupled rotor and wing problem. These are the coupled natural frequency avoid constraints at the airplane mode and helicopter mode rotor speeds, and the whirl flutter damping constraint. These coupled rotor/wing constraints require all of the wing design variables because the analysis requires the normal modes of the wing/fuselage. All of the rotor design variables are required except those that define the blade twist (these are not required because of linearized pertubational aerodynamics in the whirl flutter analysis).

Table 1. System Design Variables and Responses Matrix

	Rotor Design Variables: X_{rotor}	Wing Design Variables: X_{wing}
System Weight: W_{rotor}, W_{wing}, W_{fuel}	$X_{1\text{-}18}$	t/c, $X_{20\text{-}24,26,51}$
Hub Loads: V	$X_{1\text{-}18}$	
Rotor System Constraints: G_{rotor}	$X_{1\text{-}18}$	
Rotor/Wing Constraints: $G_{rotor/wing}$	c_{tip}, Ω_{cruise}, $X_{2,3,8\text{-}18}$	t/c, $X_{20\text{-}51}$
Wing System Constraints: G_{wing}		t/c, $X_{20\text{-}51}$

If not for the coupled rotor/wing constraints the rotor and wing sub-problems could be cleanly separated at the two sub-systems, with the only the tip chord, wing thickness, and rotor speed as shared variables. Including the coupled rotor/wing constraints, however, the wing sub-problem would contain its own set of 33 design variables in addition to 15 of the 18 design variables from the rotor. The wing sub-problem would contain nearly as many design variables as the original problem. Scaling the problem up in size would only exacerbate this situation because scaling adds mostly to the structural design variables.

Collaborative optimization cannot be exploited in this form; however, approximations can be used to reduce the number of rotor design variables that appear in the wing sub problem. Approximations are possible because the coupled rotor wing analysis used only the fundamental, or rigid-body, rotor modes, which depend on a few basic aggregate properties. Coupled rotor/wing analysis also used kinematic couplings that relate blade pitch, lead-lag, and pitch motions. However, these couplings strongly depend on the geometry of the control system, which was fixed in the hub concept selection, and vary only slightly from the elastic motions, which change with blade stiffness. The following discussion describes approximations that are sufficiently accurate so that the coupled rotor/wing analysis could be run using only a few basic design variables to define the rotor properties.

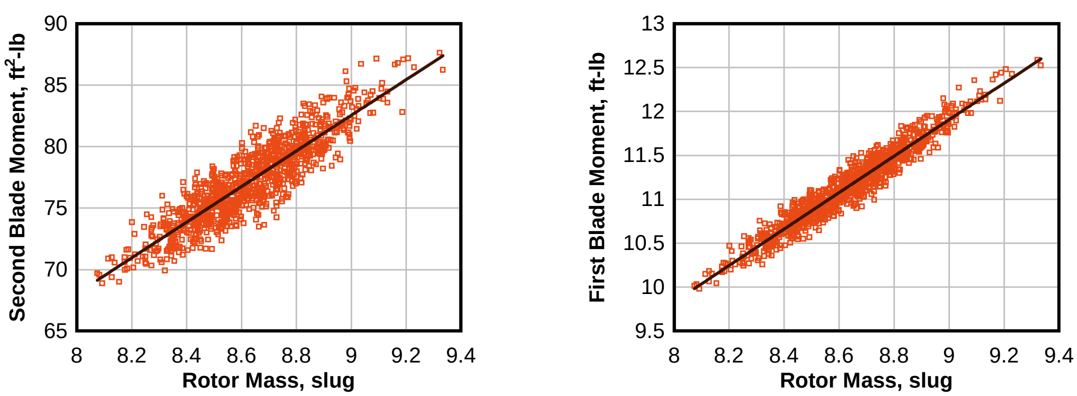

Figure 2. Use of Rotor System Mass to Approximate Blade Inertia

Rotor design variables include 11 that define the internal structural design of the blade. These, in addition to 2 variables that define the blade thickness, are used in the coupled rotor/wing analysis in the aggregate ñ as rotor mass and first and second moments of inertia. The tip chord variable also influences these parameters, but only slightly because its effect on weight is embedded in the total blade weight. The mass properties that are required for the fundamental blade modes can be approximated using only the blade weight, which was available in the primary objective function by using the rotor weight. Figure 2 shows survey data (random designs) of blade inertia properties with trend lines. Maximum errors when using the trend lines are 6 percent for the second mass moment and 1 percent for the first mass moment.

