Advanced Mathematics – WS2021 – Lab 8 – Power series & PDE

Submission is for next Monday, January the 18th! Do your best! ☺ Good luck!

Exercise 1 – Power Series

Solve the problem below via power series.

$$2x^2y'' + 3xy' + (x-1)y = 0$$

Hint: use the method of Frobenius that defines a new approach using $y = \sum_{k=0}^{\infty} a_k x^{k+\mu}$ *with* $\mu \geq 0$.

Exercise 2 – Numerical Integration in Matlab

Implement the Runge-Kutta method of order 4 for numerical integration.

$$\begin{aligned}
\boldsymbol{k}_1 &= f(x_i, \boldsymbol{y}_i) \\
\boldsymbol{k}_2 &= f\left(x_i + 0.5h, \boldsymbol{y}_i + 0.5h\boldsymbol{k}_1\right) \\
\boldsymbol{k}_3 &= f\left(x_i + 0.5h, \boldsymbol{y}_i + 0.5h\boldsymbol{k}_2\right) \\
\boldsymbol{k}_4 &= f\left(x_{i+1}, \boldsymbol{y}_i + h\boldsymbol{k}_3\right) \\
\boldsymbol{y}_{i+1} = \boldsymbol{y}(x_{i+1}) &= \boldsymbol{y}(x_i) + \frac{h}{6}(\boldsymbol{k}_1 + 2\boldsymbol{k}_2 + 2\boldsymbol{k}_3 + \boldsymbol{k}_4)
\end{aligned}$$

Solve the problem $y'' + xy = 0$ in the interval $x \in [0, 6]$ with the initial values $x_0 = 0$ and $y_0 = 0.355\,028\,053\,88$, $y_0' = 0.2588194079$ with the stepwidth $h = 0.01$ and visualize the result. Implement a Runge-Kutta-solver in MALTAB which is called by:

```
[X,Y] = RungeKutta4(fxy, x0, h, xmax, y0)
```

- The differential equation should be provided by a function handle `fxy`
- The arguments `y0` can be scalar or column vector
- Using the routine without output argument should lead to visualization
- Check all input arguments for type and dimension and provide helpfull messages

Exercise 3 – PDE of a bivariate function $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x}, \boldsymbol{y})$

$$(1 + \tanh x)u_{xx} + (1 - \tanh x)u_{yy} + \frac{4}{e^{-x} + e^{x}}u_{xy} + (1 + \tanh x)u_y = 0$$

Classify the PDE. Determine the characteristics Ψ and Φ. Use them to transform the PDE into normal-form and simplify the expression.