Advanced Mathematics

Lab 10

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Exercise 1 – Exploring Data

The Earths magnetic field is known to have reversed direction over geologic time. The file *GK2007.txt* contains data from the Gee and Kent (2007) magnetic time scale. It lists all normal and reversely magnetized chrons and gives the duration of each interval (in Myr).

Make box-and-whisker plots for the entire data set as well as for normal and reverse polarities separately. Note that the last letter in the chron, "n", means normalized, and "r" stands for reversed polarized. Make a histogram of all intervals using a bin width of 0.1 Myr. Comment.

As we seen in the following figure, in order to find the range of box-and-whisker we computed the range between $M - 2.698\sigma \le data \le M + 2.698\sigma$. By doing this, we can ensure that 99.3% od data are in the confidence interval.

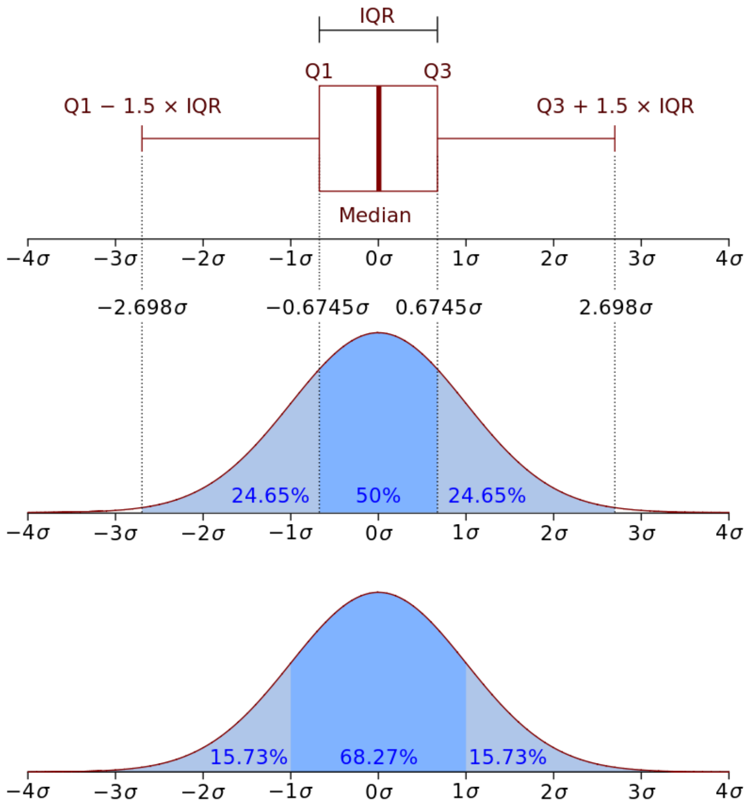

Figure 1 Boxplot and a probability density function (pdf) of a Normal N(0,1σ2) Population

At figure 2, we can demonstrate that most of the numbers of polarities gather between 0 and 1 Myr, just only a few numbers outside of this range. It also represents the data's distribution.

