第十六章　二端口网络

主要内容：

二端口网络参数和方程

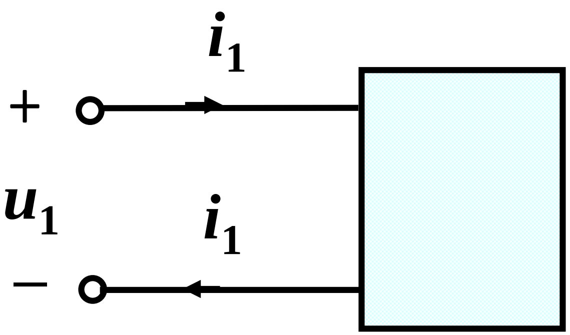
二端口网络等效电路

二端口网络的连接

§ 16–1　二端口网络

一、概念:

1. 端口 (port)

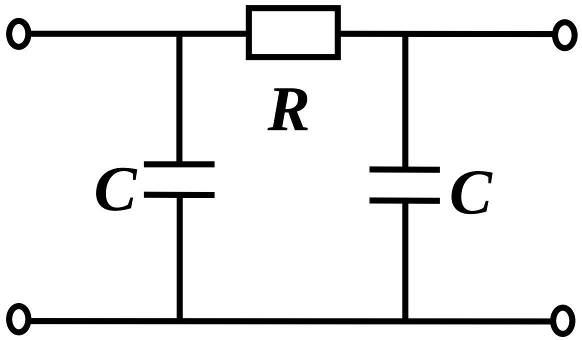


端口由一对端钮构成，且满足如下条件：从一个端钮流入的电流等于从另一个端钮流出的电流。

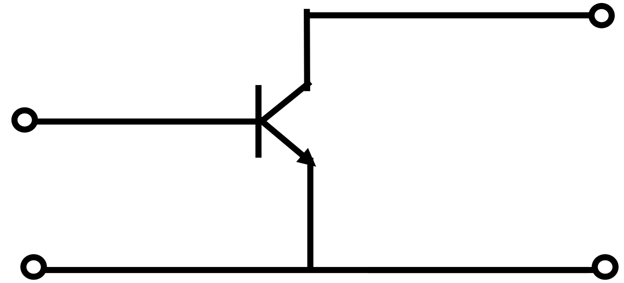
2. 二端口（two-port)