Frequencies of the fundamental blade modes can be approximated using rotor speed and blade weight. A first order surface based on rotor speed and weight provided an excellent approximation of the first lag mode. Figure 3 shows how the first lag mode from the survey data forms a plane when plotted verses rotor mass and rotor speed. The first order surface provided an approximation with the maximum error less than 0.03-per-rev. As for the first flap frequency, it ranged only from 1.035-per-rev to 1.039-per-rev in the design space survey, so a constant number was used. The design space survey also showed that the variations in kinematic coupling from blade elasticity were not significant, having only a small effect on the coupled rotor/wing problem. Therefore a fixed nominal value was used in the wing design sub-problem. Kinematic coupling itself has a very strong effect on stability. This drives the selection of control system geometry.

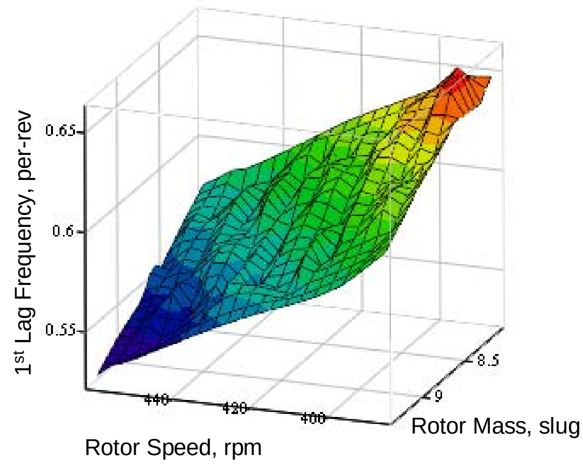

Figure 3. Use of Rotor System Mass and Rotor Speed to Approximate First Lag Frequency

Note that rotor system mass is part of the primary objective ñ reduced structural weight. Rotor mass is a system response that is computed within the rotor system design and manipulated by the master problem, but it can be used in the coupled rotor/wing problem as shown above. Therefore the rotor mass was a system-level variable that was passed to the wing sub-problem as well as the rotor-sub-problem. In collaborative optimization this role is referred to as a target variable; as such it is presented to the sub-problems as a target that must be matched, like a shared variable.

With these approximations, which introduce only very small errors, the wing sub-problem was formulated to contain only 3 additional variables associated with the rotor, rather than 15. Furthermore, increasing the size of the rotor sub-problem will not increase the number of shared variables. Table 2 restates of the presentation of Table 1, where the various rotor design variables, $X_{2,3,8\text{-}18}$, are now consolidated in the weight of the rotor system, W_{rotor}, which is part of the primary object function.

Table 2. System Design Variables and Responses Matrix

	Rotor Design Variables: X_{rotor}	Wing Design Variables: X_{wing}
System Weight: W_{rotor}, W_{wing}, W_{fuel}	$X_{1\text{-}18}$	t/c, $X_{20\text{-}24,26,51}$
Hub Loads: V	$X_{1\text{-}18}$	
Rotor System Constraints: G_{rotor}	$X_{1\text{-}18}$	
Rotor/Wing Constraints: $G_{rotor/wing}$	c_{tip}, Ω_{cruise}, W_{rotor}	t/c, $X_{20\text{-}51}$
Wing System Constraints: G_{wing}		t/c, $X_{20\text{-}51}$

IV. Multiple Objectives

Multi-criteria optimization involves compromise so that all objectives are reduced to the lowest level possible. A Pareto-optimal solution is such that no further decrease in one objective can occur without increasing another objective. A weighted-sum approach has been commonly used in rotorcraft optimization with success; however the Pareto-optimal solution is different for each selection of weights. The current problem statement has seven objective functions divided into two sets. The first set is the sum of the weights of the wing, rotor, and fuel for the cruise segment. The second set is the six vibratory hub loads, which are calculated in the rotor sub-problem only. This section discusses the treatment of multiple objectives.

