Advanced Mathematics – WS2021 – Lab 9 – PDE

Submission is for next Monday, January the 25th! Do your best! ☺ Good luck!

Exercise 1 – PDE of a bivariate function $u = u(x, y)$

$$x^2 u_{xx} + (xy^2 - y)(xy^2 + y)u_{yy} - 2x^2y^2u_{xy} = 0$$

Classify the PDE. Determine the characteristics liens for the domains $|xy| > 0$.

Hint: Bernoulli ODE, solvable by substitution w(x) := 1/y

Exercise 2 – Wave equation

a- Find the normal modes of the wave equation on 0 ≤ x ≤ π/2, t ≥ 0 given by:

$$\frac{\partial^2}{\partial t^2}u = c^2\frac{\partial^2}{\partial x^2}u \; with \; u(0,t) = u(\frac{\pi}{2}, t) = 0, t > 0$$

b- If the solution in part a- represents a vibrating string, then what frequencies will you hear if it is plucked?

c- If the length of the string is longer/shorter what happens to the sound?

d- When you tighten the string of a musical instrument such as a guitar, piano, or cello, the note gets higher. What has changed in the differential equation?

Exercise 3 – Laplace equation in other coordinate systems

The Laplace operator in two-dimensional curvilinear coordinates (v, w) is given by:

$$\Delta_{vw}\Phi = \frac{1}{\cosh v}\left[\frac{1}{\cosh v}\frac{\partial}{\partial v}\left\{\cosh v\frac{\partial\Phi}{\partial v}\right\} + \frac{1}{\cos w}\frac{\partial}{\partial w}\left\{\cos w\frac{\partial\Phi}{\partial w}\right\}\right]$$

a- Apply the separation method to get two ordinary differential equations. The constants should be chosen in such a way, that the function φ(v, w) = sin w · sinh v is one of the solutions.

b- Consider now the differential equation in v for the constant of φ(v, w) and determine an independent solution via reduction of order.