第十章　含有耦合电感的电路

主要内容：

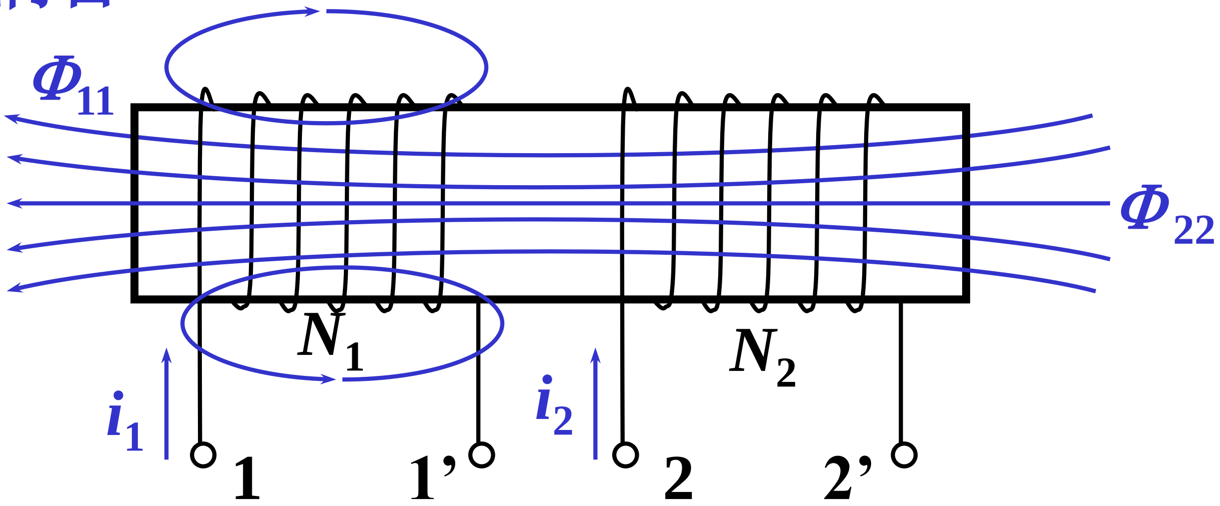
1、互感

2、含有耦合电感电路的分析计算

3、理想变压器

§ 10–1 互 感

一、磁耦合



施感电流i_1 → Φ_{11} → Ψ_{11}, Ψ_{21}

施感电流i_2 → Φ_{22} → Ψ_{22}, Ψ_{12}

二、两个线圈耦合时的磁通链

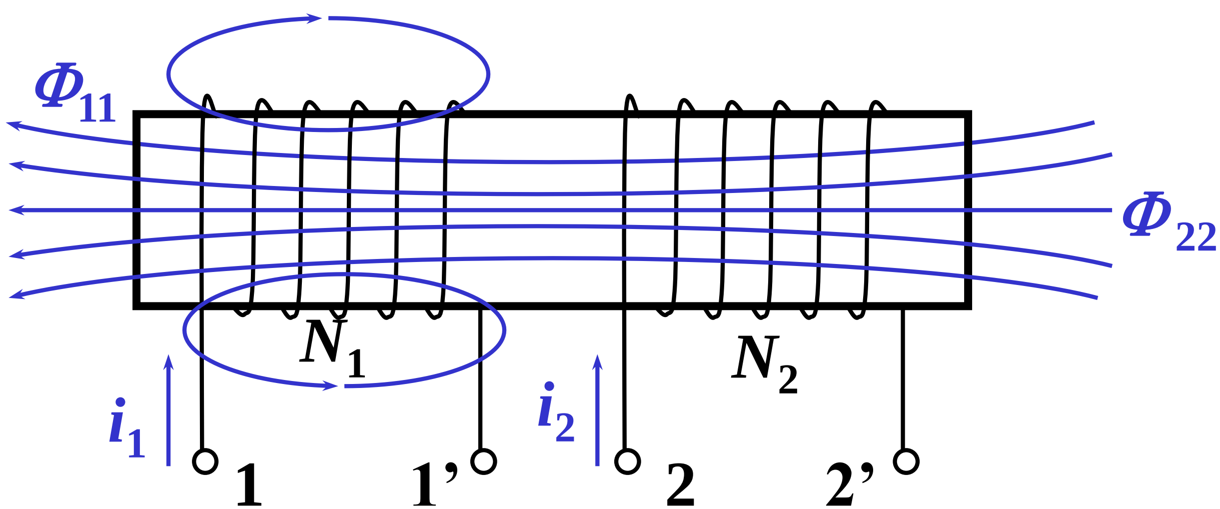
自感磁通链$_1$　　$\psi_{11}=L_1 i_1$　　$\psi_{22}=L_2 i_2$

互感磁通链$_1$　　$\psi_{12}=M_{12} i_2$　　$\psi_{21}=M_{21} i_1$

线圈1中的磁通链$_1$　　$\Psi_1=\Psi_{11}+\Psi_{12}=L_1 i_1 \pm M_{12} i_2$

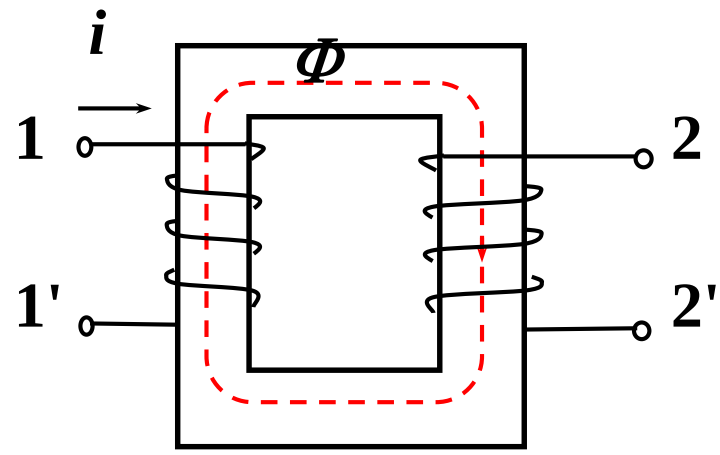
线圈2中的磁通链$_1$　　$\Psi_2=\Psi_{22}+\Psi_{21}=L_2 i_2 \pm M_{21} i_1$

同名端：当两个电流分别从两个线圈的对应端子流入 ，其所产生的磁场相互加强时，则这两个对应端子称为同名端。

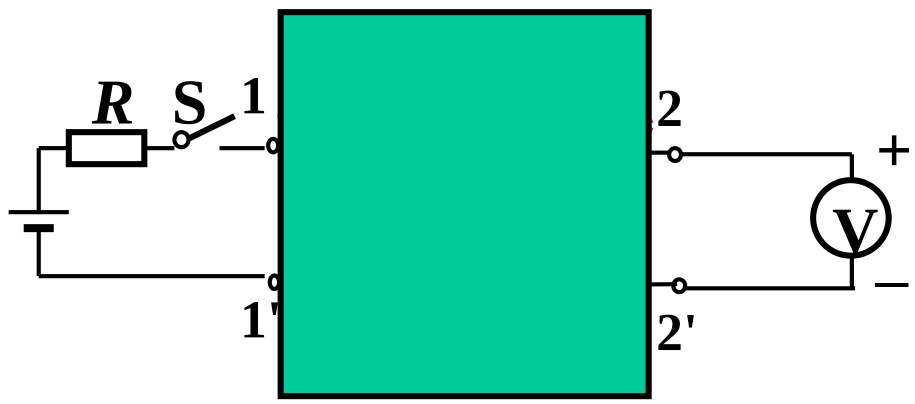


确定同名端的方法：

(1) 使用右手螺旋法则，根据线圈的绕向和相对位置来判断。



(2) 用实验的方法来判断。



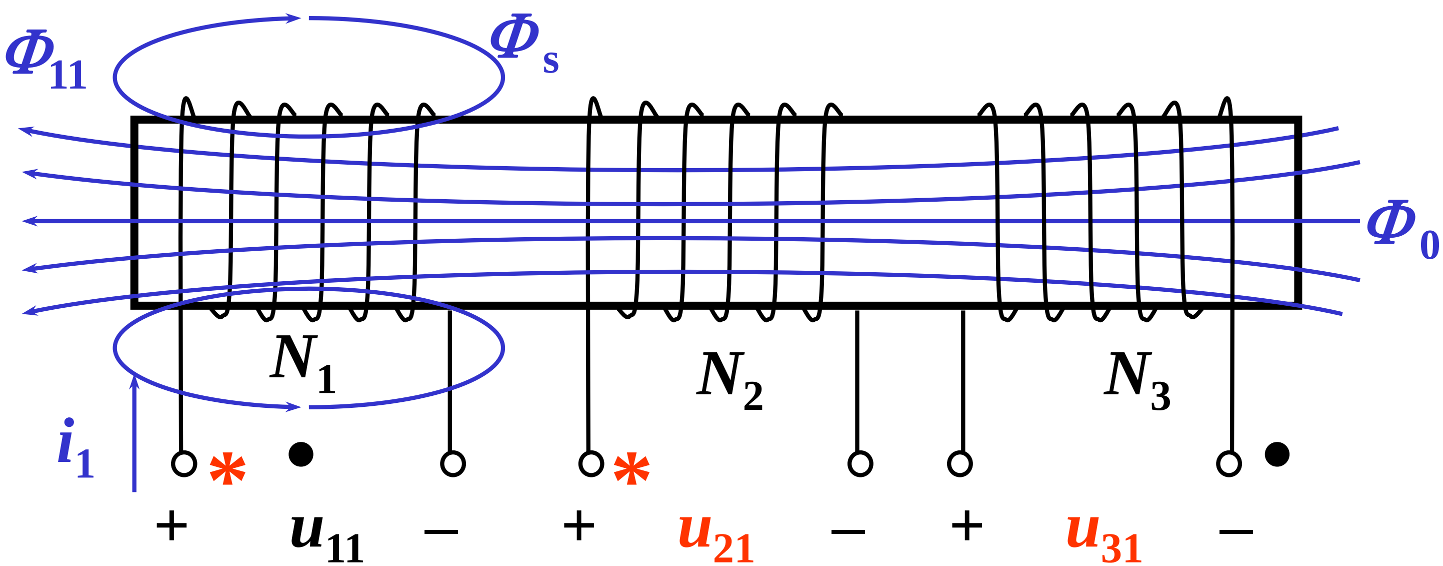
如图电路，当闭合开关S时，*i*增加，

$$\frac{di}{dt} > 0, \quad u_{22'} = M\frac{di}{dt} > 0$$ 电压表正偏。

多个线圈耦合的情况

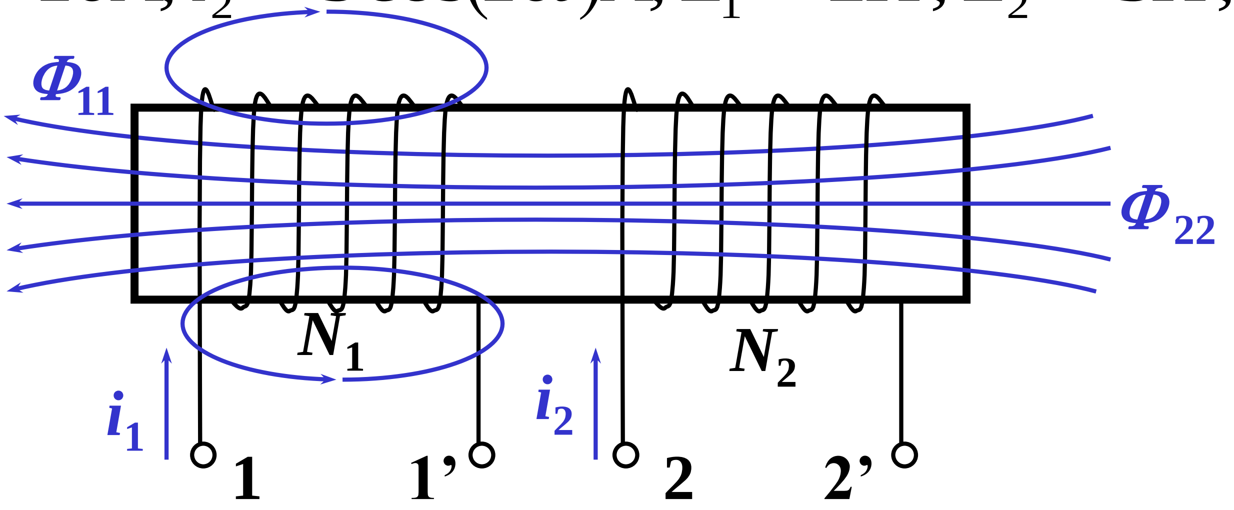
$$\Psi_k = \Psi_{kk} + \sum_{j\neq k} \Psi_{kj}$$