Ideally, all objectives can be recast into a form that represents the same high-level value metric. The sum of weights in the current objective does this in that valuable payload can be gained by reductions in any of the system or fuel weights. Vibratory hub loads objectives, however, cannot be added to the weights, at least not directly.

It has been noted that vibration treatment in rotorcraft accounts for two to five percent of the empty weight[4]. This is through dynamic tuning of structural stiffness of through direction treatment devices such as vibration absorbers. The hub loads objectives can be consolidated into one measure using the desirability function, which is an approach to multi-criterion optimization[5]. Desirability is measured by how closely an objective is to its desired value. In the minimization problem, an arbitrary lower bound is used to define this desired value. An arbitrary upper bound is also used in the formulation of the desirability function. Equation 1 shows the individual desirability functions d_j as well as the geometric mean D, which is the combined desirability function. The exponent r is a parameter that defines the shape of the desirability functions and is set to the value 1 for a linear mapping. Both the individual and combined desirability functions map the responses from zero to one.

$$d_j = \left(\frac{y_j - y_j^L}{y_j^U - y_j^L} \right)^r \qquad D = \left(\prod_j d_j \right)^{1/J} \tag{1}$$

The desirability formulation provides a method to combine vibratory hub loads objectives with the weight objectives by considering the lowest possible hub loads to add no weight and the worst hub loads to add a high fraction of empty weight. It was assumed in this problem that the worst hub loads would represent 3.5 percent empty weight, 318.5 pounds. The multi-objective statement is as shown in Eq. (2), where K is 318.5 and D is the combined desirability of the vibratory hub loads.

$$F = (W_{rotor} + W_{wing} + W_{fuel}) + K \cdot D \tag{2}$$

V. Statement of the Collaborative Optimization Problem

The previous sections laid the groundwork for solution of the integrated tiltrotor design problem, especially by providing approximations to provide a decomposition strategy that can reduce effort, especially in the wing sub-problem. A treatment of multiple objectives was also presented. This section presents the collaborative optimization problem statement.

Three objective functions were tested to determine the effectiveness of the multiple objectives approach. Two contained changes from the nominal objective. The first change was an increase in the weight associated with the combined hub loads desirability. The second change was in the 3-per-rev vertical hub shear desirability function as an increase to the upper limit. The three objective functions are shown in Eqs. (3), (4), and (5).

$$\begin{gathered} F^A = (W_{rotor} + W_{wing} + W_{fuel}) + K^A \cdot D^A \\ K^A = 318.5 \\ D^A = \left(\left(\frac{V_{3P}^Z - 600}{3200 - 600} \right) \cdot d_2 \cdot \ldots \cdot d_6 \right)^{\frac{1}{6}} \end{gathered} \tag{3}$$

$$\begin{gathered} F^B = (W_{rotor} + W_{wing} + W_{fuel}) + K^B \cdot D^B \\ K^B = 650.0 \\ D^B = \left(\left(\frac{V_{3P}^Z - 600}{3200 - 600} \right) \cdot d_2 \cdot \ldots \cdot d_6 \right)^{\frac{1}{6}} \end{gathered} \tag{4}$$

$$\begin{gathered} F^C = (W_{rotor} + W_{wing} + W_{fuel}) + K^C \cdot D^C \\ K^C = 318.5 \\ D^C = \left(\left(\frac{V_{3P}^Z - 600}{5120 - 600} \right) \cdot d_2 \cdot \ldots \cdot d_6 \right)^{\frac{1}{6}} \end{gathered} \tag{5}$$

These objective functions formed the basis of the master problem. Master problem design variables were the three weights and the combined hub loads desirability, and the three shared variables. The master problem optimizer operated on these design variables subject to the compatibility constraints. Compatibility constraints were formulated as the sum of discrepancies squared. Equation (6) is the compatibility constraint for the rotor sub-system and Eq. (7) is for the wing sub-system. Note that for this problem the shared variables appear only in the coordination constraints. This is how shared variables are coordinated between sub-systems.

