

Lecture 8. Logical Operators — Implication (Representations陈述)

"q is necessary for p"

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Example: "Good food is necessary to keep us alive"

According to the statement, if we won't have good food then, we'll soon die.

But is it the only factor to keep us alive? **No.**

Example: My friend died at the age of 16 and he has no shortage of good food.
(Even though he is having good food but still he died.)

Therefore, when we say A is necessary for B then falsity of A guarantees the falsity of B but we cannot guarantee the truth of B from the truth of A.

Similarly, when we say q is necessary for p, then we can only guarantee that when q is false then, p is definitely false.

Why p is not necessary for q

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Proof by contradiction: Let say p is necessary for q.

if p is really necessary for q then if p is false then it is mandatory for q to be false in order to make the whole compound proposition $p \rightarrow q$ true. **but it is not the case.**

According to the truth table, it is not mandatory for q to be false when p is false. Hence "p is not necessary for q."

"p is sufficient for q"

"p is sufficient for q"

Example: "It is sufficient for you to travel by car in order to reach your destination on time."

Definitely, if you travel by car, you'll reach your destination on time. No doubt.
but if you won't travel by car, does it mean you'll never reach your destination on time. May be by flight or other means of transport, you'll reach your destination much earlier.

Therefore, when we say that when A is sufficient for B then, truth of A guarantees the truth of B but we cannot guarantee the falsity of B from the falsity of A.

Similarly, when we say "p is sufficient for q", then we can only guarantee that when p is true then, q is definitely true.

Why q is not sufficient for p

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Proof by contradiction: Let say q is sufficient for p.

if q is really sufficient for p then if q is true then it is mandatory for p to be true in order to make the whole compound proposition $p \rightarrow q$ true. **but it is not the case.**

According to the truth table it is not mandatory for p to be true when q is true. Hence "q is not sufficient for p."

"q unless $\neg p$ "

"q unless $\neg p$ "

└ means 'except if'

Evolution steps:

"if p then q" (if p is true then q must be true)

↓
"q is true when p is true"

↓
"q is true except when p is false"

↓
"q is true unless p is false" = "q unless $\neg p$ "