当一个电路与外部电路通过两个端口连接时称此电路为二端口网络。

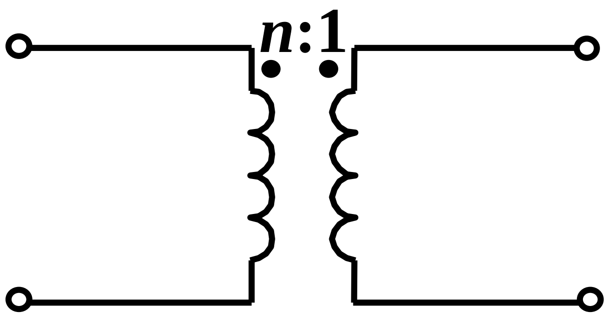
例1：几种常见的二端口网络：



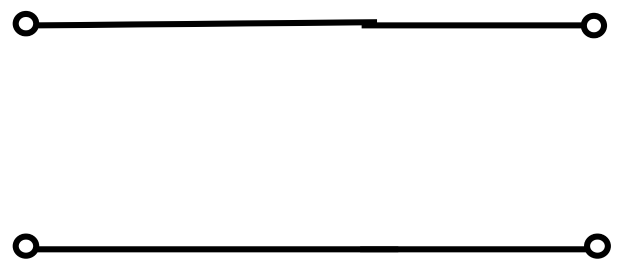
滤波器



三极管

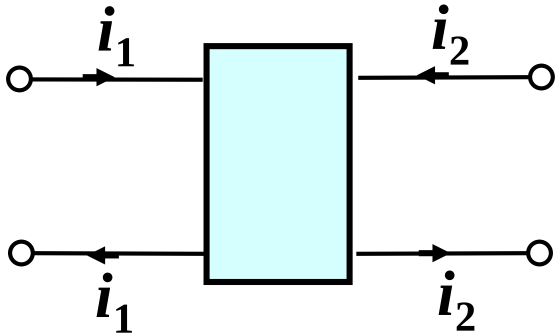


变压器

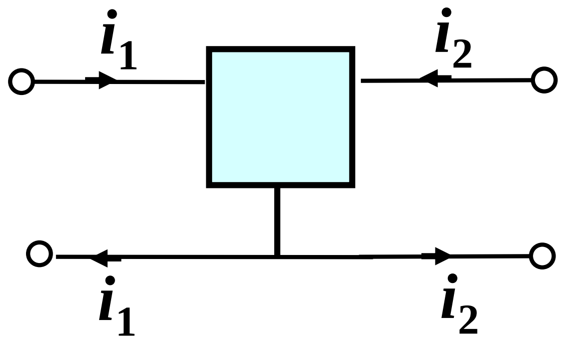


传输线

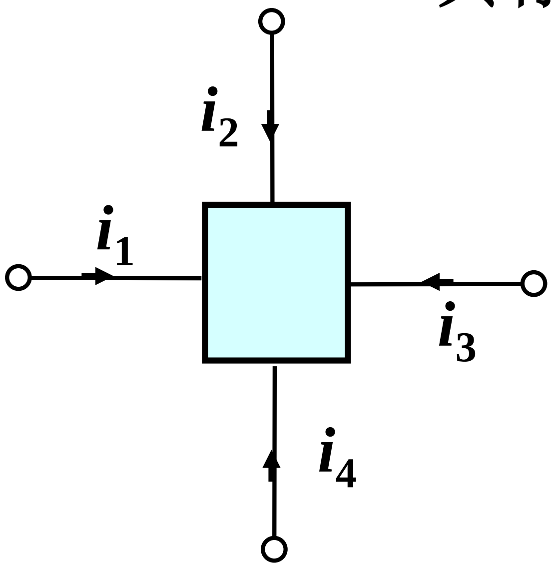
例2. 二端口网络与四端网络



二端口

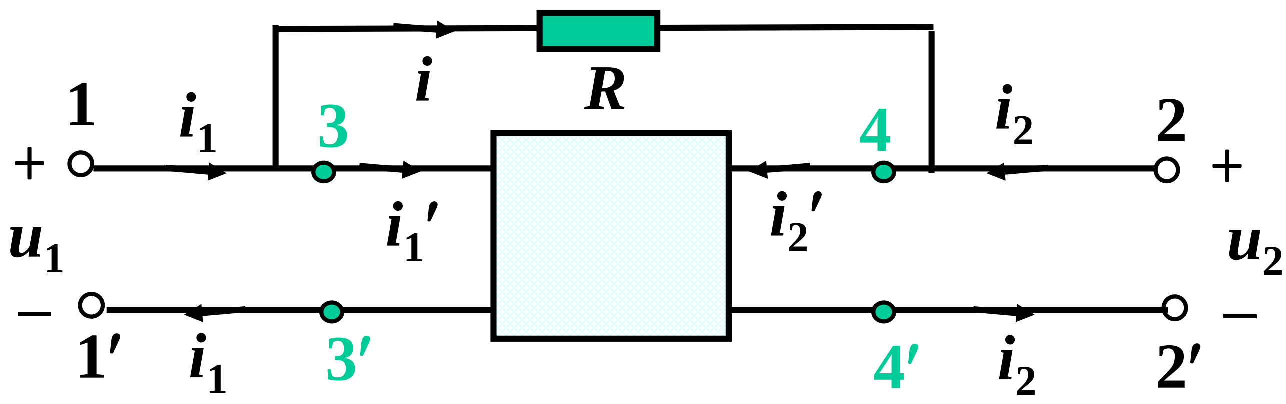


具有公共端的二端口



四端网络

例4. 二端口的两个端口间若有外部连接，则会破坏原二端口的端口条件。



1-1’ 2-2’是二端口

3-3’ 4-4’不是二端口，是四端网络

$$\left.\begin{aligned} i_1' &= i_1 - i \neq i_1 \\ i_2' &= i_2 + i \neq i_2 \end{aligned}\right\}$$ 端口条件破坏

约定

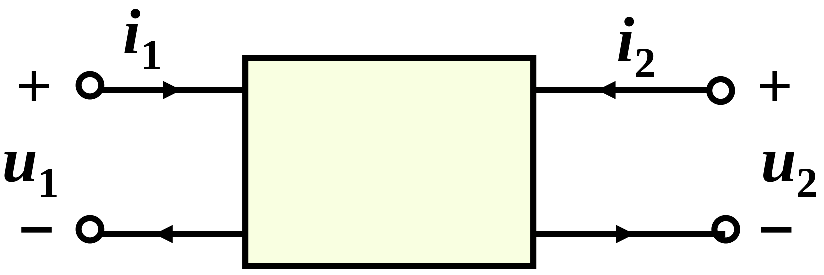
1. 讨论范围

含线性 *R*、*L*、*C*、*M*与线性受控源

不含独立源

2. 参考方向

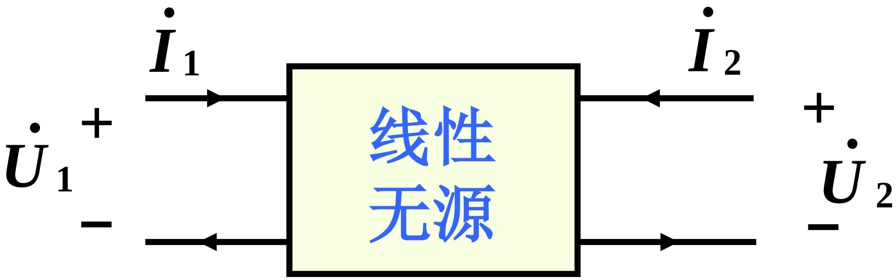
§16–2　二端口的参数方程

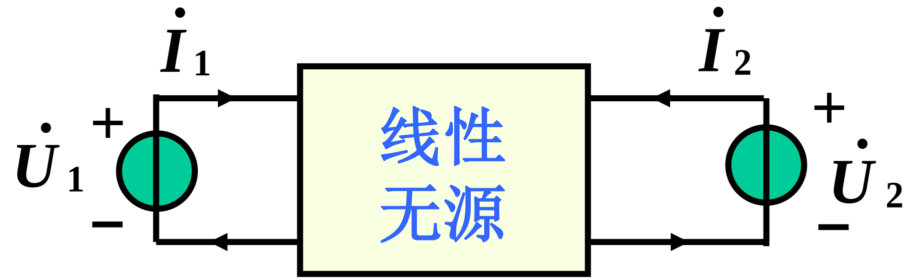


端口物理量**4个**　　i_1　i_2　u_1　u_2

端口电压电流有六种不同的方程来表示，即可用六套参数描述二端口网络。

一、Y参数方程（短路参数）：





$$\begin{cases} \dot{I}_1 = \mathbf{Y}_{11}\dot{U}_1 + \mathbf{Y}_{12}\dot{U}_2 \\ \dot{I}_2 = \mathbf{Y}_{21}\dot{U}_1 + \mathbf{Y}_{22}\dot{U}_2 \end{cases}$$

