A dynamic equilibrium model of commuting, residential and work location choices

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Motivation

- A large group of countries are undergoing a process of strong urbanization and spatial concentration of economic activity
- $\rightarrow$ increased productivity through agglomeration but...
  - ...rural areas have been declining in terms of population, labor demand and house prices
  - ...urban areas experience large increases in traffic congestion and house prices
  - $\rightarrow$ demographic composition of locations change and increased regional inequality
- Various policies are considered to ameliorate downsides of this development
- Dynamic effects of such spatial policies not well understood due to the complexity of households' joint choice of employment, work and residential locations and commuting

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We develop a dynamic equilibrium model that simulatenously tracks the following mechanisms:

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- Model fits key aspects of data well, incl. house price trends

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We simulate effects on house prices, job mobility, residential sorting and commuting in two counterfactual equilibria:

1. 5% increase in supply of housing stock in central Copenhagen
    - Stylized illustration of actually planned policy of constructing the artificial island Lynetteholm which we have analyzed for the Danish Ministry of Transportation
2. [FOCUS TODAY] Extended use of telecommuting for highly skilled (HS) workers
    - HS workers workers move out of city → prices drop in center → LS move there
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Overview

Literature

Model

Estimation

Data

Results

Housing market equilibrium

Counterfactuals

Conclusion

Overview

Related literature

- GE models of location choice: **Buchinsky2014**, **Ahlfeldtetal2015**
- Life-cycle location choice models w/o commuting: **KennanWalker2011**, **oswald2019**, **Dahl2002** and **Tunali2000**, **diamond2016**
- Interactions btw infrastructure, home and work locations: **tsivanidis2019**, **heblich2020**, **dingel2020**, **teulings2018**, **severen2021**, **chernoff2021**, **allen2019** and **monte2018**
- Search models: i.a. **manning2017**
- WTP for local non-traded amenities: **sieg2004**, **Bayer2016**, **Kuminoffetal2013**

The Choice Model

Individuals solve the following optimization problem:

$$\max_{\{d_{it}^{rl}, d_{it}^{wl}, h_{it}\}_{t=t_0}^{T}} \sum_{t=t_0}^{T} \rho_t(x_{it})^t \mathbb{E}_t\Big[u_t(x_{it}, wl_{it}, rl_{it}, d_{it}', \mathrm{h}_{it})\Big]$$

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s_{it}	$\{x_{it}, wl_{it}, rl_{it}\}$

Utility specification

$$u_t(x, d, d^{rl}, d^{wl}, h) =$$

$$\kappa(inc, s) \underbrace{\left[inc(s) - \psi_{uc} p^h(d^{rl}) \cdot h\right]}_{\text{consumption}} - swcost_r^p(x) + \underbrace{\phi_{h1}(s) h + \phi_{h2} h^2}_{\text{utility housing}}$$

$$+ \text{ttimecost}(x, d) + \text{amenities}(d^{rl}) + u_r$$

▸ Functional forms

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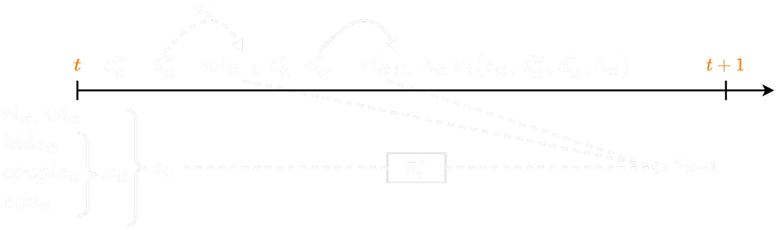
ttimecost	depends on travel-time between d^{rl} and d^{wl} and on year
amenities	depend on regional attributes of residential location, d^{rl}
u_r	utility of retiring for those eligible (≥ 60)

Functional forms

Model structure

Model is based on ▸ Bellman equations whose content is illustrated in the following:

Figure 1: Timeline of decisions and states



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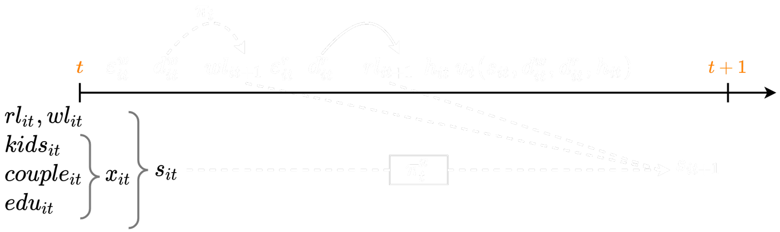

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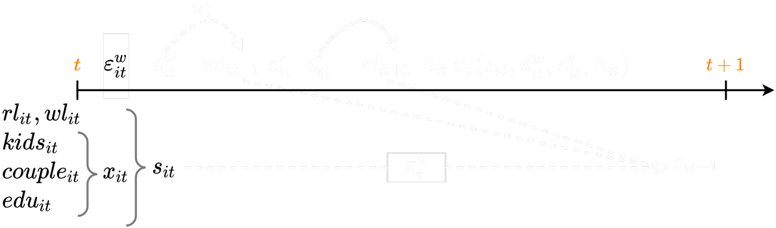

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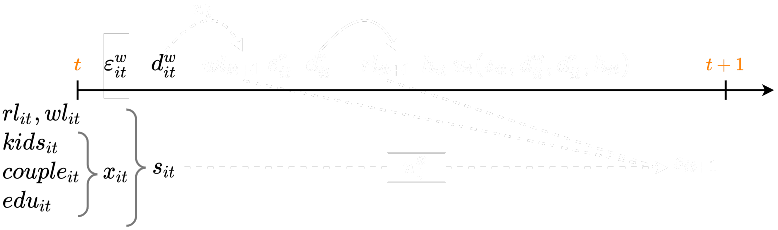

- R regions, $d^w_{it} \in R+1$ (voluntary and involuntary unemployment: ø)

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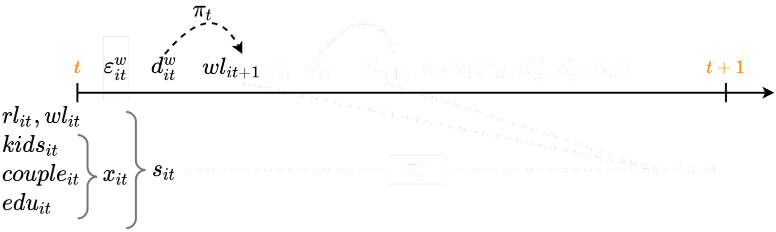

- $\pi_t(d^w_t, wl_t, x_t)$ informs about transition from desired work location to work location outcome

