

Regression

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2024-09-30

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Reference:

"https://jamleecute.web.app/regularized-regression-ridge-lasso-elastic/#google_vignette"

Data prepare

```
library(rsample)
```

Warning: 'rsample' R 4.2.3

```
library(dplyr)
```

Warning: 'dplyr' R 4.2.3

'dplyr'

'package:stats':

filter, lag

'package:base':

intersect, setdiff, setequal, union

```
library(magrittr)  
library(AmesHousing)
```

Warning: 'AmesHousing' R 4.2.3

```
data <- make_ames()  
set.seed(123)  
ames_split <- initial_split(data, prop = 0.7)  
ames_train <- training(ames_split)  
ames_test <- testing(ames_split)
```

Multicollinearity

```
library(Hmisc)
```

Warning: 'Hmisc' R 4.2.2

lattice

survival

Formula

ggplot2

Warning: 'ggplot2' R 4.2.3

'Hmisc'

'package:dplyr':

src, summarize

'package:base':

format.pval, units

```
data <- ames_train

numeric_vars <- supply(data, is.numeric)
data_numeric <- data[, numeric_vars]
high_cor <- data.frame(var1 = character(),
                       var2 = character(),
                       correlation = numeric(),
                       p_value = numeric())

# p
for(i in 1:(ncol(data_numeric)-1)) {
  for(j in (i+1):ncol(data_numeric)) {
    cor_test <- cor.test(data_numeric[[i]], data_numeric[[j]])
    if(abs(cor_test$estimate) > 0.6) {
      high_cor <- rbind(high_cor, data.frame(
        var1 = colnames(data_numeric)[i],
        var2 = colnames(data_numeric)[j],
        correlation = cor_test$estimate,
        p_value = cor_test$p.value
      ))
    }
  }
}
```

```
#
high_cor <- high_cor[order(-abs(high_cor$correlation)),]

#
print(high_cor)
```

	var1	var2	correlation	p_value
cor10	Garage_Cars	Garage_Area	0.8884495	0.000000e+00
cor1	Total_Bsmt_SF	First_Flr_SF	0.8120419	0.000000e+00
cor7	Gr_Liv_Area	TotRms_AbvGrd	0.8093677	0.000000e+00
cor8	Gr_Liv_Area	Sale_Price	0.6985090	4.195282e-300
cor9	Bedroom_AbvGr	TotRms_AbvGrd	0.6598809	1.124915e-256
cor4	Second_Flr_SF	Gr_Liv_Area	0.6469628	1.477759e-243
cor11	Garage_Cars	Sale_Price	0.6400904	7.794268e-237
cor12	Garage_Area	Sale_Price	0.6370741	6.139591e-234
cor2	Total_Bsmt_SF	Sale_Price	0.6276502	4.283657e-225
cor6	Gr_Liv_Area	Full_Bath	0.6185420	7.835941e-217
cor5	Second_Flr_SF	Half_Bath	0.6087904	2.776851e-208
cor3	First_Flr_SF	Sale_Price	0.6085229	4.718873e-208
cor	Year_Built	Year_Remod_Add	0.6045163	1.252026e-204

```
lm(Sale_Price ~ Gr_Liv_Area + TotRms_AbvGrd, data = ames_train)
```

Call:

```
lm(formula = Sale_Price ~ Gr_Liv_Area + TotRms_AbvGrd, data = ames_train)
```

Coefficients:

(Intercept)	Gr_Liv_Area	TotRms_AbvGrd
42563	137	-10563

```
lm(Sale_Price ~ Gr_Liv_Area, data = ames_train)
```

Call:

```
lm(formula = Sale_Price ~ Gr_Liv_Area, data = ames_train)
```

Coefficients:

(Intercept)	Gr_Liv_Area
14045.9	110.7

```
lm(Sale_Price ~ TotRms_AbvGrd, data = ames_train)
```

Call:

```
lm(formula = Sale_Price ~ TotRms_AbvGrd, data = ames_train)
```

Coefficients:

(Intercept)	TotRms_AbvGrd
15486	25538

Ridge Regression

```
library(glmnet)
```

Warning: 'glmnet' R 4.2.3

Matrix

Warning: 'Matrix' R 4.2.2

Loaded glmnet 4.1-8

```
library(ggplot2)
library(caret)
```

Warning: 'caret' R 4.2.3

'caret'

'package:survival':

cluster

```

library(RColorBrewer)
X <- ames_train[, -which(names(ames_train) == "Sale_Price")]
y <- log(ames_train$Sale_Price) # Log transformation of Sale_Price

# Identify numeric and categorical variables
numeric_vars <- names(X)[sapply(X, is.numeric)]
categorical_vars <- names(X)[sapply(X, is.factor)]

# Create dummy variables for categorical features
dummy <- dummyVars(" ~ .", data = X[, categorical_vars])
X_dummy <- predict(dummy, newdata = X[, categorical_vars])

# Standardize numeric variables
X_numeric_standardized <- scale(X[, numeric_vars])

# Combine standardized numeric variables with dummy variables
X_processed_lm <- cbind(X_numeric_standardized, X_dummy)
X_processed <- cbind(X[, numeric_vars], X_dummy)
# Fit ridge regression model
ridge_model <- glmnet(as.matrix(X_processed), y, alpha = 0)

coef_matrix <- coef(ridge_model)[-1, ]
lambda_values <- ridge_model$lambda
plot_data <- data.frame(
  lambda = rep(log(lambda_values), each = nrow(coef_matrix)),
  coefficient = as.vector(coef_matrix),
  variable = rep(rownames(coef_matrix), times = ncol(coef_matrix))
)

max_coef <- plot_data %>%
  group_by(variable) %>%
  summarise(max_abs_coef = max(abs(coefficient))) %>%
  arrange(desc(max_abs_coef))

#
plot_data$variable <- factor(plot_data$variable, levels = max_coef$variable)

n_vars <- length(unique(plot_data$variable))
colors <- c(
  brewer.pal(9, "Set1"),
  brewer.pal(8, "Set2"),
  brewer.pal(12, "Set3"),