Ψ_{kj}与 Ψ_{kk} 同向取“+”，反之取“-”。



例10-1：互感耦合电路中,求两耦合线圈中的磁通链。

$$i_1 = 10A, i_2 = 5\cos(10t)A, L_1 = 2H, L_2 = 3H, M = 1H$$

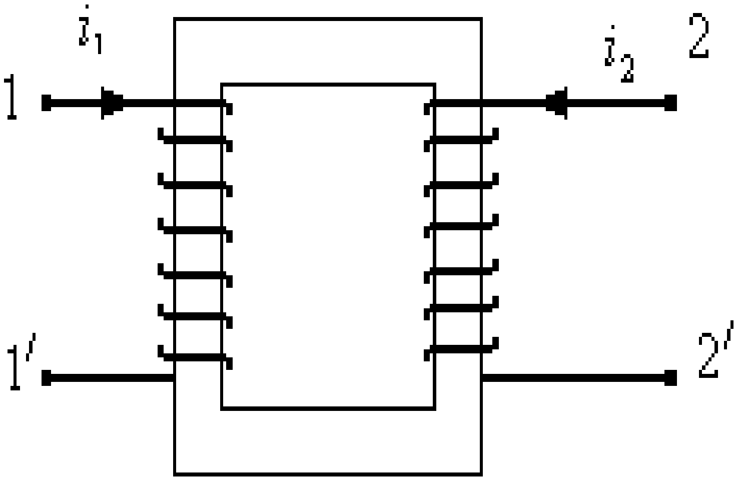


解

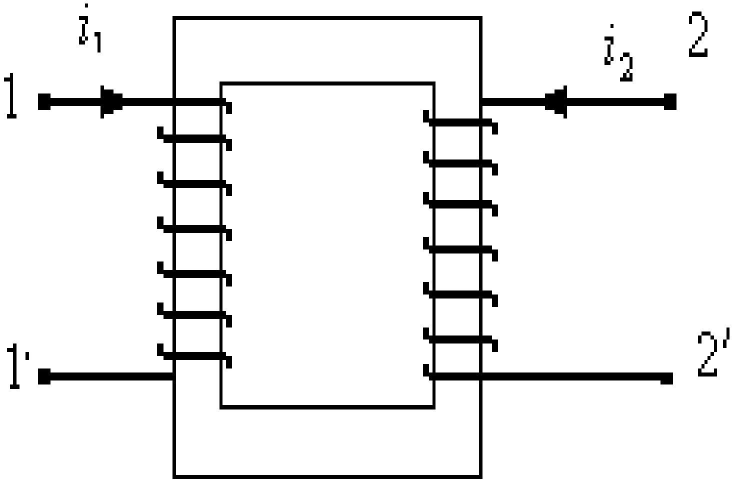
$$\Psi_1 = \Psi_{11} + \Psi_{12} = L_1 i_1 + M i_2 = 20 + 5\cos(10t) Wb$$

$$\Psi_2 = \Psi_{21} + \Psi_{22} = L_2 i_2 + M i_1 = 10 + 15\cos(10t) Wb$$

例10-2：线圈的绕向及相互位置如下图，判断同名端



M 前面为正

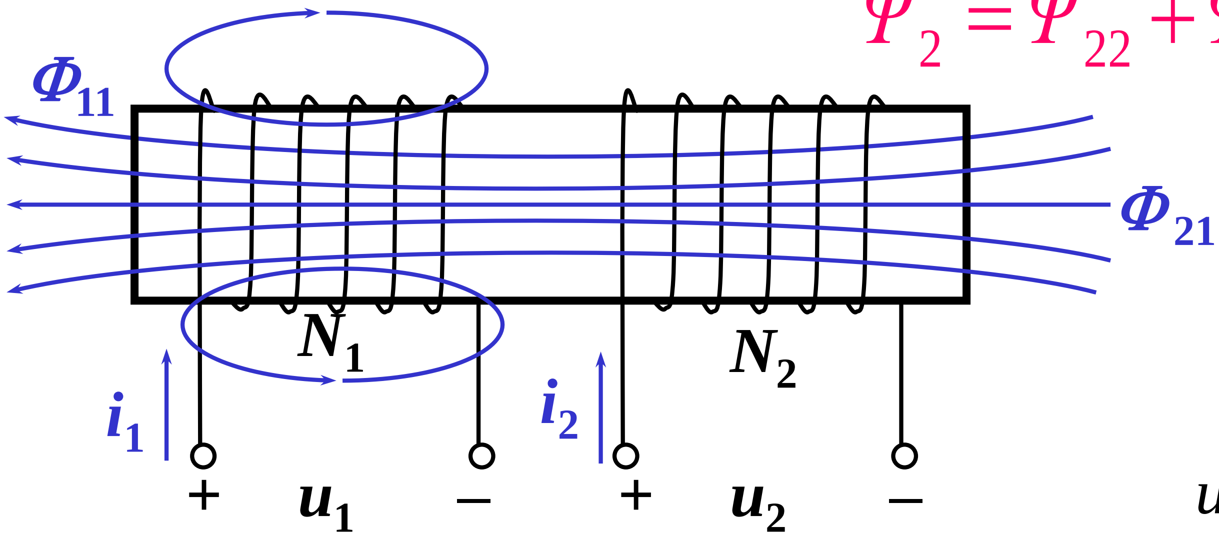


M 前面为负

三、耦合线圈中的感应电压

$$\Psi_1 = \Psi_{11} + \Psi_{12} = L_1 i_1 \pm M_{12} i_2$$

$$\Psi_2 = \Psi_{22} + \Psi_{21} = L_2 i_2 \pm M_{21} i_1$$



$$u_1 = \frac{d\psi_1}{dt} = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} \pm M \frac{\mathrm{d}i_2}{\mathrm{d}t}$$

$$u_2 = \frac{d\psi_2}{dt} = L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} \pm M \frac{\mathrm{d}i_1}{\mathrm{d}t}$$

自感电压

$$u_{11} = \frac{\mathrm{d}\Psi_{11}}{\mathrm{d}t} = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t}$$

互感电压

$$u_{12} = \frac{\mathrm{d}\Psi_{12}}{\mathrm{d}t} = M_{12} \frac{\mathrm{d}i_2}{\mathrm{d}t}$$

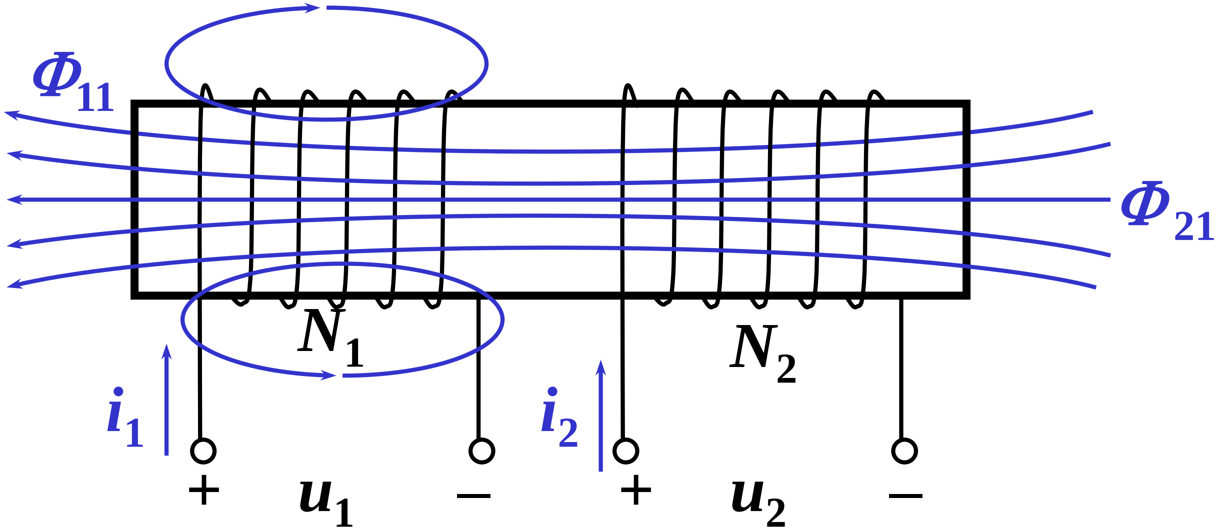
相量形式:

$$\dot{U}_1 = \mathrm{j}\omega L_1 \dot{I}_1 \pm \mathrm{j}\omega M_{12} \dot{I}_2$$

$$\dot{U}_2 = \mathrm{j}\omega L_2 \dot{I}_2 \pm \mathrm{j}\omega M_{21} \dot{I}_1$$

例10-3：互感耦合电路中,求两耦合线圈的端电压。

$$i_1 = 10A, i_2 = 5\cos(10t)A, L_1 = 2H, L_2 = 3H, M = 1H$$



解

$$u_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = -50\sin(10t)V$$ 互感电压

$$u_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} = -150\sin(10t)V$$ 自感电压

“点”符号与同名端

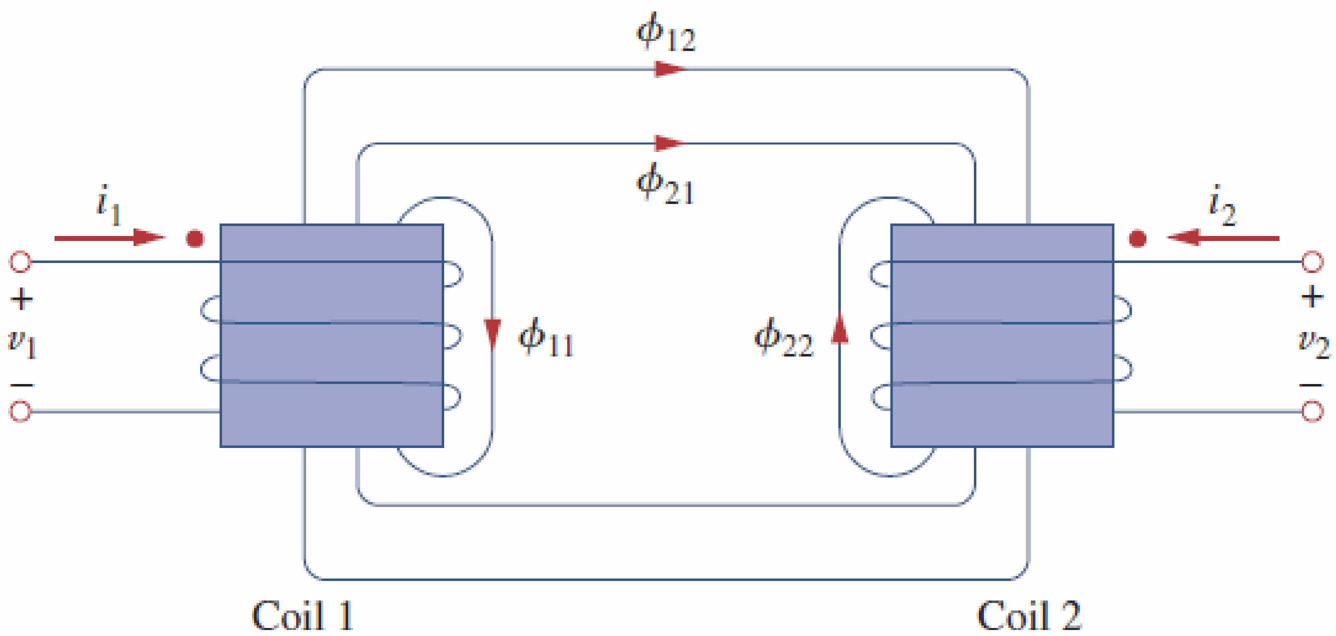


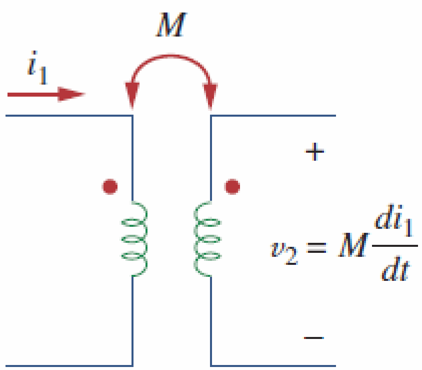
Figure 13.4
Illustration of the dot convention.

Dot convention

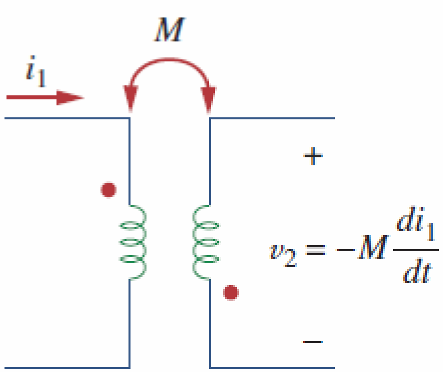
If a current **enters** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **positive** at the dotted terminal of the second coil.

Alternatively,

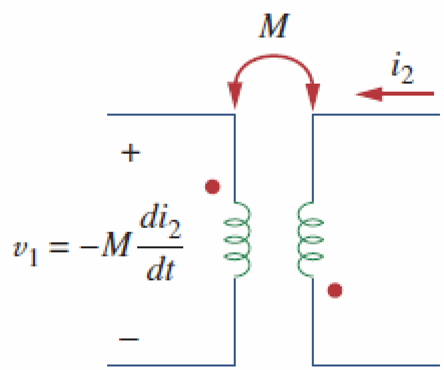
If a current **leaves** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **negative** at the dotted terminal of the second coil.