矩阵形式 $\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix}$

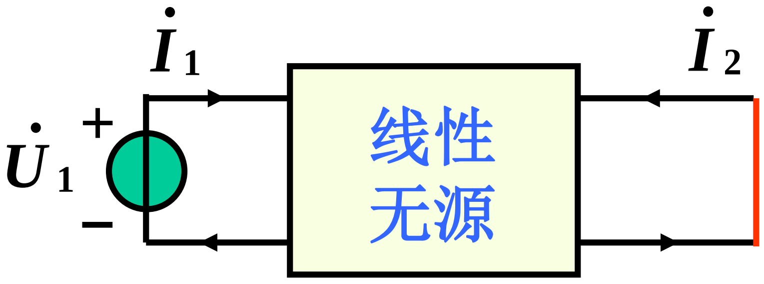
令 $\mathbf{Y} = \begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} \end{bmatrix}$ 称为Y 参数矩阵.

$$\begin{cases}\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2\end{cases}$$

Y参数的实验测定

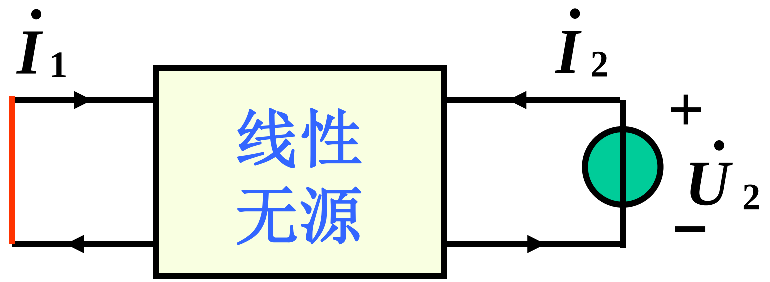
$$Y_{11} = \left.\frac{\dot{I}_1}{\dot{U}_1}\right|_{\dot{U}_2=0}$$ 自导纳

$$Y_{21} = \left.\frac{\dot{I}_2}{\dot{U}_1}\right|_{\dot{U}_2=0}$$ 转移导纳



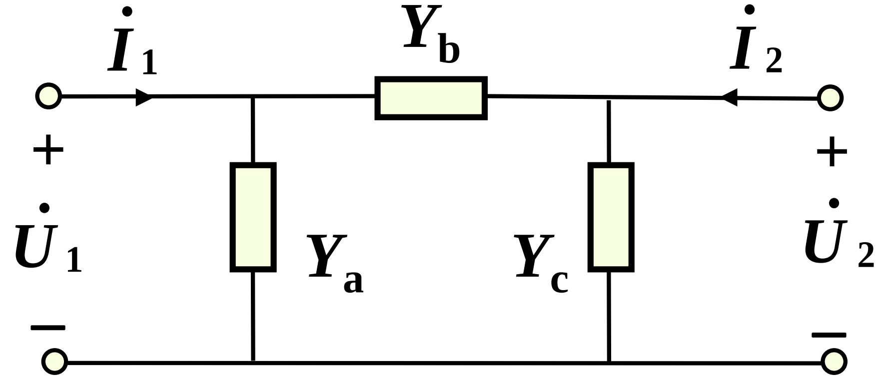
$$Y_{12} = \left.\frac{\dot{I}_1}{\dot{U}_2}\right|_{\dot{U}_1=0}$$ 转移导纳

