Geometry Lab - Computer Vision & Robotics - Sept 20 - Blog

David Cao

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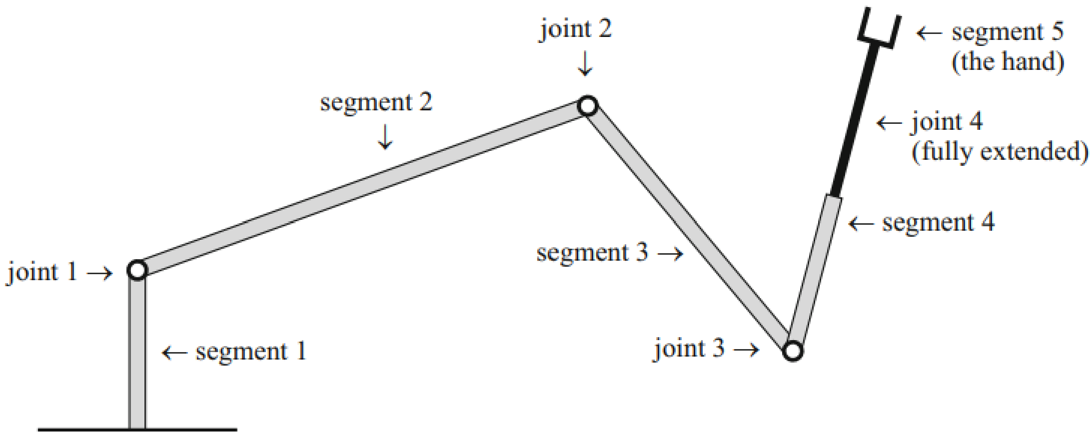
Forward Kinematic Problem #1

1 Problem statement:

Given a robot with a list of of segments, joints, and prismatic joint, can we give an explicit description of the areas that the robot can reach:

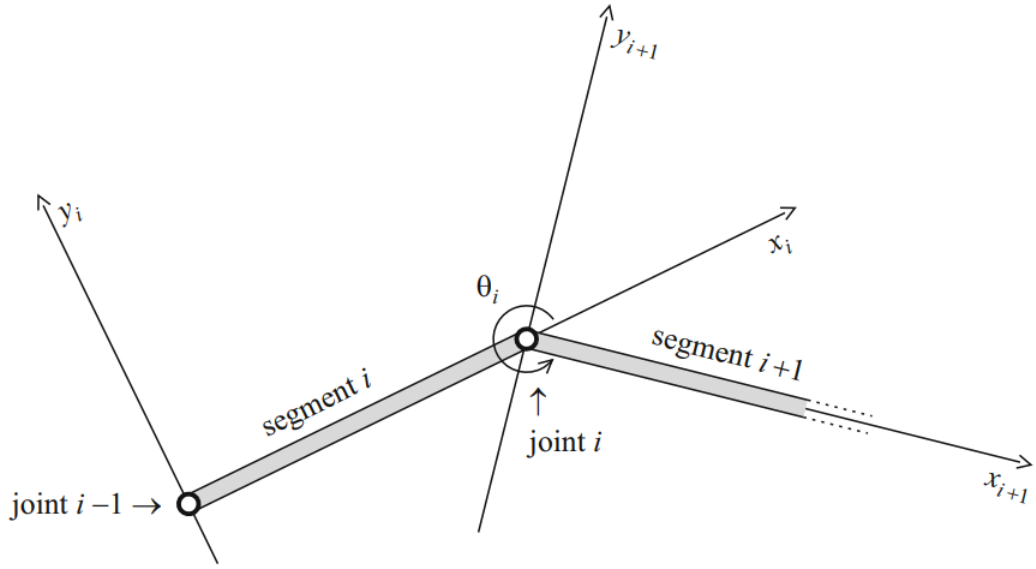


Example 1

2 Reading summary:

§2 of the reading introduces a new representation of the reachable position using linear algebra.

First, at At a revolute joint i, we introduce an (x_{i+1}, y_{i+1}) coordinate system in the following way: We take the positive xi+1-axis to lie along the direction of segment i + 1 (in the robot's current position). Then the positive y_{i+1}-axis is determined to form a normal right-handed rectangular coordinate system. A illustration of this local coordinate system is shown below:



Giving the angle between them θ_i and length of the segment l_i, such representation allows us to relate coordinate (x_{i+1}, y_{i+1}) with (x_i, y_i) using linear algebra:

$$\begin{bmatrix} x_i \\ y_1 \end{bmatrix} = \begin{bmatrix} cos(\theta_i) & -sin(\theta_i) \\ sin(\theta_i) & cos(\theta_i) \end{bmatrix} \cdot \begin{bmatrix} x_{i+1} \\ y_{i+1} \end{bmatrix} + \begin{bmatrix} l_i \\ 0 \end{bmatrix}$$

Or better using 3 by 3 matrix:

$$\begin{bmatrix} x_i \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} cos(\theta_i) & -sin(\theta_i) & l_i \\ sin(\theta_i) & cos(\theta_i) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_{i+1} \\ y_{i+1} \\ 1 \end{bmatrix} = A_i \cdot \begin{bmatrix} x_{i+1} \\ y_{i+1} \\ 1 \end{bmatrix}$$

Matrix notation allows us to express the final coordinate in a more concise way:

$$\begin{bmatrix} x_i \\ y_1 \\ 1 \end{bmatrix} = A_1 A_2 A_3 \cdot \begin{bmatrix} x_{i+1} \\ y_{i+1} \\ 1 \end{bmatrix}$$

Using the trigonometric addition formulas, this equation can be written as

$$\begin{bmatrix} x_i \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} cos(\theta_1+\theta_2+\theta_3) & -sin(\theta_1+\theta_2+\theta_3) & l_3 cos(\theta_1+\theta_2) + l_2 cos(\theta_1) \\ sin(\theta_1+\theta_2+\theta_3) & cos(\theta_1+\theta_2+\theta_3) & l_3 sin(\theta_1+\theta_2) + l_2 sin(\theta_1) \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_4 \\ y_4 \\ 1 \end{bmatrix}$$

When calculating the matrix product, we can assume that the end point of the robot (x_4, y_4) is the origin, and through matrix multiplication get the relative coordinate of the point (x_4, y_4) with respect to (x_0, y_0). Since we can assume that $x_0 = y_0 = 0$, the result of the matrix multiplication is:

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} l_3cos(\theta_1+\theta_2)+l_2cos(\theta_1) \\ l_3sin(\theta_1+\theta_2)+l_2sin(\theta_1) \\ 1 \end{bmatrix}$$

Besides using θ and l, using parametrization $x = cos(\theta)$ and $y = sin(\theta)$, we can represent the result in a polynomial form:

$$\begin{bmatrix} l_3(c_1c_2 - s_1s_2) + l_2c_1 \\ l_3(s_1c_2 + s_2c_1) + l_2s_1 \end{bmatrix}$$

Moreover, we can also use rational parametrization to represent reach:

$$x = \frac{1-t^2}{1+t^2}$$
$$y = \frac{2}{1+t^2}$$

3 Discussion and discovery: