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# CAN CONSUMERS DISTINGUISH PERSISTENT FROM TRANSITORY INCOME SHOCKS?*

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The degree to which consumers can distinguish persistent from transitory income shocks is paramount for consumption-saving dynamics. In particular, even a small amount of imperfect information causes a severe bias in conventional estimators of the marginal propensity to consume. We provide a novel method that can identify consumers' degree of information by using panel data on income and consumption, even allowing for measurement error. Employing our method to data from the Panel Study of Income Dynamics, we find that households have almost perfect information. This robust result indicates that the conventional estimators of the marginal propensity to consume are on firm ground.

The degree to which households can distinguish persistent from transitory income shocks has important implications for our understanding of consumption-saving behaviour (Blundell, 2014, p. 312). When households cannot perfectly distinguish persistent from transitory income shocks it amplifies the co-movements of consumption growth with current and future income growth. Conventional estimators of the marginal propensity to consume (MPC), such as those put forward in Blundell *et al.* (2008) (henceforth BPP) and Kaplan and Violante (2010), mistakenly interpret this as evidence of a higher MPC.[1] The assumption of perfect information is seldom stated explicitly, and has never been tested empirically. We are thus the first to provide a method for identifying households' degree of information.

We assume that households need to solve a filtering problem to infer the persistent component of their income path. To do this they use their actual income path and a noisy private signal. This allows us to consider a continuum of cases. When the private signal is noiseless, households perfectly know the persistent component of their income, and can perfectly distinguish persistent from transitory shocks. When the private signal is infinitely noisy, households have the same information set as an econometrician.

Under the stark assumption of quadratic utility (certainty equivalence), we show that households' degree of information can be identified in closed form, using covariances of consumption growth and income growth. This also holds under unknown measurement error in consumption.

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[1] Throughout, we refer to the marginal propensity to consume out of a transitory shock as the MPC and the marginal propensity to consume out of a permanent shock as the MPCP. For more on this type of estimators, see also Blundell *et al.* (2013) and the surveys in Jappelli and Pistaferri (2010) and Meghir and Pistaferri (2011).

With unknown measurement error in income, we can derive lower and upper bounds on the degree of information.[2]

With a more standard assumption of constant relative risk aversion (CRRA) utility, we show that households' degree of information can be estimated using a simulated method of moments (SMM) estimator targeting the same covariances of consumption growth and income growth, on which our closed-form estimator relies. We show this in a Monte Carlo study using the canonical buffer-stock model of Deaton (1991; 1992) and Carroll (1992; 1997; 2012), extended to allow for both persistent rather than fully permanent shocks, and an MA(1) term in the income process. We jointly estimate the degree of information together with the preferences and the income process parameters, and therefore also target autocovariances of income growth and life-cycle profiles of consumption and income. Surprisingly, the closed-form estimator derived based on the certainty equivalence model also delivers reasonable estimates when used on data simulated from this more general model.

We finally apply both of our estimators to data on income and consumption from the Panel Study of Income Dynamics (PSID). We use exactly the same sample as BPP. We find that the PSID households know the persistent component of their own income path perfectly, and that they can thus fully distinguish persistent from transitory shocks. This result is robust to a wide range of specifications of the income process and allowing for measurement error in both income and consumption. We also find no evidence of differences in the degree of information across different educational groups.

To understand our empirical result, it is instructive to discuss identification. First, note that the parameters of the income process are separately identified from the autocovariances of income growth and the life-cycle profile of income (Hryshko, 2012), while the preference parameters are separately identified from the life-cycle profile of consumption (Gourinchas and Parker, 2002). The moments pinning down the degree of information are therefore primarily the covariances between consumption growth and current and future income growth. Under the assumption of perfect information, the covariances implied by the estimated model are numerically slightly larger than those in the PSID data. But reducing the degree of information only amplifies them, and thus deteriorates the fit of the model. Only when we force households to be more patient than what is consistent with the life-cycle profile of consumption in the PSID do we find evidence of imperfect information. The reason is that the co-movements between consumption growth and income growth are otherwise too weak.

One caveat with respect to our empirical result is that the MPC estimated in BPP (and therefore in our sample) is lower than the evidence from surveys and quasi-experimental studies suggests.[3] Why this discrepancy exists remains an open question, but it could indicate that the covariances of consumption growth and income growth that we use to estimate the degree of information could change substantially if we had better data. Commault (2017) proved that the BPP estimator is biased when the income process contains an MA(1) term. Our results are, however, robust to including an MA(1) term in the income process.[4]

After discussing the related literature below, the rest of the article is organised as follows. Subsection 1.5 introduces our analytical framework with certainty equivalence and presents our

[2] Point identification cannot be achieved because, as previously noted in the literature, the variance of the transitory shocks is not point identified with measurement error in income.

[3] See, e.g., Shapiro and Slemrod (2003), Johnson *et al.* (2006), Agarwal and Souleles (2007), Parker *et al.* (2013), Misra and Surico (2014) and Fagereng *et al.* (2017).

[4] Using our model, we further show that the bias implied by the MA(1) term is quantitatively small in practice.

closed-form identification results. Section 2 presents our more general consumption-saving model and our simulated method of moments estimator together with Monte Carlo results. Section 3 presents our empirical investigation using PSID data. Section 4 concludes.

**Related literature.** The notion that households might not have perfect information about the components of their own income process goes back to at least Muth (1960). Pischke (1995) analysed the case in which the households have access to less information than the econometrician about the aggregate state of the economy, but observe their own income shocks perfectly.[5] Wang (2004; 2009) show in a continuous time model with constant absolute risk aversion (CARA) preferences that imperfect information about permanent income implies larger precautionary saving. Blundell and Preston (1998) and Cunha *et al.* (2004), also both briefly discuss the possibility that households cannot perfectly distinguish persistent from transitory shocks.

Our study is also related to the seminal papers by Guvenen (2007) and Guvenen and Smith (2014) studying consumption-saving behaviour when households are gradually learning their individual-specific income growth rate. They assume that households have only imperfect information about their own latent income growth rate in the form of an initial private signal, and otherwise need to infer it from the realisation of their income path. This implies that the standard case, with homogeneous growth rates and perfect information about the persistent component of the income process, is not nested in their specification.[6] Our article does not consider heterogeneous growth rates, but focuses on the learning of the persistent component of income given a period-by-period private signal. Hereby the standard case with perfect information is nested as a special case in our framework.

Finally, our article is related to Pistaferri (2001) and Kaufmann and Pistaferri (2009), who use subjective belief data to identify respectively permanent and transitory shocks. Their identification strategy relies on the assumption that the households know their permanent income component.

## 1. Framework and Identification

In this section, we use a certainty equivalence consumption-saving model to build intuition on how households behave when they cannot perfectly distinguish permanent from transitory income shocks. We consider a continuum of cases from households perfectly knowing their permanent income, and thus the composition of their income shocks, to households having the same information set as an econometrician observing only realised income. We show that point identification of households' degree of information can be achieved with panel data on income and consumption allowing for an unknown degree of measurement error in consumption. When also allowing for measurement error in income, we can derive lower and upper bounds on households' degree of information.

### 1.1. *Household Problem*

We assume that households have quadratic utility, like Hall (1978), face a permanent-transitory income process, and observe only actual income, $y_t$, and a noisy private signal of permanent income, $z_t$. Specifically, households solve

[5] Goodfriend (1992) investigates aggregation bias when households face an income process consisting of an individual component and an aggregate component observed with a lag.

[6] In turn, the models in Guvenen (2007) and Guvenen and Smith (2014) cannot answer the question posed in the current article.

$$U = \max_{c_t, c_{t+1}, \ldots} \tilde{\mathbb{E}}_t \sum_{k=0}^{\infty} \beta^k \left[ \alpha c_{t+k} - \gamma \frac{c_{t+k}^2}{2} \right], \quad \alpha > 0, \gamma > 0 \tag{1}$$

s.t.

$$a_t = R(a_{t-1} + y_t - c_t), \ R > 0, \tag{2}$$

$$\lim_{t \to \infty} R^{-t} a_t \geq 0, \tag{3}$$

$$y_t = p_t + \xi_t, \qquad \xi_t \sim \mathcal{N}(0, \sigma_\xi^2), \quad \sigma_\xi > 0, \tag{4}$$

$$p_t = p_{t-1} + \psi_t, \quad \psi_t \sim \mathcal{N}(0, \sigma_\psi^2), \ \sigma_\psi > 0, \tag{5}$$

$$z_t = p_t + \epsilon_t, \qquad \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2), \quad \sigma_\epsilon \geq 0, \tag{6}$$

where (2) is the budget constraint and (3) is the No-Ponzi game condition, (4)–(5) are the permanent-transitory income process, $z_t$ in (4) is the private signal, and the expectations operator $\tilde{\mathbb{E}}_t$ is conditional on the history of actual income and the private signal, i.e.,

$$\tilde{\mathbb{E}}_t[\bullet] \equiv \mathbb{E}[\bullet \mid y_t, y_{t-1}, \ldots, z_t, z_{t-1}, \ldots]. \tag{7}$$

If $\sigma_\epsilon = 0$ we are in the standard case with perfect information, $z_t = p_t$, and households can perfectly distinguish permanent from transitory income shocks. For $\sigma_\epsilon > 0$ households need to solve a filtering problem to form beliefs about their permanent income and the composition of income shocks. Because the income process is linear-Gaussian it is optimal for a household to infer its level of permanent income by the Kalman filter.[7] We denote a household's mean belief of $p_t$ by $\hat{p}_t$, and the variance of its mean belief by $\hat{q}_t$.

A useful property of the Kalman filter is that the distribution of mean-belief errors, $\hat{p}_t - p_t$, is mean-zero and is Gaussian with the same variance as the mean belief, i.e.:

$$\kappa_t \equiv \hat{p}_t - p_t \sim \mathcal{N}(0, \hat{q}_t), \tag{8}$$

and uncorrelated with future shocks,

$$\forall k > 0: \ \text{cov}(\kappa_t, \psi_{t+k}) = \text{cov}(\kappa_t, \xi_{t+k}) = \text{cov}(\kappa_t, \epsilon_{t+k}) = 0. \tag{9}$$

In Subsection 1.2, we present central properties of the Kalman filter updating process before turning to the implied consumption-saving behaviour.

