

1. Development of hybrid first principles – artificial intelligence models for high-temperature power systems

2025년 7월 7일 월요일 오전 11:39

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Research Paper
Development of hybrid first principles – artificial intelligence models for transient modeling of power plant superheaters under load-following operation

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ARTICLE INFO

Keywords:
Hybrid model
Superheaters
First-principles
Bayesian machine learning
Bayesian machine learning

ABSTRACT

This paper develops different approaches for synergistic integration of physics-based models with data-driven models for modeling of high temperature power plant superheaters. Two types of data-driven models are developed, namely, an nonlinear static-dynamic neural network model as well as models obtained using a Bayesian machine learning approach. Series, integrated, and parallel coupling of first-principles models and data-driven models are proposed. Algorithms for solving the training problem for the data-driven models and for the simulation of the hybrid models are developed. The developed approaches are applied to high temperature power plant superheaters that face considerable modeling and measurement challenges. Measurement challenges arise due to harsh operating conditions that not only make placement of sensors difficult, but also of the placed sensors very limited. Modeling challenges arise due to complex three-dimensional spatial distribution of flow gas and mass fluxes, especially under load-following operation and uncertainties in heat transfer characteristics because of multiple factors such as stochastic spatial temporal variation of sub-cooled steam on the superheater tubes in coal-fired power plants, internal scale formation in the tubes, etc. First-principles two-dimensional differential algebraic equation models of two industrial superheaters, one from a natural gas combined cycle plant and another from a coal-fired power plant, are developed. The results from the models are compared against industrial operational data. For all cases studied in this work, it is observed that both for steady-state and dynamic data, the hybrid models have much higher accuracy than when only the respective first-principles models are used. For example, during prediction of the average static tube wall temperature of superheater tubes at the combined cycle power plant, the root mean squared error observed for the standalone first-principles model improved from 12.3 °C to 0.5 °C upon integration with data-driven models. Similarly, while predicting the main steam outlet temperature in the coal-fired plant, the hybrid models resulted in reduction of the root mean squared error value from 19.3 °C to around 2 °C. In addition, the hybrid model yield highly resolved spatial-temporal temperature profiles that can be helpful for developing monitoring approaches of these critical components under load-following operation.

1. Introduction

Superheaters are critical components in power plants for producing the main steam at the desired temperature. Under load-following operation, these high temperature components can suffer from considerable damage that can eventually lead to failure and thus can

compromise the reliability of the power plants. This work investigates the use of hybrid models for spatial-temporal prediction of temperature in power plant superheaters.

Process systems can be modeled using first principles (FP) models based on conservation laws, material and thermodynamic properties, or using data-driven models based on measured operational data [1]. First-principles models can provide very good prediction even for cases where

ABSTRACT

본 논문은 고온 발전소 과열기(superheater)의 모델링을 위해 물리 기반 모델(physics-based model)과 데이터 기반 모델(data-driven model)을 시너지 있게 통합하는 다양한 접근 방식을 개발하였다.

특히, 고온 과열기의 기온 모델이 개발되었는데, 하나는 **물리-비선형 정적-동적 신경망 모델**(static-dynamic neural network model), 다른 하나는 **베이지안 머신러닝**(Bayesian machine learning) 기법을 통해 얻은 모델이다.

본 연구에서는 물리 모델(first-principles model, FP)과 데이터 기반 모델 간의 **특성(features), 통합(integration), 병렬(parallel)** 결합 구조를 제안하였으며, 각각의 구조에 대해 훈련(training) 알고리즘과 하이브리드 모델 시뮬레이션(simulation)을 위한 알고리즘이 개발되었다.

이러한 접근 방식은 고온 발전소 과열기 시스템에 적용되었으며, 해당 시스템은 모델링과 계측(measurement)에 있어 많은 어려움을 겪고 있다. 계측상의 문제는 고온-고압 운전 환경으로 인해 센서 설치가 어려우며, 설치된 센서의 수명도 매우 짧은다는 데에서 비롯된다.

모델링상의 문제는 특히 부식 후속(load-following) 운전 조건 하에서 배연가스(flue gas)와 증기 유량의 공간적 분포가 복잡하고 불균일하기 때문에 발생한다. 또한, 열전달 특성에 불확실성이 존재하는데, 이는 다중과 같은 다양한 요인에 기인한다.

예를 들어 어떤 특정 발전소에서는 과열기 튜브 외벽에 생기는 재(ash) 침전물의 시간간격 특성을 본고, 튜브 내벽의 산화물 스케일(scale) 형성 등이다.

이에 따라, 천연가스 복합화력발전소와 석탄화력발전소의 실제 산업용 과열기를 대상으로 하는 2개의 물리(2D differential-algebraic equation models)를 개발하였다.

개발된 모델 결과는 산업 현장의 운전 데이터와 비교하였으며, 모든 사례에 대해 다음과 같은 결론을 얻었다.

정상 상태(steady-state)와 동적(dynamic) 데이터 모두에서, 단독 물리 모델을 사용할 때보다 하이브리드 모델을 사용할 경우 예측 정확도가 크게 향상되었다.

예를 들어, 복합화력발전소에서 과열기 튜브의 외벽 평균 온도를 예측할 때, 물리 모델만 사용할 경우의 평균 평균 오차(RMSE)는 12.3°C였으나, 데이터 기반 모델과 결합한 하이브리드 모델은 0.5°C로 줄어든다.

또한 석탄화력발전소에서는 어떤 특정 증기 온도를 예측한 경우, 하이브리드 모델은 RMSE를 19.5°C에서 약 2°C로 줄였다.

뿐만 아니라, 하이브리드 모델은 시간간격으로 매우 세분화된(spatial-temporally resolved) 온도 분포를 제공할 수 있으며, 이는 부식 수명 문제 하에서 이와 같은 주요 구성 요소의 상태를 모니터링하는 방법을 개발하는 데 유용하게 활용될 수 있다.

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Nomenclature		per	parallel
A	total heat transfer area, m ²	rh	reheater
c_p	specific heat capacity, J/(kg·K)	st	steam
f	function of first principles model equations	sc	series
f_{gas}	function of data-driven model equations	spw	spray flow
h	convective heat transfer coefficient, W/(m ² ·K)	nr	natural tube
m	mass, kg	w	tube wall
\dot{m}	total mass flow rate, kg/s	Abbreviation	
q	heat flux, kW/m ²	AI	artificial intelligence
t	time, s	ALAMO	automatic learning of algebraic models
T	temperature, °C	ALVEN	algebraic learning via elastic net
u	model input variables	ANN	artificial neural network
v	velocity, m/s	BIGSAM	Bayesian identification of dynamic sparse algebraic models
v_v	valve opening	BML	Bayesian machine learning
x	first principles model state variables	CNN	convolutional neural network
y	horizontal direction dimension/output variable	DAE	differential algebraic equation
z	vertical direction dimension	DARN	decoupled A-B Network
Greek Symbols		DNN	deep neural network
Δ	residuals	EM	expectation maximization
θ	parameters	FP	first principles
ρ	density, kg/m ³	GRU	gated recurrent unit
Subscripts		HP	high pressure
A,B,C,D	steam inlet pipe tags	HRSO	heat recovery steam generator
avg	average	LSHM	long-short term memory
ME	machine learning	MAIS	maximum absolute instantaneous error
corr	correction	NARX	nonlinear auto-regressive network with exogenous inputs
fl	flue gas	NOCG	natural gas combined cycle
in	time step/inside	NLD	nonlinear dynamic
m	inlet	NLS	nonlinear static
int	integrated	NN	neural network
j	tube assembly/flow circuit number	ODE	ordinary differential equation
meas	measured	PDE	partial differential equation
mod	model	PC	pulverized coal
n	number	RMSSE	root mean square error
o	outside	RNN	recurrent neural network
opt	optimal	SDNY	sparse identification of nonlinear dynamics
out	outlet		

there are no or limited data availability in certain range of operating conditions or for cases where data acquisition is not feasible given the current state of measurement technology. However, constructing accurate FP physics-based mechanistic models for complex nonlinear noisy dynamic systems may lead to excessive computational expense as well as challenging for online adaptation. Furthermore, it can also be difficult, if not impossible, to develop efficient physics-based models for certain complex phenomena that are poorly understood. In contrast, artificial intelligence (AI) machine learning (ML) models are typically much faster in computation and can be relatively easier to develop, simulate, and adapt online, even for complex and ill-defined dynamic systems [2]. One of the primary limitations of the conventional data-driven models stems from the lack of predictive or extrapolative capabilities especially when there is information gap in the data used for model development. For example, high-temperature systems such as steam superheater systems have complex dynamics with limited measurements for key state variables like tube metal wall and flue gas temperatures [3]. Modeling such systems using either a standalone physics-based or AI model can yield inaccurate results. Hybrid models, however, synergistically combine FP models with AI models and can be leveraged to yield a monitoring framework that is adaptive, real-time, and accurate [4]. This work develops several approaches where first-principles mechanistic

models developed for high temperature power plant superheaters [5] have been systematically coupled with two different ML models, i.e., artificial neural networks (ANN) and Bayesian Machine Learning [7].

Over the last few decades, hybrid FP + AI models have found numerous implementations [9,10]. One of the most common applications of hybrid models has been in the form of lumped parameter models where AI models account for certain static process parameters to be subsequently utilized in the FP model [11,12]. Other implementations of hybrid FP + AI models include modeling of the residual from a FP model by using data-driven models applied on chemical reactor [13], energy systems [14], and biological processes [15].

Physics informed machine learning approaches aimed at conservation of system physics represented by mass [16,17], energy [18], and momentum balance [19], and digital twin development for coal fired power plants [20,21] and biomass energy systems [22]. These models can lead to improved interpretability and efficient extrapolation capabilities [23,24]. There are significant opportunities, however, for exploiting the synergistic hybridization of FP models with data-driven models. For such hybrid models to be successful and yield desired outcomes, various other aspects need to be considered other than coupling approaches. In particular, it is important to consider what information needs to be exchanged

between FP models and data-driven models and at what interval, how to select the specific data-driven model for the data-driven model, and when and how to adapt the hybrid model. Furthermore, if it is desired that both the FP and AI models represent nonlinear transients, then optimal synthesis of such hybrid models needs to consider complexity, computational expense, and accuracy.

For representing certain systems, more than one type of data-driven models may be simultaneously required in a hybrid framework [25]. The FP and data-driven models can be arranged in series or parallel or integrated or as combination of the configurations. If the FP model cannot adequately represent some phenomenon/phenomena of the process that is not separable (i.e., neither the FP model impacts the results of the data-driven model, nor the data-driven model impacts the results of the FP model) and some or all of the inputs are used for both the FP and data-driven models, the parallel configuration is favored [26,27]. The deficiencies in the FP model may be directly expressed by a data-driven model as a function of model inputs [28]. For non-separable discrepancies where either the FP model outputs serve as additional inputs to the data-driven model(s) or the data-driven model outputs serve as the additional inputs to the FP model inputs, but not both, then a series configuration is preferred. Series (or serial) configurations of FP with data-driven models have shown improved extrapolation properties as compared to the parallel architecture [29]. For non-separable discrepancies, where the FP model outputs serve as the (additional) inputs to the data-driven model and vice versa, then an integrated configuration is preferred. In the integrated approach, the results from the FP model are affected due to expected improved estimates of parameters/ states, etc. from the data-driven model. It should be noted that the results from the FP model are not affected in the parallel configuration or in series configuration if the FP model precedes the data-driven model. The integrated approach has been often used in the existing literature for estimating time-varying parameters [11,112] by a data-driven model to be utilized in the FP model. If the parameters to be estimated by the data-driven model are time-varying, then the integrated approach can lead to challenging training and/or adaptation problems for dynamical systems. Obviously, various combinations of series, parallel, and integrated configurations may be directly depending on the discrepancies of the data-driven model and FP models and their parameters. Furthermore, in all such approaches discussed in the literature, where certain process parameters for the FP model are estimated by a data-driven model, the training data for the respective AI models are generated by a first-order error backpropagation-based approach. While typically such algorithms for regression/optimization are computationally faster, the resulting training dataset may lack sufficient richness due to local approximation and absence of accurate Hessian information [30]. Therefore, for online applications, the hybrid models thus developed may require frequent retraining to ensure satisfactory predictive capabilities for highly nonlinear transient process systems for prolonged continuous durations of operation.

The objective of this work is to develop hybrid modeling approaches for predicting transient behavior of high temperature superheater components in power plant boilers. As the integration of renewables in the current energy grid is becoming prevalent worldwide to achieve low-carbon emissions, operation of existing fossil-fired power plants need to be flexible [31]. The current power generation fleet which is mainly designed for base load operations is facing several operational challenges due to these load-changing operations [32,33]. As a result, failure rate of the boilers especially that of the high temperature superheater components have gone up [34,35]. Advanced data-driven models utilizing extensive operational datasets have been well developed and utilized in industrial plants for rotary equipment such as gas and steam turbines especially for monitoring of these equipment items. For static equipment such as boilers, limited tools are available commercially. For developing a monitoring framework for these components, reasonably accurate estimates of temperature profile with sufficient spatial resolution are needed since the metal temperature affects creep and fatigue

damages [36] in boiler tubes and subsequent failure. However, it is challenging to develop accurate FP models for the superheaters due to multiple reasons. First of all, it is difficult to develop accurate models of the heat transfer characteristics in the superheater tubes due to build up of oxide scale inside the tubes [3]. The scale thickness keeps growing with time, but the scale can spall [37,38], which is highly stochastic. Furthermore, there is high uncertainty in the scale thermal characteristics [39,40]. In addition, in coal-fired power plants, there is deposit of coal fly ash on the outside of the tubes [41]. While frequent soot blowing is often used to clean these ash deposits, there may be found ash that can be difficult to remove [42]. Thus, there is high uncertainty in the thickness and thermal characteristics of the ash deposit for coal-fired power plants. Second, modeling challenges for superheaters arise due to inhomogeneous spatial distribution of flue gas and steam flowrates, especially under load-following operation [43]. Due to the presence of thousands of tubes in this region and specific configuration of a given superheater, firing profile of the burners, etc., accurate dynamic modeling of flow inhomogeneity is challenging. Placing sensors in the superheater region is very challenging due to harsh operating conditions that not only make placement of sensors difficult [44], but life of the placed sensors are very limited especially in coal-fired boilers [45] where the superheater environment is highly corrosive due to the fly ash-laden flue gas. Thus, developing hybrid models for the superheaters that can be eventually used for developing a monitoring framework is of high interest to utility operators.