$$Y_{22} = \left.\frac{\dot{I}_2}{\dot{U}_2}\right|_{\dot{U}_1=0}$$ 自导纳

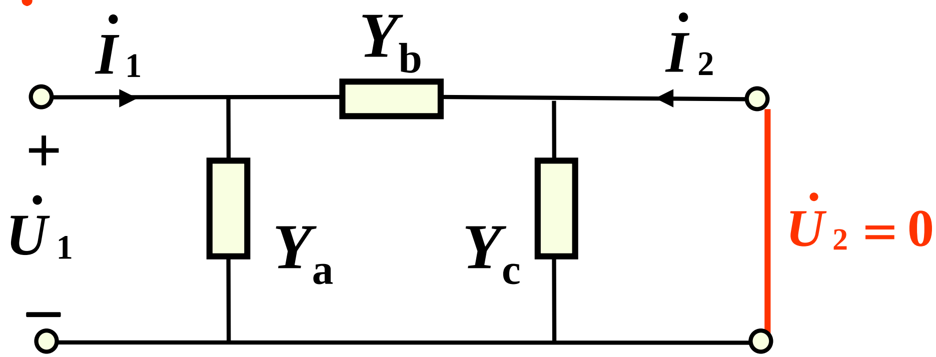


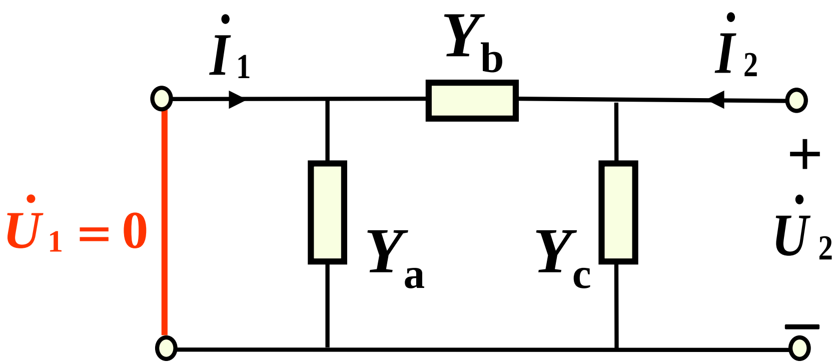
Y　短路导纳参数

例16-1. 求Y 参数。



解:





$$Y_{12} = Y_{21} = -Y_b$$

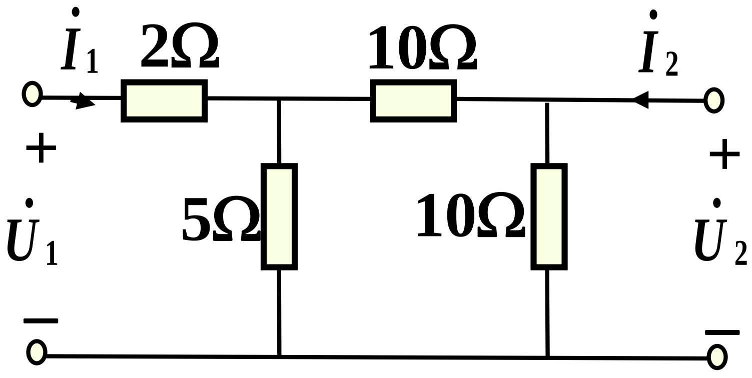
$$Y_{11} = \frac{\dot I_1}{\dot U_1}\bigg|_{\dot U_2=0} = Y_a + Y_b$$

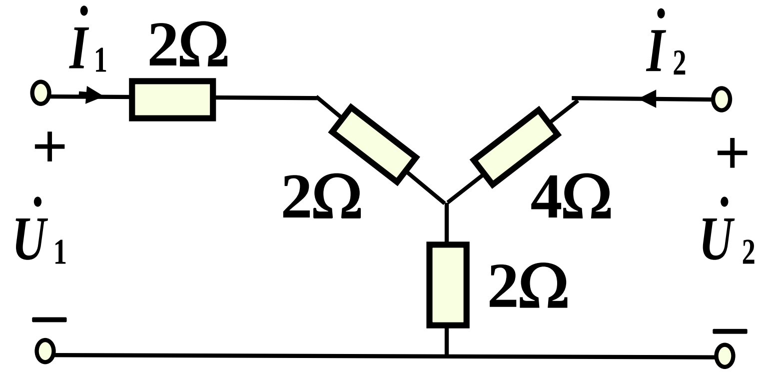
$$Y_{21} = \frac{\dot I_2}{\dot U_1}\bigg|_{\dot U_2=0} = -Y_b$$

$$Y_{12} = \frac{\dot I_1}{\dot U_2}\bigg|_{\dot U_1=0} = -Y_b$$

$$Y_{22} = \frac{\dot I_2}{\dot U_2}\bigg|_{\dot U_2=0} = Y_b + Y_c$$

例16-2. 求Y 参数。





$$\mathrm{Y}_{12} = \mathrm{Y}_{21}$$

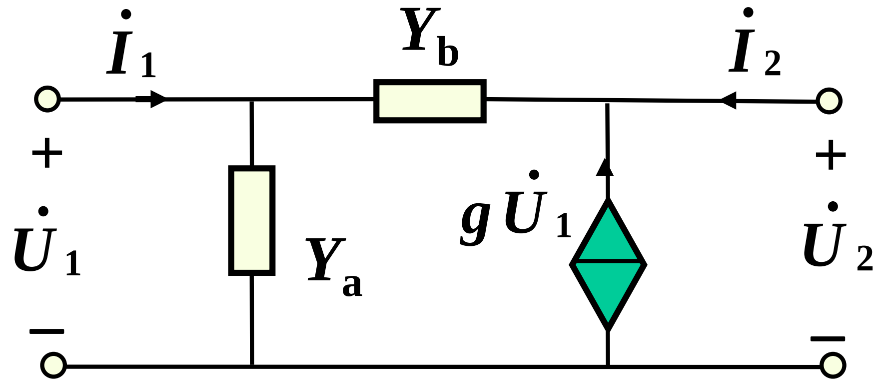
$$Z_{1-1'} = 2 + (5 // 10) = \frac{16}{3}\Omega \qquad Y_{11} = \frac{1}{Z_{1-1'}} = \frac{3}{16}\mathrm{s}$$

$$Z_{2-2'} = 10 //[10 + (5 // 2)] = \frac{16}{3}\Omega \qquad Y_{22} = \frac{1}{Z_{2-2'}} = \frac{3}{16}\mathrm{s}$$