```

```

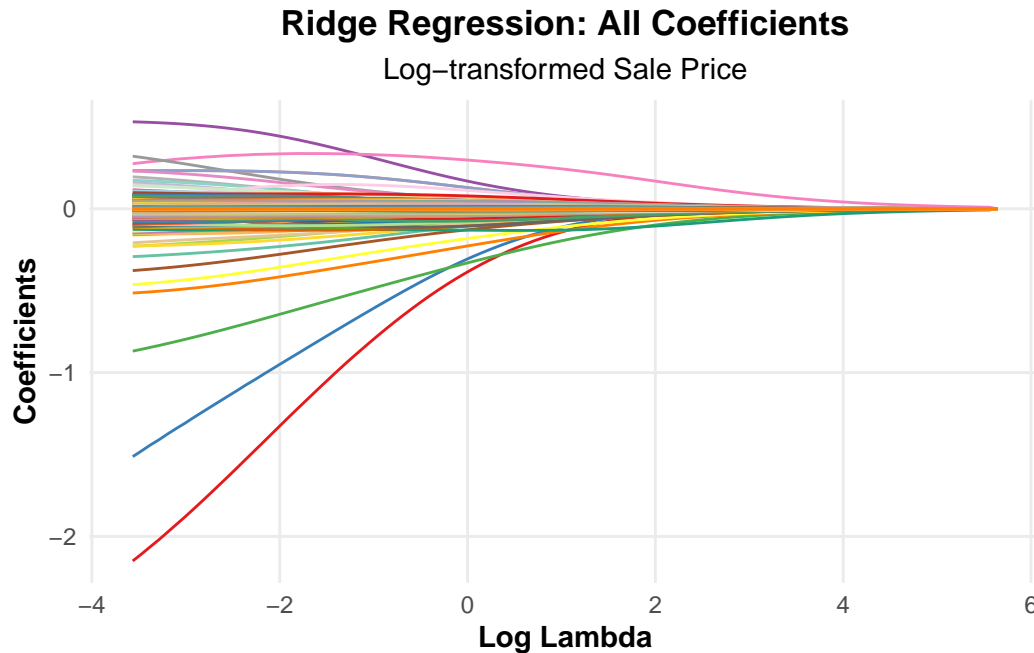
    brewer.pal(8, "Dark2"),
    brewer.pal(8, "Paired")
  )
  if(n_vars > length(colors)) {
    colors <- c(colors, colorRampPalette(colors)(n_vars - length(colors)))
  }

p <- ggplot(plot_data, aes(x = lambda, y = coefficient, group = variable, color = variable))
  geom_line(size = 0.5) +
  theme_minimal() +
  labs(title = "Ridge Regression: All Coefficients",
        subtitle = "Log-transformed Sale Price",
        x = "Log Lambda",
        y = "Coefficients") +
  theme(
    plot.title = element_text(hjust = 0.5, face = "bold"),
    plot.subtitle = element_text(hjust = 0.5),
    axis.title = element_text(face = "bold"),
    legend.position = "none",
    panel.grid.minor = element_blank(),
    panel.background = element_rect(fill = "white", color = NA),
    plot.background = element_rect(fill = "white", color = NA)
  ) +
  scale_color_manual(values = colors) +
  scale_x_continuous(
    breaks = seq(floor(min(plot_data$lambda)),
                  ceiling(max(plot_data$lambda)),
                  by = 2)
  )
)

```

Warning: Using `size` aesthetic for lines was deprecated in ggplot2 3.4.0.
 i Please use `linewidth` instead.

```
print(p)
```



```
cv_ride <- cv.glmnet(as.matrix(X_processed), y, alpha = 0, nfolds = 10)
ridge_best_lambda <- cv_ride$lambda.min

plot_data <- data.frame(
  lambda = cv_ride$lambda,
  mse = cv_ride$cvm
)

p <- ggplot(plot_data, aes(x = log(lambda), y = mse)) +
  geom_point() +
  geom_line() +
  geom_vline(xintercept = log(ridge_best_lambda), linetype = "dashed", color = "red") +
  annotate("text", x = log(ridge_best_lambda), y = max(plot_data$mse),
    label = paste("Best lambda =", round(ridge_best_lambda, 4)),
    hjust = -0.1, vjust = 1, color = "red") +
  labs(x = "Log Lambda",
    y = "Mean-Squared Error",
    title = "MSE vs Log Lambda for Ridge Regression") +
  theme_minimal() +
  theme(
    panel.background = element_rect(fill = "white", color = NA),
    plot.background = element_rect(fill = "white", color = NA),
    panel.grid.major = element_line(color = "grey90"),
```