Recently, a few hybrid modeling approaches have been implemented for process modeling, controls, and digital-twin development for thermal systems such as boiler and steam turbine for its service power plants [26,46]. Other notable applications of hybrid models include tasks such as controlling fouling in biomass boilers [47] and performance enhancement of an ejector refrigeration system [48]. The hybrid models utilized in most of these works were a combination of lumped parameter physical/mechanistic models and data-driven linear empirical models utilizing operational data. In particular, hybrid grey-box models have found implementations in industrial furnace systems [49], heat recovery steam generator (HRSG) [50] and various components of steam power plants (boiler, turbine, condenser, feed water system, etc.) [51,52]. The proposed models in these works generally used a combination of reduced order parametric models representing the physical system and static feedforward AI-based models for estimating unknown process parameters in thermal systems. Furthermore, grey-box modeling approaches for feedwater heater [53] and steam boiler system [54] have been developed by using detailed nonlinear FP differential algebraic equations (DAE) model based on mass and energy balances. The unknown system parameters were estimated for both the systems using nonlinear least-squares optimization utilizing operational measurement data. Similar approaches have been investigated for an electric hot water boiler system [55] and a superheater/boiler system in a combined cycle plant (HRSG) unit [56], using detailed mechanistic dynamic FP model based on differential mass and energy balance equations along with dynamic estimation of unknown heat transfer parameters. Dynamic hybrid models combining non-linear physics based models with gated recurrent unit (GRU) [14] or long-short term memory (LSTM) [57] type recurrent neural network (RNN) model have also been investigated for modeling heat absorption and fouling prediction in boiler systems. Recently, a hybrid modeling framework using dynamic models have been proposed for digital twin development in a coal-fired power plant [58]. The proposed model structure consists of FP based discrete-time models for different sections of the plant such as pulverizer, furnace, drum, etc. along with LSTM type of RNN models for estimation of model parameters. Although many of such hybrid structure utilizing dynamic AI models show satisfactory performance while representing critical process variables, the availability of limited transient operating data may lead to significant overfitting during model training, thus resulting in inferior predictive capabilities. Hybrid modeling techniques using lumped parameter physical models in addition to LSTM type RNNs

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[59,60] and fuzzy logic based AI approaches [61] have also been used for superheater steam temperature control in industrial systems. More recently, a detailed dynamic distributed parameter model has been proposed for studying transient behavior of boiler systems under rapid load following operations [62]. The proposed models were validated using test data from an actual operating system showing its utility for simulating system behavior. Although most of the works referenced above have developed hybrid models of various components of thermal systems, most of these works have neither considered the spatial variability of process variables in the FP models nor the dynamics of time-varying parameters/variables that are represented by the AI models. Furthermore, a limited amount of operational dynamic data has been used for model validation in these works. An accurate prediction of the dynamic process variables and time-varying parameters for high temperature superheater systems is critical for developing a monitoring framework especially during load-following operation due to rapid changes in heat transfer performance and flow distribution during the transients.

The literature review on hybrid modeling of power plants identifies several gaps that we seek to fill in:

- Hybrid models of the superheater section of power plants using detailed distributed transient FP models along with dynamic data-driven models are currently not available.
- Modeling of time-varying parameters/variables by an AI model as part of the hybrid model for power plants has not currently been investigated. Moreover, the development of algorithms for generating training data during internal coupling between FP and AI models to address parameter uncertainties while ensuring sufficient data richness has not been explored.
- No hybrid model developed in existing literature provides estimates for the time-varying tube wall temperature profile in the superheater section of power plants.
- Main steam temperature is one of the most important variables for the power plant operation. A temperature that is higher than desired can cause significant damage to the steam turbine, while a temperature that is lower than desired can cause efficiency loss [39,43]. While the main steam temperature is a controlled variable and therefore measured, a predictive model can be helpful in developing a model-based efficient main steam temperature controller [39,63]. However, accurate prediction of the main steam temperature is difficult due to uncertain heat transfer characteristics, non-uniformity in flow direction, time-varying thermal hysteresis of the systems, etc. Hybrid models can be very useful for not only estimating the main steam temperature, but also other critical transient process variables such as metal wall temperatures of superheater tubes or flue gas temperatures, simultaneously. However, to the best of our knowledge, no work in existing literature discusses the development of a hybrid model for prediction of all these different types of transient temperature profiles (main steam, tube wall, flue gas) for pulverized coal (PC) or natural gas combined cycle plant (NGCC) power plants.
- Modeling spatio-temporal variability of temperature is critical for superheaters, where hybrid models can be particularly useful due to modeling and data challenges discussed before. We could not currently find any literature that has addressed this area.
- Model validation using experimental/industrial data has been generally done in the existing literature by using the steady-state data or dynamic data for a short operating range. It is desired that the models be validated in the expected range of power plant load and ramp rate as expected during the load-following operation.

This paper presents a systematic procedure for the selection of appropriate hybrid approaches for modeling of thermal systems in which dynamic industrial steam superheater systems for two different types of boiler systems were considered as case studies. The first system

considered is part of an HRSG in a NGCC plant, and the other system is part of a PC power plant. In the present work, the FP models of both the industrial steam superheater systems which have been developed in our previous work [3] as two-dimensional (2-D) distributed DAE models considering differential mass and energy balance equations along with a detailed accounting of heat transfer through the metal tube wall have been considered. For the ML models to be used for online applications, it is important to take into consideration contrasting advantages, and disadvantages of various types of AI-based modeling techniques. As potential candidate data-driven models in the hybrid FP + AI framework discussed in existing literature, different varieties of deep neural networks (DNNs) are typically considered, such as LSTM [59,64] or GRU [14] types of RNNs, as well as convolutional neural networks [65] (CNNs). However, such conventional DNN models require significant large numbers of parameters, thus leading to potential overfitting during model training especially in absence of large datasets [59,67]. Alternative data-driven approaches commonly used in hybrid FP + AI models involve derivative-free algorithms such as fuzzy logic based decision making [61], etc. However, such approaches may suffer from slow convergence in absence of sufficient exploration and exploitation of the solution space due to absence of gradient information during parameter estimation. Both of these AI-based models/approaches prove to be considerably challenging especially for the power plant superheaters for which large sized dynamic input/output data may not be available for model development. Our previous works have shown that sparse deterministic and probabilistic data-driven models significantly outperform several state-of-the-art approaches while modeling complex nonlinear chemical systems from noisy dynamic data typical of real plants. Drawing from those studies, in this work, the deterministic data-driven models are represented by hybrid AI-nonlinear series-parallel static-dynamic neural networks [6] (referred to as NN for the rest of the paper), while the probabilistic models are obtained from the BREGMA (Bayesian Identification of Dynamic Sparse Algebraic Models) approach [7] (referred to as BMD for the rest of the paper). These models/algorithm consider a significantly lesser number of parameters (for sparse system identification) as compared to typical DNNs and hence correspond to much faster computation if the hybrid model needs to be re-trained at frequent intervals during certain online applications. More details on the structures and training algorithms for these specific data-driven models can be found in our previous papers [6,7]. While the NN models typically ensure faster computation and accurate model fit for highly nonlinear transient data, the BMD models offer capabilities for handling noisy data and uncertainty quantification along with estimation of interpretable model parameters.

In summary, the major contributions of this paper are as follows:

- Three different structures for integrating FP (physics-based) mechanistic models with data-driven models have been investigated – namely series, parallel and integrated configurations. In addition, series-parallel and integrated-parallel configurations are also investigated. Motivations for these structures are based on the evaluation of model discrepancies and analysis of how the FP and data-driven model can complement each other for achieving the desired model prediction performance.
- Novel hybridization approaches for exchange of appropriate information between FP and AI models have been developed with improved predictive and extrapolative capabilities while modeling industrial steam superheater systems under load-following conditions. Rigorous optimization-based algorithms are developed for generation of training data for AI models during internal synergistic coupling of FP and AI models, thus enhancing data richness for improved predictive capabilities. Another key focus is on developing predictive models for spatio-temporal variation of the main steam outlet temperature. In addition, predictive capabilities of the hybrid models for other variables of interest such as tube wall and flue gas temperatures are also evaluated. Accurately predicting the nonlinear

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transient temperature profiles can be useful for developing a framework for monitoring damage accumulation in the systems under consideration.

- Two different types of sparse dynamic data-driven models with contrasting advantages and disadvantages are developed and analyzed in the hybrid FP data-driven modeling framework, namely the series-parallel all-nonlinear neural network models [6] and the BME models developed through BREGM algorithm [7]. These two approaches have been compared in terms of computational expense, predictive accuracy, model interpretability, sparsity, bandwidth, and extrapolative capabilities while modeling the superheater systems. Two principal statistical measures that are considered for quantifying the relative performance of the proposed models are the root mean squared error (RMSE) and the maximum absolute instantaneous error (MAIE) calculated between the actual plant measurements and the predictions from the hybrid model.
- The proposed hybrid models have been developed and validated using dynamic data from the superheater of two operating power plants: the first one being a part of an HRSG of a NOGC power plant and the other one being part of a PC-fired boiler unit. Prolonged durations of time-varying operational data under variable load conditions have been utilized for developing and validating the proposed hybrid modeling framework(s). As a measure of acceptability of the developed hybrid models, the maximum values of the RMSE and MAIE are set at 5 °C and 10 °C for both the NOGC and PC-fired power plants.

The rest of the paper is organized as follows. Section 2 includes brief discussions on the development of the individual components of the hybrid FP + AI models. The various model structures developed by synergistic coupling of FP and AI models are discussed in Section 3, whereas extensive descriptions of the two industrial superheater systems considered in this work have been included in Section 4. Section 5 presents a methodological workflow describing the interactions among different (FP and AI) models, while the results obtained from both the standalone FP as well as hybrid FP + AI models are presented in Section 6. Finally, the conclusions are presented in Section 7.

2. Hybrid model components

In this section, we provide brief discussions on the individual components of the hybrid models considered in this work. It may be noted that the various hybrid model components proposed in this work are generic and are flexible to be applied to model other complex nonlinear dynamic processes.

2.1. Dynamic FP model

The FP model implemented for modeling dynamic process systems can be typically derived from fundamental laws of physics, chemistry, thermodynamics, reaction kinetics and transport phenomena, such as mass, momentum, and energy balances. Such high-fidelity FP models can be developed in various mathematical forms. For example, as a system of algebraic equations or as a system of ordinary or partial differential equations (ODEs or PDEs), or a combination of both differential and algebraic equations leading to a generic nonlinear system of DAEs (DAEs) [78]. The physics-based mechanistic models developed for accurate process modeling have also traditionally shown significant efficiencies in predicting the thermo-physical properties of pure substances and their mixtures, especially for both reactive and non-reactive process systems.

The key advantages of using typical FP models over classical data-driven approaches for dynamic process systems are their predictive capability for process variables for which measurement data may not be available to develop data-driven model or data may be limited or may not be available for the desired spatial and temporal resolution.

However, first-principles based modeling of complex nonlinear transient systems may face considerable difficulties. These generally stem either from the complex interactions underlying the physical phenomena or the lack of knowledge on system-specific mechanisms/nonlinearities, intermediate, high dimensionality, time-delay, and process uncertainties [13]. However, the overall predictive performance of a standalone FP model for such complex systems can be improved by combining physics-based knowledge with several AI approaches, thus leading to the development of different variants of hybrid FP + AI models (such as the various structures and approaches discussed in Section 3 of this paper).

2.2. ANN models

Conventional static feedforward neural networks are typically associated with inaccurate representations of temporal systems having time-lagged input variables [68,69]. On the other hand, although typical deep RNNs such as LSTM, GRU, etc. have shown improved performance while modeling transient systems [70,71], the presence of large number of hidden layers, and hence significantly large numbers of parameters, can lead to potential overfitting during model training [66,67]. A third type of neural network structure commonly used for modeling dynamic systems considers a linear dynamic model integrated with a nonlinear static neural network [72,73]. Such models may suffer from potential overfitting or extensive computational expense during parameter estimation, especially while modeling systems with highly nonlinear process dynamics [74] and may lead to relatively high instantaneous errors. Our previous work [6] has shown that series and parallel all-nonlinear architectures combining a nonlinear static (NLS) NN with a nonlinear dynamic (NLD) NN significantly outperform several existing state-of-the-art models such as LSTM, GRU, Decoupled A-B Network (DABNet) [73] in terms of prediction accuracy and computational expense while modeling complex nonlinear dynamic systems.

For the hybrid all-nonlinear series/parallel static-dynamic neural networks considered in this work, the NLS model is represented by a three-layered feedforward neural network [75] whereas, the NLD model refers to a Nonlinear Autoregressive with exogenous inputs (NARX) type of RNN [76,77]. Obviously, any other type of static and/or dynamic neural network model can readily be considered as well in this framework. For the all-nonlinear series configuration, two different architectures are possible: the NLS followed by the NLD model (denoted by NLS-NLD type network) and the NLD followed by the NLS model (denoted by NLD-NLS type network). The all-nonlinear parallel configuration involves the NLS and NLD models arranged in parallel with one another, subject to the same set of model inputs. The block-oriented structure of the all-nonlinear series and parallel network models have been shown in Fig. 1. For simplicity, the optimal model among the hybrid series and parallel all-nonlinear static-dynamic neural networks for every different implementation case study under consideration has been referred to as 'N9' for the rest of the paper.