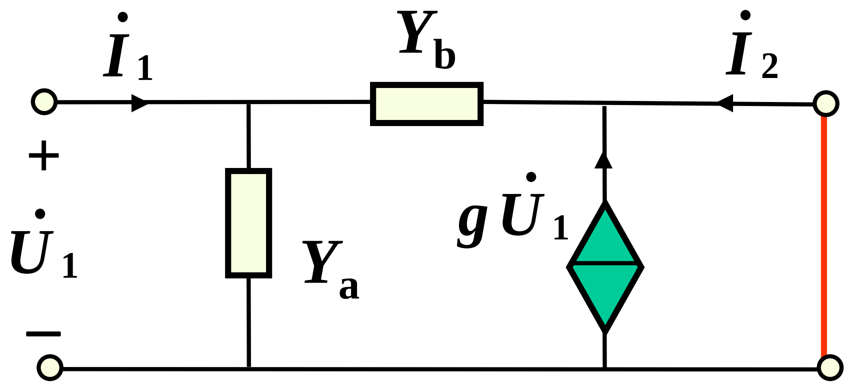
$$\mathrm{Y}_{11} = \mathrm{Y}_{22} = \frac{3}{16}\mathrm{s}$$

电气对称

例16-3



解一

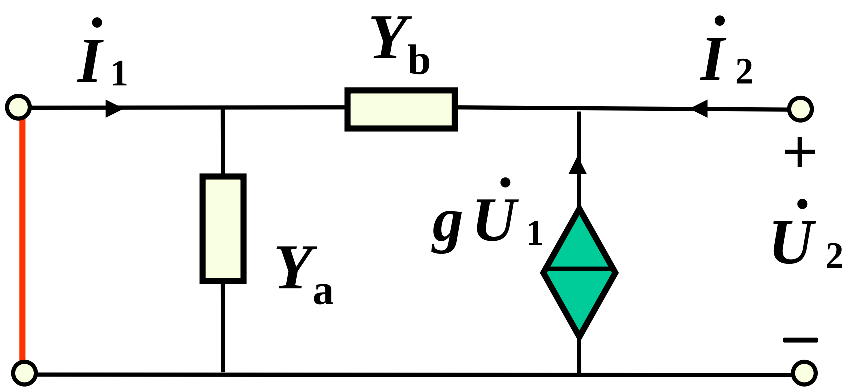


$\dot{U}_2 = 0$

$$Y_{11} = \left.\frac{\dot{I}_1}{\dot{U}_1}\right|_{\dot{U}_2=0} = Y_a + Y_b$$

$$Y_{21} = \left.\frac{\dot{I}_2}{\dot{U}_1}\right|_{\dot{U}_2=0} = -Y_b - g$$

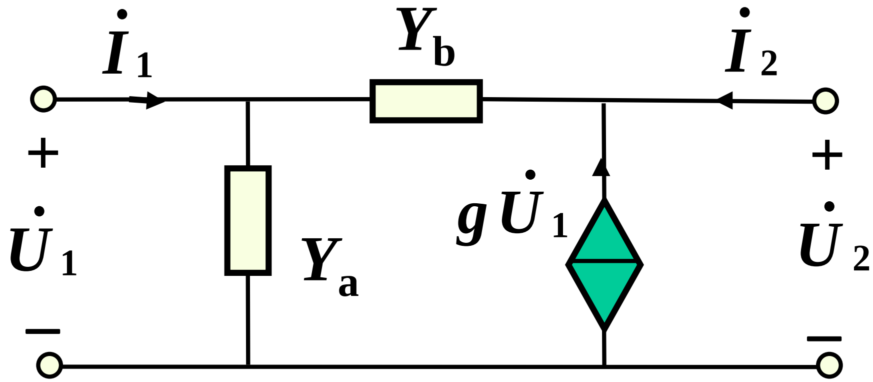
$\dot{U}_1 = 0$



$$Y_{12} = \left.\frac{\dot{I}_1}{\dot{U}_2}\right|_{\dot{U}_1=0} = -Y_b$$

$$Y_{22} = \left.\frac{\dot{I}_2}{\dot{U}_2}\right|_{\dot{U}_1=0} = Y_b$$

解二



$$\dot{I}_1 = Y_a\dot{U}_1 + Y_b(\dot{U}_1 - \dot{U}_2)$$
$$\dot{I}_2 = Y_b(\dot{U}_2 - \dot{U}_1) - g\dot{U}_1$$

$$\dot{I}_1 = (Y_a + Y_b)\dot{U}_1 - Y_b\dot{U}_2$$
$$\dot{I}_2 = (-g - Y_b)\dot{U}_1 + Y_b\dot{U}_2$$

$$\mathbf{Y} = \begin{bmatrix} Y_a + Y_b & -Y_b \\ -g - Y_b & Y_b \end{bmatrix}$$