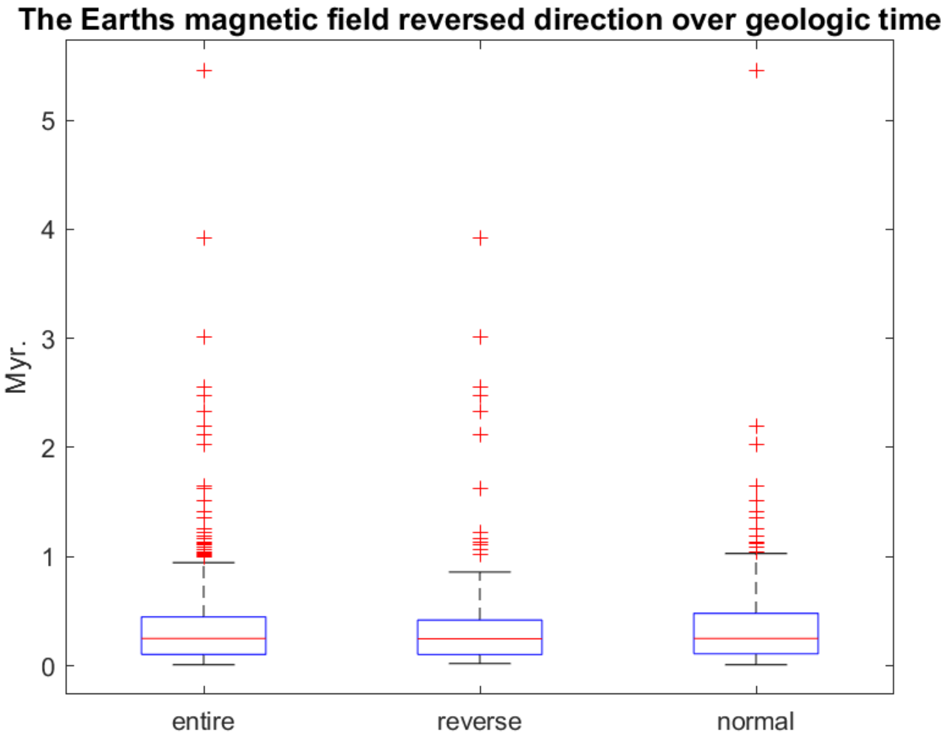

Figure 2 Boxplot of entire, reverse, normal polarities

According to figure 3, histograms convey an accurate impression of the data distribution, so we can assume that around a period of 0.1 Myr, normalized and reversed polarized occur. However, due to some physical reasons the period might increase or even longer. We only can ensure the distribution of the polarized happen, but we can't connect the relation between each period.

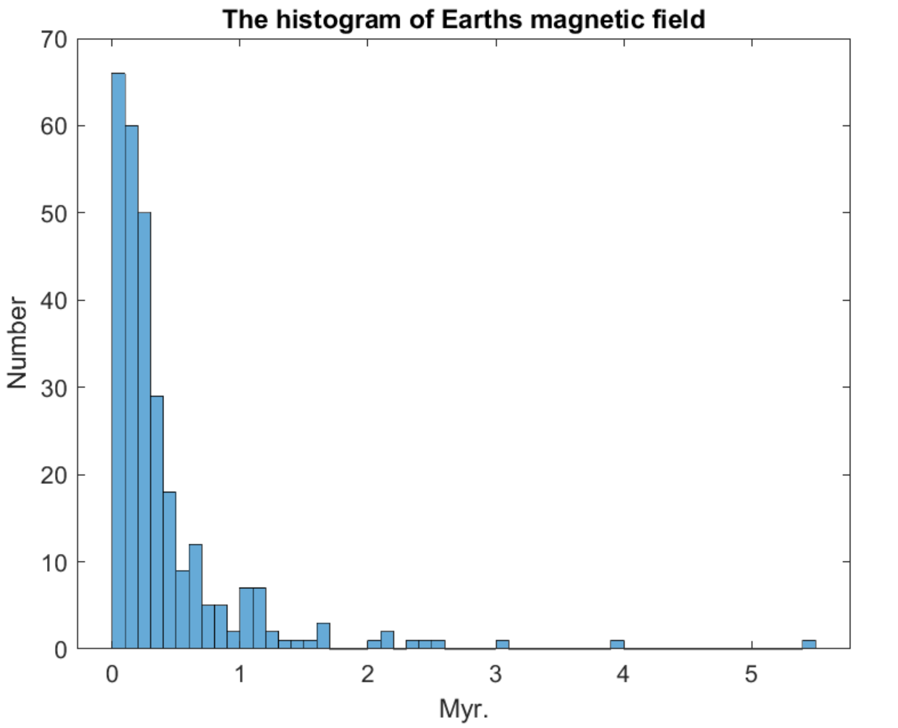

Figure 3 the histogram of Earth magnetic field

Exercise 2 – Error Analysis

The subsidence of young (< 80 Myr) oceanic crust due to lithospheric cooling has been shown to follow approximately a linear $\sqrt{\text{age}}$ relationship, given by

$$z = z_r + \frac{2\rho_m \alpha_v T_m}{(\rho_m - \rho_w)} \sqrt{\frac{\kappa t}{\pi}}.$$

Given estimates of thermal diffusivity $\kappa = 1.00 \pm 0.04$ mm$^2 s^{-1}$, water density $\rho_w = 1.027 \pm 0.001$ g cm^{-3}, mantle density $\rho_m = 3.30 \pm 0.01$ g cm^{-3}, volumetric thermal expansion coefficient $\alpha_v = (3.00 \pm 0.02) \cdot 10^{-5}$ °K^{-1}, average ridge depth $z_r = 2500 \pm 200$ m, and mantle temperature $T_m = 1300 \pm 25$ °K, determine the predicted depth and its uncertainty for a location where rocks of age $t = 29.7 \pm 0.5$ Myr were recovered. Which term dominates the final uncertainty?