### 1.2. *Updating Beliefs*

Given period $t$ information the best predictions of $t+1$ variables are

$$\hat{p}_{t+1|t} = \hat{p}_t, \tag{10}$$

$$\hat{y}_{t+1|t} = \hat{p}_{t+1|t}, \tag{11}$$

$$\hat{z}_{t+1|t} = \hat{p}_{t+1|t}, \tag{12}$$

[7] For a general treatment of the Kalman filter, see, e.g., Hamilton (1994).

$$\hat{q}_{t+1|t} = \hat{q}_t + \sigma_\psi^2. \tag{13}$$

The mean belief is optimally updated as

$$\hat{p}_{t+1} = \hat{p}_{t+1|t} + \boldsymbol{K}_{t+1}\boldsymbol{\Delta}_{t+1}, \tag{14}$$

where $\boldsymbol{\Delta}_{t+1}$ is the vector of prediction errors given by

$$\boldsymbol{\Delta}_{t+1} \equiv \begin{bmatrix} y_{t+1} \\ z_{t+1} \end{bmatrix} - \begin{bmatrix} \hat{y}_{t+1|t} \\ \hat{z}_{t+1|t} \end{bmatrix}, \tag{15}$$

and $\boldsymbol{K}_{t+1}$ is the optimal Kalman gain vector given by

$$\begin{aligned} \boldsymbol{K}_{t+1} &\equiv \hat{q}_{t+1|t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}' \boldsymbol{S}_{t+1}^{-1} \qquad (16) \\ &= \frac{\hat{q}_t + \sigma_\psi^2}{(\sigma_\xi^2 + \sigma_\epsilon^2)(\hat{q}_t + \sigma_\psi^2) + \sigma_\xi^2\sigma_\epsilon^2} \begin{bmatrix} \sigma_\epsilon^2 & \sigma_\xi^2 \end{bmatrix}, \end{aligned}$$

with

$$\boldsymbol{S}_{t+1} \equiv \begin{bmatrix} 1 \\ 1 \end{bmatrix} \hat{q}_{t+1|t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}' + \begin{bmatrix} \sigma_\xi^2 & 0 \\ 0 & \sigma_\epsilon^2 \end{bmatrix}. \tag{17}$$

The variance of the mean belief is updated optimally as

$$\hat{q}_{t+1} = \left(1 - \boldsymbol{K}_{t+1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) \hat{q}_{t+1|t}. \tag{18}$$

**Steady state.** We can solve for the steady-state value of $\hat{q}_t$ in (18) as

$$q^\star = \left(\sqrt{\frac{\sigma_\xi^2\sigma_\epsilon^2}{(\sigma_\xi^2 + \sigma_\epsilon^2)\sigma_\psi^2} + \frac{1}{4}} - \frac{1}{2}\right)\sigma_\psi^2. \tag{19}$$

The steady-state Kalman gain vector consequently is

$$\boldsymbol{K}^\star \equiv \begin{bmatrix} K_1^\star & K_2^\star \end{bmatrix} = \begin{bmatrix} q^\star\sigma_\xi^{-2} & q^\star\sigma_\epsilon^{-2} \end{bmatrix}. \tag{20}$$

Lemma 1 presents some important properties of the steady-state variance of the mean belief and Kalman gain vector. Note in particular that when all the noise in the private signal disappears, $\sigma_\epsilon \to 0$, we have perfect information and get $\hat{p}_t = p_t$ and $\hat{q}_t = 0$. On the other hand, the variance of the mean belief, $\hat{q}_t$, reaches an upper bound when the private signal becomes infinitely noisy, $\sigma_\epsilon \to \infty$. Note also that the sum of the elements of the Kalman gain vector, $\mathcal{K}^\star \equiv K_1^\star + K_2^\star$, is one in the perfect information case, and gradually declines towards a positive constant as $\sigma_\epsilon \to \infty$.

LEMMA 1. *$q^\star$ and $\boldsymbol{K}^\star$ have the following properties:*

(*1*) *The steady-state variance of the mean belief is bounded by the minimum of the transitory shock variance and the variance of the noise in the private signal, and the steady-state Kalman gains are always positive and sum to weakly less than one,*

$$q^\star \leq \min\{\sigma_\xi^2, \sigma_\epsilon^2\}, \tag{21}$$

$$K_1^\star, K_2^\star \geq 0, \tag{22}$$

$$\mathcal{K}^\star \equiv K_1^\star + K_2^\star \leq 1. \tag{23}$$

(2) *In the limit as all noise in the private signal disappears (perfect information), $\sigma_\epsilon \to 0$, the variance of the mean belief collapses and all of the Kalman gain is placed on the noiseless private signal,*

$$\lim_{\sigma_\epsilon \to 0} q^\star = 0, \tag{24}$$

$$\lim_{\sigma_\epsilon \to 0} \boldsymbol{K}^\star = \begin{bmatrix} 0 & 1 \end{bmatrix}, \tag{25}$$

$$\lim_{\sigma_\epsilon \to 0} \mathcal{K}^\star = 1, \tag{26}$$

*such that*

$$\hat{p}_{t+1} - \hat{p}_t = z_{t+1} - \hat{p}_t = p_{t+1} - p_t. \tag{27}$$

(3) *In the opposite limit, where the private signal becomes infinitely noisy, $\sigma_\epsilon \to \infty$, the variance of the mean belief reaches an upper bound, and no weight is placed on the private signal*

$$\lim_{\sigma_\epsilon \to \infty} q^\star = \left( \sqrt{\frac{\sigma_\xi^2}{\sigma_\psi^2} + \frac{1}{4}} - \frac{1}{2} \right) \sigma_\psi^2 \equiv \overline{q}^\star > 0, \tag{28}$$

$$\lim_{\sigma_\epsilon \to \infty} \boldsymbol{K}^\star = \begin{bmatrix} \overline{q}^\star \sigma_\xi^{-2} & 0 \end{bmatrix}, \tag{29}$$

$$\lim_{\sigma_\epsilon \to \infty} \mathcal{K}^\star = \overline{q}^\star \sigma_\xi^{-2} \in (0, 1), \tag{30}$$

*such that*

$$\hat{p}_{t+1} - \hat{p}_t = \overline{q}^\star \sigma_\xi^{-2} (y_{t+1} - \hat{p}_t). \tag{31}$$

(4) *The variance of the mean belief and the elements of the Kalman gain changes monotonically with the noise in the private signal as*

$$\frac{\partial q^\star}{\partial \sigma_\epsilon^2} > 0, \tag{32}$$

$$\frac{\partial K_1^\star}{\partial \sigma_\epsilon^2} > 0, \tag{33}$$

$$\frac{\partial K_2^\star}{\partial \sigma_\epsilon^2} < 0, \tag{34}$$

$$\frac{\partial \mathcal{K}^\star}{\partial \sigma_\epsilon^2} < 0. \tag{35}$$

PROOF. Follow from (19) and (20). See Online Appendix A. □

Lemma 2 presents the law of motion for the mean-belief error. An important implication is that the mean-belief errors under imperfect information have a positive autocovariance and are correlated with both past and current shocks. In particular, we see that permanent shocks lead to under-prediction of permanent income ($\frac{\partial \kappa_t}{\partial \psi_t} < 0$) and transitory shocks lead to over-prediction of permanent income ($\frac{\partial \kappa_t}{\partial \xi_t} > 0$). We also have mean reversion in the mean-belief error, $\mathbb{E}[\kappa_t|\kappa_{t-1}] < \kappa_{t-1}$ because $(1 - \mathcal{K}^\star) < 1$ when $\sigma_\epsilon > 0$.

LEMMA 2. *The law of motion for the steady-state mean-belief error is*

$$\kappa_t = (1 - \mathcal{K}^\star)\kappa_{t-1} + (\mathcal{K}^\star - 1)\psi_t + K_1^\star \xi_t + K_2^\star \epsilon_t \tag{36}$$

*and consequently the autocovariance of the mean-belief errors is*

$$\mathrm{cov}(\kappa_t, \kappa_{t-1}) = (1 - \mathcal{K}^\star)q^\star \geq 0. \tag{37}$$

PROOF. Follow from (14) and (8)–(9). See Online Appendix A. □

Note further that despite the fact that the mean-belief errors have a positive autocovariance, the income growth forecast errors, defined as $e_t \equiv (y_{t+1|t} - y_t) - (y_{t+1} - y_t)$, still have zero autocovariance as shown in Lemma 3.

LEMMA 3. *The income growth forecast error is mean zero and has excessive variance, but zero autocovariance.*

(*1*) $e_t \equiv (y_{t+1|t} - y_t) - (y_{t+1} - y_t) = \kappa_t - \psi_{t+1} - \xi_{t+1}$,
(*2*) $\mathbb{E}[e_t] = 0$,
(*3*) $\mathrm{var}(e_t) = q^\star + \sigma_\psi^2 + \sigma_\xi^2 \geq \sigma_\psi^2 + \sigma_\xi^2$,
(*4*) $\mathrm{cov}(\mathrm{e_t}, \mathrm{e_{t+k}}) = 0$, *for all* $k > 0$.

PROOF. See Online Appendix A. □

This implies that households' degree of information cannot easily be inferred from survey data on their income growth forecasts. In principle, the variance of the income growth forecast errors could be used, but it would require strong assumptions on the measurement error in the reported forecasts. Here, we instead pursue identification through the use of consumption data.

### 1.3. *Consumption Saving*

We can now derive an analytical formula for the change in consumption under imperfect information. The result is provided in Theorem 1.

THEOREM 1. *Consider a household solving the problem in (1) where the variance of the mean belief has converged to* $q^\star$. *If* $\beta R = 1$ *then*

$$\Delta c_t = \phi_\psi(\psi_t - \kappa_{t-1}) + \phi_\xi \xi_t + \phi_\epsilon \epsilon_t, \tag{38}$$

*where*

$$\phi_\psi \equiv R^{-1}(R - 1 + q^\star(\sigma_\xi^{-2} + \sigma_\epsilon^{-2})), \tag{39}$$

$$\phi_\xi \equiv R^{-1}(R - 1 + q^\star \sigma_\xi^{-2}), \tag{40}$$

$$\phi_\epsilon \equiv R^{-1} q^\star \sigma_\epsilon^{-2}. \tag{41}$$

PROOF. See Online Appendix A. □

Using the consumption growth result in Theorem 1, we can analytically derive the autocovariances of consumption growth and the covariances between consumption growth and past, current and future income growth. These covariances are useful for pinpointing moments that identify the variance of households' private signal, $\sigma_\epsilon^2$, i.e. their degree of information. Corollary 1 shows that the standard result of consumption growth being a random walk is preserved under imperfect information about permanent income.[8] However, the variance of consumption growth, given $R > 1$, is increasing in the noise of the private signal, $\sigma_\epsilon$, through its positive effect on the variance of the mean belief, $q^\star$.

COROLLARY 1. *The variance and autocovariances of consumption growth are*

$$\operatorname{cov}(\Delta c_t, \Delta c_{t+k}) = \begin{cases} \sigma_\psi^2 + \dfrac{(R-1)^2}{R^2}\sigma_\xi^2 + \dfrac{R^2-1}{R^2}q^\star & \text{if } k = 0, \\ 0 & \text{else}, \end{cases}$$

PROOF. Follows from Theorem 1 and Lemma 2. See Online Appendix A. □

Corollary 2 shows the covariances of consumption growth with past, current and future income growth. The standard results that consumption growth is uncorrelated with past income growth (i.e., no excess sensitivity[9]) and future income growth beyond the first lead (i.e., no indication of advance information[10]) is preserved.

The covariance of consumption growth with current income growth, however, increases with the noise of the private signal, $\sigma_\epsilon$, through its positive effect on the variance of the mean belief, $q^\star$. The covariance of consumption growth with next-period income growth, on the other hand, becomes even more negative through the effect of $\sigma_\epsilon$ on the variance of the mean belief $q^\star$ (see Lemma 1).

COROLLARY 2. *The covariances of consumption growth and income growth are*

$$\operatorname{cov}(\Delta c_t, \Delta y_{t+k}) = \begin{cases} -R^{-1}(\sigma_\xi^2(R-1) + q^\star) & \text{if } k = 1, \\ \sigma_\psi^2 + \dfrac{R-1}{R}\sigma_\xi^2 + q^\star & \text{if } k = 0, \\ 0 & \text{else}. \end{cases}$$

PROOF. Follows from Theorem 1 and (4). □

In sum, we have thus shown in Corollary 1 and 2 that three central moments for identifying $\sigma_\epsilon$ are $\operatorname{var}(\Delta c_t)$, $\operatorname{cov}(\Delta c_t, \Delta y_t)$ and $\operatorname{cov}(\Delta c_t, \Delta y_{t+1})$. We return to identification of $\sigma_\epsilon$ after discussing the implications of $\sigma_\epsilon$ on the transmission parameters in (39)–(41).

