1.

$$\hat{\rho}(k) = \frac{1}{\hat{\sigma}^2(T-k)} \sum_{t=k+1}^{T} (Y_t - \bar{Y})(Y_{t-k} - \bar{Y})$$

$$\lim_{T\to\infty} \hat{\rho}(k) = \frac{1}{\sigma^2 T} \sum_{t=k+1}^{T} (Y_t - \mu_Y)(Y_{t-k} - \mu_Y)$$

Let $\frac{1}{T} \sum_{t=k+1}^{T} (Y_t - \mu_Y)(Y_{t-k} - \mu_Y) = S_k$

$$\sqrt{T} S_k \xrightarrow{d} N(0, \sigma_Y^4)$$

$$\sqrt{T}\,\hat{\rho}(k) \longrightarrow N(0,1)$$

$$T\,\hat{\rho}^2(k) \longrightarrow \chi^2(1) \quad \#$$

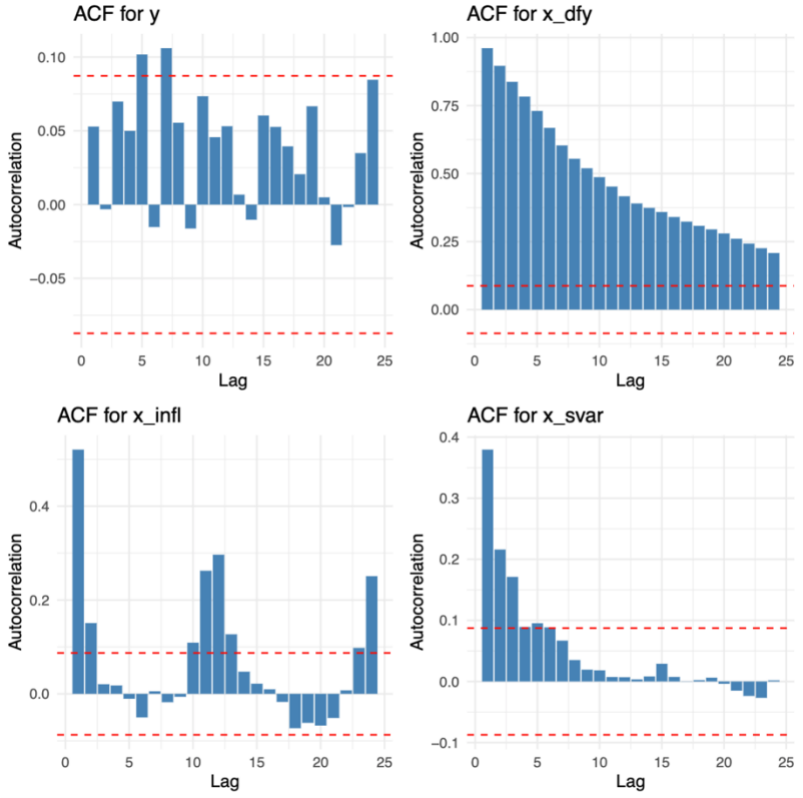
2.

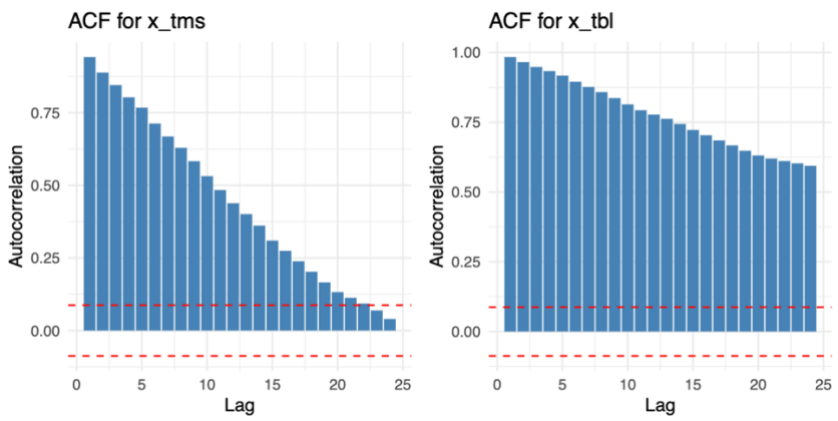
$$Cov(\hat{\rho}(k), \hat{\rho}(l)) = E\left[\frac{(Y_{t-k} - \mu)(Y_{t-l} - \mu)}{(Y_t - \mu)^2}\right] = 0$$

$$\sqrt{T}\begin{pmatrix} \hat{\rho}(1) \\ \hat{\rho}(2) \\ \vdots \\ \hat{\rho}(m) \end{pmatrix} \xrightarrow{d} N\left(\begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}, I_m\right) \quad (\text{By question 1.})$$