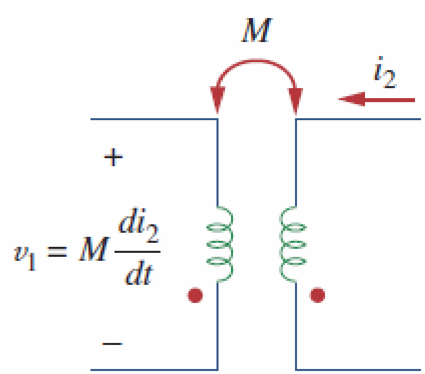
(a)



(b)

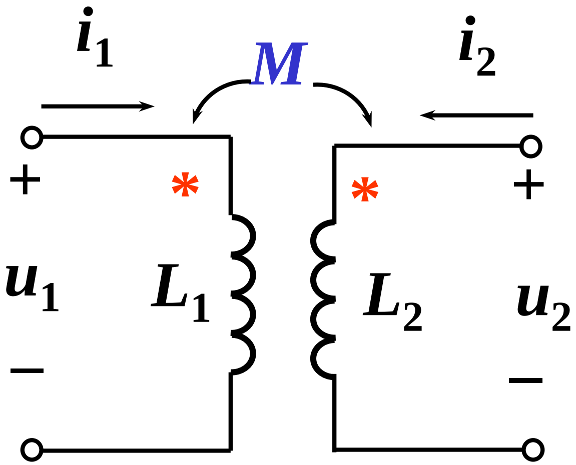


(c)



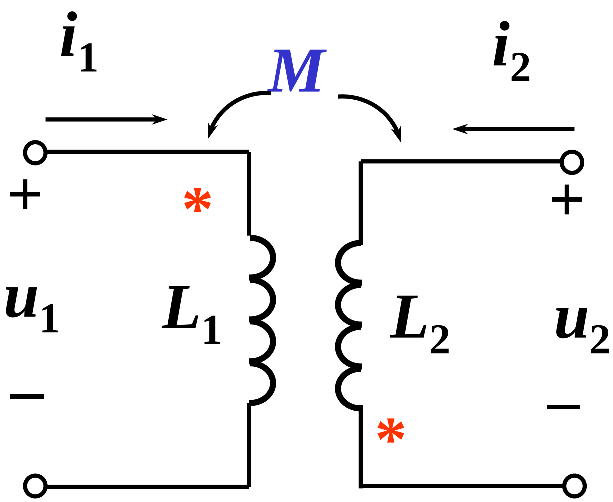
(d)

例10-4：根据图中“同名端”，写出感应电压表达式。



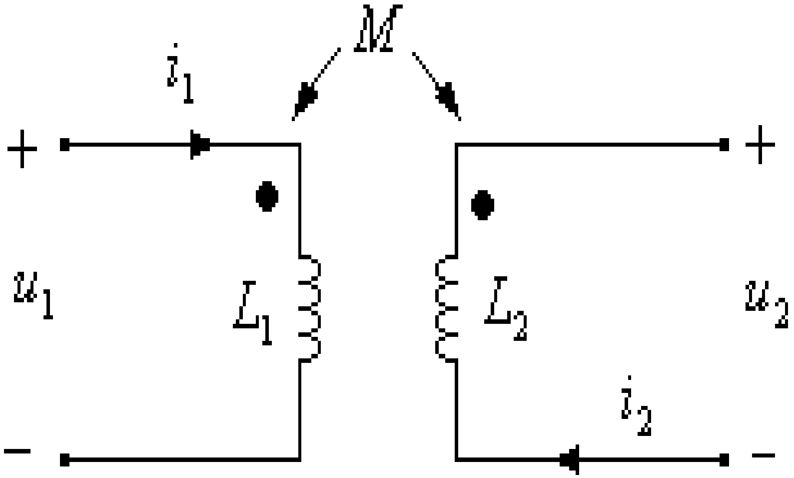
$$u_1 = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} + M \frac{\mathrm{d}i_2}{\mathrm{d}t}$$

$$u_2 = L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} + M \frac{\mathrm{d}i_1}{\mathrm{d}t}$$



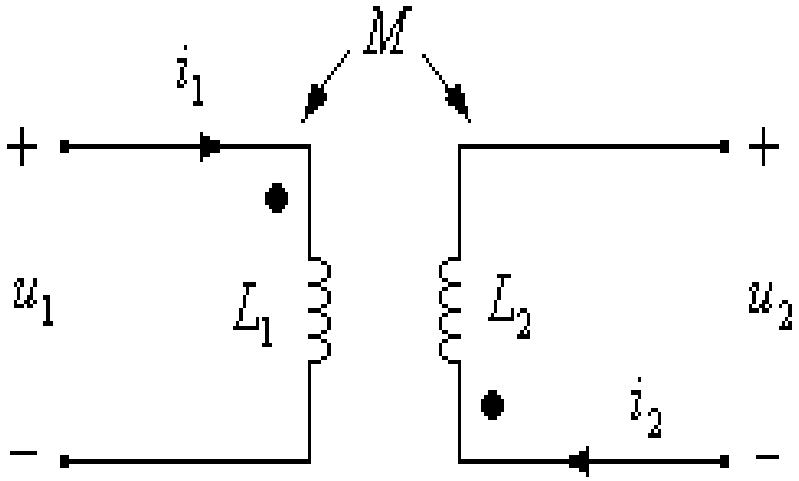
$$u_1 = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} - M \frac{\mathrm{d}i_2}{\mathrm{d}t}$$

$$u_2 = L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} - M \frac{\mathrm{d}i_1}{\mathrm{d}t}$$



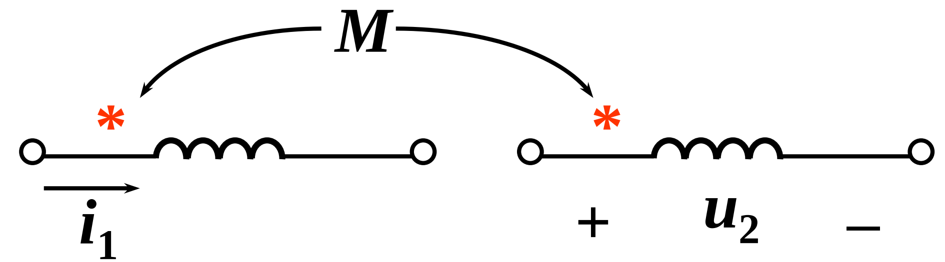
$$u_1 = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} - M \frac{\mathrm{d}i_2}{\mathrm{d}t}$$

$$u_2 = -L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} + M \frac{\mathrm{d}i_1}{\mathrm{d}t}$$

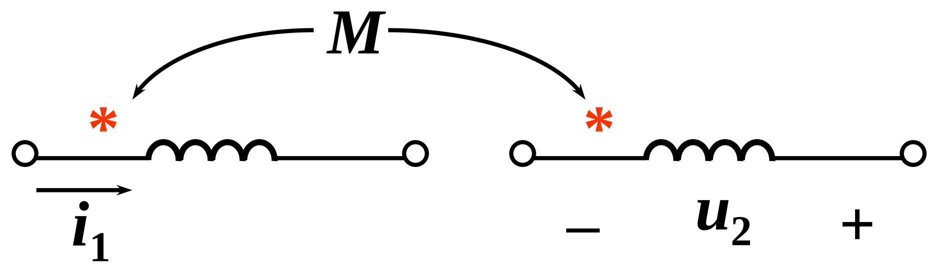


$$u_1 = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} + M \frac{\mathrm{d}i_2}{\mathrm{d}t}$$

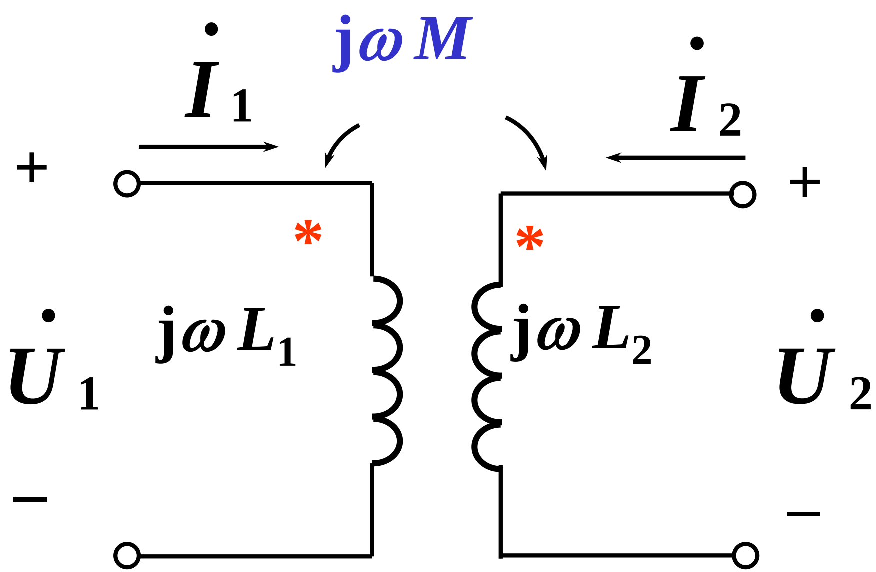
$$u_2 = -L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} - M \frac{\mathrm{d}i_1}{\mathrm{d}t}$$



$$u_2 = M\frac{\mathrm{d}i_1}{\mathrm{d}t}$$



$$u_2 = -M\frac{\mathrm{d}i_1}{\mathrm{d}t}$$

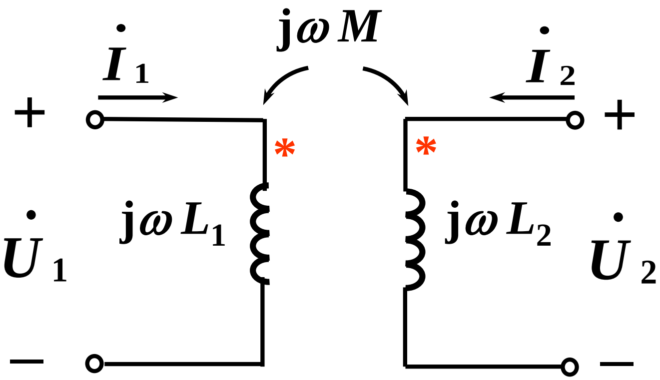


$$\dot U_1 = \mathrm{j}\omega L_1 \dot I_1 + \mathrm{j}\omega M \dot I_2$$

$$\dot U_2 = \mathrm{j}\omega M \dot I_1 + \mathrm{j}\omega L_2 \dot I_2$$

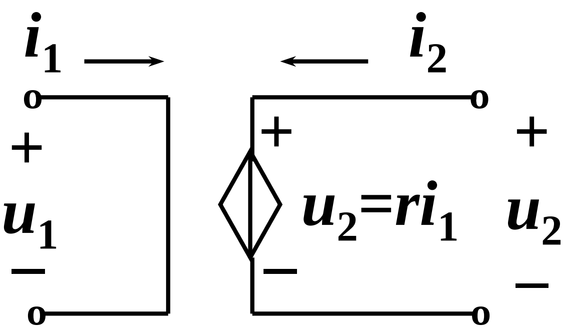
ωM：互感抗

四、耦合电感的等效受控源电路

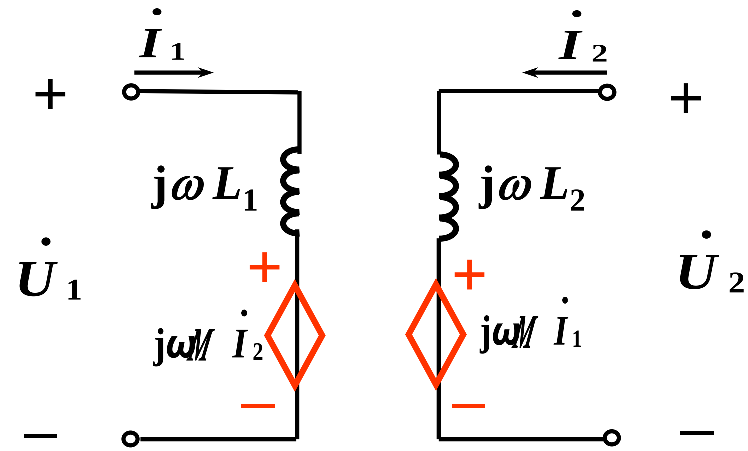


$$\dot{U}_1 = \dot{U}_{11} + \dot{U}_{12}$$

$$\dot{U}_2 = \dot{U}_{22} + \dot{U}_{21}$$



CCVS



五、耦合因数

$$K \overset{def}{=\!=} \sqrt{\frac{|\Psi_{12}\Psi_{21}|}{\Psi_{11}\Psi_{22}}} \Rightarrow K \overset{def}{=\!=} \frac{M}{\sqrt{L_1 L_2}} \le 1$$

k=1
全耦合

顺序相连（串联）的耦合电感

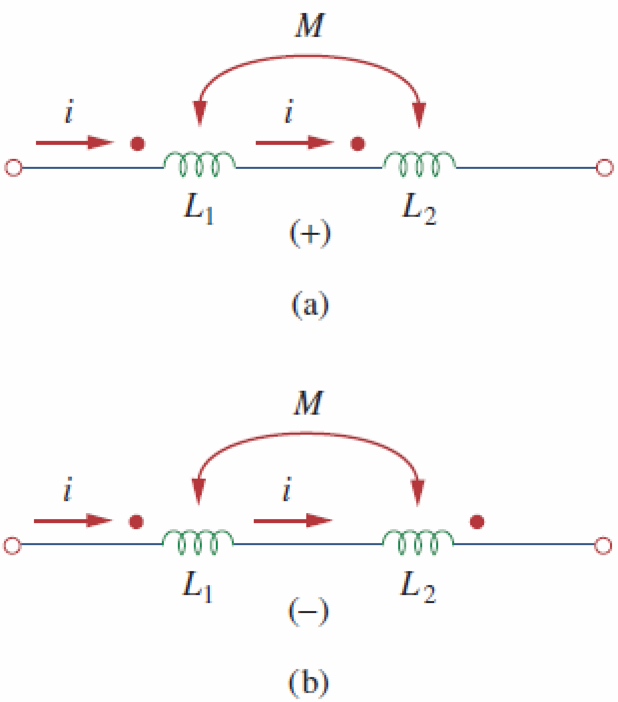


Figure 13.6
Dot convention for coils in series; the sign indicates the polarity of the mutual voltage: (a) series-aiding connection, (b) series-opposing connection.