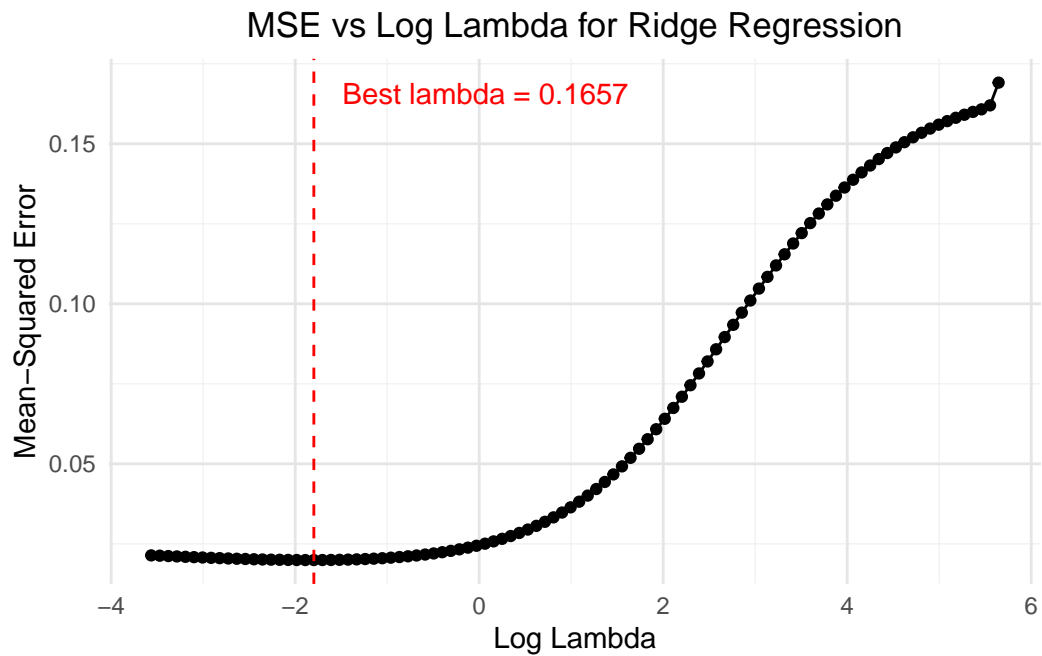


```

    panel.grid.minor = element_line(color = "grey95"),
    plot.title = element_text(hjust = 0.5)
)

print(p)

```



Lasso Regression

```

lasso_model <- glmnet(as.matrix(X_processed), y, alpha = 1)

coef_matrix <- coef(lasso_model)[-1, ]
lambda_values <- lasso_model$lambda

plot_data <- data.frame(
  lambda = rep(log(lambda_values), each = nrow(coef_matrix)),
  coefficient = as.vector(coef_matrix),
  variable = rep(rownames(coef_matrix), times = ncol(coef_matrix))
)

```

```

max_coef <- plot_data %>%
  group_by(variable) %>%
  summarise(max_abs_coef = max(abs(coefficient))) %>%
  arrange(desc(max_abs_coef))

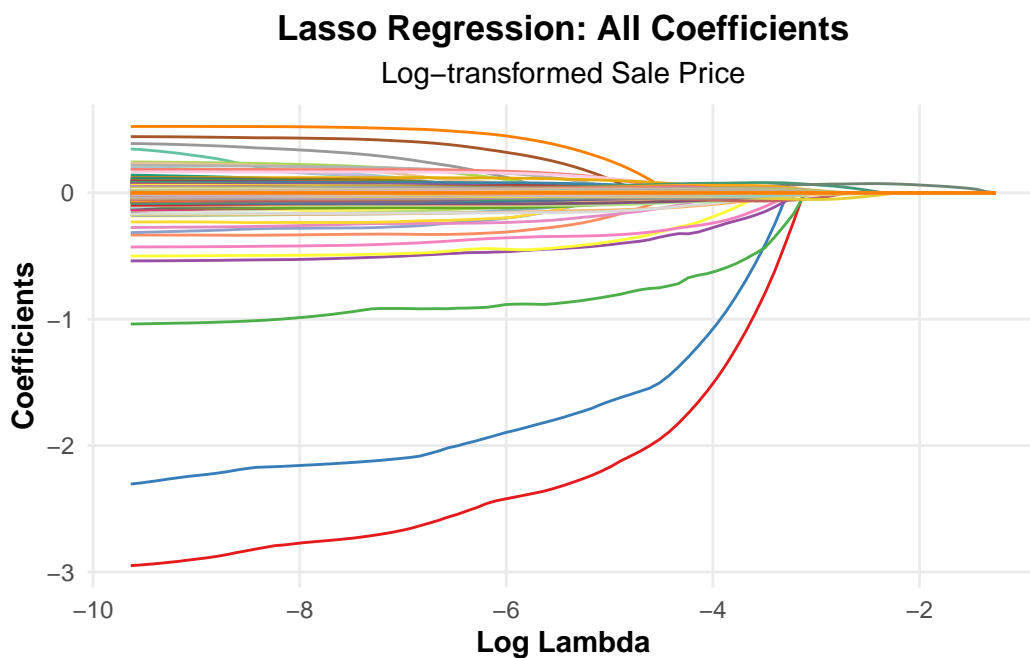
#
plot_data$variable <- factor(plot_data$variable, levels = max_coef$variable)

n_vars <- length(unique(plot_data$variable))
colors <- c(
  brewer.pal(9, "Set1"),
  brewer.pal(8, "Set2"),
  brewer.pal(12, "Set3"),
  brewer.pal(8, "Dark2"),
  brewer.pal(8, "Paired")
)
if(n_vars > length(colors)) {
  colors <- c(colors, colorRampPalette(colors)(n_vars - length(colors)))
}

p <- ggplot(plot_data, aes(x = lambda, y = coefficient, group = variable, color = variable))
  geom_line(size = 0.5) +
  theme_minimal() +
  labs(title = "Lasso Regression: All Coefficients",
        subtitle = "Log-transformed Sale Price",
        x = "Log Lambda",
        y = "Coefficients") +
  theme(
    plot.title = element_text(hjust = 0.5, face = "bold"),
    plot.subtitle = element_text(hjust = 0.5),
    axis.title = element_text(face = "bold"),
    legend.position = "none",
    panel.grid.minor = element_blank(),
    panel.background = element_rect(fill = "white", color = NA),
    plot.background = element_rect(fill = "white", color = NA)
  ) +
  scale_color_manual(values = colors) +
  scale_x_continuous(
    breaks = seq(floor(min(plot_data$lambda)),
                  ceiling(max(plot_data$lambda)),
                  by = 2)
  )

