# Geometry Lab - Computer Graphics & Robotics - Sept 20 - Blog

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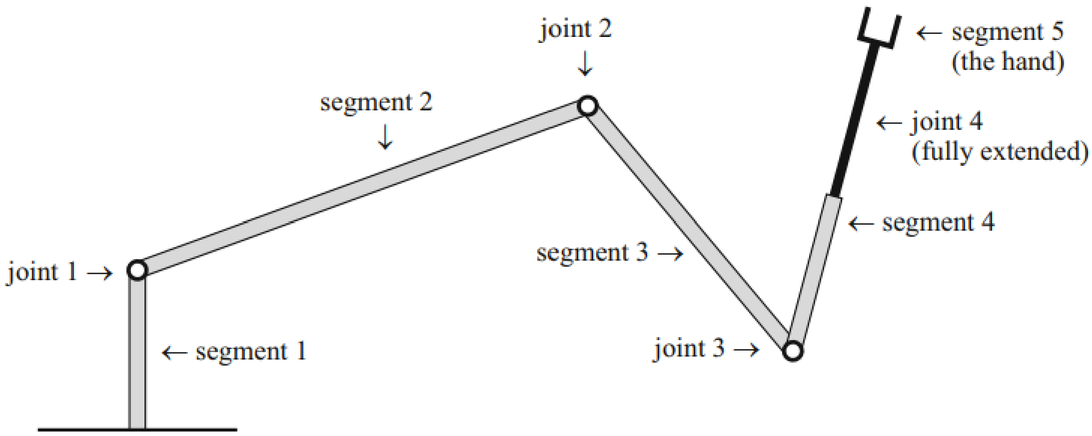
Sept 20

## Forward Kinematic Problem #1

## 1 Problem statement:

Given a robot with a list of of segments, joints, and prismatic joint, can we give an explicit description of the areas that the robot can reach:




Example 1

## 2 Reading summary:

This week's reading is the same as last week's

# 3 Exercise Problem:

## 3.1 Problem 1 statement:

3. In this exercise, we will reconsider the hand orientation for the robots in Examples 1 and 2. Namely, let $\alpha = \theta_1 + \theta_2 + \theta_3$ be the angle giving the hand orientation in the $(x_1, y_1)$ coordinate system.
   a. Using the trigonometric addition formulas, show that

$$c = \cos\alpha, \ \ s = \sin\alpha$$

   can be expressed as polynomials in $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$. Thus, the whole mapping $f$ can be expressed in polynomial form, at the cost of introducing an extra coordinate function for $\mathcal{C}$.
   b. Express $c$ and $s$ using the rational parametrization (8) of the circle.

## 3.2 Problem 2 statement:

4. Consider a planar robot with a revolute joint 1, segment 2 of length $l_2$, a prismatic joint 2 with settings $l_3 \in [0, m_3]$, and a revolute joint 3, with segment 4 being the hand.
   a. What are the joint and configuration spaces $\mathcal{J}$ and $\mathcal{C}$ for this robot?
   b. Using the method of Examples 1 and 2, construct an explicit formula for the mapping $f : \mathcal{J} \to \mathcal{C}$ in terms of the trigonometric functions of the joint angles.
   c. Convert the function $f$ into a polynomial mapping by introducing suitable new coordinates.

### Problem 2 statement:

# 4 Discussion and discovery:

## 4.1 Problem 1:

To represent $cos(\alpha)$ using polynomials, we can use trigonometric identities:

$$c = cos(\alpha) = cos(\theta_1 + \theta_2 + \theta_3)$$
$$c = cos(\theta_1)cos(\theta_2 + \theta_3) - sin(\theta_1)sin(\theta_2 + \theta_3)$$
$$c = cos(\theta_1)(cos(\theta_2)cos(\theta_3) - sin(\theta_2)sin(\theta_3)) - sin(\theta_1)(sin(\theta_2)cos(\theta_3) + cos(\theta_2)sin(\theta_3))$$
$$s = sin(\alpha) = sin(\theta_1 + \theta_2 + \theta_3)$$
$$s = sin(\theta_1)cos(\theta_2 + \theta_3) + cos(\theta_1)sin(\theta_2 + \theta_3)$$
$$s = sin(\theta_1)(cos(\theta_2)cos(\theta_3) - sin(\theta_2)sin(\theta_3)) + cos(\theta_1)(sin(\theta_2)cos(\theta_3) + cos(\theta_2)sin(\theta_3))$$

using polynomial:

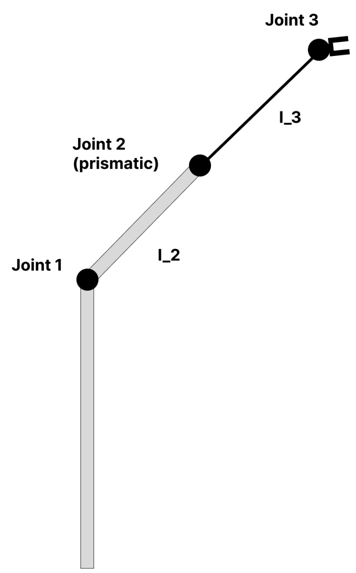
$$c_i = cos(\theta_i)$$
$$s_i = sin(\theta_i)$$
$$c_i^2 + s_i^2 = 1$$

we can represent $cos(\alpha)$ as:

$$c = c_1(c_2c_3 - s_2s_3) - s_1(s_2c_3 + c_2s_3)$$
$$s = s_1(c_2c_3 - s_2s_3) + c_1(s_2c_3 + c_2s_3)$$

### 4.2 Problem 2:

Here's a illustrative graph of problem 2's robot:




$$A_1 = \begin{bmatrix} cos(\theta_1) & -sin(\theta_1) & 0 \\ sin(\theta_1) & cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \; A_2 = \begin{bmatrix} cos(0) & -sin(0) & l_2 \\ sin(0) & cos(0) & 0 \\ 0 & 0 & 1 \end{bmatrix} \; A_3 = \begin{bmatrix} cos(\theta_2) & -sin(\theta_2) & l_3 \\ sin(\theta_2) & cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x = l_2 cos(\theta_1) + l_3 cos(\theta_1 + \theta_2)$$
$$y = l_2 sin(\theta_1) + l_3 sin(\theta_1 + \theta_2)$$

$$x = l_2(\frac{1 + t_1^2}{1 - t_1^2}) + l_3(\frac{1 + t_2^2}{1 - t_2^2})$$
$$y = l_2(\frac{2t_1}{1 - t_1^2}) + l_3(\frac{2t_2}{1 - t_2^2})$$