Advanced Mathematics

WS2021 – Lab 2 – Gradient / Curl / Divergence

Please put you name and student ID on the paper or in the mail you send me (bruce.thomas@gis.uni-stuttgart.de). Submission is for next Monday, November the 23th.

Exercises 1, 2 and 3 must be written down by hand or on a text software. Exercise 4 must be send as a Matlab code that I can easily run. Don't forget to comment the code! I have to understand it (and validate it) even without testing it! 😉

Do your best! 😊 *Good luck!*

Exercise 1 – A little bit of div and curl /20

Determine the divergence and the curl of the vector field (spherical coordinates).

$$\bar{G} = \frac{1}{r^2}\widehat{h_r} - cos\lambda sin\vartheta \widehat{h_\vartheta} + sin2\vartheta sin\lambda \widehat{h_\lambda}$$

Exercise 2 – Gradient search /20

This relationship defines a new set of coordinates. Determine the gradient in this system for an arbitrary function Φ.

$$x = \frac{\alpha}{\alpha^2 + \beta^2} \quad ; \quad y = \frac{\beta}{\alpha^2 + \beta^2} \quad ; \quad z = \zeta$$

Exercise 3 – Cylinder coordinates /30

Express the vector field in standard cylinder coordinates and determine the curl and the divergence.

$$V = \begin{pmatrix} -\omega y \\ \omega x \\ 1 - x^2 - y^2 \end{pmatrix} with\ \omega > 0$$

Exercise 4 – Matlab /30

Use the 3 data sheets (AIS_IceSheet_ice, AIUB_CHAMP01S_geoid and weathermodel_winds) on Matlab to plot and visualize the data. Load the data directly and use the metadatas to understand what is inside each file. **/10**

Example for the ice sheet:

```
load('AIS_IceSheet_ice.mat')

[X,Y] = meshgrid(ice.x,ice.y)
figure
h=pcolor(X',Y',shiftdim(ice.CHANGE(:,:,120)))
set(h,'LineStyle','none')
c = colorbar
c.Label.String = 'Ice Mass Change (kg/m^2)'
```

Then, compute grad, div and curl operations. Combine them is possible to check that div(curl) and curl(grad) are equal to 0. Give for each calculation a physical explication. **/20**

Example for the grad:

```
[gx,gy] = gradient(shiftdim(ice.CHANGE(:,:,120)))
figure
quiver(X',Y',gx,gy)
hold on
contour(X',Y',shiftdim(ice.CHANGE(:,:,120)),'ShowText','on')
hold off
```