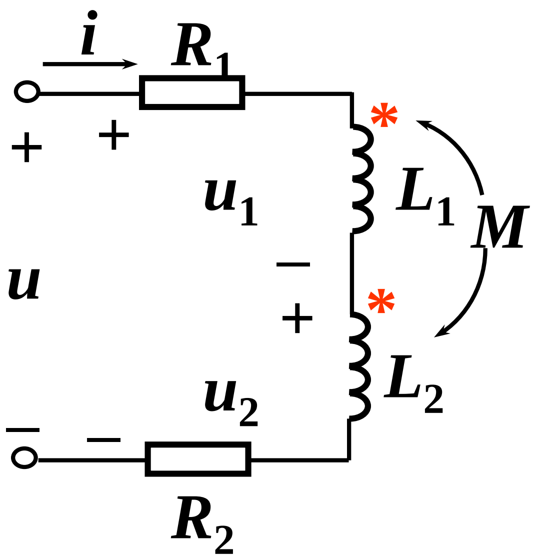
$$L = L_1 + L_2 + 2M \qquad \text{(Series-aiding connection)}$$

$$L = L_1 + L_2 - 2M \qquad \text{(Series-opposing connection)}$$

§10–2 含有耦合电感电路的计算

一、互感线圈的串联

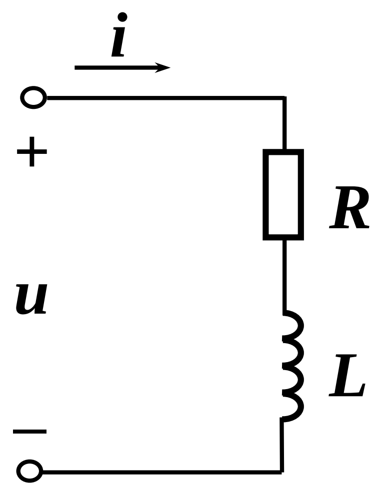
a、顺向串联



$$u = u_1 + u_2$$

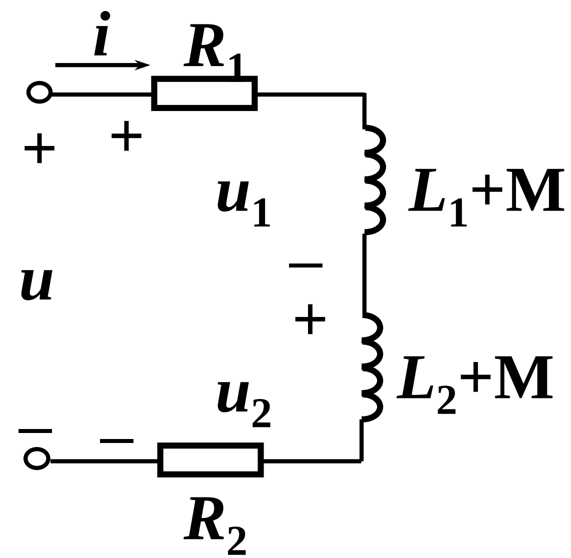
$$= R_1 i + L_1 \frac{\mathrm{d}i}{\mathrm{d}t} + M \frac{\mathrm{d}i}{\mathrm{d}t} + L_2 \frac{\mathrm{d}i}{\mathrm{d}t} + M \frac{\mathrm{d}i}{\mathrm{d}t} + R_2 i$$

$$= (R_1 + R_2) i + (L_1 + L_2 + 2M) \frac{\mathrm{d}i}{\mathrm{d}t}$$

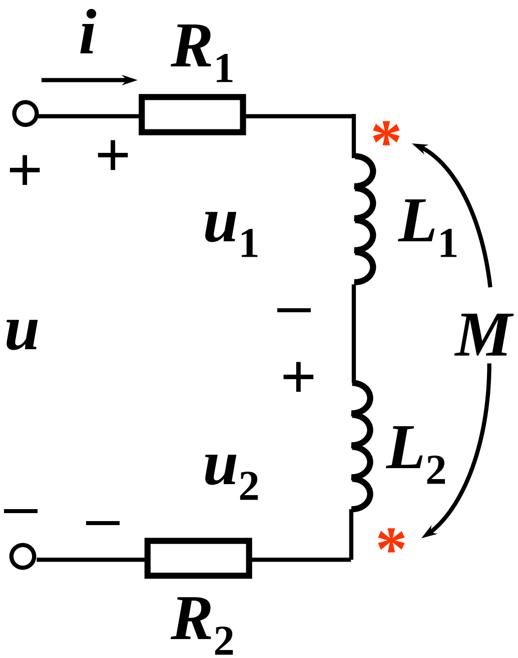


$$R_{eq} = R_1 + R_2$$

$$L_{eq} = L_1 + L_2 + 2M$$



b、反向串联



$$u = u_1 + u_2$$

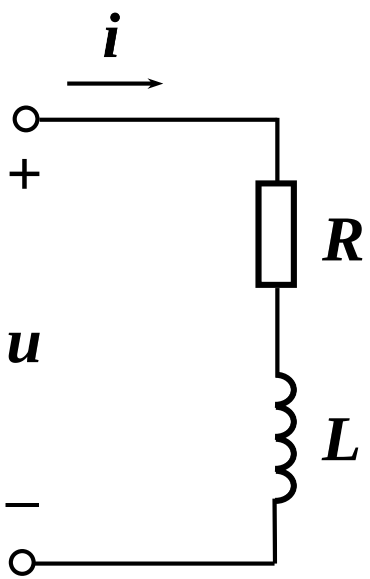
$$= R_1 i + L_1 \frac{\mathrm{d}i}{\mathrm{d}t} - M\frac{\mathrm{d}i}{\mathrm{d}t} + L_2\frac{\mathrm{d}i}{\mathrm{d}t} - M\frac{\mathrm{d}i}{\mathrm{d}t} + R_2 i$$

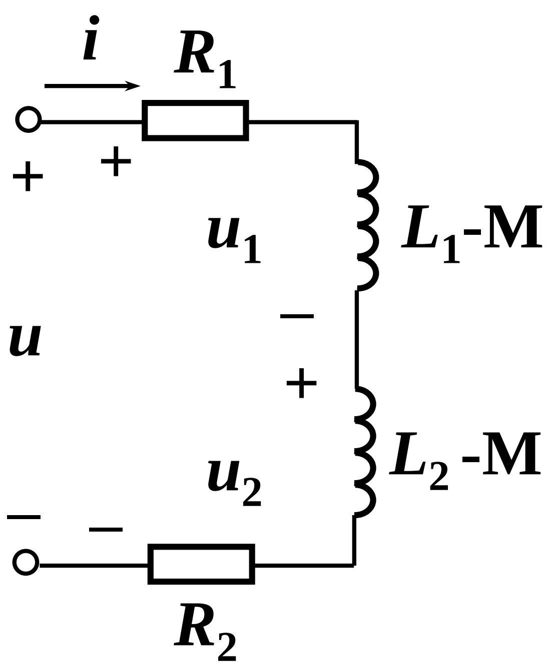
$$= (R_1 + R_2)i + (L_1 + L_2 - 2M)\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$R_{eq} = R_1 + R_2$$

