第九章　正弦稳态电路的分析

主要内容：

1、阻抗、导纳的概念

2、电路的相量图

3、用相量法分析正弦稳态电路

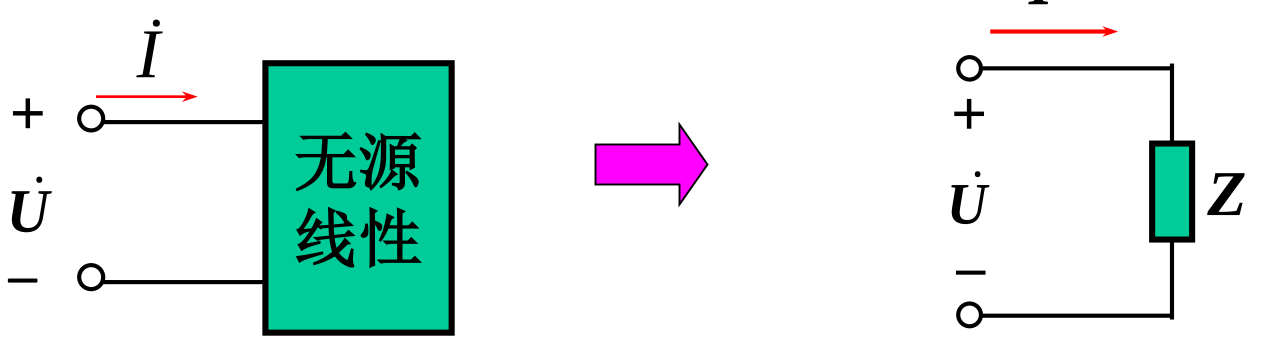
4、正弦稳态电路中的功率分析

§ 9-1 阻抗和导纳

一、阻抗

1. 阻抗的定义

正弦激励下



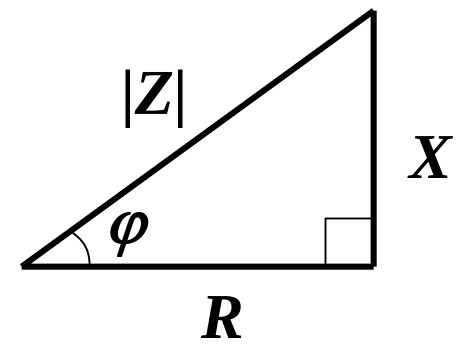
$$Z \overset{def}{=\!=} \frac{\dot{U}}{\dot{I}} = \frac{U\angle\varphi_u}{I\angle\varphi_i} = \frac{U}{I}\angle\varphi_u - \varphi_i = |Z|\angle\varphi_Z$$

阻抗模 $|Z| = \dfrac{U}{I}$　　阻抗角 $\varphi_Z = \varphi_u - \varphi_i$

2. 阻抗的代数形式

阻抗 $Z=\dfrac{\dot{U}}{\dot{I}}=|Z|\angle\varphi_Z=R+\mathrm{j}X$

$$\begin{cases} R=\mathrm{Re}\left[Z\right]=\left|Z\right|\cos\varphi_Z & \text{电阻} \\ X=\mathrm{Im}\left[Z\right]=\left|Z\right|\sin\varphi_Z & \text{电抗} \end{cases}$$



阻抗三角形

$$\left|Z\right|=\sqrt{R^2+X^2}\quad,\quad \varphi_Z=\arctan\left(\frac{X}{R}\right)$$

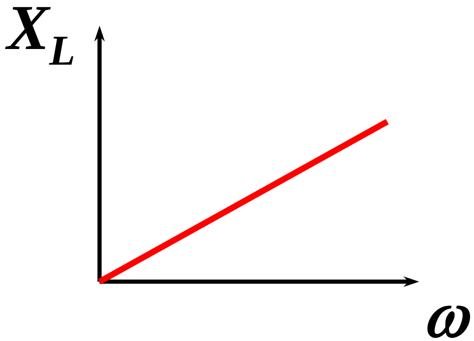
若一端口N_0仅含单个元件

$$\begin{cases} R:\quad Z_R=R & \\ L:\quad Z_L=j\omega L & \text{感抗} \\ C:\quad Z_C=\dfrac{1}{j\omega C}=-j\dfrac{1}{\omega C} & \text{容抗} \end{cases}$$

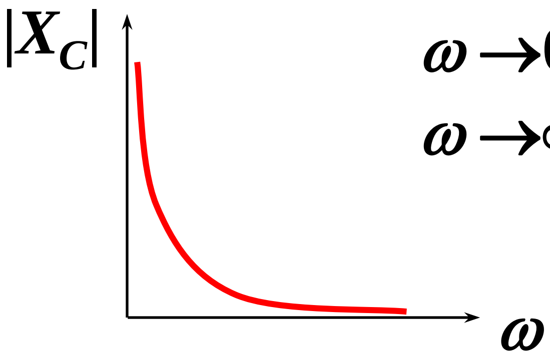
阻抗的物理意义：

(1) 表示限制电流的能力；

(2) 感抗：ωL　　和频率成正比, $\omega\to 0$ 短路, $\omega\to\infty$ 开路；



(3) 容抗:$-1/\omega C$，其绝对值和频率成反比，

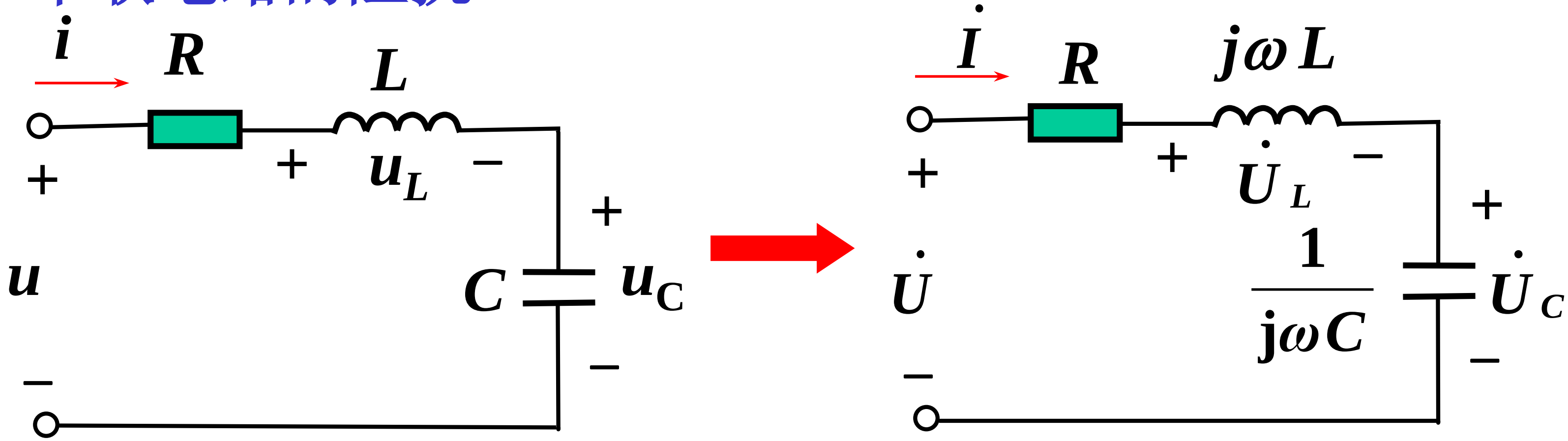


$\omega\to 0$， $|X_C|\to\infty$ 直流开路(隔直)

$\omega\to\infty$， $|X_C|\to 0$　高频短路(旁路作用)

(4) 由于电抗的存在，使电流、电压的相位不同，存在相位差。

3. RLC串联电路的阻抗



由KVL: $\dot{U}=\dot{U}_R+\dot{U}_L+\dot{U}_C=R\dot{I}+\mathrm{j}\omega L\dot{I}-\mathrm{j}\dfrac{1}{\omega C}\dot{I}$

$$=(R+\mathrm{j}\omega L-\mathrm{j}\frac{1}{\omega C})\dot{I}$$

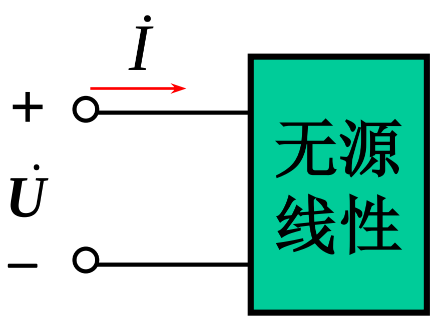
$$Z=\frac{\dot{U}}{\dot{I}}=R+j\omega L+\frac{1}{j\omega C}=R+j(\omega L-\frac{1}{\omega C})$$

$$=R+jX=|Z|\angle\varphi_Z$$

电抗 $X=X_L-X_C=\omega L-\dfrac{1}{\omega C}$

电抗 $$X = X_L - X_C = \omega L - \frac{1}{\omega C}$$

$$\begin{cases} 1.\ X > 0,\ \omega L > \dfrac{1}{\omega C},\ \varphi_Z > 0,\ Z\ \text{呈感性，电压超前电流} \\ 2.\ X < 0,\ \omega L < \dfrac{1}{\omega C},\ \varphi_Z < 0,\ Z\ \text{呈容性，电压滞后电流} \\ 3.\ X = 0,\ \omega L = \dfrac{1}{\omega C},\ \varphi = 0,\ Z\ \text{呈电阻性，电压、电流 同相} \end{cases}$$



$$Z = \frac{\dot{U}}{\dot{I}}$$

$$Z(j\omega) = R(\omega) + jX(\omega)$$

电阻分量　　电抗分量

二、导纳

1. 导纳的定义

$$Y=\frac{1}{Z}=\frac{\dot{I}}{\dot{U}}=\frac{I}{U}\angle\varphi_i-\varphi_u=|Y|\angle\varphi_y$$

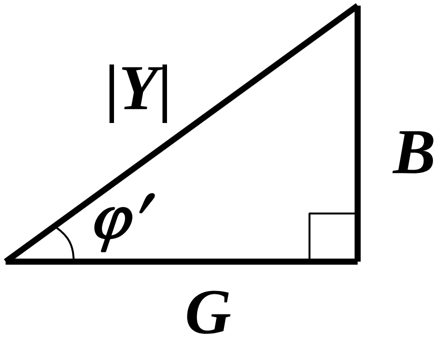
导纳模 $|Y|=\dfrac{I}{U}$ 导纳角 $\varphi_y=\varphi_i-\varphi_u$

2. 导纳的代数形式

$$Y=\frac{1}{Z}=|Y|\angle\varphi_y=G+jB$$

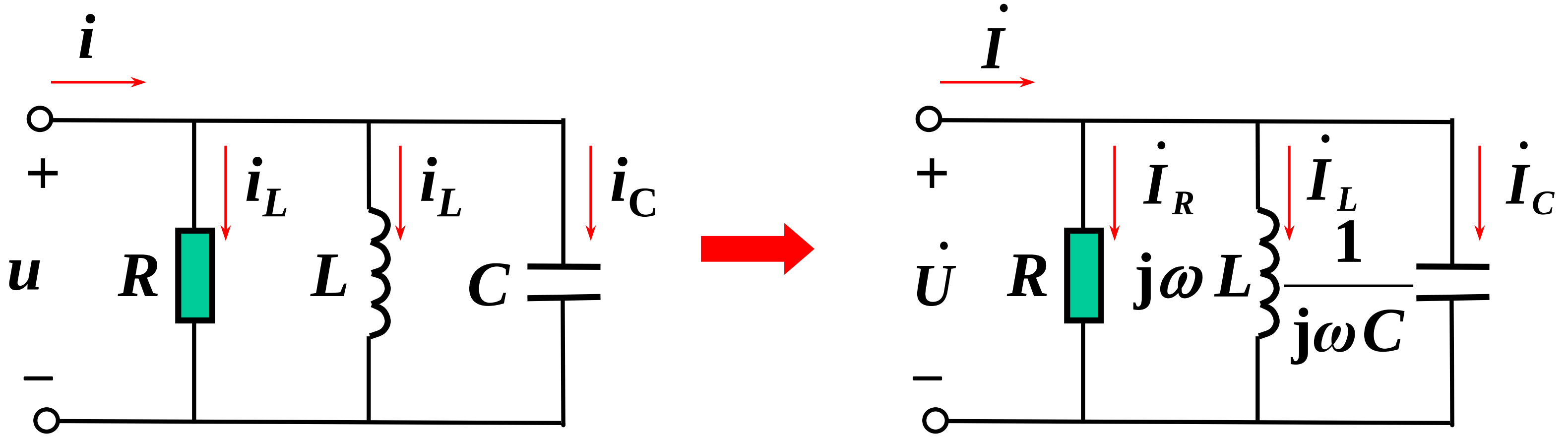
$$\begin{cases} G=\mathrm{Re}[Y]=|Y|\cos\varphi_y & \text{电导} \\ B=\mathrm{Im}[Y]=|Y|\sin\varphi_y & \text{电纳} \end{cases}$$

$$|Y|=\sqrt{G^2+B^2},\quad \varphi_y=\arctan\left(\frac{B}{G}\right)$$



导纳三角形

3. RLC并联电路的导纳



由KCL：
$$\dot{I}=\dot{I}_R+\dot{I}_L+\dot{I}_C=G\dot{U}-\mathrm{j}\frac{1}{\omega L}\dot{U}+\mathrm{j}\omega C\dot{U}$$
$$=(G-\mathrm{j}\frac{1}{\omega L}+\mathrm{j}\omega C)\dot{U}$$

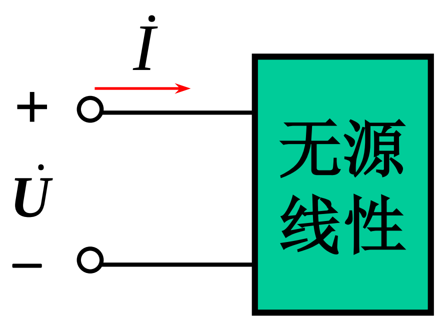
$$Y=\frac{\dot{I}}{\dot{U}}=\frac{1}{R}+\frac{1}{j\omega L}+j\omega C=\frac{1}{R}+j\left(\omega C-\frac{1}{\omega L}\right)$$

$$=G+jB=|Y|\angle\varphi_y$$

电纳 $B=\omega C-\dfrac{1}{\omega L}=B_C-B_L$

电纳 $$B=\omega C-\frac{1}{\omega L}=B_C-B_L$$

$$\begin{cases} 1.\ B>0\ ,\ \omega C>\dfrac{1}{\omega L}\ ,\ \varphi_y>0,\ Y\text{ 呈容性，电流超前电压} \\ 2.\ B<0\ ,\ \omega C<\dfrac{1}{\omega L}\ ,\ \varphi_y<0,\ Y\text{ 呈感性，电流滞后电压} \\ 3.\ B=0\ ,\ \omega C=\dfrac{1}{\omega L}\ ,\ \varphi_y=0, Y\text{ 呈电阻性，电压电流同相} \end{cases}$$



$$Y=\frac{1}{Z}=\frac{\dot{I}}{\dot{U}}$$

$$Y(j\omega)=G(\omega)+jB(\omega)$$

电导分量　　电纳分量

三、阻抗和导纳的关系

$$\because \quad \mathrm{Y} = \frac{1}{\mathrm{Z}}$$

$$\therefore \quad Z(j\omega)Y(j\omega) = 1$$

$$|Z(j\omega)||Y(j\omega)| = 1 \quad , \quad \varphi_Z + \varphi_y = 0$$

注意：

N_0 含有受控源时，可有 $\mathrm{Re}[Z(j\omega)] < 0$，或 $|\varphi_Z| > 90^0$ 情况

四、阻抗和导纳的串并联

1、n个阻抗串联：

$$Z_{eq} = Z_1 + Z_2 + \cdots + Z_n = \sum_{k=1}^{n} Z_k$$

$$\dot{U}_k = \frac{Z_k}{Z_{eq}} \dot{U} \quad , \quad k = 1,2,3\cdots,n$$

2、n个导纳并联：

$$Y_{eq} = Y_1 + Y_2 + \cdots + Y_n = \sum_{k=1}^{n} Y_k$$

$$\dot{I}_k = \frac{Y_k}{Y_{eq}} \dot{I} \quad , \quad k = 1,2,3\cdots,n$$

3、两个阻抗的串联和两个导纳的并联：

① 两个阻抗的串联：

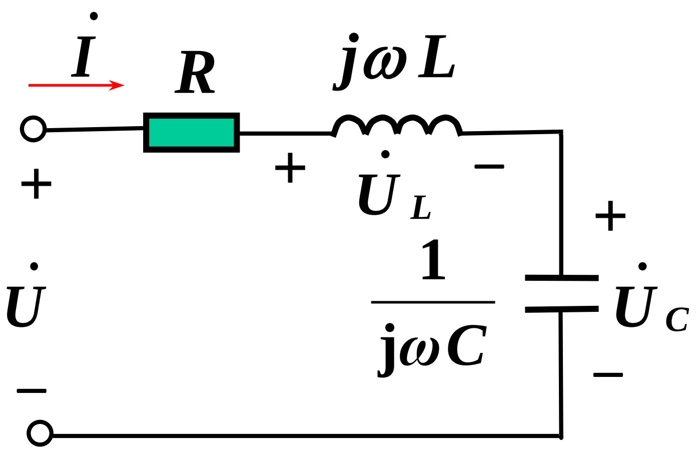
$$Z = Z_1 + Z_2, \ \dot{U}_1 = \frac{Z_1}{Z}\dot{U}, \ \dot{U}_2 = \frac{Z_2}{Z}\dot{U}$$

