Reynolds average turbulence modelling using deep neural networks with embedded invariance

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There exists significant demand for improved **Reynolds-averaged Navier–Stokes (RANS) turbulence models** that are informed by and can represent a richer set of turbulence physics. This paper presents a method of using **deep neural networks** to **learn a model for the Reynolds stress anisotropy tensor from high-fidelity simulation data**. A novel neural network architecture is proposed which uses a multiplicative layer with an **invariant tensor basis to embed Galilean invariance into the predicted anisotropy tensor**. It is demonstrated that this neural network architecture provides **improved prediction accuracy** compared with a generic neural network architecture that does not embed this invariance property. The Reynolds stress anisotropy predictions of this invariant neural network are propagated through to the velocity field for two test cases. For both test cases, **significant improvement versus baseline RANS linear eddy viscosity and nonlinear eddy viscosity models** is demonstrated.

GOAL: Improve prediction of anisotropy tensor from mean shear/rot.

Methods:

- Using six fluid direct numerical simulations as training data sets,
- Train a standard multi-layer perceptron and a Gal. inv. Neural net. to:
- Predict three simulations with different configurations &
- Compare the results with eddy viscous closure and direct num. sim.

Results:

- Neural networks predict anisotropy tensor with higher accuracy than eddy viscous closure models
- It is possible to construct Gal. inv. Net. With even higher accuracy

Acronyms

DNS: Direct numerical simulation
DNS-b = (Anisotropy tensor from DNS) + RANS
LEVM: Linear eddy viscosity model
MLP: Multi-layer perceptron
QEVM: Quadratic eddy viscosity model
RANS: Reynolds-averaged Navier-Stokes
ReLU: Rectified linear unit
RMSE: Root-mean square error
TBNN: Tensor basis neural network

Definitions/Useful properties

$$S_{ij}\,[\mathrm{s}^{-1}] = \frac{1}{2}\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right)$$

$$R_{ij}\,[\mathrm{s}^{-1}] = \frac{1}{2}\left(\frac{\partial \overline{u_i}}{\partial x_j} - \frac{\partial \overline{u_j}}{\partial x_i}\right)$$

$$(s_{ij}, r_{ij}) = \frac{k}{2\epsilon}\,(S_{ij}, R_{ij})$$

$$b_{ij} = \frac{\overline{u_i{\prime} u_j{\prime}}}{2k} - \frac{\delta_{ij}}{3}$$

$$\epsilon\,[\mathrm{m}^2.\mathrm{s}^{-3}] = \nu\,[\mathrm{m}^2.\mathrm{s}^{-1}] \times \sum_{k=1}^{3} \overline{\left(\frac{\partial u_i{\prime}}{\partial x_k}\right)^2}$$

$$k\,[\mathrm{m}^2.\mathrm{s}^{-2}] = \frac{1}{2}\sum_{k=1}^{3} \overline{(u_k{\prime})^2}$$

$$b_{ij} = \sum_{n=1}^{10} g^{(n)}\left\{\overrightarrow{\lambda}\right\} \times T_{ij}^{(n)} \;\Rightarrow\; \text{Galilean invariance}$$

$$\text{Galilean invariance} \;\overset{\text{Incompressibility}}{\Rightarrow}\; b_{ij} = \sum_{n=1}^{10} g^{(n)}\left\{\overrightarrow{\lambda}\right\} \times T_{ij}^{(n)}$$