Gaining some intuition for why the covariance of consumption growth with current income growth increases with the noise of the private signal is beneficial for understanding our identification results. In the proof of Theorem 1, we show that the change in consumption can be written

[8] The random walk result is only broken if we assume that the household does not observe its own income perfectly.

[9] Excess sensitivity can, e.g., be due to liquidity constraints (Flavin, 1981) or precautionary saving (Commault, 2017) which we have both ruled out here. In the general CRRA model studied below we allow for both these elements in the model.

[10] Primiceri and van Rens (2009) and Kaufmann and Pistaferri (2009) argue in favour of advance information about one year ahead shocks. BPP do not find any evidence for advance information.

in terms of a perceived transitory income shock, $y_t - \hat{p}_t$, and a perceived permanent income shock, $\hat{p}_t - \hat{p}_{t-1}$. Specifically,

$$\Delta c_t = \frac{R-1}{R}(y_t - \hat{p}_t) + (\hat{p}_t - \hat{p}_{t-1}), \tag{42}$$

where the first term shows that the household only consumes the annuity value of the perceived transitory income shock, and the second term shows that the household adjusts its consumption level one to one with the perceived permanent shock.

Under perfect information, perceived and actual shocks coincides ($y_t - \hat{p}_t = \xi_t$, $\hat{p}_t - \hat{p}_{t-1} = \psi_t$), and the derivation of $\mathrm{cov}(\Delta c_t, \Delta y_t) = \sigma_\psi^2 + \frac{R-1}{R}\sigma_\xi^2$ is straightforward, remembering $\Delta y_t = \xi_t + \psi_t - \xi_{t-1}$.

The situation is more complicated when information is imperfect. We show in the proof of Corollary 2 that the covariance of consumption growth with current income growth can also be written as

$$\mathrm{cov}(\Delta c_t, \Delta y_t) = \underbrace{\phi_\psi}_{<1} \sigma_\psi^2 + (\underbrace{\phi_\xi + \phi_\psi K_1^\star}_{>\frac{R-1}{R}})\sigma_\xi^2. \tag{43}$$

This seems to indicate an indeterminate result as the coefficient on the permanent variance is reduced, while the coefficient on the transitory variance is increased. In the final analysis, however, the second effect always dominates, and the covariance of consumption growth with current income growth increases with the noise of the private signal as shown in Corollary 2. When the private signal contains valuable information ($\sigma_\epsilon < \infty$) the household updates its belief regarding its permanent level of income partly orthogonal to changes in actual income. By (42), this induces changes in consumption which are partly orthogonal to changes in income. When the private signal looses its informational content ($\sigma_\epsilon \to \infty$), the household begins to update its beliefs regarding its level of permanent income solely based on the information contained in actual income changes (see (31)). Fewer and fewer changes in consumption are then orthogonal to changes in income, which can explain why the covariance of consumption growth with current income growth increases with the noise of the private signal.

## 1.4. *Transmission Parameters*

The parameters $\phi_\psi$, $\phi_\xi$ and $\phi_\epsilon$ in the consumption growth equation (38) are informative with respect to how consumption responds to different shocks. In particular, $\phi_\psi$ and $\phi_\xi$ have similar interpretations as the transmission parameters estimated in BPP.

Corollary 3 shows that in the limit with perfect information, $\sigma_\epsilon \to 0$, the household optimally responds one to one to permanent shocks, $\phi_\psi = 1$, and only marginally to transitory shocks, $\phi_\xi = \frac{R-1}{R}$. This is the standard result from BPP. In the limit where the information sets of the household and the econometrician coincide, $\sigma_\epsilon \to \infty$, we instead have that $\phi_\psi = \phi_\xi$ such that the transmission parameters to the permanent and transitory shocks are the same. Furthermore, when the private signal is infinitely noisy the household cannot use it to improve its estimate of its permanent income and the random draw of the private signal does therefore not affect consumption choices, explaining why $\lim_{\sigma_\epsilon \to \infty} \phi_\epsilon = 0$.

Note, however, that when $\sigma_\epsilon > 0$ the interpretation of $\phi_\psi$ and $\phi_\xi$ as transmission parameters is not correct in the usual sense because the household cannot distinguish transitory and permanent shocks. Specifically, we always have, irrespective of $\sigma_\epsilon$, that the marginal propensity to consume

(MPC) out of a windfall known to be transitory (such as a lottery win or a tax rebate) is $\frac{R-1}{R}$ and the marginal propensity to consume out of a windfall known to be permanent (MPCP) is 1. On the other hand both $\phi_\psi$ and $\phi_\xi$ vary systematically with $\sigma_\epsilon$. Imperfect information thus opens up a wedge between the MPC and MPCP, which are of great importance to policymakers, and the respective transmission parameters.

COROLLARY 3. *The transmission parameters $\phi_\psi$, $\phi_\xi$ and $\phi_\epsilon$ vary with $\sigma_\epsilon$ as follows:*

(*1*) *For $\phi_\psi$ we have*

$$\lim_{\sigma_\epsilon \to 0} \phi_\psi = 1,$$
$$\lim_{\sigma_\epsilon \to \infty} \phi_\psi = R^{-1}(R - 1 + \overline{q}^\star \sigma_\xi^{-2}),$$
$$\frac{\partial \phi_\psi}{\partial \sigma_\epsilon} < 0.$$

(*2*) *For $\phi_\xi$ we have*

$$\lim_{\sigma_\epsilon \to 0} \phi_\xi = \frac{R-1}{R},$$
$$\lim_{\sigma_\epsilon \to \infty} \phi_\xi = R^{-1}(R - 1 + \overline{q}^\star \sigma_\xi^{-2}),$$
$$\frac{\partial \phi_\xi}{\partial \sigma_\epsilon} > 0.$$

(*3*) *For $\phi_\epsilon$ we have*

$$\lim_{\sigma_\epsilon \to 0} \phi_\epsilon = \frac{1}{R},$$
$$\lim_{\sigma_\epsilon \to \infty} \phi_\epsilon = 0,$$
$$\frac{\partial \phi_\epsilon}{\partial \sigma_\epsilon} < 0.$$

PROOF. Follows from Theorem 1 and Lemma 1. See Online Appendix A. □

BPP showed that the transmission parameters are identified under the assumption that the household has perfect information about its permanent income, $\sigma_\epsilon = 0$. Using the same moment condition, the parameter $\phi_\xi$ can still be recovered under the assumption of imperfect information. This follows from (44) in Corollary 4. Note, however, that Theorem 1 shows that $\phi_\xi$ is in general *not* equal to the marginal propensity to consume out of a transitory windfall, such as a tax rebate, which might be of considerable importance to policy makers.

The coefficient $\phi_\psi$ can, however, not be recovered using the moment condition suggested by BPP when information is imperfect. The reason is that the lagged shocks have a non-zero covariance with the lagged prediction error, $\kappa_{t-1}$, present in the equation for the change in consumption, (38). Specifically, the BPP estimate of $\phi_\psi$ is upwards biased as seen in (45) in Corollary 4.

COROLLARY 4. *Using the moments proposed by BPP to estimate the transmission parameters, we get*

$$\hat{\phi}_{\xi}^{BPP} \equiv \frac{\mathrm{cov}(\Delta c_t, -\Delta y_{t+1})}{\mathrm{cov}(\Delta y_t, -\Delta y_{t+1})} = \phi_{\xi}, \tag{44}$$

*and*

$$\begin{aligned} \hat{\phi}_{\psi}^{BPP} &\equiv \frac{\mathrm{cov}(\Delta c_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})}{\mathrm{cov}(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})} \\ &= \phi_{\psi}\left[1 + (1 - \mathcal{K}^{\star})\left(1 + K_1^{\star}\frac{\sigma_{\xi}^2}{\sigma_{\psi}^2}\right)\right] > \phi_{\psi}. \end{aligned} \tag{45}$$

PROOF. Follows from Theorem 1 and (4). See Online Appendix A. □

This further implies the surprising result in Corollary 5 that while the actual $\phi_{\psi}$ is decreasing in the noise of the private signal, $\frac{\partial \phi_{\psi}}{\partial \sigma_{\epsilon}} < 0$ (Corollary 3), the estimated $\hat{\phi}_{\psi}^{BPP}$ is increasing in it, $\frac{\partial \hat{\phi}_{\psi}^{BPP}}{\partial \sigma_{\epsilon}} > 0$.

COROLLARY 5. *We have*

$$\frac{\partial \hat{\phi}_{\xi}^{BPP}}{\partial \sigma_{\epsilon}^2} = \frac{\partial \phi_{\xi}}{\partial \sigma_{\epsilon}^2} = \frac{1}{R} Q^{\star},$$

$$\frac{\partial \hat{\phi}_{\psi}^{BPP}}{\partial \sigma_{\epsilon}^2} = \frac{R-1}{R}\frac{\sigma_{\xi}^2}{\sigma_{\psi}^2} Q^{\star},$$

*where*

$$Q^{\star} \equiv \frac{\sigma_{\xi}^2(2q^{\star} + \sigma_{\psi}^2)}{(\sigma_{\epsilon}^2 + \sigma_{\xi}^2)(4\sigma_{\epsilon}^2 + (\sigma_{\epsilon}^2 + \sigma_{\xi}^2)\sigma_{\psi}^2)}.$$

PROOF. See Online Appendix A. □

### 1.5. *Identification of* $\sigma_{\epsilon}^2$

We now turn to identification of households' degree of information with panel data on income and consumption. Lemma 4 shows how to estimate the variance of the private signal, $\sigma_{\epsilon}^2$, given estimates of the variance of the mean belief $\hat{q}^{\star}$, and the income shocks variances, $\hat{\sigma}_{\psi}^2$ and $\hat{\sigma}_{\xi}^2$.

LEMMA 4. *Given estimates* $\hat{q}^{\star}$, $\hat{\sigma}_{\psi}^2 > 0$ *and* $\hat{\sigma}_{\xi}^2 > 0$ *the variance of the private signal is*

$$\hat{\sigma}_{\epsilon}^2 = q^{\star -1}(\hat{q}^{\star}, \hat{\sigma}_{\psi}^2, \hat{\sigma}_{\xi}^2), \tag{46}$$

*where*

$$q^{\star -1}(q^{\star}, \sigma_{\psi}^2, \sigma_{\xi}^2) \equiv \begin{cases} 0 & \text{if } q^{\star} \le 0, \\ \dfrac{q^{\star}\sigma_{\xi}^2(q^{\star} + \sigma_{\psi}^2)}{\sigma_{\xi}^2\sigma_{\psi}^2 - q^{\star}(q^{\star} + \sigma_{\psi}^2)} & \text{if } q^{\star} \in \left(0, \left(\sqrt{\sigma_{\xi}^2/\sigma_{\psi}^2 + \dfrac{1}{4}} - \dfrac{1}{2}\right)\sigma_{\psi}^2\right). \end{cases} \tag{47}$$

*If* $q^\star \geq \left(\sqrt{\sigma_\xi^2/\sigma_\psi^2 + \frac{1}{4}} - \frac{1}{2}\right)\sigma_\psi^2$ *there does not exist any* $\sigma_\epsilon^2$ *for the given* $\sigma_\xi^2$ *and* $\sigma_\psi^2$ *that is consistent with* $q^\star$.

PROOF. $q^{\star\,-1}(\bullet)$ is the solution in $\sigma_\epsilon^2$ to (19). For $\sigma_\epsilon > 0$, we always have $\sigma_\xi^2\sigma_\psi^2 - q^\star(q^\star + \sigma_\psi^2) > 0$. □

Theorem 2 next shows that the degree of noise in the private signal, $\sigma_\epsilon^2$, is point identified with panel data on income and consumption even when consumption is subject to measurement error.

