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Flexible Generalized Low-Rank Regularizer for Tensor RPCA

Anonymous submission

Abstract

Tensor Robust Principal Component Analysis (TRPCA) has emerged as a powerful technique for tensor recovery. In this paper, we design a novel Flexible Generalized low-rank regularizer called FGTNN. Equipped with this, we develop an FGTRPCA framework, which has the following two desirable properties. **1) generalizability**: Many existing TRPCA methods can be viewed as special cases of our framework; **2) flexibility**: Using FGTRPCA as a general platform, we can also develop a series of new TRPCA models by tuning a continuous parameter to improve performance. This grants its flexibility on top of generalizability. Moreover, for modeling gradient tensors, we adopt the FGTNN in the gradient domain and propose a novel tensor-correlated Flexible Generalized Joint Prior (t-FGJP) regularizer, which leverages the inherent local smoothness of certain tensor data and leads to the novel smooth FGTRPCA (SFGTRPCA) models. Besides, we device efficient optimization algorithms based on the Alternating Direction Method of Multipliers (ADMM) framework to implement the proposed models. Experimental results on various denoising and recovery tasks demonstrate the superiority of our models.

Introduction

Tensor data are ubiquitous, many real-world data are often inherently multidimensional, with information stored in multi-way arrays known as tensors, e.g., images, videos, network flow data, etc. In recent years, significant advancements across various interdisciplinary domains have been made in tensor analysis, such as machine learning (Wen, Chen, and Chen 2024; Phothilimthana et al. 2024), data mining (Zhang et al. 2023a; Huang et al. 2024), and computer vision (Zhao et al. 2024; Liu et al. 2024a). However, due to the limitations of signal acquisition equipment, such as sensor sensitivity, photon effects, and calibration errors, tensor data collected in the real world often exhibit significant corruption (Wang et al. 2023a). Consequently, tensor denoising has become a crucial task in tensor analysis.

In this paper, we focus on the Tensor Robust Principal Component Analysis (TRPCA) problem (Huang et al. 2015), which seeks to recover the underlying low-rank tensor $\mathcal{L}$ and sparse tensor $\mathcal{E}$ from their sum $\mathcal{M}$ (see Figure 1 for an intuitive illustration) and solves the following problem

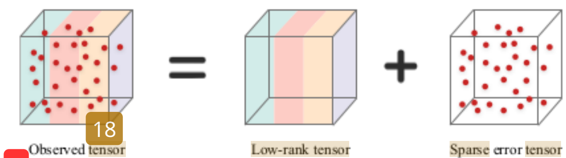


Figure 1: An illustration of TRPCA, which aims to recover the low-rank and sparse components from the observation.

$$\min_{\mathcal{L},\mathcal{E}\in\mathbb{R}^{d_1\times d_2\times d_3}} \operatorname{rank}(\mathcal{L}) + \lambda\|\mathcal{E}\|_1 \ \text{ s.t. } \mathcal{M} = \mathcal{L} + \mathcal{E}, \quad (1)$$

where $\lambda > 0$ is a regularization parameter, $\operatorname{rank}(\mathcal{L})$ denotes the rank of clean tensor $\mathcal{L}$ and $\|\mathcal{E}\|_1$ is ℓ_1-norm (sum of the absolute values of all the entries) to measure the sparsity of the noise tensor $\mathcal{E}$. One of the primary challenges is the definition of tensor rank, which is more sophisticated than matrix rank. There are many classical candidates for tensor rank based on different tensor decompositions. For instance, inspired by the tensor singular value decomposition (t-SVD) (Kilmer et al. 2013) proposed the tensor tubal rank that can be efficiently computed using the fast Fourier Transform (FFT). Since the non-convexity and discontinuity of the rank function, solving the problem (1) is usually NP-hard. Therefore, (Lu et al. 2020) proposed a new tensor nuclear norm as a convex surrogate of the tensor tubal rank and proposed a new TRPCA method defined as follows

$$\min_{\mathcal{L},\mathcal{E}} \|\mathcal{L}\|_* + \lambda\|\mathcal{E}\|_1 \ \text{ s.t. } \mathcal{M} = \mathcal{L} + \mathcal{E}, \quad (2)$$

where $\|\cdot\|_*$ represents the tensor nuclear norm (TNN). Moreover, the recent works (Kilmer et al. 2021) have proved the optimal representation and compression theories of t-SVD, making model (2) significantly more notable in characterizing the intrinsic low-rank structures of tensors. As a result, the model (2) under t-SVD has garnered considerable interests recently (Hou et al. 2024; Liu et al. 2024c; Qin et al. 2024).

Despite the impressive performance of TRPCA, it still exhibits several limitations. Specifically, when minimizing the TNN, TRPCA employs tensor singular value thresholding, which uniformly shrinks all singular values. In real-world