② 两个导纳的并联：

$$Y = Y_1 + Y_2, \ \dot{I}_1 = \frac{Y_1}{Y}\dot{I}, \ \dot{I}_2 = \frac{Y_2}{Y}\dot{I}$$

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}, \ \dot{I}_1 = \frac{Z_2}{Z_1 + Z_2}\dot{I}, \ \dot{I}_2 = \frac{Z_1}{Z_1 + Z_2}\dot{I}$$

例9-1：图示RLC串联电路，求其阻抗。



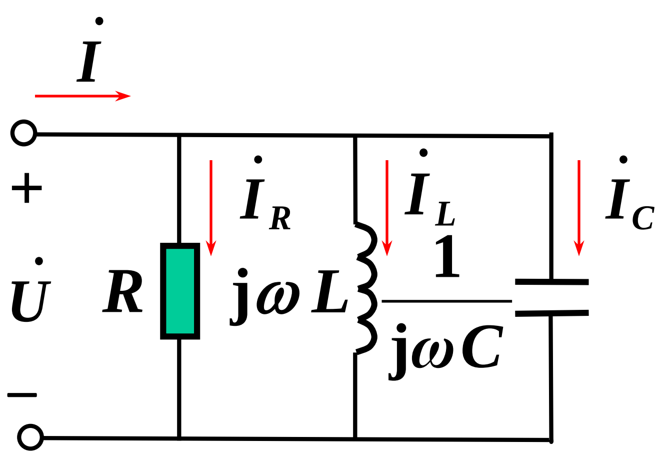
$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$= R + j(\omega L - \frac{1}{\omega C})$$

$$= R + jX$$

$$Y = \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2}$$

例9-2：图示RLC并联电路，求其导纳。



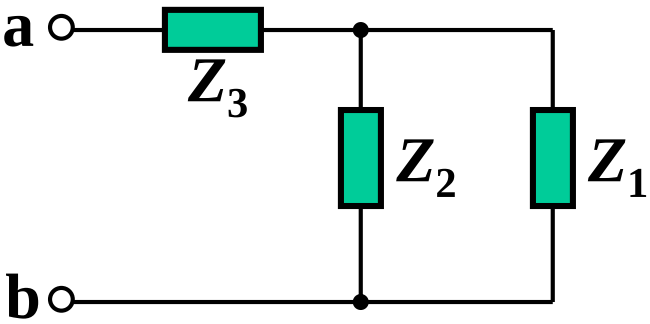
$$Y = G + \frac{1}{j\omega L} + j\omega C$$

$$= G + j（\omega C - \frac{1}{\omega L}）$$

$$= G + jB$$

$$Z = \frac{G}{G^2 + B^2} - j\frac{B}{G^2 + B^2}$$

例9-3： 已知 $Z_1=10+j6.28\Omega$, $Z_2=20-j31.9\ \Omega$, $Z_3=15+j15.7\ \Omega$ 。求 Z_{ab}。



解：

$$Z_{ab}=Z_3+Z_1\,\|\,Z_2=Z_3+\frac{Z_1Z_2}{Z_1+Z_2}$$

$$=15+j15.7+\frac{(10+j6.28)(20-j31.9)}{10+j6.28+20-j31.9}$$

$$=15+j15.7+\frac{11.81\angle 32.13^{o}\times 37.65\angle -57.91^{o}}{39.45\angle -40.5^{o}}$$

$$=15+j15.7+11.27\angle 14.72^{\circ}$$

$$=15+j15.7+10.9+j2.86$$

$$=25.9+j18.56=31.86\angle 35.6^{o}\ \ \Omega$$

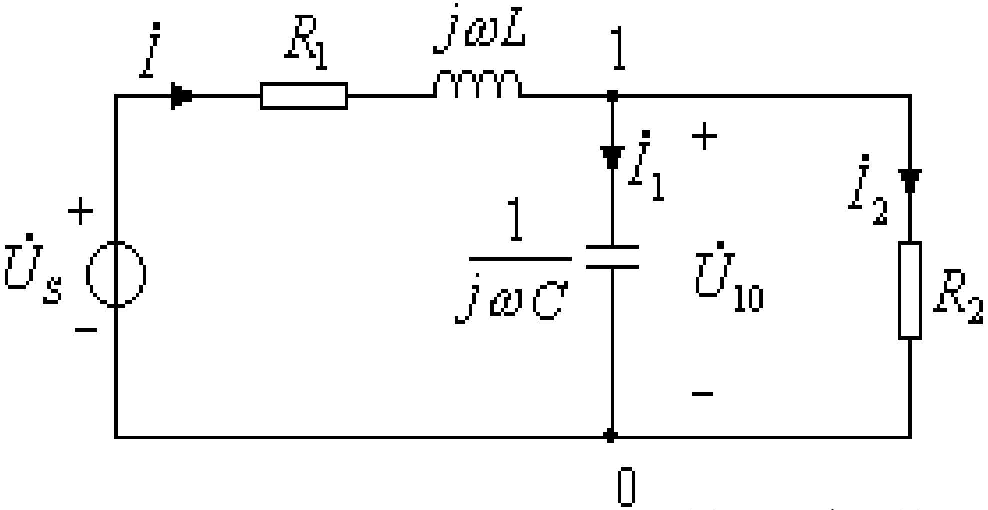
```
In[5]:= ArcTan[6.28 / 10] * 180 / Pi
Out[5]= 32.1288

In[6]:= ArcTan[-31.9 / 20] * 180 / Pi
Out[6]= -57.914

In[8]:= 32.13 - 57.91 - (-40.5)
Out[8]= 14.72
```

例9-4： 图示电路中，$\omega = 314\ \ rad/s$，

$R_1 = 10\Omega$， $L = 0.5H$， $R_2 = 1000\Omega$， $C = 10\mu F$， $U_S = 100V$



求各支路电流和电压。

解：$\dot{U}_S = 100\angle 0°$，

$Z_{R1} = 10\ \Omega$， $Z_{R2} = 1000\ \Omega$，

$Z_L = j\omega L = j157\ \ \Omega$，$Z_C = -j\dfrac{1}{\omega C} = -j318.47\ \ \Omega$

$$Z_{10} = Z_{R2} // Z_C = 303.45\angle -72.33° = (92.11 - j289.13)\ \Omega$$

$$Z_{eq} = Z_{10} + Z_{R1} + Z_L = (102.11 - j132.13)\ \Omega = 166.99\angle -52.30°\ \ \Omega$$

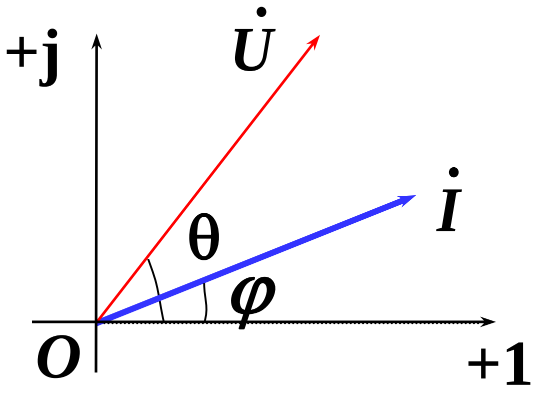
$$\therefore \quad \dot{I} = \frac{\dot{U}}{Z_{eq}} = 0.60\angle 52.30°\ \ A \qquad \dot{U}_{10} = Z_{10}\dot{I} = 182.07\angle -20.03°\ \ V$$

$$\dot{I}_1 = \frac{\dot{U}_{10}}{Z_C} = 0.57\angle 69.97°\ \ A \qquad \dot{I}_2 = \frac{\dot{U}_{10}}{R_2} = 0.18\angle -20.03°\ \ A$$

§9–3　电路的相量图

相量图

相量和复数一样可以在平面上用向量表示



电路的相量图

反映电路的KCL、KVL和电压、电流相位关系的相量图.

§9–4 正弦稳态电路的分析

电阻电路与正弦稳态电路相量法分析比较：

电阻电路：

$$\begin{cases} \text{KCL}: \quad \sum i = 0 \\ \text{KVL}: \quad \sum u = 0 \\ \text{元件约束关系：} \quad u = Ri \\ \qquad\qquad \text{或} \quad i = Gu \end{cases}$$

正弦电路相量分析：

$$\begin{cases} \text{KCL}: \quad \sum \dot{I} = 0 \\ \text{KVL}: \quad \sum \dot{U} = 0 \\ \text{元件约束关系：} \quad \dot{U} = Z\dot{I} \\ \qquad\qquad \text{或} \quad \dot{I} = Y\dot{U} \end{cases}$$

可见，二者依据的电路定律是相似的。只要作出正弦稳态电路的*相量模型*，便可将电阻电路的分析方法推广到正弦稳态电路的相量分析中。

一、利用 $\dot{U}=Z\dot{I}$ 或 $\dot{I}=Y\dot{U}$ 求解

1．同一元件的阻抗与导纳互为倒数，同一对端口之间的阻抗与导纳互为倒数，即

$$Z=\frac{1}{Y},\ Y=\frac{1}{Z}$$

2．记住基本元件的阻抗和导纳

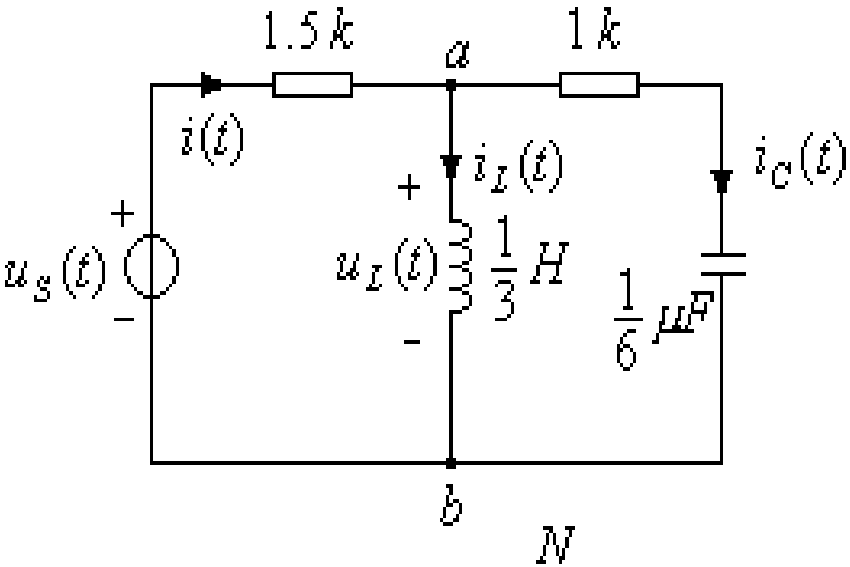
$$Z_R=R,\quad Z_L=j\omega L,\quad Z_C=\frac{1}{j\omega C}=-j\frac{1}{\omega C}$$

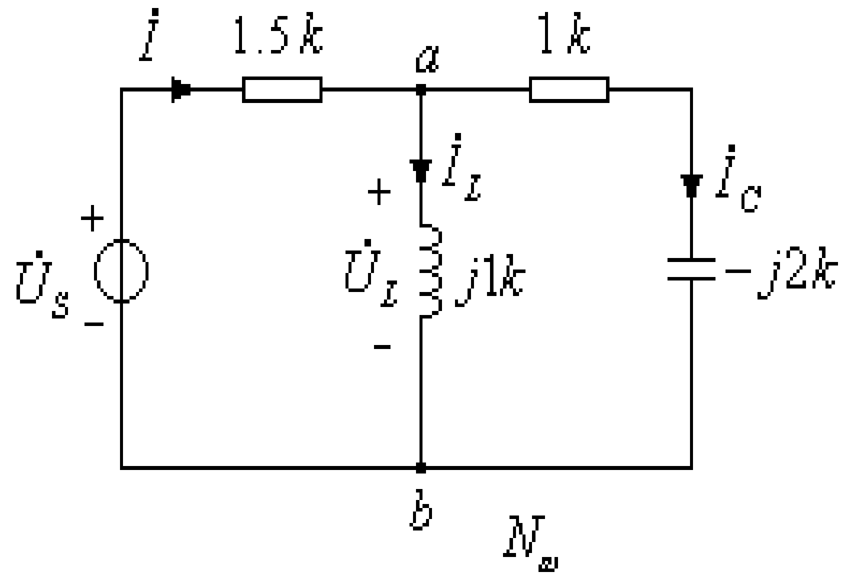
3．串联部分总阻抗和并联部分的总导纳

$$Z=\sum_{k=1}^{n}Z_k \qquad Y=\sum_{k=1}^{n}Y_k$$

例9-5：电路如下图所示，$U_S = 40\sqrt{2}\cos 3000t \ V$ ，

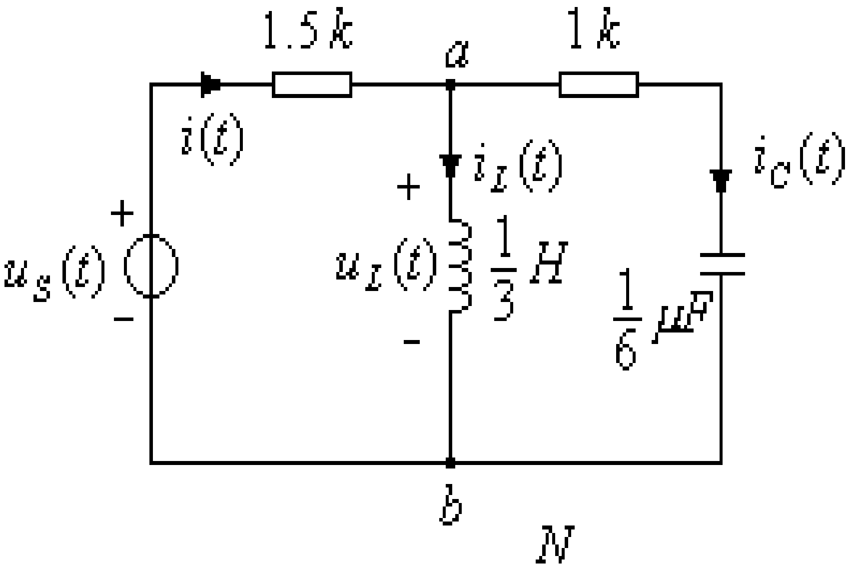
求 $i(t), i_C(t), i_L(t)$

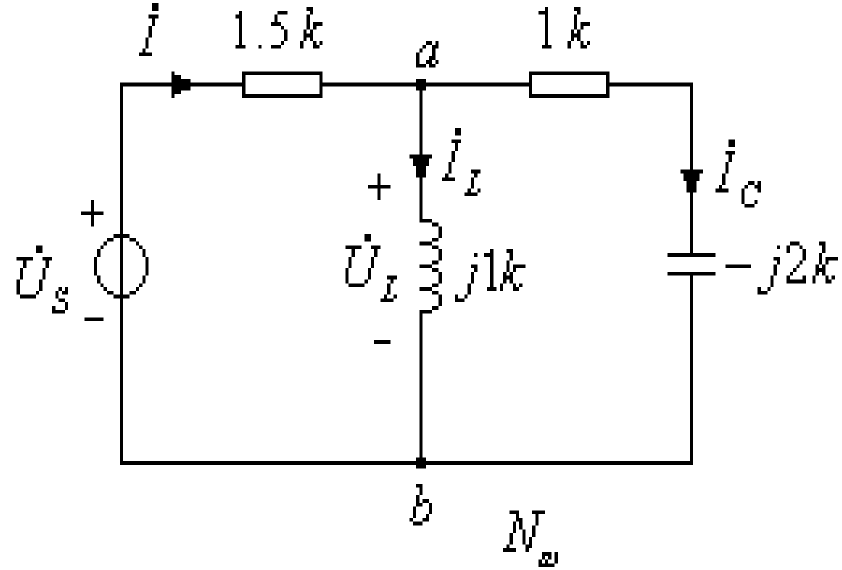




解：已知：$\dot{U}_S = 40\angle 0° \ V$

$$Z = 1.5 + Z_{ab} = 1.5 + \frac{(j1)(1-j2)}{j1+1-j2} = 2 + j1.5 = 2.5\angle 36.9° \quad k\Omega$$





$$\dot{I} = \frac{\dot{U}_S}{Z} = \frac{40\angle 0°}{2.5\angle 36.9°} = 16\angle -36.9° \ \ mA$$

$$\dot{I}_C = \dot{I}\frac{j1}{1+j1-j2} = \dot{I}\times\frac{j}{1-j} = \frac{1}{2}(j-1)\dot{I} = 11.3\angle 98.1° \ \ mA$$

$$\dot{I}_L = \dot{I}\frac{1-j2}{1+j1-j2} = 25.3\angle -55.3° \ \ mA$$

$$\dot{I} = 16\angle -36.9^\circ \ \ mA$$

$$\dot{I}_C = 11.3\angle 98.1^\circ \ \ mA$$

$$\dot{I}_L = 25.3\angle -55.3^\circ \ \ mA$$

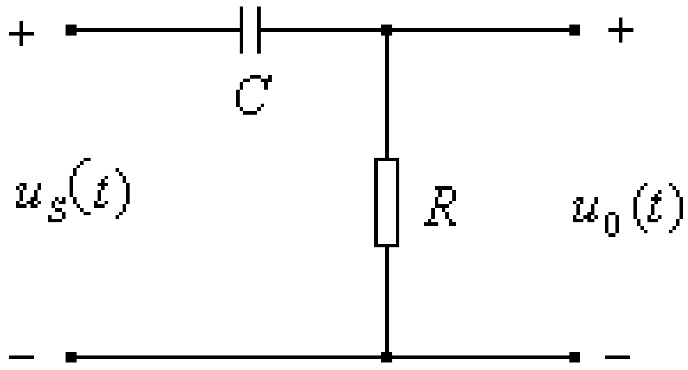
$$\therefore \ i(t) = 16\sqrt{2}\cos(3000\ t - 36.9^\circ) \ \ mA$$