$$J_{rotor} = \begin{matrix} \left(W_{rotor} - \tilde{W}_{rotor}\right)^2 + \left(W_{fuel} - \tilde{W}_{fuel}\right)^2 + \left(D - \tilde{D}\right)^2 + \\ \left(c_{tip} - \tilde{c}_{tip}\right)^2 + \left(\Omega_{cruise} - \tilde{\Omega}_{cruise}\right)^2 + \left(t/c - \widetilde{t/c}\right)^2 \end{matrix} \tag{6}$$

$$J_{rotor} = \begin{matrix} \left(W_{rotor} - \tilde{W}_{rotor}\right)^2 + \left(W_{wing} - \tilde{W}_{wing}\right)^2 + \\ \left(c_{tip} - \tilde{c}_{tip}\right)^2 + \left(\Omega_{cruise} - \tilde{\Omega}_{cruise}\right)^2 + \left(t/c - \widetilde{t/c}\right)^2 \end{matrix} \tag{7}$$

These compatibility constraints are the objective functions for the respective sub-problems. Sub-problem design variables are the shared variables plus the other design variables that are associated uniquely with the sub-problem. Constraints are the disciplinary constraints of each sub-problem.

The collaborative optimization statement is shown in Fig. 4. The key feature is the passing of targets (shared variables and target responses) from the master problem to each sub-problem, and the passing from the sub-problems of the values of the discrepancy functions. Note that collaborative optimization generally does not pass the actual value of the discrepancy function but rather passes sensitivity derivative information to formulate an approximation for the compatibility constraints in the master problem. This work proposed to execute a full optimal search in each sub-problem each time the master problem requires a value for the coordination constraints. This represents a literal interpretation of Fig. 4. Note that this approach magnifies the number of function evaluations because the sub-problems optimal searches are performed as many times as required to solve the master problem. Clearly surrogates for the system responses must be employed; this will be discussed next.

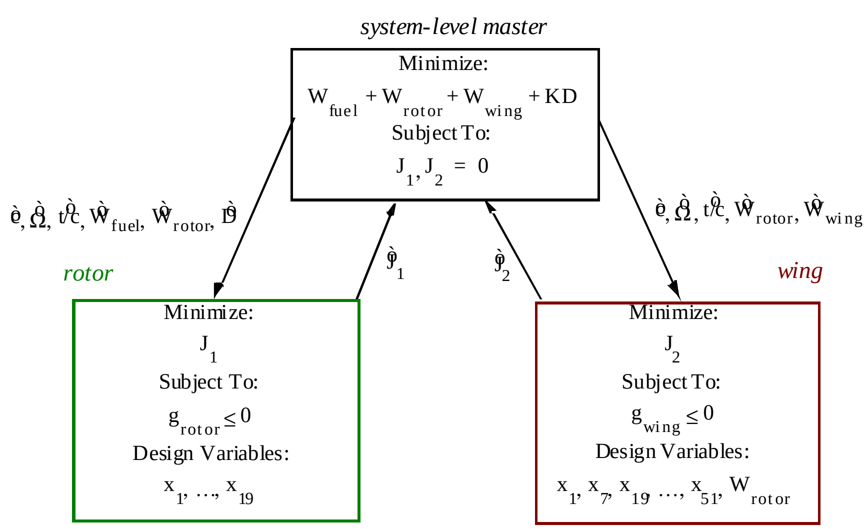

Figure 4. Collaborative Optimization of the Rotor/Wing Problem

This approach does not required sensitivity information from the sub-problems because it does not construct approximations of the coordination constraints in the master problem. When executed with genetic algorithm based optimal search for the master problem and the sub-problems, the collaborative optimization approach becomes a global optimal search.

VI. Neural Network Surrogates for System Responses

Neural networks have been used to generalize input output relationships for nonlinear functions. They provide quick-running surrogates for the system responses. Neural networks have been used successfully in rotorcraft optimization[6]. Studies in Ref. 7 also demonstrated the utility of neural networks in rotorcraft optimization; however inaccuracies in function approximation often resulted in the search identifying an infeasible design. This section will demonstrate the use of neural network surrogates in an approach that mitigates inaccuracy. This work employed a back-propagation neural network.

Execution of the rotor sub-problem optimal search required several days of computer run-time (HP C3600 workstation, 550 MHz). Most of the CPU time was consumed by rotor analysis for performance, loads, and natural frequencies. Solution using neural network surrogates required a small fraction of the time. Several sub-problem searches were executed both with exact analysis and neural network surrogates to examine the performance of the surrogates.