Ex 2.

$$z = z_r + \frac{2\rho_m \alpha_v T_m}{(\rho_m - \rho_w)} \sqrt{\frac{kt}{\pi}}$$

$k = 1 \pm 0.04 \ \frac{mm^2}{s}$ $\qquad$ $\alpha_v = (3 \pm 0.02) \cdot 10^{-5} \ \frac{1}{°k}$

$\rho_w = 1.027 \pm 0.001 \ g/cm^3$ $\qquad$ $z_r = 2500 \pm 200$ m

$\rho_m = 3.3 \pm 0.01 \ g/cm^3$ $\qquad$ $T_m = 1300 \pm 25$ °k

$t = 29.7 \pm 0.5$ Myr $= (29.7 \pm 0.5)(3.1536 \cdot 10^{13})$ s

$z_r = (2500 \pm 200) \cdot 10^2$ cm , $\quad k = (1 \pm 0.04) \cdot 10^{-2} \ \frac{cm^2}{s}$

$$\delta z = \sqrt{(\delta z_r)^2 + \left(\frac{2\rho_m T_m}{\rho_m - \rho_w}\sqrt{\frac{kt}{\pi}}\,\delta\alpha_v\right)^2 + \left(\frac{2\rho_m \alpha_v}{\rho_m - \rho_w}\sqrt{\frac{kt}{\pi}}\,\delta T_m\right)^2 + \cdots}$$

$$\overline{\left(2\alpha_v T_m \sqrt{\frac{kt}{\pi}}\left(\frac{-\rho_w}{(\rho_m - \rho_w)^2}\right)\delta\rho_m\right)^2 + \left(2\alpha_v T_m \sqrt{\frac{kt}{\pi}}\left(\frac{\rho_m}{(\rho_m - \rho_w)^2}\right)\delta\rho_w\right)^2 +}$$

$$\overline{\left(\frac{2\rho_m \alpha_v T_m}{(\rho_m - \rho_w)} \cdot \sqrt{\frac{t}{\pi}} \cdot \frac{1}{2}\frac{1}{\sqrt{k}} \cdot \delta k\right)^2 + \left(\left(\frac{2\rho_m \alpha_v T_m}{\rho_m - \rho_w}\right) \cdot \sqrt{\frac{k}{\pi}} \cdot \frac{1}{2}\frac{1}{\sqrt{t}}\,\delta t\right)^2}$$

$$\Rightarrow \delta z = \sqrt{(\delta z_r)^2 + \left(\frac{2\rho_m}{\rho_m - \rho_w}\sqrt{\frac{kt}{\pi}}\right)^2\left((T_m\,\delta\alpha_v)^2 + (\alpha_v\,\delta T_m)^2\right) +}$$

$$\overline{\left(\frac{2\rho_m \alpha_v T_m}{\rho_m - \rho_v} \cdot \sqrt{\frac{1}{\pi}} \cdot \frac{1}{2}\right)^2 \cdot \left(\frac{t}{k}\delta k^2 + \frac{k}{t}\delta t^2\right)}$$

$$\therefore \left(\frac{2\rho_m}{\rho_m - \rho_w}\sqrt{\frac{kt}{\pi}}\right)^2 = \left(\frac{2 \cdot 3.3}{3.3 - 1.027}\sqrt{\frac{10^{-2} \cdot 29.7 \cdot 3.1536 \cdot 10^{13}}{\pi}}\right)^2 = 2.5136 \cdot 10^{12}$$

$$\left(2\alpha_v T_m \sqrt{\frac{kt}{\pi}} \cdot \frac{1}{(\rho_m - \rho_w)^2}\right)^2 = 6.7952 \times 10^8$$

$$\left(\frac{\cancel{2}\rho_m \alpha_v T_m}{\rho_m - \rho_w} \cdot \sqrt{\frac{1}{\pi}} \cdot \cancel{\frac{1}{2}}\right)^2 = 1.02 \times 10^{-3}$$

