Q1.

1.

$$\ell(\beta) = \sum_{i=1}^{n} [Y_i \log G(X_i'\beta) + (1-Y_i) \log (1 - G(X_i'\beta))]$$

$$S_i(\beta) = \frac{\partial \ell_i(\beta)}{\partial \beta} = \left[\frac{Y_i g(X_i'\beta)}{G(X_i'\beta)} - (1-Y_i) \frac{g(X_i'\beta)}{1 - G(X_i'\beta)} \right] X_i = [Y_i - G(X_i'\beta)] \lambda_i X_i \quad , \quad \lambda_i = \frac{g(X_i'\beta)}{G(X_i'\beta)(1 - G(X_i'\beta))}$$

$$I(\beta) = E[S_i(\beta) S_i(\beta)'] = E[(Y_i - G(X_i'\beta))^2 \lambda_i^2 X_i X_i']$$

$$= E\left[\frac{|g(X_i'\beta)|^2}{G(X_i'\beta)(1 - G(X_i'\beta))} X_i X_i' \right] \#$$

(b)

$$H_i(\beta) = \frac{\partial S_i(\beta)}{\partial \beta'} = -g'(X_i'\beta) \lambda_i X_i X_i' + \underbrace{[Y_i - G(X_i'\beta)]}_{\text{expectation} = 0} \frac{\partial \lambda_i}{\partial \beta'} X_i$$

$$E[H_i(\beta)] = -E\left[\frac{|g(X_i'\beta)|^2}{G(X_i'\beta)(1 - G(X_i'\beta))} X_i X_i' \right]$$

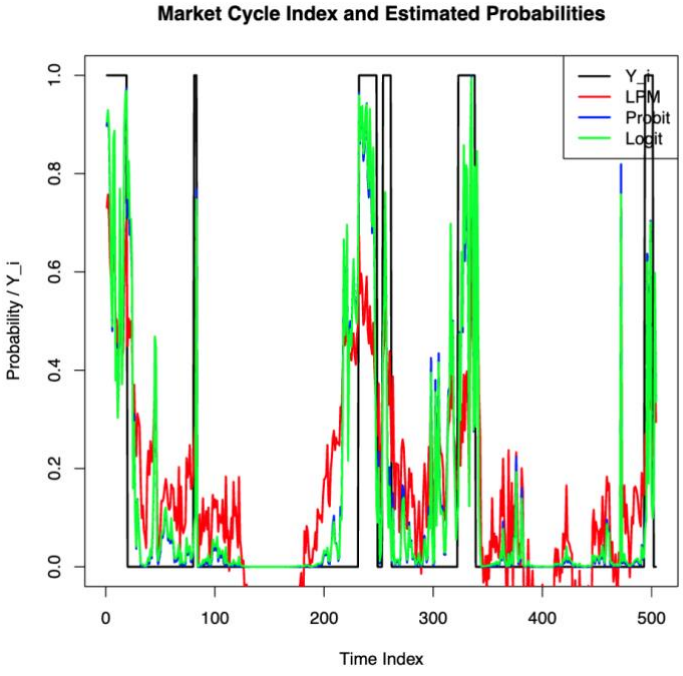
$$= -I(\beta) \#$$

equation hold.

Q2.

(a)

It is obvious that Probit and Logit show similar results, which are better than Linear Probability model.



(b)

Both value is near to 0 for both logit and probit model.

```
Probit model optimization converged.
Logit model optimization converged.
Score function at MLE for the probit model:
                   [,1]
Intercept -0.0316631735
x_dfy      0.0055677644
x_infl     0.0010700294
x_svar    -0.0118361506
x_tms      0.0001058626
x_tbl      0.0334732127
x_dfr      0.0183582654
x_dp       0.1135020855
x_ltr      0.0100104231
x_ep       0.1084157421
x_bmr      0.0173087298
x_ntis     0.0037558696
```

```
Score function at MLE for the logit model:
                   [,1]
Intercept  9.167080e-03
x_dfy      1.201280e-03
x_infl     3.991171e-04
x_svar     1.976480e-04
x_tms     -6.553894e-04
x_tbl     -1.381212e-04
x_dfr     -2.052281e-03
x_dp      -3.783134e-02
x_ltr      5.836199e-05
x_ep      -2.658905e-02
x_bmr     -1.890811e-03
x_ntis    -2.159018e-03
```

All code can be found on: https://github.com/YuJu0819/quant-method/tree/main/hw10