$$L_{eq} = L_1 + L_2 - 2M$$

$$\therefore M \le \frac{1}{2}(L_1 + L_2)$$



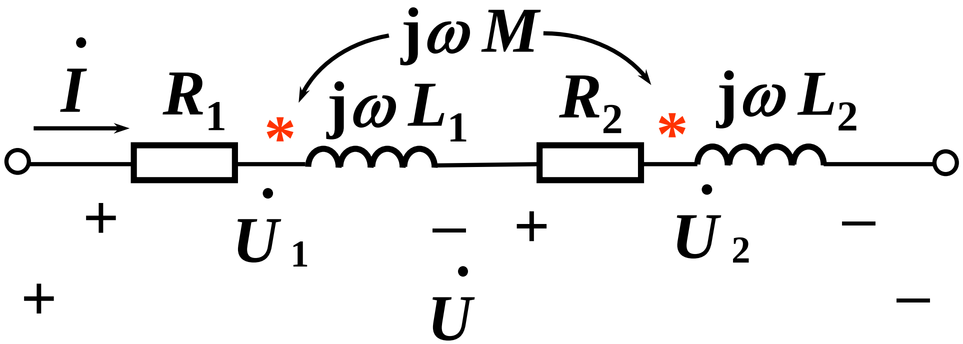


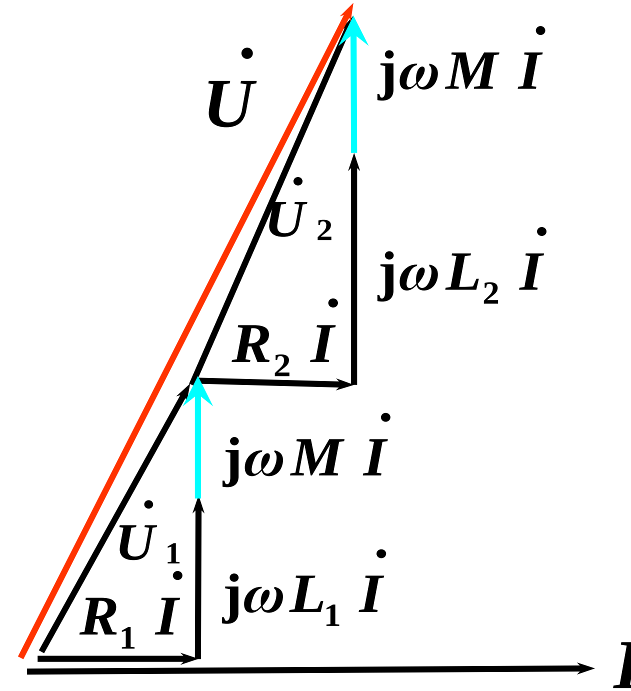
互感的测量方法：

•顺接一次，反接一次，就可以测出互感：

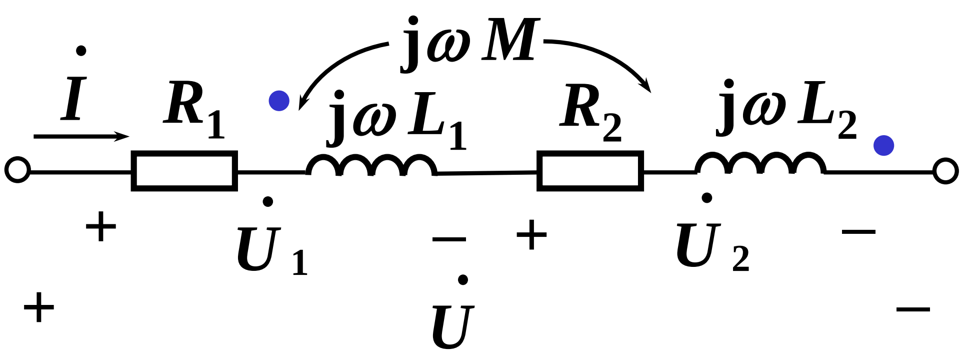
$$\boxed{M = \frac{L_{顺} - L_{反}}{4}}$$

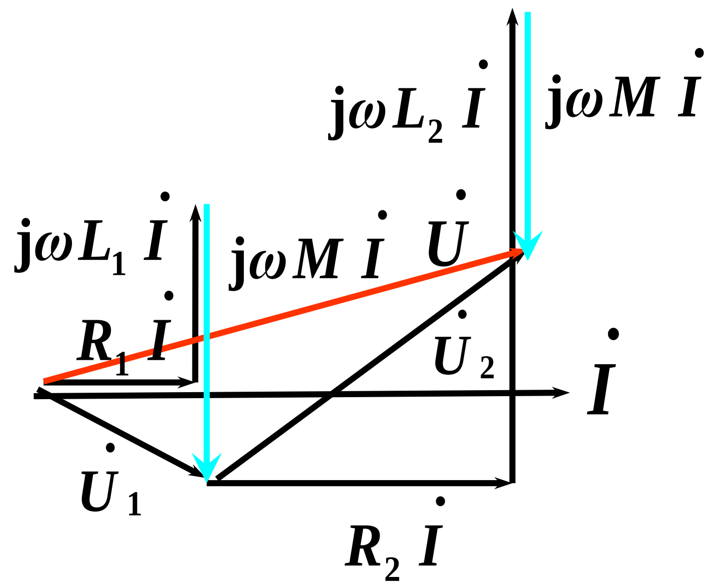
在正弦激励下：





$$\dot{U} = (R_1 + R_2)\dot{I} + j\omega(L_1 + L_2 + 2M)\dot{I}$$

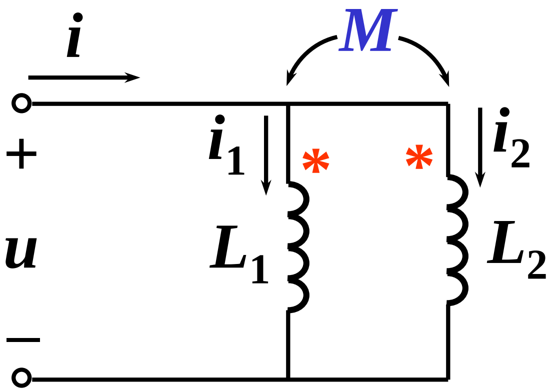




$$\dot{U} = (R_1 + R_2)\dot{I} + j\omega(L_1 + L_2 - 2M)\dot{I}$$

二、互感线圈的并联

a、同侧并联



$$\begin{cases} u = L_1 \dfrac{\mathrm{d}i_1}{\mathrm{d}t} + M \dfrac{\mathrm{d}i_2}{\mathrm{d}t} \\ u = L_2 \dfrac{\mathrm{d}i_2}{\mathrm{d}t} + M \dfrac{\mathrm{d}i_1}{\mathrm{d}t} \\ i = i_1 + i_2 \end{cases}$$

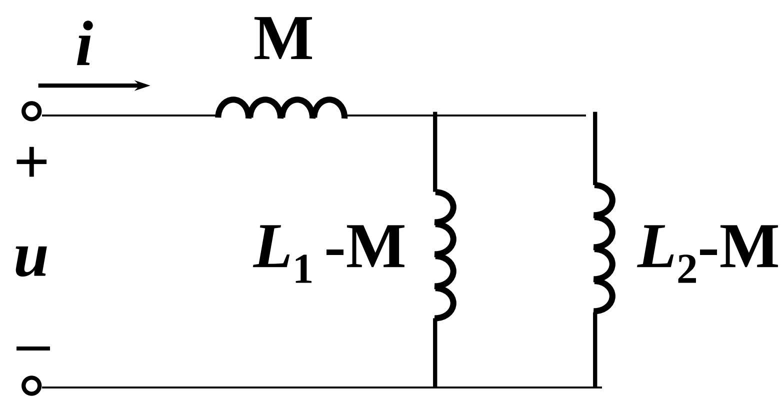
$$u = (L_1 - M)\frac{\mathrm{d}i_1}{\mathrm{d}t} + M\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$u = (L_2 - M)\frac{\mathrm{d}i_2}{\mathrm{d}t} + M\frac{\mathrm{d}i}{\mathrm{d}t}$$

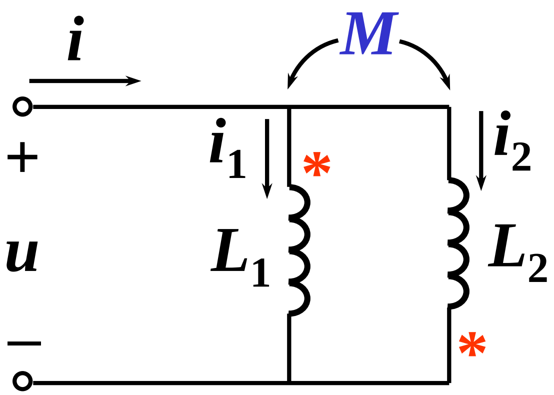
$$u = \frac{(L_1 L_2 - M^2)}{L_1 + L_2 - 2M}\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$L_{eq} = \frac{(L_1 L_2 - M^2)}{L_1 + L_2 - 2M} \geq 0$$

$$M \leq \sqrt{L_1 L_2}$$



b、异侧并联



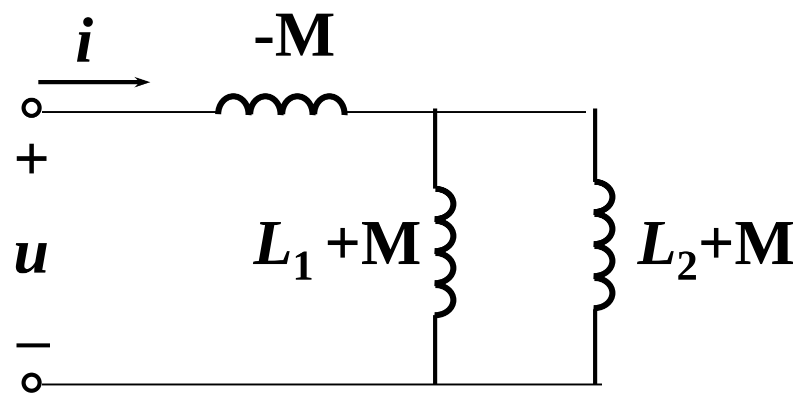
$$\begin{cases} u = L_1 \dfrac{\mathrm{d}i_1}{\mathrm{d}t} - M\dfrac{\mathrm{d}i_2}{\mathrm{d}t} \\ u = L_2 \dfrac{\mathrm{d}i_2}{\mathrm{d}t} - M\dfrac{\mathrm{d}i_1}{\mathrm{d}t} \\ i = i_1 + i_2 \end{cases}$$

$$u = (L_1 + M)\frac{\mathrm{d}i_1}{\mathrm{d}t} - M\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$u = (L_2 + M)\frac{\mathrm{d}i_2}{\mathrm{d}t} - M\frac{\mathrm{d}i}{\mathrm{d}t}$$

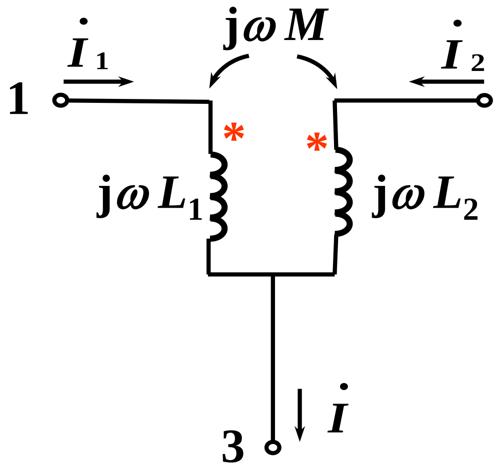
$$u = \frac{(L_1 L_2 - M^2)}{L_1 + L_2 + 2M}\frac{\mathrm{d}i}{\mathrm{d}t}$$

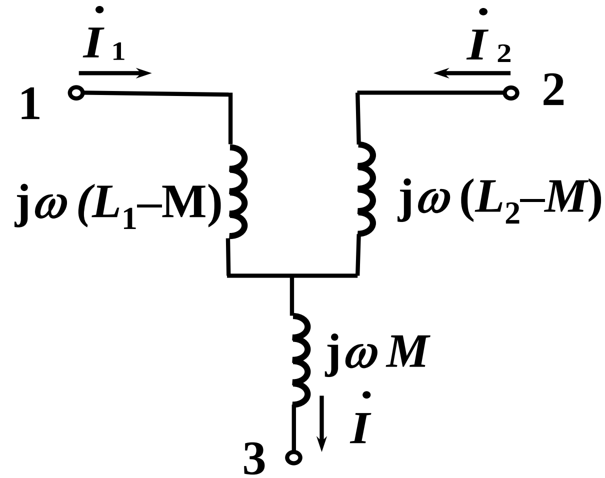
$$L_{eq} = \frac{(L_1 L_2 - M^2)}{L_1 + L_2 + 2M} \geq 0$$



三、有公共端的耦合电感的T型等效电路

(a) 同名端接在一起

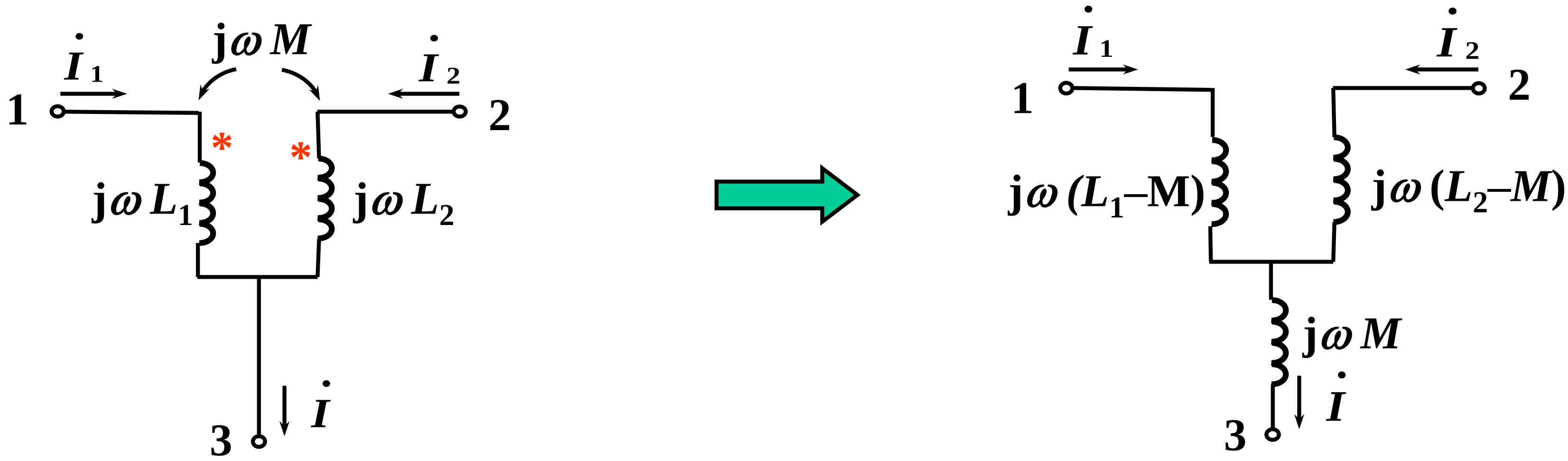




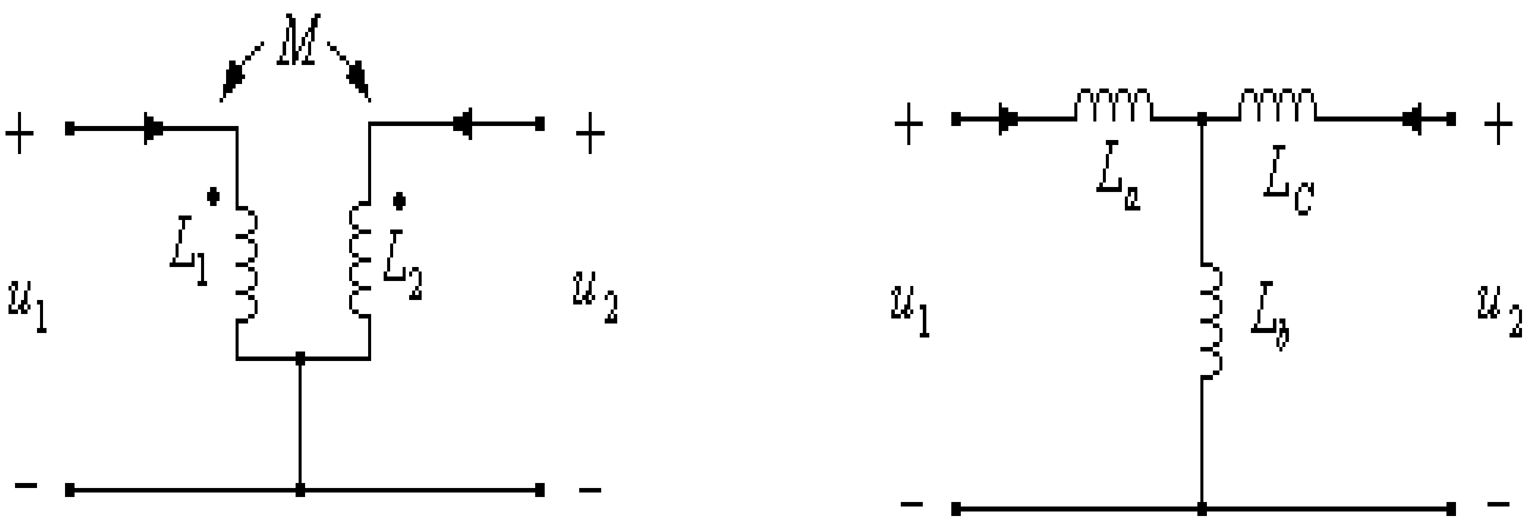
$$\begin{cases} \dot{U}_{13} = \mathrm{j}\omega L_1 \dot{I}_1 + \mathrm{j}\omega M \dot{I}_2 \\ \dot{U}_{23} = \mathrm{j}\omega L_2 \dot{I}_2 + \mathrm{j}\omega M \dot{I}_1 \\ \dot{I} = \dot{I}_1 + \dot{I}_2 \end{cases}$$