非互易二端口网络（网络内部有受控源）四个独立参数。

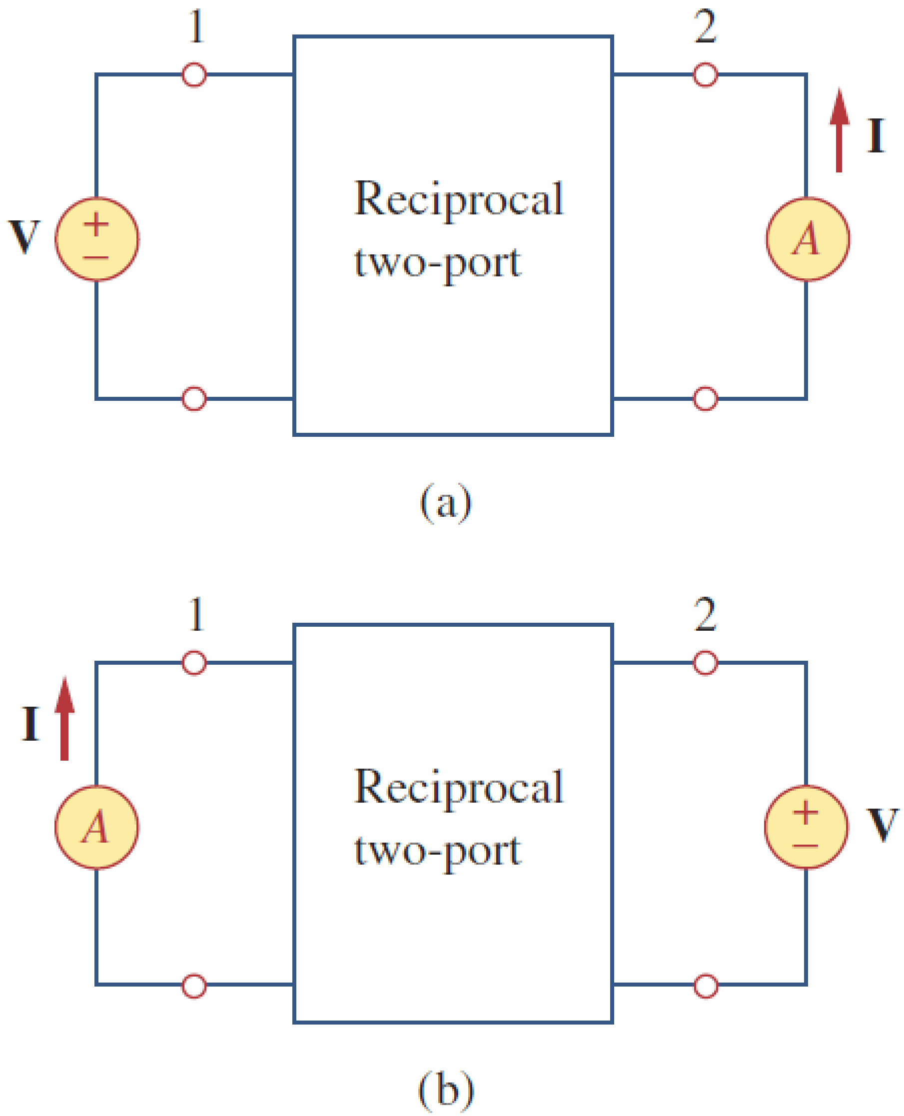
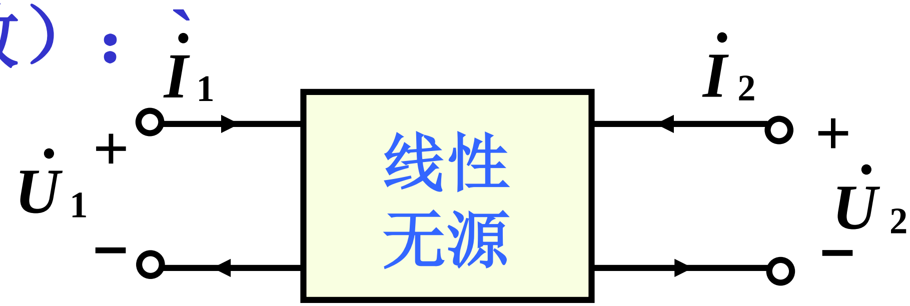


Figure 19.4
Interchanging a voltage source at one port with an ideal ammeter at the other port produces the same reading in a reciprocal two-port.

二、Z参数方程（开路参数）：



由Y 参数方程 $\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}$ 可解出$\dot{U}_1, \dot{U}_2$.

即：$\begin{cases} \dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 \end{cases}$

其矩阵形式为

$$\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix}$$

$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$ 称为Z参数矩阵

Z参数的实验测定

$$Z_{11}=\frac{\dot{U}_1}{\dot{I}_1}\bigg|_{\dot{I}_2=0} \qquad Z_{12}=\frac{\dot{U}_1}{\dot{I}_2}\bigg|_{\dot{I}_1=0}$$

$$Z_{21}=\frac{\dot{U}_2}{\dot{I}_1}\bigg|_{\dot{I}_2=0} \qquad Z_{22}=\frac{\dot{U}_2}{\dot{I}_2}\bigg|_{\dot{I}_1=0}$$