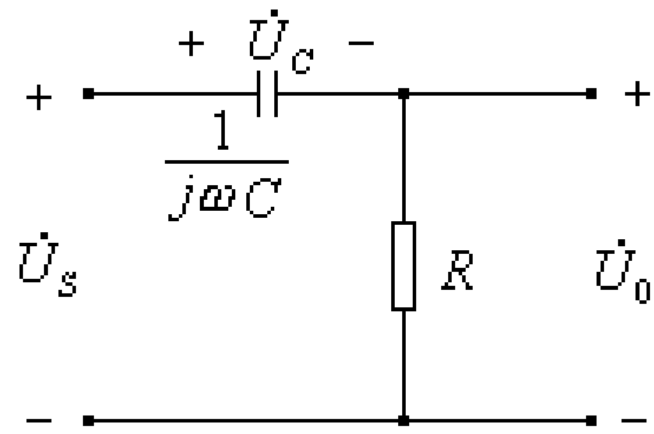
$$i_C(t) = 11.3\sqrt{2}\cos(3000\ t + 98.1^\circ) \ \ mA$$

$$i_L(t) = 25.3\sqrt{2}\cos(3000\ t - 55.3^\circ) \ \ mA$$

例9-6：电路如下图所示，$u_S = \sqrt{2} U_S \cos \omega t$

求$u_0(t)$对$u_S(t)$ 的相位关系。





解：$\because \quad \dot{U}_S = U_S \angle 0° \ V$

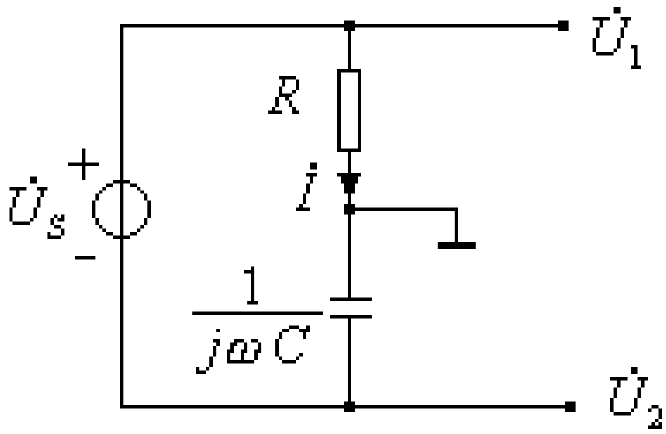
$$\dot{U}_0 = \dot{U}_S \frac{R}{R - j\frac{1}{\omega C}} = \dot{U}_S \frac{R}{\sqrt{R^2 + (\frac{1}{\omega C})^2}} \angle \arctan \frac{1}{\omega CR} = U_0 \angle \varphi_0$$

$$\therefore \quad \varphi_0 = \arctan \frac{1}{\omega CR}$$

$$\varphi = \varphi_0 - \varphi_S = \arctan \frac{1}{\omega CR} > 0$$

$\because \omega \to \infty \quad \varphi \to 0; \ \omega \to 0 \quad \varphi \to 90°$ $\quad \therefore \ 0° < \varphi = \arctan \frac{1}{\omega CR} < 90°$

例9-7：求下图所示相量模型中 $\dot{U}_1$ 与 $\dot{U}_2$的相位关系。



解：令 $\dot{I} = I\angle\varphi_i$

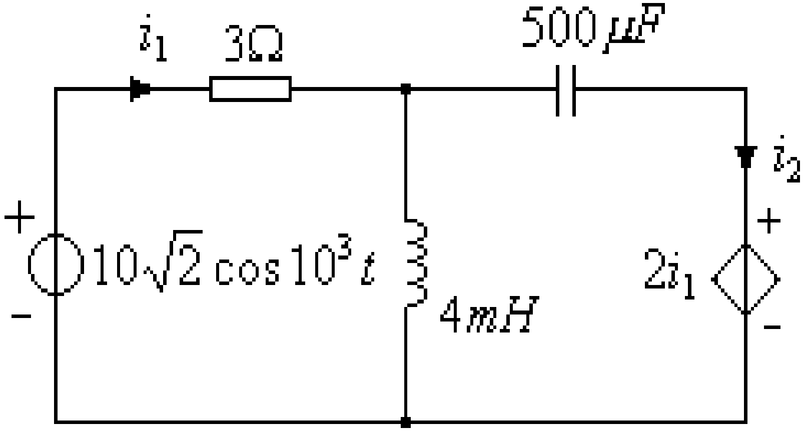
$$\dot{U}_1 = R\dot{I} = RI\angle\varphi_i = U_1\angle\varphi_i$$

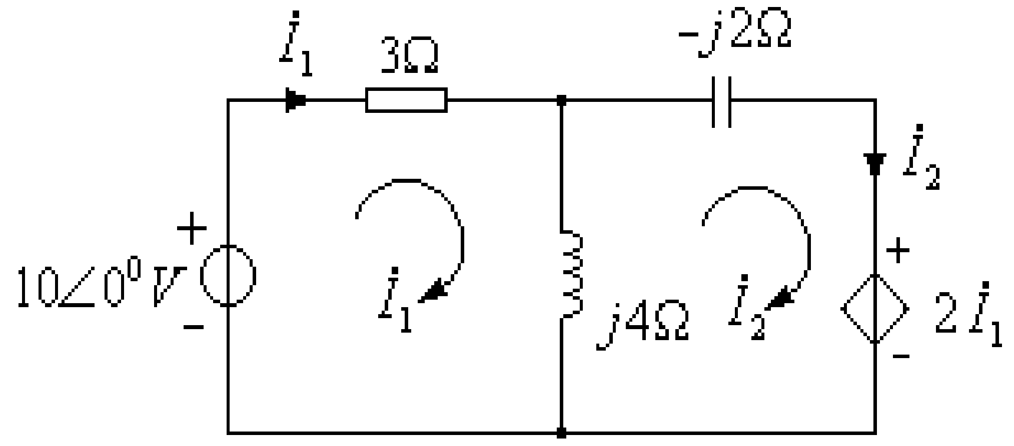
$$\dot{U}_2 = -(\frac{1}{j\omega C})\dot{I} = j\frac{1}{\omega C}\dot{I} = \frac{I}{\omega C}\angle\varphi_i + 90^\circ = U_2\angle\varphi_i + 90^\circ$$

$$\therefore \quad \varphi = \varphi_i + 90^\circ - \varphi_i = 90^\circ$$

二、相量模型的网孔分析法和结点分析法

例9-8：电路如下图所示，求解 $i_1(t)$ 和 $i_2(t)$ 。





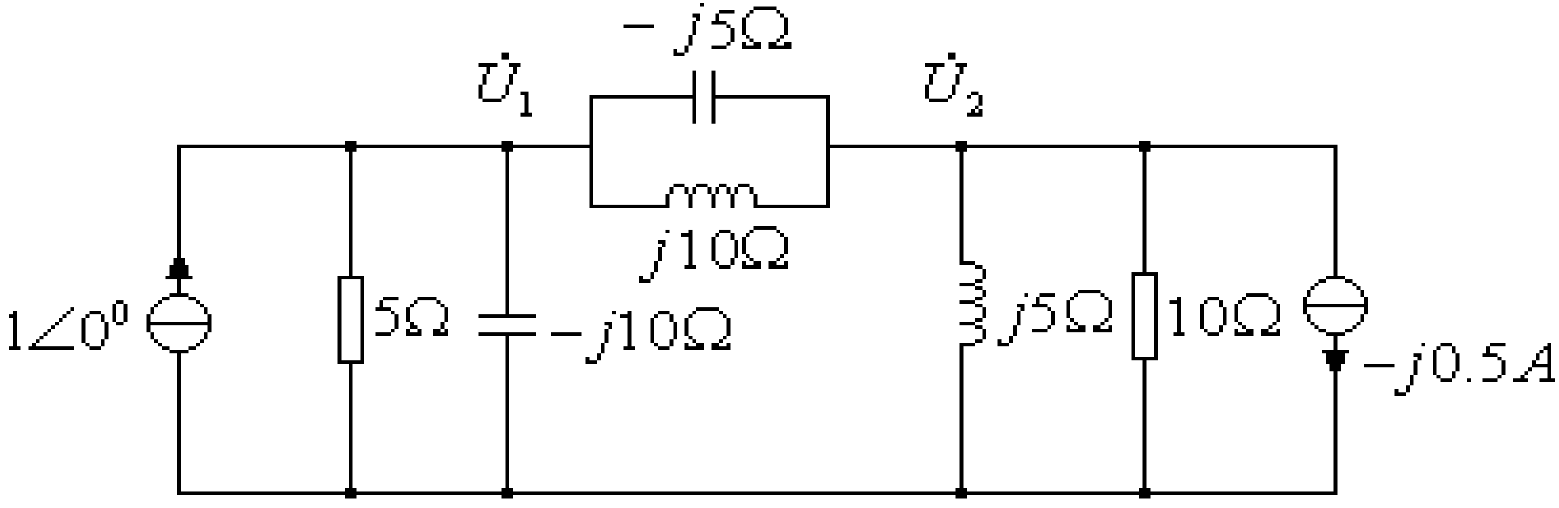
解：

$$Z_L = j\omega L = j4\Omega, \quad Z_C = \frac{1}{j\omega C} = -j2\Omega$$

$$\begin{cases} (3+4j)\dot{I}_1 - j4\dot{I}_2 = 10\angle 0^\circ \\ -j4\dot{I}_1 + (j4 - j2)\dot{I}_2 = -2\dot{I}_1 \end{cases} \quad \therefore \begin{cases} \dot{I}_1 = \dfrac{10}{7-j4} = 1.24\angle 29.7^\circ \ A \\ \dot{I}_2 = \dfrac{20+j30}{13} = 2.77\angle 56.3^\circ \ A \end{cases}$$

$$\begin{cases} i_1(t) = 1.24\sqrt{2}\cos(10^3 t + 29.7^\circ) \quad A \\ i_2(t) = 2.77\sqrt{2}\cos(10^3 t + 56.3^\circ) \quad A \end{cases}$$

例9-9：电路相量模型如下图所示，试列出结点电压相量方程。



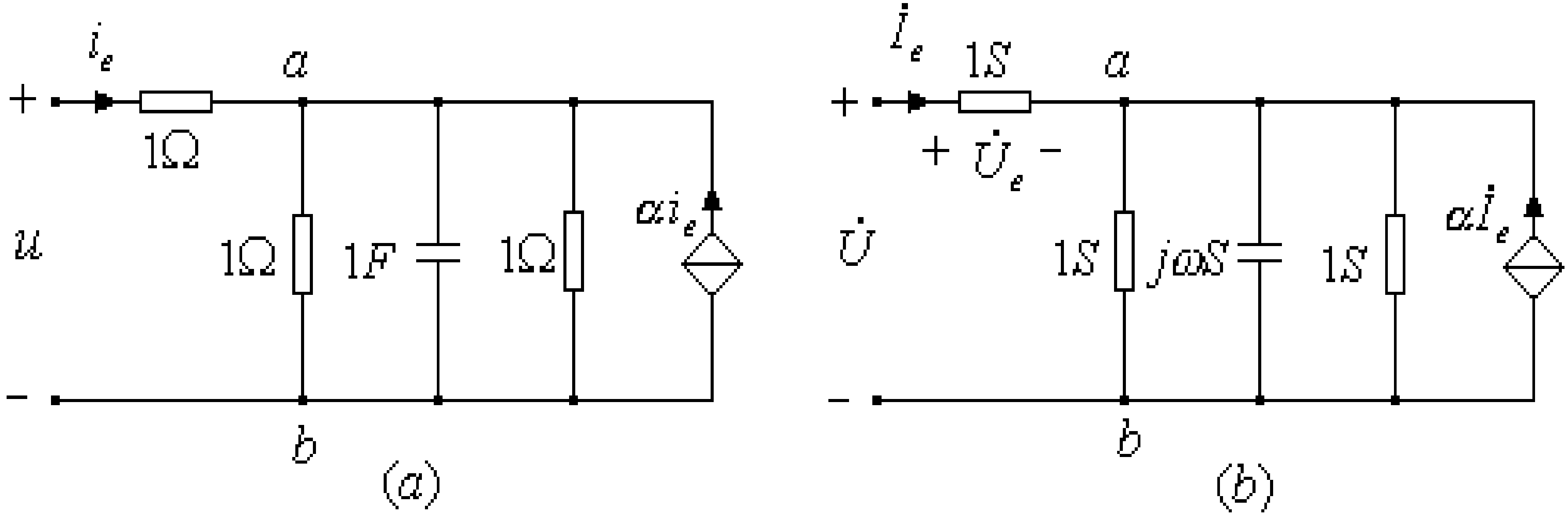
解：

$$(\frac{1}{5}+\frac{1}{-j10}+\frac{1}{j10}+\frac{1}{-j5})\dot{U}_1-(\frac{1}{-j5}+\frac{1}{j10})\dot{U}_2=1\angle 0^\circ$$

$$-(\frac{1}{-j5}+\frac{1}{j10})\dot{U}_1+(\frac{1}{10}+\frac{1}{j5}+\frac{1}{j10}+\frac{1}{-j5})\dot{U}_2=-(-j0.5)$$

$$\therefore\ \begin{cases}(0.2+j0.2)\dot{U}_1-j0.1\dot{U}_2=1\angle 0^\circ\\ -j0.1\dot{U}_1+(0.1-j0.1)\dot{U}_2=j0.5\end{cases}\Rightarrow\begin{cases}2(1+j)\dot{U}_1-j\dot{U}_2=10\\ -j\dot{U}_1+(1-j)\dot{U}_2=j5\end{cases}$$

