Advanced Mathematics & Map Projections and Geodetic Coordinate Systems

WS2021 – Lab 1 – General Presentation

Exercises 1, 2 and 3 must be written down by hand or on a text software (Word). Try to be precise and do some figures to explain your work. If you write it down, you can send me pictures or scan in a PDF.

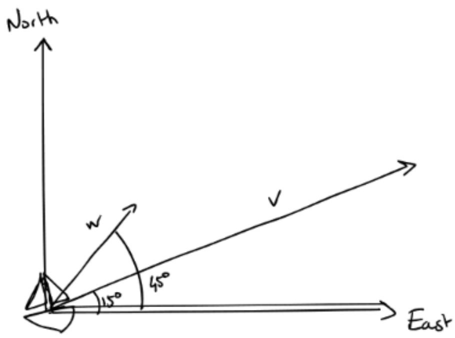
Exercises 4 and 5 must be send as a Matlab code that I can easily run. Don't forget to comment the code! I have to understand it (and validate it) even without testing it! 😉

Please put you name and student ID on the paper or in the mail you send me. I will adjust the grades depending on the difficulty of the exercises. So, no worries, do your best! 😉 *Good luck!*

Exercise 1 – Vectors in Space

1) If $a + b = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ and $a - b = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$, compute and draw a and b.
2) Express vectors $\overrightarrow{PQ}$ (P is the initial point and Q is the terminal point):
   - $P(0,10,5)\ \ and\ \ Q(1,1,-3)$
   - $Q(0,7,-6)\ and\ M(-1,3,2), where\ M\ is\ the\ midpoint\ of\ the\ line\ segment \overline{PQ}$
   - $Find\ initial\ point\ P\ of\ vector\ \overrightarrow{PQ} = (-9,1,2)\ with\ the\ initial\ point\ at\ Q(10,0,-1)$
3) Use the given vectors $\vec{a}$ and $\vec{b}$ to find and express the vectors $\vec{a} + \vec{b}$, $4\vec{a}$ and $-5\vec{a} + 3\vec{b}$:
   - $\vec{a} = (3,-2,4)$ and $\vec{b} = (-5,6,9)$
   - $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = -2\hat{\imath} - 3\hat{\jmath} + \hat{k}$
4) What is the sum of the twelve vectors that go from the center of a clock to the hours 1:00, 2:00, …, 12:00? Now, if the 2:00 vector is removed, what happens? Give the components of that 2:00 vector $v = (cos\,\theta, sin\,\theta)$? *Need a hint? Contact me!* 😉
5) Marta Vieira da Silva, Wendie Renard and Francesco Totti are practicing soccer for an upcoming game. Marta runs straight for 10 m from the goal to point P on the field. Then, she turns left in right angle and runs again 10 m but reaches Totti. Finally, she kicks back the ball at an upward angle of 45° to Wendie at the goal and with a speed of 10 m.s^{-1}. Write down the velocity of the ball using unit vectors. *Need a hint? Contact me!* 😉

Exercise 2 – The Dot Product



1) Find the dot product $\vec{u} \cdot \vec{v}$:
   - $\vec{u} = (3,-4)$ and $\vec{v} = (4,3)$
   - $\vec{u} = (4,5,-6)$ and $\vec{v} = (0,-2,-3)$
2) Find the dot product $(\vec{u} \cdot \vec{v})\vec{w}$ and $(\vec{u} \cdot \vec{w})\vec{v}$:
   - $\vec{u} = (0,1,2)$ and $\vec{v} = (-1,0,1)$ and $\vec{w} = (1,0,-1)$
   - $\vec{u} = \hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{v} = \hat{\jmath} + 3\hat{k}$ and $\vec{w} = -\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$
3) The Mangareva sailboat leaves the port of Nouméa traveling 15 north of east. Thanks to powerful sails, it goes at a v=20 knots speed along that path (check the figure). But the w ocean current moves the boat northeast at a speed of 2 knots. Considering that, how fast is the boat moving in the direction 15 north east? *Need a hint? Contact me!* 😉

Exercise 3 – The Cross Product

1) Find the cross product $\vec{u} \times \vec{v}$ and sketch the vectors $\vec{u}$, $\vec{v}$ and $\vec{u} \times \vec{v}$.
   - $\vec{u} = (3,2,-1)$ and $\vec{v} = (1,1,0)$
   - $\vec{u} = (0,2,3)$ and $\vec{v} = (3,0,1)$
2) Simplify the following vector $\hat{\jmath} \times (\hat{k} \times \hat{\jmath} + 2\hat{\jmath} \times \hat{\imath} - 3\hat{\jmath} \times \hat{\jmath} + 5\hat{\imath} \times \hat{k})$
3) You want to dispose a triangle sail in your garden to protect you from the sun. You need to find its area knowing the vertices of the triangle. Use the cross product. *Need a hint? Contact me!* 😉
   - Barbecue zone B=(1,0,0)
   - Garden zone G=(0,1,0)
   - Pétanque game (boules) zone P=(0,0,1)

Exercise in Matlab – Work on a matrix

Let's work with matrix $M = \begin{pmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{pmatrix}$

Compute its determinant for M and for its inverse M^{-1}. Compute matrix M*M and M.*M. What do you see?

Use vectors $v_1 = \begin{pmatrix} 32 \\ 23 \\ 33 \\ 31 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 32.1 \\ 22.9 \\ 33.1 \\ 30.9 \end{pmatrix}$ and compute linear systems $Ax = v_1$and $Ax = v_2$. Is it surprising?

Exercise in Matlab – Forest fire

This algorithm can illustrate the propagation of any phenomenon: forest fire, epidemic, etc.

We have an infinite checkerboard. We assume that the probability p of propagation of fire from a cell to its immediate neighbors (horizontally or vertically) is fixed. At the start, only one square burns, then the fire spreads gradually. You must write a program that simulates the course of the fire.

The checkerboard will be described by a matrix of integers of size N × N, each integer being able to take three values corresponding to the three states: normal, burning, burned. Each cell will be identified by its row index and its column index, between 1 and N. We will take N of the order of 80.

At the start, all the cells will be normal, and we will start the fire from the central box. We will stop the simulation when there are no more boxes burning. The final state will then be displayed.

What is the influence of the parameter p on the course of the fire? *Need a hint? Contact me!* 😉