Paper figures

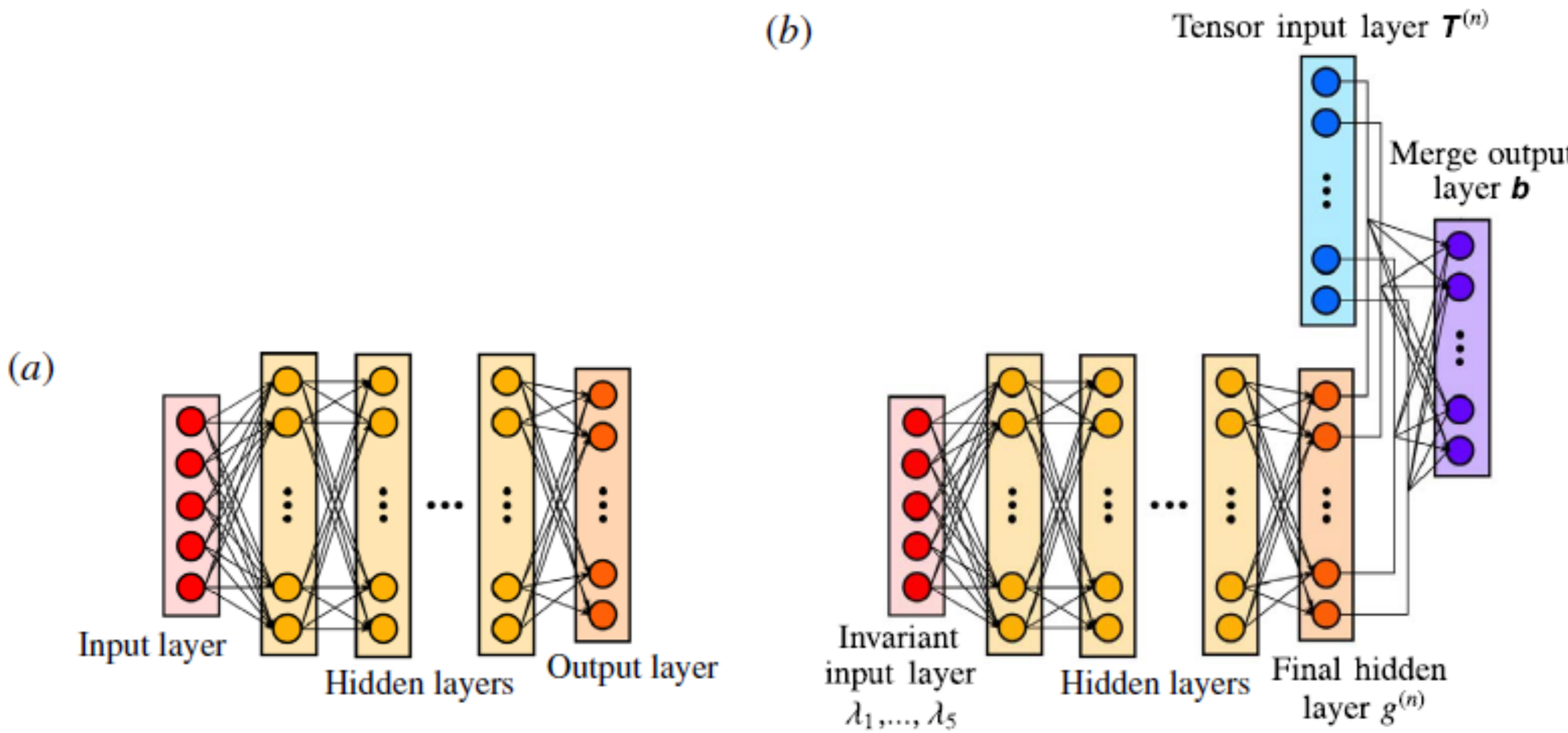

FIGURE 1. (Colour online) Schematic of neural network architectures.