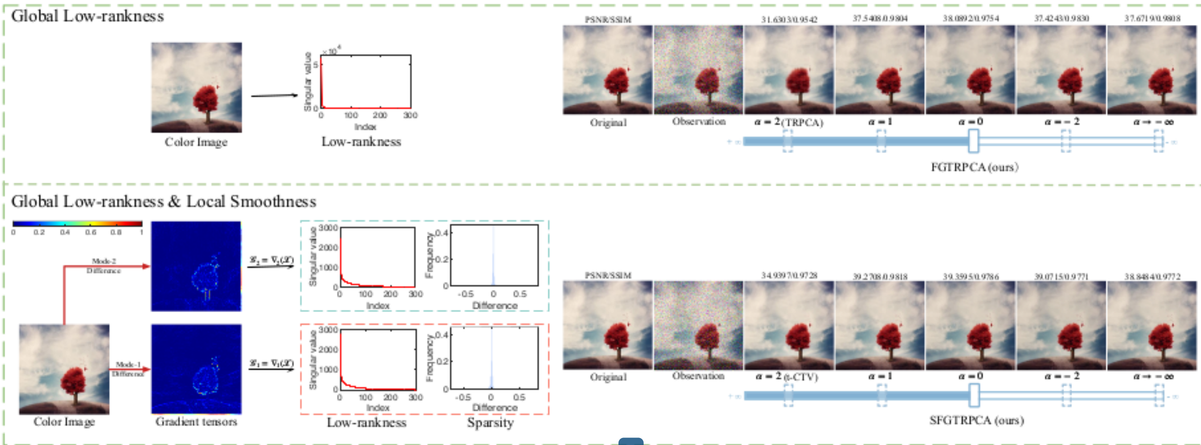


Figure 2: Take a color image sample from ZJU (Hu et al. 2012) dataset as an example. The two frames illustrate the recovery performance of our proposed FGTRPCA and SFGTRPCA with different values of shape parameter α (see Eq. (5)) under different structure priors of color images.

applications, singular values often carry distinct physical meanings, with prior knowledge suggesting that larger singular values are typically associated with more significant information. The uniform shrinkage approach of TRPCA fails to account for these differences among singular values, potentially leading to suboptimal results.

While many existing advanced methodologies (Gao et al. 2020; Jiang et al. 2020; Wang et al. 2023b; Zhang et al. 2023b; Yan and Guo 2024; Liu et al. 2024b) develop various TNN-based low-rank regularizers to penalty less for large singular values and more for small singular values, which can efficiently preserve the essential information and filter out the irrelevant details, their discrete and fixed models make it unflexible to diverse scenarios. In this paper, we design a novel Flexible Generalized low-rank regularizer (FGTNN) to adaptively assign different penalties to distinct singular values and impose the constraint on the sparse component. We have shown that several existing TRPCA models can be reformulated as special cases of FGTRPCA. Apart from that, we develop a wider family of new TRPCA models by tuning a continuous parameter to improve performance. Through this, our model significantly improves flexibility and efficiency in complex situations.

Besides the global low-rankness prior, the local smoothness prior represented by total variation (TV) is also generally applied in tensor recovery fields (Ko et al. 2020; Qiu et al. 2021). This prior states how similar objects/scenes (with shapes) are adjacently distributed (Peng et al. 2022b). Most previous works encoded the two priors with two independent regularizers and incorporated them into a unified model, which achieved better performance (Peng et al. 2020, 2022a). However, they have two drawbacks: (1) it is challenging to fine-tune the regularization parameter between the two terms; (2) the theoretical guarantee for exact recovery remains unproven for the related methods.

Given the circumstances above, (Wang et al. 2023a) proposed the tensor Correlated Total Variation (t-CTV) norm which integrates the two priors into a single regularization term, eliminating the need for tuning separate parameters. Additionally, this work provided theoretical guarantees for the exact recovery of similar tensor methods that jointly model both priors. Analogously, the integration regularization term was also based on TNN in the gradient domain. Consequently, (Huang et al. 2024) proposed a reweighted regularizer based on ℓ_p norm as a surrogate for t-CTV term. In this paper, we extend our proposed regularizer to the gradient domain and propose a novel tensor-correlated flexible generalized joint prior (t-FGJP) regularizer.

Our main contributions can be delineated as follows:

- We propose a flexible generalized low-rank regularizer (FGTNN) that accounts for the varying importance of different singular values in low-rank tensors and develop a novel FGTRPCA framework. Most previous works on TRPCA only consider single methods, while our framework regards several existing methods as special cases, and can be extended to a wider family of new TRPCA methods by tuning a continuous parameter.
- Considering the local smoothness prior, we extend FGTNN to the gradient domain and propose a novel tensor-correlated Flexible Generalized Joint Prior (t-FGJP) regularizer. The proposed t-FGJP also maintains the flexibility of discriminatively controlling different singular values of the gradient tensors and is applied to the proposed smooth FGTRPCA (SFGTRPCA) model.
- We design ADMM-based (Boyd et al. 2011) algorithmic frameworks tailored for each of the aforementioned models. Our extensive experiments on various tensor denoising and recovery tasks demonstrate the advantagements of our models.

Notations and preliminaries

To begin with, we introduce some essential notations and definitions utilized throughout the paper. We use lowercase letters, boldface lowercase letters, and boldface uppercase letters to denote scalars, e.g., x, vectors, e.g., $\mathbf{x}$, and matrices, e.g., $\mathbf{X}$, respectively. $mbf1_{d_1 \times d_2}$ and $\mathcal{W}_0 = \mathbf{1}_{d_1 \times d_2 \times d_3}$ represent a matrix of size $d_1 \times d_2$ and a tensor of size $d_1 \times d_2 \times d_3$ with all entries as ones. Tensors are presented by bold calligraphic letters, e.g., $\mathcal{X}$. For a 3-order tensor $\mathcal{X} \in \mathbb{R}^{d_1 \times d_2 \times d_3}$, we denote $\mathcal{X}_{ijk}$ as its (i, j, k)-th entry, $\mathcal{X}(i, :, :)$ as its horizontal slice, $\mathcal{X}(:, j, :)$ as its lateral slice, $\mathcal{X}(:, :, k)$ as its frontal slice, respectively. For convenience, the frontal slice $\mathcal{X}(:, :, k)$ is often denoted as $\mathbf{X}^{(k)}$. The tensor nuclear norm (TNN), tensor ℓ_1 norm (TL1N), tensor Frobenius norm and tensor infinity norm of $\mathcal{X}$ are defined by $\|\mathcal{X}\|_*$, $\|\mathcal{X}\|_1 = |\mathcal{X}_{ijk}|$, $\|\mathcal{X}\|_F = \sqrt{\sum_{ijk} |\mathcal{X}_{ijk}|^2}$ and $\|\mathcal{X}\|_\infty = \max_{ijk} |\mathcal{X}_{ijk}|$, respectively. The transpose of $\mathcal{X}$ is defined as $\mathcal{X}^T \in \mathbb{R}^{d_2 \times d_1 \times d_3}$ (Lu et al. 2020).