例9-10：单口网络如下图所示，试求输入阻抗及输入导纳。

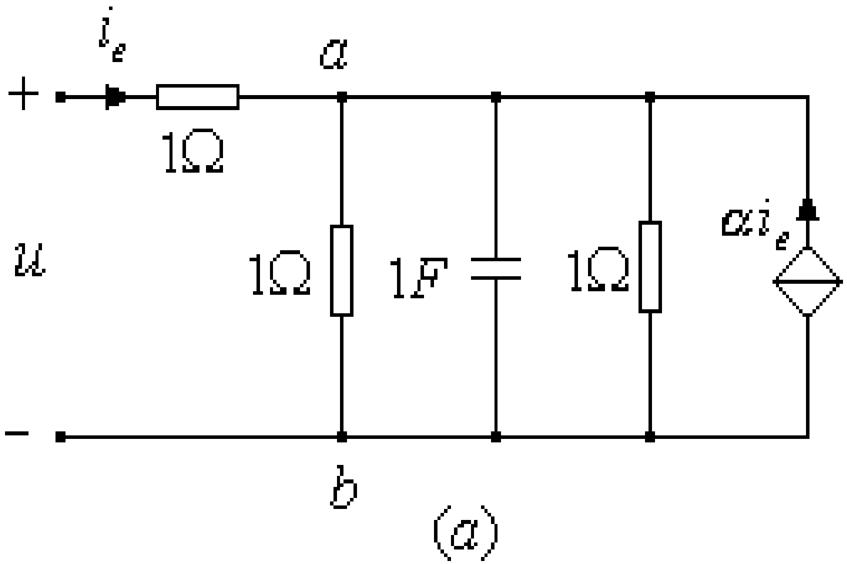


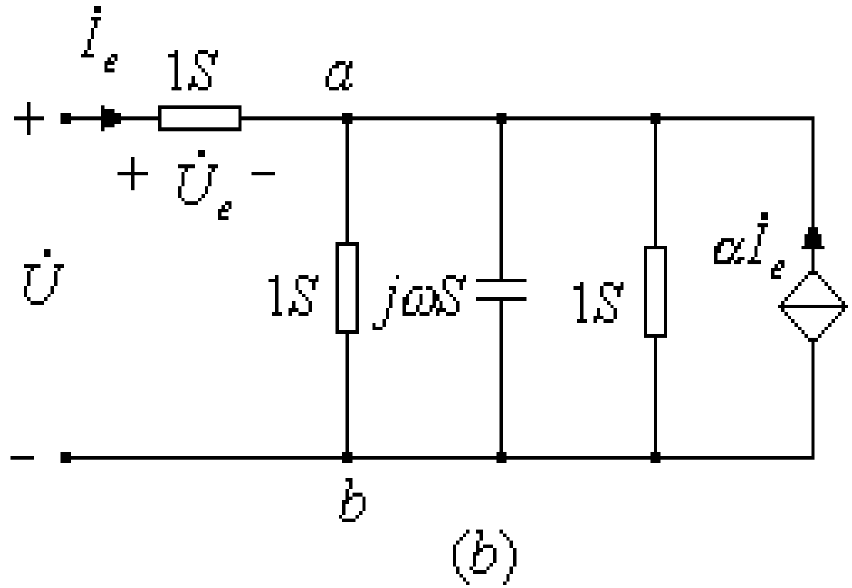
解：$\begin{cases}(3+j\omega)\dot U_a=\dot U+\alpha\dot I_e\\ \dot U-\dot U_a=\dot I_e\end{cases}$

$$\Rightarrow[(3+j\omega)-1]\dot U=[3+j\omega+\alpha]\dot I_e$$

$$\therefore Z=\frac{\dot U}{\dot I_e}=\frac{3+\alpha+j\omega}{2+j\omega}\Omega$$

$$Y=\frac{1}{Z}=\frac{2+j\omega}{3+\alpha+j\omega}=\frac{6+2\alpha+\omega^2}{(3+\alpha)^2+\omega^2}+j\frac{(1+\alpha)\omega}{(3+\alpha)^2+\omega^2}$$





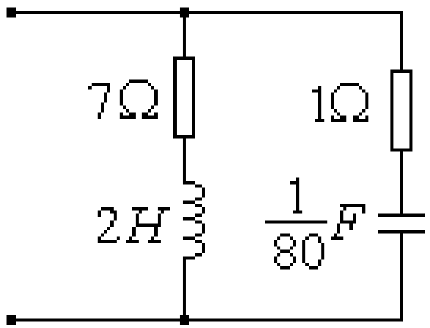
设 $\dot{I}_e = 1A$ $\quad \dot{U}_e = 1V$

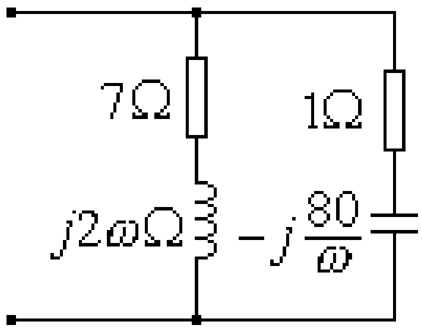
$$\dot{U} = \dot{U}_e + \frac{(1+\alpha)}{2+j\omega}\dot{I}_e \qquad \dot{U} = 1 + \frac{1+\alpha}{2+j\omega} = \frac{3+\alpha+j\omega}{2+j\omega}$$

$$\therefore Z = \frac{\dot{U}}{\dot{I}_e} = \frac{3+\alpha+j\omega}{2+j\omega} = \frac{6+2\alpha+\omega^2}{4+\omega^2} - j\frac{(1+\alpha)\omega}{4+\omega^2}$$

三、运用戴维南定理及诺顿定理求解

例9-11：单口网络及相量模型如下图所示，试求在 $\omega = 4\,rad/s$ 的等效相量模型和 $\omega = 10\ rad/s$ 的等效相量模型。





解：

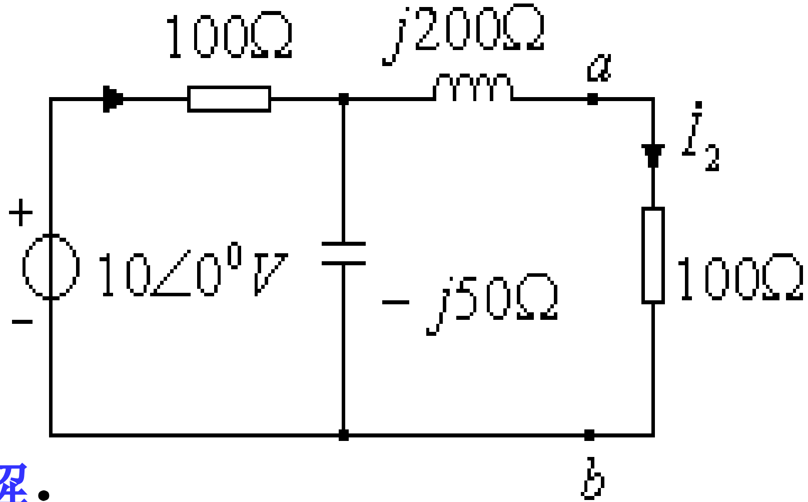
$$\omega = 4\,rad/s$$

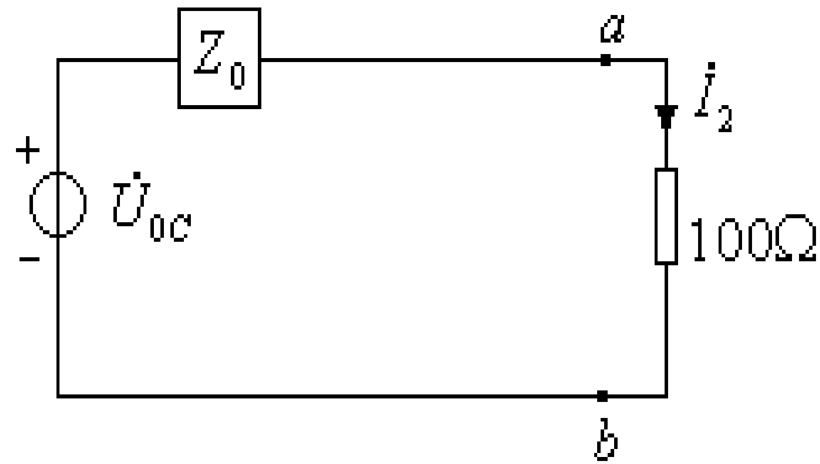
$$Z(j4) = (7 + j8)//(1 - j20) = (14.04 + j4.56)\ \Omega$$

$$\omega = 10\ rad/s$$

$$Z(j10) = (7 + j20)//(1 - j8) = (4.35 - j11.02)\ \Omega$$

例9-12 用戴维南定理求下图所示相量模型中的电流相量 $\dot{I}_2$。





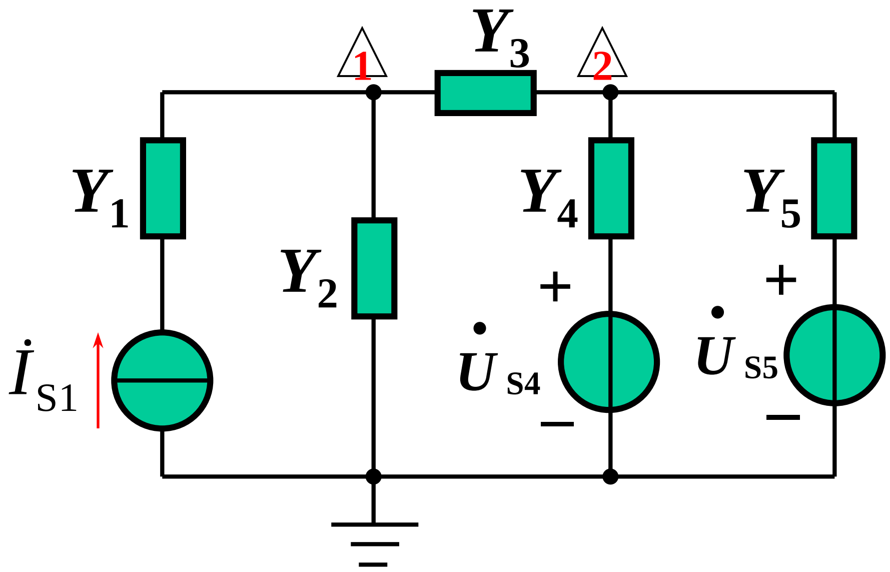
解：

$$\dot{U}_{0C}:\ \dot{U}_{0C(ab)} = 10\angle 0^\circ \times \frac{-j50}{100-j50} = 10\times\frac{-j}{2-j} = 4.47\angle -63.4^\circ\ V$$

$$Z_0:\quad Z_0 = j200 + 100 // (-j50) = j200 + \frac{-j100}{2-j} = 20 + j160\ \ \Omega$$

$$\dot{I}_2:\quad \dot{I}_2 = \frac{\dot{U}_{0C}}{Z_0+100} = \frac{4.47\angle -63.4^\circ}{20+j160+100} = 0.0224\angle -116.53^\circ\ \ A$$

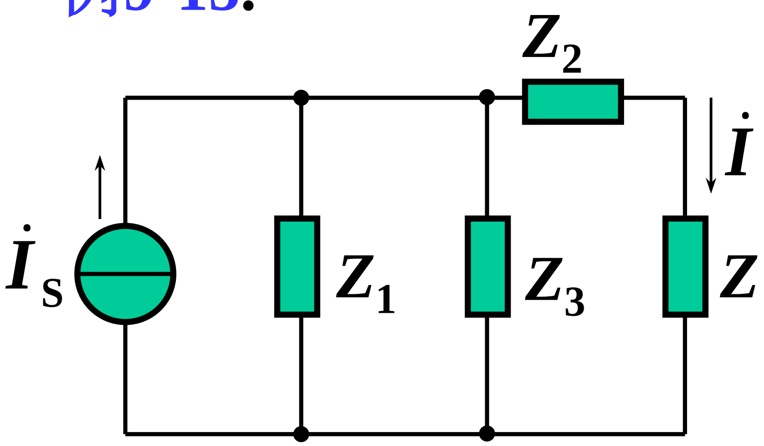
例9-14.　列写电路的节点电压方程



解:

$$\begin{cases} (Y_2+Y_3)\dot{U}_1 - Y_3\dot{U}_2 = \dot{I}_{S1} \\ -Y_3\dot{U}_1 + (Y_3+Y_4+Y_5)\dot{U}_2 = Y_4\dot{U}_{S4} + Y_5\dot{U}_{S5} \end{cases}$$

例9-15.

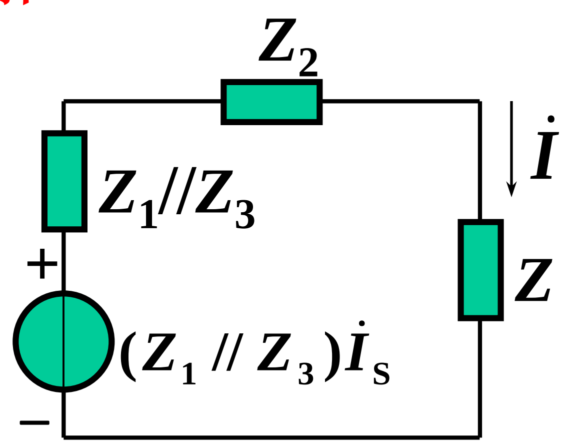


已知：$\dot{I}_S = 4\angle 90^\circ\ \text{A}$，$Z_1 = Z_2 = -j30\Omega$

$Z_3 = 30\Omega$，$Z = 45\Omega$

求：$\dot{I}$.

解：



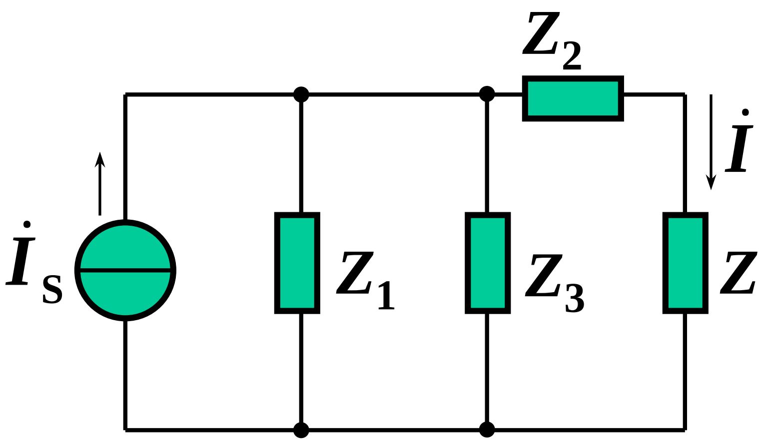
法一：电源变换

$$Z_1 // Z_3 = 15 - j15$$

$$\dot{I} = \frac{\dot{I}_S(Z_1 // Z_3)}{Z_1 // Z_3 + Z_2 + Z} = \frac{j4(15-j15)}{15-j15-j30+45}$$

$$= \frac{5.657\angle 45^\circ}{5\angle -36.9^\circ}$$

$$= 1.13\angle 81.9^\circ\ \text{A}$$

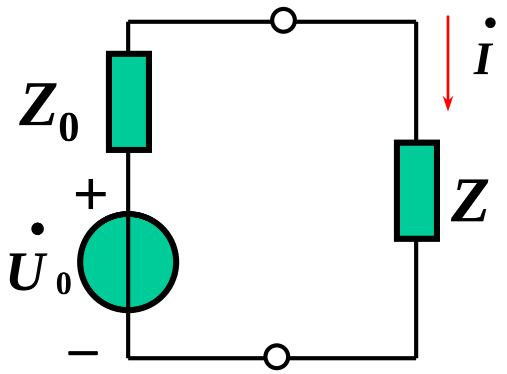


已知：$\dot{I}_S = 4\angle 90^\circ\,\text{A}$，$Z_1 = Z_2 = -\text{j}30\Omega$

$Z_3 = 30\Omega$，$Z = 45\Omega$

求：$\dot{I}$.