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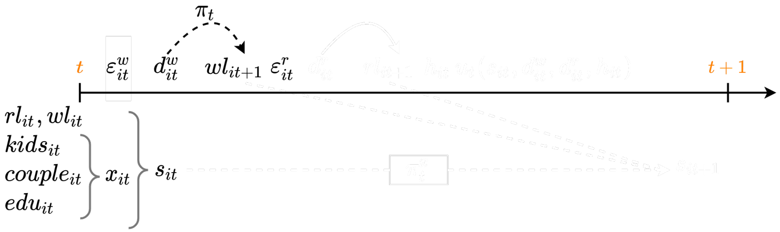

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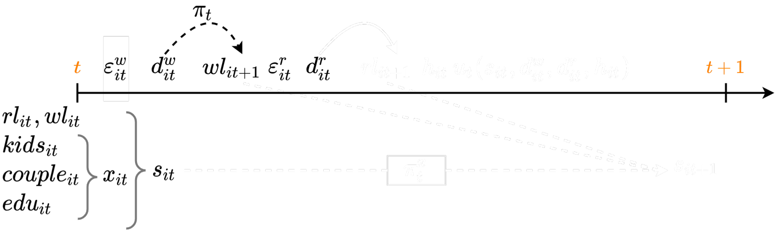

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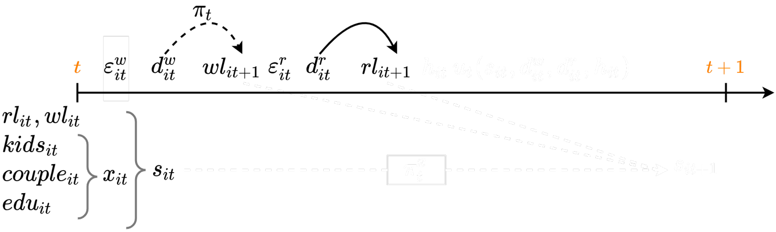

- $d^r_{it} \in R$ and perfect control over residential decision: $d^r_{it} = rl_{it+1}$

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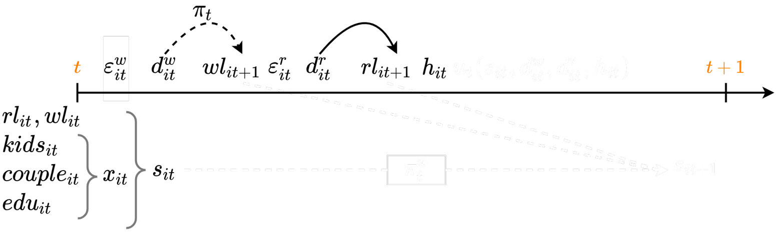

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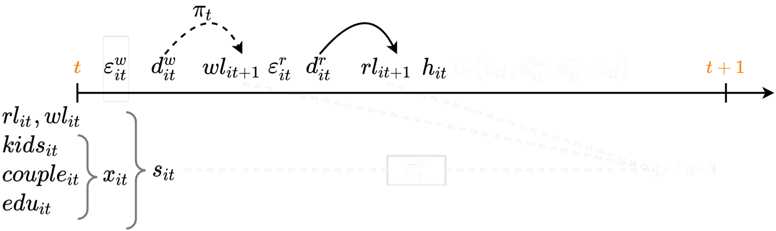

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- $\Rightarrow$ Next period value function $V_{t+1}(x, d, \varepsilon)$ independent of h

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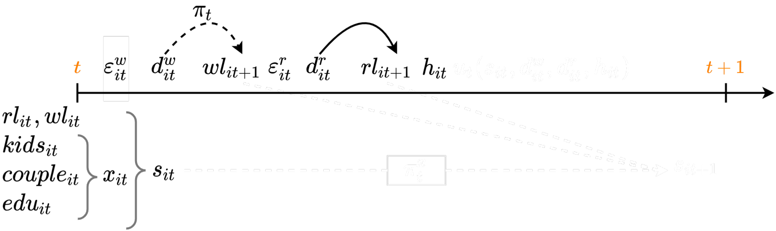

- $\Rightarrow$ Optimal housing demand given by FOC

$$\frac{\partial u_t(\cdot)}{\partial h} = \phi_{h1}(s) + 2\phi_{h2}h - \kappa(s)p^h(d^{rl}) = 0 \quad \Rightarrow$$

$$h_t^* = \frac{\phi_{h1}(s) - \kappa(s)p^h(d^{rl})}{-2\phi_{h2}}$$

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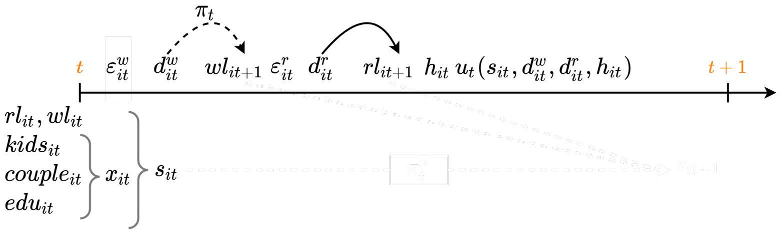

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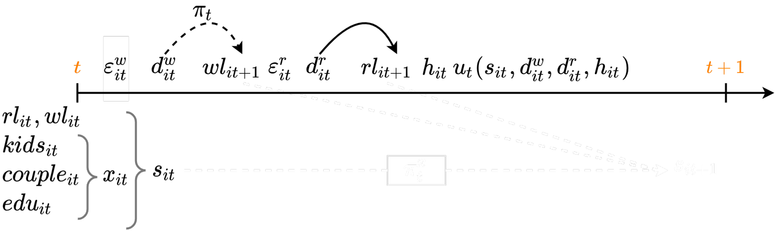

- Substituting expression of optimal housing demand into the utility function defined above, we obtain the *indirect utility function* $u(s_{it}, w_{it+1}, rl_{it+1})$
- Pure discrete choice model conditional on housing demand

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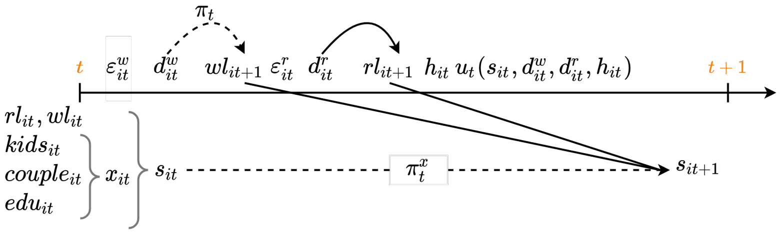