Training the hybrid all-nonlinear static-dynamic networks can be difficult and computationally expensive, especially while trying to simultaneously estimate the optimal parameters of both static and dynamic networks in a monolithic approach. Our previous work [6] discussed the development of sequential decomposition-based algorithms by exploiting the network structure for training the static-dynamic models which have shown to achieve convergence 50–100 times [6] faster than the monolithic (simultaneous) parameter estimation approaches. The codes for training and simulating the hybrid all-nonlinear static-dynamic neural network models are also publicly available at: <https://github.com/shahbazer/shahbazer-hybrid-Static-Dynamic-NN-Models>.

2.3. BME models

The black box nature of most data-driven models leads to the major

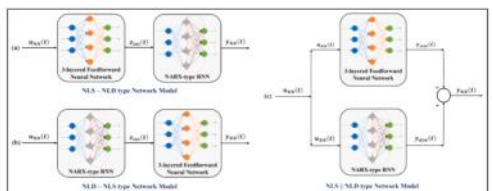


Fig. 1. Block-oriented structures for (a) NLS-NLD (series), (b) NLD-NLS (series) and (c) NLS||NLD (parallel) types of hybrid all-nonlinear static-dynamic models.

drawback of lack of model interpretability consequent upon the complex structures of the underlying models. While our previous work [6] has addressed the issue of sparsity in the use of conventional ANN/NN, several works make that attempt to address the challenge of model interpretability including sparse identification of nonlinear dynamics (SINDy) [79], Automatic Learning of Algebraic Models (ALAMOD) [80], and Algebraic Learning via elastic net (ALVEN) [80]. While these approaches give functional relationships between input and output variables, they have mainly been tested on synthetic data that are noise-free. In our previous work [7], we have shown, for some specific case studies, the ALAMOD and SINDy Algorithms are sensitive to the presence of noise in the training data. This significantly influences the model size and prediction accuracy for the algorithms. On the contrary, our developed BREGM algorithm [7] demonstrates significant robustness to noise as model size, model form, and prediction accuracy remain consistent whether model is developed using 'true' or noisy data. The BREGM algorithm achieves model interpretability (i.e., an understanding of the causation and effect of the input variables on the output variable) by approximating system nonlinearities by a combination of a set of well-defined basis functions. These basis functions are obtained from linear and nonlinear transformations of the input variables and their interactive effects with the measurements. The selection of the optimal subset of the resulting large family of basis functions is done by using a branch and bound algorithm. Candidate models are compared using an information criterion that rewards model fitness, penalizes model size and complexity, while also interpreting a measure of parameter tunability (i.e., the ability to identify unique values for the model parameter from the available data) into the process. Robustness of this algorithm to noise is enhanced by explicitly accounting for noise and their possible correlation during parameter estimation which employs the two-step procedure of an expectation maximization (EM) algorithm. The efficiency of this algorithm makes it suitable for model development by using the real plant data such as from the industrial superheater systems that are typically corrupted with noise of unknown characteristics. The codes for BREGM algorithm for developing sparse static-state and dynamic data-driven models are publicly available at: <https://github.com/shahbazer/BREGM-Code>.

3. Configurations of hybrid FP + AI models

This section discusses such hybrid FP + AI model architecture proposed in this paper and the motivation behind transfer of appropriate information to and from such FP and AI models while constructing

accurate hybrid models for various dynamic process systems. It is to be noted that a particular hybrid model structure is neither superior nor exclusive to the corresponding implementation discussed in this paper. Obviously, one or more structures proposed in this work can be applied to a particular objective subject to availability of necessary data. Although theoretically all structures are expected to be accurate for unsaturated operations, this paper presents a systematic approach of selecting the optimal hybrid FP + AI model for representing key variables of interest.

3.1. Series hybrid FP + AI model

A possible configuration of the series hybrid FP + AI model is shown in Fig. 2. Obviously, the AI model can generate the FP model, or the FP model can be sandwiched between two AI models or often may stand and there can be many such nests of FP and AI models that can be considered. Additional synthetic inputs to the hybrid model may be the same (in (c)) and/or different (in (a)) as compared to the inputs to the FP model (see Fig. 3). As such, (a) refers to the model parameters for the FP and AI models, respectively, whereas $x_{\text{model}}(t)$ represent the final outputs from the overall hybrid model. It should be noted that for this specific structure of the hybrid model, the model structure, i.e., the arrangement and interconnections between the FP and AI models, remain consistent during both training and validation (simulation) of the hybrid FP + AI models. This may not be the case for the other structures proposed in this work. The dynamic target outputs for the AI models are considered as the residuals $\Delta y(t)$ calculated between the measured values $y_{\text{meas}}(t)$ and the FP model predicted values $y_{\text{FP}}(t)$ of the FP output variable, as defined by Eq. (3), for the final superheater systems under consideration.

$$\Delta y(t) = y_{\text{meas}}(t) - y_{\text{FP}}(t) \quad (3)$$

3.2. Integrated hybrid FP + AI model

The flow of information between the FP and AI models in the hybrid framework does not need to be one directional. For some applications, it may become necessary that information gets exchanged iteratively between the two models, as shown in Fig. 3. Such type of integrated cross-coupling configurations may be necessary to handle uncertainties in process parameters manifested in the FP model. However, unlike the series hybrid FP + AI model discussed in Section 3.1, the integrated hybrid FP + AI model structure may slightly change during training

3. Hybrid FP + AI model architectures

이 장에서는 본 논문에서 제안한 **직렬하이브리드 FP(AI) 및 병렬 하이브리드 FP + AI (통합형) 모델 구조**와, 다양한 동적 공정을 시스템(dynamic process systems)에 대해 **정확한 하이브리드 모델을 구축하는 FP 모델과 AI 모델 간의 최적화된 정보 흐름 구조**를 연구하기 위해 소개하고 논의한다.

특정 하이브리드 모델 구조는 본 논문에서 논의한 구현에만 최적화되거나 특별한 것이 아닐을 주목할 필요가 있다. 본 연구에서 제안한 하이브리드 구조는 필요한 데이터가 확보된다면 특정 목적에 적용할 수 있다.

이론적으로는 모든 구조가 예기치 않은 운전 조건에서도 정확한 것으로 예상되지만, 본 논문에서는 주요 관련 변수를 나타내며, 실제 최적의 하이브리드 모델 (first-principles model) + **연속적(AI) 모델**을 선택하는 체계적인 접근 방식을 제시한다.

3.1. 직렬 하이브리드 FP + AI 모델 (Series hybrid FP + AI model)

직렬 하이브리드 FP + AI 모델의 가능한 구성은 Fig. 2에 예시되어 있다. AI 모델은 FP 모델의 출력에 사용할 수도 있고, FP 모델이 두 AI 모델 사이에 위치할 수도 있으며, 그 반대의 경우도 가능하다.

이와 같이 여러 개의 FP와 AI 모델이 직렬로 연결된 다양한 구조가 고려될 수 있다.

하이브리드 모델에 대한 추가 합성 입력(synthetic inputs)은 동일한 입력 (in (c)) 또는 다른 (in (a))에 비해 FP 모델에 대한 입력과 다른 umodel 할 수 있다 (Fig. 3 참조).

모형 파라미터 θ-FP와 θ-AI는 각각 FP 모델과 AI 모델의 파라미터를 의미하며, y-FP와 y-AI는 각각 하이브리드 모델의 최종 출력을 나타낸다.

이 하이브리드 모델 구조에서는 훈련(training)과 검증(validation), 시뮬레이션 시 단계 공정에서 FP 모델과 AI 모델의 배치 및 상호작용 방식이 동일하게 유지된다는 점에 유의해야 한다.

이러한 본 논문에서 제안한 구조 중 최적의 구성을 선택할 수 있다.

AI 모델의 동적 목표 출력(dynamic target outputs)은 측정값 y_{meas}(t)과 FP 모델 예측값 y_{FP}(t) 간의 차이인 잔차 Δy(t)로 간주된다.

이 관계는 최종 출력(superheater) 시스템에 대해 다음과 같이 정의된다.

3.2. 통합형 하이브리드 FP + AI 모델 (Integrated hybrid FP + AI model)

하이브리드 프레임워크에서 FP 모델과 AI 모델 간의 정보 흐름은 반드시 단방향일 필요는 없다.

일부 응용에서는 Fig. 3에 나타내며, 배치, 주, 주말 사이에서 내부 역으로 변화가 고려되어야 할 필요가 있을 수 있다.

이러한 형태의 통합 교차 연결 구조(integrated cross-coupling configurations)는 FP 모델에서 고려한 운영 파라미터의 불확실성을 잘 처리하는 데 도움이 된다.

교차 연결은 훈련 단계에서 발생하는 하이브리드 FP + AI 모델과 같지, 통합 하이브리드 FP + AI 모델의 구조는 훈련(training)과 시뮬레이션(simulation) 단계에서 약간 달라질 수 있다.

이와 Fig. 3(a)의 같은 형태로도 표현 가능하다. FP 모델에서 AI 모델로 인한 정보 흐름은 역방향에 따라 결정된다.

특정 하이브리드 모델 구조가 본 논문에서 제시된 구현 사례에만 국한되거나 독립적인 것은 아니라는 점에 주목할 필요가 있다.

즉, 이 논문에서 제안된 여러 구조들 중 하나 이상은, 필요한 데이터만 갖춰진다면 다양한 목적에 맞게 적용할 수 있다.

이론적으로는 어떤 구조든 예기치 않은 운전 상황에서도 정확한 결과를 낼 것으로 기대되지만,

본 논문에서는 관심 있는 주요 변수들을 잘 표현할 수 있도록 **최적의 하이브리드 모델** — 즉, 물리 기반 모델(FP, first-principles model)과 인공지능 모델(AI)을 결합한 모델 — 을 **선택하는 체계적인 방법**을 제시한다.

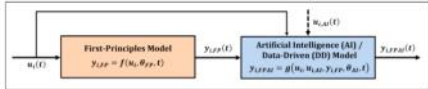


Fig. 3. Block-oriented configuration of series hybrid FP + AI model during training and simulation.

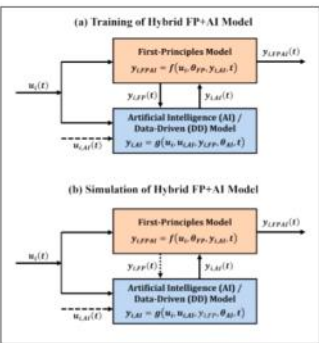


Fig. 4. Block-oriented configuration of integrated hybrid FP + AI model during (a) training and (b) simulation.

series simulation depending on what information gets exchanged from the FP to the AI models (represented by the dotted arrow in Fig. 3). While a solid arrow in Fig. 3(a) signifies that the transfer of information from the FP to AI models is mandatory during development (training) of optimal hybrid models, such transfer may not always be necessary during model simulation, as represented by the dotted arrow in Fig. 3(b). For the case when the AI model utilizes the instantaneous outputs of the simulation FP model as additional (synthetic) inputs during model training followed by subsequent transfer of information back to the FP model, the algorithm follows the same sequence even during simulation, i.e., when the optimally trained model is considered for future prediction subject to unknown inputs. In such examples, at every time step, the original FP model is solved first, followed by transferring partial or total information about the FP model output to the AI models. Thereafter, the data-driven models utilize all information obtained from the FP model, along with additional inputs, if necessary, before sending back corrected estimates of certain parameters with which the FP model is standardized for a second time (within the same iteration/time index) leading to the final output (y_{FP+AI}) from the overall hybrid model.

The integrated arrangement of FP and AI models may also have certain, specific applications, where the AI models do not require instantaneous estimates of any variable from the FP model as additional inputs during their training. In such examples, even though partial information from the FP model may get transferred to the AI model during training, such information is only utilized for data preparation for the AI models, i.e., just as additional inputs. Therefore, although the sequential integrated framework builds true during training development of the hybrid model, the model architecture transforms to a series configuration during model simulation. At every time step during simulation of the optimal hybrid model, the AI models are first simu-

lated, and their outputs are then used as additional inputs to the FP model, along with additional inputs, if necessary, before sending back corrected estimates of certain parameters with which the FP model is standardized for a second time (within the same iteration/time index) leading to the final output (y_{FP+AI}) from the overall hybrid model.

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lated followed by the FP model which can directly utilize the corrected parametric estimates transferred from the simulation of the optimal AI model. The second approach (i.e., considering y_{FP} only for data preparation for AI models instead of additional inputs) also reduces the number of function evaluations during forward simulation of the integrated hybrid FP + AI models when exposed to unknown inputs, since it involves the FP model to be solved only once, thus leading to relatively faster computations as compared to the first approach (i.e., considering y_{FP} as additional inputs for the AI models) where the FP model has to be solved twice during each iteration/time step.

3.2. Parallel hybrid FP + AI model

The parallel arrangement of FP and AI models became necessary for various applications, where even though a mechanistic (FP) model is available, its predictive capabilities are limited due to inaccurate descriptions of certain effects, nonlinearities, or dynamic behavior. The fundamental difference of this type of configuration with the previous two structures described in Sections 3.1 and 3.2 is that the parallel hybrid FP + AI model provides the flexibility of optimal synthesis of both FP and AI models independently without requiring sequential transfer of any information from one to another or between each other, as shown in Fig. 4. However, partial or total information may be transferred from the FP to AI models during training the parallel hybrid FP + AI model (indicated by the dotted arrow in Fig. 4(a)), but such information will only be used in preparing input/output data for the data-driven models. The outputs of the data-driven models, for this configuration, act as additive corrections or augmentations of those obtained from the FP model, thus leading to the generation of the overall outputs from the parallel hybrid FP + AI model represented by $y_{FP+AI}(t)$. It is to be noted that depending on the specific application of the hybrid model, the parallel structure can accommodate various model

forms substituting the FP and/or AI models. For example, the series hybrid FP + AI model (as described in Section 3.1) can readily replace the FP model in the parallel framework, thus leading to a series-parallel hybrid FP + AI model. Similarly, if the FP block in the parallel structure gets substituted by the integrated hybrid FP + AI model (as discussed in Section 3.2), it may lead to the 'integrated parallel hybrid FP + AI model' architecture. The corresponding architectures of series-parallel and integrated-parallel hybrid models during simulation have been shown in Fig. 5, where y_{FP} , y_{FP+AI} , and y_{FP+AI} refer to the outputs from the series, integrated, and parallel hybrid FP + AI models, respectively. The training algorithms also follow similar structural representations.

