Q1.

a) grad div curl

grad: vector quantity from a scalar field; tangent vector pointing in direction of max change of scalar field at point P (w/ distance); magnitude = rate in increase in that direction.

div: scalar quantity from a vector field; the flux generation per unit volume at each point of the field.+ve = source, -ve = sink.

curl: vector quantity from a vector field; represents the vorticity of the field around P; direction along rotation axis, norm = magnitude of rotation.

b) curl $\vec{F} = x^3y^2\,\vec{i} + x^2y^3z^4\,\vec{j} + x^2z^2\,\vec{k}$

$$curl\ \vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3y^2 & x^2y^3z^4 & x^2z^2 \end{vmatrix}$$

$$= \left(\tfrac{\partial}{\partial y}\left(x^2z^2\right) - \tfrac{\partial}{\partial z}\left(x^2y^3z^4\right) \right)\vec{i} + \left(\tfrac{\partial}{\partial z}\left(x^3y^2\right) - \tfrac{\partial}{\partial x}\left(x^2z^2\right) \right)\vec{j} + \left(\tfrac{\partial}{\partial x}\left(x^2y^3z^4\right) - \tfrac{\partial}{\partial y}\left(x^3y^2\right) \right)\vec{k}$$

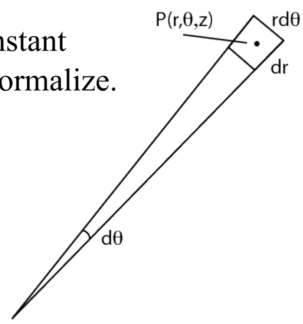
$$= \left(0 - 4x^2y^3z^3\right)\vec{i} + \left(0 - 2xz^2\right)\vec{j} + \left(2xy^3z^4 - 2x^3y\right)\vec{k}$$

Thus the curl is

$$= \left(- 4x^2y^3z^3\right)\vec{i} + \left(- 2xz^2\right)\vec{j} + \left(2xy^3z^4 - 2x^3y\right)\vec{k}$$

c) grad in cylindrical coords

i) the incremental distances for sides of an infinitesimal patch in the z=constant place have lengths dr and $rd\theta$ so need to divide q_2 coord axis by $r=q_1$ to normalize.



ii) $f = xyz.$

$x=r\cos\theta$

$y=r\sin\theta$

$z = z$
$f = r\cos\theta\, r\sin\theta\, z$

Solution. We have $f = r^2 z \sin\theta \cos\theta$. Thus

$$\nabla f = \mathbf{u}_1 2rz \sin\theta \cos\theta + \mathbf{u}_2 rz(\cos^2\theta - \sin^2\theta) + \mathbf{u}_3 r^2 \sin\theta \cos\theta.$$

d) div(curl A) = 0

i) curl produces vector output that measures "vorticity" at a point with rotation axis oriented along each coordinate axis, so result is perpendicular to coordinate axis. div measures change in vector components along their own coordinate axes: no component exists after curl operation.

ii) Laplacian operator. Measures the amount to which the specific value of the scalar field at P differs from the predicted value based on surrounding points, i.e. measures the local smoothness (and thus predictability) of the field

iii) Harmonic field. Solutions (harmonic functions) to the field for one surface – outside of the source region- are solutions to the field everywhere else (outside the source region).

iv) Stokes Theorem. Reduces volume integral to surface integral so can deduce properties (e.g. mean density distribution) throughout a volume from measurements at the surface.