Advanced Mathematics – WS2021 – Lab 5 – ODE

Please put you name and student ID on the paper or in the mail you send me (bruce.thomas@gis.uni-stuttgart.de). Submission is for next Monday, December the 14th. Ask me any questions you have! Do your best! 😊 *Good luck!*

Exercise 1 – Second-order Linear ODE's with constant coefficients

Solve these ODE by hand and on Matlab. For the Matlab part, I just want a commented code that works. In the comments, write each answer you have for each equation. Use this link if you need help: https://de.mathworks.com/help/matlab/math/choose-an-ode-solver.html.

(1.1) $y'' - 3y' + 2y = 0$

(1.2) $y'' + 2y' + 2y = 0$

(1.3) $y'' + 4y' + 4y = 0 \quad with\ y(0) = 1\ and\ y'(0) = 1$

(1.4) $y'' + 2y' - 3y = 0 \quad with\ y(0) = 1\ and\ y'(0) = -1$

(1.5) $y'' - 2y' + 5y = 0 \quad with\ y(0) = 1\ and\ y'(0) = -1$

(1.6) $y'' + 2y' + y = 4xe^x$

(1.7) $y'' + y = \cos(x)$

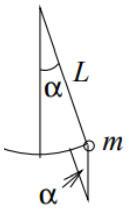
(1.8) $|x|y' + (x-1)y = x^3 \quad give\ solutions\ for\ x \in]0, +\infty[\ then\ for\ x \in]-\infty, 0[$

Exercise 2 – Euler's equidimensional equation

(2.1) $x^2y'' + pxy' + qy = 0 \quad with\ p\ and\ q\ constants$

Show that setting $x = e^t$ changes it into an equation with constant coefficients. Use this to find the general solution to 2.1 with p=1 and q=1.

Exercise 3 – Pendulum



Show that the angle α of the pendulum swinging with small amplitude approximately obeys to a second-order ODE with constant coefficients.

Use L = length, m = mass, damping = mcdα/dt, for some constant c. If the motion is undamped, i.e., c=0, express the period in terms of L, m, and the gravitational constant g.