法二：戴维南等效变换



$$\dot{U}_0 = \dot{I}_S(Z_1 \mathbin{//} Z_3) = 84.855\angle 45^\circ\,\text{V}$$

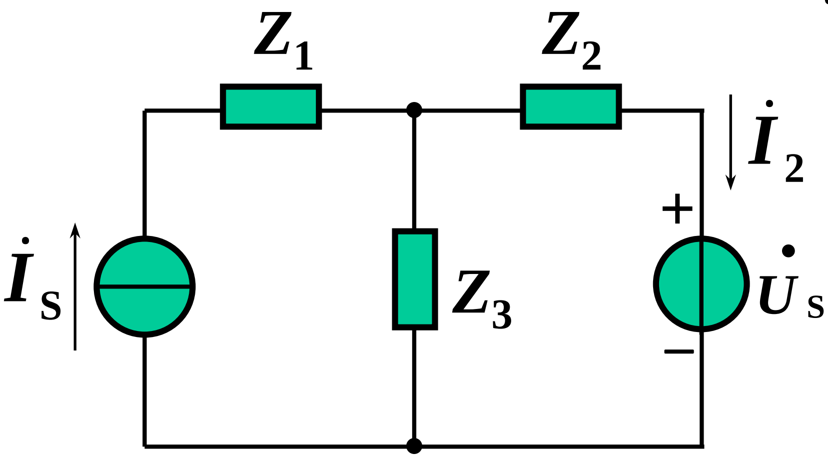
$$Z_0 = Z_1 \mathbin{//} Z_3 + Z_2 = 15 - \text{j}45\Omega$$

$$\dot{I} = \frac{\dot{U}_0}{Z_0 + Z} = 1.13\angle 81.9^\circ\,\text{A}$$

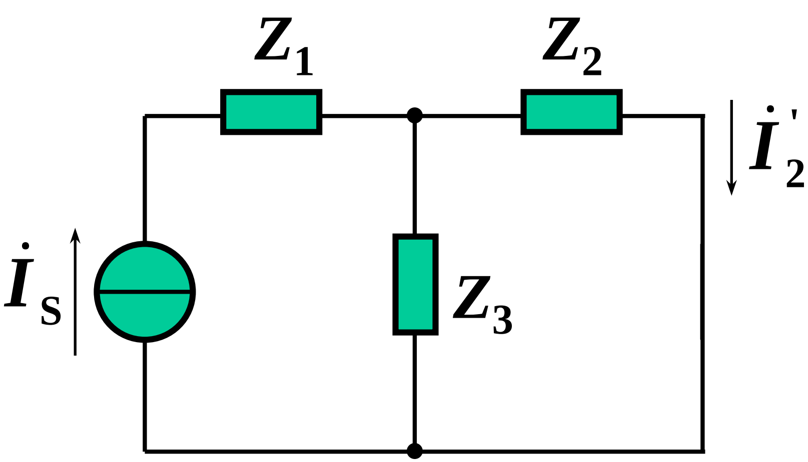
例9-16.

已知：$\dot{U}_S = 100\angle 45^\circ\,\text{V},\ \dot{I}_S = 4\angle 0^\circ\,\text{A},$

$Z_1 = Z_3 = 50\angle 30^\circ\,\Omega,\ Z_2 = 50\angle -30^\circ\,\Omega.$



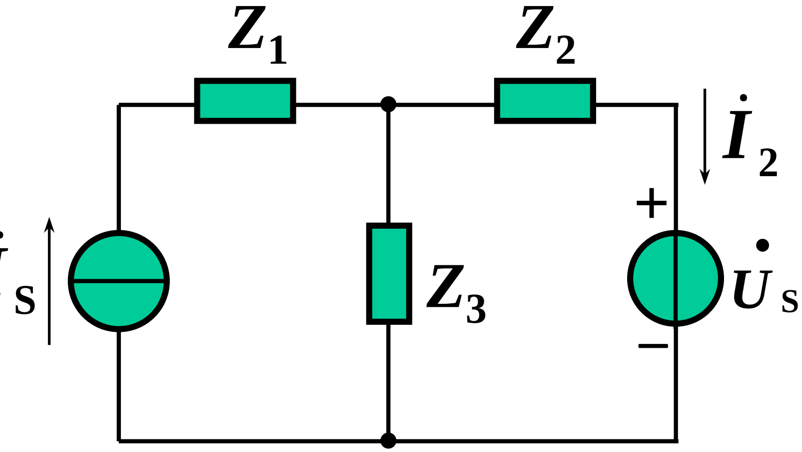
解：(1) $\dot{I}_S$ 单独作用($\dot{U}_S$ 短路)：



$$\dot{I}_2' = \dot{I}_S \frac{Z_3}{Z_2 + Z_3}$$

$$= 4\angle 0^\circ \times \frac{50\angle 30^\circ}{50\angle -30^\circ + 50\angle 30^\circ}$$

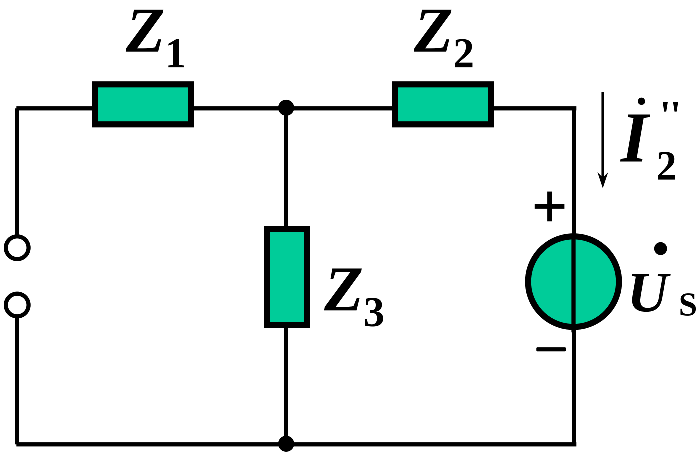
$$= \frac{200\angle 30^\circ}{50\sqrt{3}} = 2.31\angle 30^\circ\,\text{A}$$



已知：$\dot{U}_S = 100\angle 45^\circ \text{V}$, $\dot{I}_S = 4\angle 0^\circ \text{A}$,

$Z_1 = Z_3 = 50\angle 30^\circ \Omega$, $Z_3 = 50\angle -30^\circ \Omega$.

(2) $\dot{U}_S$ 单独作用($\dot{I}_S$ 开路)：



$$\dot{I}_2'' = -\frac{\dot{U}_S}{Z_2 + Z_3}$$

$$= \frac{-100\angle 45^\circ}{50\sqrt{3}} = 1.155\angle -135^\circ \text{A}$$

$$\dot{I}_2 = \dot{I}_2' + \dot{I}_2''$$

$$= 2.31\angle 30^\circ + 1.155\angle -135^\circ$$

$$= (2 + \text{j}1.155) + (-0.817 - \text{j}0.817)$$

$$= 1.183 + \text{j}0.338$$

$$= 1.23\angle 15.9^\circ \text{A}$$

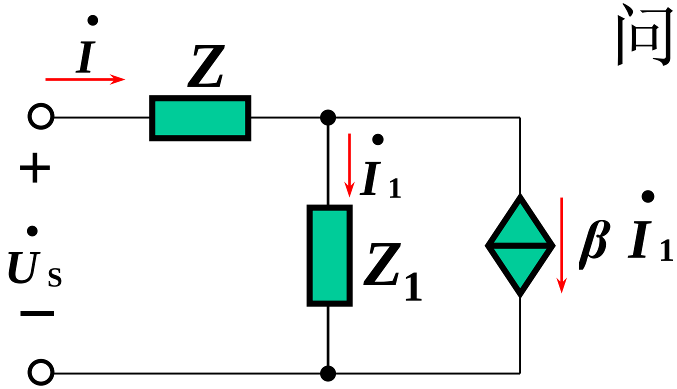
```
In[39]:= N [-100 / (50 Sqrt[3])]
Out[39]= -1.1547

In[42]:= -1.1547 * Cos[Pi / 4] + (-1.1547) * Sin[Pi / 4] j
Out[42]= -0.816496 - 0.816496 i

In[43]:= Sqrt[2 × 0.816496^2]
Out[43]= 1.1547

In[44]:= ArcTan[1]
Out[44]= π/4
```

例9-17. 已知：$Z=10+j50\Omega$, $Z_1=400+j1000\Omega$。



问：β等于多少时，$\dot{I}_1$和$\dot{U}_S$相位差90°？

分析：找出 $\dot{I}_1$和$\dot{U}_S$关系：$\dot{U}_S = Z_{转}\dot{I}_1$，

$Z_{转}$实部为零，相位差为90°.

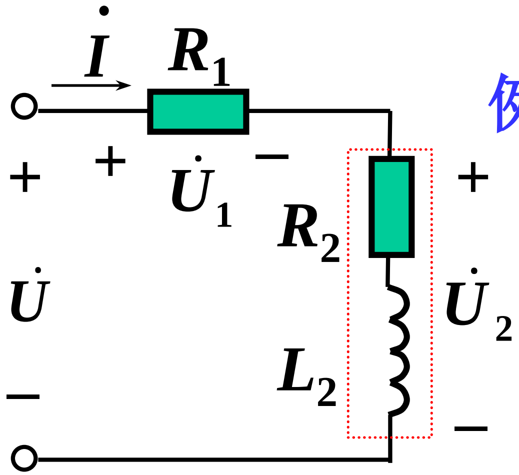
解：

$$\dot{U}_S = Z\dot{I} + Z_1\dot{I}_1 = Z(1+\beta)\dot{I}_1 + Z_1\dot{I}_1$$

$$\frac{\dot{U}_S}{\dot{I}_1} = (1+\beta)Z + Z_1 = 410 + 10\beta + j(50 + 50\beta + 1000)$$

令 $410 + 10\beta = 0$ ，$\beta = -41$

$$\frac{\dot{U}_S}{\dot{I}_1} = -j1000$$ 故电流领先电压 90°.

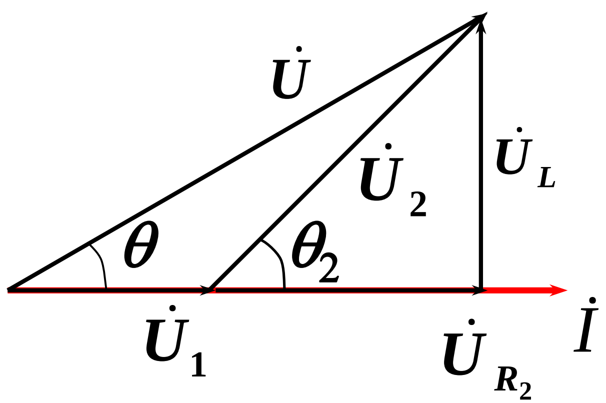


例9-18. 已知：U=115V，U_1=55.4V，U_2=80V，R_1=32Ω，f=50Hz

求：线圈的电阻R_2和电感L_2。

解：画相量图进行定性分析 $U^2 = U_1^2 + U_2^2 + 2U_1U_2\cos\theta_2$

$$\therefore \theta_2 = 64.9^\circ$$



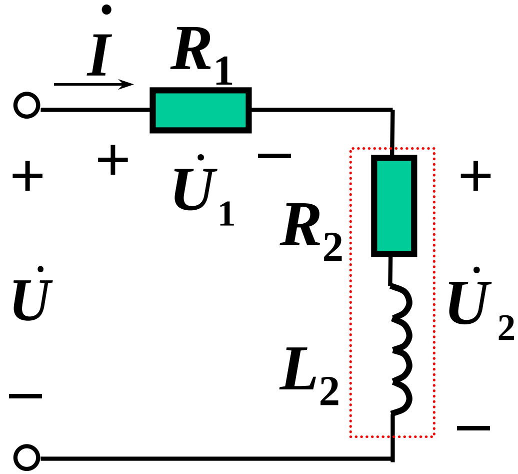
$$I = U_1 / R_1 = 55.4/32 = 1.73\text{A}$$

$$|Z_2| = U_2 / I = 80/1.73 = 46.2\Omega$$

$$R_2 = |Z_2|\cos\theta_2 = 19.6\Omega$$

$$X_2 = |Z_2|\sin\theta_2 = 41.8\Omega$$

$$L = X_2/(2\pi f) = 0.133\text{H}$$



已知：U=115V，U_1=55.4V，U_2=80V，

R_1=32Ω，f=50Hz

求：　线圈的电阻R_2和电感L_2。

$$I = U_1 / R_1 = 55.4 / 32 = 1.73\text{A}$$

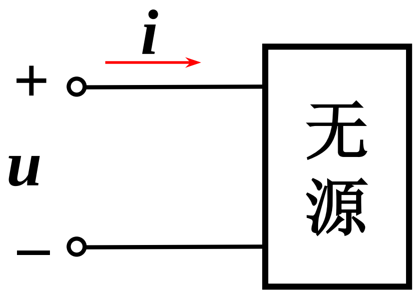
$$\frac{115}{\sqrt{(32 + R_2)^2 + (\omega L_2)^2}} = I = 1.73$$

$$\frac{80}{\sqrt{R_2^2 + (\omega L_2)^2}} = I = 1.73$$

解得：

$$R_2 = 19.58\Omega, \quad L_2 = \frac{41.86}{2\pi f} = 0.133\text{H}.$$

§9–5　正弦稳态电路的功率



$$u(t)=\sqrt{2}U\cos(\omega t+\varphi_u)$$

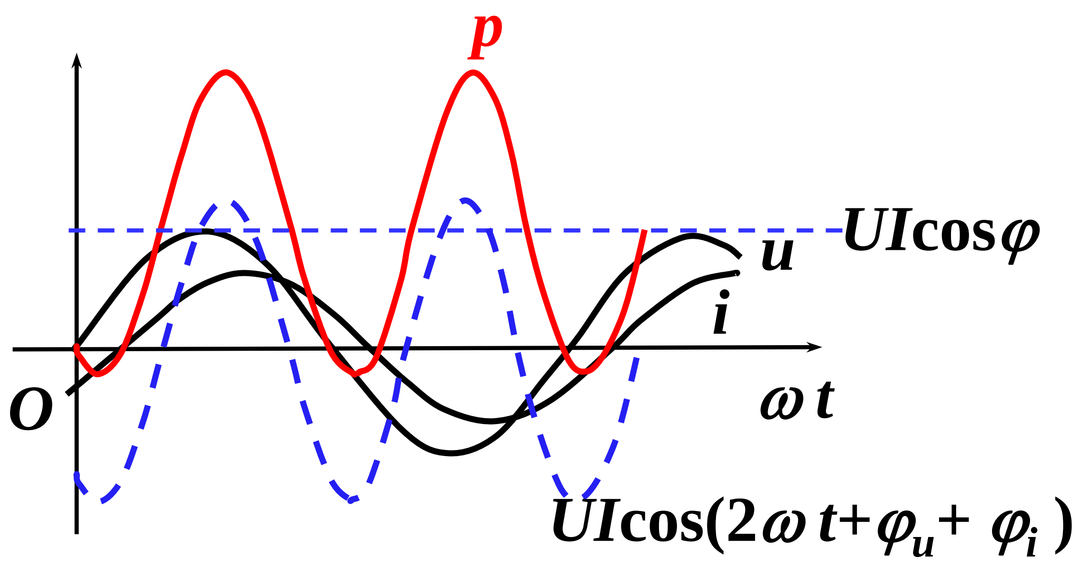
$$i(t)=\sqrt{2}I\cos(\omega t+\varphi_i)$$

$$\varphi=\varphi_u-\varphi_i$$

一、瞬时功率

$$\begin{aligned}p=ui&=\sqrt{2}U\cos(\omega t+\varphi_u)\cdot\sqrt{2}I\cos(\omega t+\varphi_i)\\&=2UI\cos(\omega t+\varphi_u)\cos(\omega t+\varphi_i)\\&=UI\cos(\varphi_u-\varphi_i)+UI\cos(2\omega t+\varphi_u+\varphi_i)\\&=UI\cos\varphi+UI\cos(2\omega t+\varphi_u+\varphi_i)\end{aligned}$$

$$p = ui = UI\cos\varphi + UI\cos(2\omega t + \varphi_u + \varphi_i)$$



- *p*>0，电路吸收功率；*p*<0，电路发出功率；
- *p*以2*ω*角频率变化
- 恒定分量：$UI\cos\varphi$

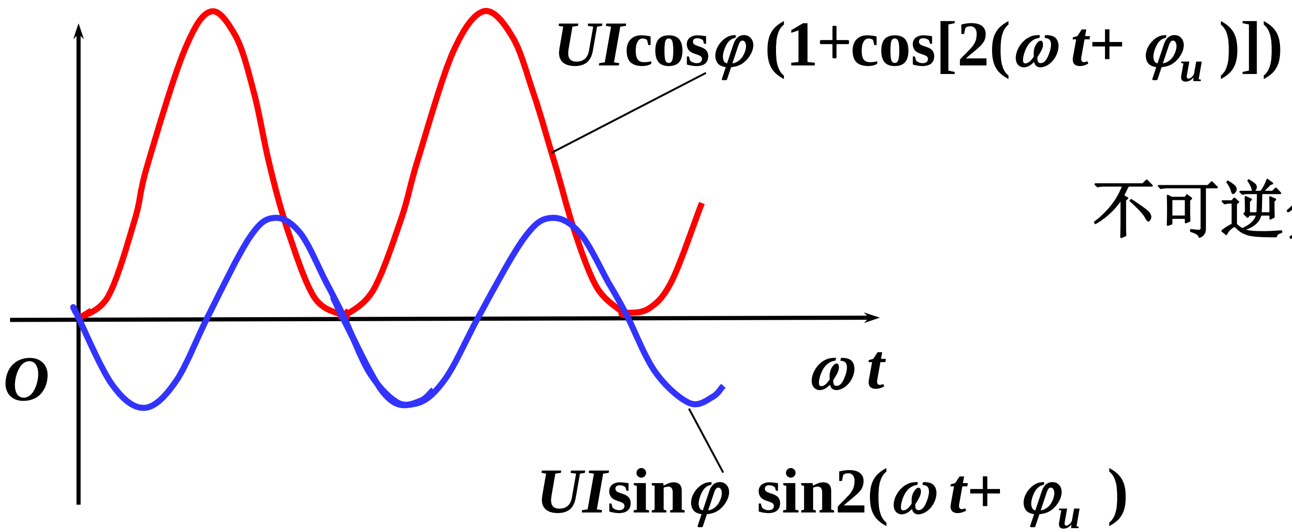
  正弦分量：$UI\cos(2\omega t+\varphi_u+\varphi_i)$

$$p = ui = UI\cos\varphi + UI\cos(2\omega t + \varphi_u + \varphi_i)$$

$$= UI\cos\varphi + UI\cos(2\omega t + 2\varphi_u - \varphi\ )$$

$$= UI\cos\varphi + UI\cos\varphi\cos(2\omega t + 2\varphi_u) + UI\sin\varphi\sin(2\omega t + 2\varphi_u)$$

$$= UI\cos\varphi\,\{1 + \cos[2(\omega t + \varphi_u)]\} + UI\sin\varphi\sin[2(\omega t + \varphi_u)]$$