```

```
)
print(p)
```



```
cv_lasso <- cv.glmnet(as.matrix(X_processed), y, alpha = 1, nfolds = 10)
lasso_best_lambda <- cv_lasso$lambda.min

plot_data <- data.frame(
  lambda = cv_lasso$lambda,
  mse = cv_lasso$cvm
)

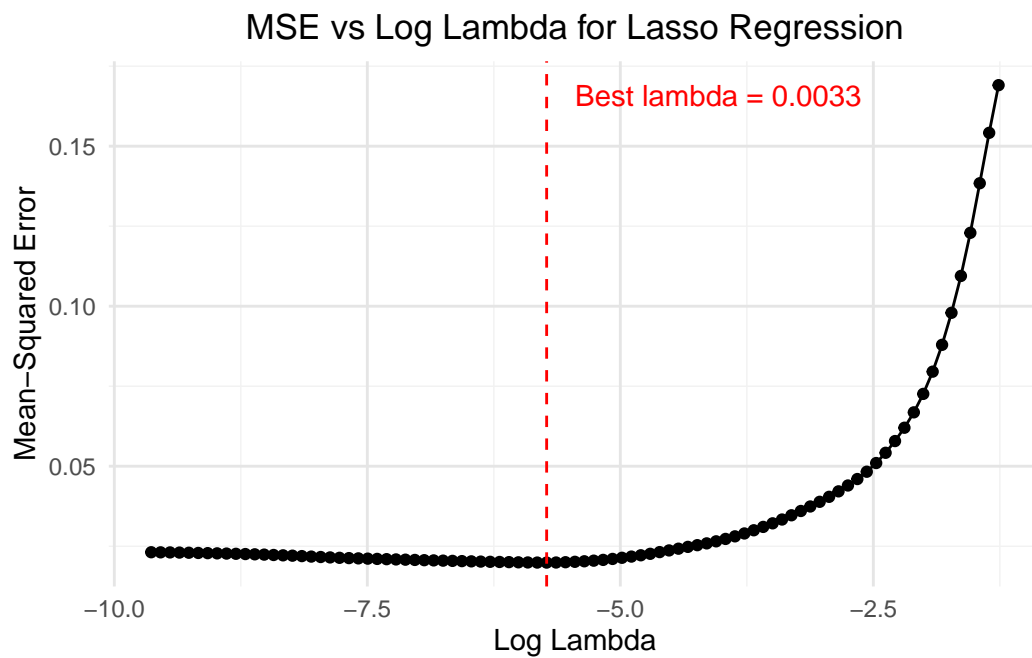
p <- ggplot(plot_data, aes(x = log(lambda), y = mse)) +
  geom_point() +
  geom_line() +
  geom_vline(xintercept = log(lasso_best_lambda), linetype = "dashed", color = "red") +
  annotate("text", x = log(lasso_best_lambda), y = max(plot_data$mse),
    label = paste("Best lambda =", round(lasso_best_lambda, 4)),
    hjust = -0.1, vjust = 1, color = "red") +
  labs(x = "Log Lambda",
    y = "Mean-Squared Error",
    title = "MSE vs Log Lambda for Lasso Regression") +
  theme_minimal() +
```

```

theme(
  panel.background = element_rect(fill = "white", color = NA),
  plot.background = element_rect(fill = "white", color = NA),
  panel.grid.major = element_line(color = "grey90"),
  panel.grid.minor = element_line(color = "grey95"),
  plot.title = element_text(hjust = 0.5)
)

print(p)