It was discovered that the neural network surrogates for the coupled rotor/wing responses were not satisfactory. Inaccuracies of neural networks sometimes allowed the optimal search to produce an infeasible design (as determined by checking the optimal design with exact analysis). This problem was much worse when the coupled rotor/wing constraints were involved. The mapping of inputs to responses was initially a direct link from design variables to constraint values. For the coupled rotor/wing constraints the mapping was changed so that some pre and post-processing operations were performed outside of the neural network. This changed the mapping from a direct computation of constraints from design variables to an intermediate mapping of wing beam properties to frequencies and damping. While extra steps were required, they were not CPU intensive operations, so little time was added to the overall computation of system responses. While this scheme greatly improved the overall accuracy of constraints from design variables, the same accuracy problem was still present. That is, the optimal search still tended to find a design that, when evaluated with exact analysis was infeasible. The extent to which it was infeasible, however, was much less so than before.

Compatibility constraints in the master problem, which are merely the optimal value of the discrepancy functions of the sub-problems, require a numerical value at each step or response evaluation in the master problem. This part of the data interchange was seen in Fig. 4 as J_{rotor} and J_{wing}. Small values of these constraints mean that the sub-problems can produce a feasible design that matched the target values. No information is required, however, about the design that produced these values. This observation was the basis for the use of neural network surrogates in the sub-problem execution.

Rotor sub-problem executions were tested with 14 sets of targets, both with exact function evaluations and with neural network surrogates, and both with a genetic algorithm search (wing sub-problems were similarly tested with

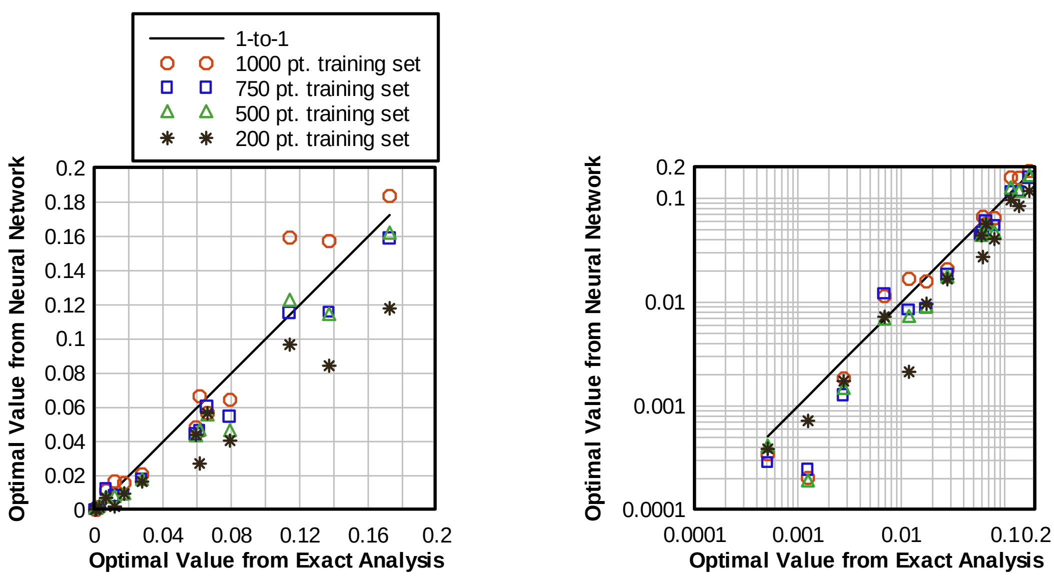

Figure 5. Comparison of Optimal Values using Exact Analysis and Neural Network Surrogates

the same conclusions). Four sets of neural networks were trained using different sizes of uniform random training sets: 1000, 750, 500, and 200 points. Corresponding discrepancy values of the 14 test cases were compared in Fig. 5, which shows the same data on linear and log-log scales. Discrepancies were actually normalized so that they could be meaningfully subtracted. It was determined that a satisfactory target for the normalized discrepancy is 0.0001.