4. Applications of hybrid models to industrial steam superheater systems

This section briefly describes the geometrical configurations and the detailed dynamic first principles model development for two industrial steam superheater systems considered in this work for the validation of the hybrid modeling framework. The first system, referred to as 'Plant A' for the remainder of this paper, is a final superheater system that is part of an EGBC unit in an NGCC plant. The second system, referred to as 'Plant B' for the remainder of this paper, is another final superheater system which is part of a PC-fired boiler unit. Two different types of superheater systems have been selected in this study to demonstrate the applicability of the proposed hybrid models to model any generic system using different types of fuels for combustion. Plant A superheater system uses natural gas fuel while Plant B utilizes solid pulverized coal for combustion. The use of a generic or solid fuel for combustion leads to a direct impact on the performance and efficiency of the system. The operating temperature of gas-fired systems are usually lower with steady operating conditions because of less impurities in natural gas. On the

other hand, plant B (solid fuel) FP and AI models are used as additional inputs to the FP model, along with additional inputs, if necessary, before sending back corrected estimates of certain parameters with which the FP model is standardized for a second time (within the same iteration/time index) leading to the final output (y_{FP+AI}) from the overall hybrid model.

The integrated arrangement of FP and AI models may also have certain, specific applications, where the AI models do not require instantaneous estimates of any variable from the FP model as additional inputs during their training. In such examples, even though partial information from the FP model may get transferred to the AI model during training, such information is only utilized for data preparation for the AI models, i.e., just as additional inputs. Therefore, although the sequential integrated framework builds true during training development of the hybrid model, the model architecture transforms to a series configuration during model simulation. At every time step during simulation of the optimal hybrid model, the AI models are first simu-

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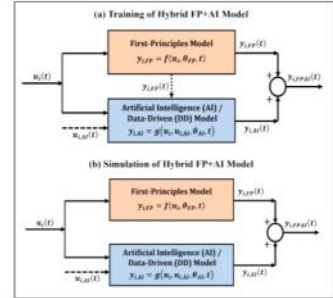


Fig. 4. Block-oriented configuration of parallel hybrid FP + AI model during (a) training and (b) simulation.

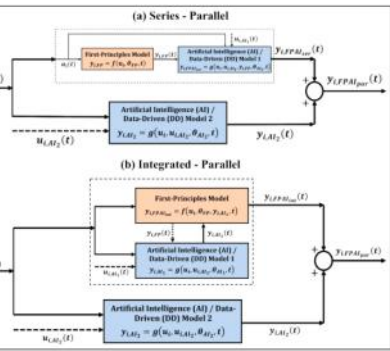


Fig. 5. Block-oriented comparison of (a) series-parallel and (b) integrated-parallel hybrid FP + AI models during simulation.

other hand, the solid coal-fired system experiences high operating temperature with more dynamic variability in operating conditions because of large amount of reparation and uncertainties associated with a solid combustion fuel. Hence, to study the performance of hybrid

models over a variety of different operating conditions and system characteristics, two different types of superheater systems have been used in this work. Operational data is available for both the units from the industrial sector plants which have been utilized for the

purpose of this study. The first unit is a solid coal-fired system, while the second unit is a gas-fired system. The first unit is a solid coal-fired system, while the second unit is a gas-fired system. The first unit is a solid coal-fired system, while the second unit is a gas-fired system.

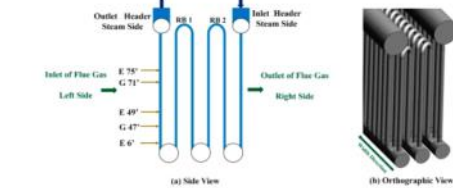


Fig. 6. Final superheater system (Plant A) layout. (a) side view (inset locations on the left side indicate elevation in ft) and (b) Orthographic view (three-dimensional) schematic of tube assembly.

development and validation of the proposed hybrid FP + AI models.

calculate the mean for the inlet flue gas temperature which is then used in the hybrid model as input data.

4.1. Plant A final superheater system (HRSG-NGCC Unit)

The power plant (Plant A) system is a final superheater system which is part of an HRSG unit in an NGCC plant having a rated capacity of 600 MW. The layout of the selected system is shown in Fig. 6. The main steam produced from the superheater system is transported to an outlet steam header connected to the main steam pipeline which connects further to the high-pressure (HP) steam turbine. The main steam outlet conditions of the superheater are 538 °C and 10 MPa under normal conditions.

In Fig. 6(a), the side view of the system is represented schematically. The system consisting of a six-pass tube assembly configuration where the main steam enters through the inlet header on the upper right side of the diagram and exits through the outlet header dependent on the upper left side of the diagram. Furthermore, the actual system contains a total of 74 flow circuits along the width direction as shown in Fig. 6(b) with an orthographic view of the superheater system. Each of these flow circuits is formed by the combination of six six-pass tube assembly shown in Fig. 6(a), arranged behind one another in a staggered arrangement. An extensive amount of temperature measurement data was available for this system from thermocouples placed at the locations marked in Fig. 6(a). In total out of the 74 flow circuits, three thermocouples were placed on 10 flow circuits making the system well instrumented as compared to typical power plant superheater systems. The thermocouples placed on these 10 flow circuits at various locations (refer Fig. 6(a)) are categorized into two types. The first one are thermocouples for measuring outer diameter tube wall temperatures. These thermocouples are denoted by T1, T2, T3, T4, T5, T6, T7, T8, T9, T10, T11, T12, T13, T14, T15, T16, T17, T18, T19, T20, T21, T22, T23, T24, T25, T26, T27, T28, T29, T30, T31, T32, T33, T34, T35, T36, T37, T38, T39, T40, T41, T42, T43, T44, T45, T46, T47, T48, T49, T50, T51, T52, T53, T54, T55, T56, T57, T58, T59, T60, T61, T62, T63, T64, T65, T66, T67, T68, T69, T70, T71, T72, T73, T74, T75, T76, T77, T78, T79, T80, T81, T82, T83, T84, T85, T86, T87, T88, T89, T90, T91, T92, T93, T94, T95, T96, T97, T98, T99, T100, T101, T102, T103, T104, T105, T106, T107, T108, T109, T110, T111, T112, T113, T114, T115, T116, T117, T118, T119, T120, T121, T122, T123, T124, T125, T126, T127, T128, T129, T130, T131, T132, T133, T134, T135, T136, T137, T138, T139, T140, T141, T142, T143, T144, T145, T146, T147, T148, T149, T150, T151, T152, T153, T154, T155, T156, T157, T158, T159, T160, T161, T162, T163, T164, T165, T166, T167, T168, T169, T170, T171, T172, T173, T174, T175, T176, T177, T178, T179, 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present more challenges as compared to that obtained from NOCC plants in terms of significantly higher systematic and random noise and considerable spatial distribution.

4.3. Dynamic First-Principles (FP) superheater model

The dynamic first-principles superheater model used in the hybrid modeling framework is developed in our previous work [35]. The model used differential mass and energy balance equations for the flow streams along with consideration of metal tube wall temperature dynamics based on thin-walled assumptions. As illustrated in Fig. 8, the model consists of a control volume block approach. This provides a modular unit which can be combined in any form based on any industrial system geometry and configuration [35]. The approach can be easily extended to multiple superheaters, reheaters arranged in series/parallel.

In Fig. 8, a cross-flow arrangement is shown for the system with flow direction y as green arrow flowing in the horizontal y direction whereas steam flow is depicted by a blue arrow in the vertical z direction. However, this considered flow pattern is generic and may be modified as per the system specifics. Also, the model is developed with the assumption that property changes for both the flow streams occur in their respective flow directions only. Hence, no change in the thermophysical properties is considered in the width direction as shown in Fig. 8 for the two streams in the developed model. The differential equations are solved for each stream only in their respective flow direction neglecting the other directions. The dominant least transfer mechanism considered for the model is convection only without any radiative effects.

The modeling equations for both the steam and flue gas are as follows:

Mass Balance Equation:

$$\frac{d\dot{m}_i}{dt} + \frac{d(\dot{m}_i \rho_i)}{dt} = 0 \quad (2)$$

$$\frac{d\dot{m}_i}{dt} + \frac{d(\dot{m}_i \rho_i)}{dt} = 0 \quad (3)$$

Energy Balance Equations:

$$\frac{dT_{s,i}}{dt} + \frac{dT_{s,i}}{dt} = - \frac{Q_{s,i}}{h_{s,i} \rho_{s,i} V_{s,i}} \quad (4)$$

$$\frac{dT_{f,i}}{dt} + \frac{dT_{f,i}}{dt} = - \frac{Q_{f,i}}{h_{f,i} \rho_{f,i} V_{f,i}} \quad (5)$$

$$\rho_{s,i} C_{p,s,i} \frac{dT_{s,i}}{dt} = h_{s,i} \rho_{s,i} (T_{s,i} - T_{s,i}) - h_{s,i} \rho_{s,i} (T_{s,i} - T_{s,i}) \quad (6)$$

Furthermore, the least transfer coefficients and other associated parameters considered for the model are calculated using rigorous property correlations for both the steam and flue gas side. The steam side heat transfer coefficient is calculated using Gnielinski correlation for smooth tubes. The flue gas side heat transfer coefficient is calculated using Gnielinski correlation for flue annus tube bundles. Non-uniformity in the inlet to the superheater on the flue gas side is taken into consideration by accounting the void fraction and tube arrangement factor based on inline and staggered tube arrangement. The complete set of modeling equations for the same can be found in our previous publication [35].

The entire superheater system for both Plant A and Plant B shown in Figs. 4 and 7 are divided into individual control volumes as shown in Fig. 8 and the modeling equations from Eqs. (2) to (6) are solved for each of these control volumes. Since the modeling equations were PDEs, the method of lines is used to convert these PDEs into ODEs. The reformulated ODEs are coupled with the algebraic equations making the entire system a DAE model. Fig. 9 shows the layout of both the superheater systems in control volumes and these entire systems have been solved in MATLAB using the DAE solver 'ode32s' [36].

5. Methodology for hybrid modeling of steam superheater system

This section describes the various hybrid approaches considered in this work by hybridizing the FP and AI models. For both superheater systems

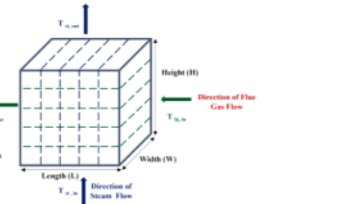


Fig. 8. Control volume block of the dynamic first-principles superheater model.

[35]

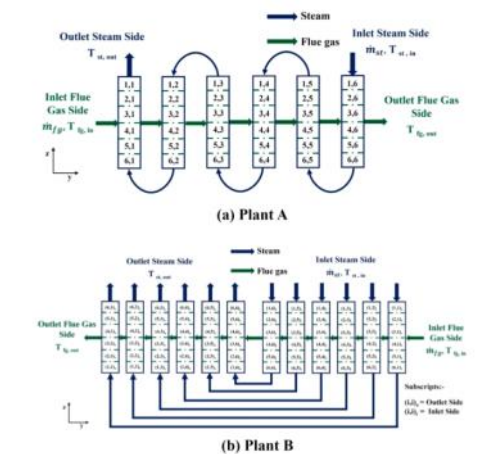


Fig. 9. Fluid superheater system layout using control volumes (inset index of individual control volumes represents actual pass number of the superheater system) (a) Plant A and (b) Plant B.

(i.e., NOCC and coal-fired power plants) discussed in Section 4, the key process variables include the transient outlet temperature profile of steam and flue gas, as well as the entire tube metal wall temperature of superheater tubes. It should be noted that the steam from the superheater often directly goes to the steam turbine and therefore that temperature directly affects the power generation that predictive modeling of the main steam temperature and its spatio-temporal variation are important for ensuring highly efficient operation of the power plant especially under load-following operation. Spatio-temporal distribution of tube wall temperature affects the thermal stress in the boiler and therefore tube life. Thus, a predictive model of these temperatures can be useful for eventually developing a monitoring tool. But flue gas temperature from the superheater and its spatio-temporal distribution are valuable for understanding least transfer efficiency affected by plant operation, local variation in least transfer resistance due to possible fouling, non-uniform distribution of the flue gas flow especially under load-

[36]

$$\partial T_{f,i} / \partial t + v_{f,i} \partial T_{f,i} / \partial x = \alpha_{wall,i} / (\rho_{f,i} C_{p,i}) \quad (4)$$

$$\partial T_{f,i} / \partial t + v_{f,i} \partial T_{f,i} / \partial y = \alpha_{wall,i} / (\rho_{f,i} C_{p,i}) \quad (5)$$

4.3. Hybrid First-Principles (FP, 물리 기반) superheater (열교환기) 모델

하이브리드 모델링 프레임워크에 사용된 동적 first-principles superheater 모델은 선행 연구 [35]에서 개발되었다. 이 모델은 유체스 트림(flue stream)에 대한 질량 및 에너지 수치 미방정식을 사용하 며, 얇은 벽 가정(thin-walled assumption)을 기반으로 한 금속 튜브 벽의 온도분포를 계산한다.