```



```

#  LASSO
lasso_model <- glmnet(as.matrix(X_processed), y, alpha = 1)

#  lambda
nzero_coef <- data.frame(
  lambda = lasso_model$lambda,
  nzero = lasso_model$df
)

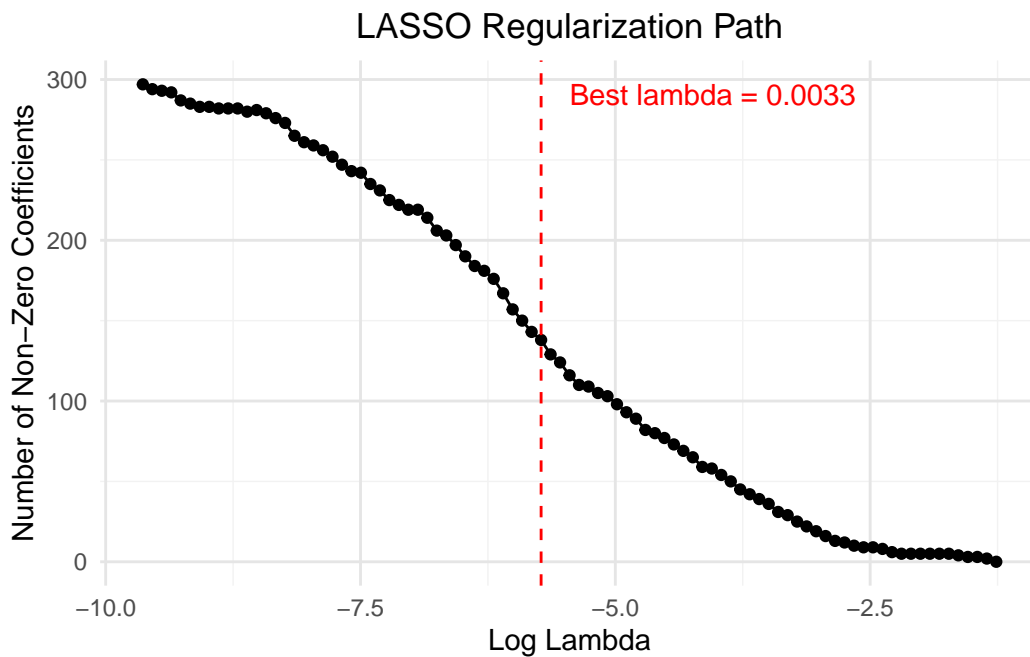
#
p <- ggplot(nzero_coef, aes(x = log(lambda), y = nzero)) +
  geom_line() +

```

```

geom_point() +
labs(x = "Log Lambda",
     y = "Number of Non-Zero Coefficients",
     title = "LASSO Regularization Path") +
theme_minimal() +
theme(
  panel.background = element_rect(fill = "white", color = NA),
  plot.background = element_rect(fill = "white", color = NA),
  panel.grid.major = element_line(color = "grey90"),
  panel.grid.minor = element_line(color = "grey95"),
  plot.title = element_text(hjust = 0.5)
)
p <- p +
  geom_vline(xintercept = log(cv_lasso$lambda.min), linetype = "dashed", color = "red") +
  annotate("text", x = log(cv_lasso$lambda.min), y = max(nzero_coef$nzero),
         label = paste("Best lambda =", round(lasso_best_lambda, 4)), hjust = -0.1, vjust =
  )
print(p)

