* Hello everyone!

Lab1 WS2021 Adv Maths 1/

Express vectors $\overrightarrow{PQ}$ P initial point
Q terminal point

- $\begin{cases} P(3,0,2) \\ Q(-1,-1,4) \end{cases} \Rightarrow \overrightarrow{PQ} = -4\hat{i} - \hat{j} + 2\hat{k}$

$= (-4, -1, 2)$

- $\begin{cases} P(-2,5,-8) \\ M(1,-7,4) \end{cases}$ midpoint $\overline{PQ}$ $\Rightarrow \overrightarrow{PQ} = \overrightarrow{PM} + \overrightarrow{MQ} = 2\overrightarrow{PM}$

$= 2 \times (3 \quad -12 \quad 12)$

$= (6, -24, 24)$

- Terminal point Q

$\begin{cases} \overrightarrow{PQ} = (7,-1,3) \\ P(-2,3,5) \end{cases} \Rightarrow Q?\ (x, y, z)$

$7\hat{i} - \hat{j} + 3\hat{k} = x\hat{i} + 2\hat{i} + y\hat{j} - 3\hat{j} + z\hat{k} - 5\hat{k}$

$\Rightarrow Q(5, 2, 8)$

2/

Cube 4 corners $(0,0,0)$

$\hat{\imath}\ (1,0,0)$

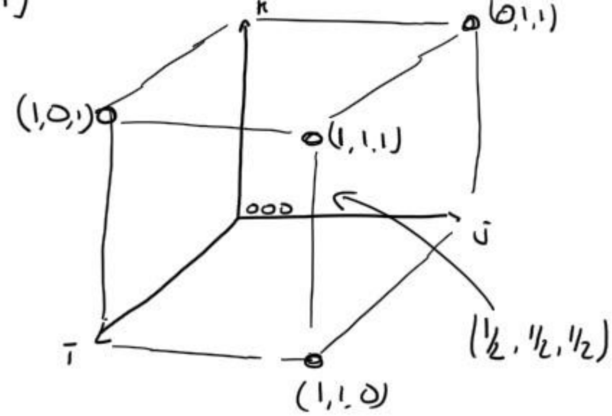
$\hat{\jmath}\ (0,1,0)$

$k\ (0,0,1)$

4 others corners?

center of cube?

center of faces?



$\vec{a}$, $\vec{b}$ express $\begin{cases} \vec{a}+\vec{b} \\ 4\vec{a} \\ -5\vec{a}+3\vec{b} \end{cases}$

$$\begin{cases} \vec{a} = (-1,-2,4) \\ \vec{b} = (-5,6,-7) \end{cases} \Rightarrow \begin{aligned} \vec{a}+\vec{b} &= (-6,\ 4,\ -3) \\ 4\vec{a} &= (-4,\ -8,\ 16) \\ -5\vec{a}+3\vec{b} &= (-10,\ 28,\ -41) \end{aligned}$$

$$\begin{cases} \vec{a} = -\hat{k} \\ \vec{b} = -\hat{\imath} \end{cases} \Rightarrow \begin{aligned} \vec{a}+\vec{b} &= -\hat{\imath} - \hat{k} \\ 4\vec{a} &= -4\hat{k} \\ -5\vec{a}+3\vec{b} &= -3\hat{\imath} + 5\hat{k} \end{aligned}$$

Dot product $\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$

$\begin{cases} \vec{u}\,(3,0) \\ \vec{v}\,(2,2) \end{cases}$ $\vec{u} \cdot \vec{v} = 3 \times 2 + 0 \times 2 = 6$

$\begin{cases} \vec{u}\,(2,2,-1) \\ \vec{v}\,(-1,2,2) \end{cases}$ $\vec{u} \cdot \vec{v} = -2 + 4 - 2 = 0$

$\vec{u}\,(2,0,-3)$
$\vec{v}\,(-4,-7,1)$
$\vec{w}\,(1,1,-1)$

$\underbrace{(\vec{u} \cdot \vec{v})}_{\text{scalar}}\vec{w} = (-8+0-3)\,\vec{w} = -11\,\vec{w}$
$= (-11,-11,11)$

$(\vec{u} \cdot \vec{w})\vec{v} = 5\,\vec{v} = (-20,-35,5)$

$\vec{u} = \hat{\imath} + \hat{\jmath}$
$\vec{v} = \hat{\imath} - \hat{k}$
$\vec{w} = \hat{\imath} - 2\hat{k}$