THEOREM 2. *Consider a panel data set where income,* $\tilde{y}_t$*, is observed without measurement error, and consumption,* $\tilde{c}_t$*, is observed with additive i.i.d. measurement error with variance* $\sigma_c^2$*.*
*The variance of the mean belief,* $q^\star$*, is point identified as*

$$\hat{q}^\star = -Rcov(\Delta\tilde{c}_t, \Delta\tilde{y}_{t+1}) - (R-1)\hat{\sigma}_\xi^2. \tag{48}$$

*where*

$$\hat{\sigma}_\xi^2 = \max\{cov(\Delta\tilde{y}_t, -\Delta\tilde{y}_{t+1}), 0\} \tag{49}$$

$$\hat{\sigma}_\psi^2 = \max\{cov(\Delta\tilde{y}_t, \Delta\tilde{y}_{t-1} + \Delta\tilde{y}_t + \Delta\tilde{y}_{t+1}), 0\} \tag{50}$$

$$\hat{\sigma}_c^2 = \max\{cov(\Delta\tilde{c}_t, -\Delta\tilde{c}_{t+1}), 0\}. \tag{51}$$

*The degree of noise in the private signal,* $\sigma_\epsilon^2$*, is point identified as*

$$\hat{\sigma}_\epsilon^2 = q^{\star -1}(\hat{q}^\star, \hat{\sigma}_\psi^2, \hat{\sigma}_\xi^2), \tag{52}$$

*where* $q^{\star-1}(\bullet)$ *is defined in Lemma 4.*

PROOF. See Online Appendix A. □

With unknown measurement in income, the transitory shock variance is not point identified. However, Corollary 6 shows that the estimate of $\sigma_\epsilon^2$ is monotonically decreasing in the variance of measurement error of income. Assuming no measurement error in income thus provides a lower bound on $\sigma_\epsilon^2$. In practice, the amount of measurement in income can be bounded by the observed variance of income growth, and we can thus also provide an upper bound.

COROLLARY 6. *Consider the same case as Theorem 2, but assume that there additionally also is additive i.i.d. measurement error in income,* $\tilde{y}_{it}$*, with variance* $\sigma_y^2$*.*
*For any* $\tilde{\sigma}_y^2 \in [0, cov(\Delta\tilde{y}_t, -\Delta\tilde{y}_{t+1}))$ *we can estimate the degree of information by*

$$\hat{\sigma}_\epsilon^2(\tilde{\sigma}_y^2) = q^{\star-1}(\hat{q}^\star(\hat{\sigma}_\xi^2(\tilde{\sigma}_y^2)), \hat{\sigma}_\psi^2, \hat{\sigma}_\xi^2(\tilde{\sigma}_y^2)), \tag{53}$$

*where* $q^{\star-1}(\bullet)$ *is defined in Lemma 4, and*

$$\hat{\sigma}_\xi^2(z) = cov(\Delta\tilde{y}_t, -\Delta\tilde{y}_{t+1}) - z, \tag{54}$$

$$\hat{q}^\star(x) = -Rcov(\Delta\tilde{c}_t, \Delta\tilde{y}_{t+1}) - (R-1)x. \tag{55}$$

*We have that the estimated degree of information will be increasing in the measurement error of income*

$$\frac{\partial\hat{\sigma}_\epsilon^2(\tilde{\sigma}_y^2)}{\partial\tilde{\sigma}_y^2} = \frac{\partial q^{\star-1}(\hat{q}^\star(\tilde{\sigma}_y^2), \hat{\sigma}_\psi^2, \hat{\sigma}_\xi^2(\tilde{\sigma}_y^2))}{\partial\tilde{\sigma}_y^2} > 0. \tag{56}$$

PROOF. See Online Appendix A. □

## 2. Buffer-Stock Model and Monte Carlo

In this section, we present a more general model in which households have CRRA preferences and face a life-cycle income process with potentially persistent rather than fully permanent shocks and an MA(1) term. In the limit where the households' information about their permanent income is perfect, the model nests the canonical buffer-stock model of Deaton (1991; 1992) and Carroll (1992; 1997; 2012). After describing the model details, we show how to estimate the degree of information with the Simulated Method of Moments (SMM) using panel data on consumption and income, and present an encouraging Monte Carlo study.

### 2.1. *General Model*

Specifically, we consider the following specification for log income

$$p_t = \Gamma_t + \alpha p_{t-1} + \psi_t, \quad \alpha \in [-1, 1], \tag{57}$$

$$y_t = p_t + \xi_t + \omega\xi_{t-1}. \tag{58}$$

where $\Gamma_t$ captures the life-cycle profile of income, the AR(1) coefficient $\alpha$ allows for persistent rather than fully permanent shocks, and the MA(1) coefficient $\omega$ allows for additional short-run dynamics independent of the dynamics of the persistent component of income.

Because the income process is still linear-Gaussian it is still optimal for the household to use the Kalman filter. The transition and measurement equations can be written as

$$\begin{bmatrix} p_t \\ \eta_t \\ \xi_t \end{bmatrix} = \underbrace{\begin{bmatrix} \alpha & 0 & 0 \\ 0 & 0 & \omega \\ 0 & 0 & 0 \end{bmatrix}}_{\equiv \boldsymbol{F}} \begin{bmatrix} p_{t-1} \\ \eta_{t-1} \\ \xi_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} \Gamma_t + \mu_\psi \\ \mu_\xi \\ \mu_\xi \end{bmatrix}}_{\equiv \boldsymbol{\mu}} + \underbrace{\begin{bmatrix} \sigma_\psi & 0 & 0 \\ 0 & \sigma_\xi & 0 \\ 0 & \sigma_\xi & 0 \end{bmatrix}}_{\equiv \boldsymbol{\mathcal{W}}} \begin{bmatrix} \pi_{1t} \\ \pi_{2t} \\ 0 \end{bmatrix}, \tag{59}$$

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}_{\equiv \boldsymbol{H}} \begin{bmatrix} p_t \\ \eta_t \\ \xi_t \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & \sigma_\epsilon \end{bmatrix}}_{\equiv \boldsymbol{\mathcal{R}}} \begin{bmatrix} 0 \\ \pi_{3t} \end{bmatrix}, \tag{60}$$

$$\pi_t^j \sim \text{i.i.d.}\,\mathcal{N}(0, 1), \;\; j \in \{1, 2, 3\}.$$

The prediction step becomes

$$\begin{bmatrix} \hat{p}_{t+1|t} \\ \hat{\eta}_{t+1|t} \\ \hat{\xi}_{t+1|t} \end{bmatrix} = \boldsymbol{F} \begin{bmatrix} \hat{p}_t \\ \hat{\eta}_t \\ \hat{\xi}_t \end{bmatrix} + \boldsymbol{\mu}, \tag{61}$$

$$\hat{\boldsymbol{Q}}_{t+1|t} = \boldsymbol{F}\hat{\boldsymbol{Q}}_t\boldsymbol{F}' + \boldsymbol{\mathcal{W}}\boldsymbol{\mathcal{W}}'. \tag{62}$$

The optimal Kalman gain vector becomes

$$\boldsymbol{K}_{t+1} = \hat{\boldsymbol{Q}}_{t+1|t}\boldsymbol{H}'\boldsymbol{S}_{t+1}^{-1}, \tag{63}$$

where

$$\boldsymbol{S}_{t+1} = \boldsymbol{H}\hat{\boldsymbol{Q}}_{t+t|t}\boldsymbol{H}' + \boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}'. \tag{64}$$

The update step becomes

$$\begin{bmatrix} \hat{p}_{t+1} \\ \hat{\eta}_{t+1} \\ \hat{\xi}_{t+1} \end{bmatrix} = \begin{bmatrix} \hat{p}_{t+1|t} \\ \hat{\eta}_{t+1|t} \\ \hat{\xi}_{t+1|t} \end{bmatrix} + \boldsymbol{K}_{t+1}\boldsymbol{\Delta}_{t+1}, \tag{65}$$

$$\hat{\boldsymbol{Q}}_{t+1} = (\boldsymbol{I} - \boldsymbol{K}_{t+1}\boldsymbol{H})\,\hat{\boldsymbol{Q}}_{t+1|t}. \tag{66}$$

where

$$\boldsymbol{\Delta}_{t+1} = \begin{bmatrix} y_{t+1} \\ z_{t+1} \end{bmatrix} - \begin{bmatrix} \hat{p}_{t+1|t} + \hat{\eta}_{t+1|t} \\ \hat{p}_{t+1|t} \end{bmatrix}. \tag{67}$$

The vector of belief errors is

$$\boldsymbol{\kappa}_t \equiv \begin{bmatrix} \kappa_t^p \\ \kappa_t^\eta \\ \kappa_t^\xi \end{bmatrix} \equiv \begin{bmatrix} \hat{p}_t \\ \hat{\eta}_t \\ \hat{\xi}_t \end{bmatrix} - \begin{bmatrix} p_t \\ \eta_t \\ \xi_t \end{bmatrix} \sim \mathcal{N}(0, \hat{Q}_t).$$

We denote the diagonal matrix of the sorted (ascending) eigenvalues of $\hat{\boldsymbol{Q}}_t$ by $\boldsymbol{D}$ and the associated matrix with the eigenvectors as the columns by $\boldsymbol{V}$. We make the following conjecture, which we test numerically in practice when solving the model.

CONJECTURE 1. *The smallest eigenvalue of $\hat{\boldsymbol{Q}}_t$ is zero for all $t$ if it is zero for $\hat{\boldsymbol{Q}}_0$.*

The household retires in period $T_R$, and hereafter receives retirement benefits equal to a fixed ratio of the permanent income,

$$t > T_R : \; y_t = p_{T_R} + \log \lambda. \tag{68}$$

The full recursive formulation of the household's problem then becomes

$$\begin{aligned}
V_t(M_t, \hat{p}_t, \hat{\xi}_t) &= \max_{C_t} \frac{C_t^{1-\rho}}{1-\rho} + \beta \tilde{\mathbb{E}}_t\left[V_{t+1}(M_{t+1}, \hat{p}_{t+1}, \hat{\xi}_{t+1})\right] \qquad (69)\\
&\text{s.t.}\\
A_t &= M_t - C_t,\\
\begin{bmatrix} p_{t+1} \\ \eta_{t+1} \\ \xi_{t+1} \end{bmatrix} &= \boldsymbol{F}\left(\begin{bmatrix} \hat{p}_t \\ 0 \\ \hat{\xi}_t \end{bmatrix} + \boldsymbol{V}\boldsymbol{D}^{\frac{1}{2}}\begin{bmatrix} \iota_{1t+1} \\ \iota_{2t+1} \\ 0 \end{bmatrix}\right) + \boldsymbol{\mu} + \boldsymbol{\mathcal{W}}\begin{bmatrix} \iota_{3t+1} \\ \iota_{4t+1} \\ 0 \end{bmatrix},\\
\begin{bmatrix} y_{t+1} \\ z_{t+1} \end{bmatrix} &= \begin{bmatrix} p_{t+1} + \eta_{t+1} \\ p_{t+1} \end{bmatrix} + R\begin{bmatrix} 0 \\ \iota_{5t+1} \end{bmatrix},\\
\begin{bmatrix} \hat{p}_{t+1} \\ 0 \\ \hat{\xi}_{t+1} \end{bmatrix} &= \boldsymbol{F}\begin{bmatrix} \hat{p}_t \\ 0 \\ \hat{\xi}_t \end{bmatrix} + \boldsymbol{\mu} + \boldsymbol{K}_{t+1}\boldsymbol{\Delta}_{t+1},\\
M_{t+1} &= R \cdot A_t + \exp(y_{t+1}),\\
A_t &\geq 0,\\
\iota_{jt} &\sim \text{i.i.d.}\mathcal{N}(0,1), \; j \in \{1,2,3,4,5\}.
\end{aligned}$$

where $R$ is the return factor and the households are not allowed to borrow. We always assume $\hat{\boldsymbol{Q}}_0 = \mathbf{0}$.