```



Elastic Net

```
# Elastic Net
elastic_net_model <- glmnet(as.matrix(X_processed), y, alpha = 0.75)

#
coef_matrix <- coef(elastic_net_model)[-1, ]

# lambda
lambda_values <- elastic_net_model$lambda

#
plot_data <- data.frame(
  lambda = rep(log(lambda_values), each = nrow(coef_matrix)),
  coefficient = as.vector(coef_matrix),
  variable = rep(rownames(coef_matrix), times = ncol(coef_matrix))
)

#
max_coef <- plot_data %>%
  group_by(variable) %>%
  summarise(max_abs_coef = max(abs(coefficient))) %>%
  arrange(desc(max_abs_coef))

#
plot_data$variable <- factor(plot_data$variable, levels = max_coef$variable)

#
n_vars <- length(unique(plot_data$variable))
colors <- c(
  brewer.pal(9, "Set1"),
  brewer.pal(8, "Set2"),
  brewer.pal(12, "Set3"),
  brewer.pal(8, "Dark2"),
  brewer.pal(8, "Paired")
)
if(n_vars > length(colors)) {
  colors <- c(colors, colorRampPalette(colors)(n_vars - length(colors)))
}

# ggplot
```

```

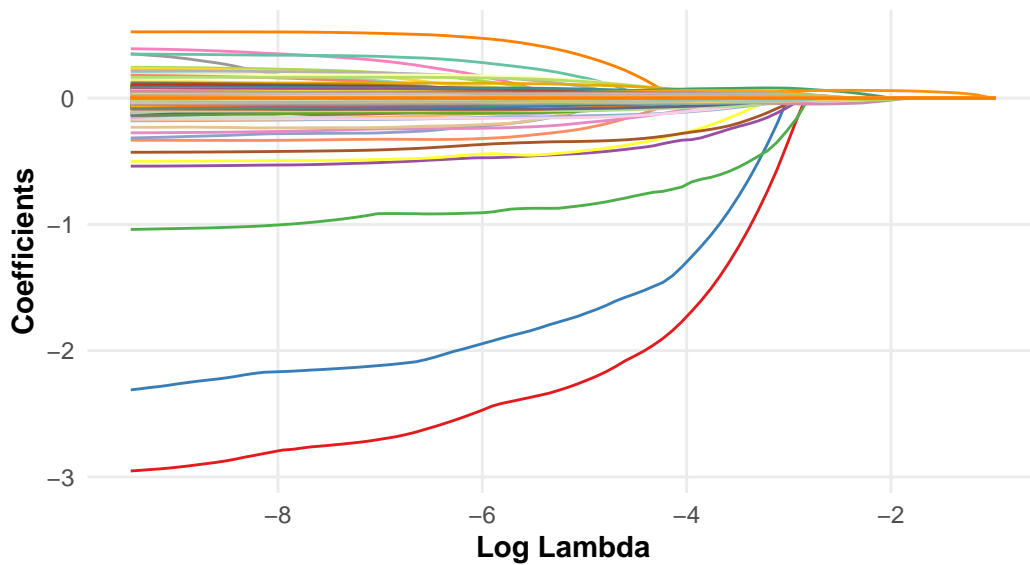
p <- ggplot(plot_data, aes(x = lambda, y = coefficient, group = variable, color = variable))
  geom_line(size = 0.5) +
  theme_minimal() +
  labs(title = "Elastic Net Regression: All Coefficients",
        subtitle = "Log-transformed Sale Price (alpha = 0.75)",
        x = "Log Lambda",
        y = "Coefficients") +
  theme(
    plot.title = element_text(hjust = 0.5, face = "bold"),
    plot.subtitle = element_text(hjust = 0.5),
    axis.title = element_text(face = "bold"),
    legend.position = "none",
    panel.grid.minor = element_blank(),
    panel.background = element_rect(fill = "white", color = NA),
    plot.background = element_rect(fill = "white", color = NA)
  ) +
  scale_color_manual(values = colors) +
  scale_x_continuous(
    breaks = seq(floor(min(plot_data$lambda)),
                  ceiling(max(plot_data$lambda)),
                  by = 2)
  )

#
print(p)

```

Elastic Net Regression: All Coefficients

Log-transformed Sale Price (alpha = 0.75)



```
# Elastic Net
elastic_net_model <- glmnet(as.matrix(X_processed), y, alpha = 0.25)

#
coef_matrix <- coef(elastic_net_model)[-1, ]

# lambda
lambda_values <- elastic_net_model$lambda

#
plot_data <- data.frame(
  lambda = rep(log(lambda_values), each = nrow(coef_matrix)),
  coefficient = as.vector(coef_matrix),
  variable = rep(rownames(coef_matrix), times = ncol(coef_matrix))
)

#
max_coef <- plot_data %>%
  group_by(variable) %>%
  summarise(max_abs_coef = max(abs(coefficient))) %>%
  arrange(desc(max_abs_coef))

#
```



```

plot_data$variable <- factor(plot_data$variable, levels = max_coef$variable)

#
n_vars <- length(unique(plot_data$variable))
colors <- c(
  brewer.pal(9, "Set1"),
  brewer.pal(8, "Set2"),
  brewer.pal(12, "Set3"),
  brewer.pal(8, "Dark2"),
  brewer.pal(8, "Paired")
)
if(n_vars > length(colors)) {
  colors <- c(colors, colorRampPalette(colors)(n_vars - length(colors)))
}

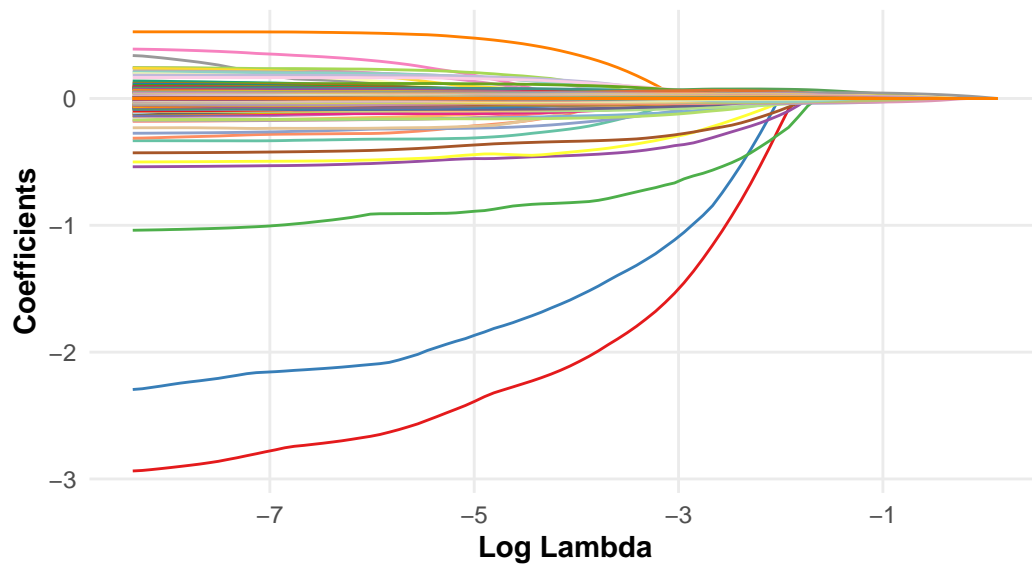
# ggplot
p <- ggplot(plot_data, aes(x = lambda, y = coefficient, group = variable, color = variable))
  geom_line(size = 0.5) +
  theme_minimal() +
  labs(title = "Elastic Net Regression: All Coefficients",
        subtitle = "Log-transformed Sale Price (alpha = 0.25)",
        x = "Log Lambda",
        y = "Coefficients") +
  theme(
    plot.title = element_text(hjust = 0.5, face = "bold"),
    plot.subtitle = element_text(hjust = 0.5),
    axis.title = element_text(face = "bold"),
    legend.position = "none",
    panel.grid.minor = element_blank(),
    panel.background = element_rect(fill = "white", color = NA),
    plot.background = element_rect(fill = "white", color = NA)
  ) +
  scale_color_manual(values = colors) +
  scale_x_continuous(
    breaks = seq(floor(min(plot_data$lambda)),
                  ceiling(max(plot_data$lambda)),
                  by = 2)
  )

#
print(p)