$(\vec{u} \cdot \vec{w})\vec{v} = 1\,\vec{v} = (1,0,-1)$

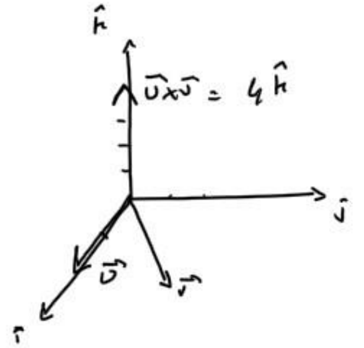
Cross Product $\vec{u} \times \vec{v}$

$u(u_1, u_2, u_3)$

$v(v_1, v_2, v_3)$

$$\boxed{\vec{u} \times \vec{v} = (u_2 v_3 - u_3 v_2)\,\hat{i} - (u_1 v_3 - u_3 v_1)\,\hat{j} + (u_1 v_2 - u_2 v_1)\,\hat{k}}$$

$$\begin{cases} \vec{u} = (2, 0, 0) \\ \vec{v} = (2, 2, 0) \end{cases}$$



$$\vec{u} \times \vec{v} = 0\,\hat{i} - 0\,\hat{j} + 4\,\hat{k}$$

$$\begin{cases} \vec{u}\ (2, 3, 0) \\ \vec{v}\ (0, 1, 2) \end{cases} \qquad \vec{u} \times \vec{v} = (6, -4, 2)$$

5)

Simplify

$$\Big(\underbrace{\hat{i}\times\hat{i}}_{=0} - \underbrace{2}\,\underbrace{\hat{i}\times\hat{j}}_{\hat{k}} - \underbrace{4}\,\underbrace{\hat{i}\times\hat{k}}_{-\hat{j}} + \underbrace{3}\,\underbrace{\hat{j}\times\hat{k}}_{\hat{i}}\Big)\times\hat{i}$$

$$\hat{i}\times\hat{i} = 0$$

$$\hat{i}\times\hat{j} = \hat{k} \qquad \hat{j}\times\hat{i} = -\hat{k}$$

$$\hat{j}\times\hat{k} = \hat{i} \qquad \hat{k}\times\hat{j} = -\hat{i}$$

$$\hat{k}\times\hat{i} = \hat{j} \qquad \hat{i}\times\hat{k} = -\hat{j}$$

$$-2\hat{i}\times\hat{j} = (-2\hat{i})\times\hat{j} = \hat{i}\times(-2\hat{j})$$

$$-2\hat{j} - 4\hat{k}$$

Vectors and Matrix

You have these 3 column vectors and the following matrix:

$$\overrightarrow{u_1} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \overrightarrow{u_2} = \begin{pmatrix} -5 \\ 2 \\ 1 \end{pmatrix}, \vec{u}_3 = \begin{pmatrix} -1 \\ -3 \\ 7 \end{pmatrix}, A = \begin{pmatrix} 2 & 3 & 4 \\ 7 & 6 & 5 \\ 2 & 8 & 7 \end{pmatrix}$$

Compute $\overrightarrow{u_1} + 3\overrightarrow{u_2} - \overrightarrow{u_3}/5$.

Compute the scalar product between vectors $\overrightarrow{u_1}$ and $\overrightarrow{u_2}$.

Compute the product $A\overrightarrow{u_1}$.

MATLAB SOLUTION

```
u1 = [ 1 ; 2 ; 3 ]
u2 = [ -5 ; 2 ; 1 ]
u3 = [ -1 ; -3 ; 7 ]
A = [ 2 3 4 ; 7 6 5 ; 2 8 7 ]
c1 = u1+3*u2-u3/5
c2 = u1'*u2
c3 = A*u1
```

The magic square

The matrix of Dürer is $D = \begin{pmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{pmatrix}$

Check that this matrix is magic, that means that the sum of each line, each column and the diagonal is the same.

Is the sum of 2 matrix D magic?

Is the product of 2 matrix D magic? (check the matrix product and the product elements by elements)

Is the division of 2 matrix D magic? (check the matrix division and the division elements by elements)

Add a 5th column of your choice on matrix A.

MATLAB SOLUTION

```
A = [16 3 2 13; 5 10 11 8; 9 6 7 12; 4 15 14 1]

% A is magic <=>
l_magic = sum(A)
c_magic = sum(A')
d_magic = sum(diag(A))

% A+A is magic!
sum_magic = A+A
l_sum_magic = sum(sum_magic)
c_sum_magic = sum(sum_magic')
d_sum_magic = sum(diag(sum_magic))

% A.*A is not magic
prod_magic = A.*A
l_prod_magic = sum(prod_magic)
c_prod_magic = sum(prod_magic')
d_prod_magic = sum(diag(prod_magic))

% A*A is not magic
prodM_magic = A*A
l_prodM_magic = sum(prodM_magic)
c_prodM_magic = sum(prodM_magic')
d_prodM_magic = sum(diag(prodM_magic))

% A./A is magic, it's the identity!
div_magic = A./A

% A/A is not magic
divM_magic = A/A
l_prodM_magic = sum(divM_magic)
c_prodM_magic = sum(divM_magic')
d_prodM_magic = sum(diag(divM_magic))

% add %th column
A(:,5) = [0 0 0 9]
```