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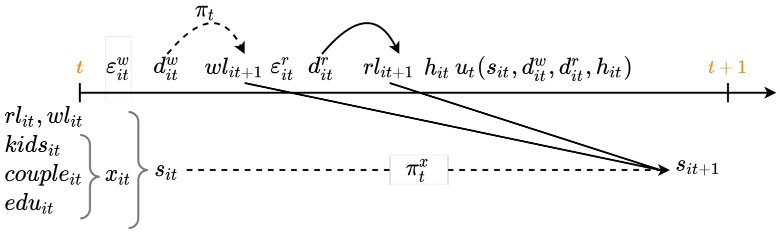

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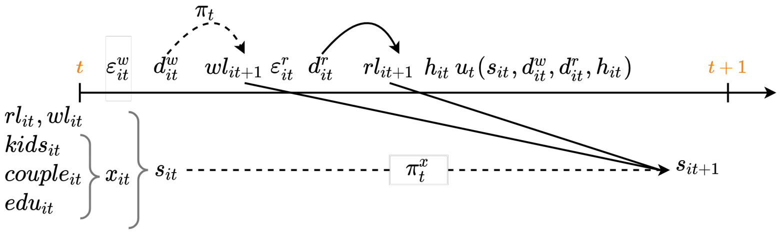

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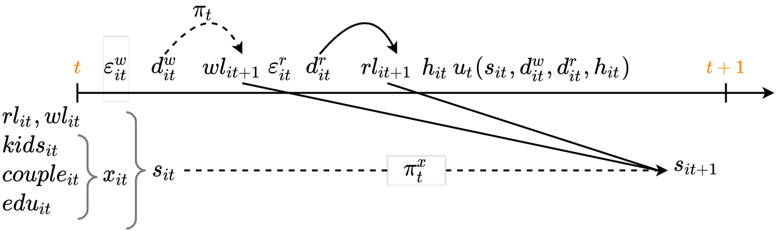

- Location states transition as a deterministic function of decisions
- edu_i is time-constant
- Non-location states:

$$(cs_{it+1}, ms_{it+1}) \sim \mu_{cs,ms}(\cdot|cs_t, ms_t, edu, age_t) \equiv \pi^x(x_t, x_{t+1}) \tag{1}$$

Overview

Estimation strategy

We estimate the model sequentially in three separate steps:

1. Estimate the parameters governing the pre-tax income equations, income tax system and transition probabilities of children and marital status
2. Estimate a reduced form housing demand equation
3. Estimate the remaining structural parameters by maximum likelihood applying the parameters obtained in 1) and 2) *conditional* on house prices

We solve the model via backwards induction for each evaluation of the likelihood function

On top of this comes an equilibrium solver in the counterfactual simulations.

▸ Functional forms ▸ Log-likelihood function

Overview

Data sources

We use full population Danish administrative panel data and focus estimation on the period 2000-2004 and 2009-2013

- Exclude years around housing boom in financial crisis. No attempt to model temporary price hike during this period

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- Each individual's choice $d_{i,t} \equiv \{rl_{i,t+1}, wl_{i,t+1}, h_{i,t+1}\}$ and state $s_{i,t}$ on an annual basis

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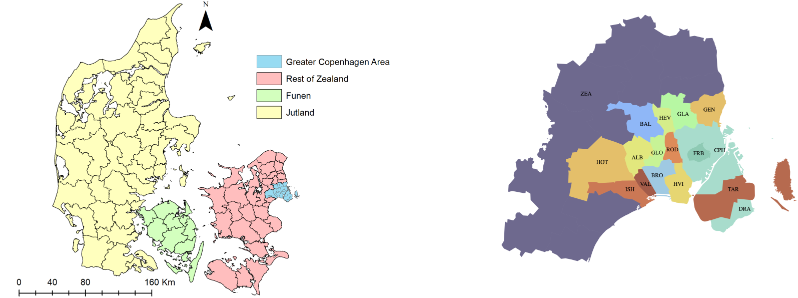
- Local labor market attributes (we use # employees by *edu* to proxy for labor demand)
- Travel time between all regions

Figure 2: Definition of regions

(a) Entire Denmark **(b)** Greater Copenhagen Area



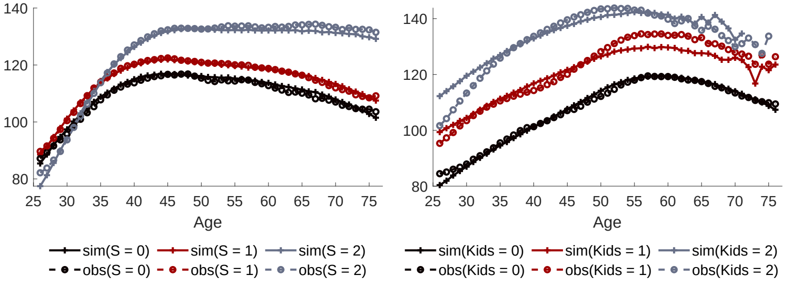
Note: The abbreviations denote the following regions: Copenhagen (CPH), Frederiksberg (FRB), Ballerup (BAL), Broendby (BRO), Dragoer (DRA), Gentofte (GEN), Gladsaxe (GLA), Glostrup (GLO), Herlev (HEV), Albertslund (ALB), Hvidovre (HVI), Hoeje-Taastrup (HOT), Roedovre (ROD), Ishoej (ISH), Taarnby (TAR), Vallensbaek (VAL), rest of Zealand (ZEA). Figure 2b only displays subset of ZEA.

Overview

Model fit: Housing demand

Figure 3: House size in square meters over the life cycle

(a) By schooling **(b)** By children



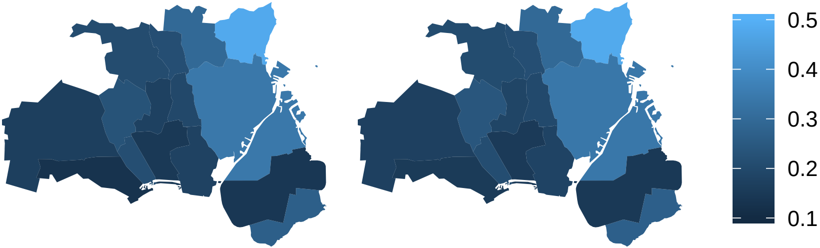
- Clear demographic differences in housing demand over the life cycle
- The reduced form model captures the crucial dependence between household composition and housing demand and difference over the life cycle
- Though some challenges capturing differences in demand at the beginning of the life cycle

▸ Spatial ▸ Marital status ▸ Params housing ▸ Params user costs

Model fit: Spatial educational sorting

Figure 4: Residential sorting by home region

(a) Share highly educated (obs.) **(b)** Share highly educated (sim.)