Proximity to the unity line (1-to-1) represents the ability of the neural network-based searches to achieve the same discrepancy for the given targets as the same search with exact analysis. Note that discrepancy function values range several orders of magnitude, which suggests that the neural network based solutions need only be accurate to an order of magnitude. Furthermore it was seen that, overall, smaller training sets produced results with proximity to the unity line as close as with 1000 points. This suggests that a smaller training set might be just as effective. The 200 point training set perhaps would have been too small; additional studies would be required to discern how small the training set can be. The 1000 point training set was used for the remainder of this work. Finally, it should be noted that the neural network based genetic algorithm searches were executed with more generations than the exact analysis versions. A genetic algorithm search typically moves quickly to an improved design but can require much more effort to reduce the object further. Most points are on the right side of the unity line. It is likely, however, that points would move to the left, towards the unity line, if the exact analysis version were executed with more generations.

VII. Collaborative Optimization Execution

The master problem controlled the collaborative optimization search process. Genetic algorithm searches were used for the master as well as the rotor and wing sub-problems. Global search was maintained at all times by proscribing the range (side constraints) of all design variables in the genetic algorithm control files. Each time the master problem search required numerical values for the compatibility constraints two optimal searches were launched simultaneously ñ the rotor and wing sub-problems. After these were completed, optimal values of the discrepancy functions were passed back to the master problem to complete a master problem function evaluation cycle. While neural network surrogates made this approach feasible, it should be noted that information passing through the computing system network was not trivial. Integration of the optimal search and neural network codes would have made the total collaborative optimization run time very fast ñ perhaps a few hours.

Each of the three objective functions were tested and compared to all-in-one executions, which used exact analysis for response calculations[7]. Computation time prevented the all-in-one cases from being fully converged ñ a fact that highlights the need for a more efficient procedure. Approximately 4500 function evaluations were allowed for each all-in-one case. Table 3 presents the optimal values of the shared variables and targets, and the objective functions of the collaborative optimization (**CO**) and all-in-one cases (**AIO**) for the three objective functions **A**, **B**, and **C**.

Table 3. Collaborative Optimization (CO) All-In-One (AIO) Optimization Comparisons

Targets, Objectives, Constraints	**CO-A**	**CO-B**	**CO-C**	**AIO-A**	**AIO-B**	**AIO-C**
ctip	9.00	9.25	9.00	9.0	9.0	10.5
Ω	390	394	390	404	404	404
t/c	0.23	0.22	0.24	0.23	0.23	0.23
W_{rotor}	538	532	543	542	538	542
W_{fuel}	1612	1619	1624	1634	1631	1656
W_{wing}	1486	1491	1488	1466	1489	1449
D (Hub Loads Desirability)	0.322	0.264	0.350	0.392	0.316	0.400
F (Objective)	3738.6	3813.6	3766.5	3879	3920	3814

All-in-one optimization was expected to produce superior results because there were no approximations of system interactions, no surrogate analysis, and multi-disciplinary interactions were treated simultaneously within one big optimal search. The collaborative optimization, however, produced superior results for the objective functions because of the CPU limitation noted above. Each of the four parts of the objective functions for the **CO** cases (ΣW and hub loads desirability) was equal to the **AIO** case or lower.

Similarities among cases were the tip chord moving near the lower bound of 9 inches, the rotor speed near the low end of its range (386-460 rpm), and the wing thickness near 23 percent, which is a typical value for current tiltrotors. This last result is not surprising in that the design problem was similar to current tiltrotors. In particular, the 275 knots cruise speed was about the same as current designs. A faster design cruise speed would have driven the wing thickness lower.

Effects of the multi-objective formulations were also seen, both in the **CO** and **AIO** cases. Compared to objective **A**, case **B** emphasized the hub loads more, which produced optimal designs with lower hub loads desirability values but higher total system and fuel weights.

Results from the collaborative optimization could be considered complete; however, this depends on the reliability of the neural networks that were used in the sub-problems. It was noted that sub-problem execution with the surrogates was effective to evaluate compatibility constraints of the master problem, but the actual design might not be reliable because of constraint violations when evaluated exactly. Sub-problems from the master problem optimal solution were run again using exact analysis rather than neural network surrogates. It was shown that both can produce the same small value of objective function (within an order of magnitude); this was verified again.

When the sub-problem objective is small, as is the case in compatibility constraints in a master problem optimal solution, the master problems targets are matched closely. This fact allowed removal of the shared variables from the sub-problem design variable set. Note that in Genetic Algorithms this means processing less genetic material, which generally speeds the search procedure.