$$\Rightarrow \delta z = \sqrt{(200 \times 10^2)^2 + (2.5136 \cdot 10^{13}) \cdot (6.76 \times 10^{-8} + 5.625 \times 10^{-7}) + \cdots}$$

$$\overline{(6.7952 \times 10^8) \cdot (1.055 \cdot 10^{-4} + 1.089 \times 10^{-5}) + \cdots}$$

$$\overline{(1.02 \times 10^{-3}) \cdot (1.5 \times 10^{10} + 2.6545 \times 10^{9})}$$

$$\Rightarrow \delta z = 208.307 \ [m]$$

$$Z_0 = (2500 \times 10^{2}) + \frac{2 \cdot (3.3) \cdot (3 \times 10^{-5}) \cdot 1300}{(3.3 - 1.027)} \cdot \sqrt{\frac{10^{-2} \cdot 29.7 \times 3.1536 \times 10^{13}}{\pi}}$$

$$= 4.4553 \cdot 10^{5} \text{ [cm]} = 4455.3 \text{ [m]}$$

$$\underline{Z = 4455.3 \pm 208.307 \text{ [m]}} \ \#$$

Exercise 3 – Least square method

Apply the generalized least square method on the Ice Sheet data set to study the ice change. There are no other instructions on purpose, to let you think and research. You must send me a commented code in Matlab with a physical analysis of the result.

```matlab
%% Ex 3
load AIS_IceSheet_ice.mat
delta_ice = zeros(154,1);

for i = 1 : 154
    delta_ice(i) = nanmean(ice.CHANGE(:, :, i), 'all');
end
% y = Ax + e
A = [ice.time_dec(:), ones(154,1)];
y = delta_ice;
x_hat = (A'*A)^(-1) * A' * y;
y_hat = A * x_hat;
e_hat = y - y_hat;
m = size(A,1);
n = size(A,2);
sigma0 = sqrt(e_hat' * e_hat / (m-n)); % standard deviation

[e_hat1, TF] = rmoutliers(e_hat); % remove the outliers
sigma0_rmout = sqrt(e_hat1' * e_hat1 / (m-n)); % standard deviation after removing the outliers
ice.time_dec(19) = []; % remove the outliers
A1 = [ice.time_dec(:), ones(153,1)];
y(19) = []; % remove the outliers
y1 = y;
y_hat(19)=[]; % remove the outliers
x1 = (A1'*A1)^(-1) * A1' * y1;

figure
scatter(ice.time_dec, y1,'.')
hold on
box on
p1 = plot(ice.time_dec, x_hat(1)*ice.time_dec+x_hat(2), 'r');
p2 = plot(ice.time_dec, x1(1)*ice.time_dec + x1(2), 'g');
legend([p1, p2],'OLS', 'remove the outliers')
xlabel('Time')
ylabel('Ice changed')
title('Ice changed from 2002 to 2018')
```

First, I set up the linear regression $ice\ changed = aT + b$. Then, create adjustment with observation equations $y = Ax + e$.

$$y = \begin{bmatrix} \Delta ice_1 \\ \vdots \\ \Delta ice_{154} \end{bmatrix}_{154\times1}$$

$$diff(aT, a) = T$$

$$diff(b, b) = 1$$

$$A = \begin{bmatrix} T_1 & 1 \\ \vdots & \vdots \\ T_{154} & 1 \end{bmatrix}_{154\times2}$$

$$x = \begin{bmatrix} a \\ b \end{bmatrix}_{2\times1} \ (we\ need\ to\ find\ out\ a\ and\ b)$$

$$\hat{x} = (A^T A)^{-1} * A^T y$$

$$\hat{y} = y - A\hat{x}$$

$$\hat{e} = y - \hat{y}$$

$$m = 154\ (number\ of\ unknown)$$

$$n = 2\ (number\ of\ observation)$$

$$\sigma_0 = \sqrt{\frac{\hat{e}^T * \hat{e}}{m - n}} = 11.1758$$

$$x = \begin{bmatrix} a \\ b \end{bmatrix}_{2\times1} = \begin{bmatrix} -7.9221 \\ 1.5912 * 10^4 \end{bmatrix}_{2\times1}$$