Residential sorting driven mainly by:

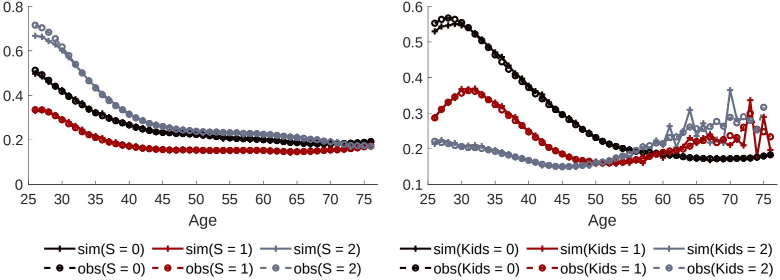
- Regional variation in house prices and regional-specific amenities
- Individual differences in housing demand
- Individual differences in marginal utility of money (main channel of educational sorting)
- Distance to local labor markets

▸ Params cafes ▸ Params amenities FE

Model fit: Sorting over the life cycle

Figure 5: Share living in Copenhagen over the life cycle

(a) By schooling **(b)** By children



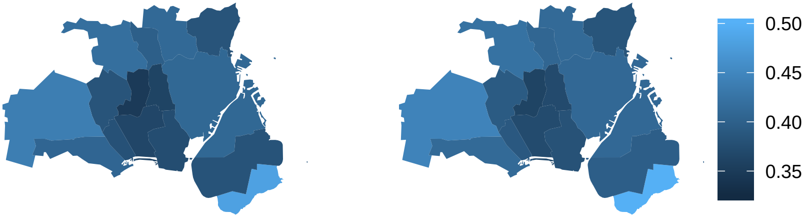
- Only for the youngest cohorts a slight under-prediction
- Reason: not modelling educational choice. Poorer fit only evident for highly-educated

▸ Marital status ▸ Moving ▸ Params moving costs

Model fit: Commuting across space

Figure 6: Commute times (hours) by residential location

(a) (obs.) **(b)** (sim.)



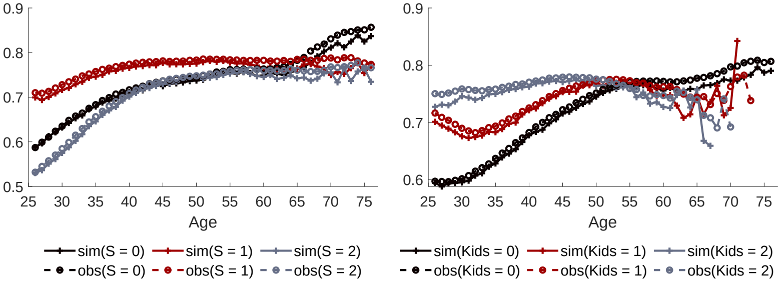
▸ Work in Cph ▸ Job moves ▸ Params commute costs ▸ Params job finding/dismissal

Model fit: Commute time over the life cycle

Figure 7: Commute time (hours)

(a) By schooling

(b) By children



- Model captures shorter commute by highly educated as they can afford housing close to dense labor markets
- Above age 60 harder to explain the strong selection among working individuals at that age

▸ Marital status

Demand and supply of housing

- Equilibrium prices, P^h: adjust so total expected demand $D_t(rl, P^h)$ for housing measured in square meters equals the (inelastic) supply $S_t(rl)$ in each residential region

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- Housing supply $S_t(rl)$: micro aggregated observed square meters of housing h_{it} for people who live in region $rl_{it} = rl$ at the beginning of each period t

$$S_t(rl) = \sum_{i=1}^{N} h_{it} 1(rl_{it} = rl)(1 - ms_{it}/2)$$

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- *Expected* housing demand $D_t(rl, P^h)$: population average of housing demand weighted by choice probabilities of either staying or moving to region rl at the end of period t.

$$D_t(rl, P^h) = \sum_{i=1}^{N} h(rl, x_{it}; P^h(rl))\Pi_t(rl|wl_{it+1}, rl_{it}, x_{it}; P^h)(1 - ms_{it}/2),$$

where $\Pi_t(rl|wl_{it+1}, rl_{it}, x_{it}; P^h)$ is the choice probability that individuals in state $s_{it} = (wl_{it+1}, rl_{it}, x_{it})$ choose to live in region rl, given the vector of regional house prices, P^h

Equilibrium house prices

To compute the housing market equilibrium, P^h is set to solve

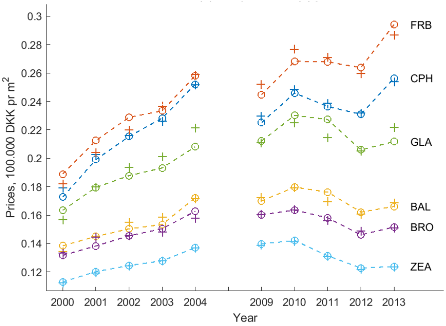
$$\begin{aligned} S_t(1) &= D_t(1, P^h) \\ &\vdots \\ S_t(R) &= D_t(R, P^h) \end{aligned}$$

where

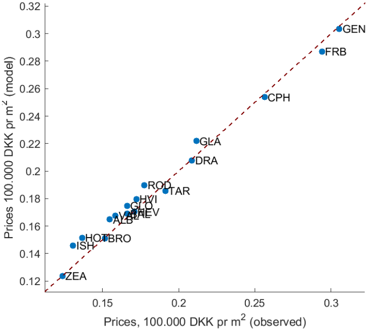
- $P^h = (P^h(1), .., P^h(R))$ s the R-dimensional vector of regional square meter prices in each residential region $rl = \{1, \ldots R\}$
- $S_t(rl)$ the inelastic, exogenously fixed supply of total square meters of housing in region rl
- $D_t(rl, P^h)$ is the demand for available square meters of housing in region rl
- We can easily solve the R equilibrium equations with R unknowns using Newton's Method.

Figure 8: Observed and predicted equilibrium house prices per m^2

(a) Selected regions, 2000-2004 and 2009-2013



(b) All regions, 2013



▸ Baseline fit

Overview

Counterfactual: increased access to telecommuting - effects on locations

Lowering commute time by 50% for highly-educated

Table 1: Counterfactual II: % change of home and work locations by schooling (2013)

	$rl(s_0)$	$wl(s_0)$	$rl(s_1)$	$wl(s_1)$	$rl(s_2)$	$wl(s_2)$
Center of CPH	3.77	0.44	4.52	0.39	-10.79	3.95
West of CPH	3.46	0.05	4.13	0.01	-13.23	-0.41
North of CPH	7.31	-0.06	7.51	-0.05	-9.62	-0.21
East of CPH	2.08	-0.11	4.81	-0.51	-10.40	1.07
RestOfZealand	-3.95	-0.44	-3.09	-0.76	11.69	7.64
Non-employment	-	-0.27	-	-0.20	-	-8.38

