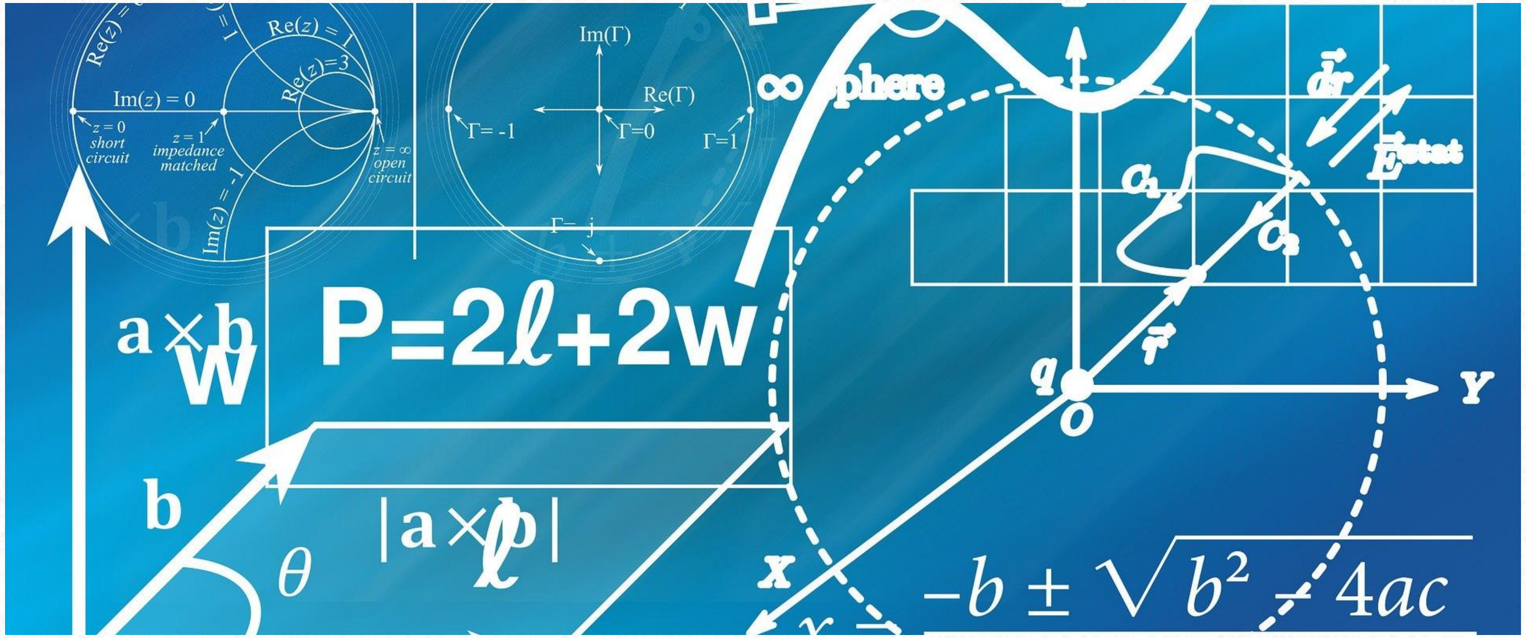
Kilauea © Bruce Thomas



Advanced Mathematics, Map Projections & Geodetic Coordinate Systems – Lab 1

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Lab Structure

Bruce Thomas
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Plannning

09:45 to 11:15 for Adv. Maths

11:30 to 12:15 for Map Proj.

Goals

Practice applying techniques with data.

Developing competence in Matlab and Python for data analysis and presentation.

Exam

All exercises must be successfully completed (e.g. executable and explained code) to be able to sit the exam.

They are to be submitted by the following Monday by email.

Matlab

High performance language for technical computing.

Programming language and interface.

Suited for matrix manipulation and program solving related to linear algebra. Easy plotting of data and functions.

Great expansion possible by addition of toolboxes and functions.

Python

High-level general-purpose programming language.

Interpreted, interactive and object-oriented programming language.

Computation, visualization and programming in an easy-to-use and familiar environment.

Open-source and large community development.
Extensive support libraries.

Vectors in Space

- The 3-dimensional coordinate system is built around a set of 3 axes that intersect at right angles at the origin.
- (x,y,z) are used to describe the location of a point in space with standard unit vectors $(\hat{\imath} + \hat{\jmath} + \hat{k})$

- The distance between 2 points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- Scalar multiplication

$$k\vec{v} = (kx_1, ky_1, kz_1)$$

- Vector addition

$$\vec{u} + \vec{v} = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$$

GIVE IT A TRY!