Definition 1. *(**T-SVD**) (Kilmer and Martin 2011) For a tensor $\mathcal{X} \in \mathbb{R}^{d_1 \times d_2 \times d_3}$, it can be factorized by t-SVD as*

$$\mathcal{X} = \mathcal{U} * \mathcal{S} * \mathcal{V}^T, \tag{3}$$

where $\mathcal{U} \in \mathbb{R}^{d_1 \times d_1 \times d_3}$, $\mathcal{V} \in \mathbb{R}^{d_2 \times d_2 \times d_3}$ are orthogonal tensors, i.e., $\mathcal{U} * \mathcal{U}^T = \mathcal{U}^T * \mathcal{U} = \mathcal{V} * \mathcal{V}^T = \mathcal{V}^T * \mathcal{V} = \mathcal{I}$, and $\mathcal{S} \in \mathbb{R}^{d_1 \times d_2 \times d_3}$ is an f-diagonal tensor, i.e., its frontal slices are the diagonal matrices, and "$*$" is the t-product.

Definition 2. *(**Tensor Nuclear Norm, TNN**) (Lu et al. 2020) For $\mathcal{X} \in \mathbb{R}^{d_1 \times d_2 \times d_3}$ and $d = \min(d_1, d_2)$, the Tensor Nuclear Norm of $\mathcal{X}$ is defined as*

$$\|\mathcal{X}\|_* = \frac{1}{d_3} \sum_{k=1}^{d_3} \sum_{i=1}^{d} \sigma_i \left(\bar{\mathbf{X}}^{(k)} \right), \tag{4}$$

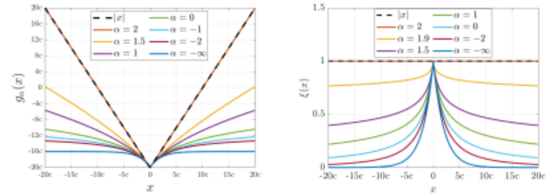
where $\bar{\mathcal{X}}$ is the result by applying FFT on $\mathcal{X}$ along the third dimension, i.e., $\bar{\mathcal{X}} = fft(\mathcal{X}, [], 3)$. $\bar{\mathbf{X}}^{(k)}$ is the k-th slice of $\bar{\mathcal{X}}$, $\sigma_i \left(\bar{\mathbf{X}}^{(k)} \right)$ is the i-th singular value of $\bar{\mathbf{X}}^{(k)}$, and $d = min(d_1, d_2)$.

Proposed methods

In this part, we first introduce the proposed low-rank regularizer FGTNN and FGTRPCA model. Secondly, we devise an efficient optimization algorithm to implement FGTRPCA. Then considering the local smoothness prior, t-FGJP is proposed to capture the low-rank structure in the gradient domain. Lastly, on this basis, we propose the smooth FGTRPCA (SFGTRPCA) model.

Flexible Generalized TNN

According to Definition 2, the original TNN uniformly shrinks each singular value of the low-rank tensor $\mathcal{L}$ when minimizing the tensor nuclear norm. In fact, larger singular values correspond to more significant information in the tensor. Therefore, it is essential to apply less shrinkage to the larger singular values while increasing the shrinkage for the smaller ones to preserve critical information better. For this purpose, we introduce a flexible generalized tensor nuclear norm (FGTNN) defined below.



(a) $g_\alpha(x)$ (b) $\xi(x)$

Figure 3: Our proposed $g_\alpha(x)$ and its corresponding weight function $\xi(x)$.

Definition 3. *(**FGTNN**) For a tensor $\mathcal{X} \in \mathbb{R}^{d_1 \times d_2 \times d_3}$, $d = \min(d_1, d_2)$, the Flexible Generalized Tensor Nuclear Norm (FGTNN) is defined as follows*

$$\|\mathcal{X}\|_{G,\alpha,*} = \frac{1}{d_3} \sum_{k=1}^{d_3} \sum_{i=1}^{d} g_\alpha \left(\sigma_i \left(\bar{\mathbf{X}}^{(k)} \right) \right), \tag{5}$$

where $g_\alpha(x)$ is

$$g_\alpha(x) = 2c \cdot \frac{|\alpha - 2|}{\alpha} \left(\left(\frac{|x|/c}{|\alpha - 2|} + 1 \right)^{\alpha/2} - 1 \right), \tag{6}$$

where $\alpha \in \mathbb{R}$ is a continuous-valued parameter that controls the shape of $g_\alpha(x)$ and $c > 0$ is a constant.

Remark 1. *Our proposed FGTNN mainly exhibits two desirable properties. **1) generalizability**: By introducing a continuous parameter α, low-rank regularizers in many existing popular methods such as TRPCA, LRTF, ETR, and DATRPCA can be viewed as special cases of FGTNN with different values of α.(see Table 1 for more details); **2) flexibility**: We can develop plenty of new low-rank regularizers by tuning α and achieve better performance. Compared to the method with fixed-form low-rank regularizer, our model gains flexibility and can adapt to more complex scenarios.*

Remark 2. *Figure 3(a) intuitively presents the characteristics of $g_\alpha(x)$. We observe that $g_\alpha(x)$ increases slower than $|x|$ for various α, which means less shrunk to large singular values, preserving the critical information within the tensor to a greater extent. More importantly, α is related to the shape of $g_\alpha(x)$. When $\alpha \to -\infty$, $g_\alpha(x)$ follows an approximately exponential form; When $\alpha = 0$, $g_\alpha(x)$ takes a logarithmic form; When $\alpha = 2$, $g_\alpha(x)$ turns to $|x|$; And in the other case of α, $g_\alpha(x)$ is represented in a approximate power form.*

Moreover, we extend the proposed new low-rank regularizer to sparse component and define the flexible generalized tensor ℓ_1 norm (FGTL1N).