The standard Euler equation applies,

$$C_t^{-\rho} = \beta R \tilde{\mathbb{E}}_t \left[ C_{t+1}^{-\rho} \right], \tag{70}$$

and the model can be solved using the Endogenous Grid Method (EGM) proposed by Carroll (2006) extended to allow for multiple states. The model is, however, still computationally demanding as it contains three continuous states and the expectation is a five dimensional integral.[11]

## 2.2. *Estimation*

We imagine having panel data on income and consumption for $i = 1, \ldots, N$ individuals in $t = 1, \ldots, T$ periods. We define $w_{it} \equiv (y_{it}, c_{it})$ such that $\mathbf{w}$ denotes the stacked data. In addition to the standard deviation of the private signal, $\sigma_\epsilon$, we also wish to estimate the preferences and the income process parameters. We denote the vector of parameters to be estimated by $\boldsymbol{\theta} \in \Theta \subset \mathbb{R}^{\dim \theta}$. In our empirical investigation below we have $\boldsymbol{\theta} = (\sigma_\epsilon, \beta, \sigma_c, \sigma_\xi, \sigma_\psi, g_0, g_1, \alpha, \omega)$, where we allow income growth to be age dependent through $\Gamma_t = g_0 + g_1(age_t - 25)/100$.

We use the SMM pioneered by McFadden (1989) to estimate $\boldsymbol{\theta}$. Let $\boldsymbol{\Lambda}(\mathbf{w})$ be a $K \times 1$ vector of moments calculated based on observed data. For each value of $\boldsymbol{\theta}$, we solve the model and simulate income and consumption trajectories for the $N$ households forward from age 25 through 65. We use data from age 30 to mimic the PSID data used in the empirical investigation below.[12] We can then calculate the same moments using the simulated data. Denote as $\overline{\boldsymbol{\Lambda}}(\boldsymbol{\theta}) = \frac{1}{J}\sum_{j=1}^{J} \boldsymbol{\Lambda}_j(\boldsymbol{\theta})$ the average of the same $K$ moments calculated from $J$ simulated data sets from the model for a given value of $\boldsymbol{\theta}$.

We estimate $\boldsymbol{\theta}$ as

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta} \in \boldsymbol{\theta}} (\boldsymbol{\Lambda}(\mathbf{w}) - \overline{\boldsymbol{\Lambda}}(\boldsymbol{\theta}))' \boldsymbol{W} (\boldsymbol{\Lambda}(\mathbf{w}) - \overline{\boldsymbol{\Lambda}}(\boldsymbol{\theta})),$$

where $\boldsymbol{W}$ is a $K \times K$ positive semidefinite weighting matrix. As a baseline we choose $\boldsymbol{W}$ as the inverse of the covariance matrix of $\boldsymbol{\Lambda}(\mathbf{w})$.

We explicitly utilise our theoretical results from the certainty equivalence case in the previous section for identification of $\sigma_\epsilon$. Particularly, we use the following 82 ($K = 82$) moments to uncover the parameters in $\boldsymbol{\theta}$:

- **Moments 1–31:** Age profile of log income from age 30 through 60 (demeaned), $\text{mean}(y)|_{t=a} - \text{mean}(y)$, $a = 30, \ldots, 60$. These moments are included primarily for identification of the income growth parameters, $g_0$ and $g_1$.
- **Moments 32–62:** Age profile of log consumption from age 30 through 60 (demeaned), $\text{mean}(c)|_{t=a} - \text{mean}(c)$, $a = 30, \ldots, 60$. These moments are included primarily for identification of the discount factor, $\beta$.
- **Moments 63–68:** The autocovariances of log income growth, $\text{cov}(\Delta y_t, \Delta y_{t+k})$ for $k = 0, \ldots,$ 5. These moments are primarily included for identification of the transitory and permanent income shock variances, $\sigma_\xi^2$ and $\sigma_\psi^2$, and the AR(1) and MA(1) parameters, $\alpha$ and $\omega$.

[11] Numerically, we approximate the integral with Gauss-Hermite quadrature. When assuming $\omega = 0$ the dimensionality of the state space reduces to two and makes the expectation three dimensional. Furthermore, when assuming $\alpha = 1$ the state space reduces to just one dimension and the expectation to just two dimensions.

[12] Households are initialised with $\hat{q}_0 = 0$, $A_t = 0$ and $p_0 = 1$.

Table 1. *Parameter Values Used to Simulate Data.*

| $\sigma_\epsilon$ | $\beta$ | $\sigma_c$ | $\sigma_y$ | $\sigma_\xi$ | $\sigma_\psi$ | $g_0$ | $g_1$ | $\alpha$ | $\omega$ | $R$ | $\lambda$ | $\rho$ | $T_R$ | $T$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.96 | 0.25 | 0.1 | 0.15 | 0.15 | 0.06 | −0.05 | 0.9 | 0.15 | 1.03 | 0.5 | 1.5 | 40 | 55 |

- **Moments 69–74:** The autocovariances of log consumption growth, $\text{cov}(\Delta c_t, \Delta c_{t+k})$ for $k = 0, \ldots, 5$. These moments are primarily included for identification of the measurement error in consumption, $\sigma_c^2$.
- **Moments 75–82:** The covariances of log consumption growth with log income growth, $\text{cov}(\Delta c_t, \Delta y_{t+k})$ for $k = -2, \ldots, 5$. These moments are primarily included for identification of the noise in the private signal, $\sigma_\epsilon^2$.

## 2.3. *Monte Carlo*

We now present results from a Monte Carlo study mimicking our empirical analysis on PSID in Section 3. Specifically, we simulate $N = 2{,}000$ households from age 25 through 65 from the general CRRA model.

We estimate $\boldsymbol{\theta}$ ($\sigma_\epsilon$, $\beta$, $\sigma_c$, $\sigma_\xi$, $\sigma_\psi$, $g_0$, $g_1$, $\alpha$, $\omega$) by SMM using the moments specified above. The parameter values used to simulate synthetic data for estimation are given in Table 1. While estimating $\boldsymbol{\theta}$ we keep $\sigma_y^2$ at its true value. We simulate data 200 times and perform the estimation on each of the data sets.

Figure 1 reports the marginal distributions of the estimated parameters. For all parameters, we find that our estimator on average uncovers the true value. This is reassuring because it indicates that we can uncover the parameters of the model with the suggested moments.

We now turn to the closed-form results derived for the CEQ model in Corollary 6. While the data is still simulated from the CRRA model with the income process in (57)–(58), and is identical to that used to generate Figure 1, we see in Figure 2 that the degree of noise in the private signal, $\sigma_\epsilon$, is on average estimated surprisingly close to the true value. While the measurement error in consumption is also estimated close to its true value, estimates of the permanent and transitory income shock variances are severely over- and under-estimated, respectively, due to the misspecification of the income process. In total, the biases in the income shock variances seem to net out in the estimation of $\sigma_\epsilon$, which is very encouraging for the use of this simple estimation strategy.

In the Online Appendix (see Figures C.3–C.5) we provide additional Monte Carlo results where we use the exact parameters we estimate on the PSID in the next section.[13] We find that our estimator also performs well in this case and in additional scenarios where we, as here, assume $\sigma_\epsilon = 0.1$ and vary the true value of either $\beta$, $\sigma_\psi$, $\sigma_\xi$, or $\alpha$. Interestingly, we find that estimating the general CRRA model using the closed-form results derived for the CEQ model in Corollary 6 overestimates the degree of noise, $\sigma_\epsilon$, when the true value is $\sigma_\epsilon = 0$. This is in line with our empirical results using the PSID below.

[13] In the Online Appendix we focus on the model without an MA(1) term since we find that it does not matter much for economic behaviour, but increases computational time dramatically.

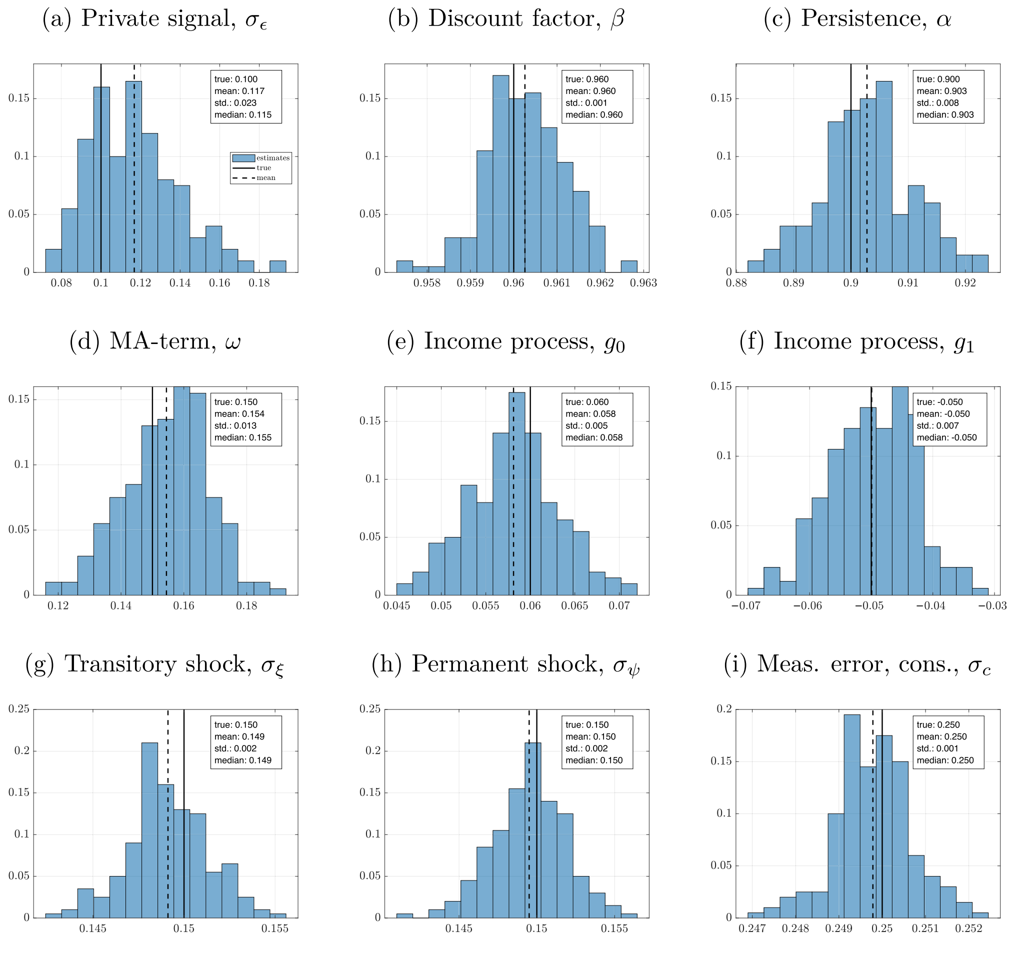


Fig. 1. *Monte Carlo Results.*

*Notes:* Figure 1 reports histograms of Monte Carlo estimates of the nine parameters $\boldsymbol{\theta} = (\sigma_\epsilon, \beta, \sigma_c, \sigma_\xi, \sigma_\psi, g_0, g_1, \alpha, \omega)$ from 200 replications. True parameter values are reported in Table 1 and indicated with solid black lines. Dashed lines represent the average estimate across the 200 replications.

