Lab 4 Maths

$$y'' + 3\frac{y'}{1+x} + \frac{y}{(1+x)^2} = 0$$

coeffizent $\frac{1}{1+x} = y_1$

$$2\frac{1}{(1+x)^3} + 3\left(-\frac{1}{(1+x)^2}\right)\frac{1}{1+x} + \frac{1}{(1+x)^3} = 0 \quad \checkmark$$

$$y_2 = u\frac{1}{1+x} \qquad \text{function } u$$

$$y_2' = u'\frac{1}{1+x} - u\frac{1}{(1+x)^2}$$

$$y_2'' = u''\frac{1}{1+x} - 2u'\frac{1}{(1+x)^2} + 2u\frac{1}{(1+x)^3}$$

↳ add these into the ODE $\quad y'', y', y \xrightarrow{\text{replace them}} y_2'', y_2', y_2$

$$\dots \quad \boxed{u''(1+x) + u' = 0}$$

substitute the $u' = v \quad \Rightarrow \quad v'(1+x) + v = 0$

$$U''(1+x) + u' = 0 \qquad v = u'$$

$$\frac{dp}{dx} = -\frac{p}{1+x}$$

$$\ln p = \int \frac{1}{p}\,dp = -\int \frac{dx}{1+x} = -\ln|1+x| + \text{constant}$$

$$e^{\ln p} = p = e^{c - \ln|1+x|} = c\,\frac{1}{1+x}$$

$$U = \int p\,dx = c\,\ln|1+x|$$

$$y_2 = c\,\frac{1}{1+x}\,\ln|1+x| \qquad c \text{ is constant}$$

$$\left[\; y = c_1 y_1 + c_2 y_2 = c_1\frac{1}{1+x} + c_2\frac{1}{1+x}\ln|1+x|\right.$$

↳ Euler-Cauchy ODE

ODE2

homogeneous diff equa with cte coeff

initial values

$$y'' - 2y' - 4y = 0 \qquad y(0) = 1 \qquad y'(0) = -1$$

$$\hookrightarrow \quad \mu^2 - 2\mu - 4 = 0 \qquad \mu_{1,2} = \frac{2 \pm \sqrt{4+16}}{2} = 1 \pm \sqrt{5}$$

$$y(x) = A e^{x+\sqrt{5}x} + B e^{x-\sqrt{5}x}$$

$$y'(x) = A(1+\sqrt{5}) e^{x+\sqrt{5}x} + B(1-\sqrt{5}) e^{x-\sqrt{5}x}$$

$$\begin{cases} y(0) = 1 = A + B \quad \Rightarrow \quad A = 1 - B \\ y'(0) = -1 = A(1+\sqrt{5}) + B(1-\sqrt{5}) \end{cases}$$

$$\Rightarrow (1-B)(1+\sqrt{5}) + B(1-\sqrt{5}) = -1$$

$$\Rightarrow \begin{cases} B = \frac{1}{\sqrt{5}} + \frac{1}{2} \\ A = \frac{1}{2} - \frac{1}{\sqrt{5}} \end{cases}$$

$$y(x) = \left(\frac{1}{2} - \frac{1}{\sqrt{5}}\right) e^{x+\sqrt{5}x} + \left(\frac{1}{2} + \frac{1}{\sqrt{5}}\right) e^{x-\sqrt{5}x}$$

$$\overset{4x-1}{}$$

$$(4x-1)y''' + x(4x^4-2x^3-16)y'' + (2x^3-16x^5+16)y' + 8(4x-1)x^3y=0$$

$y = x^{k}$ k is cte

$$\begin{cases} y' = k x^{k-1} \\ y'' = k(k-1)x^{k-2} \\ y''' = \boxed{k(k-1)(k-2)}\, x^{k-3} \end{cases}$$

$$(4x^{k-2} - x^{k-3})k(k-1)(k-2)$$
$$+ (4x^{k+3} - 2x^{k+2} - 16x^{k-1})k(k-1)$$
$$+ (2x^{k+2} - 16x^{k+4} + 16x^{k-1})k$$
$$+ 8(4x^{k+4} - x^{k+3}) = 0$$

insert into ODE

$$\Rightarrow (4x^{k-2} - x^{k-3})k(k-1)(k-2) + \dots$$

ODE can be fulfilled only if x^{k-3} term vanishes

$$k(k-1)(k-2) = 0$$

- if $k=0 \Rightarrow 8(4x^4 - x^3) \neq 0$
- if $k=1 \Rightarrow (2x^3 - 16x^5 + 16) + 8(4x^5 - x^4) \neq 0$
- if $k=2$?

check the guess if $\boxed{y=x^2}$ ok ✓

ODE $=0$

particular solution $y_p = x^2 z(x)$

$y = x^2$ stv ODE $= 0$

there is a term like x^2 in the y solution

$y_{particular} = x^2 z(x)$ $\quad z(x)$ function of x

$$y' = 2xz + x^2 z'$$

$$y'' = 2z + 4xz' + x^2 z''$$

$$y''' = 6z' + 6xz'' + x^2 z'''$$

$y_p = x^2$ $\quad z(x) = 1 \Rightarrow$ ODE $= 0$

into the ODE $\quad$ all terms of z must cancel out

$$x^2(4x-1)\underbrace{z'''}_{p''} + 2x\overbrace{z''}^{p'}(2x^6 - x^5 - 8x^2 + 12x - 3)$$
$$+ \underbrace{z'}_{p}(-16x^7 + 16x^6 - 6x^5 - 48x^2 + 24x - 6) = 0$$

$z' = p$

sol: $y = x^2 z(x)$

$y_1 = e^{bx}$ is also successful