Definition 4. *(**FGTL1N**) For a tensor $\mathcal{X} \in \mathbb{R}^{d_1 \times d_2 \times d_3}$, the Flexible Generalized Tensor ℓ_1 Norm (FGTL1N) is defined as follows:*

$$\|\mathcal{X}\|_{G,\alpha,1} = \sum_{i=1}^{d_1} \sum_{j=1}^{d_2} \sum_{k=1}^{d_3} g_\alpha \left(|\mathcal{X}_{ijk}| \right), \tag{7}$$

Author and year	Method	Value of α	$g_\alpha(x)$	Low-rank Regularizer
(Lu et al. 2020)	TRPCA	$\alpha = 2$	$\|x\|$	$\|\mathcal{X}\|_{G,2,*}$
(Chen et al. 2021)	LRTF	$\alpha = 0$	$2c \ln\left(\frac{1}{2}\|x\|/c + 1\right)$	$\|\mathcal{X}\|_{G,0,*}$
(Ji and Feng 2023)	ETR	$\alpha = -2$	$2c\,\frac{2\|x\|/c}{\|x\|/c+4}$	$\|\mathcal{X}\|_{G,-2,*}$
(Wang et al. 2023b)	DATRPCA	$\alpha \to -\infty$	$2c\left(1 - \exp\left(-\|x\|/2c\right)\right)$	$\|\mathcal{X}\|_{G,-\infty,*}$

Table 1: The FGTNN regularizer view for many special cases.

Flexible Generalized TRPCA

By incorporating both FGTNN and FGTL1N into TRPCA framework, we have the following Flexible Generalized TRPCA (FGTRPCA) model

$$\min_{\mathcal{L},\mathcal{E}\in\mathbb{R}^{d_1\times d_2\times d_3}} \|\mathcal{L}\|_{G,\alpha,*} + \lambda\|\mathcal{E}\|_{G,\alpha,1} \ \text{ s.t. } \mathcal{M} = \mathcal{L} + \mathcal{E}. \tag{8}$$

Note that FGTNN includes a series of specific functions that are nonlinear and complex, thus making it hard to obtain the optimal solution of the FGTRPCA model. In this paper, we design an efficient algorithm optimization framework based on the ADMM framework (Boyd et al. 2011) to implement the FGTRPCA model.

Proposition 1. *For $g_\alpha(x)$, there exists a convex conjugate function $\phi : \mathbb{R} \to \mathbb{R}$ which satisfied*

$$g_\alpha(x) = \min_{w\in\mathbb{R}_+} (w|x| + \phi(w)), \tag{9}$$

and for fixed x, the minimum is reached at $w = \xi(x)$, which is defined as

$$w = \xi(x) = \begin{cases} 1, & \text{if } \alpha = 2 \\ 2c/(|x| + 2c), & \text{if } \alpha = 0 \\ \exp\left(-|x|/2c\right), & \text{if } \alpha = -\infty \\ \left(\frac{|x|/c}{|\alpha-2|} + 1\right)^{\alpha/2-1}, & \text{otherwise.} \end{cases} \tag{10}$$

Remark 3. *According to Proposition 1, the function in Eq. (6) can be optimized by an adaptive alternating weighted minimization scheme. From the perspective of weights, smaller weights represent smaller shrinkages to singular values. As shown in Figure 3(b), TNN assigns the same weights to each singular value, i.e., TNN treats each singular value equally. For our proposed FGTNN, large singular values will be adaptively assigned small weights and shrunk less.*

According to Proposition 1, FGTNN can be transformed into

$$\|\mathcal{L}\|_{G,\alpha,*} = \min_{\mathbf{W}} \frac{1}{d_3}\sum_{k=1}^{d_3}\sum_{i=1}^{d}(W_{ki}\sigma_i(\bar{\mathbf{L}}^{(k)}) + \phi(W_{ki})), \tag{11}$$

where the W_{ki} is the k,i-th element of matrix $\mathbf{W} \in \mathbb{R}^{d_3\times d}$. The minimum is reached at $W_{ki} = \xi(\sigma_i(\bar{\mathbf{L}}^{(k)}); c)$. Similarly, as for FGTL1N, we have

$$\|\mathcal{E}\|_{1,*} = \min_{\mathcal{W}} \sum_{i=1}^{d_1}\sum_{j=1}^{d_2}\sum_{k=1}^{d_3}(\mathcal{W}_{ijk}|\mathcal{E}_{ijk}| + \phi(\mathcal{W}_{ijk})), \tag{12}$$

where the $\mathcal{W}_{ijk}$ is the i,j,k-th element of tensor $\mathcal{W} \in \mathbb{R}^{d_1\times d_2\times d_3}$. The minimum is reached at $\mathcal{W}_{ijk} = \xi(|\mathcal{E}_{ijk}|; c)$.

Notably, problem (8) will be transformed into weighted tensor nuclear norm minimization problem (11) and weighted tensor ℓ_1 norm minimization problem (12), we first present the definitions of two important concepts: Weighted tensor nuclear norm (WTNN) and weighted tensor ℓ_1 norm (WTL1N).