不可逆分量

可逆分量，周期性变化

二、平均功率（有功功率）

$$P=\frac{1}{T}\int_0^T p\,dt=\frac{1}{T}\int_0^T UI[\cos\varphi+\cos(2\omega t+2\varphi_u+\varphi_i)]d\,t$$

$$=UI\cos\varphi$$

P 的单位：W 瓦特

$\lambda=\cos\varphi$ 称为功率因数

$\varphi=\varphi_u-\varphi_i$：功率因数角。对无源网络，为其等效阻抗的阻抗角。

$$\cos\varphi\begin{cases}1, & \text{纯电阻}\\ 0, & \text{纯电抗}\end{cases}$$

$X>0,\ \varphi>0$，感性， 滞后功率因数

$X<0,\ \varphi<0$，容性， 超前功率因数

三、无功功率

$$Q \overset{\text{def}}{=} UI\sin\varphi$$ 单位：var (乏)。

$$p = ui = UI\cos\varphi + UI\cos(2\omega t + \varphi_u + \varphi_i)$$

$Q>0$，表示网络吸收无功功率；

$Q<0$，表示网络发出无功功率。

Q 的大小反映网络与外电路交换功率的大小。是由储能元件L、C的性质决定的

四、视在功率

$$S \overset{\text{def}}{=} UI \qquad \text{单位：VA (伏安)}$$

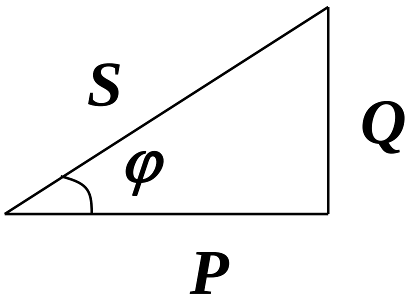
反映电气设备的容量。

有功，无功，视在功率的关系：

有功功率：$P=UI\cos\varphi$ 单位：W

视在功率：$S=UI$ 单位：VA

无功功率：$Q=UI\sin\varphi$ 单位：var



功率三角形

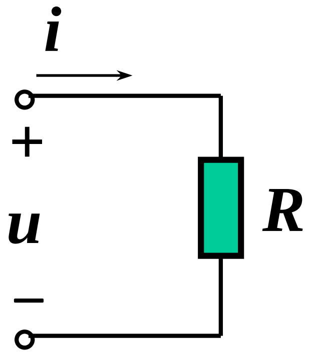
$$P = S\cos\varphi, \qquad Q = S\sin\varphi$$

$$S = \sqrt{P^2 + Q^2}, \quad \varphi = \arctan(\frac{Q}{P})$$

五、RLC元件的功率分析

1. 电阻R

$$\varphi=\varphi_u-\varphi_i=0$$



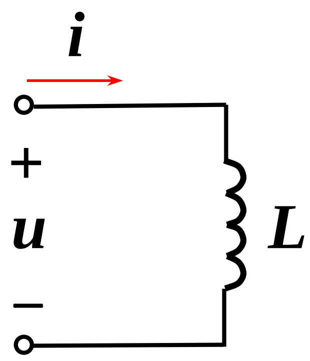
$$p=UI\{1+\cos[2(\omega t+\varphi_u)]\}\geq 0$$

$$P_R=UI\cos\varphi=UI\cos0°=UI=I^2R=U^2/R$$

$$Q_R=UI\sin\varphi=UI\sin0°=0$$

2. 电感L

$$\varphi=\varphi_u-\varphi_i=\frac{\pi}{2}$$

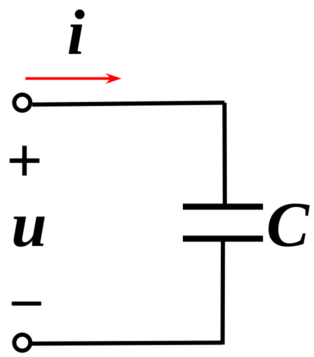


$$p=UI\sin\varphi\sin[2(\omega t+\varphi_u)]$$

$$P_L=UI\cos\varphi=UI\cos90°=0$$

$$Q_L=UI\sin\varphi=UI\sin90°=UI$$

3. 电容C



$$\varphi = \varphi_u - \varphi_i = -\frac{\pi}{2}$$

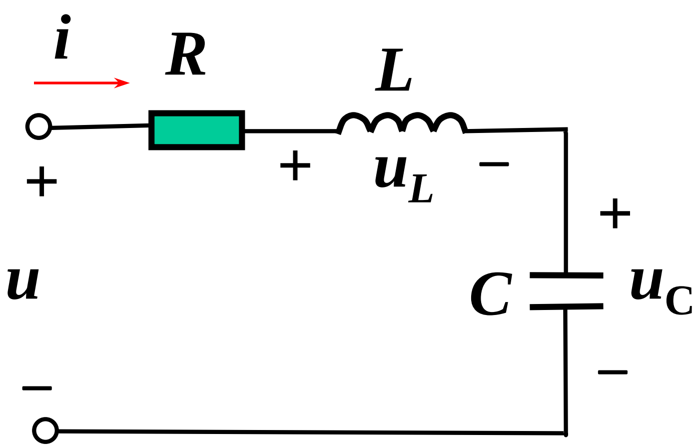
$$p = UI\sin\varphi\sin\left[2(\omega t + \varphi_u)\right]$$

$$= -UI\sin\left[2(\omega t + \varphi_u)\right]$$

$$P_C = UI\cos\varphi = UI\cos(-90°) = 0$$

$$Q_C = UI\sin\varphi = UI\sin(-90°) = -UI$$

4. RLC串联电路



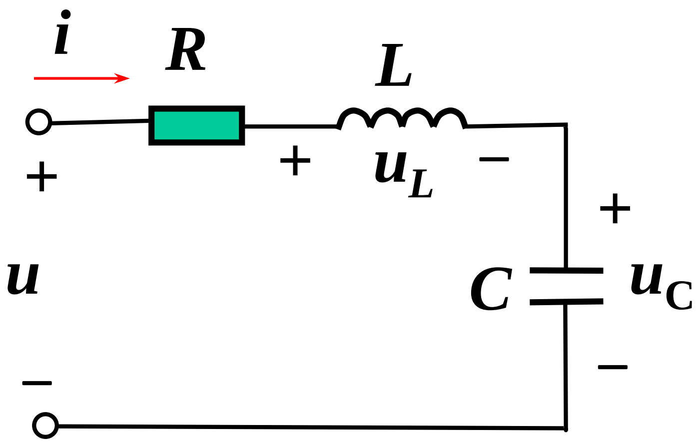
$$\varphi = \varphi_u - \varphi_i = \varphi_Z$$

$$Z = R + j(\omega L - \frac{1}{\omega C}),\ \varphi_z = \arctan(\frac{X}{R})$$

$$U = |Z|I,\ R = |Z|\cos\varphi_Z,\ X = |Z|\sin\varphi_Z$$

4. RLC串联电路

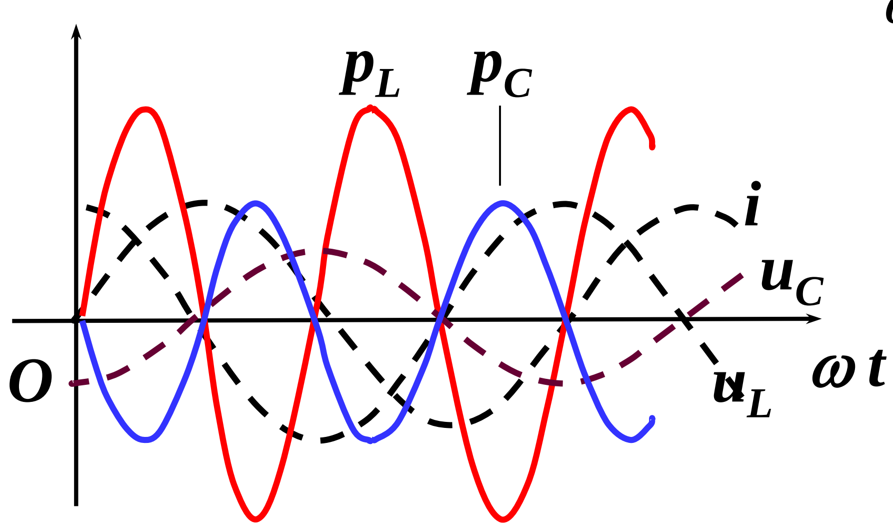
$$\varphi = \varphi_u - \varphi_i = \varphi_Z$$



$$Z = R + j(\omega L - \frac{1}{\omega C}),\ \varphi_z = \arctan(\frac{X}{R})$$

$$U = |Z|\, I,\ R = |Z|\cos\varphi_Z,\ X = |Z|\sin\varphi_Z$$

$$\begin{cases} P = U\,I\cos\varphi = |Z|\,I^2\cos\varphi = R\,I^2 \\ Q = U\,I\sin\varphi = |Z|\,I^2\sin\varphi = X\,I^2 = (\omega L - \frac{1}{\omega C})I^2 = Q_L + Q_C \end{cases}$$



例9-19：单口网络电压、电流为关联参考方向，

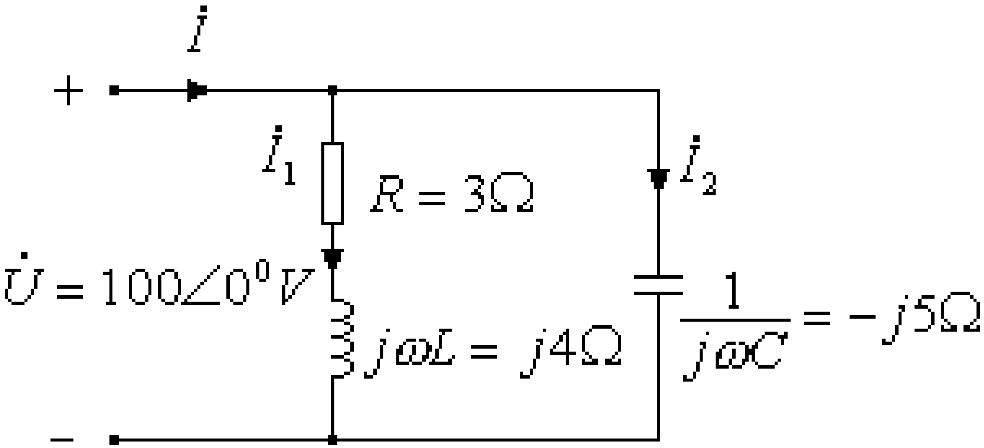
$$u = 300\sqrt{2}\cos(314t + 10°)\ V \quad i = 50\sqrt{2}\cos(314t - 45°)\ A$$

求网络吸收的平均功率。

解： $U = 300\ V,\ I = 50\ A,\ \varphi = \varphi_Z = 10° - (-45°) = 55°$

$$P = UI\cos\varphi = 300 \times 50 \times \cos 55^0 = 8610\ W$$

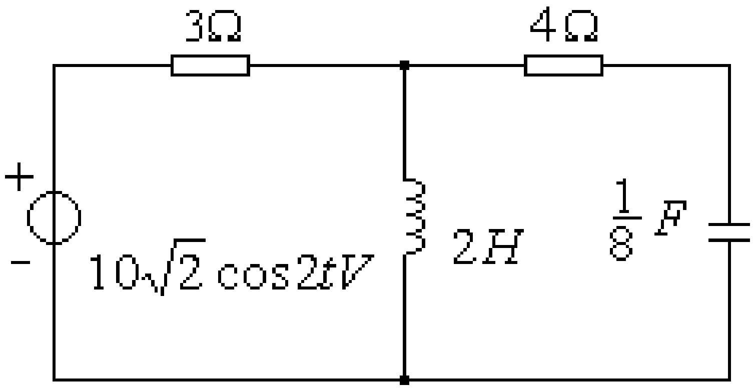
例 9-20：电路及其相量模型如下图所示，求单口网络的功率。

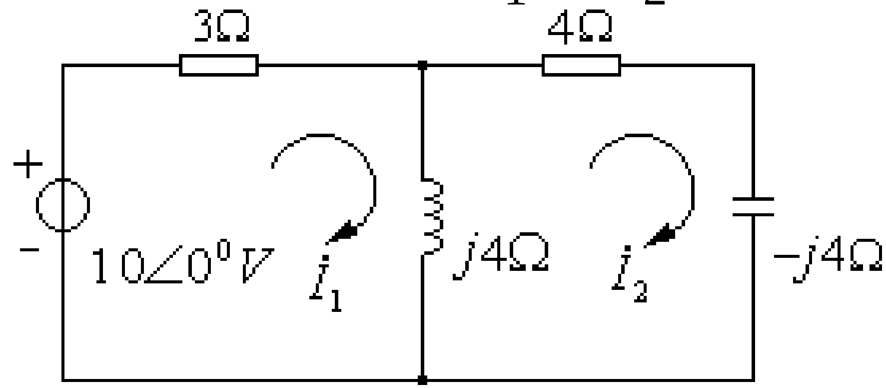


已知

$$\begin{cases} \dot{I} = 12.65\angle 18.5°A \\ \dot{I}_1 = 20\angle -53.1°A \\ \dot{I}_2 = 20\angle 90°A \end{cases}$$

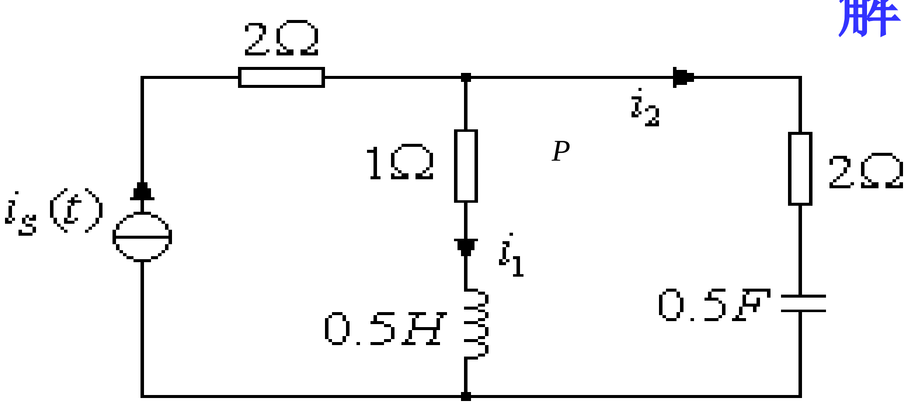
例 9-21：求下图所示电路中电源提供的功率。$I_1 = I_2 = 1.24\ A$





解：$P = I_1^2 \times 3 + I_2^2 \times 4 = 1.24^2 \times 3 + 1.24^2 \times 4 = 10.8\ W$