A. Rotor Sub-Problem

Execution of the rotor sub-problems (cases **CO-A** and **CO-C**) with exact analysis allowed 700 function evaluations, which makes 1700 total function evaluations of the rotor problem when added to the 1000 point training data set. Note again that the all-in-one cases were allowed approximately 4500 function evaluations, yet would required more to achieve satisfactory results. The effects of the objective function formulation as well as the effects of using neural network surrogates were examined with these cases.

First considering the desirability function, note that the combined hub loads desirability target was presented only to the rotor sub-problem. Furthermore note that the only difference in objective functions between case **A** and **C** was the first of the six individual desirability functions. The change in emphasis of d_1 was by increasing the presumed range for the vertical 3-per-rev hub force, Eq. (5). Table 4 summarizes the key comparisons, which were similar for the **CO** and **AIO** cases. Note that in both, the case **C** combined desirability was greater that case **A**, and the case **C** vertical 3-per-rev hub force desirability was lass than case **A**. This shows that even the collaborative optimization approach was able to discriminate the different weighting in the combined hub loads desirability. Trends of 3-per-rev hub forces, however, were not consistent between optimization approaches. Case **AIO-C** produced a lower hub force than **AIO-A**, but case **CO-C** produced a higher hub force that **CO-A**. In both cases the collaborative optimization produced lower levels for this, and the other objectives.

Table 4. Collaborative Optimization (CO) All-In-One (AIO) Optimization Comparisons

Targets, Objective, Constraints	**CO-A**	**CO-C**	**AIO-A**	**AIO-C**
D (combined hub loads desirability)	0.322	0.350	0.392	0.400
d1(3-per-rev vertical hub force desirability)	0.272	0.176	0.62612	0.314
V_{3P}^{Z} (3-per-rev vertical hub force, lb)	1308.34	1394.2	2227.9	2018.1

Differences in designs when using neural network surrogates verses exact analysis were examined using the optimal solution from case **CO-A**. The neural network optimal design was evaluated with exact analysis and shown in this case to be essentially feasible, having slightly violated the constraint on rotor torque. This observation is consistent with similar experiments for rotor optimization[7]. Figure 6 demonstrates how the neural network based execution produced a similar design to the execution with exact analysis, as exemplified by the blade twist and thickness distributions. Blade stiffness and mass distributions exhibited the same of similarity.

B. Wing Sub-Problem

Wing sub-problem executions were similarly examined using case **CO-A**. Neural-network and exact analysis executions of the same sub-problem produced designs that were more different than what was seen in the rotor sub-problem. When evaluated exactly, the optimal design from using neural networks was found to be feasible; however the actual weight was 15 pounds lower than the target. This suggests that the final design does not actually match the

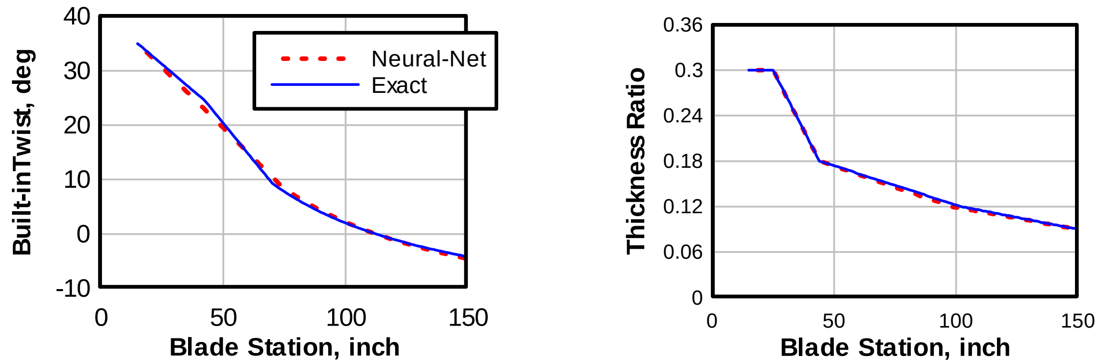

Figure 6. Comparison of Optimal Aerodynamic Designs using Exact Analysis and Neural Network Surrogates (case CO-A, Rotor)

system targets, so additional work is required. This shows that it is necessary to take the optimal target values and run the sub-problem again using exact analysis.