Definition 5. *(Weighted Tensor Nuclear Norm, WTNN)(Wang et al. 2023b) For a tensor $\mathcal{X} \in \mathbb{R}^{d_1\times d_2\times d_3}$ and a weight matrix $\mathbf{W} \in \mathbb{R}^{d\times d_3}$, $d = \min(d_1, d_2)$, the WTNN of $\mathcal{X}$ is defined as*

$$\|\mathcal{X}\|_{\mathbf{W},*} = \frac{1}{d_3}\sum_{k=1}^{d_3}\sum_{i=1}^{d} W_{ik}\sigma_i\left(\bar{\mathbf{X}}^{(k)}\right). \tag{13}$$

Definition 6. *(Weighted Tensor ℓ_1 Norm, WTL1N)(Wang et al. 2023b) For a tensor $\mathcal{X} \in \mathbb{R}^{d_1\times d_2\times d_3}$ and a weight tensor $\mathcal{W} \in \mathbb{R}^{d_1\times d_2\times d_3}$, the WTL1N of $\mathcal{X}$ is defined as*

$$\|\mathcal{X}\|_{\mathcal{W},1} = \sum_{i=1}^{d_1}\sum_{j=1}^{d_2}\sum_{k=1}^{d_3}|\mathcal{W}_{ijk}\mathcal{X}_{ijk}|. \tag{14}$$

By incorporating Eq. (11) and Eq. (12) into model (8), and according to the definition of WTNN and WTL1N, we have

$$\min_{\mathcal{L},\mathcal{E},\mathbf{W},\mathcal{W}} \|\mathcal{L}\|_{\mathbf{W},*} + \lambda\|\mathcal{E}\|_{\mathcal{W},1} + \Phi_M(\mathbf{W}) + \Phi_T(\mathcal{W}) \quad \text{s.t. } \mathcal{M} = \mathcal{L} + \mathcal{E}, \tag{15}$$

where $\Phi_M(\mathbf{W})$ and $\Phi_T(\mathcal{W})$ are defined such that $(\Phi_M(\mathbf{W}))_{ki} = \phi(W_{ki})$ and $(\Phi_T(\mathcal{W}))_{ijk} = \phi(\mathcal{W}_{ijk})$. In the next part, we will present the optimization for implementing FGTRPCA.

Optimization for FGTRPCA

The Lagrangian function of the FGTRPCA model is

$$L(\mathcal{L},\mathcal{E},\mathbf{W},\mathcal{W},\mathcal{Z},\mu) = \|\mathcal{L}\|_{\mathbf{W},*} + \lambda\|\mathcal{E}\|_{\mathcal{W},1} + \Phi_M(\mathbf{W}) + \Phi_T(\mathcal{W}) + \frac{\mu}{2}\left\|\mathcal{L} + \mathcal{E} - \mathcal{M} + \frac{\mathcal{Z}}{\mu}\right\|_F^2 - \frac{\mu}{2}\|\mathcal{Z}/\mu\|_F^2, \tag{16}$$

where $\mathcal{Z} \in \mathbb{R}^{d_1\times d_2\times d_3}$ denotes the Lagrangian multiplier and μ is a positive parameter. Each variable can be updated alternately in the scheme of the ADMM framework.

Step1: Update $\mathcal{L}$ by fixing the other variables:

$$\mathcal{L}_{t+1} = \arg\min_{\mathcal{L}} \frac{1}{\mu_t}\|\mathcal{L}\|_{\mathbf{W},*} + \frac{1}{2}\|\mathcal{L} - (\mathcal{M} - \mathcal{E}_t - \mathcal{Z}_t/\mu_t)\|_F^2. \tag{17}$$

The closed-form solution of (17) can be easily obtained with the following proximity operator.

Lemma 1 *Given $\mathcal{X} \in \mathbb{R}^{d_1\times d_2\times d_3}$ with t-SVD $\mathcal{X} = \mathcal{U} * \mathcal{S} * \mathcal{V}^*$ and a weight matrix $\mathbf{W} \in \mathbb{R}^{d\times d_3}$, where $\mathbf{w}_k$ is the k-th column of $\mathbf{W}$ and $d = min\{d_1, d_2\}$. Considering the following Weighted Tensor Nuclear Norm minimization (WTNNM) problem*

$$\mathbf{Prox}_{\|\cdot\|_{\mathbf{W},*}}(\mathcal{X}) = \arg\min_{\mathcal{L}} \|\mathcal{L} - \mathcal{X}\|_F^2 + \|\mathcal{L}\|_{\mathbf{W},*}, \tag{18}$$

where $\|\cdot\|_{\mathbf{W},*}$ denotes the WTNN, and $\mathbf{Prox}_{\|\cdot\|_{\mathbf{W},*}}$ is defined as a proximal operator. For non-descending weights $0 \le W_{1k} \le W_{2k} \le \cdots \le W_{dk}(k = 1, \ldots, d_3)$, the problem (18) has the global solution which is defined as

$$\mathcal{L}^* = \mathbf{Prox}_{\|\cdot\|_{\mathbf{W},*}}(\mathcal{X}) = \mathcal{U} * ifft(\mathcal{P}_{\mathbf{W}}(\bar{\mathcal{S}}), [], 3) * \mathcal{V}^*, \tag{19}$$

where $\mathcal{P}_{\mathbf{W}}(\bar{\mathcal{S}})$ is a tensor to meet the conditions of its k-th frontal slice is $\mathbf{P}_{\mathbf{w}_k}(\bar{\mathbf{S}}^{(k)})$ for $k = 1, \ldots, d_3$. $\bar{\mathbf{S}}^{(k)}$ is the k-th frontal slice of $\bar{\mathcal{S}}$, and $\mathbf{P}_{\mathbf{w}_k}(\bar{\mathbf{S}}^{(k)})$ denotes a diagonal matrix which can be computed as $(\mathbf{P}_{\mathbf{w}_k}(\bar{\mathbf{S}}^{(k)}))_{ii} = (\bar{\mathbf{S}}^{(k)}_{ii} - w_{ki})_+$, where $(x)_+ = x$ if $x > 0$ and $(x)_+ = 0$ otherwise. w_{ki} is the i-th element of the $\mathbf{w}_k$.