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- Easier for highly educated to keep high-paying jobs in city centre while living in attractive suburban areas
- Lower-income households better chance to reside closer to dense labor markets
- All regions more mixed on sociodemographics

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Lowering commute time by 50% for highly-educated

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- Highly-educated more likely to work in Cph and RoZ as they only have to commute half of the week
- Main part of extra workers in RoZ come from reduction in non-employment
- Less educated only slight increase in tendency to work in more urbanized areas

Counterfactual: increased access to telecommuting - effects on prices

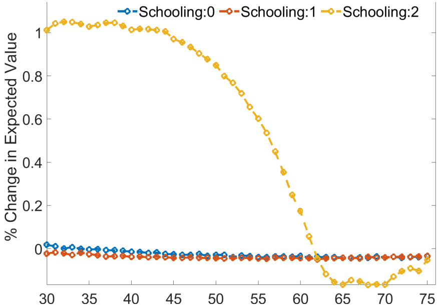
Table 2: Counterfactual II: % change in equilibrium prices 2009-2013

	2009	2010	2011	2012	2013
Center of CPH	-0.53	-1.01	-1.35	-1.64	-1.95
West of CPH	-0.36	-0.64	-0.93	-1.15	-1.60
North of CPH	-0.57	-1.08	-1.46	-1.58	-1.75
East of CPH	-0.06	-0.10	-0.79	-1.04	-1.66
RestOfZealand	0.79	1.55	2.08	2.45	3.03

- Dynamic model $\rightarrow$ Gradual changes in prices
- Lower demand for living in Cph for highly-educated $\rightarrow$ prices fall by 0.53% immediately
- Over time prices lower by 1.6-2.0% in the GCA while higher by 3.0% in RoZ
- $\rightarrow$ affordable for lower-income hhs to live in GCA

Counterfactual: increased access to telecommuting - welfare effects

Figure 9: Simulated change in welfare (%)



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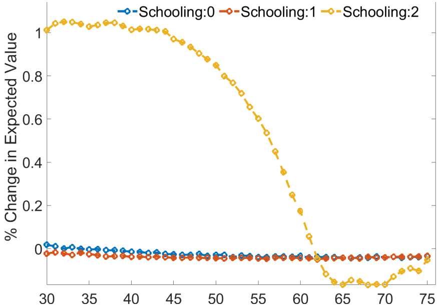

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- As they age, their marginal utility of money declines → welfare effects drops to 0 around retirement
- At retirement-eligible age: most not working → no benefit of telecommuting but facing increased house prices in their popular regions

Counterfactual: increased access to telecommuting - welfare effects

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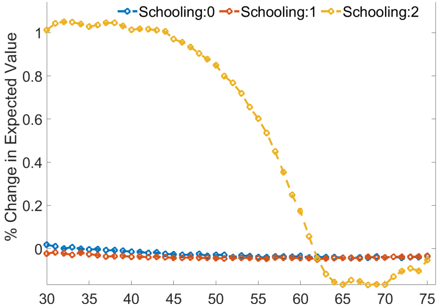

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- ≈ 0 welfare effect for less educated: gain from lower commute times, but only by paying higher house prices

Overview

Literature

Model

Estimation

Data

Results

Housing market equilibrium

Counterfactuals

Conclusion

Conclusion

- Dynamic equilibrium model of joint home and work location decisions as well as housing demand for individuals
- Focused on the Greater Copenhagen Area in Denmark and analyzed the counterfactual effects of encouraging more telecommuting for highly educated
- Provides understanding of how location and movement patterns are driven by cost of living and commuting and are very heterogeneous
- Counterfactual: highly educated move out of the city to peripheral regions $\rightarrow$ consume larger homes at a reduced price
- $\rightarrow$ freed up space in the center $\rightarrow$ lower educated people could afford living closer by their jobs in the new equilibrium
- Reducing commute times allows locations to become more specialized in either jobs or residence
- Welfare gains positive in total, but unequally distributed across household types with higher educated being better off and lower educated indifferent

Overview

Appendix: Bellman equations

Appendix: Functional forms

Appendix: Econometrics

Appendix: Model fits

Appendix: Model parameters

The residential choice

Let EV^r be the *ex ante* expected value of residence r,
... but conditional on the employment location outcome wl'
... and before learning about the residential location shocks $\epsilon^r(d^r)$.

EV^r is given by the usual log-sum formula

$$EV^r(wl, rl, wl', x) =$$

$$\sigma_r \log \left(\sum_{d^r} \exp\{[u(wl, rl, wl', d^r, x) + \beta EV(wl', d^r, x)]/\sigma_r\} \right) .$$

Residence location choice probabilities are given by logit formulas

$$P^r(d^r|wl, rl, wl', x) = \frac{\exp\{[u(wl, rl, wl', d^r, x) + \beta EV(wl', d^r, x)]/\sigma_r\}}{\sum_{d^r} \exp\{[u(wl, rl, wl', d^r, x) + \beta EV(wl', d^r, x)]/\sigma_r\}}.$$

◂ Timeline

The work location choice

- Let $EV^w(wl, rl, x)$ be the *ex ante* expected value of location (wl, rl)
  ...before learning about the work location shocks $\epsilon^w(d^w)$
  ...and the outcome of the job search process

$$EV^w(wl_t, rl_t, x) = \sigma_w \log \left(\sum_{d^w} \exp\left\{ v^w(wl, rl, x, d^w)/\sigma_w \right\} \right).$$

- $v^w(wl, rl, x, d^w)$ is the expected choice-specific value corresponding to the particular choice of job location d^w.

$$v^w(wl, rl, x, d^w) = \sum_{wl} \pi(d^w, wl, x, wl) EV^r(wl, rl, wl, x).$$

- $\pi(d^w, wl, x, wl')$ governs how job search location d^w translates into the realized one wl'

- The work location *choice* probabilities are given by logit formulas

$$P^w(d^w | wl, rl, x) = \frac{\exp\{v^w(wl, rl, x, d^w)/\sigma_w\}}{\sum_{d^w} \exp\{v^w(wl, rl, x, d^w)/\sigma_w\}}.$$

◂ Timeline

Utility function

Parsimonious utility function:

$$u = u_m + u_r + u_h + \underbrace{amenities - swcost_r^p - ttimecost}_{u_o} \tag{2}$$

- u_m: *monetary utility* (disposable income net of housing expenditures)
- u_r: utility of retirement for the eligible individuals ($t \geq 60$)
- u_h: *housing utility* obtained from the utilization of a chosen home size
- *amenities*: *regional-specific* attractiveness of housing options
- $swcost_r^p$: *psychological costs* of changing the location of residence
- *ttimecost*: *cost of commuting* between the chosen locations of work and residence