Z参数又称开路阻抗参数

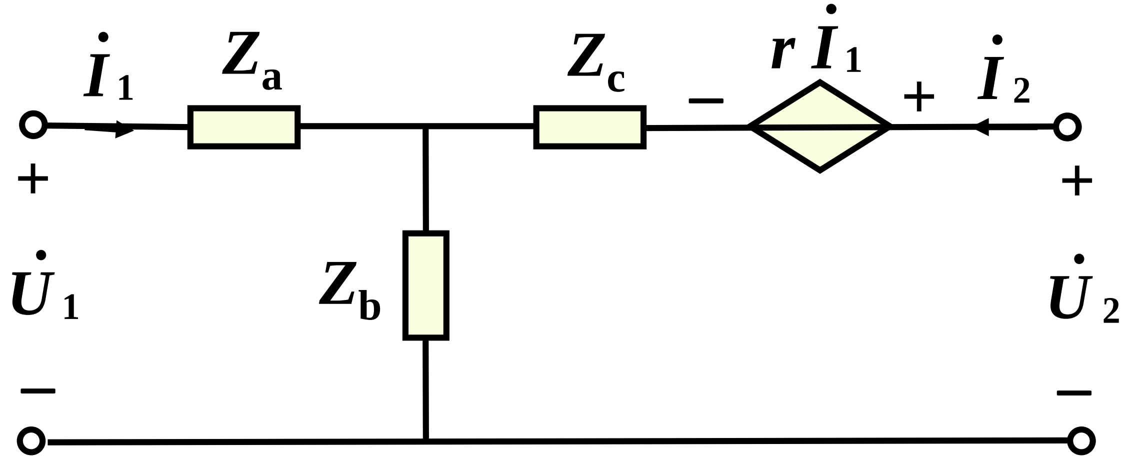
互易二端口 $\quad Z_{12}=Z_{21}$

对称二端口 $\quad Z_{11}=Z_{22} \quad (Z_{12}=Z_{21})$

若 矩阵 Z 与 Y 非奇异

则 $\quad \mathbf{Y}=\mathbf{Z}^{-1} \qquad \mathbf{Z}=\mathbf{Y}^{-1}$

例16-4



$$\dot{U}_1 = Z_a\dot{I}_1 + Z_b(\dot{I}_1 + \dot{I}_2)$$

$$\dot{U}_2 = r\dot{I}_1 + Z_c\dot{I}_2 + Z_b(\dot{I}_1 + \dot{I}_2)$$

$$\mathrm{Z} = \begin{bmatrix} Z_a + Z_b & Z_b \\ r + Z_b & Z_b + Z_c \end{bmatrix}$$

三、T参数方程（传输参数）：

$$\begin{cases}\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 & (1)\\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 & (2)\end{cases}$$

由(2)得：

$$\dot{U}_1 = -\frac{Y_{22}}{Y_{21}}\dot{U}_2 + \frac{1}{Y_{21}}\dot{I}_2 \qquad (3)$$

将(3)代入(1)得：

$$\dot{I}_1 = \left(Y_{12} - \frac{Y_{11}Y_{22}}{Y_{21}}\right)\dot{U}_2 + \frac{Y_{11}}{Y_{21}}\dot{I}_2$$

即：

$$\begin{cases}\dot{U}_1 = T_{11}\dot{U}_2 - T_{12}\dot{I}_2\\ \dot{I}_1 = T_{21}\dot{U}_2 - T_{22}\dot{I}_2\end{cases}$$

$$\dot{U}_1 = -\frac{Y_{22}}{Y_{21}}\dot{U}_2 + \frac{1}{Y_{21}}\dot{I}_2 \qquad \dot{I}_1 = \left(Y_{12} - \frac{Y_{11}Y_{22}}{Y_{21}}\right)\dot{U}_2 + \frac{Y_{11}}{Y_{21}}\dot{I}_2$$

可得

$$T_{11} = -\frac{Y_{22}}{Y_{21}} \qquad T_{12} = \frac{-1}{Y_{21}}$$

$$T_{21} = \frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}} \qquad T_{22} = -\frac{Y_{11}}{Y_{21}}$$

其矩阵形式

$$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_2 \\ -\dot{I}_2 \end{bmatrix}$$

(注意负号)

$$\mathbf{T} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$$

称为T 参数矩阵

T 参数的实验测定

$$T_{11}=\frac{\dot U_1}{\dot U_2}\bigg|_{\dot I_2=0} \qquad T_{21}=\frac{\dot I_1}{\dot U_2}\bigg|_{\dot I_2=0} \quad \text{开路参数}$$

$$T_{12}=\frac{\dot U_1}{-\dot I_2}\bigg|_{\dot U_2=0} \qquad T_{22}=\frac{\dot I_1}{-\dot I_2}\bigg|_{\dot U_2=0} \quad \text{短路参数}$$

互易二端口 $Y_{12}=Y_{21}$

$$T_{11}=-\frac{Y_{22}}{Y_{21}} \qquad T_{12}=\frac{-1}{Y_{21}}$$

$$T_{21}=\frac{Y_{12}Y_{21}-Y_{11}Y_{22}}{Y_{21}} \qquad T_{22}=-\frac{Y_{11}}{Y_{21}}$$

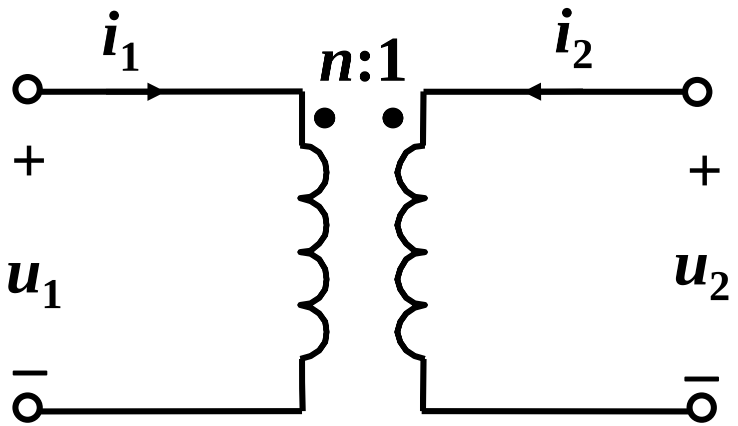
$T_{11}T_{22}-T_{12}T_{21}$

$$=\frac{\cancel{Y_{11}}Y_{22}}{\cancel{Y_{21}^2}}+\frac{Y_{12}\cancel{Y_{21}}}{\cancel{Y_{21}^2}}-\frac{\cancel{Y_{11}}Y_{22}}{\cancel{Y_{21}^2}}$$