By recalling the definition of WTNN in Definition 5, we have $\frac{1}{\mu_t}\|\mathcal{L}\|_{\mathbf{W},*} = \|\mathcal{L}\|_{\frac{1}{\mu_t}\mathbf{W},*}$. Based on Lemma 1, the solution of the subproblem (17) can be described as

$$\mathcal{L}_{t+1} = \mathbf{Prox}_{\|\cdot\|_{\frac{1}{\mu_t}\mathbf{W},*}}(\mathcal{M} - \mathcal{E}_t - \mathcal{Z}_t/\mu_t). \tag{20}$$

Step2: Update $\mathcal{E}$ by fixing other variables:

$$\mathcal{E}_{t+1} = \arg\min_{\mathcal{E}} \frac{\lambda}{\mu_t}\|\mathcal{E}\|_{\mathcal{W},1} + \frac{1}{2}\|\mathcal{E} - (\mathcal{M} - \mathcal{L}_t - \mathcal{Z}_t/\mu_t)\|_F^2. \tag{21}$$

To get the closed-form solution of the above problem, we utilize the tensor soft-thresholding operator (**TST**) defined below to update $\mathcal{E}_{t+1}$.

$$\mathcal{E}_{t+1} = \mathbf{TST}(\mathcal{M} - \mathcal{L}_t - \mathcal{Z}_t/\mu_t, \frac{\lambda}{\mu_t}\mathcal{W}_t), \tag{22}$$

where the ijk-th entry of **TST** is defined by

$$(\mathbf{TST}(\mathcal{X}, \mathcal{W}))_{ijk} = \text{sign}(\mathcal{X}_{ijk})(|\mathcal{X}_{ijk}| - \mathcal{W}_{ijk})_+. \tag{23}$$

Step3: Update the elements of $\mathbf{W}$ and $\mathcal{W}$ by an adaptive way according to Proposition 1

$$W_{ki} = \xi(\sigma_i(\bar{\mathbf{L}}^{(k)}_{t+1}); c), \mathcal{W}_{ijk} = \xi(|(\mathcal{E}_{t+1})_{ijk}|; c). \tag{24}$$

Step4: Update the Lagrangian multiplier tensor $\mathcal{Z}$ and the parameter μ by

$$\mathcal{Z}_{t+1} = \mathcal{Z}_t + \mu_t(\mathcal{L}_{t+1} + \mathcal{E}_{t+1} - \mathcal{M}), \tag{25}$$

$$\mu_{t+1} = \rho\mu_t, \tag{26}$$

where $\rho = 1.1$. The convergence conditions are defined as

$$\left\{ \begin{array}{l} \|\mathcal{L}_{t+1} - \mathcal{L}_t\|_\infty \\ \|\mathcal{E}_{t+1} - \mathcal{E}_t\|_\infty \\ \|\mathcal{M} - \mathcal{L}_{t+1} - \mathcal{E}_{t+1}\|_\infty \end{array} \right\} \le \epsilon, \tag{27}$$

The whole optimization procedure is summarized in Algorithm 1.

Algorithm 1: **FGTRPCA algorithm**

Input: Observation tensor data $\mathcal{M} \in \mathbb{R}^{d_1\times d_2\times d_3}$, and the parameter λ.

Initialize $\mathcal{L}_0 = \mathcal{E}_0 = \mathcal{Z}_0 = 0$, $\mathbf{W}_0 = \mathbf{1}_{d_3\times d}$, $\mathcal{W}_0 = \mathbf{1}_{d_1\times d_2\times d_3}$, $\mu_0 = 10^{-2}$, $\rho = 1.1$, $\epsilon = 10^{-6}$, and $t = 0$.
while not converge **do**
  Update the low-rank tensor $\mathcal{L}$ by Eq. (17).
  Update the sparse tensor $\mathcal{E}$ by Eq. (21).
  Update the weights $\mathbf{W}$ and $\mathcal{W}$ by Eq. (24).
  Update the Lagrangian multiplier $\mathcal{Z}$ by Eq. (25).
  Update the parameter μ by Eq. (26).
  Check the convergence condition in Eq. (27).
end while

Output: $\mathcal{L} = \mathcal{L}_{t+1}, \mathcal{E} = \mathcal{E}_{t+1}$

Smooth FGTRPCA

Considering a structured tensor that exhibits both global low-rankness and local smoothness, we devise a novel regularizer that aims to represent both two properties simultaneously on the gradient tensors, instead of employing a combination of two distinct regularizers for encoding the two properties. We first introduce the definition of the gradient tensor and present our proposed tensor-correlated Flexible Generalized Joint Prior (t-FGJP) regularizer.

Definition 7. ***(Gradient Tensor)****(Wang et al. 2023a) For a tensor $\mathcal{X} \in \mathbb{R}^{d_1\times d_2\times d_3}$, its gradient tensor along the k-th mode is defined as*

$$\mathcal{G}_k := \nabla_k(\mathcal{X}) = \mathcal{X} \times_k \mathbf{D}_{n_k}, k = 1, 2, 3, \tag{28}$$

where $\mathbf{D}_{n_k}$ is a row circulant matrix of $(-1, 1, 0, ..., 0)$.