◂ Econometrics ◂ Utility

Housing demand

Quadratic utility of housing

$$u_m + u_h = \kappa(inc_t)(inc_t - hcost_t) + \Phi(x_t)h_{t+1} + \frac{1}{2}\phi_{h2}\, h_{t+1}^2,$$

where $\phi_{h2} < 0$ (diminishing returns to house size)

Marginal utility of money depends on income, year and x

$$\kappa(inc_t) = \kappa_0 + \sum_{\tilde{y}=1}^{Y} \kappa_{year,\tilde{y}} \mathbb{1}_{\{year=\tilde{y}\}} + \kappa_y\, inc_t + \kappa_{ms}\, ms_t +$$

$$+ \sum_{k=1}^{2} \kappa_{c,k} \mathbb{1}_{\{cs_t=k\}} + \kappa_a\, age_t + \sum_{j=1}^{2} [(\kappa_{s,j} + \kappa_{as,j}\, age_t) \mathbb{1}_{\{edu_t=j\}}]$$

Housing demand

$\Phi(x_t)$ allows for heterogeneity in marginal utility of housing

$$\Phi(x_t) = \phi_0 + \sum_{y=1}^{Y} \phi_{year,y} \mathbb{1}_{\{year=y\}} + \phi_a age_t + \phi_{a2} age_t^2 + \phi_{ms} ms_t$$
$$+ \sum_{k=1}^{2} \phi_{c,k} \mathbb{1}_{\{cs_t=k\}} + \sum_{j=1}^{2} \phi_{s,j} \mathbb{1}_{\{edu_t=j\}} + \sum_{rl=1}^{R} \phi_{rl} \mathbb{1}_{\{rl_{t+1}=rl\}}.$$

Housing costs are given by

$$hcost_t(rl_{t+1}, h_{t+1}) = \psi_{uc} p^h(rl_{t+1}) h_{t+1},$$

Implied housing demand (linear regression).

$$h_{t+1} = \frac{\kappa(inc_t) p^h(rl_{t+1}) \psi_{uc} - \Phi(x_t)}{\phi_{h2}}.$$

◂ Econometrics ◂ Utility

Regional amenities

Amenities of regions come as a bundle of attributes that each contributes to the experienced utility of a region.

$$\begin{aligned} amenities(rl_{t+1}) = & (\alpha_0^{cafe} + \alpha_a^{cafe} age_t + \sum_{k=1}^{2} \alpha_{c,k}^{cafe} \mathbb{1}_{\{cs_t = k\}}) cafes_{rl_{t+1}} \\ & + \sum_{rl=1}^{R} \boldsymbol{\alpha}_{rl} \mathbb{1}_{\{rl_{t+1} = rl\}} \end{aligned}$$

- We can include a rich set of amenities almost without any additional computational cost associated with solving the model.
- This may require more parameters as number of amenities increases, but the number of parameters in the chosen specification is independent of the number of regions

◂ Econometrics ◂ Utility

Job arrival and dismissal

Probability of getting a new job

$$\pi_t^n(d^w, wl, x) = \Bigg[1 + exp\Bigg(- \Big(\beta_0^{\pi(n)} + \beta_a^{\pi(n)} age + \beta_\varnothing^{\pi(n)} 1_{wl=\varnothing} + \beta_{jobdens}^{\pi(n)} jobdens(d^w) + \sum_{k=1}^{2} (\beta_s^{\pi(n)}(k) 1_{edu=k})\Big)\Bigg)\Bigg]^{-1}, \tag{3}$$

Probability of keeping current job

$$\pi^k(wl, x) = \Bigg[1 + exp\Bigg(- \Big(\beta_0^{\pi(k)} + \beta_a^{\pi(k)} age + \sum_{k=1}^{2} (\beta_s^{\pi(k)}(k)\{1_{edu=k}))\Big)\Bigg)\Bigg]^{-1}. \tag{4}$$

◂ Econometrics ◂ Utility

Utility cost of moving, $swcost_r^p(x_t)$

The utility cost of moving residence

$$swcost_r^p(x_t) = \mathbb{1}_{\{rl_t \neq rl_{t+1}\}}[\gamma_0 + \gamma_a age_t + \sum_{k=1}^{2} \gamma_{c,k} \mathbb{1}_{\{cs_t = k\}} + \gamma_{ms} ms_t + \sum_{j=1}^{2} \gamma_{s,j} \mathbb{1}_{\{edu_t = j\}}]$$

◂ Econometrics ◂ Utility

Travel time costs

The commuting cost between d^{rl} and d^{wl}

$$ttimecost = (\eta_0 + \sum_{y=1}^{Y} \eta_{year,y} \mathbb{1}_{\{year=y\}}) f^{tt}(rl_{t+1}, wl_{t+1})$$

where

- The $f^{tt}(\cdot)$ function denotes the shortest travel-time by any means of transportation between locations.
- Commute cost are zero when unemployed

Commute cost are assumed to be a function of

- Travel time between the two destinations.
- Year (allowing disutility of commuting to change over time)

◂ Econometrics ◂ Utility

Overview

Log-likelihood function

- The MLE is obtained as

$$\hat{\theta} = \text{argmax}_{\theta} \; \frac{1}{N} \sum_{i} \sum_{t} \{ \log P_t^r(rl_{it+1} | wl_{it}, rl_{it}, wl_{it+1}, x_{it}; \theta) + \log \sum_{d^w} P_t^w(d^w | wl_{it}, rl_{it}, x_{it}; \theta) \pi_t(d^w, wl_{it}, x_{it}, wl_{it+1}; \theta) \}, \tag{5}$$

  where N is the number of individuals.
- We solve the model via backwards induction for each evaluation of the likelihood function.

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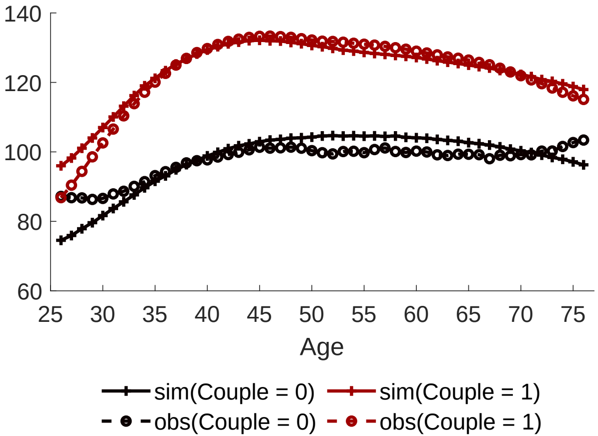
Overview