그림 8에 나타낸 바와 같이, 이 모델은 제어 체적 블록(control volume block) 접근법을 사용한다. 이러한 방식은 유체스 트림 시스템의 형 상 및 구성에 따라 자유롭게 결합할 수 있는 모듈형 유닛을 제공 한다[35]. 이 접근법은 열교환기 또는 열교환 배열과 다수의 superheater/reheater(열교환기/재열기)로도 손쉽게 확장 가능하다.

그림 8에서는 횡단(cross-flow) 배열의 시스템에 대해 나타내었으 며, 연소가스(flue gas)는 녹색 화살표로 수평 y 방향으로 흐르며, 증기(steam)는 파란 화살표로 수직 z 방향으로 흐른다. 그러나 이 경우에서는 일반적인 횡단형과, 시스템의 특성 구성에 따라 수정할 수 있다. 또한, 이 모델은 두 유체스 트림 각각의 흐름 방향에 대해서만 풀 (solved)된 변화만을 고려하도록 개발되었다. 즉, 그림 8에서 나타내 어진 것과 같이, 방향에서는 열역학적 물성의 변화가 고려되지 않는다.

전반명에서는 각각의 유체스 트림에 대해 그림 8의 흐름 방향에서만 풀 리며, 다른 방향은 무시된다. 이 모델에서 고려되는 주요 열전달 메커니즘은 대류(convection)인 흐름 에, 복사 효과(radiative effects)는 고려되지 않는다.

증기 및 연소가스(flue gas)에 대한 모델링 방정식은 다음과 같다:

$$\rho_i \frac{d\dot{m}_i}{dt} + \frac{d(\dot{m}_i \rho_i)}{dt} = 0 \quad (2)$$

$$\rho_i \frac{d\dot{m}_i}{dt} + \frac{d(\dot{m}_i \rho_i)}{dt} = 0 \quad (3)$$

에너지 수치 방정식(Energy Balance Equation):

5.1. Methodology for hybrid modeling of steam superheater system

이 절에서는 FP(First-Principles, 물리 기반) 모델과 AI 모델을 하이브리 드화하여 고체한 다양한 하이브리드 구조에 대해 설명한다. 4절에서 다룬 두 superheater 시스템(대수 방정식) 모델에 따른, 전체 시스템은 MATLAB의 DAE 해法器 'ode15s' [36]를 사용 하여 해석되었다.

그림 9는 제어 체적 단위로 나눈 두 superheater 시스템의 구성을 보여 주며, 전체 시스템은 MATLAB의 DAE 해法器 'ode15s' [36]를 사용 하여 해석되었다.

증기 및 연소가스의 출구 온도 변화 프로파일 (transient outlet temperature profiles)

superheater 튜브의 외부 금속 벽 온도 (tube metal wall temperatures)

그러나 superheater 내 주요 성능 변수들의 과도 응답(transient) 모 델링에 대해 최적의 하이브리드 FP+AI 구조(적합, 통합 또는 병렬)를 결정하는 것은 불가능할 수 있다. 본 연구에서 수행된 전체 방 법론적 워크플로우는 그림 10에 제시되어 있다.

이후에서는 특정 변수에 대한 최종 하이브리드 구조/모델 선택에 사용 된 구체적인 방법을 설명한다. 4절에서 논의한 바와 같이, 4.2절의 하이브리드 FP+AI 모델 구조 중 병렬(parallel) 하이브리드 구조의 수 요성을 중시하는 FP 모델을 각각의 모듈을 각각의 모듈로 개별 및 시 열 재조합할 수 있다는 유연성이다. 이를 위해 주어진 데이터 키에 대 해 적절(isort) 및 통합(integrated) 구조에 비해 상대적으로 더 빠른 연산이 가능하다. 따라서 본 연구에서는 모든 데이터에 대해 병렬 하이브리드 FP+AI 모델 구조를 기본(baseline) 모델로 고려하고, 그 성 능에 만족하지 않을 경우에만 다른 구조를 사용하도록 결정한다.

Plant A의 superheater 시스템에 대해 증기, 연소가스, 그리고 튜브 벽의 과도 온도 프로파일(transient temperature profiles)을 모델링 하는 과정에서, 병렬 하이브리드 모델은 MAE(mean absolute instantaneous error) 값이 기존 상한인 10⁻³ C를 초과하는 등 현저히 높은 수준으로 나타남을 확인하였다.

superheater에서 나온 증기는 종종 비로 증기 터빈으로 전달되며, 이러 서 이 증기는 발전 플랜트에 직접적인 영향을 미친다. 이에 따라 증기 기 온도(main steam temperature) 및 그 시공간적(ispato-temporal) 변 화에 대한 예측 모델링은 특히 부하(following) 운전 하에서 발전소의 효율을 손실을 방지하기 위해 매우 중요하다. 튜브 벽 온도의 시공간 분포는 터빈의 내 열응력(thermal stress)에 영향을 미치며, 결 격적으로 튜브의 수명에도 영향을 미친다. 따라서 이러한 증기 온도 또한 예측된 변수 중 하나인 증기 온도에 유용하게 활용될 수 있다.

superheater 출구 연소가스 온도(outlet flue gas temperature) 및 그 시공간 분포 또한 열전달 효율을 이해하는 데 있어 중요한 정보를 제공 한다. 이는 열교환기의 운전, 조절(following)으로 인한 즉각적인 열전 달 시정 변화, 특히 추운 증기 온도 시 연소가스 튜브를 비 균일 분포 등 다양한 요인에 의해 영향을 받는다.

Table 1
List of model input and output variables in the series hybrid FP + AI models for the regenerative system of Plant A.

Variable used in FP	Actual input/output variables for the series hybrid FP + AI model in terms of process variables in Plant A
$T_{in}(t)$	$T_{in,air} = T_{in,air,0} + T_{in,air,1} \cdot T_{in,air,2} \cdot T_{in,air,3}$
$T_{out}(t)$	$T_{out,air} = T_{out,air,0} + T_{out,air,1} \cdot T_{out,air,2} \cdot T_{out,air,3}$
$T_{out}(t)$	$T_{out,water} = T_{out,water,0} + T_{out,water,1} \cdot T_{out,water,2} \cdot T_{out,water,3}$

plant historian and the series "FP Model Results" represents the results obtained by simulating the standalone dynamic FP model (refer Section 3.1) for Plant A in absence of any data-driven augmentation. As shown in Fig. 11, the performance of the FP model has been validated for predicting the main steam outlet temperature ($T_{out,ms}$ refer Fig. 6) by comparing the measured data (purple dots) with the FP model results (red line). It can clearly be observed from Fig. 11 that although the FP model trends well with the measured data in the steady-state zones used above, the FP model predictions show some deviations with respect to the measurements during dynamic operations, especially during 6–10 h and 14–18 h. Overall, the standalone FP model yields an RMSE of 1.0 °C and a MAE of 3.6 °C for the entire duration.

The application of the series hybrid FP + AI model structure (refer Fig. 2) to Plant A provides updated results for the main steam outlet temperature ($T_{out,ms}$ refer Fig. 5). The input/output variables as shown in Fig. 1 corresponding to the model process variables for the regenerative system in Plant A have been listed in Table 1, where $T_{in,air}(t)$ signifies the corrected output variables obtained after passing the outputs from the FP model, i.e., $y_{FP}(t)$ through the data-driven model. The series hybrid FP + AI model follows a sequential approach where information flows in a single direction, i.e., from the FP to the AI model.

In Table 1, out of the eight output variables obtained for Plant A considering the specific implementation of the series hybrid model, while all may be useful to know for proper unit operation, the accurate prediction of the main steam outlet temperature is critical since it not only impacts plant operation but also directly impacts turbine and tube life [16]. The validation results of the series hybrid models for the main steam outlet temperature are shown in Fig. 12. As shown in Fig. 11, the standalone FP model has some deviations while predicting the main

Table 2
Comparison among FP, FP + NN, and FP + BML models in terms of RMSE, MAE, number of parameters and computational time for training the main steam outlet temperature for Plant A.

Type of Model	Number of Model Parameters	Training Computational Time for respective AI models	Mean Stream Outlet Temperature (°C)
FP	–	–	RMSE MAE
FP + NN	162	2.5 min	0.9 1.8
FP + BML	136	25.7 s	0.8 0.9

steam outlet temperature with respect to the measured data, resulting in an approximate RMSE of 1.0 °C. The black dotted vertical lines in Fig. 12 signify the total duration of dynamic time-series data (3–25 h) used for training both AI (neural nets, NN) and BML models. It can be observed that the hybrid FP + NN and FP + BML models show improvement in predictive capabilities over the standalone FP model not only for the dynamic data used for training the respective AI models, but also for the validation data. It is to be mentioned here that the all-nonlinear parallel (NLS || NLS2) multi-dynamic NN model (as shown in Fig. 10c) has been chosen to represent the NN model in the hybrid FP + NN structure, due to its relative superiority among the other two candidate architectures. The RMSE values in the prediction of the main steam outlet temperature, for instance, have been reduced from 1.0 °C, with respect to the standalone FP model, to 0.9 °C and 0.8 °C, for the hybrid FP + NN and FP + BML models, respectively. The comparison among the standalone FP and both types of hybrid models in terms of RMSE and MAE, and the overall computational time for training the hybrid FP + AI models, have been shown in Table 2.

Fig. 13 shows a second series (15–20 h) of the validation data shown in Fig. 12 for comparing the performances of the two AI models for the prediction of the main steam outlet temperature. It can be observed from Fig. 13 (b) that the FP + BML model shows a relatively larger mismatch with the measurements, and leads to much higher instantaneous errors as compared to the FP + NN model as shown in Fig. 13 (a) and Table 2. One of the primary reasons for such mismatch observed for the hybrid FP + BML model probably stems from the type

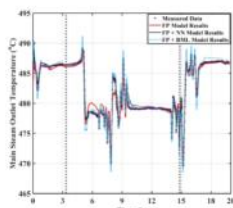


Fig. 12. Comparison of results among standalone FP, FP + NN and FP + BML series models with the measured data for the main steam outlet temperature for Plant A.

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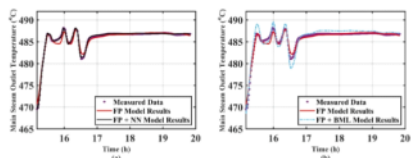


Fig. 13. Comparison of results among standalone FP and hybrid (a) FP + NN and (b) FP + BML models with the measured data for the main steam outlet temperature of Plant A.

of candidate basis functions considered during optimal model synthesis, in contrast to the activation functions used in the optimal NN model. While all standard NN activation functions are strictly bounded, the BML (i.e., REGENet) algorithm also considers unbounded candidate basis functions such as quadratic and bilinear combinations of input/output variables, thus leading to higher instantaneous errors while modeling highly complex dynamic data. Furthermore, the consideration of two different types of AI models in the hybrid framework also shows the tradeoff between prediction accuracy and computational expense involved during optimal model development (manuscript). While the consideration of BML approach in the hybrid series FP + AI structure yields a relatively lesser number of model parameters to be estimated during training, on the contrary, the FP + NN type of hybrid model is associated with a significantly lesser (about) training computational time and superior predictive capabilities. The complete list of all optimal candidate basis functions estimated for this case study by the BML algorithm, as well as their applicability to model process variables, is provided in Tables A1 and A2 in Appendix A. From the list of selected candidate basis functions obtained from the optimal inseparable BML models as presented in these tables, it is evident that the consideration of $y_{FP}(t)$ as additional synthetic inputs for the AI models in the series architecture has a significant impact on the predictive performance of the resulting hybrid models, as compared to the parallel hybrid FP + AI structure discussed in Section 3.3. Furthermore, on implementation of the parallel hybrid FP + AI models (refer Section 3.3) for predicting $T_{out,ms}$, it is observed that the resulting optimal models leads to MAE values as high as around 10 °C which exceeds the acceptable limits considered in this work. It is worth mentioning that the transient temperature profiles observed for the MOC power plant show relatively lesser uncertainties and hence improved/sufficient richness in measured data, as compared to the PC power plant. Therefore, the results obtained from the series hybrid model in this case are considered satisfactory for accurately predicting the main steam outlet temperature.

• Results from the Integrated Hybrid FP + AI Model (Plant B)

The final regenerative system of Plant B (PC-fired boiler, refer Fig. 7) has been simulated and validated using similar methodology described for Plant A. The control volume discretization scheme of Plant B is shown in Fig. 9 (b), and like Plant A, inlet flowrates and temperatures of steam and flue gas obtained from pre-plant operational data are considered as inputs to the standalone FP model. However, for this system, only the main steam inlet data, i.e., the steam inlet temperature ($T_{in,ms}$) and main

flue gas inlet temperature with respect to the measured data, resulting in an approximate RMSE of 1.0 °C. The black dotted vertical lines in Fig. 12 signify the total duration of dynamic time-series data (3–25 h) used for training both AI (neural nets, NN) and BML models. It can be observed that the hybrid FP + NN and FP + BML models show improvement in predictive capabilities over the standalone FP model not only for the dynamic data used for training the respective AI models, but also for the validation data. It is to be mentioned here that the all-nonlinear parallel (NLS || NLS2) multi-dynamic NN model (as shown in Fig. 10c) has been chosen to represent the NN model in the hybrid FP + NN structure, due to its relative superiority among the other two candidate architectures. The RMSE values in the prediction of the main steam outlet temperature, for instance, have been reduced from 1.0 °C, with respect to the standalone FP model, to 0.9 °C and 0.8 °C, for the hybrid FP + NN and FP + BML models, respectively. The comparison among the standalone FP and both types of hybrid models in terms of RMSE and MAE, and the overall computational time for training the hybrid FP + AI models, have been shown in Table 2.