Definition 8. ***(t-FGJP)*** *For a tensor $\mathcal{X} \in \mathbb{R}^{d_1\times d_2\times d_3}$, the proposed t-FGJP norm is defined as*

$$\|\mathcal{X}\|_{\text{t-FGJP}} := \frac{1}{\gamma}\sum_{k\in\Gamma}\|\mathcal{G}_k\|_{G,\alpha,*}, \tag{29}$$

where Γ represents a priori set of directions along which $\mathcal{X}$ equips both global low-rankness and local smoothness priors and $\gamma := \#\{\Gamma\}$ denotes the cardinality of Γ. By incorporating both t-FGJP and FGTL1N into the TRPCA framework, we propose a smooth FGTRPCA (SFGTRPCA) model defined as

$$\min_{\mathcal{L},\mathcal{E}} \|\mathcal{L}\|_{\text{t-FGJP}} + \lambda\|\mathcal{E}\|_{G,\alpha,1} \ \text{s.t.}\ \mathcal{M} = \mathcal{L} + \mathcal{E}. \tag{30}$$

The SFGTRPCA optimization problem is similar to the FGTRPCA problem. Details of the optimization algorithm and the entire procedure are available in the supplementary material due to space limitations.

Experiments

In this section, we present several real-world experiments to substantiate the effectiveness of our models. More results can be found in the supplementary material.

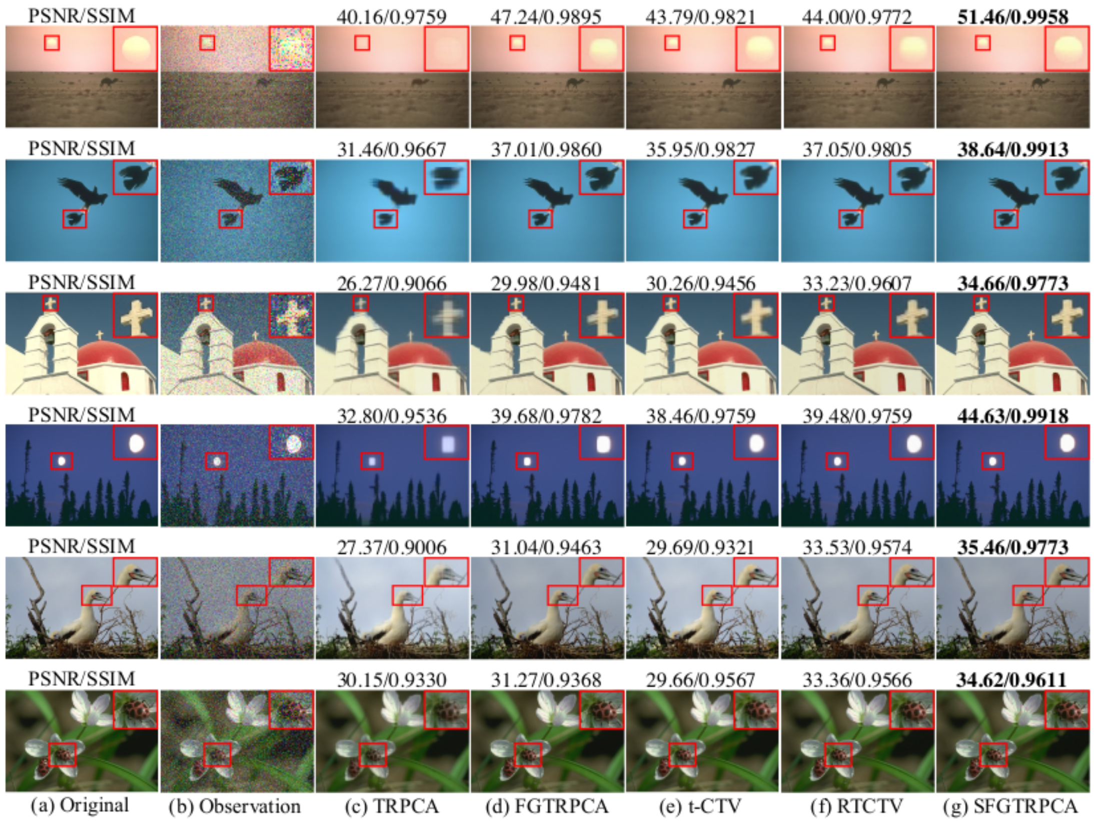


Figure 4: Recovery results on 6 color images from the BSD dataset with 20% noise ratio.

Settings

Datasets: For comprehensive comparison, we use 4 widely used tensor data types including color images, grayscale videos, hyperspectral images (HSIs), and multispectral images (MSIs). For color images, we choose 3 widely used datasets including Berkeley Segmentation Dataset[1] (BSD) (Martin et al. 2001), Kodak (Kodak 1993) dataset[2], and ZheJiang University (ZJU) (Hu et al. 2012) dataset[3]. For grayscale videos, we use 14 grayscale video sequences from the YUV dataset[4] and select the first 100 frames for each sequence. For HSIs, we select Cuprite[5], DCMall[5], Urban[5], Indian Pines[5], and Pavia University[5] (PaviaU) for experiments and select the first 50 bands from each HSI dataset. For MSIs, we randomly select 10 MSIs from the CAVE dataset (Yasuma et al. 2008).

Baselines: Our baselines are divided into two categories based on different priors. (1) Low-rankness: TRPCA (Lu et al. 2020), ETRPCA (Gao et al. 2020), and PTRPCA (Yan and Gao 2024); (2) Joint Low-rankness & Smoothness: t-CTV (Wang et al. 2023a) and RTCTV (Huang et al. 2024). We utilize the parameters recommended by the authors. For the key parameter α in our models, we search from a candidate set and employ $\alpha = 1$. More detailed parameter settings can be seen in supplementary material.

Evaluation metrics: The peak signal-to-noise ratio (PSNR) and structural similarity (SSIM) are used to evaluate the recovery performance.

Noising Data Construction: For each channel of the color image, each frame of the grayscale video, and each band of HSI and MSI, we add random salt and pepper noise at varying noise ratios of 10%, 20%, and 30%.