Model fit: Housing demand by marital status

Figure 10: House size in square meters over the life cycle



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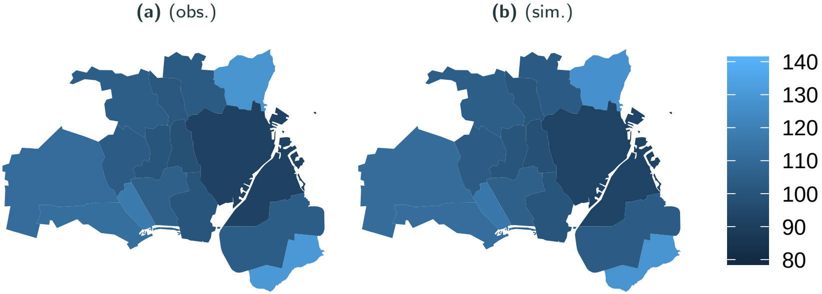
Model fit: Housing demand by region

Figure 11: House size by home region



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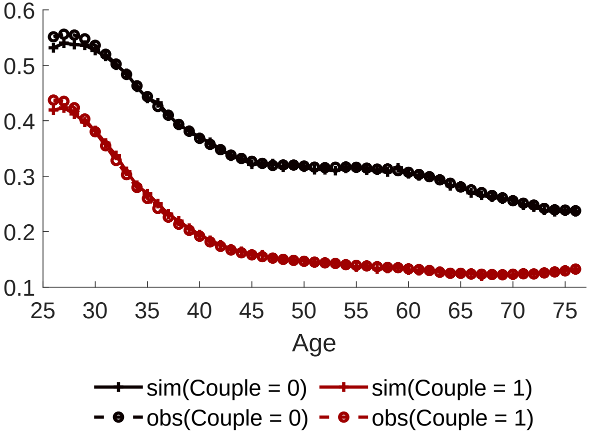
Model fit: Share living in Copenhagen by marital status

Figure 12: Share living in Copenhagen over the life cycle by marital status



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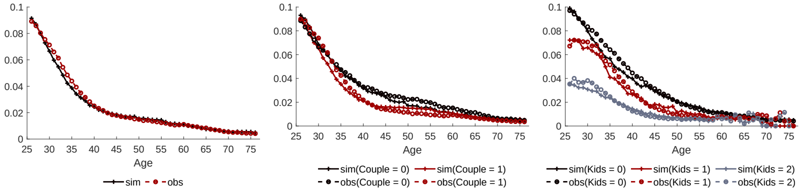
Model fit: Moving propensity

Figure 13: Share moving residential location over the life cycle

(a) Overall **(b)** By marital status **(c)** By children



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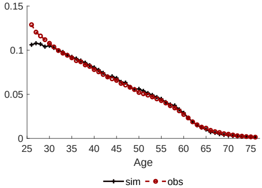
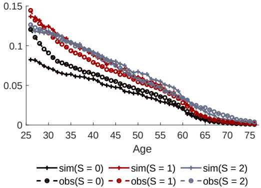
Model fit: Sorting on workplace over the life cycle

Figure 14: Job moves

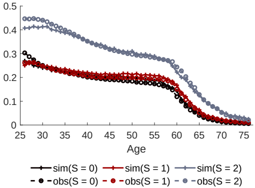
(a) Move job



(b) Move job by schooling



(c) Working in CPH by schooling



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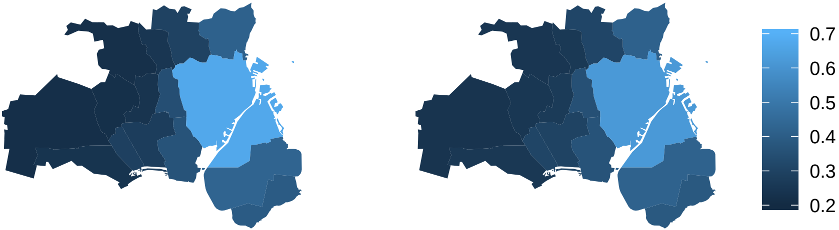
Model fit: Working in Cph by home location

Figure 15: Work in Copenhagen by residential location

(a) (obs.) **(b)** (sim.)



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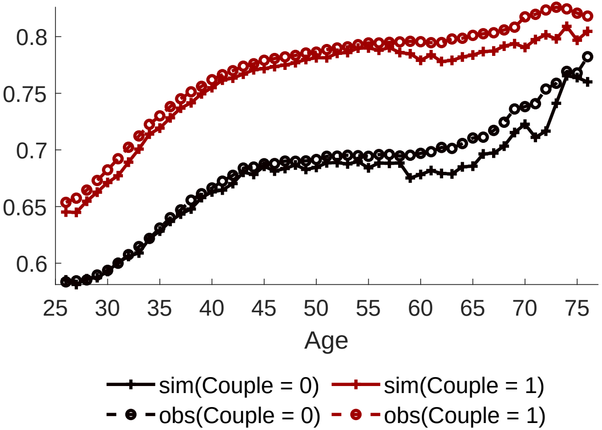
Model fit: Commuting by marital status

Figure 16: Commute time (hours) by marital status



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Overview

Model parameters: Housing demand

Table 3: First Stage Parameter Estimates, Reduced form Housing Demand

	Coeff. Estimates	Standard Error	Z-statistic
Const., $\tilde{\phi}_0$	70.2740	0.16419	428.0
Married, $\tilde{\phi}_{ms}$	27.8578	0.03868	720.2
Children, $\tilde{\phi}_c$ (1)	5.7386	0.05149	111.4
Children, $\tilde{\phi}_c$ (2)	14.6098	0.04911	297.5
Age, $\tilde{\phi}_a$	2.1723	0.00348	624.8
$\text{Age}^2/1000$, $\tilde{\phi}_{a2}$	-19.1718	0.03074	-623.6
Price pr. sqm, $\tilde{\kappa}_0$	-296.2954	0.91043	-325.4
Price pr. sqm $\times$ Income, $\tilde{\kappa}_y$	20.2790	0.07002	289.6
Price pr. sqm $\times$ Age, $\tilde{\kappa}_a$	0.0209	0.00853	2.4
Price pr. sqm $\times$ Age x Schooling, $\tilde{\kappa}_{a,s}$ (1)	1.0073	0.00476	211.5
Price pr. sqm $\times$ Age x Schooling, $\tilde{\kappa}_{a,s}$ (2)	2.9563	0.00529	558.4
Price pr. sqm $\times$ Schooling, $\tilde{\kappa}_s$ (1)	-51.7247	0.24317	-212.7
Price pr. sqm $\times$ Schooling, $\tilde{\kappa}_s$ (2)	-95.3167	0.25080	-380.1
Price pr. sqm $\times$ Children, $\tilde{\kappa}_c$ (1)	0.4389	0.29673	1.5
Price pr. sqm $\times$ Children, $\tilde{\kappa}_c$ (2)	13.4067	0.28757	46.6
Price pr. sqm $\times$ Married, $\tilde{\kappa}_{ms}$	-63.4794	0.22117	-287.0