It has been observed that the series hybrid FP + AI model (as discussed in Section 3.3) in this case (Plant B) yields satisfactory performance for the training data. However, the resulting optimal series hybrid model showed significantly higher mismatches and inferior predictive capabilities during simulation, leading to MAE values as high as 21 °C for $T_{out,ms}$ and therefore not acceptable. Modeling such energy systems can benefit from synergistic cross-coupling between the FP and AI models (i.e., integrated model as shown in Fig. 3). In the integrated hybrid model, the flue gas inlet heat flux (q_{in}), given by Eq. (9), has been modified in the formulation of the FP model to account for the uncertainties in the heat transfer rate by incorporating an additive correction term for the heat transfer coefficient ($h_{w,gs}$) as defined in Eq. (10). In Eq. (10), $h_{w,gs}$ refers to the original heat transfer coefficient calculated from the least transfer correlations involved in the FP model,

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and T_{in} and T_{out} represent the inlet flue gas temperature and the outside tube wall temperature respectively.

$$q_1 = h_{m,0}(T_{in} - T_{out}) \quad (9)$$

$$q_2 = h_{m,0}(T_{in} - T_{out}) + h_{m,0}(T_{in} - T_{out}) \quad (10)$$

The convective heat transfer coefficient ($h_{m,0}$) values are represented by the AI models (refer Fig. 1(a)). A continuous duration spanning around 27 days of plant operational data has been considered, out of which only 9 days (constraining windows spanning for 24 h continuous time series data measured every minute, leading to approximately 33 % of the total data) are chosen randomly. For these data, a single parameter constrained optimization problem given by Eq. (11) is solved by using a second-order constrained optimization approach. The squared error between the results from the FP model, i.e., T_{out} (refer Fig. 1(b)), and the respective plant measurements is minimized by optimizing $h_{m,0}$.

$$\min_{h_{m,0}} (T_{out,model}(t) - T_{out,plant}(t))^2 \quad (11)$$

$$s.t. T_{out,model} = f(x, u, \theta, h_{m,0}, t)$$

where, x denotes the state variables considered in the FP model and $f(x, u, \theta, h_{m,0}, t)$ denotes the output of the FP model. The convective term for the heat transfer coefficient ($h_{m,0}$) is expressed as a function of the input boundary conditions (i.e., T_{in} , T_{out} , T_{in} , T_{out} , T_{in} , T_{out}) plant (see Fig. 8(a)) as model inputs. In addition to these four inputs, the atmospheric spray flowrate (q_{spray}), as well as the valve openings on four inlet pipes connecting to the steam inlet header ($v_{v1}, v_{v2}, v_{v3}, v_{v4}$), are considered as additional inputs for the AI models. The list of the input/output variables considered for the integrated hybrid FP + AI model can be found in Table 8 in Appendix B. It should be noted here that out of the five additional inputs considered for the AI models only four inputs which are the valve openings ($v_{v1}, v_{v2}, v_{v3}, v_{v4}$) are directly measured in the plant using instruments. The atmospheric spray flowrate (q_{spray}), although available from plant operating data is not a measured value and is calculated using a developed correlation by the plant based on valve openings and plant load.

The respective temporal variations of these additional input variables have been shown in Figs. 8(a) and 8(b) in Appendix B. It should be noted that, the improved prediction of the heat transfer coefficient is also expected to improve the predictions of the other variables of interest such as tube wall temperature and flue gas temperature, that are important variables for performance analysis, yet remain largely unmeasured in many power plants.

Fig. 14 shows the comparison between the integrated hybrid FP + NN and FP + BML models for the main steam outlet temperature in Plant B. It can be observed that the integrated hybrid FP + AI models accurately capture the overshoots and undershoots in the temporal profile of the main steam outlet temperature with desired accuracy.

The comparison among the different performance measures among the standalone FP, FP + NN, and FP + BML models has been provided in Table 9. Both types of hybrid FP + AI models significantly improve the prediction accuracy within the acceptable limits of RMSE and MAE considered in this work, as can be seen in Table 9. It should be noted that the training computational time reported in Table 9 does not include the computational expense associated while generating the training data for the AI models by solving the constrained optimization problem. The training and prediction results obtained from the NN and BML models for $h_{m,0}$ have been shown in Fig. 8(c) and 8(d) in Appendix B. It should be noted that the negative values of the optimal $h_{m,0}$ correct the over-prediction of the flue gas side heat transfer coefficient by the standalone

FP model.

From Table 9, it is evident that both hybrid integrated FP + AI models show similar predictive performance for the main steam outlet temperature. However, the BML model requires less than one-third of the number of parameters used in the NN model. It is to be mentioned here that for all case studies, consistently higher computational expense associated with the training of the BML model can be attributed to the reverse search for the globally optimal subset of well-defined basis functions using the branch and bound approach in the BREGM algorithm [7]. In addition to this, the approach explicitly accounts for noise that may be present in the training data as it simultaneously carries out uncertainty quantification alongside model parameter estimation. Furthermore, the mathematical model forms of the optimal parametric NN and BML models obtained for the estimation of $h_{m,0}$ have been included in Table 82 in Appendix B.

6.1.2. Results for the Spatio-Temporal distribution of steam outlet temperature

• Results from the Integrated Parallel Hybrid FP + AI Model (Plant B)

As discussed in Section 4.2, the final superheater system for Plant B under consideration consists of tubular flow circuits studied one behind another across the heat transfer space. In the actual system configuration, there are 127 flow circuits in Plant B. However, constructing an accurate first-principles model for each flow circuit (location) along the width direction (refer Fig. 6) for the given system can be computationally impractical for most cases. Fig. 15 in Appendix C shows the temperature distribution in the width direction for the measured data of the steam outlet temperature for Plant B. It is to be mentioned that the legend "Average Measured Data" for the steam outlet temperature, as represented in Fig. 15, refers to the measured main steam outlet temperature available directly from plant historian. Additionally, in all the results shown for the integrated parallel hybrid model in Fig. 15 and Fig. 16, the legend "Max Measured Data" refers to the maximum value (in terms of absolute magnitude) observed and "Min Measured Data" refers to the minimum value (in terms of absolute magnitude) observed for any particular variable among the actual operating data.

Fig. 15 shows the temperature distribution for the measured data of the steam outlet temperature available from the industrial partner power plant for a continuous operational period of 27 days. In Fig. 15, the average measured data (purple dots) are shown, along with the maximum measured data (brown dots) and minimum measured data (cyan dots) representing the wide range of temperature distribution for the steam outlet temperature along the width direction for the superheater in Plant B. Only two datasets for dynamic temperature profiles (termed as Fac 1 and Fac 2) corresponding to the maximum and minimum deviation from the average measured value, i.e., the steam outlet temperature respectively, have been considered. The standalone FP model developed for both the superheater systems (Plants A and B, refer Section 4.2) neglects variability of transport variables in the width direction. Inclusion of another dimension, i.e., the width direction may lead to excessive computational expense and hence, can be difficult for online adaptation. However, accurately predicting the temperature profiles at each tube assembly/location in the width direction will be useful for developing a health monitoring framework for the boiler components for damage prediction in each tube and predict potential tube failures in future. Therefore, such limitations present in a standalone FP model can be mitigated with appropriate data-driven models. For such limitations in the FP model, a parallel hybrid configuration of FP and AI models may be beneficial.

In Plant B, the integrated hybrid approach where the AI model is used to represent the correction factor for the heat transfer coefficient is coupled with another AI model in parallel, thus yielding the integrated parallel hybrid FP + AI model. The integrated parallel model developed is then implemented for modeling the dynamic temperature profiles

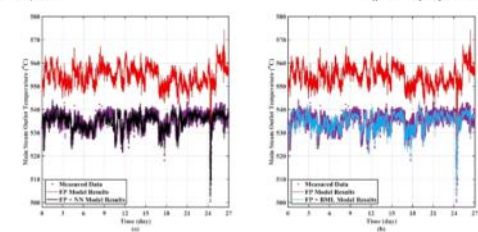


Fig. 14. Comparison of results between standalone FP, hybrid (a) FP + NN and (b) FP + BML models for main steam outlet temperature with respect to the superheaters across at Plant B.

Table 9
Comparison between hybrid integrated FP + NN and FP + BML models in terms of overall prediction RMSE, MAE, training and testing computational time for the main steam outlet temperature of Plant B.

Type of model	RMSE (°C)	MAE (°C)	Training time (min)	Testing time (min)
FP + NN	1.5	0.5	120	1.7
FP + BML	0.5	0.5	20	22.0

along the width direction of the superheater system in Plant B. Similar to the steam hybrid FP + AI model (refer Fig. 13), depending on the specific type of parallel hybrid structure, the target dynamic outputs of the second AI models are given by Eqs. (12).

$$h_{m,0}(t) = f_{m,0}(t) - f_{m,0}(t) \quad (12)$$

where, $h_{m,0}$ denotes the deviations, $f_{m,0}(t)$ denotes the measured dynamic temperatures at all tube assembly flow circuits along the width direction, and $f_{m,0}(t)$ denotes each of the respective (integrated) model predicted values of the average temperatures of the i^{th} output variable (refer Fig. 1). The model inputs for the second data-driven model, as shown in Fig. 5, consists of the inputs to the respective FP model ($u(t)$), in addition to various other input measurements not used in standalone FP models such as the atmospheric spray flowrate, valve openings, and certain inlet steam temperature measurements at specific locations (T_{in}) for the integrated parallel model (Plant B). The list of the input/output variables considered for the integrated parallel hybrid FP + AI model can be found in Table 8 in Appendix C. The variables involved in the integrated parallel hybrid models represent the same input/output operational variables in the parallel framework, as compared to the individual integrated hybrid FP + AI models and, therefore, are not re-defined in Table C1.

The temporal variations of steam inlet temperature, over a continuous period of 27 days of plant operation, at a couple of locations (indicated as "Fac 1" and "Fac 2" on the side side of the superheater system at Plant B have been shown in Fig. 82 in Appendix C. The

training data for the AI models in this case are prepared by considering packets of 12-h operational data. For example, the available data for the entire duration of plant operations under consideration (say, 27 days) are distributed into smaller packets, each spanning a continuous duration of approximately 12 h based on the steady-state plant load characteristics. Therefore, around 60 % of the total number of those 12-h packets have been considered for training the data-driven models, followed by a 5-fold cross validation step to remove any potential bias in data selection. The predictive performance of the hybrid integrated-parallel FP + AI models have been analyzed while modeling the temporal steam outlet temperature at individual tube assemblies over a continuous duration of around 27 days, similar to the integrated hybrid models. The steam outlet temperature profiles also portray a considerably higher degree of fluctuations during transients due to variation of plant load frequently cycling among low (400 MW), medium (2000 MW) and high (700 MW) load values, as shown in Fig. 82 in Appendix B.

Fig. 15 shows the comparison between the integrated parallel hybrid FP + NN and FP + BML models in predicting the steam outlet temperature distribution along the width direction of the superheater system in Plant B. As before, only two of these locations (tube assemblies) are chosen for demonstration which correspond to the highest and lowest measured temperature measurements among all tube assemblies. From Fig. 15, it can be observed that the hybrid FP + NN model relatively outperforms the hybrid FP + BML model in terms of overall RMSE values and associated computational expense during model training, as shown in Table 9. Similar to the prediction of main steam outlet temperature for the superheater system at Plant A, for this case as well, it is observed that the all-weather parallel (AFL) (NN) main-steam NN model (as shown in Fig. 1 (c)) performed superior compared to the NLS-NLS or NLS-NLS models and hence, has been chosen to represent the NN model in the hybrid integrated-parallel FP + NN structure.

From Table 9, the RMSE values obtained from the hybrid FP + NN model for predicting the steam outlet temperature at both locations fall within the permissible limits considered in this work. However, the corresponding results obtained from the hybrid FP + BML model slightly exceed the acceptable RMSE value of 5 °C. It should also be noted that the MAE values observed for this specific case study example are significantly higher than the desired permissible limit defined for this study. Nevertheless, the integrated parallel hybrid FP + AI models

본 연구는 고온 증기 시스템(슈퍼히터)의 동적 거동을 정확히 예측하기 위한 하이브리드 First-Principles(FP) + AI 모델을 개발하고, 그 성능을 NGCC 및 PC 보일러 시스템 두 곳에서 실증하였다.

주요 기여는 다음과 같다:

1. **세 가지 하이브리드 구조(직렬, 병렬, 통합형)**를 비교 분석하였으며, 각 구조는 시스템 특성과 데이터 품질에 따라 장단점을 지닌.
2. AI 모델로는 정적-동적 시계열 신경망(NN)과 BML(Bayesian Machine Learning)을 활용하여 정확도와 불확실성 처리 성능을 비교함.
3. 실제 플랜트 데이터를 기반으로, 하이브리드 모델은 기존 FP 단독 모델보다 예측 정확도(RMSE 기준)가 10배 이상 향상됨.
예: FP 모델의 RMSE가 12.3 °C → FP+AI 모델은 0.5 °C
4. 하이브리드 모델은 시간적/공간적 분포 예측에도 효과적이며, 향후 실시간 감시 시스템이나 디지털 트윈 기반 운영 최적화에도 응용 가능성이 있음.

이러한 결과는 하이브리드 모델이 고온 열교환기 구성 요소의 정확한 상태 예측 및 모니터링을 위한 강력한 도구가 될 수 있음을 보여준다.

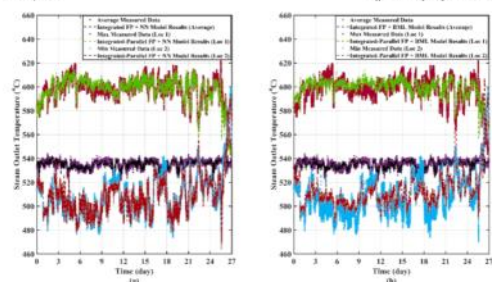


Fig. 15. Comparison of results between hybrid integrated-parallel (a) FP + NN and (b) FP + BML models for predicting steam outlet temperature along width direction with respect to Plant B.

Table 4
Comparison between FP + NN and FP + BML models in terms of RMSE, MAE, and computational expense for predicting steam outlet temperature in Plant B at individual flow circuits along the width direction.