## 3. Empirical Investigation

In this section, we investigate empirically whether consumers can distinguish persistent from transitory income shocks. We use data from the PSID, and the model and estimation approach validated by the Monte Carlo study in the previous section.

### 3.1. *Data*

We use the same PSID data as BPP, including their exact variable definitions and sample selection criteria. We use their imputed total non-durable consumption as our consumption measure and the income measure is total family income net of financial income and taxes. Both measures are deflated using the CPI. We exclude the low-income sample (SEO) and focus on stable, married

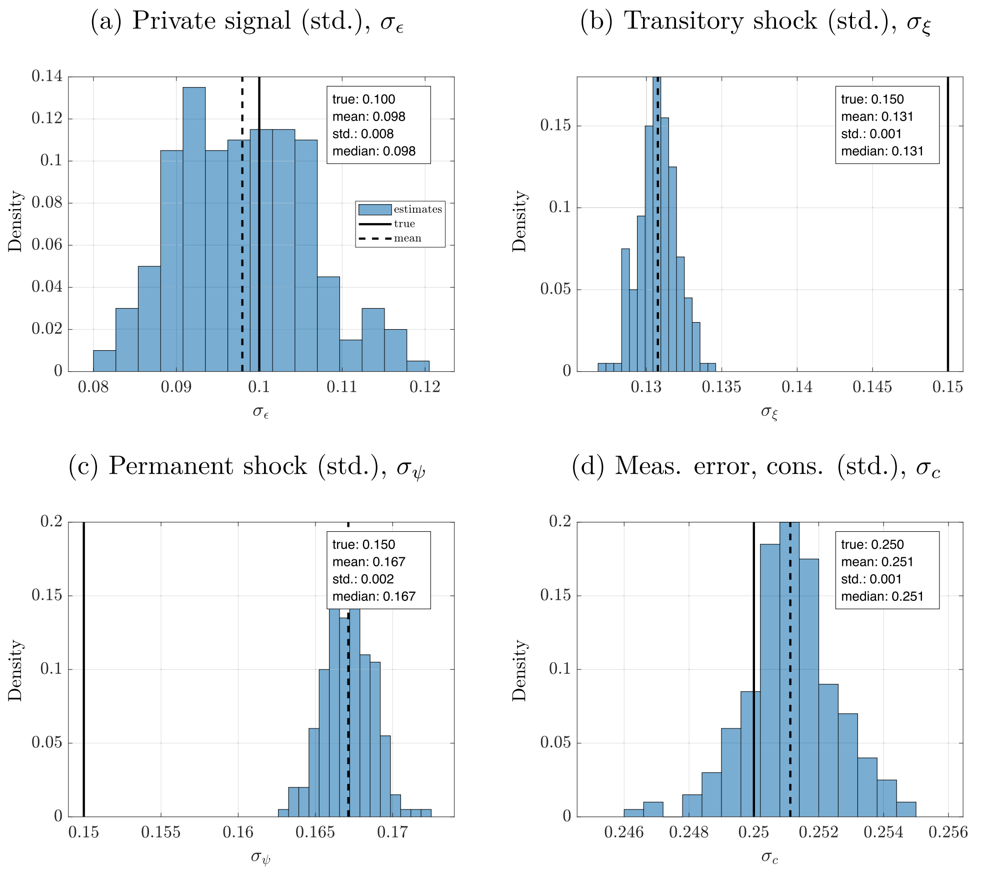


Fig. 2. *Monte Carlo Results. CEQ.*

*Notes:* Figure 2 reports histograms of Monte Carlo estimates of the four parameters $\boldsymbol{\theta} = (\sigma_\epsilon, \sigma_\xi, \sigma_\psi, \sigma_c)$ using the CEQ model, but with data simulated from the CRRA model studied in Figure 1. We use 200 replications. True parameter values are reported in Table 1 and indicated with solid black lines. Dashed lines represent the average estimate across the 200 replications.

households of opposite sexes in which the head is male and aged 30–65. We keep households in which the husband was born between 1920 and 1959. We discard a few income observations due to top-coded tax or income. In turn, we have 17,604 household-time observations from 1,765 households. We refer the reader to the Appendix of BPP for more details on the data and sample selection criteria.

We follow BPP and remove predictable variation due to demographics from the income and consumption series through regression. As with BPP, we include dummies for educational level, race, number of children, number of family members, region of residence, employment status and external income source. All these dummies are allowed to be varying over time. Finally, we include time and age dummies in the regressions.[14] Throughout the remainder of this article we

[14] We include age dummies rather than birth cohort dummies as was done in BPP. This is, however, identical because year dummies are also included. The age dummies provide us with an estimate of the life-cycle profiles of income and consumption, which we utilise as moments in the estimation of the model.

Table 2. *Estimates, CEQ Model.*

| Parameter | | Whole sample (1) | No college (2) | College (3) |
|---|---|---|---|---|
| $\sigma_\epsilon$ | Private signal | 0.031 | 0.043 | 0.011 |
| | | (0.026) | (0.039) | (0.025) |
| $\sigma_c$ | Measurement error in consumption | 0.264 | 0.296 | 0.230 |
| | | (0.008) | (0.013) | (0.007) |
| $\sigma_\psi$ | Persistent shock | 0.165 | 0.172 | 0.158 |
| | | (0.005) | (0.008) | (0.008) |
| $\sigma_\xi$ | Transitory shock | 0.135 | 0.143 | 0.126 |
| | | (0.005) | (0.007) | (0.007) |

*Notes:* Bootstrapped standard errors based on 5,000 bootstrap replications reported in brackets.

refer to $y_t$ and $c_t$ as log income and log consumption, respectively, with all predictable variation removed.

## 3.2. *Estimation Results: CEQ Model*

As a first step, we apply the closed-form estimator derived in Corollary 6 under the assumption of a certainty equivalence (CEQ) model. We assume that the real interest rate is 3% ($R = 1.03$) as in, e.g., Gourinchas and Parker (2002), and that measurement error in income accounts for 25% ($\tau = 0.25$) of the total variance of log income growth in the PSID data ($\sigma_y^2 = 0.5 \cdot \tau \cdot 0.0907 = 0.0113$). This strategy is identical to that employed in Meghir and Pistaferri (2004) and is in part based on the result in Bound *et al.* (1994) that around 22% of the overall income growth variance in the PSID is attributed to measurement error.[15]

Table 2 reports the reduced-form estimates of the CEQ model. The first column reports estimates for the whole sample and Columns (2) and (3) report estimation results for the no-college and college groups. In all cases we estimate a small positive standard deviation of the private signal, $\sigma_\epsilon$, which, however, are clearly insignificant at the 5% level.[16] The point estimate is higher for the group with no college education, albeit not statistically different.

The other estimated parameters are in the ranges typically found in the literature. The measurement error in consumption is substantial, with a variance around 0.07, which is in the same range as reported in BPP.[17] The transitory and permanent income shock variances are estimated to be around 0.018 and 0.027, which are also close to the estimates reported in BPP.

In Online Appendix C, Table C.1 and C.2, we report the effect of assuming either no measurement error in income ($\tau = 0.0$) or a very high measurement error ($\tau = 0.50$). As expected, we find that the estimated transitory income shock variance is falling in the assumed degree of measurement error in income, while the noise in the private signal is increasing in it. In the high measurement error case we estimate $\sigma_\epsilon = 0.040$ with a standard deviation of 0.046, i.e., still small and insignificant.

[15] The results are robust to these calibrations.

[16] Figure C.6 in Online Appendix C reports the distributions of bootstrapped estimates showing that we get $\sigma_\epsilon = 0$ in 30% of the samples.

[17] The measurement error variance in BPP is allowed to be time varying and the authors write that the imputation error variance is estimated in the range 0.05 to 0.10.

Table 3. *Estimates, General Model.*

| Parameter | | Whole sample | | | No college | College |
|---|---|---|---|---|---|---|
| | | (1) | (2) | (3) | (4) | (5) |
| $\sigma_\epsilon$ | Private signal | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | | – | – | – | – | – |
| $\beta$ | Discount factor | 0.961 | 0.963 | 0.964 | 0.960 | 0.969 |
| | | (0.001) | (0.002) | (0.002) | (0.003) | (0.002) |
| $\sigma_c$ | Measurement error in consumption | 0.271 | 0.274 | 0.273 | 0.302 | 0.238 |
| | | (0.005) | (0.004) | (0.004) | (0.007) | (0.004) |
| $\sigma_\psi$ | Persistent shock | 0.127 | 0.173 | 0.154 | 0.145 | 0.151 |
| | | (0.005) | (0.007) | (0.009) | (0.014) | (0.012) |
| $\sigma_\xi$ | Transitory shock | 0.154 | 0.128 | 0.152 | 0.172 | 0.138 |
| | | (0.004) | (0.006) | (0.008) | (0.010) | (0.012) |
| $g_0$ | Income growth, constant | 0.027 | 0.098 | 0.070 | 0.058 | 0.076 |
| | | (0.002) | (0.022) | (0.016) | (0.021) | (0.022) |
| $g_1$ | Income growth, age | −0.081 | −0.030 | −0.044 | −0.042 | −0.045 |
| | | (0.010) | (0.014) | (0.014) | (0.019) | (0.020) |
| $\alpha$ | AR(1) component | – | 0.818 | 0.861 | 0.877 | 0.858 |
| | | – | (0.026) | (0.028) | (0.044) | (0.036) |
| $\omega$ | MA(1) component | – | – | 0.144 | 0.195 | 0.102 |
| | | – | – | (0.041) | (0.045) | (0.067) |
| Objective | | 144.191 | 68.536 | 62.006 | 44.866 | 67.166 |
| $p$-value for $\sigma_\epsilon = 0$ | | 0.495 | 0.500 | 0.499 | 0.500 | 0.500 |

*Notes:* Asymptotic standard errors reported in brackets for all parameters in the interior of their domain. Details are in Online Appendix B.

As the estimates above are derived under the (arguably unrealistic) assumptions of the CEQ model, we now turn to the more general model with CRRA preferences and a more general income process.

### 3.3. *Estimation Results: General CRRA Model*

We fix a few parameters of the model while estimating the remaining. In particular, we set the real interest rate to 3% ($R = 1.03$) and fix the degree of measurement error in income to account for 25% of the total variance of log income growth in the PSID data as above. The replacement rate in retirement is fixed at 50% ($\lambda = 0.5$) and retirement happens with certainty at age 65 and agents die with certainty at age 85. Finally, we fix the constant relative risk aversion coefficient at $\rho = 1.5$ (Attanasio and Weber, 1995). We perform robustness checks below in Subsection 3.5 showing that changing these calibrated parameters does not change our results.

We estimate the parameters, $\boldsymbol{\theta} = (\sigma_\epsilon, \beta, \sigma_c, \sigma_\xi, \sigma_\psi, g_0, g_1, \alpha, \omega)$, by SMM as discussed in Section 2. Table 3 reports the estimated parameters. The first three columns are based on the whole PSID sample. In Column (1), we consider a model with a permanent–transitory income process with a unit root and no MA(1) term ($\alpha = 1$ and $\omega = 0$). In column (2), we allow for a non-unit root ($\alpha$ free, $\omega = 0$), and in Column (3) we additionally allow for a MA(1) term ($\alpha$ free, $\omega$ free). Using the preferred model from Column (3), Columns (4) and (5) are, respectively, for the sub-sample with no college degree and with a college degree.