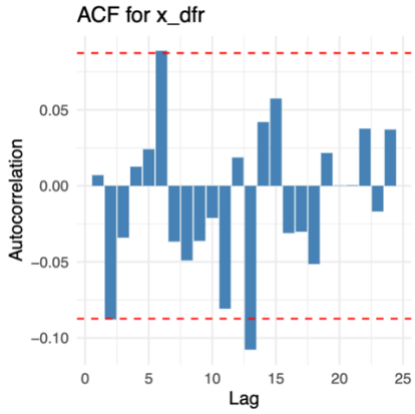
$$T \sum_{k=1}^{m} \hat{\rho}^2(k) = \chi^2(m)$$

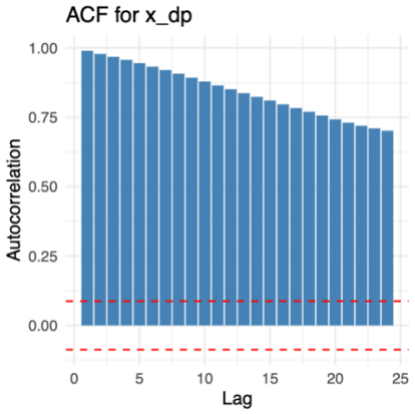
2.




ACF for x_tms
Autocorrelation
0.75
0.50
0.25
0.00
0
5
10
15
20
25
Lag


ACF for x_tbl
Autocorrelation
1.00
0.75
0.50
0.25
0.00
0
5
10
15
20
25
Lag


ACF for x_dfr
Autocorrelation
0.05
0.00
−0.05
−0.10
0
5
10
15
20
25
Lag


ACF for x_dp
Autocorrelation
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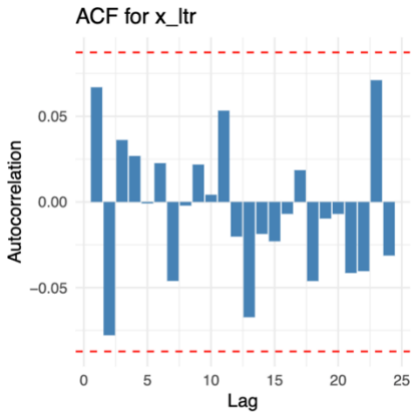
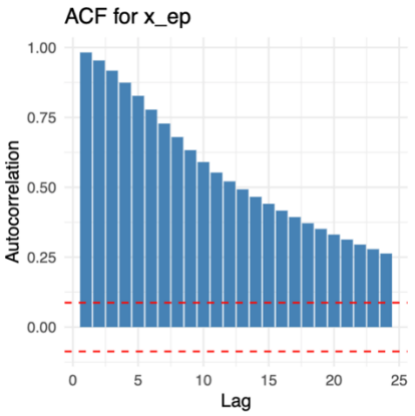
0.75
0.50
0.25
0.00
0
5
10
15
20
25
Lag


ACF for x_ltr
Autocorrelation
0.05
0.00
−0.05
0
5
10
15
20
25
Lag


ACF for x_ep
Autocorrelation
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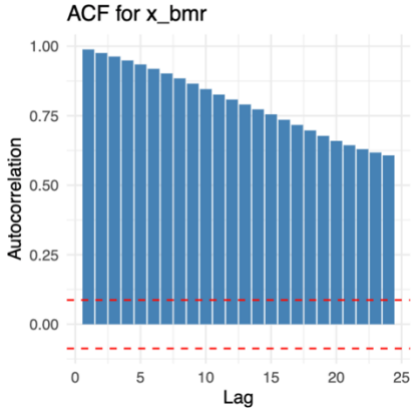
0.75
0.50
0.25
0.00
0
5
10
15
20
25
Lag


ACF for x_bmr
Autocorrelation
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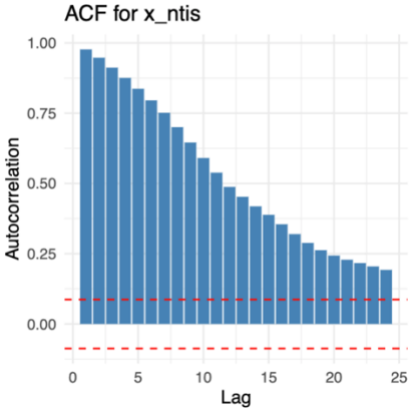
0.50
0.25
0.00
0
5
10
15
20
25
Lag