```

Elastic Net Regression: All Coefficients

Log-transformed Sale Price (alpha = 0.25)



```
library(tidyverse)
```

Warning: 'tidyverse' R 4.2.3

Warning: 'tibble' R 4.2.3

Warning: 'tidyr' R 4.2.3

Warning: 'readr' R 4.2.3

Warning: 'purrr' R 4.2.2

Warning: 'stringr' R 4.2.3

Warning: 'forcats' R 4.2.3

Warning: 'lubridate' R 4.2.3

```
-- Attaching core tidyverse packages ----- tidyverse 2.0.0 --
v forcats 1.0.0      v stringr 1.5.0
v lubridate 1.9.3    v tibble 3.2.1
v purrr 1.0.1       v tidyr 1.3.0
v readr 2.1.5

-- Conflicts ----- tidyverse_conflicts() --
x tidyr::expand()      masks Matrix::expand()
x tidyr::extract()     masks magrittr::extract()
x dplyr::filter()      masks stats::filter()
x dplyr::lag()         masks stats::lag()
x purrr::lift()        masks caret::lift()
x tidyr::pack()        masks Matrix::pack()
x purrr::set_names()   masks magrittr::set_names()
x Hmisc::src()         masks dplyr::src()
x Hmisc::summarize()   masks dplyr::summarize()
x tidyr::unpack()      masks Matrix::unpack()
i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become
```

```
set.seed(123)

# fold_id
nfolds <- 10
fold_id <- sample(rep(1:nfolds, length.out = nrow(X_processed)))

#
tuning_grid <- expand_grid(
  alpha = seq(0, 1, by = .1),
  mse_min = NA,
  mse_1se = NA,
  lambda_min = NA,
  lambda_1se = NA
)

# alpha
for(i in seq_along(tuning_grid$alpha)) {
  #
  fit <- cv.glmnet(
    x = as.matrix(X_processed),
    y = y,
    alpha = tuning_grid$alpha[i],
    foldid = fold_id,
    nfolds = nfolds
  )
}
```

```

)

# MSE lambda
tuning_grid$mse_min[i] <- min(fit$cvm)
tuning_grid$mse_1se[i] <- fit$cvm[fit$lambda == fit$lambda.1se]
tuning_grid$lambda_min[i] <- fit$lambda.min
tuning_grid$lambda_1se[i] <- fit$lambda.1se
}

#
print(tuning_grid)

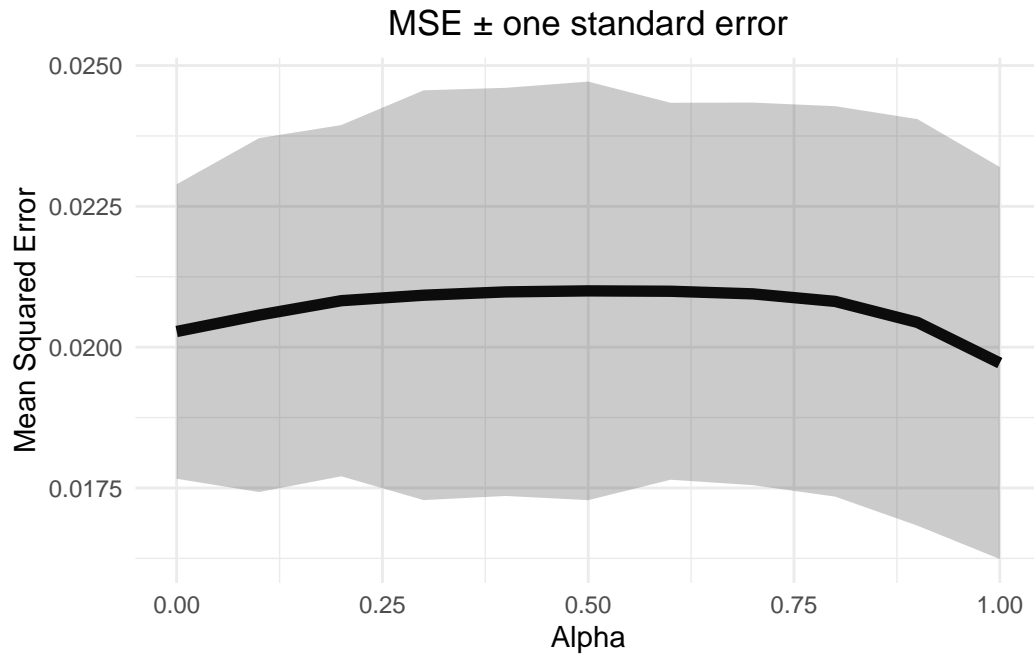
```