Type of model	RMSE	MAE	RMSE	MAE	Training computational time for respective models
	°C	°C	°C	°C	
FP + NN	0.1	0.3	0.4	0.3	2 min
FP + BML	0.1	0.3	0.4	0.3	94 min

naturally enhance (as evident from the RMSE values with respect to measured data, refer Table 5) the overall fidelity of the hybrid model by incorporating an additional dimension (variability along width) as compared to the FP model. The higher instantaneous mismatch in the temporal temperature profiles may be attributed to the potential presence of uncertainties in the data under consideration with respect to the real-time power plant system (Plant B), stemming from both process noise as well as measurement noise. Furthermore, several other process variables, such as variabilities in coal composition, moisture content, and burner conditions, measurement of which are not available from the industrial partner plant, can significantly impact the temporal temperature profiles.

6.2. Modeling of tube wall and flue gas temperature

In addition to the steam outlet temperature profiles, the developed hybrid FP + AI models also provide capabilities for representing other process variables such as outer tube metal wall temperature of superheater tubes and temperature profile of the flue gas stream.

6.2.1. Results for the average tube wall and flue gas temperature

Accurate prediction of the transient tube wall temperatures in

thermal systems can help to prevent overheating and local hotspots, thus reducing potential failure of superheater tubes. On the other hand, precisely estimating the temporal flue gas temperature profile can ensure efficient heat recovery in boiler systems and avert abnormal/external fouling of the tubes, and damage to the refractory. For brevity, the results are presented for only a subset of locations (tags), namely RB 1 for results related to tube wall temperature and O 71 for those related to flue gas outlet temperature (please see Fig. 6 (a)).

• Results from the Series Hybrid FP + AI Models (Plant A)

Similar to the results obtained from the standalone FP model for the main steam outlet temperature (see Fig. 11), the predictive performance of the FP model for the average tube wall and flue gas temperatures at locations RB 1 and O 71 (refer Fig. 6 (a)), respectively are shown in Fig. 16 in Appendix A. As mentioned in Section 4.1, Plant A contains 74 flow circuits along the width direction (refer Fig. 4 (b)), out of which 18 flow circuits have temperature data available for tube wall and flue gas temperatures at various locations (refer Fig. 6 (a)). In both Fig. 16 (a) and 16 (b), large discrepancies are observed between the average measured data and the predictions obtained from the FP model. In Fig. 16 (a), the FP model underpredicts the average measured data for the tube temperature at RB 1 with an RMSE of around 12.5 °C, whereas, in Fig. 16 (b), the FP model overpredicts the observed average flue gas temperature at O 71 with an approximate RMSE of 0.1 °C, spanning for the overall duration of the 20 d used for model simulation. The significantly large plant-model mismatch observed for both these variables can be attributed to various uncertainties associated with the standalone FP model, due to one or more modeling assumption(s) such as ignoring ash fouling of outer tube wall surface and oxide scale deposition inside the tube. However, as discussed before, it is difficult to develop accurate mechanistic models of some these phenomena such as ash fouling due to uncertainty and complexity of the mechanisms of ash deposit on and removal from the tube outer surfaces. The series hybrid FP + AI

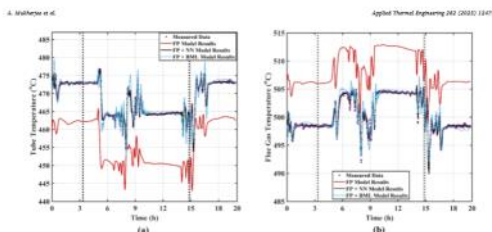


Fig. 16. Comparison of results among standalone FP, FP + NN and FP + BML models with the measured data for Plant A, (a) average tube wall temperature at RB 1 and (b) average flue gas temperature at O 71 (see Fig. 6).

model described in Section 5.1 has been implemented to address such plant-model mismatches. Also similar to Fig. 11, in the legend “Measured Data” represents the arithmetic average of the actual measured industrial operating data obtained from the plant historian and the Standalone FP Model Results” represent the results obtained by simulating the standalone dynamic distributed FP model (refer Section 4.1) for Plant A in absence of any data-driven augmentation.

Similar to the prediction of the main steam outlet temperature using the series hybrid FP + AI as discussed in Section 6.1.1, the optimal series hybrid FP + AI models also significantly improved the predictive performance of a standalone FP model while estimating the average tube wall temperatures and flue gas temperatures at different locations (see Fig. 6) of the superheater system at Plant A. Fig. 16 shows the comparison among standalone FP vs FP + NN vs FP + BML models for the average tube wall temperature at RB 1 and the average flue gas temperature at O 71 of the superheater system at Plant A. As shown previously in Fig. 11, the standalone FP model shows a consistent negative bias with respect to the average temperature measurements at RB 1 and a consistent positive bias with respect to the average temperature measurements at O 71, thus leading to approximate overall RMSE of 12.5 °C and 0.1 °C, respectively, for both locations during the entire time series data spanning around 20 d. The accurate prediction of metal temperature of the superheater tubes can be critical for developing a boiler health monitoring framework for assessing and predicting potential tube failure. Therefore, underpredicting or overpredicting the tube wall and flue gas temperatures may lead to inaccurate failure predictions for superheater tubes and other boiler components. From Fig. 16 (a), it can be seen that results from both hybrid FP + AI models are closer to the real-time plant measurements of the average tube wall temperature at RB 1, thus yielding RMSE values of 0.5 °C and 1.4 °C, for

the FP + NN model and the FP + BML model, respectively. Similarly, significant improvement in performance of both hybrid FP + AI models for the average flue gas temperature at O 71 is observed in Fig. 16 (b), with an overall RMSE of 0.4 °C and 0.9 °C for the FP + NN model and the FP + BML model, respectively. Furthermore, not only does the hybrid FP + NN model yields superior RMSE values as compared to the FP + BML model while modeling the transient average tube wall and flue gas temperature profiles, but it also significantly outperforms the latter in terms of accurately capturing the overshoots and undershoots in the temporal temperature profiles, thus leading to much lesser values of MAE, as shown in the plots in Fig. 17 and Fig. 18 in Appendix A, illustrating a second portion of training data considered between the 2nd and 10th hours of plant operation. While the FP + NN yields the worst-case instantaneous error, i.e., MAE value of 4.0 °C and 0.9 °C for tube wall and flue gas temperature respectively for the entire 20 d duration, the FP + BML model yields MAE values as high as 6.0 °C and 4.0 °C, respectively, for these two output variables, as seen in Table 5. The comparison between the candidate models in terms of various performance measures while predicting the average tube temperature (RB 1) and average flue gas temperature (O 71) has also been shown in Table 5. Comparisons among the candidate hybrid models along with the standalone FP model for predicting the average tube wall temperatures and average flue gas temperatures at other locations (see Fig. 6 (a)) in the Plant A superheater has been included in Fig. A3 through Fig. A9 in Appendix A. Comparison for the validation data at the other locations have been shown in Table A3 in Appendix A. From Table A3, it is evident that both types of hybrid FP + AI models significantly improve the predictive capabilities as compared to the standalone FP model. Furthermore, for this case study, the FP + NN model yields superior RMSE and MAE values for both the training

Table 5
Comparison among FP, FP + NN, and FP + BML models in terms of RMSE, MAE, number of parameters and training computational time for average tube temperatures (RB 1) and average flue gas outlet temperature (O 71) for Plant A.

Type of model	Number of Model Parameters	Training computational time for respective models	Average Tube Wall Temperature at RB 1 (°C)		Average Flue Gas Outlet Temperature at O 71 (°C)	
			RMSE	MAE	RMSE	MAE
FP	4	–	12.5	25.8	0.1	18.0
FP + NN	182	2.8 min	0.5	4.0	0.4	2.8
FP + BML	138	22.7 s	1.4	6.0	0.9	4.8

and validation data compared to the PP + BML model.

The complete list of all optimal candidate basis functions estimated for this case study by the BML algorithm, as well as their applicability to model variables, is shown in Table A1 and A2 in Appendix A. The computational time associated with the training of the NN model (2.3 min) for implementation in the hybrid PP + AI framework is also significantly lower than that of the BML model (21.7 h), due to the branch and bound algorithm involved in the BML algorithm for selection of optimal set of basis functions. However, it is to be noted that the hybrid PP + BML model exhibits superior model sparsity and hence, requires significantly lesser number of parameters than the PP + NN model while modelling the final superheated system at Plant A. This, in turn, leads to a slightly higher value of the overall Akaike information criteria (AIC) (AIC) obtained for the optimal PP + NN model (AIC = 204.7) when compared to that of the hybrid PP + BML model (AIC = 207.6) with respect to the system under consideration. Minimizing the AICs during parameter estimation in data-driven models provides overfitting during model training, thus leading to a parsimonious model structure.

6.2.2. Results for the Spatio-Temporal distribution of tube wall temperature

The superheated system in Plant A consists of 74 flow circuits (see Fig. 6 (b)) along the width direction. Hence, the Plant A is discretized in Section 6.1.1, constructing an accurate first-principles model for each flow circuit along the width direction (refer Fig. 6 (b)) for the system can also be computationally expensive.

• Results from the Series Parallel Hybrid PP + AI Model (Plant A)

Table C3 in Appendix C shows the temperature distribution in the width direction for tube wall temperature at location E 77 (refer Fig. 6 (a)) for Plant A. As seen in Fig. C3, the measured temperature data for different flow circuits at location E 77 shows a significant variance in the temperature measurements.

For Plant A, the series hybrid model considered to predict the

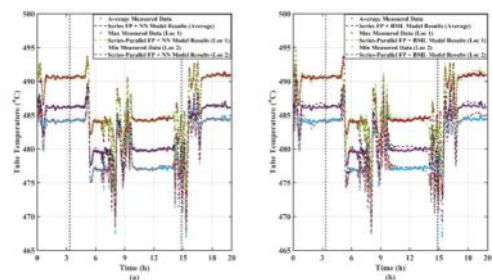


Fig. 17. Comparison of results between hybrid series-parallel (a) PP + NN and (b) PP + BML models for tube temperature at E 77 location (refer Fig. 6 (a)) for Plant A.

Types of Models	Location 1 (E 77)		Location 2 (E 77)	
	RMSE (°C)	MAE (°C)	RMSE (°C)	MAE (°C)
PP + NN	2.4	2.0	0.3	2.8
PP + BML	1.0	0.7	1.0	0.7

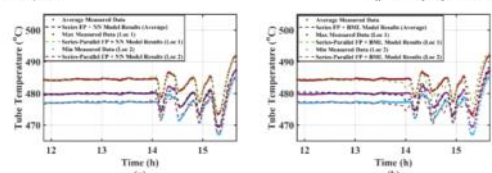
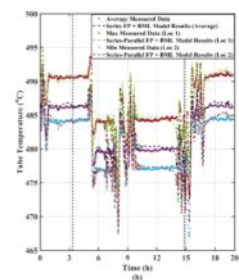


Fig. 18. Comparison of results between hybrid series-parallel (a) PP + NN and (b) PP + BML models for tube temperature at E 77 location (refer Fig. 6 (a)) for Plant A.

flow circuits along each location, elevation (see Fig. 6 (b)), out of which two tube assemblies are chosen for analysis which recorded the highest and lowest temperature values at the respective elevations across all 18 circuits. Fig. 17 shows the comparison between the hybrid series-parallel PP + NN (Fig. 17 (a)) or PP + BML (Fig. 17 (b)) models for prediction of the tube temperature distribution along the width direction at E 77. The Mark dotted vertical lines in Fig. 17 indicate the duration of training data. The comparison of RMSE and MAE for the temporal temperature distribution of tube wall at E 77 between both types of series-parallel hybrid models for both Loc 1 and Loc 2 is shown in Table 6.

As can be seen in Table 6, both types of hybrid PP + AI models improve the overall predictive performance yielding acceptable values of RMSE and MAE. Although both types of series-parallel hybrid models satisfactorily match the trends in the temporal temperature profiles at both locations (referred to as Loc 1 and Loc 2) as shown in Fig. 17, the PP + NN model slightly overpredicts the PP + BML model, especially in terms of accurately capturing the overshoots and undershoots during transients as seen in the second section plot in Fig. 13 for the E 77 location. Furthermore, the PP + NN model also yields slightly superior RMSE and MAE as compared to the PP + BML model not only while predicting the tube temperature at E 77, but also at the other elevations for which tube temperature and flue gas temperature measurements are available, as can be seen in Table C3 in Appendix C. The plots comparing model performance for the validation data of other locations are shown in Fig. C5 through Fig. C10 in Appendix C.

Comparisons between the NN and BML models used in the series-parallel hybrid PP + AI framework in terms of the number of parameters (model sparsity) and overall training computational time has been included in Table C4 in Appendix C. Furthermore, the optimal set of candidate basis functions selected by both BML models as well as their applicability to model variables with respect to Plant A has been listed in Table A1 and A2 in Appendix A.

7. Conclusions

All three types of hybrid models developed in this paper, i.e., series, integrated, and parallel structures, significantly outperform the standard PP model while modelling the final superheated system of the combined cycle (Plant A) and coal-fired (Plant B) power plants. For instance, while the standard PP model developed for the superheated

system at Plant A yielded RMSE and MAE values as high as 2.3 °C and 25.6 °C, respectively, while predicting the transient average tube wall temperature at E 77, the series hybrid PP + AI model yields RMSE values of 0.5 °C (PP + NN) and 1.4 °C (PP + BML), and MAE values of 4.0 °C (PP + NN) and 6.0 °C (PP + BML), respectively. For this case study, the series hybrid PP + NN model consistently outperforms the PP + BML model in terms of predictive accuracy and computational expense while predicting the dynamic temperature profiles of all output variables considered, but the NN model has considerably greater (125 vs 150) number of model parameters. On the contrary, both hybrid PP + AI models show similar predictive performance in terms of RMSE (around 1.9–2.0 °C) and MAE (around 6.5–8.5 °C) values for the main steam outlet temperature of Plant B through the integrated framework, but the optimal BML model has less than one-third of the number of parameters compared to the optimal NN model. It is worth mentioning that the computational time involved for the training of the NN models in all hybrid PP + AI structures discussed in this paper is considerably lower than that of the BML models mainly due to the time taken for solving the mixed integer nonlinear programming problem for selection of optimal set of candidate basis functions and parameter estimation in the BML algorithm. The computational expense associated with the BML models can eventually be reduced by leveraging parallel computing approaches in the BML algorithm. On the other hand, the sequential training algorithms for the all nonlinear static-dynamic NN models ensure faster convergence within the hybrid framework, even for highly nonlinear transient data.

