Exercise 1

Two archers take turns shooting at a target. The first to touch wins. The player who starts has the probability p 1 of hitting each turn and the second the probability p 2 (with p 1 , p 2 > 0)

a) What is the probability that the first player wins?

b) Show that the game is almost certain to end.

c) For which value of p 1 is there a value of p 2 for which the game is fair?

a) A_n = event that the target is reached at rank n

J_1 wins $\rightarrow$ disjoint union A_{2k+1}

$$P(A_{2k+1}) = (1-p_1)^k (1-p_2)^k p_1$$

$$P(J_1 \text{ wins}) = \sum_{k=0}^{\infty} (1-p_1)^k (1-p_2)^k p_1 = \frac{p_1}{p_1 + p_2 - p_1 p_2}$$

b) J_2 wins $\rightarrow$ $P(A_{2k}) = (1-p_1)^k (1-p_2)^k p_2$

$$P(J_2 \text{ wins}) = \frac{p_2 - p_1 p_2}{p_1 + p_2 - p_1 p_2}$$

$P(J_1 \text{ wins}) + P(J_2 \text{ wins}) = 1$ $\leftarrow$ almost certain that the game ends

c) The game is fair when

$$P(J_1 \text{ wins}) = 1/2 = P(J_2 \text{ wins})$$

$$2p_1 = p_1 + p_2 - p_1 p_2$$

$$\Rightarrow p_2 = \frac{p_1}{1-p_1} \quad \text{if } p_1 \neq 1$$

$$p_1 \leq 1/2$$

Exercise 2

Machines produce sheet metal plates for stacking; it is estimated that 0.1% are unusable plates. The use of these plates consists in stacking n of them, numbered from 1 to n by taking them randomly. For n = 2000, what is the law followed by the random variable N "number of unusable plates among the 2000"? (an adapted probability law will be used); what is the probability that N is less than or equal to equal to 3 ? What is the probability that N is strictly less than 3?

$n = 2000$

law of N? → Poisson law parameter 2

$P(N \leq 3) = 0.86$

you can also use:

→ central limit theorem $p = 0.76$

↳ do a correction of continuity

⇒ $p = 0.85$

Exercise 3

In the last presidential elections in France, candidate A obtained 20% of the vote. We take randomly in polling stations in large towns, packets of 200 ballots: we denote X the random variable "number of votes for A in the various offices ".

1. What is the probability law of X?

2. How can we approach him?

3. What then is the probability that: X is greater than 45? X between 30 and 50?

a) ballot taken randomly → 0.2 proba.

b) Sample with replacement
put again the ballots

↳ binomial law $\begin{cases} n = 200 \\ p = 0.2 \end{cases}$

$np = 40$ ⇒ normal approximation

$\begin{cases} \text{expectation} \quad m = 40 \\ \text{standard deviation} \quad \sqrt{40 \times 0.8} = 4\sqrt{2} \end{cases}$

! use table of Gauss

$$P[X \geq 45] = 1 - P[X \leq 44]$$

$$\simeq 1 - F\left(\frac{44.5 - 40}{4\sqrt{2}}\right) \simeq 21\%$$

$$P(30 \leq X \leq 50)$$

$$\simeq F\left(\frac{50.5 - m}{\sigma}\right) - F\left(\frac{29.5 - m}{\sigma}\right)$$

$$\simeq 93.6\%$$

Exercise 4

A Stuttgart – Honolulu flight is operated by a 150-seat Airbus; for this flight, estimates have shown that the probability of a person confirming their ticket is p = 0.75. The company sells n tickets, n> 150. Let X be the random variable "number of people among the n possible, having confirmed their reservation for this flight".

1. What is the exact law followed by X?

2. What is the maximum number of seats that the company can sell so that, at least 95%, it be sure that everyone can get on the plane?

3. Repeat the same exercise with an aircraft with a capacity of 300 seats; vary the parameter p = 0.5; p = 0.8.

binomial distribution n, p

$$\begin{cases} \text{success} & p \\ \text{failure} & q = 1-p \end{cases}$$

$E[x] = np$

$= 0.75n$

$Var(x) = np(1-p)$

$= 0.25 * 0.75n$

because $n \geq 150$, normal distribution

$E[x] = 0.75n \quad \sigma = \sqrt{0.25 \times 0.75n}$

$$P[X > 150] \leq 0.05$$

$$P[X \leq 150] \geq 0.95$$

use table of Gauss

$$\text{if } P\left[\frac{X - 0.75n}{\sqrt{0.25 \cdot 0.75n}} \leq \frac{150 - 0.75n}{\sqrt{}}\right] \geq 0.95$$

$$F(1.645) = 0.95$$

$$\frac{150.5 - 0.75n}{\sqrt{}} \geq 1.645$$

$$\Rightarrow \quad 0 \leq n \leq 187$$

by selling less than 187 tickets, the company only takes a risk of less than 5% of having to compensate the "excess" travellers