$=1$

对称二端口 $Y_{11}=Y_{22}$ 则 $T_{11}=T_{22}$

例16-5 求T参数



$$u_1 = nu_2$$

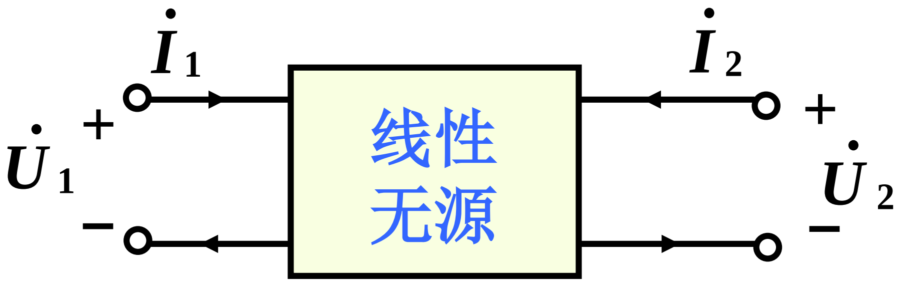
$$i_1 = -\frac{1}{n} i_2$$

即 $$\begin{bmatrix} u_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & \dfrac{1}{n} \end{bmatrix} \begin{bmatrix} u_2 \\ -i_2 \end{bmatrix}$$

则 $$\mathrm{T} = \begin{bmatrix} n & 0 \\ 0 & \dfrac{1}{n} \end{bmatrix}$$

四、H参数方程（混合参数）：

H 参数方程



$$\begin{cases} \dot{U}_1 = H_{11}\dot{I}_1 + H_{12}\dot{U}_2 \\ \dot{I}_2 = H_{21}\dot{I}_1 + H_{22}\dot{U}_2 \end{cases}$$

矩阵形式

$$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{U}_2 \end{bmatrix}$$

H 参数的实验测定

$$\left.\begin{aligned} H_{11} &= \frac{\dot{U}_1}{\dot{I}_1}\Big|_{\dot{U}_2=0} \\ H_{21} &= \frac{\dot{I}_2}{\dot{I}_1}\Big|_{\dot{U}_2=0} \end{aligned}\right\} \text{短路参数} \qquad \left.\begin{aligned} H_{12} &= \frac{\dot{U}_1}{\dot{U}_2}\Big|_{\dot{I}_1=0} \\ H_{22} &= \frac{\dot{I}_2}{\dot{U}_2}\Big|_{\dot{I}_1=0} \end{aligned}\right\} \text{开路参数}$$

互易二端口 $\mathbf{H}_{12} = -\mathbf{H}_{21}$

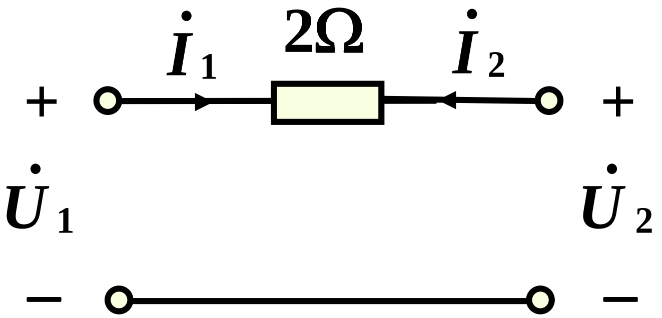
对称二端口 $\mathbf{H}_{11}\mathbf{H}_{22} - \mathbf{H}_{12}\mathbf{H}_{21} = 1$

小结　1. 六套参数，还有逆传输参数 和逆混合参数。

2 .为什么用这么多参数表示

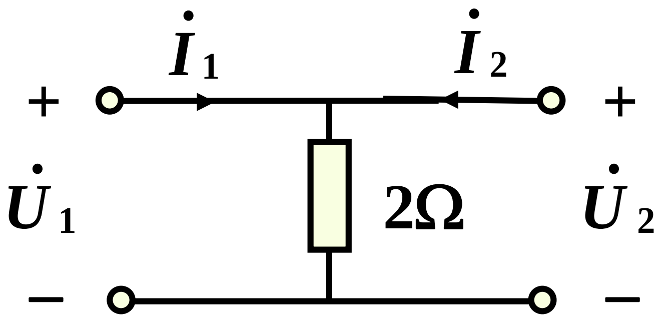
（1）为描述电路方便，测量方便。

（2）有些电路只存在某几种参数。



$$Y=\begin{bmatrix}0.5\mathrm{s} & -0.5\mathrm{s}\\ -0.5\mathrm{s} & 0.5\mathrm{s}\end{bmatrix}$$

Z参数 不存在



$$Z=\begin{bmatrix}2\Omega & 2\Omega\\ 2\Omega & 2\Omega\end{bmatrix}$$

Y 参数不存在

3.含有受控源的电路四个独立参数。

4. 二端口网络的各参数矩阵之间可以相互转换