整理得

$$\begin{cases} \dot{U}_{13} = \mathrm{j}\omega(L_1 - M)\dot{I}_1 + \mathrm{j}\omega M \dot{I} \\ \dot{U}_{23} = \mathrm{j}\omega(L_2 - M)\dot{I}_2 + \mathrm{j}\omega M \dot{I} \end{cases}$$

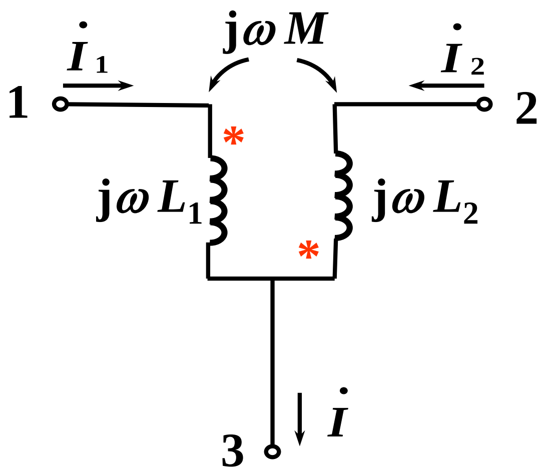


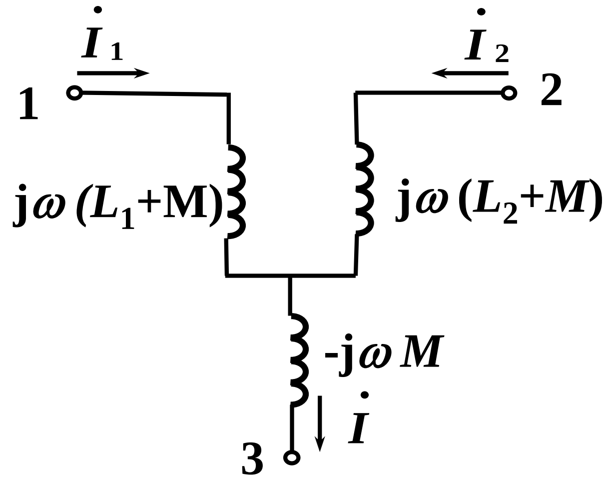
jωM
İ1
İ2
1
2
jωL1
jωL2
3
İ
jω(L1–M)
jω(L2–M)
jωM



M
+
+
L1
L2
u1
u2
–
–
La
Lc
Lb

(b) 非同名端接在一起

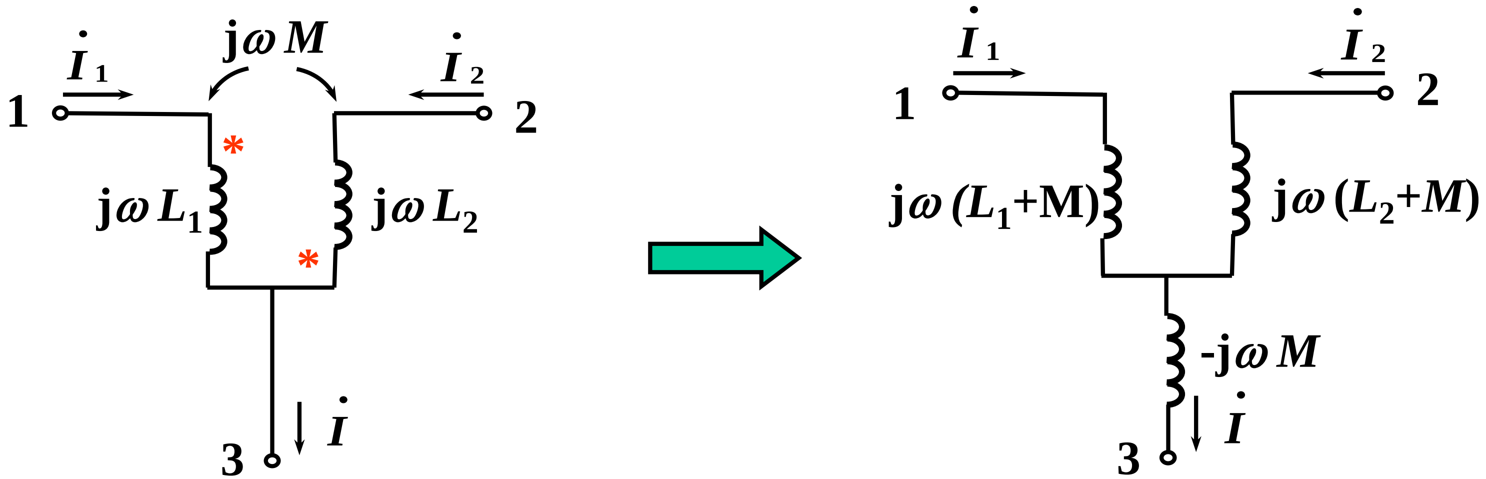




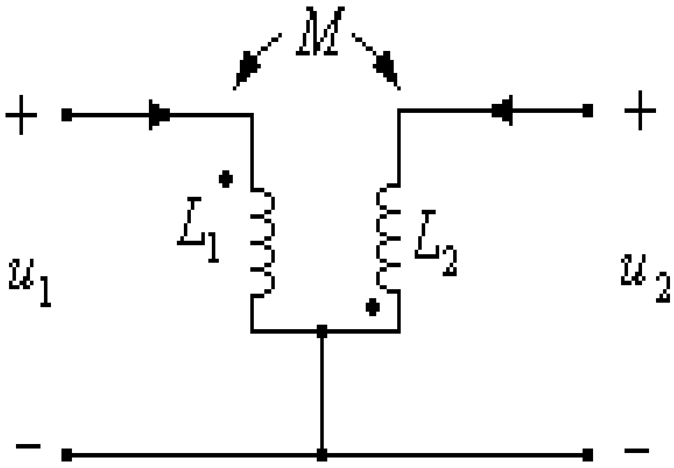
$$\begin{cases} \dot U_{13} = \mathrm{j}\omega L_1 \dot I_1 - \mathrm{j}\omega M \dot I_2 \\ \dot U_{23} = \mathrm{j}\omega L_2 \dot I_2 - \mathrm{j}\omega M \dot I_1 \\ \dot I = \dot I_1 + \dot I_2 \end{cases}$$

整理得

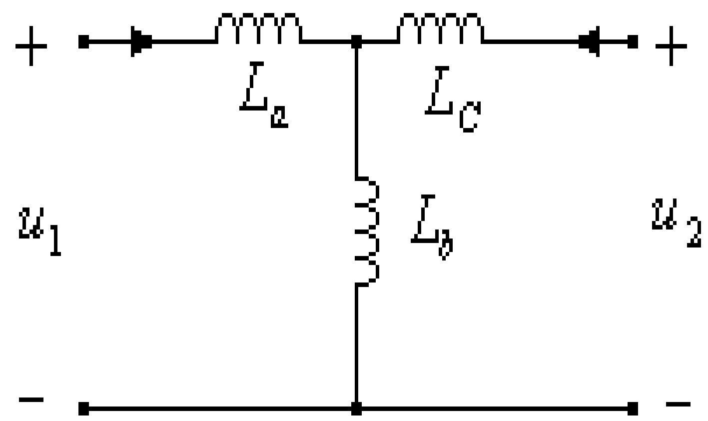
$$\begin{cases} \dot U_{13} = \mathrm{j}\omega(L_1 + M)\dot I_1 - \mathrm{j}\omega M \dot I \\ \dot U_{23} = \mathrm{j}\omega(L_2 + M)\dot I_2 - \mathrm{j}\omega M \dot I \end{cases}$$



1
2
3
jωM
jωL1
jωL2
I1
I2
I
jω(L1+M)
jω(L2+M)
-jωM



M
L1
L2
u1
u2
+
+
−
−



u1
u2
+
+
−
−

变压器

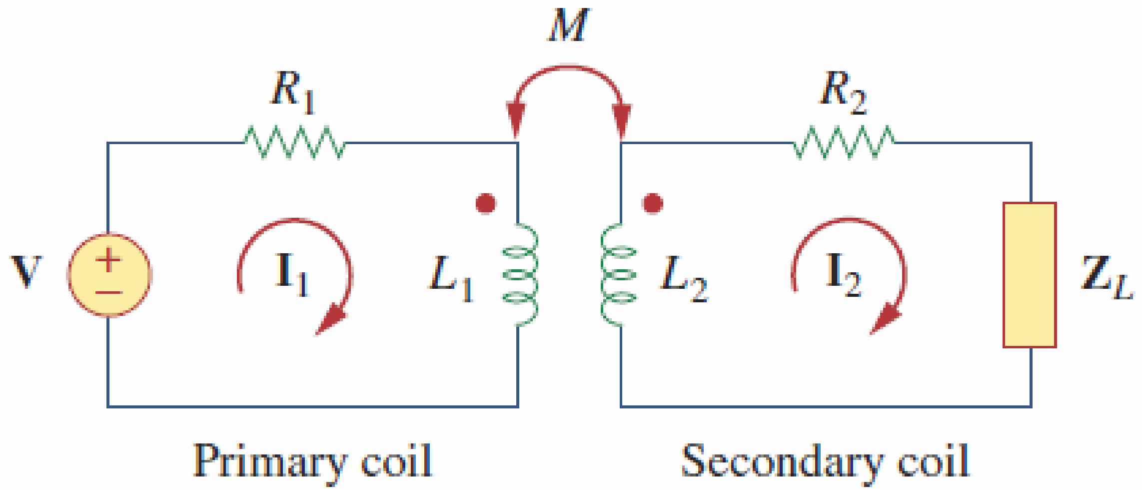
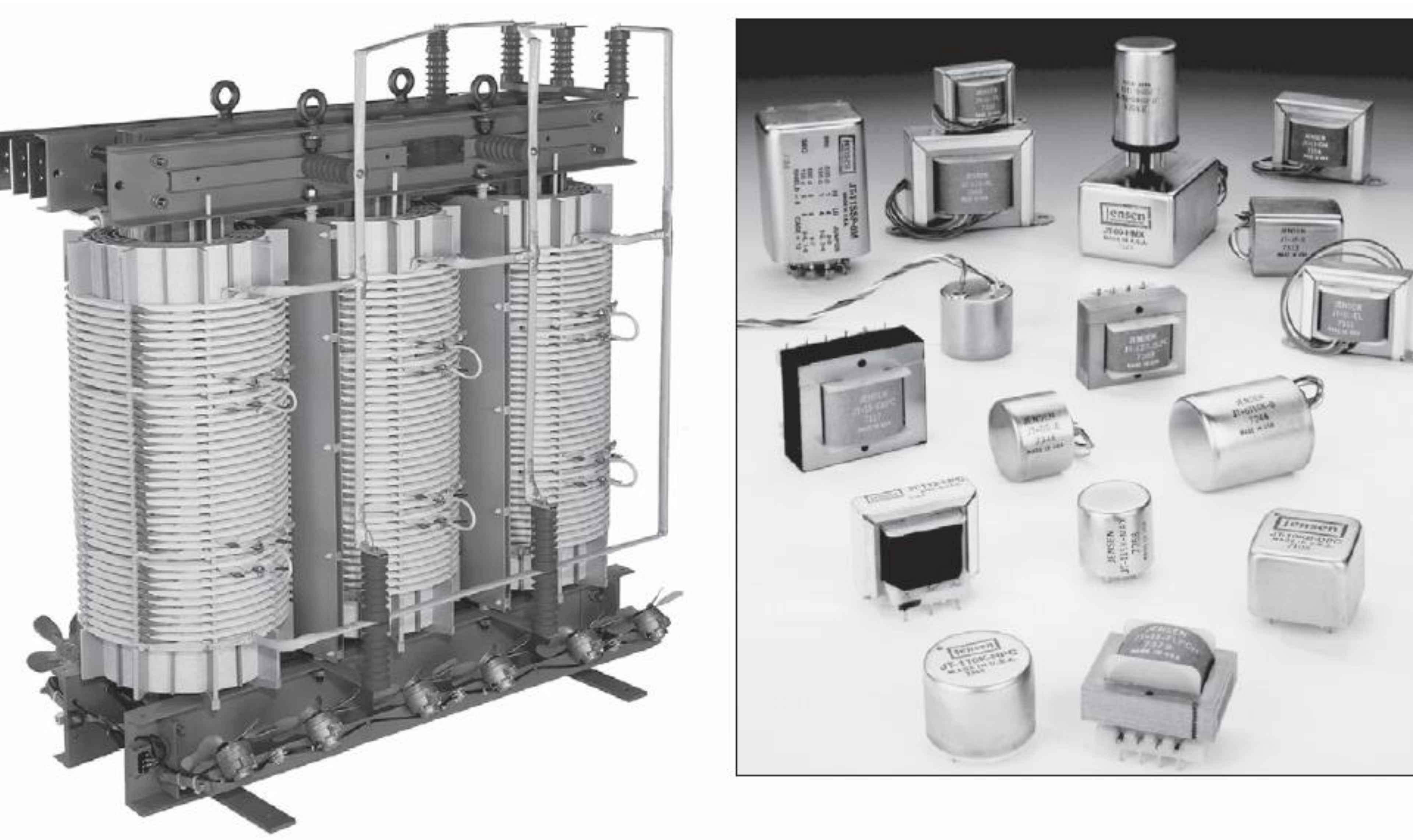


Figure 13.19
A linear transformer.