例 9-22：下图所示电路中，$i_S = 5\sqrt{2}\cos 2t\ A$，电路处于稳态，试求P、Q、S、$\lambda$



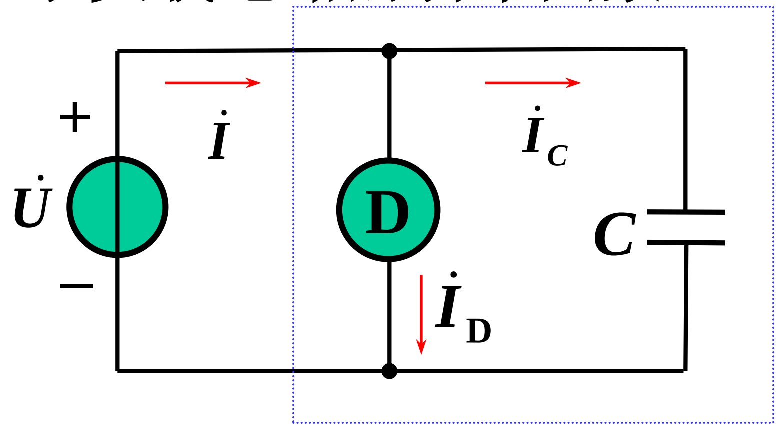
解：

$$Z = 2 + \frac{(1+j)(2-j)}{1+j+2-j} = 3 + j\frac{1}{3}\ \Omega$$

$$P = I^2 \operatorname{Re}[Z] = 75\ \ W$$

$$Q = I^2 \operatorname{Im}[Z] = \frac{25}{3}\ \ Var$$

例9-23. 已知：电动机 P_D=1000W，U=220V，f =50Hz，C =30μF。求负载电路的功率因数。D的功率因素为0.8。



解：

$$I_D = \frac{P_D}{U\cos\varphi_D} = \frac{1000}{220 \times 0.8} = 5.68\,\text{A}$$

$$\because \quad \cos\varphi_D = 0.8(\text{滞后}), \quad \therefore \varphi_D = 36.8^\circ$$

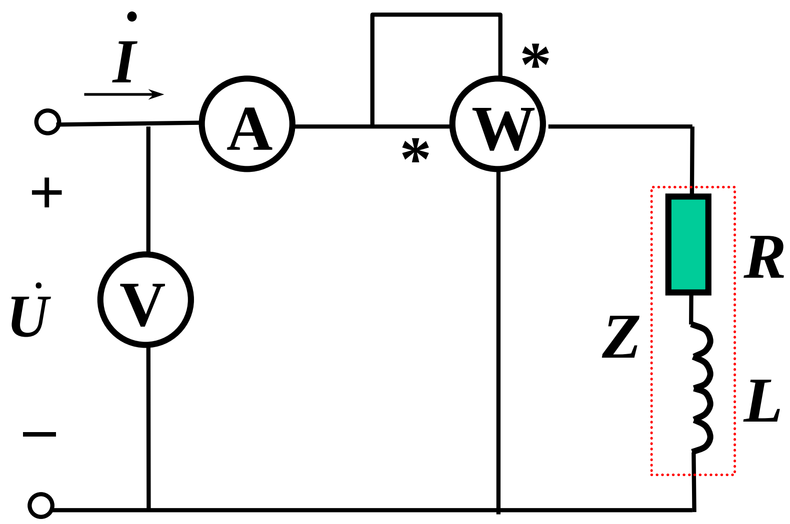
设 $\dot{U} = 220\angle 0^\circ$

$$\dot{I}_D = 5.68\angle -36.8^\circ \quad , \quad \dot{I}_C = 220\angle 0^\circ \cdot \mathrm{j}\omega C = \mathrm{j}2.08$$

$$\dot{I} = \dot{I}_D + \dot{I}_C = 4.54 - \mathrm{j}1.33 = 4.73\angle -16.3^\circ$$

$$\therefore \quad \cos\varphi = \cos[0^\circ - (-16.3^\circ)] = 0.96 \quad (\text{滞后})$$

例9-24. 已知f=50Hz，且测得U=50V，I=1A，P=30W。



解：

$$P = I^2 R \qquad \therefore R = \frac{P}{I^2} = \frac{30}{1^2} = 30\Omega$$

$$|Z| = \frac{U}{I} = \frac{50}{1} = 50\Omega$$

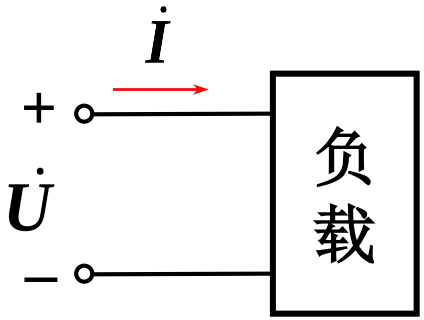
$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

$$L = \frac{1}{\omega}\sqrt{|Z|^2 - R^2} = \frac{1}{314}\sqrt{50^2 - 30^2} = \frac{40}{314} = 0.127\text{H}$$

§9–6 复 功 率

一、复功率

为了用相量$\dot U$和$\dot I$来计算功率，引入“复功率”



$$\dot U = U\angle\varphi_u \quad , \qquad \dot I = I\angle\varphi_i$$

$$P = UI\cos(\varphi_u - \varphi_i) = UI\mathrm{Re}[e^{\mathrm{j}(\varphi_u - \varphi_i)}]$$

$$= \mathrm{Re}(U\mathrm{e}^{\mathrm{j}\varphi_u} \cdot I\mathrm{e}^{-\mathrm{j}\varphi_i})$$

$$\boxed{\dot U} \qquad \boxed{\dot I^*}$$

$$\boldsymbol{P = \mathrm{Re}[\dot U \cdot \dot I^*]}$$

定义 $\overline{S} = \dot U \dot I^*$ 为复功率, 单位 VA

$$\overline{S} = \dot U \dot I^* = UI\angle(\varphi_u - \varphi_i) = UI\angle\varphi = S\angle\varphi$$

$$= UI\cos\varphi + \mathrm{j}UI\sin\varphi$$

$$= P + \mathrm{j}Q$$

复功率 $\bar{S}$ 也可以表示为以下式子:

$$\bar{S}=\dot{U}\dot{I}^*=Z\dot{I}\cdot\dot{I}^*=ZI^2$$

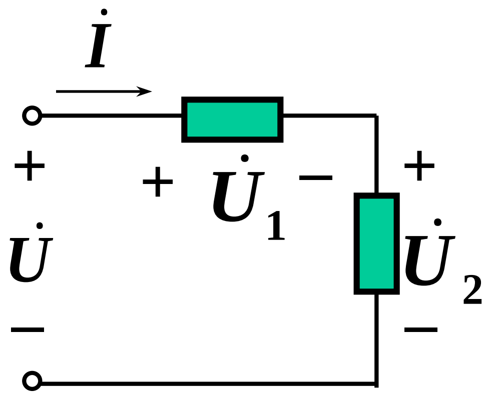
$$\bar{S}=\dot{U}\dot{I}^*=\dot{U}(\dot{U}Y)^*=\dot{U}\cdot\dot{U}^*Y^*=U^2Y^*$$

复功率守恒定理：在正弦稳态下，任一电路的所有支路吸收的复功率之和为零。

$$\sum_{k=1}^{b}\bar{S}_k=0 \qquad \sum_{k=1}^{b}\dot{U}_k\dot{I}_k^*=0$$

$$\sum_{k=1}^{b}(P_k+\mathrm{j}Q_k)=0 \qquad \begin{cases}\sum_{k=1}^{b}P_k=0\\ \sum_{k=1}^{b}Q_k=0\end{cases}$$

* 复功率守恒不等于视在功率守恒



$$\bar{S} = \dot{U}\dot{I}^* = (\dot{U}_1 + \dot{U}_2)\dot{I}^*$$

$$= \dot{U}_1\dot{I}^* + \dot{U}_2\dot{I}^* = \bar{S}_1 + \bar{S}_2$$

$$\because U \neq U_1 + U_2$$

$$\therefore S \neq S_1 + S_2$$

一般情况下：

$$S \neq \sum_{k=1}^{b} S_k$$

$$\text{Complex Power} = \mathbf{S} = P + jQ = \mathbf{V}_{\text{rms}}(\mathbf{I}_{\text{rms}})^*$$

$$= |\mathbf{V}_{\text{rms}}|\,|\mathbf{I}_{\text{rms}}| \underline{/\theta_v - \theta_i}$$

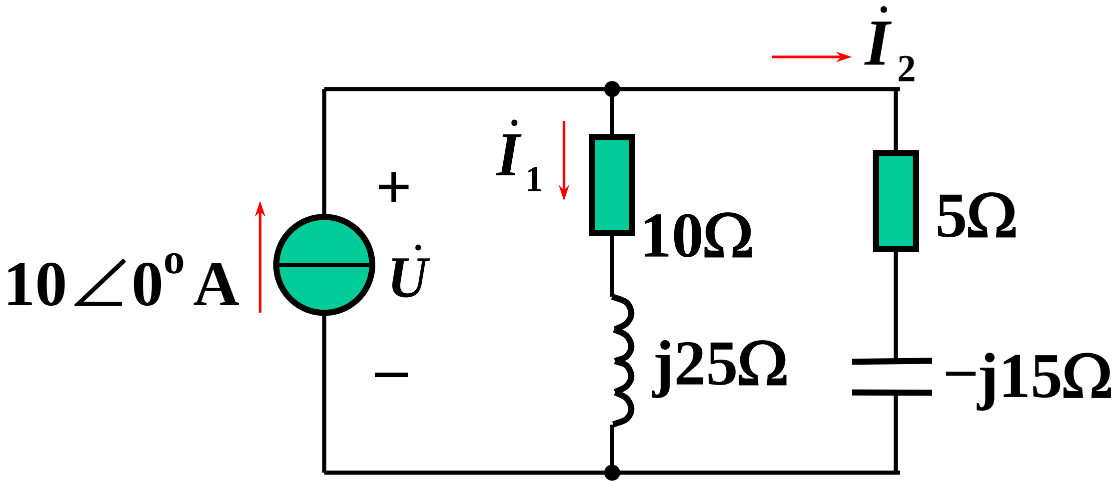
$$\text{Apparent Power} = S = |\mathbf{S}| = |\mathbf{V}_{\text{rms}}|\,|\mathbf{I}_{\text{rms}}| = \sqrt{P^2 + Q^2}$$

$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$

$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$

$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

例9-25. 已知如图，求各支路的复功率。



解一：

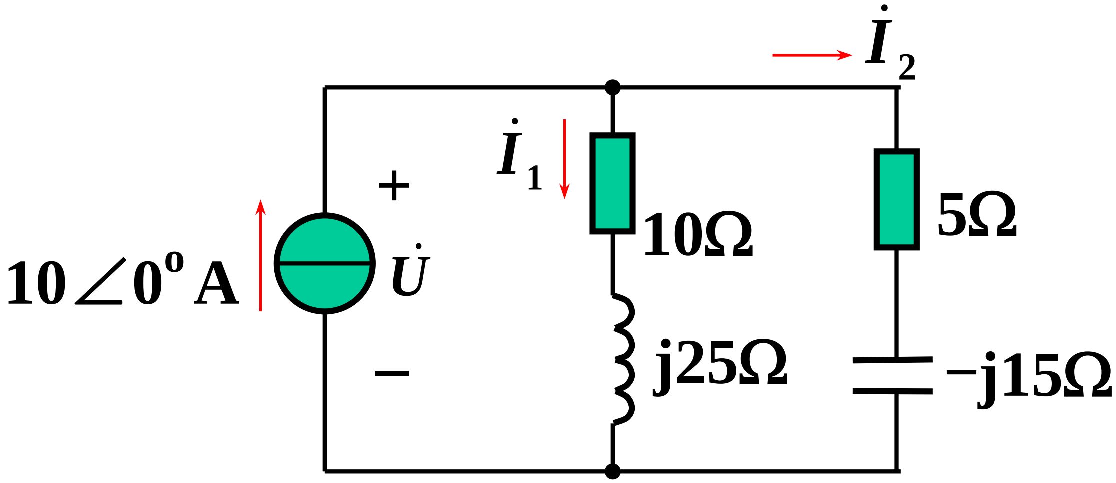
$$\dot{U}=10\angle 0^o\times[(10+j25)//(5-j15)]$$

$$=236\angle(-37.1^o)\quad \text{V}$$

$$\overline{S}_{发}=236\angle(-37.1^o)\times 10\angle 0^o=1882-j1424\quad \text{VA}$$

$$\overline{S}_{1吸}=U^2Y_1^*=236^2\left(\frac{1}{10+j25}\right)^*=768+j1920\quad \text{VA}$$

$$\overline{S}_{2吸}=U^2Y_2^*=1114-j3344\quad \text{VA}$$



解二：

$$\dot{I}_1 = 10\angle 0^{o} \times \frac{5 - j15}{10 + j25 + 5 - j15} = 8.77\angle(-105.3^{o}) \quad A$$

$$\dot{I}_2 = \dot{I}_S - \dot{I}_1 = 14.94\angle 34.5^{o} \quad A$$

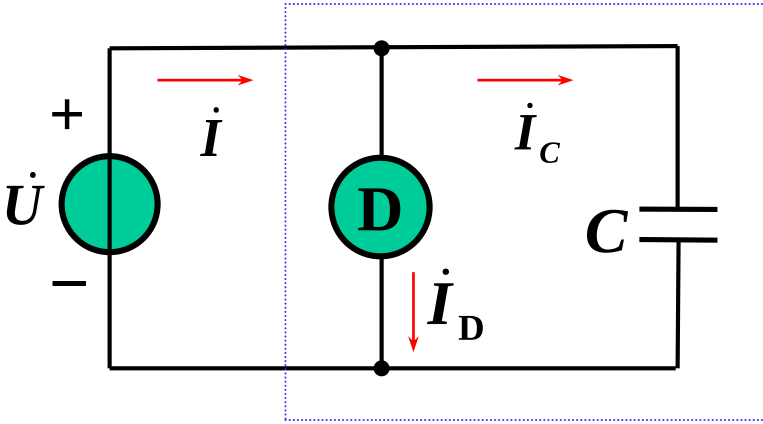
$$\bar{S}_{1吸} = I_1^{\ 2} Z_1 = 8.77^{2} \times (10 + j25) = 769 + j1923 \quad VA$$

$$\bar{S}_{2吸} = I_2^{\ 2} Z_2 = 14.94^{2} \times (5 - j15) = 1116 - j3346 \quad VA$$

$$\bar{S}_{发} = \dot{I}_S^{\ *} \cdot \dot{I}_1 Z_1 = 10 \times 8.77\angle(-105.3^{o})(10 + j25)$$

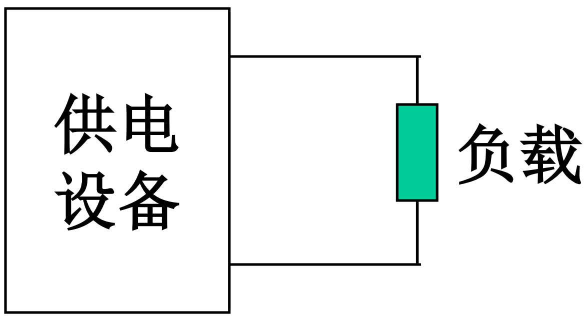
$$= 1885 - j1423 \quad VA$$

三、功率因数提高



$$\cos\varphi_D = 0.8$$

$$\cos\varphi = 0.96$$



$S=UI$

$P=S\cos\varphi$

$\cos\varphi=1,\quad P=S$

$\cos\varphi=0.7,\ P=0.7S$

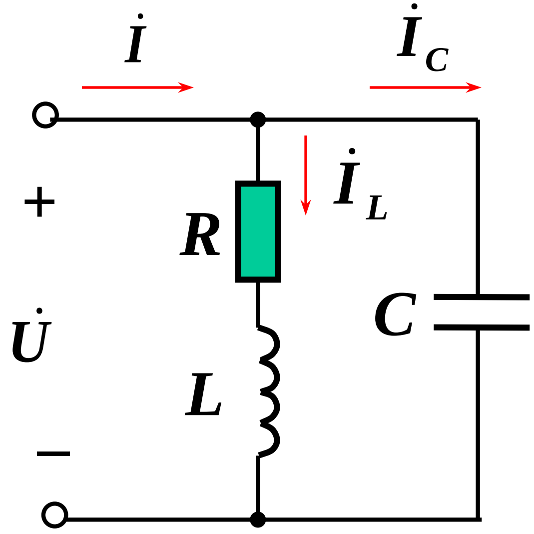
日光灯 $\cos\varphi=0.45\sim0.6$

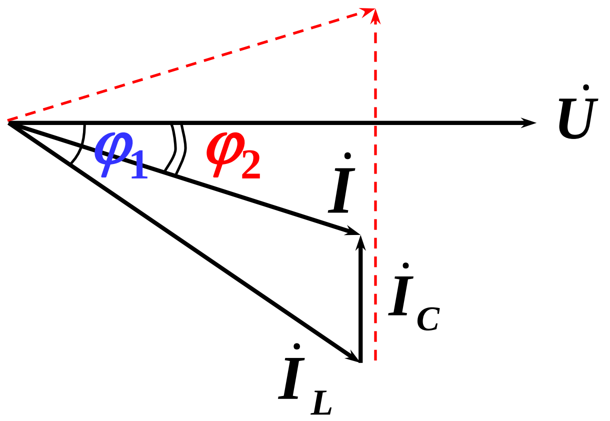
功率因数低带来的问题:

(1) 设备不能充分利用，电流到了额定值，但功率容量还有；

(2) 当输出相同的有功功率时，线路上电流大 $I=P/(U\cos\varphi)$，线路压降损耗大。

解决办法：并联电容，提高功率因数 (改进自身设备)。





功率因数提高后，线路上电流减少，就可以带更多的负载，充分利用设备的能力。

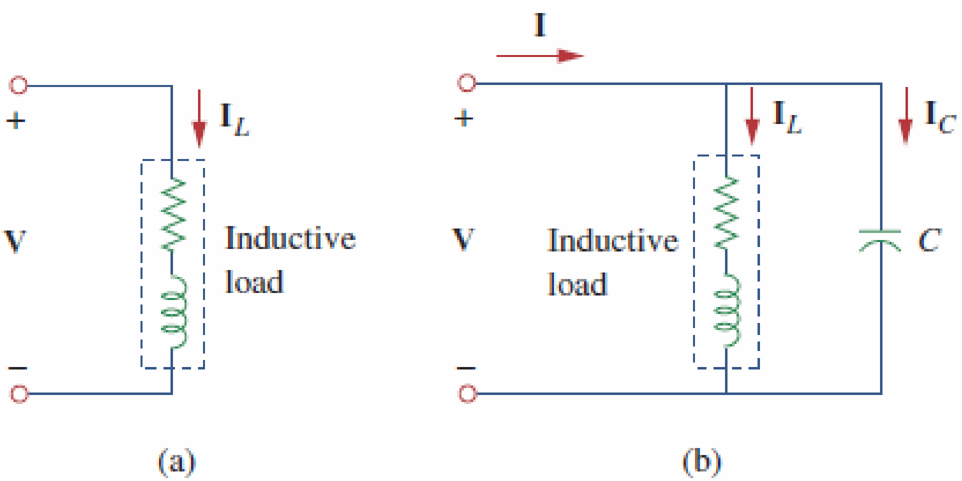


Figure 11.27
Power factor correction: (a) original inductive load, (b) inductive load with improved power factor.