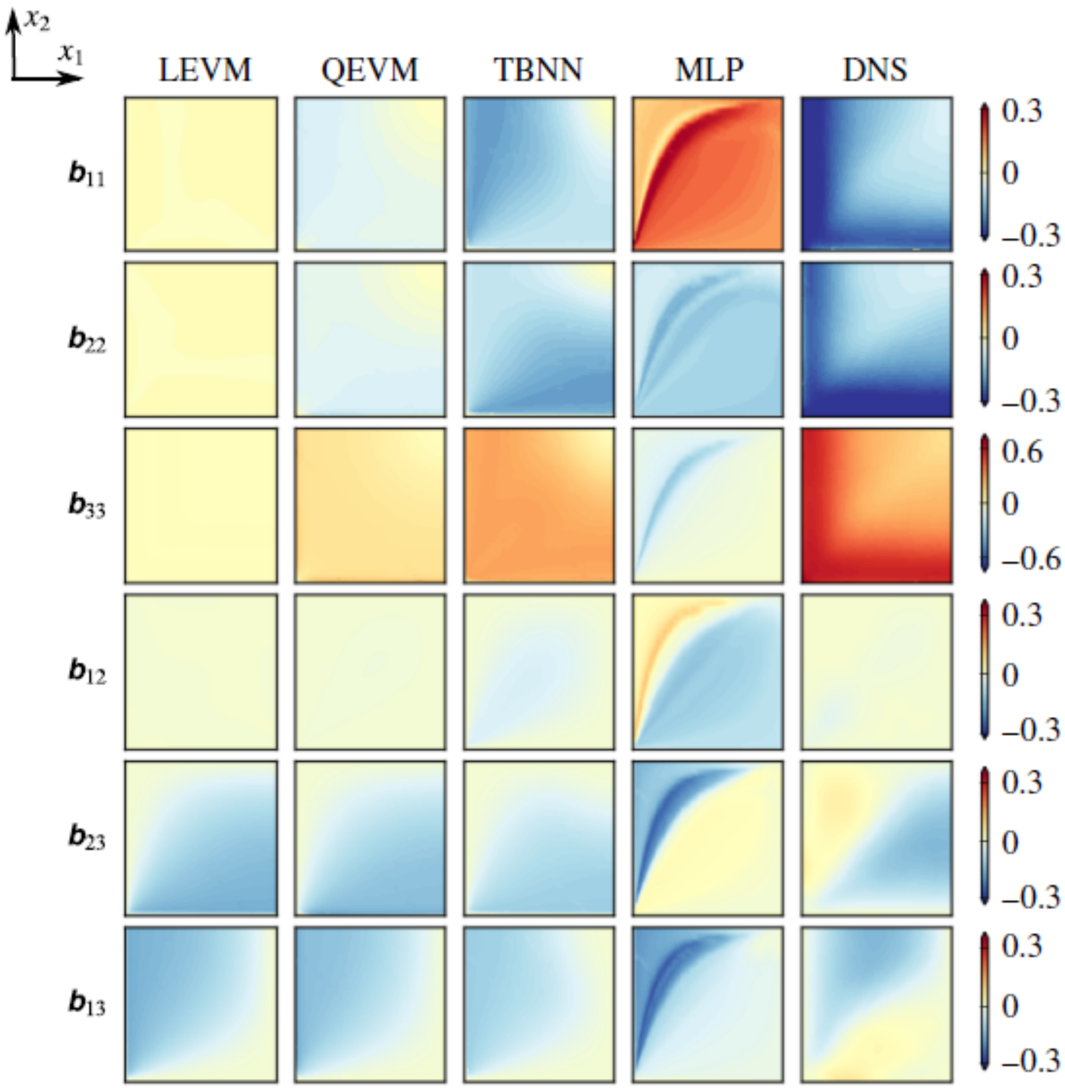

FIGURE 2. Predictions of Reynolds stress anisotropy tensor $\boldsymbol{b}$ on the duct flow test case. Only the lower left quadrant of the duct is shown, and the streamwise flow direction is out of the page. The columns show the predictions of the LEVM, QEVM, TBNN and MLP models. The true DNS anisotropy values are shown in the right-most column for comparison.