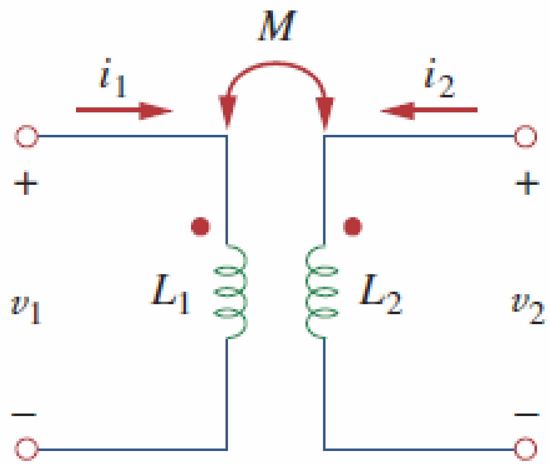
(a) (b)

gure 13.20

ifferent types of transformers: (a) copper wound dry power transformer, (b) audio transformers.

ourtesy of: (a) Electric Service Co., (b) Jensen Transformers.

理想变压器



$$\mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2$$
$$\mathbf{V}_2 = j\omega M \mathbf{I}_1 + j\omega L_2 \mathbf{I}_2$$

$$\mathbf{I}_1 = (\mathbf{V}_1 - j\omega M \mathbf{I}_2)/j\omega L_1$$

$$\mathbf{V}_2 = j\omega L_2 \mathbf{I}_2 + \frac{M\mathbf{V}_1}{L_1} - \frac{j\omega M^2 \mathbf{I}_2}{L_1}$$

$M = \sqrt{L_1 L_2}$ for perfect coupling ($k = 1$). Hence,

$$\mathbf{V}_2 = j\omega L_2 \mathbf{I}_2 + \frac{\sqrt{L_1 L_2}\mathbf{V}_1}{L_1} - \frac{j\omega L_1 L_2 \mathbf{I}_2}{L_1} = \sqrt{\frac{L_2}{L_1}}\mathbf{V}_1 = n\mathbf{V}_1$$

A transformer is said to be ideal if it has the following properties:

1. Coils have very large reactances (L_1, L_2, $M \to \infty$).
2. Coupling coefficient is equal to unity ($k = 1$).
3. Primary and secondary coils are lossless ($R_1 = 0 = R_2$).

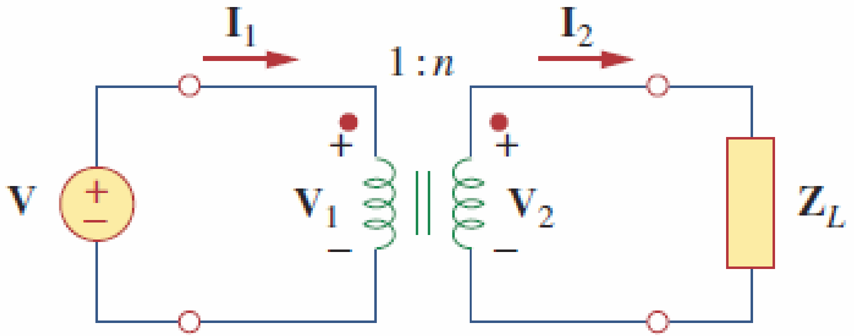


Figure 13.31
Relating primary and secondary quantities in an ideal transformer.

$$v_1 = N_1 \frac{d\phi}{dt}$$

$$v_2 = N_2 \frac{d\phi}{dt}$$

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n$$

$$\boxed{\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n}$$

$$v_1 i_1 = v_2 i_2$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = n$$

$$\boxed{\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2} = \frac{1}{n}}$$

如何确定n的正负号

1. If $\mathbf{V}_1$ and $\mathbf{V}_2$ are *both* positive or both negative at the dotted terminals, use $+n$ in Eq. (13.52). Otherwise, use $-n$.
2. If $\mathbf{I}_1$ and $\mathbf{I}_2$ *both* enter into or both leave the dotted terminals, use $-n$ in Eq. (13.55). Otherwise, use $+n$.

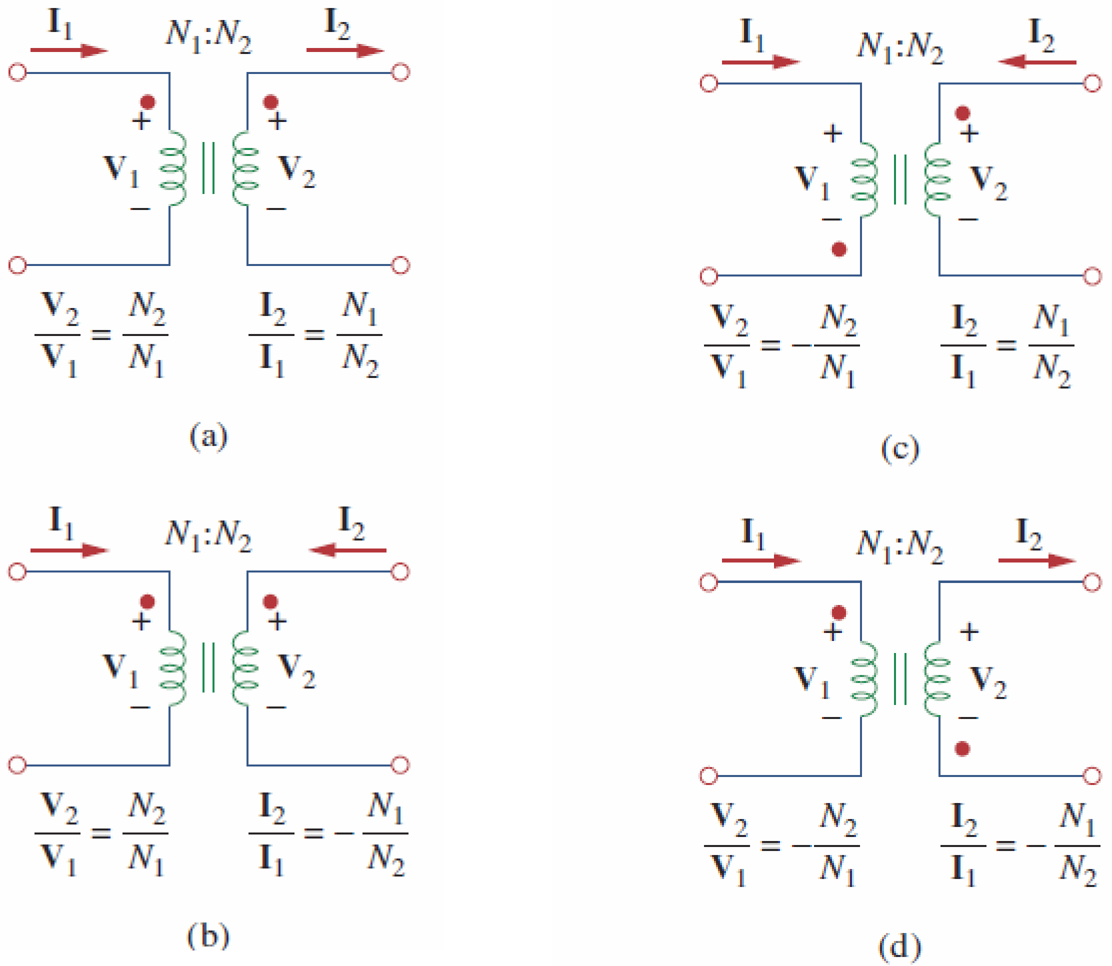
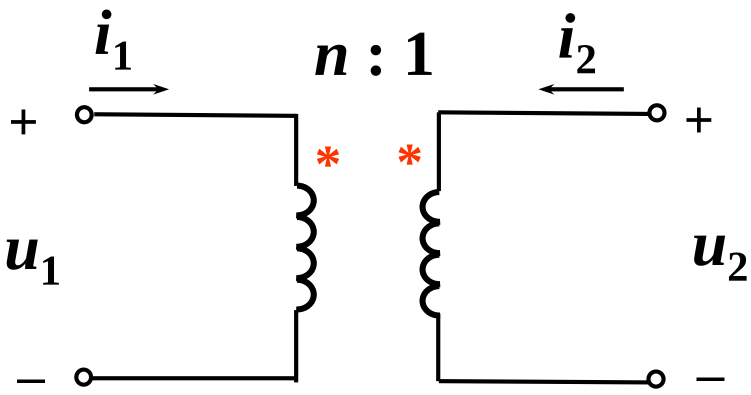
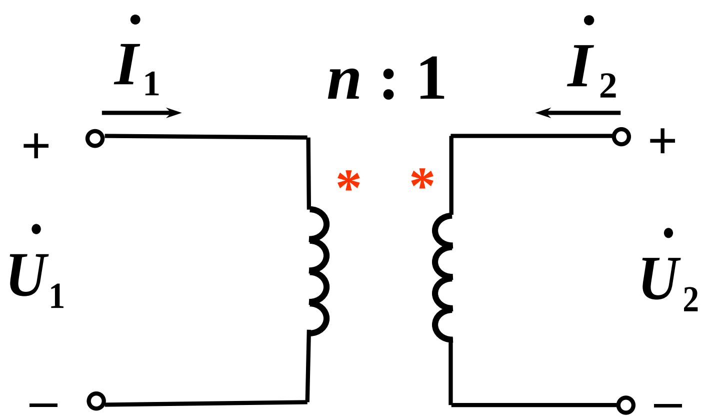


Figure 13.32
Typical circuits illustrating proper voltage polarities and current directions in an ideal transformer.

§ 10–3 理想变压器



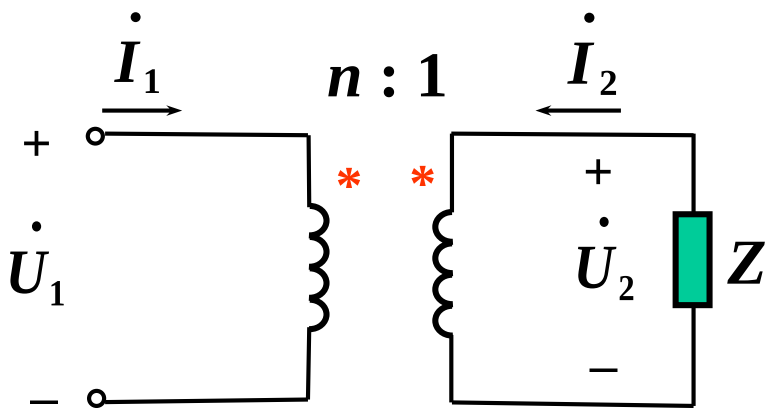
$$\begin{cases} u_1 = n u_2 \\ i_1 = -\dfrac{1}{n} i_2 \end{cases}$$

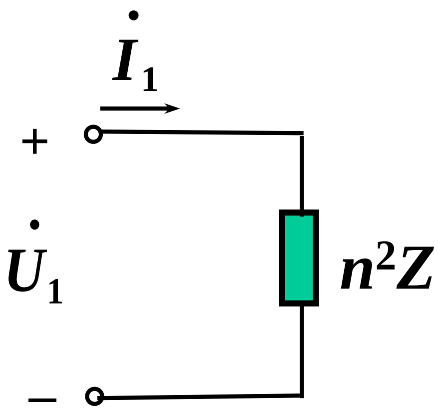


$$\begin{cases} \dot{U}_1 = n \dot{U}_2 \\ \dot{I}_1 = -\dfrac{1}{n} \dot{I}_2 \end{cases}$$