Results

Visual Quality. To clearly illustrate the advantages of our methods, Figure 4 presents 6 sample images from the BSD dataset, along with the recovery results under 20% salt and pepper noise. The PSNR and SSIM values are listed above the recovered images to enhance the credibility of the results. The results show that SFGTRPCA constructs more

[1] https://www2.eecs.berkeley.edu/Research/Projects/CS/vision/bsds/
[2] http://r0k.us/graphics/kodak/
[3] https://sites.google.com/site/zjuyaohu/
[4] http://trace.eas.asu.edu/yuv/
[5] https://lesun.weebly.com/hyperspectral-data-set.html

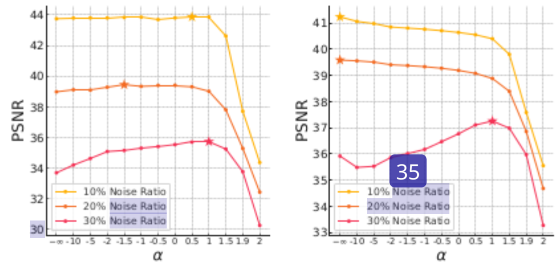
	Noise Ratio	10%		20%	
	Methods	**PSNR**	**SSIM**	**PSNR**	**SSIM**
Color images	TRPCA	31.20	0.9464	29.55	0.9115
	ETRPCA	33.26	0.9580	31.23	0.9233
	PTRPCA	33.37	0.9622	31.43	0.9350
	FGTRPCA	37.26	0.9796	33.26	0.9415
	t-CTV	32.84	0.9525	31.71	0.9348
	RTCTV	34.96	0.9689	33.46	0.9529
	SFGTRPCA	**40.96**	**0.9907**	**36.93**	**0.9782**
Grayscale videos	TRPCA	35.19	0.9636	34.16	0.9538
	ETRPCA	38.29	0.9772	36.13	0.9433
	PTRPCA	38.95	0.9807	37.31	0.9669
	FGTRPCA	41.85	0.9858	39.07	0.9743
	t-CTV	37.37	0.9721	36.52	0.9665
	RTCTV	41.11	0.9843	38.62	0.9482
	SFGTRPCA	**44.51**	**0.9911**	**41.91**	**0.9846**
HSIs	TRPCA	44.18	0.9754	42.48	0.9718
	ETRPCA	44.54	0.9747	43.20	0.9720
	PTRPCA	47.38	0.9815	45.48	0.9772
	FGTRPCA	47.30	0.9858	44.90	0.9787
	t-CTV	45.72	0.9779	44.39	0.9759
	RTCTV	48.29	0.9812	46.76	0.9789
	SFGTRPCA	**52.38**	**0.9888**	**50.16**	**0.9856**
MSIs	TRPCA	42.07	0.9898	40.41	0.9867
	ETRPCA	45.95	0.9931	44.00	0.9906
	PTRPCA	46.87	0.9939	44.84	0.9920
	FGTRPCA	49.73	0.9960	46.05	0.9921
	t-CTV	46.62	0.9938	45.21	0.9925
	RTCTV	50.19	0.9952	48.69	0.9941
	SFGTRPCA	**57.37**	**0.9977**	**53.35**	**0.9955**

Table 2: Average PSNR and SSIM results on 4 tensor types with different noise ratios. The best results are marked in bold, and the second best results are underlined.

image details and color information (Especially the contour and color of the moon in the 4-th image). Additionally, we have observed that the proposed FGTRPCA and SFGTRPCA methods significantly outperform the baseline methods under corresponding priors. Notably, the average PSNR value of SFGTRPCA surpasses the second-highest (apart from SGTRPCA) method by over 3.1 dB.

Quantitative Quality. Table 2 displays the results of all the competitors on the 4 tensor types with 10% and 20% noise. From the results, we make the following conclusions:

- Firstly, our FGTRPCA method outperforms the competitive methods under low-rankness prior in most cases. This contributes to the proposed FGTNN and FGT1N regularizers which treat singular values of the low-rank component and entries of the sparse component simultaneously from the observation tensor in an adaptive way thus preserving the key information.



(a) ZJU (color images) (b) Indian Pine (HSIs)

Figure 5: PSNR values of our SFGTRPCA algorithm on different cases. For various values of α, with the point of highest value marked by a pentagram.

- Secondly, considering the smoothness prior, our SFGTRPCA method exhibits a substantial performance gain over the comparisons in all cases, specifically color images with 10% noise (17.16% PSNR improvement) and MSIs with 10% noise (14.31% PSNR improvement). This suggests the necessity and potential of extending FGTNN to gradient tensors.
- Thirdly, it is evident that under varying noise levels in different data types, our methods consistently yield competitive scores of evaluation indices. This demonstrates that our methods better leverage the underlying low-rank and sparse structures within the tensor, exhibiting strong recovery ability and robustness.

Parameter Analysis. Obviously, the parameter α plays a crucial role in the performance of our models. This parameter directly influences flexibility and generalizability, making it essential for achieving optimal results in various scenarios. In Figure 5 we show the PSNR of the recovery results for different values of α with various noise ratios on ZJU (color images) and Indian Pines (HSIs) datasets. For some situations, performance is insensitive to variations in α, but in other cases, adjusting α improves performance. This improvement highlights the benefit of incorporating α as a hyperparameter and turning it accordingly instead of using fixed formula.

Conclusion

In this paper, we consider the TRPCA problem, which aims to recover low-rank and sparse components from observed data. In response to the limitation of TNN, We propose a flexible generalized low-rank regularizer called FGTNN to preserve critical information in the low-rank components. Using this, we develop a novel FGTRPCA framework, which contains plenty of previous works as special cases by tuning a continuous parameter. Taking into account some tensor data also exhibits local smoothness structures, we extend our regularizer to the gradient domain, prompting the introduction of the SFGTRPCA model. Experimental results demonstrate that our models significantly improve the benchmark performance on various denoising and recovery tasks with flexibility and generalizability.

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