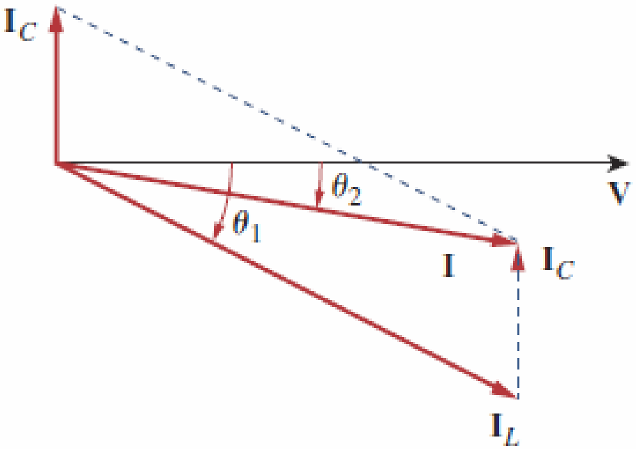


Figure 11.28
Phasor diagram showing the effect of adding a capacitor in parallel with the inductive load.

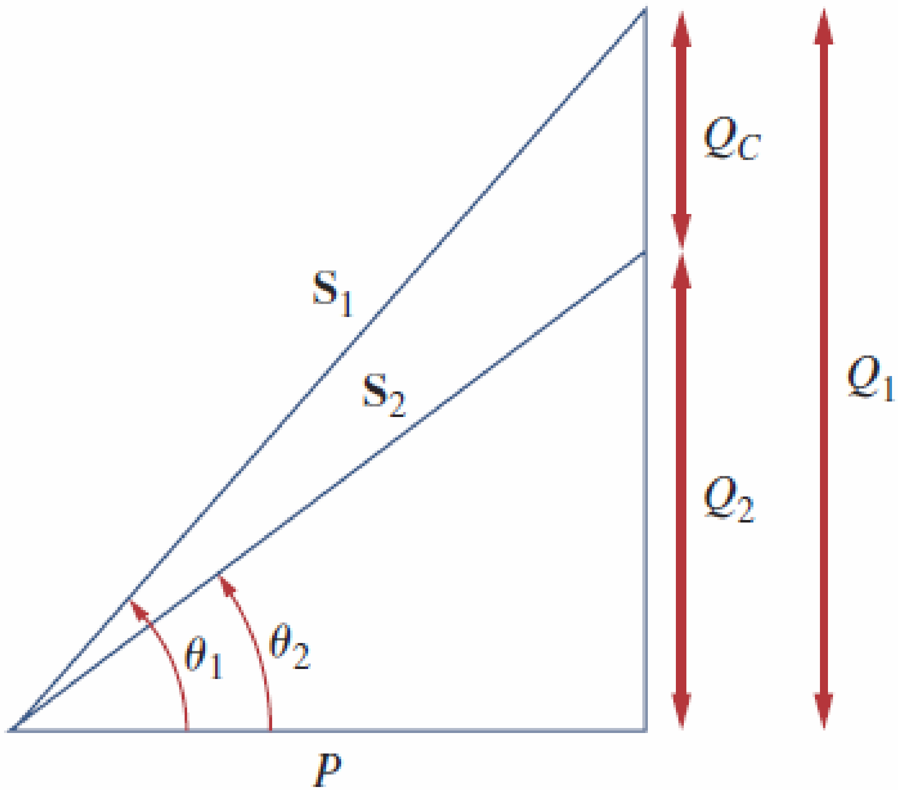


Figure 11.29
Power triangle illustrating power factor correction.

$$P = S_1 \cos\theta_1, \qquad Q_1 = S_1 \sin\theta_1 = P \tan\theta_1$$

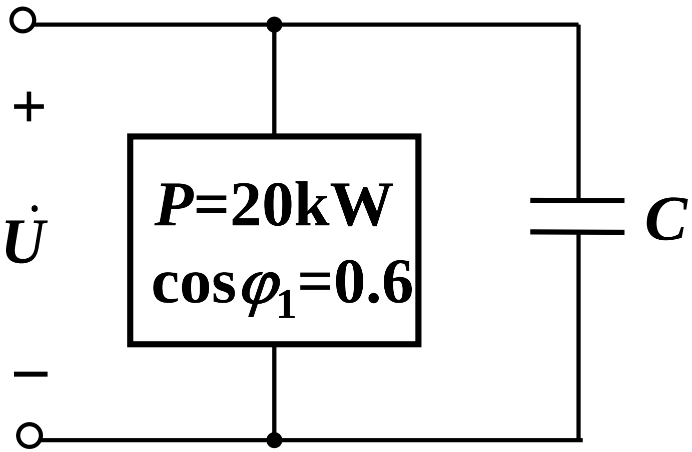
$$Q_2 = P \tan\theta_2$$

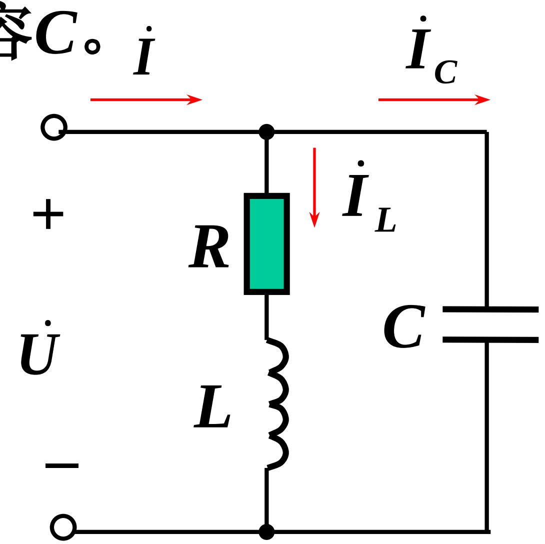
$$Q_C = Q_1 - Q_2 = P(\tan\theta_1 - \tan\theta_2)$$

$$Q_C = V_{\text{rms}}^2 / X_C = \omega C V_{\text{rms}}^2.$$

$$\boxed{C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{\text{rms}}^2}}$$

例9-26. 已知：f=50Hz, U=380V, P=20kW, cosφ_1=0.6(滞后)。要使功率因数提高到0.9，求并联电容C。

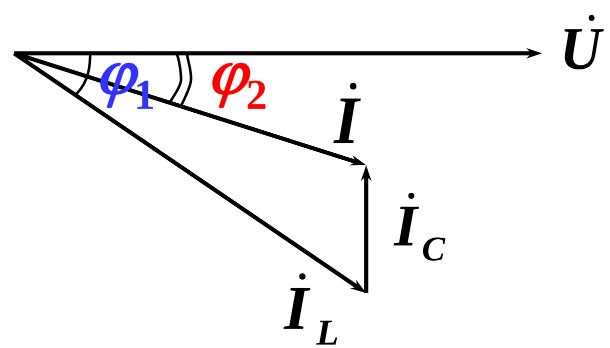




解：

由$\cos\varphi_1 = 0.6$ 得 $\varphi_1 = 53.13^{\circ}$

由$\cos\varphi_2 = 0.9$ 得 $\varphi_2 = 25.84^{\circ}$

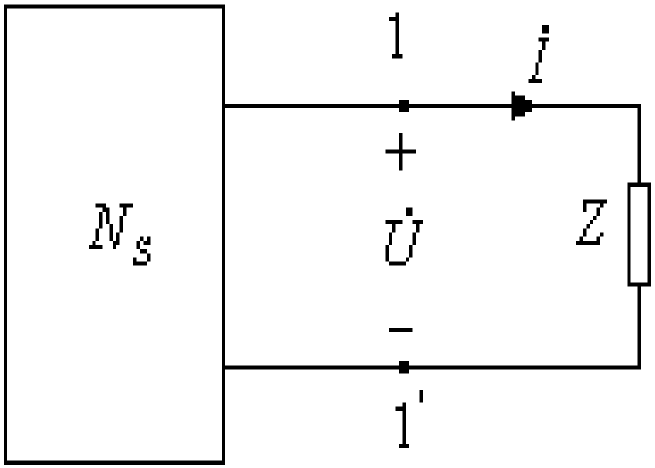


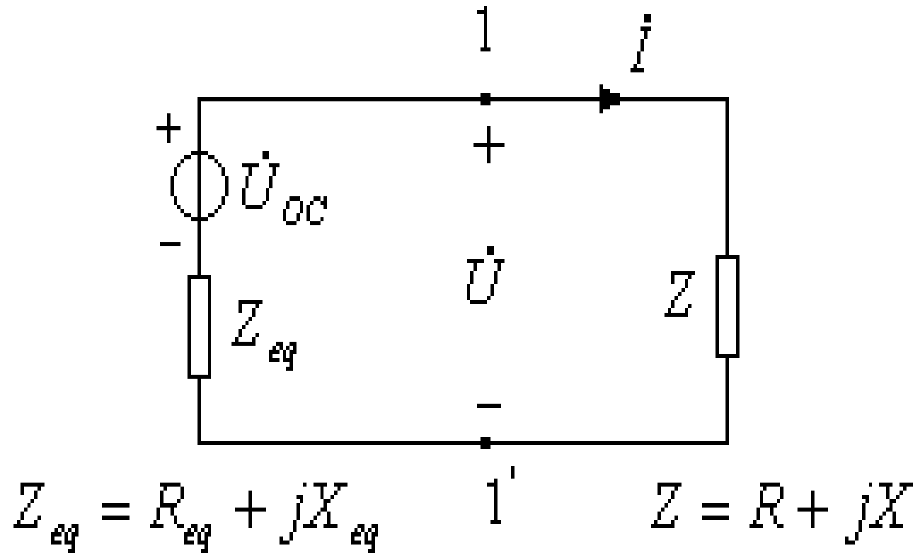
$$C = \frac{P}{\omega U^2}(\text{tg}\varphi_1 - \text{tg}\varphi_2)$$

$$= \frac{20\times10^3}{314\times380^2}(\text{tg}53.13^{\circ} - \text{tg}25.84^{\circ})$$

$$= 375\,\mu\text{F}$$

§9–7　最大功率传输





$$\dot I = \frac{\dot U_{oc}}{Z_{eq} + Z_L}, \quad I = \frac{U_{oc}}{\sqrt{(R_{eq} + R_L)^2 + (X_{eq} + X_L)^2}}$$

$$P = RI^2 = \frac{U_{OC}^2 \cdot R}{(R + R_{eq})^2 + (X + X_{eq})^2}$$

$$\frac{\partial P}{\partial X}=\frac{U_{OC}^{2}R\left[-2(X+X_{eq})\right]}{\left[(R+R_{eq})^{2}+(X+X_{eq})^{2}\right]^{2}}=0 \quad\Rightarrow\quad X=-X_{eq}$$

$$\frac{\partial P}{\partial R}=\frac{(R+R_{eq})^{2}-2R(R+R_{eq})}{(R+R_{eq})^{4}}U_{OC}^{2}=0 \quad\Rightarrow\quad R=R_{eq}$$

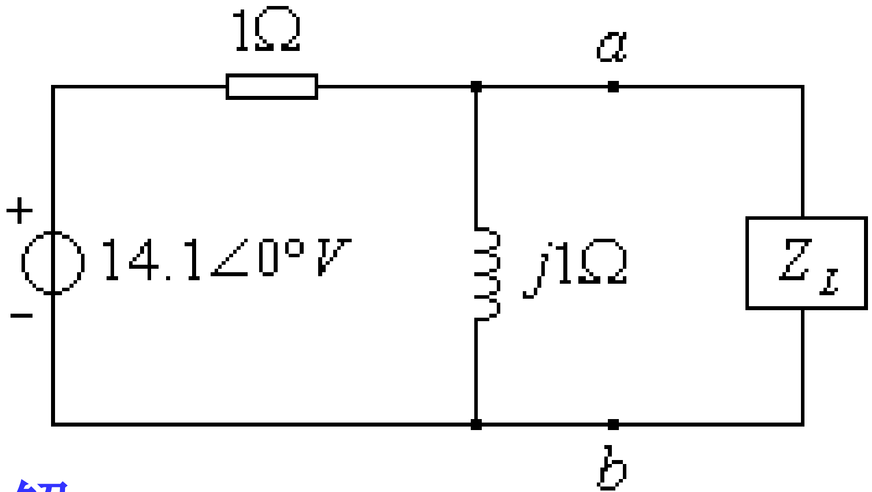
$$Z=R_{eq}-jX_{eq}=Z_{eq}^{*}$$

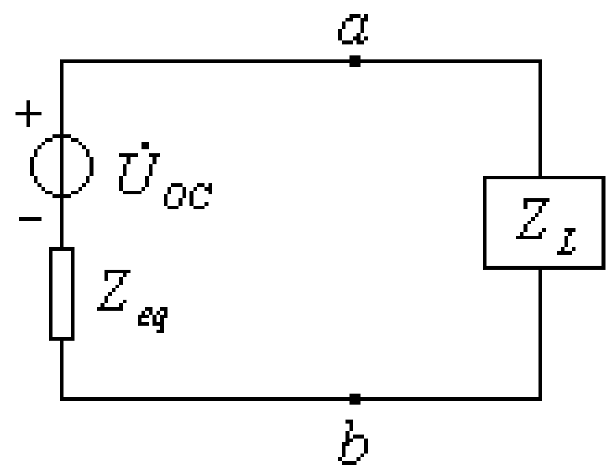
$$P_{\max}=\frac{U_{OC}^{2}}{4R_{eq}}$$

$$Y=Y_{eq}^{*}$$

$$P_{\max}=\frac{I_{SC}^{2}}{4G_{eq}}$$

例9-27： 电路如下图所示，若 Z_L 的实部、虚部均能变动，若使获得最大功率，Z_L 应为何值，最大功率是多少？





解：

$$\dot{U}_{OC}=14.1\angle 0^\circ\times\frac{j}{1+j}=10\sqrt{2}\angle 0^\circ\times\frac{1\angle 90^\circ}{\sqrt{2}\angle 45^\circ}=10\angle 45^\circ\ V$$

$$Z_{eq}=\frac{1\times j}{1+j}=\frac{1}{\sqrt{2}}\angle 45^\circ=0.5+j0.5\ \Omega$$

$$Z_L=0.5-j0.5\ \Omega,$$

$$P_{L\max}=\frac{10^2}{4\times 0.5}=50\ W$$