The low estimated noise in the private signal, $\sigma_\epsilon \approx 0$, suggests that PSID households have a high degree of information about their own permanent income. In fact, we cannot reject the one-sided hypothesis that $\mathcal{H}_0 : \sigma_\epsilon = 0$ against $\mathcal{H}_A : \sigma_\epsilon > 0$ with any standard confidence levels.[18]

[18] Because our hypothesis is on the boundary of the parameter space, we employ a modified quasi likelihood ratio (QLR) test. Particularly, define the objective function to be minimised as $Q = (\boldsymbol{\Lambda}(\mathbf{w}) - \overline{\boldsymbol{\Lambda}}(\boldsymbol{\theta}))' V^{-1} (\boldsymbol{\Lambda}(\mathbf{w}) - \overline{\boldsymbol{\Lambda}}(\boldsymbol{\theta}))$ where

This is true for all specifications and across all robustness checks, reported below. Because $\sigma_\epsilon$ is estimated to be very close to the boundary of the parameter space we do not report standard errors for this parameter in Table 3 and report asymptotic standard errors for the remaining parameters under the assumption that $\sigma_\epsilon = 0$.

While the estimated variance of the private signal is zero (i.e., perfect information) for both educational groups, we estimate a slightly higher discount factor for the college group. The no-college group has a higher degree of measurement error in consumption and face more volatile transitory shocks. The income growth rates also differ. Since we do not find important differences in the estimated degree of information across educational groups, our preferred specification, and the focus in the remainder of the article, is the results from the whole sample in Column (3) of Table 3.[19]

In the following subsections, we respectively consider the implied model fit, the transmission parameters implied by the data and the estimated models, and finally some central robustness checks.[20]

### 3.4. *Model Fit and Sensitivity Analysis*

Figure 3 reports the used moments calculated from the PSID with bootstrapped 95% confidence bands along with the same moments calculated using simulated data from the preferred model.[21] Given the amount of structure that the estimated model places on the data, the model seems to fit the PSID data quite well. The moments calculated using synthetic data simulated from the estimated model are very close to the moments calculated using the PSID. In particular, all moments except two are within the bootstrapped 95% confidence intervals. The only two moments outside the 95% confidence bands are two moments of the income age profile.

To investigate which moments the degree of information, $\sigma_\epsilon$, is sensitive to, Figure 4 illustrates the effect of increasing $\sigma_\epsilon$ on the moments involving consumption. While decreasing the degree of information, we keep all other parameters at their estimated values in Column (3) in Table 3. We see that $\mathrm{cov}(\Delta c_t, \Delta c_{t+k})$ changes insignificantly when varying the degree of information. The same is true for the consumption age profile. The moments that seem to be changing the most are the covariances between consumption and income growth, $\mathrm{cov}(\Delta c_t, \Delta y_{t+k})$. Especially for $k = 0$ and $k = 1$. This is in line with the theoretical results from the certainty equivalence model.

With $\sigma_\epsilon = 0$, the covariance between current consumption growth and current income growth is slightly larger in the estimated model than in the PSID. A higher $\sigma_\epsilon$, however, only worsens the fit. Likewise, the covariance between current consumption growth and future income growth is slightly more negative in the model, and introducing imperfect information makes this moment even more negative in the model. The intuition for the result of perfect information thus is that

the inverse of $V = \mathrm{var}(\boldsymbol{\Lambda}(\mathbf{w}))$ is the optimal weighting matrix (see Online Appendix B on the calculation of $V$). The test statistic $QLR = Q(\hat{\boldsymbol{\theta}}_{\sigma_\epsilon=0}) - Q(\hat{\boldsymbol{\theta}})$ where $Q(\hat{\boldsymbol{\theta}}_{\sigma_\epsilon=0})$ and $Q(\hat{\boldsymbol{\theta}})$, are the estimated objective functions under the null and with all parameters estimated, respectively, follows a mixture of two $\chi^2$ distributions. The $p$-value related to the null hypothesis can thus be found as $1 - F(QLR)$ where $F(z) = \frac{1}{2} + \frac{1}{2}\chi_1^2(z)$ and $\chi_1^2(z)$ is the CDF of a $\chi^2$ distribution with 1 degree of freedom evaluated at $z$ (Andrews, 2001, Theorem 4).

[19] Note, however, that the improvement in fit from adding the MA(1) term in Column (3) is limited, and makes the model computationally much more demanding. In Online Appendix C, we show in Figure C.11 that the value of the MA(1) term have almost no effect on the moments, except, naturally, for the covariances between current and future income growth, $\mathrm{cov}(\Delta y_t, \Delta y_{t+k})$.

[20] In the Online Appendix, we show the transitory transmission parameters estimated as in BPP in both the data (using the estimated income shock variances) and in simulations from the model with varying degrees of information.

[21] The fit of the remaining specifications are shown in the Online Appendix C.

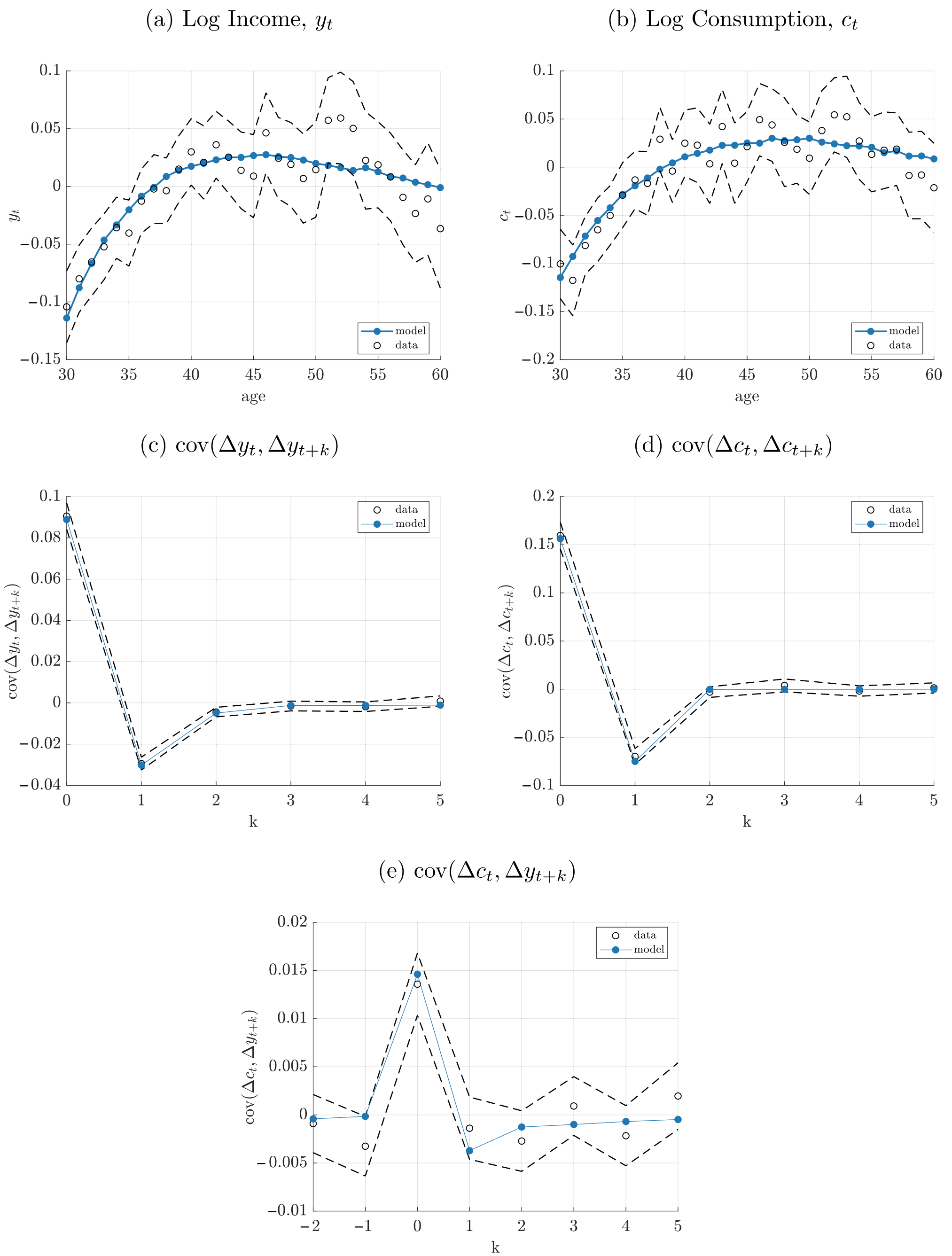


Fig. 3. *Model Fit ($\alpha$ free, $\omega$ free).*

*Notes:* Figure 3 illustrates the average age profiles of log income and log consumption together with the covariance moments. Both age profile series are normalised by the overall mean of each series. Hollow dots are calculated using the PSID, $\mathbf{\Lambda}(\mathbf{w})$, dashed lines are 95% confidence intervals, and solid dots are calculated using simulated data from the model, $\overline{\mathbf{\Lambda}}(\hat{\boldsymbol{\theta}})$.

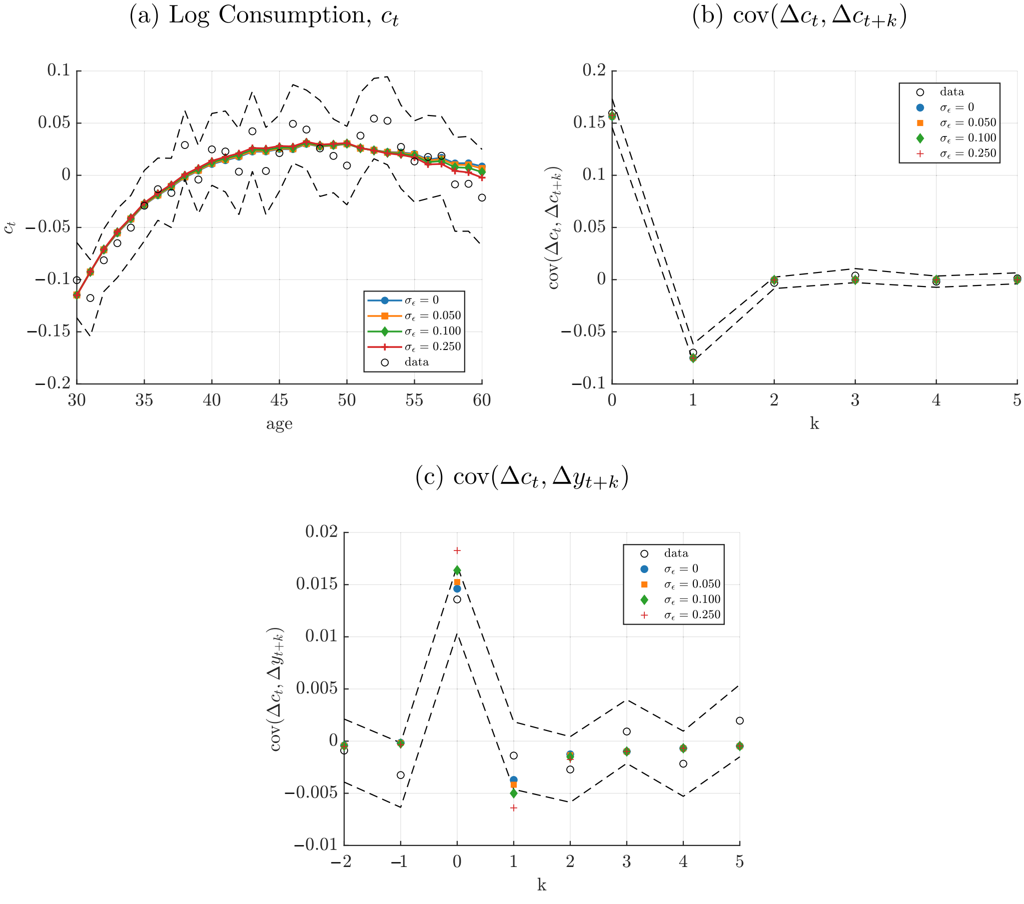


Fig. 4. *Model Sensitivity ($\alpha$ free, $\omega$ free).*

*Notes:* Figure 4 illustrates the average age profile log consumption together with covariance moments. The age profile is normalised by the overall mean. Hollow dots are calculated using the PSID, $\mathbf{\Lambda}(\mathbf{w})$, dashed lines are 95% confidence intervals, and solid dots are calculated using simulated data from the model, $\overline{\mathbf{\Lambda}}(\hat{\boldsymbol{\theta}})$.

reducing the degree of information first and foremost increases the transmission of income growth to consumption growth, but the model already fits this transmission with perfect information, $\sigma_\epsilon = 0$.