理想变压器的性质：

(a) 阻抗变换性质



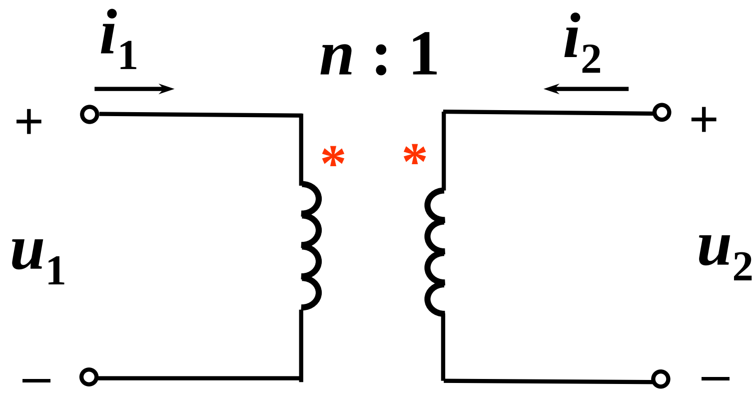


$$\frac{\dot{U}_1}{\dot{I}_1}=\frac{n\dot{U}_2}{-1/n\dot{I}_2}=n^2\left(-\frac{\dot{U}_2}{\dot{I}_2}\right)=n^2Z$$

(b) 功率性质：

理想变压器的特性方程为代数关系，因此无记忆作用。

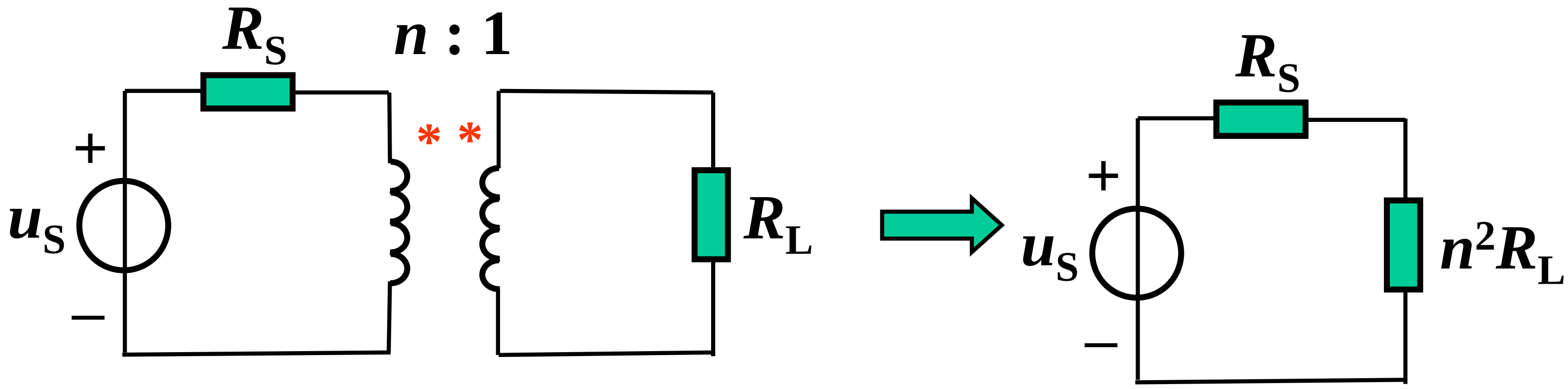
$$\begin{cases} u_1 = nu_2 \\ i_1 = -\dfrac{1}{n} i_2 \end{cases}$$



$$p = u_1 i_1 + u_2 i_2 = u_1 i_1 + \frac{1}{n} u_1 \times (-n i_1) = 0$$

由此可以看出，理想变压器既不储能，也不耗能，在电路中只起传递信号和能量的作用。

例10-9. 已知电源内阻R_S=1kΩ，负载电阻R_L=10Ω。为使R_L上获得最大功率，求理想变压器的变比n。



解:

当 $n^2R_L=R_S$时匹配，即

$$10n^2=1000$$

$$\therefore\ n^2=100,\quad n=10\,.$$

阻抗匹配(impedance matching)

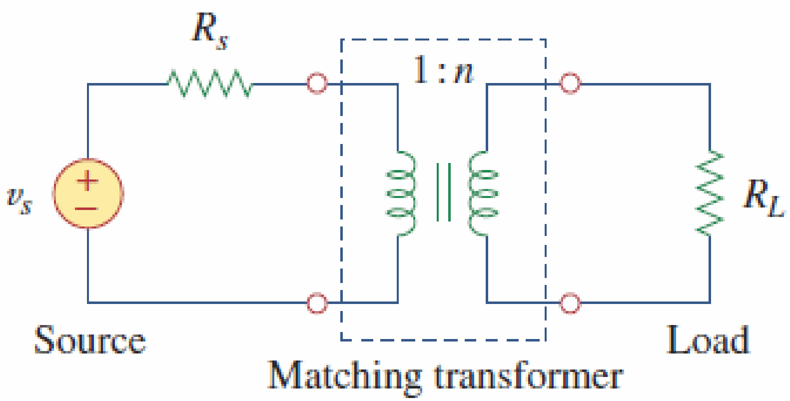


Figure 13.64
Transformer used as a matching device.

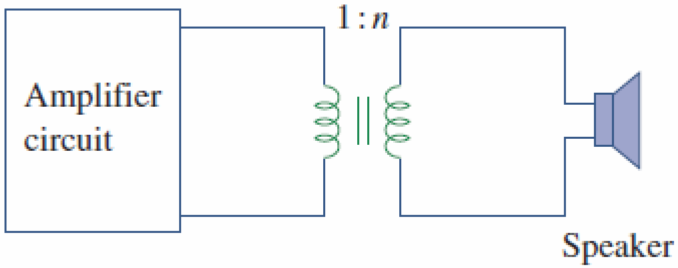


Figure 13.65
Using an ideal transformer to match the speaker to the amplifier; for Example 13.16.

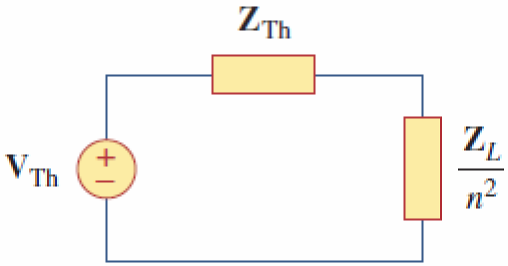
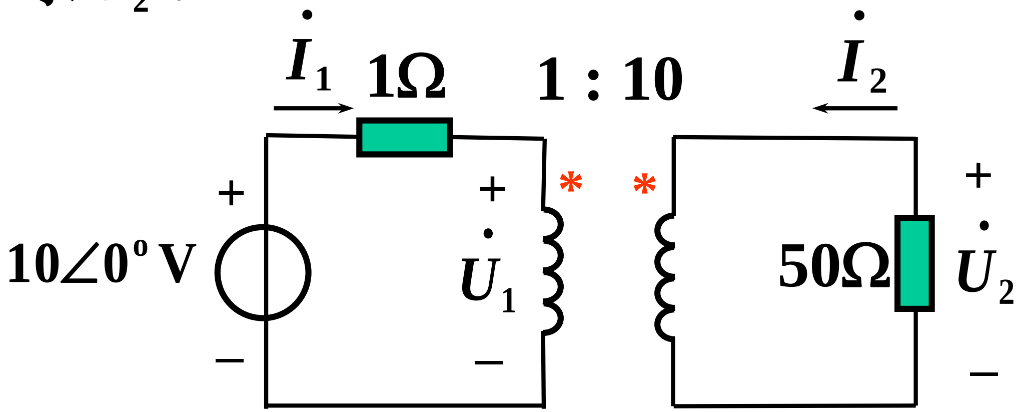


Figure 13.66
Equivalent circuit of the circuit in Fig. 13.65; for Example 13.16.

例10-4. 求$\dot{U}_2$.



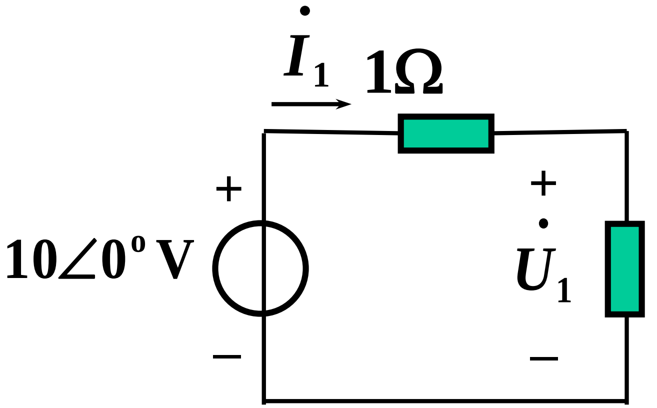
方法1：列方程

$$\begin{cases} 1\times \dot{I}_1 + \dot{U}_1 = 10\angle 0^\circ \\ 50\dot{I}_2 + \dot{U}_2 = 0 \\ \dot{U}_1 = \dfrac{1}{10}\dot{U}_2 \\ \dot{I}_1 = -10\dot{I}_2 \end{cases}$$

解得

$$\dot{U}_2 = 33.33\angle 0^\circ\,\mathrm{V}$$

方法2：阻抗变换



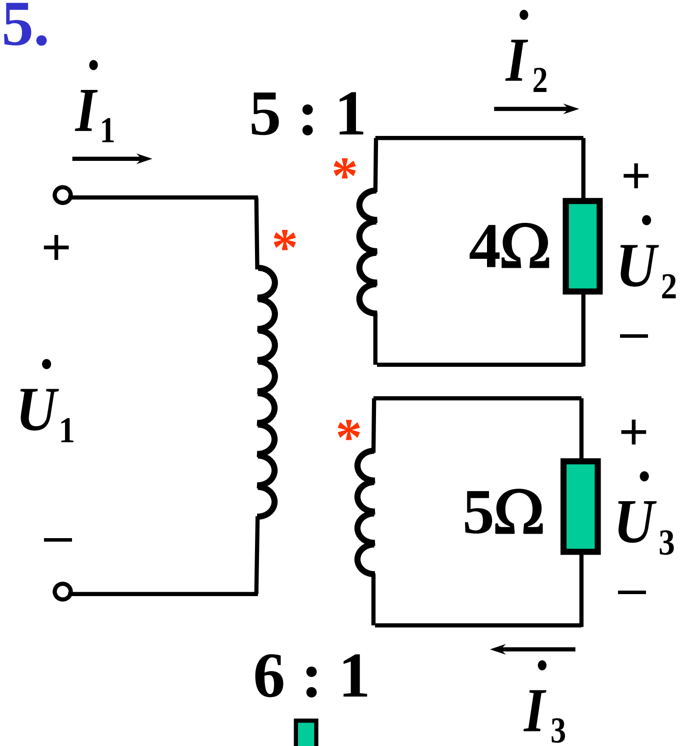
$$(\frac{1}{10})^2 \times 50 = \frac{1}{2}\Omega$$

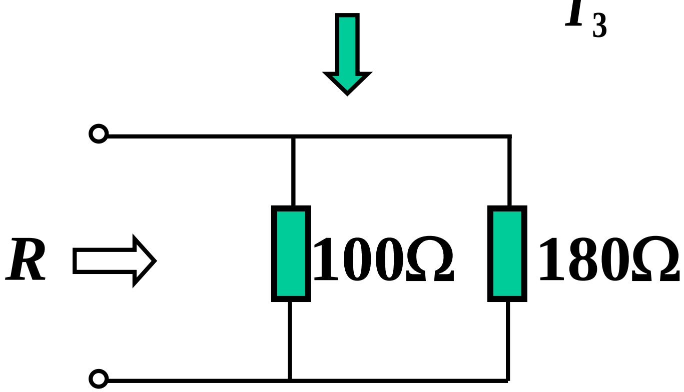
$$\dot{U}_1 = \frac{10\angle 0^{o}}{1+1/2} \times \frac{1}{2} = \frac{10}{3}\angle 0^{o}\ \mathrm{V}$$

$$\dot{U}_2 = n\dot{U}_1 = 10\dot{U}_1$$

$$= 33.33\angle 0^{o}\ \mathrm{V}$$

例10-5.





$\therefore\ R = 100 // 180 = 64.3\Omega$

理想变压器副边有两个线圈，变比分别为5:1和6:1。

求原边等效电阻*R*。

两个副边并联原边

解: $\dot{I}_1 = \frac{1}{5}\dot{I}_2 + \frac{1}{6}\dot{I}_3$

$\dot{U}_1 = 5\dot{U}_2\ ,\qquad \dot{U}_1 = 6\dot{U}_3$

$$R = \frac{\dot{U}_1}{\dot{I}_1} = \frac{\dot{U}_1}{\frac{1}{5}\dot{I}_2 + \frac{1}{6}\dot{I}_3} = \frac{\dot{U}_1}{\frac{1}{5}\frac{\dot{U}_2}{4} + \frac{1}{6}\frac{\dot{U}_3}{5}}$$

$$= \frac{\dot{U}_1}{\frac{\dot{U}_1}{5^2 \times 4} + \frac{\dot{U}_1}{6^2 \times 5}} = \frac{1}{\frac{1}{5^2 \times 4} + \frac{1}{6^2 \times 5}}$$