Dependent variable: House size in square meters
Other controls: Regional dummies, $\tilde{\phi}_{rl}$ and time effects $\tilde{\phi}_{year}$ and $\tilde{\kappa}_{year}$

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Model parameters: User costs of housing demand curvature

Table 4: User Cost of Housing and Curvature Parameter of Housing Demand

	Coeff. Estimates	Standard Error	Z-statistic
Coef. on $h^2, \phi_{h2} \times 1000$	-0.0465	0.00036	-127.4
Baseline user cost of housing, ψ_0	0.0239	0.00024	99.9
Time effect, ψ_{2001}	-0.0052	0.00015	-34.9
Time effect, ψ_{2002}	-0.0045	0.00016	-28.8
Time effect, ψ_{2003}	-0.0063	0.00015	-41.0
Time effect, ψ_{2004}	-0.0090	0.00016	-57.3
Time effect, ψ_{2009}	-0.0035	0.00015	-23.6
Time effect, ψ_{2010}	-0.0076	0.00015	-50.2
Time effect, ψ_{2011}	-0.0089	0.00015	-57.7
Time effect, ψ_{2012}	-0.0088	0.00015	-57.1
Time effect, ψ_{2014}	-0.0110	0.00016	-67.4

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Model parameters: Amenities (time-varying)

Table 5: Taste Variation in Regional Amenities

	Coeff. Estimates	Standard Error	Z-statistic
Taste for cafes and bars, α^{cafe}			
Constant, α_0^{cafe}	0.0118	0.00005	257.9
Age, α_a^{cafe}	-0.0002	0.00000	-257.5
Children, α_c^{cafe} (1)	-0.0047	0.00005	-97.8
Children, α_c^{cafe} (2)	-0.0074	0.00003	-254.4

Other controls: Regional dummies, $\alpha_{rl_{t+1}}$, shown in online appendix.

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Model parameters: Amenities (fixed effects)

Table 6: Time-Constant Regional Amenities

	Coeff. Estimates	Standard Error	Z-statistic
α_{rl} (1)	-0.0976	0.00045	-219.3
α_{rl} (2)	-0.0827	0.00095	-87.0
α_{rl} (3)	-0.1244	0.00107	-115.8
α_{rl} (4)	-0.2733	0.00207	-131.9
α_{rl} (5)	-0.1956	0.00126	-155.7
α_{rl} (6)	-0.0721	0.00081	-89.4
α_{rl} (7)	-0.1329	0.00127	-104.5
α_{rl} (8)	-0.1086	0.00116	-93.6
α_{rl} (9)	-0.1246	0.00121	-102.7
α_{rl} (10)	-0.0713	0.00088	-81.5
α_{rl} (11)	-0.1310	0.00101	-129.9
α_{rl} (12)	-0.0801	0.00094	-85.3
α_{rl} (13)	-0.1888	0.00139	-135.9
α_{rl} (14)	-0.1037	0.00099	-104.6
α_{rl} (15)	-0.2247	0.00163	-137.8
α_{rl} (16)	-0.0274	0.00127	-21.6

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Model parameters: Moving costs

Table 7: Utility Cost of Moving Residence

	Coeff. Estimates	Standard Error	Z-statistic
Const., γ_0	1.8750	0.00521	360.0
Age, γ_a	0.0579	0.00012	495.1
Children, γ_c (1)	0.4934	0.00382	129.2
Children, γ_c (2)	1.1926	0.00450	265.0
Married, γ_{ms}	-0.0368	0.00291	-12.6
Schooling, γ_s (1)	0.0163	0.00309	5.3
Schooling, γ_s (2)	-0.1803	0.00317	-56.9

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Model parameters: Commuting

Table 8: Commute Cost

	Coeff. Estimates	Standard Error	Z-statistic
Cost of travel time, η_0	0.1789	0.00149	120.3
Time effect, η_{2001} (1)	-0.0014	0.00174	-0.8
Time effect, η_{2002} (2)	-0.0081	0.00178	-4.5
Time effect, η_{2003} (3)	-0.0210	0.00184	-11.4
Time effect, η_{2004} (4)	-0.0451	0.00181	-24.9
Time effect, η_{2009} (5)	0.0406	0.00191	21.2
Time effect, η_{2010} (6)	0.0323	0.00190	17.0
Time effect, η_{2011} (7)	0.0450	0.00194	23.2
Time effect, η_{2012} (8)	0.0484	0.00195	24.8
Time effect, η_{2013} (9)	0.0202	0.00197	10.2

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Model parameters: Job arrival and dismissal

Table 9: Job Arrival and Dismissal

	Coeff. Estimates	Standard Error	Z-statistic
Probability of keeping job: $\pi_t^k(wl_t, x_t; \beta^k)$			
Const., $\beta_0^{\pi(keep)}$	0.3066	0.01085	28.3
Age, $\beta_a^{\pi(keep)}$	0.0558	0.00030	186.7
Schooling, $\beta_s^{\pi(keep)}$ (1)	0.9288	0.00536	173.4
Schooling, $\beta_s^{\pi(keep)}$ (2)	1.0818	0.00575	188.0
Probability of new job: $\pi_t^n(d_t^w, wl_t, x_t : \beta^n)$			
Const., $\beta_0^{\pi(new)}$	-1.0998	0.00466	-235.9
Age, $\beta_a^{\pi(new)}$	-0.0431	0.00010	-415.9
Schooling, $\beta_s^{\pi(new)}$ (1)	0.1980	0.00253	78.3
Schooling, $\beta_s^{\pi(new)}$ (2)	0.2264	0.00278	81.5
Job density $\beta_{jobdensity}^{\pi(new)}$	0.2608	0.00045	583.9
Prev. unempl., $\beta_{unemp}^{\pi(new)}$	1.0474	0.00236	443.9

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Model parameters: Baseline fit

Table 10: Baseline fit: Change in home and work locations (share by schooling) and equilibrium prices (100,000 DKK)

(data - baseline)	$rl(s_0)$	$rl(s_1)$	$rl(s_2)$	$wl(s_0)$	$wl(s_1)$	$wl(s_2)$	P^{eq}
Center of CPH	0.000	0.006	-0.048	0.003	0.033	-0.009	-0.023
West of CPH	-0.005	-0.003	0.012	0.119	0.120	0.111	0.010
North of CPH	-0.004	0.000	0.006	0.019	0.019	0.007	0.000
East of CPH	0.001	-0.001	0.003	0.024	0.025	0.024	0.004
RestOfZealand	0.008	-0.002	0.027	-0.142	-0.222	-0.170	0.012
Unemployment	-	-	-	-0.024	0.024	0.037	-

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