

Geometry Lab - Computer Graphics & Robotics - Sept 20 - Blog

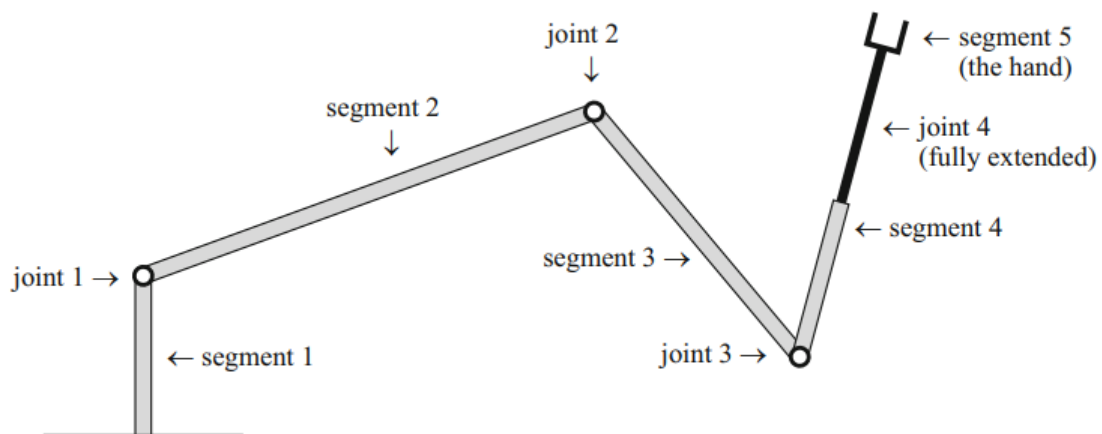
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Sept 20

Forward Kinematic Problem #1

1 Problem statement:

Given a robot with a list of segments, joints, and prismatic joint, can we give an explicit description of the areas that the robot can reach:



Example 1

2 Reading summary:

This week's reading is the same as last week's

3 Exercise Problem:

3.1 Problem 1 statement:

3. In this exercise, we will reconsider the hand orientation for the robots in Examples 1 and 2. Namely, let $\alpha = \theta_1 + \theta_2 + \theta_3$ be the angle giving the hand orientation in the (x_1, y_1) coordinate system.
 - a. Using the trigonometric addition formulas, show that

$$c = \cos \alpha, \quad s = \sin \alpha$$

can be expressed as polynomials in $c_i = \cos \theta_i$ and $s_i = \sin \theta_i$. Thus, the whole mapping f can be expressed in polynomial form, at the cost of introducing an extra coordinate function for \mathcal{C} .

- b. Express c and s using the rational parametrization (8) of the circle.

3.2 Problem 2 statement:

4. Consider a planar robot with a revolute joint 1, segment 2 of length l_2 , a prismatic joint 2 with settings $l_3 \in [0, m_3]$, and a revolute joint 3, with segment 4 being the hand.
 - a. What are the joint and configuration spaces \mathcal{J} and \mathcal{C} for this robot?
 - b. Using the method of Examples 1 and 2, construct an explicit formula for the mapping $f : \mathcal{J} \rightarrow \mathcal{C}$ in terms of the trigonometric functions of the joint angles.
 - c. Convert the function f into a polynomial mapping by introducing suitable new coordinates.

Problem 2 statement:

4 Discussion and discovery:

4.1 Problem 1:

To represent $\cos(\alpha)$ using polynomials, we can use trigonometric identities:

$$\begin{aligned} c &= \cos(\alpha) = \cos(\theta_1 + \theta_2 + \theta_3) \\ c &= \cos(\theta_1)\cos(\theta_2 + \theta_3) - \sin(\theta_1)\sin(\theta_2 + \theta_3) \\ c &= \cos(\theta_1)(\cos(\theta_2)\cos(\theta_3) - \sin(\theta_2)\sin(\theta_3)) - \sin(\theta_1)(\sin(\theta_2)\cos(\theta_3) + \cos(\theta_2)\sin(\theta_3)) \\ s &= \sin(\alpha) = \sin(\theta_1 + \theta_2 + \theta_3) \\ s &= \sin(\theta_1)\cos(\theta_2 + \theta_3) + \cos(\theta_1)\sin(\theta_2 + \theta_3) \\ s &= \sin(\theta_1)(\cos(\theta_2)\cos(\theta_3) - \sin(\theta_2)\sin(\theta_3)) + \cos(\theta_1)(\sin(\theta_2)\cos(\theta_3) + \cos(\theta_2)\sin(\theta_3)) \end{aligned}$$

using polynomial:

$$c_i = \cos(\theta_i)$$

$$s_i = \sin(\theta_i)$$

$$c_i^2 + s_i^2 = 1$$

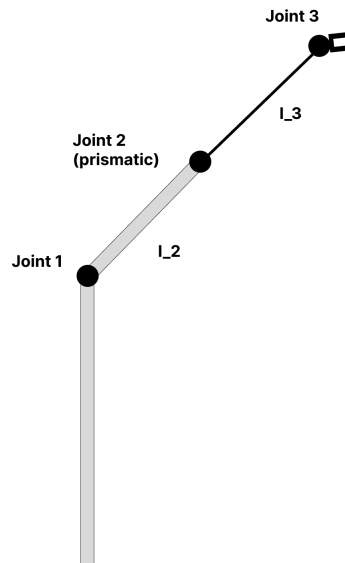
we can represent $\cos(\alpha)$ as:

$$c = c_1(c_2c_3 - s_2s_3) - s_1(s_2c_3 + c_2s_3)$$

$$s = s_1(c_2c_3 - s_2s_3) + c_1(s_2c_3 + c_2s_3)$$

4.2 Problem 2:

Here's a illustrative graph of problem 2's robot:



$$A_1 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} \cos(0) & -\sin(0) & l_2 \\ \sin(0) & \cos(0) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & l_3 \\ \sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$