Express vectors $\overrightarrow{PQ}$ (P is the initial point and Q is the terminal point):

- $P(3,0,2)\ and\ Q(-1,-1,4)$
- $P(-2,5,-8)\ and\ M(1,-7,4), where\ M\ is\ the\ midpoint\ of\ the\ line\ segment \overline{PQ}$
- $Find\ terminal\ point\ Q\ of\ vector\ \overrightarrow{PQ} = (7,-1,3)\ with\ the\ initial\ point\ at\ P(-2,3,5)$

Use the given vectors $\vec{a}$ and $\vec{b}$ to find and express the vectors $\vec{a} + \vec{b}$, $4\vec{a}$ and $-5\vec{a} + 3\vec{b}$:

- $\vec{a} = (-1,-2,4)$ and $\vec{b} = (-5,6,-7)$
- $\vec{a} = -\hat{k}$ and $\vec{b} = -\hat{\imath}$

The Dot Product

$$\vec{u}\vec{v} = (u_1 v_1 + u_2 v_2 + u_3 v_3)$$

- Commutativity $uv = uv$
- Linearity $u(v + w) = uv + uw$
- Orthogonal if $uv = 0$

- To find the measure of the angle between 2 vectors

$$uv = uvcos\theta$$

- Work W is done when a force F is applied to an object causing a displacement s

$$W = Fs = Fscos\theta$$

GIVE IT A TRY!

Find the dot product $\vec{u} \cdot \vec{v}$:

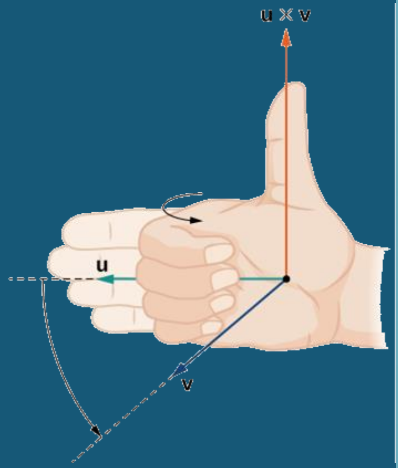
$\vec{u} = (3,0)$ and $\vec{v} = (2,2)$

$\vec{u} = (2,2,-1)$ and $\vec{v} = (-1,2,2)$

Find the dot product $(\vec{u} \cdot \vec{v})\vec{w}$ and $(\vec{u} \cdot \vec{w})\vec{v}$:

$\vec{u} = (2,0,-3)$ and $\vec{v} = (-4,-7,1)$ and $\vec{w} = (1,1,-1)$

$\vec{u} = \hat{\imath} + \hat{\jmath}$ and $\vec{v} = \hat{\imath} - \hat{k}$ and $\vec{w} = \hat{\imath} - 2\hat{k}$



The Cross Product

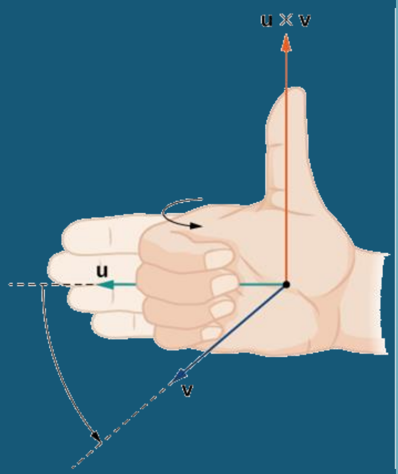
$$\vec{u}\vec{v} = (u_2v_3 - u_3v_2)i - (u_1v_3 - u_3v_1)j + (u_1v_2 - u_2v_1)k$$

- Commutativity $uv = uv$
- Linearity $u(v + w) = uv + uw$
- $uu = 0$

- To find the measure of the angle between 2 vectors

$$uv = uvsin\theta$$

Right hand rule: uv is orthogonal to both u and v



The Cross Product

GIVE IT A TRY!

Find the cross product $\vec{u} \times \vec{v}$ and sketch the vectors $\vec{u}$, $\vec{v}$ and $\vec{u} \times \vec{v}$.

$\vec{u} = (2,0,0)$ and $\vec{v} = (2,2,0)$

$\vec{u} = (2,3,0)$ and $\vec{v} = (0,1,2)$

Simplify the following vector.

$(\hat{\imath} \times \hat{\imath} - 2\hat{\imath} \times \hat{\jmath} - 4\hat{\imath} \times \hat{k} + 3\hat{\jmath} \times \hat{k}) \times \hat{\imath}$

Mathematics & Matlab

GIVE IT A TRY!

Vectors and Matrix

You have these 3 column vectors and the following matrix:

$$\overrightarrow{u_1} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \overrightarrow{u_2} = \begin{pmatrix} -5 \\ 2 \\ 1 \end{pmatrix}, \vec{u}_3 = \begin{pmatrix} -1 \\ -3 \\ 7 \end{pmatrix}, A = \begin{pmatrix} 2 & 3 & 4 \\ 7 & 6 & 5 \\ 2 & 8 & 7 \end{pmatrix}$$

Compute $\overrightarrow{u_1} + 3\overrightarrow{u_2} - \overrightarrow{u_3}/5$.

Compute the scalar product between vectors $\overrightarrow{u_1}$ and $\overrightarrow{u_2}$.

Compute the product $A\overrightarrow{u_1}$.

GIVE IT A TRY!

The magic square

The matrix of Dürer is $D = \begin{pmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{pmatrix}$

Check that this matrix is magic, that means that the sum of each line, each column and the diagonal is the same.

Is the sum of 2 matrix D magic?

Is the product of 2 matrix D magic? (check the matrix product and the product elements by elements)

Is the division of 2 matrix D magic? (check the matrix division and the division elements by elements)

Add a 5th column of your choice on matrix A.