Geometry Lab - Computer Graphics & Robotics - Oct 19 - Blog

David Cao

Oct 19

Inverse Kinematic Problem and Motion Planning #3

1 Problem statement:

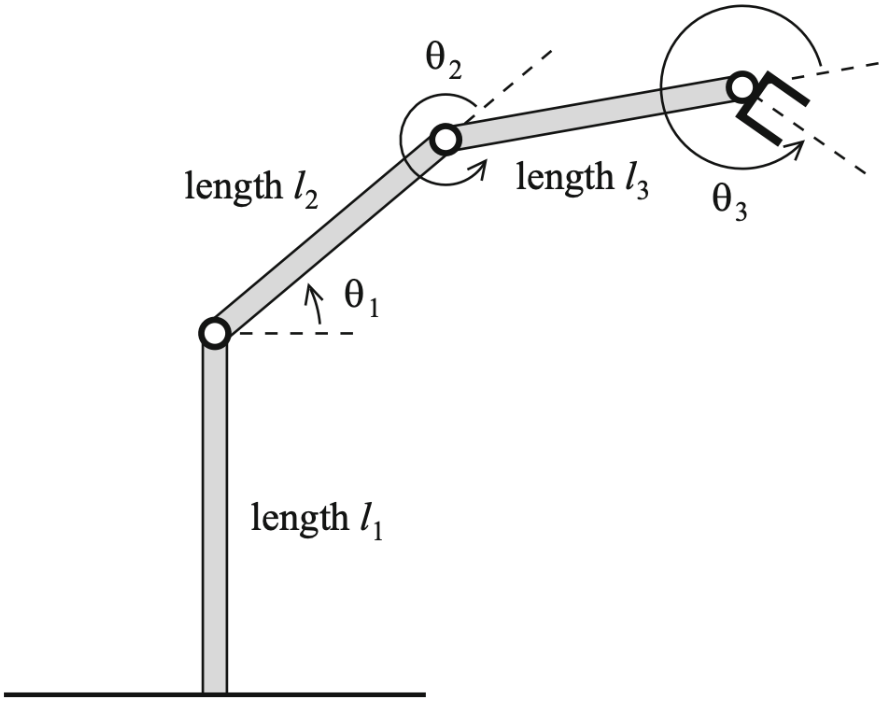
Given a point $(x_1, y_1) = (a, b) \in \mathbb{R}^2$ and an orientation, we wish to determine whether it is possible to place the hand of the robot at that point with that orientation. If it is possible, we wish to find all combinations of joint settings that will accomplish this.

2 Reading Summary

In previous readings we have discussed that we can use polynomials to represent the position of the robot arm.

Similarly, we can use the polynomial approach to inverse kinematics problem as well. Although often times the polynomial equation is difficult to solve due to non-linearity, we can utilize grobner basis to deduce the solution.

Consider this problem:



Consider the position of the robot hand to be (a, b) and we want to figure out a possible configuration of the arm to reach the position, we can represent the position of the robot arm as a polynomial equation shown below:

$$a = l_3(c_1c_2 - s_1s_2) + l_2c_1,$$
$$b = l_3(c_1s_2 + c_2s_1) + l_2s_1,$$
$$0 = c_1^2 + s_1^2 - 1,$$
$$0 = c_2^2 + s_2^2 - 1$$

Given this polynomial we can use grobner basis to simplify the equation and deduce the solution:

for c_1, s_1, c_2, s_2. To solve these equations, we first compute a grevlex Gröbner basis with

$$c_1 > s_1 > c_2 > s_2.$$

Our solutions will depend on the values of a, b, l_2, l_3, which appear as symbolic parameters in the coefficients of the Gröbner basis:

$$\begin{aligned} & c_1 - \frac{bl_2l_3}{l_2(a^2+b^2)}s_2 - \frac{a(a^2+b^2+l_2^2-l_3^2)}{2l_2(a^2+b^2)}, \\ & s_1 + \frac{al_2l_3}{l_2(a^2+b^2)}s_2 - \frac{b(a^2+b^2+l_2^2-l_3^2)}{2l_2(a^2+b^2)}, \\ & c_2 - \frac{a^2+b^2-l_2^2-l_3^2}{2l_2l_3}, \\ & s_2^2 + \frac{(a^2+b^2)^2-2(a^2+b^2)(l_2^2+l_3^2)+(l_2^2-l_3^2)^2}{4l_2^2l_3^2}. \end{aligned} \tag{2}$$

3 Exercise Problem:

2. This exercise studies the geometry of the planar robot with three revolute joints discussed in the text with the dimensions specialized to $l_2 = l_3 = 1$.

   b. Explain geometrically why the equations for c_2 and s_2 in (3) do not involve c_1 and s_1. Hint: Use the geometry of the diagram from part (a) to determine how the two values of θ_2 are related.

3. Consider the robot arm discussed in the text. Setting $l_2 = l_3 = 1$ and $a = b = 0$ in (1) gives the Gröbner basis (4). How is this basis different from the basis (3), which only assumes $l_2 = l_3 = 1$? How does this difference explain the properties of the kinematic singularity at $(0, 0)$?

4 Discussion and discovery:

4.1 Problem 2 (b):

When specifying coordinates, the coordinate is based on the position of the joint. According to the polynomial coordinates:

$$\begin{aligned} x &= cos(\theta) \\ y &= sin(\theta) \\ c_i &= cos(\theta_i) \\ s_i &= sin(\theta_i) \end{aligned}$$

We can then notice that because when describing the coordinates based on joint2, it is independent from joint 1 because the coordinate is based on the position of joint 2 which do not include any components of joint 1.
Different from joint 2, when calculating the coordinates based on joint 1, it must also include the coordinate of joint 2 because geometrically the position of joint 1 is based on the position of joint 2.

4.2 Problem 3:

The grobner basis when $a = b = 0$ is:

$$\begin{aligned} &c_1^2 + s_1^2 - 1, \\ &c_2 + 1, \\ &s_2. \end{aligned}$$

We notice that the compare to the previous grobner basis, s_1 is a free variable which showed that there are infinite amount of solutions to this problem. Geometrically, if the robot want to point to the origin the second robot arm has to point backwards to the exact position of the first arm, shown below: