

# Geometry Lab - Computer Graphics & Robotics - Sept 20 - Blog

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## Inverse Kinematic Problem and Motion Planning #2

### 1 Problem statement:

Given a point  $(x_1, y_1) = (a, b) \in \mathbb{R}^2$  and an orientation, we wish to determine whether it is possible to place the hand of the robot at that point with that orientation. If it is possible, we wish to find all combinations of joint settings that will accomplish this.

### 2 Exercise Problem:

Use the algorithm given in class on Friday to find a Gröbner basis the following ideal:  $I = \langle x^2y - 1, xy^2 - x \rangle$ . Use the lex, then the grlex order in each case, and then compare your results.

### 3 Discussion and discovery:

$$I = \langle x^2y-1, xy^2-x \rangle$$

$$\text{lex } (x > y)$$

$$S(f_1, f_2) = y(x^2y-1) - x(xy^2-x) = -y+x^2$$

$$S(f_1, f_2) \neq 0$$

$$F = \{x^2y-1, xy^2-x, x^2-y\}$$

$$S(f_1, f_2) = x^2y-1 - y(x^2-y) = y^2-1 \text{ (Not divisible)}$$

$$F = \{x^2y-1, xy^2-x, x^2-y, y^2-1\}$$

$$S(f_1, f_3) = y(x^2y-1) - x^2(y^2-1) = x^2-y$$

$$S(f_2, f_3) = x(xy^2-x) - y^2(x^2-y) = y^3-x^2$$

$$S(f_3, f_4) = xy^2-x - x(y^2-1) = x-x=0$$

$$S(f_2, f_4) = y^2(x^2-y) - x^2(y^2-1) = x^2-y^3$$

$$F = \{x^2-y, y^2-1\}$$

$$x = \pm 1, y = \pm 1$$

$$F = \{x^2y-1, xy^2-x\}$$

$$S(f_1, f_2) = -y+x^2$$

$$x^2-y \quad y^2-1$$

$$\begin{array}{r} -x^2+y^3 \\ -x^2+y \\ \hline y^3-y \\ -y^3-y \\ \hline 0 \end{array}$$