

Lecture 7. Logical Operators — Implication (Representations 陈述)

Different ways to represent conditional statements:

"if p then q"

"p implies q"

"q when p"

"q whenever p"

"q follows from p"

"p only if q"

"q is necessary for p"

"p is sufficient for q"

"q unless $\neg p$ "

"p only if q" and "if p then q"

How "if p then q" and "p only if q" can be same?

Example: "I will stay at home only if I'm sick."

Let p = "I will stay at home" and let q = "I'm sick"

Above statement is of the form **p only if q**

According to the above statement, becoming sick is the necessary condition that will make you stay at home.

This means "if you're not sick then, you cannot stay at home at any cost."

In order to falsify the above statement, q must be FALSE and p must be TRUE i.e. you are not sick and you still stay at home.

Proof idea: truth value of p and q must be same in order to falsify the statement.

if p then q: "If I'll stay at home then I'm sick"

The only way to falsify the above statement is by making p TRUE and q FALSE. Therefore, **p only if q is equivalent to if p then q**

Why "p only if q" is not equivalent to "if q then p"?

Example: "I'll stay at home only if I'm sick."

p = I'll stay at home and q = I'm sick

Proof: p only if q is false when p is TRUE and q is FALSE

If q then p: "If I'm sick then I'll stay at home."
FALSE TRUE = TRUE

As when p is TRUE and q is FALSE, "p only if q" is false but with same truth values of p and q, "if q then p" is true.

Therefore, "p only if q" is not equivalent to "if q then p".

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T