The wing sub-problem contains 36 design variables. Differences between optimal designs between the neural network and exact solutions are summarized and exemplified by the beam-wise bending stiffness and running weight shown in Fig. 6. Other properties were likewise dissimilar between neural network and exact analysis cases. This further highlights the need to run the sub-problem with exact analysis to obtain a reliable final design, especially for the wing.

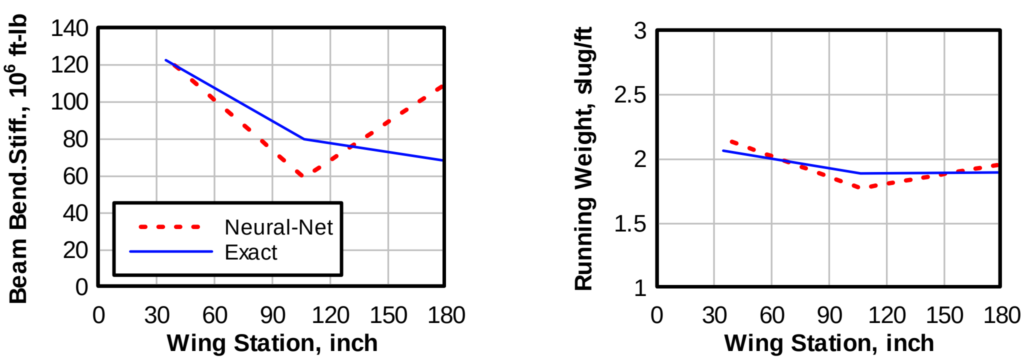

Figure 6. Comparison of Optimal Aerodynamic Designs using Exact Analysis and Neural Network Surrogates (case CO-A, Wing)

VIII. Conclusions

The collaborative optimization scheme was employed in the integrated design of rotor wing systems of a tiltrotor aircraft. The decomposition into sub-systems was between the rotor and wing systems. This decomposition required approximations for use in the wing sub-problem that consolidated most of the rotor design variables into a few, which allowed an efficient decomposition. The solution scheme was based on a literal view of the collaborative optimization statement. That is, for each cycle of response evaluation in the master problem, the system targets and shared variables were passed to the respective sub-problems to obtain values of the master problem compatibility constraints. Execution of the collaborative optimization problem employed global search with genetic algorithms both in the master problem and the rotor and wing sub-problems.

The scheme magnified computational effort because of the requirement to run sub-problems at each cycle of the master problem. However, it was found that neural network surrogates for exact analysis could be used, which allowed reasonable run time of the sub-problems. Neural network surrogates have been used in rotorcraft optimization but are often not sufficiently accurate for reliable use. Sub-problem optimal values become compatibility constraints to the master problem. It was found that when neural network surrogates are used to solve the sub-problems, the optimal values are an order of magnitude accurate compared to use of exact analysis. Sub-problem optimal values range several orders of magnitude; therefore this level of accuracy was sufficient. Furthermore, the master problem compatibility constraints require optimal values of a discrepancy function but not the optimal design itself. This property not only allowed use of neural networks, but did so in a way that mitigated inaccuracy.

Application of the scheme was demonstrated on a multi-objective function and two variations. The objective function was the sum of rotor and wing system weights, fuel weight, plus weight attributed to vibratory hub loads. The desirability function approach was used to map six hub loads into a combined desirability from zero to one, which was a factor in the hub loads weighting. Objective function variations were first to change the hub loads weighting, and second to change the scale in one of the individual hub loads desirability functions. These variations effectively changed the emphasis of all hub loads, then one select hub load. The collaborative optimization scheme successfully discriminated the variations as shown in the respective optimal values of the system responses.

In terms of the number of evaluations using exact, expensive computational analysis, the scheme was fairly efficient. Further testing is required determine the smallest size of neural network training data can be used in the sub-problem surrogates. Further testing is also required to determine how many function evaluations would be required to achieve more satisfactory answers in the all-in-one version. However, it can be conservatively estimated that the global collaborative optimization scheme with neural network surrogates would require 1000 to 2000 exact analysis evaluations verses 5000 to 8000 or more for the all-in-one approach. Overall, the scheme provided an efficient solution procedure in a collaborative framework that represents the organizational structure of an industrial design.

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