The parallel (series-parallel and integrated-parallel) hybrid PP + AI models are found to accurately capture the spatio-temporal variations in the tube wall, flue gas and steam outlet temperature profiles along the width direction of the superheated systems at Plants A and B. For all these case studies, the optimal hybrid PP + BML models consistently show superior model sparsity and interpretability, whereas the hybrid PP + NN models exhibit consistent superior performance, if not similar, in terms of predictive capabilities and computational expense during model development.

Finally, the authors would like to note that it is considerably difficult to arrive at a universal conclusion on the superiority of a particular type of hybrid PP + AI model over another since the efficiency of such synergistic integration of PP and AI models largely depend on several factors such as the process chemistry of the specific system, type of

information transferred between the models, characterization of data available for training the data-driven models, and candidate architectures, algorithms associated with the development of the hybrid framework. In general, it is our opinion that for modeling complex dynamic systems, various structures of the hybrid FP + AI models can be exploited without necessarily developing a very high-fidelity FP model that may require large resources and run time extensive computational cost. Although, the fidelity of the FP model and the capability/characterization of the AI models depend on specific system and the variables to be represented by the hybrid models. We believe that the proposed hybrid FP + AI structures and AI models robust sufficient functionalities to be implemented independently or in combination with one another, not only for modeling industrial superheaters systems, but also many other complex dynamic nonlinear systems.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The project is funded by the U.S. DOE through the project titled "Boiler Health Monitoring Using a Hybrid Plant Principals – Artificial Intelligence Model" (Grant #: DE-EE0003790). The DOE financial support is gratefully acknowledged.

Appendix A

Additional plots and tables showing temporal distributions of input variables and results from the series hybrid FP + AI Models with respect to Plant A.

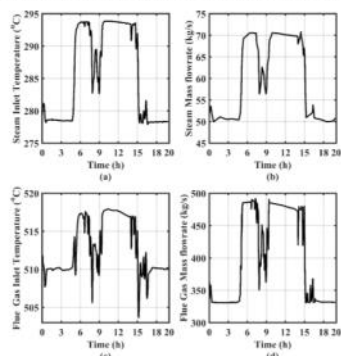


Fig. A3. Dynamic profile of input variables for Plant A final superheater system considered for the standalone FP model, i.e., (a) steam inlet temperature ($T_{s,in}$), (b) steam mass flowrate ($\dot{m}_{s,in}$), (c) flue gas inlet temperature ($T_{fg,in}$), and (d) flue gas mass flowrate (\dot{m}_{fg}) (see Fig. 5 (a)).

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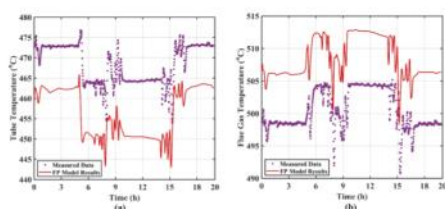


Fig. A5. Comparison of results from the standalone FP model vs measured data for Plant A for, (a) tube wall temperature at RB 1 and (b) flue gas temperature at 0.71, (see Fig. 5).

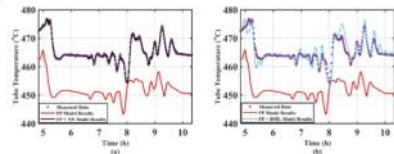


Fig. A6. Comparison of results among standalone FP and hybrid (a) FP+NN and (b) FP+RNN models with measured data for tube wall temperature at RB 1 (see Fig. 5) for Plant A.

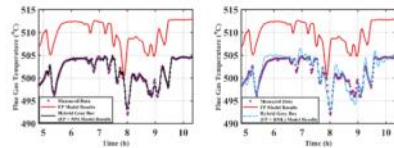


Fig. A7. Comparison of results among standalone FP and hybrid (a) FP+NN and (b) FP+RNN models with measured data for flue gas temperature at 0.71 (see Fig. 5 (a)) for Plant A.

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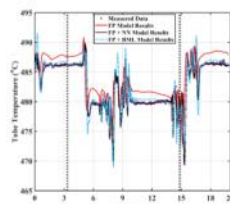


Fig. A5. Comparison of results among standalone FF, FF+NN and FF+BNN models with the measured data for tube wall temperature at 8.75° (see Fig. 6 (a)) for Plant A.

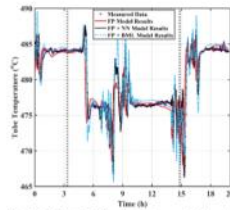


Fig. A6. Comparison of results among standalone FF, FF+NN and FF+BNN models with the measured data for tube wall temperature at 8.40° (see Fig. 6 (a)) for Plant A.

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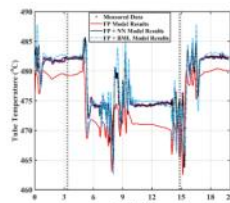


Fig. A7. Comparison of results among standalone FF, FF+NN and FF+BNN models with the measured data for tube wall temperature at 8.4° (see Fig. 6 (a)) for Plant A.

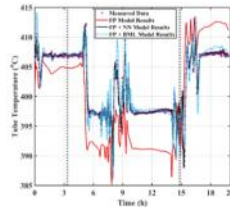


Fig. A8. Comparison of results among standalone FF, FF+NN and FF+BNN models with the measured data for tube wall temperature at 8.2° (see Fig. 6 (a)) for Plant A.

24

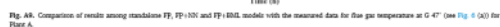


Table A1
List of optimal combinations of selected basis functions for the SVM model in series hybrid FF + AI structure while modeling the superheater system at Plant A

Table A3

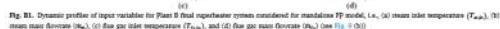
Table A2
Physical significance of all input/output variables used in the basis functions of EML model in series hybrid FF + AI structure while modeling the superheater system at Plant A.

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Table A3
Comparison among PD, PD + NSI, and PD + EML models in terms of FMIIE and MAIE for average tube wall temperature and average flue gas temperature (see Fig. 6 (a)) for Plant A

Appendix 7

Additional plots and tables showing temporal distributions of input variables and results from integrated hybrid FP + AI Models with respect to Flare B.



The quadratic correlation for relative inlet flow rate transmission with plant load is

$$y = -0.005x^2 + 0.2762x + 1543.5 \quad (B)$$

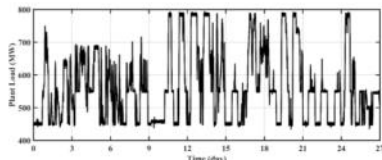


Fig. 82. Temporal profiles of the plant load among low, medium, and high values for the superheater system at Plant B.

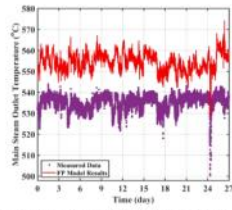


Fig. 83. Comparison of results from the standalone PF model with the measured data for main steam outlet temperature with respect to Plant B.

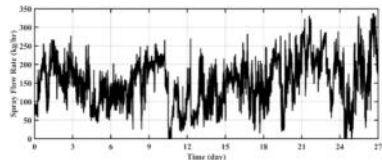


Fig. 84. Temporal profile of antitemperature spray flow rate (m_{spray}) considered as additional input for the AI models with respect to Plant B for the integrated hybrid PF + AI structure.

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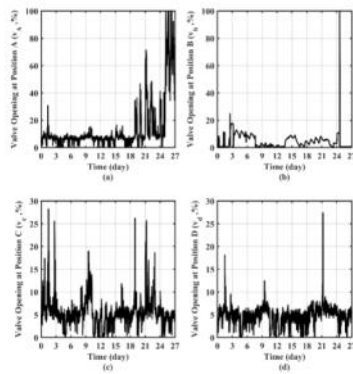


Fig. 85. Temporal profiles of input variables, i.e., valve openings at (a) Position A (v_A), (b) Position B (v_B), (c) Position C (v_C), and (d) Position D (v_D), considered as additional input for the AI models for Plant B for the integrated hybrid PF + AI structure.

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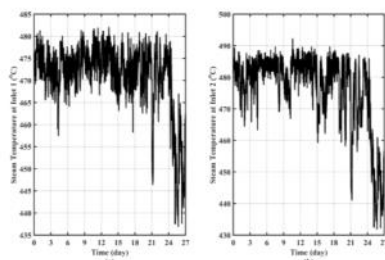


Fig. C2. Dynamic profile of the additional input variables, i.e., mean tube temperatures at (a) 'Inlet 1' and (b) 'Inlet 2' considered in the AI models for the integrated parallel hybrid FP+AI structure for Plant B.

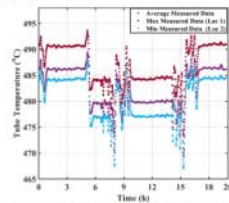


Fig. C3. Distribution of transfer tube wall temperature profile at 0.75 (under Fig. 6 (a)) with respect to Plant A across the flow circuit/locations along the width direction.

23

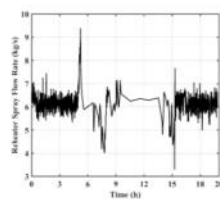


Fig. C4. Dynamic profile of reheater spray main flow rate (%) considered as an additional input in the AI models of Plant A for the series parallel hybrid FP+AI structure.

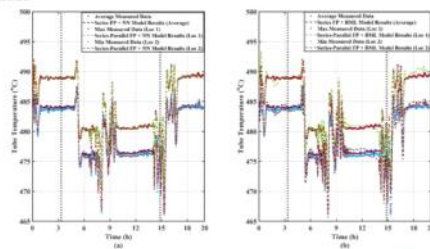


Fig. C5. Comparison of results between series parallel hybrid (a) FP+NN and (b) FP+RBF models for tube temperatures at 0.49 (see Fig. 6 (a)) for Plant A.

24

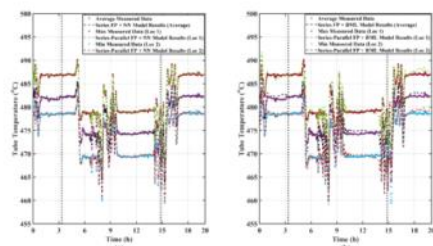


Fig. C6. Comparison of results between series parallel hybrid (a) FP+IS7 and (b) FP+BM6 models for tube temperature at 6 °C (see Fig. 6 (a)) for Plant A.

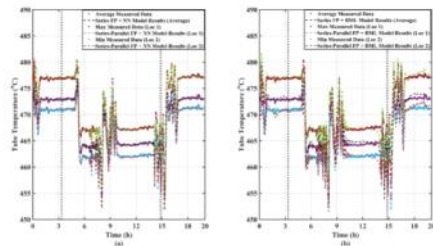


Fig. C7. Comparison of results between series parallel hybrid (a) FP+IS7 and (b) FP+BM6 models for tube temperature at 12 °C (see Fig. 6 (a)) for Plant A.

23

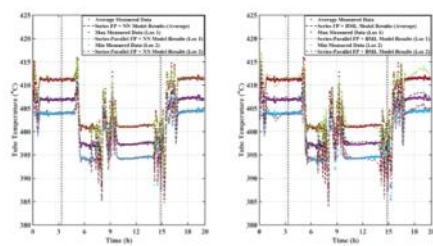


Fig. C8. Comparison of results between series parallel hybrid (a) FP+IS7 and (b) FP+BM6 models for tube temperature at 18 °C (see Fig. 6 (a)) for Plant A.

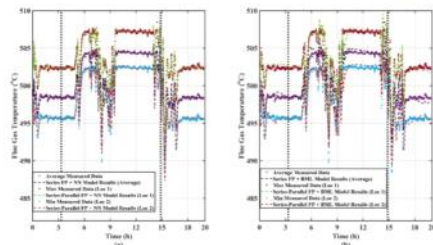


Fig. C9. Comparison of results between series parallel hybrid (a) FP+IS7 and (b) FP+BM6 models for flue gas temperature at 0 °C (see Fig. 6 (a)) for Plant A.

24



Table C1
List of model input and output variables in the integrated-parallel hybrid FP + AI models for the superheater system of Plant B

Variables used in Fig. 5 (b)

Table C2
List of model input and output variables in the series-parallel hybrid FF + AI models for the superheater systems of Plant A.

Variables used in *Fig. 5 (a)* A central tissue current variables for the series-parallel hybrid FF + AL model in terms of new

Table C3
Comparison between FF + NN and FF + BML models in terms of RMSE and MAE for tube wall and flue gas temperatures at individual locations along width direction with respect to Plant A

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Table C3 (continued)

Table C4
Comparison between NN and BML models for the series parallel hybrid FP + AI structure in terms of training computational time and number of parameters.

Types of AI Model for Series-Parallel FP + AI Structure	Number of Model Parameters	Overall Training Computational Time
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NN	298	3 min
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BSIL ₁	112	233 min
BSIL ₂	126	168 min

Table C5
List of optimal combinations of selected basis functions for the RMI models in series parallel hybrid

FP + AI structure while modeling the superheater system at Plant A.

Table C6
Physical significances of all input/output variables used in the basis functions of BML models in series

Table C6

Physical significances of all input/output variables used in the basis functions of EML models in series

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All data that have been used have been plotted, provided in tables or text.

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