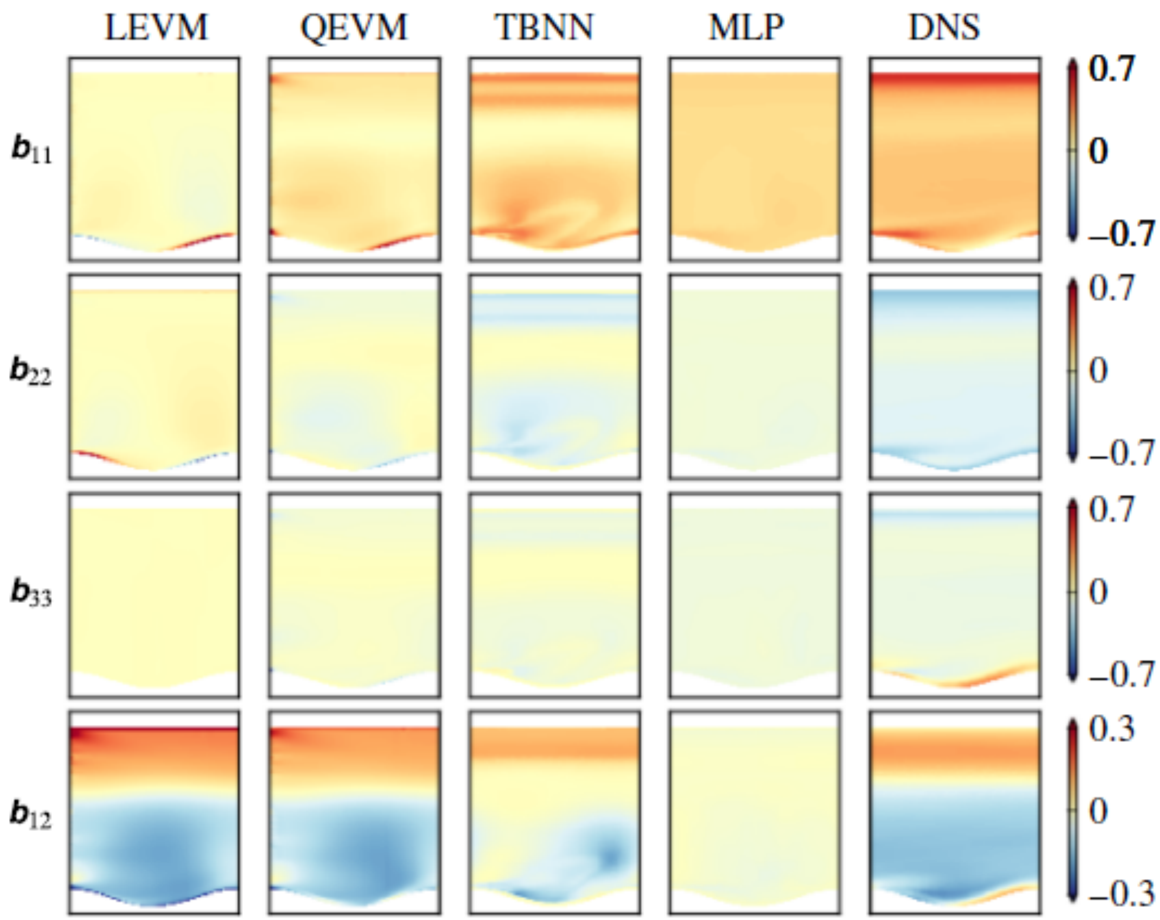

FIGURE 3. Predictions of Reynolds stress anisotropy $\boldsymbol{b}$ tensor on the wavy wall test case. The columns show the predictions of the LEVM, QEVM, TBNN and MLP models. The true DNS anisotropy values are shown in the right-most column for comparison.

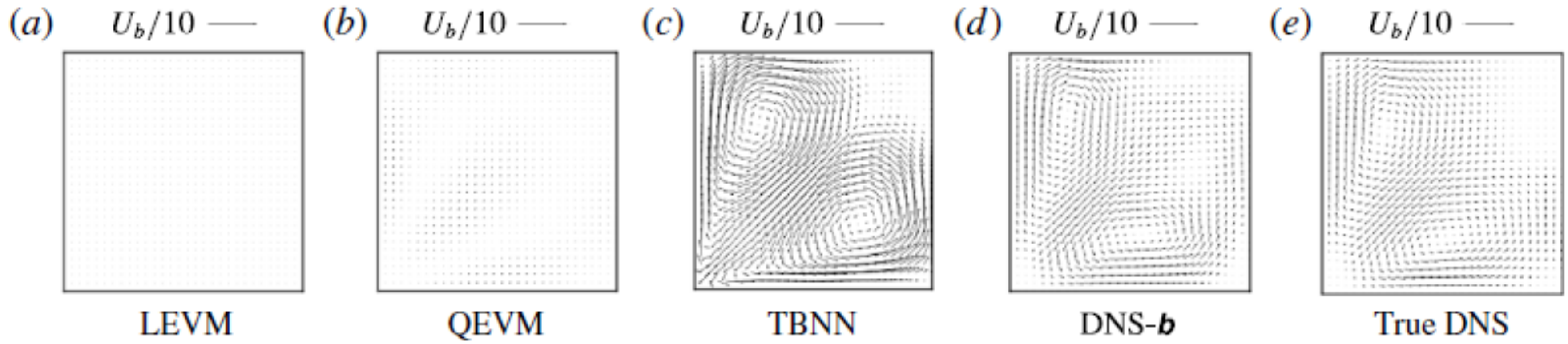

FIGURE 4. Plot of secondary flows in duct flow case. Reference arrows of length $U_b/10$ shown at the top of each plot.

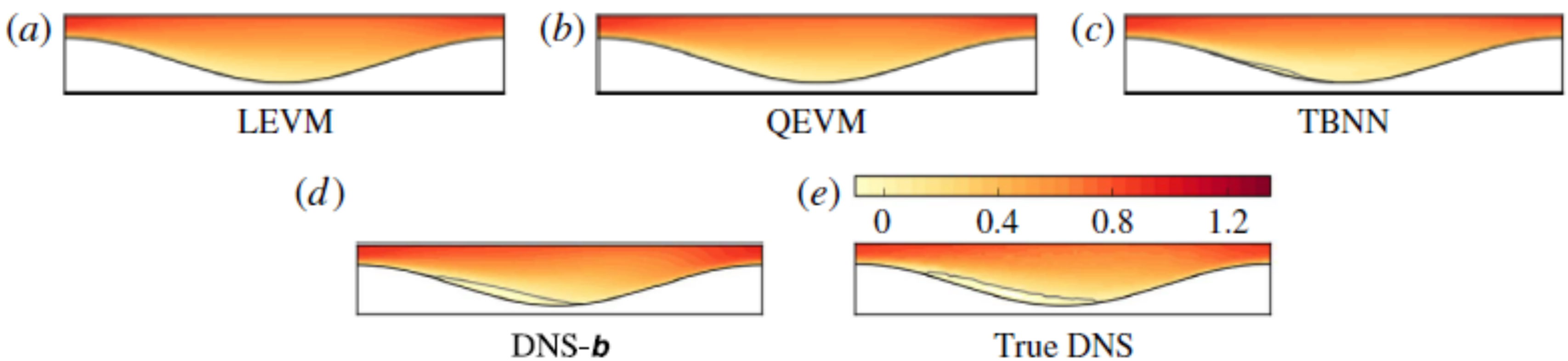

FIGURE 5. Contours of streamwise velocity normalized by bulk velocity in the wavy wall test case, zoomed into the near-wall region. Separated regions outlined in grey.