### 3.5. *Robustness Checks*

Figure 5 shows the results of an experiment in which we re-estimate the degree of information, $\sigma_\epsilon$, for various discount factors, $\beta$, keeping the remaining parameters fixed at their estimated levels in Column (3) in Table 3. Panel (a) shows the resulting objective function, while Panel (b) shows the associated estimate of the degree of noise in the private signal, $\sigma_\epsilon$. Panels (c) and (d) show, respectively, the life-cycle profile of consumption and the covariances of consumption growth and income growth when varying the discount factor, $\beta$, keeping the remaining parameters (including $\sigma_\epsilon$) fixed at their estimated levels in Table 3 Column (3).

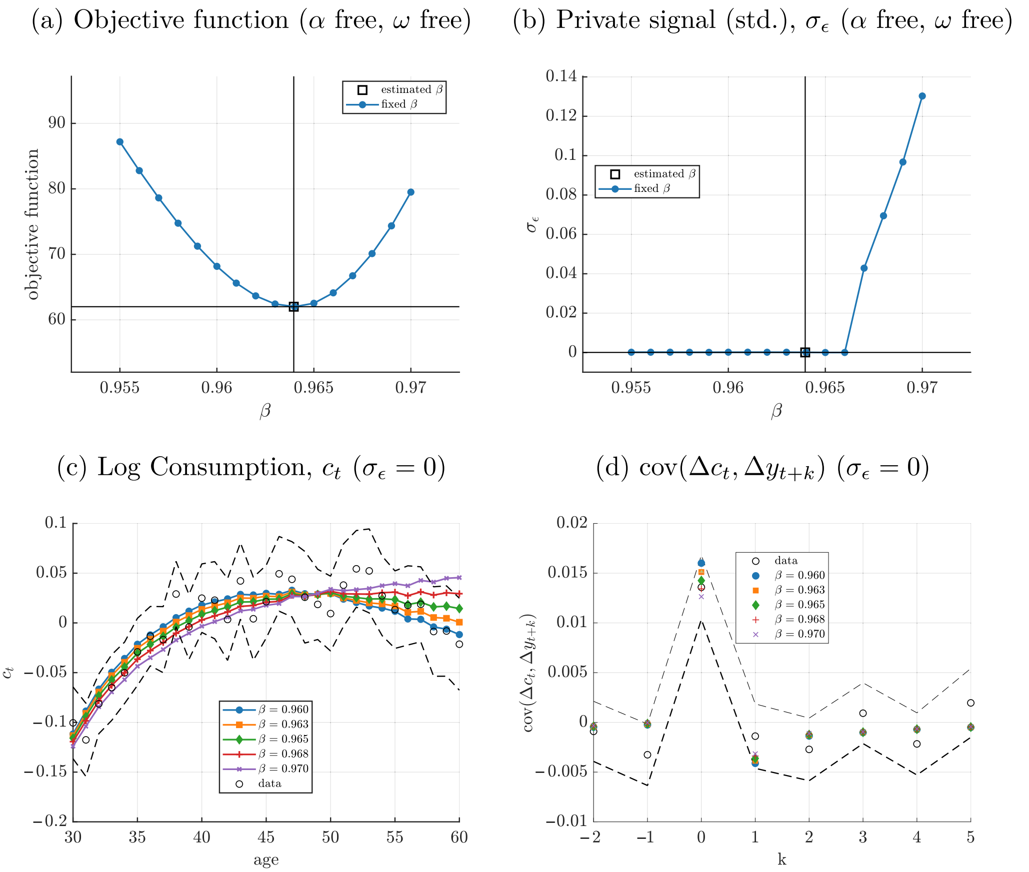


Fig. 5. *Identification. Re-estimating $\sigma_\epsilon$ for Various $\beta$.*
*Notes:* Panels (a) and (b) show the objective function and value of $\sigma_\epsilon$ when re-estimating $\sigma_\epsilon$ for various values of $\beta$ keeping the remaining parameters fixed at their estimated levels in Column (3) in Table 3. Panels (c) and (d) show the life-cycle profile of consumption and the covariances of consumption growth and income growth for various values of $\beta$ keeping the remaining parameters fixed at their estimated levels in Column (3) in Table 3.

We see that when the discount factor is fixed at a high value, we estimate a substantial degree of imperfect information. The explanation is that when households are forced to be patient the covariances between consumption growth and income growth are dampened (Panel (d)), which implies that imperfect information is then required to fit the observed covariances (remember Figure 4). Note, however, that fixing the discount factor just slightly above the estimated value leads to a substantial increase in the objective function mainly due to a worse fit of the life-cycle profile of consumption (Panel (c)).

Table 4 reports a series of additional robustness checks. First, we see in Columns (1)–(6) that neither varying the choice of risk aversion, $\rho$, the measurement error in income, $\tau$, or the replacement rate in retirement, $\lambda$, affect the main result of perfect information about permanent income. Secondly, in Columns (7)–(8), we consider an extension of the model in which the household is allowed to borrow. Specifically, we allow that it can borrow a fraction $\zeta$ of the mean

Table 4. *Estimates, Robustness ($\alpha$ free, $\omega$ free).*

| Parameter | | $\rho = 2.0$ (1) | $\rho = 4.0$ (2) | $\tau = 0.0$ (3) | $\tau = 0.5$ (4) | $\lambda = 0.25$ (5) | $\lambda = 0.75$ (6) | $\zeta = 0.2$ (7) | $\zeta = 0.4$ (8) | Diagonal W (9) | Equal W (10) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sigma_\epsilon$ | Private signal | 0.000 | 0.000 | 0.000 | 0.002 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.005 |
| | | – | – | – | – | – | – | – | – | – | – |
| $\beta$ | Discount factor | 0.960 | 0.936 | 0.965 | 0.963 | 0.967 | 0.961 | 0.964 | 0.965 | 0.963 | 0.940 |
| | | (0.002) | (0.007) | (0.002) | (0.002) | (0.001) | (0.002) | (0.002) | (0.002) | (0.002) | (0.014) |
| $\sigma_c$ | Measurement error in consumption | 0.273 | 0.274 | 0.273 | 0.273 | 0.274 | 0.273 | 0.273 | 0.273 | 0.272 | 0.262 |
| | | (0.004) | (0.004) | (0.004) | (0.004) | (0.004) | (0.004) | (0.004) | (0.004) | (0.005) | (0.009) |
| $\sigma_\psi$ | Persistent shock | 0.155 | 0.153 | 0.148 | 0.160 | 0.174 | 0.119 | 0.158 | 0.162 | 0.150 | 0.217 |
| | | (0.009) | (0.009) | (0.009) | (0.009) | (0.010) | (0.008) | (0.009) | (0.009) | (0.009) | (0.032) |
| $\sigma_\xi$ | Transitory shock | 0.152 | 0.154 | 0.189 | 0.102 | 0.140 | 0.171 | 0.150 | 0.146 | 0.156 | 0.074 |
| | | (0.008) | (0.008) | (0.007) | (0.012) | (0.010) | (0.006) | (0.009) | (0.009) | (0.008) | (0.191) |
| $g_0$ | Income growth, constant | 0.070 | 0.065 | 0.068 | 0.071 | 0.076 | 0.046 | 0.075 | 0.083 | 0.061 | 0.057 |
| | | (0.016) | (0.014) | (0.016) | (0.016) | (0.017) | (0.009) | (0.018) | (0.020) | (0.013) | (0.007) |
| $g_1$ | Income growth, age | −0.044 | −0.046 | −0.045 | −0.044 | −0.040 | −0.066 | −0.041 | −0.038 | −0.053 | −0.070 |
| | | (0.014) | (0.014) | (0.014) | (0.014) | (0.014) | (0.013) | (0.014) | (0.014) | (0.014) | (0.015) |
| $\alpha$ | AR(1) component | 0.862 | 0.870 | 0.862 | 0.862 | 0.856 | 0.915 | 0.853 | 0.842 | 0.883 | 0.935 |
| | | (0.028) | (0.027) | (0.029) | (0.027) | (0.027) | (0.025) | (0.028) | (0.028) | (0.027) | (0.026) |
| $\omega$ | MA(1) component | 0.144 | 0.147 | 0.103 | 0.292 | 0.110 | 0.185 | 0.138 | 0.127 | 0.154 | −0.500 |
| | | (0.041) | (0.039) | (0.028) | (0.079) | (0.053) | (0.029) | (0.042) | (0.046) | (0.039) | (3.912) |
| Objective | | 61.878 | 61.210 | 66.448 | 59.635 | 65.316 | 73.749 | 62.420 | 63.624 | 60.891 | 14.454 |
| $p$-value for $\sigma_\epsilon = 0$ | | 0.500 | 0.499 | 0.500 | 0.496 | 0.500 | 0.495 | 0.500 | 0.500 | – | – |

*Notes:* Asymptotic standard errors reported in brackets.

belief of its persistent income in each period, i.e.:

$$A_t \geq -\zeta \exp(\hat{p}_t). \tag{71}$$

Once again we estimate no noise in the private signal. A closer analysis of the results reveal that loosening the borrowing constraint dampens the covariances between consumption and income growth, but that the effect is minimal after re-estimating all other parameters. Finally, in Columns (9)–(10) we see that our results are not affected by either choosing a diagonal weighting matrix with the inverse of the variances of the moments on the diagonal, or the identity matrix. In Online Appendix C we show that our results survive the same robustness checks when fixing $\alpha = 1$ and/or $\omega = 0$.

## 4. Conclusions

We have developed a novel consumption-saving model with a flexible specification of the households' ability to distinguish persistent from transitory income shocks. We showed that the assumption about the households' degree of information has important implications for our interpretation of consumption-saving behaviour in general and estimated transmission parameters in particular. If households, like an econometrician, need to solve a filtering problem to distinguish persistent from transitory income shocks, it eradicates the correspondence between transmission parameters estimated as in Blundell *et al.* (2008), and the true marginal propensity to consume. Based on a theoretical analysis of a certainty equivalence model we were able to show that the households' degree of information is identifiable from panel data of income and consumption. In a Monte Carlo study we validated that our approach works for a general model with a complex income process.

Finally, we estimated the general model using the PSID. We estimate a large degree of information and cannot reject that households can perfectly distinguish persistent from transitory income shocks. This is reassuring in terms of validating the standard interpretation of estimated transmission parameters of persistent and transitory income shocks.

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Additional Supporting Information may be found in the online version of this article:

**Online Appendix**
**Replication Package**

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