In order to make the linear regression more precision, we can take off the outliers then adjust again. We can see that after skip the outliers the standard deviation became lower, which means the linear regression is more represented the trend of the ice changed.

$$\sigma_0 = \sqrt{\frac{\hat{e}^T * \hat{e}}{m - n}} = 10.7635$$

$$x = \begin{bmatrix} a \\ b \end{bmatrix}_{2\times1} = \begin{bmatrix} -7.8444 \\ 1.5756 * 10^4 \end{bmatrix}_{2\times1}$$

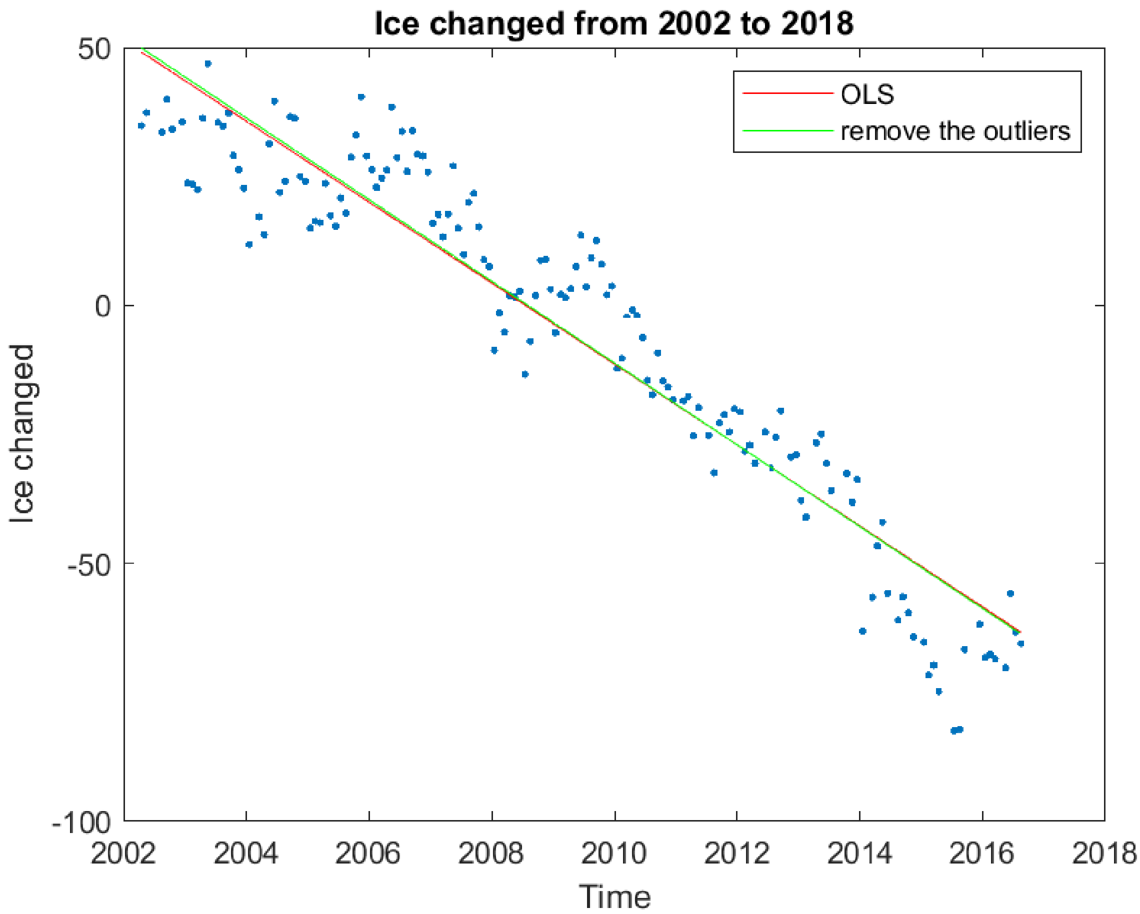

Figure 4 Ice changed from 2002 to 2008

This picture shows that as time changes, the change in the ice layer also decreases (melts). Between 2002 and 2006, the ice layer was still growing (only slowly beginning to grow negatively). Until 2008, the ice layer almost fell in a melting trend. In 2016, the ice has melted 50 meters, which means that the sea level will gradually rise, and the consequences will be unimaginable.