

## Lab 12

### Exercise 3 – Kilauea crater

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The rim of the Halemaumau crater within Kilauea caldera has been digitized and its UTM coordinates  $(x_i, d_i)$  are listed in halemaumau.txt. We wish to approximate this shape by a perfect circle with parameters  $(x_0, d_0, r)$ .

Task 1 – what is the misfit function  $E$  that we want to minimize using the least squares criterion?

Task 2 – determine the three parameters for the circular model by minimizing  $E$ .

Task 3 – estimate the area of the crater.

From the circle equation  $x^2 + d^2 + ax + bd + c = 0$ , the least squares criterion is set as  $-(x^2 + y^2) = ax + by + c$

$$A_{65 \times 3} = [x_i \quad d_i \quad 1] = \begin{bmatrix} 259749.45 & 2147112.57 & 1 \\ 259778.64 & 2147071.53 & 1 \\ \vdots & \vdots & \vdots \\ 259734.96 & 2147121.07 & 1 \end{bmatrix}$$

$$Y_{65 \times 1} = [-(x_i^2 + d_i^2)] = \begin{bmatrix} -4677562151920.23 \\ -4677401085542.04 \\ \vdots \\ -4677591123871.25 \end{bmatrix}$$

$$\hat{\mathbf{x}} = (A^T A)^{-1} A^T Y = \begin{bmatrix} \hat{a} \\ \hat{b} \\ \hat{c} \end{bmatrix} = \begin{bmatrix} -5.2038 \times 10^5 \\ -4.2948 \times 10^6 \\ 4.6791 \times 10^{12} \end{bmatrix}$$

$$\hat{e} = Y - A(A^T A)^{-1} A^T Y$$

$$E = \hat{e}^T \hat{e} = 6.05 \times 10^{10}$$

**Estimate the parameters  $(x_0, d_0, r)$  of the fitted perfect circle**

$$x_0 = 5.2038 \times \frac{10^5}{2} = 2.6019 \times 10^5$$

$$d_0 = 4.2948 \times \frac{10^6}{2} = 2.1474 \times 10^6$$

$$r = \sqrt{-4.6791 \times 10^{12} + x_0^2 + d_0^2} = 518.0077$$

**Estimate the area of the crater**

With MATLAB code *polyarea*, it shows that the area covered by the data is  $8.4363 \times 10^5$ .