Advanced Mathematics – WS2021 – Lab 4 – ODE

Please put you name and student ID on the paper or in the mail you send me (bruce.thomas@gis.uni-stuttgart.de). Submission is for next Monday, December the 7th. Do your best! 😊 Good luck!

Exercise 1 – ODE to solve

Solve these homogeneous differential equations with constant coefficients:

(1.1) $y'' - 4y' + 13y = 0 \quad with\ y\left(\frac{\pi}{6}\right) = -8\ and\ y'\left(\frac{\pi}{6}\right) = 2$

(1.2) $y'' + 22y' + 121y = 0 \quad with\ y(2) = 2\ and\ y'(0) = 4$

(1.3) $4y'' + 16y' + 18y = 0 \quad with\ y(2) = 4 + 2i\ and\ y'(0) = -1 - 4i$

Solve these differential equation using the reduction of order:

(1.4) $-xy'' + (x-2)y' + y = 0 \quad with\ y(1) = 1\ and\ y'(1) = 1$

(1.5) $(tan^2x)y'' + (tan^3x + tanx)y' - y = 0 \quad with\ the\ first\ solution\ is\ y_1 = sinx$

(1.6) $x^2(x-2)y'' - 2x(2x-3)y' + 6(x-1)y = 0$

Solve these ODE:

(1.7) $y' + y = 2e^x$

(1.8) $y' - (tanx)y = sinx \quad for\ x\epsilon]-\pi/2;\pi/2[$

(1.9) $y'' + (1 + \frac{2}{x})y' + (\frac{2}{x^2} - \frac{1}{x})y = 0$

Exercise 2 – Bernoulli and Riccati

The Bernoulli equation is an ODE of the form $y' + p(x)y = q(x)y^n \quad with\ n \neq 1$.

<u>Task 1</u>: show it becomes linear if one makes the change of dependent variable $u = y^{1-n}$ (hint: begin by dividing both sides of the ODE by y^n)

<u>Task 2</u>: solve these Bernoulli equations using the method demonstrated

(2.1) $y' + y = 2xy^2$

(2.2) $x^2y' - y^3 = xy$

The Ricatti equation is where the right handed side is a quadratic function of y. In general, it is not solvable by elementary means.

<u>Task 3</u>: however, show that if $y_1(x)$ is a solution, then the general solution is $y = y_1 + u$ where u is the general solution of the Bernoulli equation

<u>Task 4</u>: solve the Ricatti equation using the method demonstrated

(2.3) $y' = 1 - x^2 + y^2$