	alpha	mse_min	mse_1se	lambda_min	lambda_1se
1	0.0	0.02027839	0.02289314	0.181871094	0.734216744
2	0.1	0.02056912	0.02371082	0.029640457	0.109028856
3	0.2	0.02082546	0.02394396	0.014820228	0.054514428
4	0.3	0.02092240	0.02456036	0.009880152	0.039886345
5	0.4	0.02098157	0.02460525	0.006751820	0.029914759
6	0.5	0.02099941	0.02471517	0.005401456	0.023931807
7	0.6	0.02099293	0.02433927	0.004501213	0.018171476
8	0.7	0.02094646	0.02434270	0.003858183	0.015575551
9	0.8	0.02081266	0.02427863	0.003375910	0.013628607
10	0.9	0.02043994	0.02404927	0.003000809	0.012114317
11	1.0	0.01971851	0.02319865	0.002460803	0.009934303

```

# MSE ±
tuning_grid %>%
  mutate(se = mse_1se - mse_min) %>%
  ggplot(aes(alpha, mse_min)) +
  geom_line(size = 2) +
  geom_ribbon(aes(ymin = mse_min - se, ymax = mse_min + se), alpha = .25) +
  ggtitle("MSE ± one standard error") +
  xlab("Alpha") +
  ylab("Mean Squared Error") +
  theme_minimal() +
  theme(plot.title = element_text(hjust = 0.5)) #

```



```
#   alpha  lambda
best_model <- tuning_grid %>%
  filter(mse_min == min(mse_min))

cat("Best alpha:", best_model$alpha, "\n")
```

Best alpha: 1

```
cat("Best lambda (min MSE):", best_model$lambda_min, "\n")
```

Best lambda (min MSE): 0.002460803

```
cat("Best lambda (1SE rule):", best_model$lambda_1se, "\n")
```

Best lambda (1SE rule): 0.009934303

Prediction

```
library(Metrics)
```

Warning: 'Metrics' R 4.2.3

```
'Metrics'
```

```
'package:caret':
```

```
precision, recall
```

```
X_test <- ames_test[, -which(names(ames_test) == "Sale_Price")]
y_test <- log(ames_test$Sale_Price) # Sale_Price

# dummy
X_dummy_test <- predict(dummy, newdata = X_test[, categorical_vars])
X_numeric_standardized_test <- scale(X_test[, numeric_vars])
X_processed_test_lm <- cbind(X_numeric_standardized_test, X_dummy_test)
X_processed_test <- cbind(X_test[, numeric_vars], X_dummy_test)

#
X_processed_test <- X_processed_test[, colnames(X_processed)]
lm_model <- lm(y ~ ., data = as.data.frame(X_processed_lm))

# lambda Ridge Lasso
ridge_model <- glmnet(as.matrix(X_processed), y, alpha = 0, lambda = ridge_best_lambda)
lasso_model <- glmnet(as.matrix(X_processed), y, alpha = 1, lambda = lasso_best_lambda)

#
X_processed_test <- as.matrix(X_processed_test)

#
lm_predictions <- predict(lm_model, newdata = as.data.frame(X_processed_test_lm))
```

Warning in predict.lm(lm_model, newdata = as.data.frame(X_processed_test_lm)):

```

# Ridge
ridge_predictions <- predict(ridge_model, newx = X_processed_test)

# Lasso
lasso_predictions <- predict(lasso_model, newx = X_processed_test)

# MSE
lm_mse <- mean((exp(y_test) - exp(lm_predictions))^2)
ridge_mse <- mean((exp(y_test) - exp(ridge_predictions))^2)
lasso_mse <- mean((exp(y_test) - exp(lasso_predictions))^2)

# R-squared
lm_r_squared <- cor(exp(y_test), exp(lm_predictions))^2
ridge_r_squared <- cor(exp(y_test), exp(ridge_predictions))^2
lasso_r_squared <- cor(exp(y_test), exp(lasso_predictions))^2

#
cat("Linear Regression:\n")

```

Linear Regression:

```
cat(paste("MSE:", lm_mse, "\n"))
```

MSE: 1251020856.97055

```
cat(paste("R-squared:", lm_r_squared, "\n\n"))
```

R-squared: 0.824783980714906

```
cat("Ridge Regression:\n")
```

Ridge Regression:

```
cat(paste("MSE:", ridge_mse, "\n"))
```

MSE: 757893241.524645

```
cat(paste("R-squared:", ridge_r_squared, "\n\n"))
```

R-squared: 0.879322121231107

```
cat("Lasso Regression:\n")
```

Lasso Regression:

```
cat(paste("MSE:", lasso_mse, "\n"))
```

MSE: 1027655818.72105

```
cat(paste("R-squared:", lasso_r_squared, "\n"))
```

R-squared: 0.843884072404135