ACF for x_ntis
Autocorrelation
1.00
0.75
0.50
0.25
0.00
0
5
10
15
20
25
Lag

```
--------------------------------------------------
Variable: y
Length of Time Series (T): 504

Box-Pierce Test Results for m = 12:
Statistic (Q): 23.03
Degrees of Freedom: 12
P-value: 0.0275
Conclusion: Reject the null hypothesis of IIDness at the 5% level.

Box-Pierce Test Results for m = 24:
Statistic (Q): 34.2
Degrees of Freedom: 24
P-value: 0.0812
Conclusion: Fail to reject the null hypothesis of IIDness at the 5% level.

--------------------------------------------------
Variable: x_dfy
Length of Time Series (T): 504

Box-Pierce Test Results for m = 12:
Statistic (Q): 2810.91
Degrees of Freedom: 12
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.

Box-Pierce Test Results for m = 24:
Statistic (Q): 3376.78
Degrees of Freedom: 24
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.
```

```
Variable: x_bmr
Length of Time Series (T): 504

Box-Pierce Test Results for m = 12:
Statistic (Q): 4973.9
Degrees of Freedom: 12
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.

Box-Pierce Test Results for m = 24:
Statistic (Q): 7891.06
Degrees of Freedom: 24
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.

--------------------------------------------------
Variable: x_ntis
Length of Time Series (T): 504

Box-Pierce Test Results for m = 12:
Statistic (Q): 3595.62
Degrees of Freedom: 12
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.

Box-Pierce Test Results for m = 24:
Statistic (Q): 4174.72
Degrees of Freedom: 24
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.
```

```
Variable: x_ltr
Length of Time Series (T): 504

Box-Pierce Test Results for m = 12:
Statistic (Q): 9.56
Degrees of Freedom: 12
P-value: 0.6543
Conclusion: Fail to reject the null hypothesis of IIDness at the 5% level.

Box-Pierce Test Results for m = 24:
Statistic (Q): 18.36
Degrees of Freedom: 24
P-value: 0.7851
Conclusion: Fail to reject the null hypothesis of IIDness at the 5% level.

--------------------------------------------------
Variable: x_ep
Length of Time Series (T): 504

Box-Pierce Test Results for m = 12:
Statistic (Q): 3572.37
Degrees of Freedom: 12
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.

Box-Pierce Test Results for m = 24:
Statistic (Q): 4422.18
Degrees of Freedom: 24
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.
```

```
--------------------------------------------------
Variable: x_dfr
Length of Time Series (T): 504

Box-Pierce Test Results for m = 12:
Statistic (Q): 15.08
Degrees of Freedom: 12
P-value: 0.2371
Conclusion: Fail to reject the null hypothesis of IIDness at the 5% level.

Box-Pierce Test Results for m = 24:
Statistic (Q): 27.53
Degrees of Freedom: 24
P-value: 0.2804
Conclusion: Fail to reject the null hypothesis of IIDness at the 5% level.

--------------------------------------------------
Variable: x_dp
Length of Time Series (T): 504

Box-Pierce Test Results for m = 12:
Statistic (Q): 5172.4
Degrees of Freedom: 12
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.

Box-Pierce Test Results for m = 24:
Statistic (Q): 8723.79
Degrees of Freedom: 24
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.
```

```
--------------------------------------------------
Variable: x_tms
Length of Time Series (T): 504

Box-Pierce Test Results for m = 12:
Statistic (Q): 3035.22
Degrees of Freedom: 12
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.

Box-Pierce Test Results for m = 24:
Statistic (Q): 3354.73
Degrees of Freedom: 24
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.

--------------------------------------------------
Variable: x_tbl
Length of Time Series (T): 504

Box-Pierce Test Results for m = 12:
Statistic (Q): 4742.99
Degrees of Freedom: 12
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.

Box-Pierce Test Results for m = 24:
Statistic (Q): 7443.11
Degrees of Freedom: 24
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.
```

```
Variable: x_infl
Length of Time Series (T): 504

Box-Pierce Test Results for m = 12:
Statistic (Q): 235.12
Degrees of Freedom: 12
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.

Box-Pierce Test Results for m = 24:
Statistic (Q): 289.65
Degrees of Freedom: 24
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.

--------------------------------------------------
Variable: x_svar
Length of Time Series (T): 504

Box-Pierce Test Results for m = 12:
Statistic (Q): 126.84
Degrees of Freedom: 12
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.

Box-Pierce Test Results for m = 24:
Statistic (Q): 128.13
Degrees of Freedom: 24
P-value: 0
Conclusion: Reject the null hypothesis of IIDness at the 5% level.
```

All code can be found on: https://github.com/YuJu0819/quant-method/tree/main/hw11