

Q1.

1.

First col

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   5.956604   0.036371 163.775 < 2e-16 ***
experience     0.053258   0.006948   7.666 2.38e-14 ***
experience_sq -0.218720   0.034021  -6.429 1.49e-10 ***
black         -0.263903   0.018485 -14.277 < 2e-16 ***
reg76r        -0.143458   0.016336  -8.782 < 2e-16 ***
smsa76r        0.184752   0.017503  10.555 < 2e-16 ***
nearc4         0.044624   0.017011   2.623 0.00876 **
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Second col

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  16.65917    0.17639  94.446 < 2e-16 ***
experience   -0.41001    0.03369 -12.169 < 2e-16 ***
experience_sq  0.07323    0.16499   0.444 0.657201
black        -1.00614    0.08965 -11.224 < 2e-16 ***
reg76r       -0.29146    0.07922  -3.679 0.000238 ***
smsa76r       0.40388    0.08489   4.758 2.05e-06 ***
nearc4        0.33732    0.08250   4.089 4.45e-05 ***
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```

Third col

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -1.8695     4.2984  -0.435 0.663638
black         -1.4684     0.1154 -12.719 < 2e-16 ***
reg76r        -0.4597     0.1024  -4.488 7.47e-06 ***
smsa76r        0.8354     0.1093   7.647 2.76e-14 ***
nearc4         0.3471     0.1070   3.244 0.001191 **
age76         1.0614     0.3014   3.522 0.000435 ***
age_sq       -1.8760     0.5231  -3.586 0.000341 ***
```

Forth col

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -4.13048     4.29836  -0.961 0.336658
black         1.46837     0.11544  12.719 < 2e-16 ***
reg76r        0.45970     0.10243   4.488 7.47e-06 ***
smsa76r       -0.83540     0.10925  -7.647 2.76e-14 ***
nearc4        -0.34710     0.10700  -3.244 0.001191 **
age76         -0.06144     0.30140  -0.204 0.838482
age_sq        1.87598     0.52314   3.586 0.000341 ***
```

Fifth col

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   6.09943    0.90220   6.761 1.64e-11 ***
black          0.28206    0.02423  11.641 < 2e-16 ***
reg76r         0.11177    0.02150   5.199 2.14e-07 ***
smsa76r        -0.17554    0.02293  -7.655 2.59e-14 ***
nearc4         -0.07251    0.02246  -3.229 0.00126 **
age76          -0.55452    0.06326  -8.765 < 2e-16 ***
age_sq         1.31320    0.10980  11.960 < 2e-16 ***
```

## Sixth col

Coefficients:					
	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	16.65729	0.17612	94.579	< 2e-16	***
experience	-0.41327	0.03366	-12.278	< 2e-16	***
experience_sq	0.09280	0.16486	0.563	0.573549	
black	-1.00631	0.08951	-11.243	< 2e-16	***
reg76r	-0.26711	0.07947	-3.361	0.000786	***
smsa76r	0.39975	0.08477	4.716	2.52e-06	***
nearc4a	0.43035	0.08736	4.926	8.85e-07	***
nearc4b	0.12263	0.10628	1.154	0.248643	

2.

First col:

Coefficients:					
	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	4.733664	0.067603	70.022	< 2e-16	***
ed76	0.074009	0.003505	21.113	< 2e-16	***
experience	0.083596	0.006648	12.575	< 2e-16	***
experience_sq	-0.224088	0.031784	-7.050	2.21e-12	***
black	-0.189632	0.017627	-10.758	< 2e-16	***
reg76r	-0.124862	0.015118	-8.259	< 2e-16	***
smsa76r	0.161423	0.015573	10.365	< 2e-16	***
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Second col:

Coefficients:					
	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	3.75278	0.82934	4.525	6.27e-06	***
ed76	0.13229	0.04923	2.687	0.00725	**
experience	0.10750	0.02130	5.047	4.76e-07	***
experience_sq	-0.22841	0.03341	-6.836	9.84e-12	***
black	-0.13080	0.05287	-2.474	0.01342	*
reg76r	-0.10490	0.02307	-4.546	5.67e-06	***
smsa76r	0.13132	0.03013	4.359	1.35e-05	***
---					

Third col:

Coefficients:					
	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	4.06567	0.60850	6.682	2.81e-11	***
ed76	0.13295	0.05138	2.588	0.009712	**
experience	0.05596	0.02599	2.153	0.031412	*
experience_sq	-0.07957	0.13403	-0.594	0.552797	
black	-0.10314	0.07737	-1.333	0.182624	
reg76r	-0.09818	0.02876	-3.413	0.000651	***
smsa76r	0.10798	0.04974	2.171	0.030010	*

Forth col:

Coefficients:					
	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	3.26801	0.68718	4.756	2.07e-06	***
ed76	0.16109	0.04077	3.951	7.96e-05	***
experience	0.11931	0.01818	6.564	6.16e-11	***
experience_sq	-0.23054	0.03503	-6.582	5.46e-11	***
black	-0.10173	0.04531	-2.245	0.0248	*
reg76r	-0.09504	0.02165	-4.389	1.18e-05	***
smsa76r	0.11645	0.02705	4.305	1.73e-05	***

Sargan Statistic: 0.8205905  
P-value: 0.3650076

Fifth col:

Coefficients:				
	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	3.74815	0.48338	7.754	1.21e-14 ***
ed76	0.15969	0.04090	3.904	9.65e-05 ***
experience	0.04703	0.02502	1.880	0.060213 .
experience_sq	-0.03225	0.12811	-0.252	0.801256
black	-0.06403	0.06301	-1.016	0.309611
reg76r	-0.08573	0.02563	-3.345	0.000834 ***
smsa76r	0.08348	0.04125	2.024	0.043073 *

Sargan Statistic: 0.5237889  
P-value: 0.4692299

All code can be found on: <https://github.com/YuJu0819/quant-method> in folder hw9

2. As shown in text book,  $s = \frac{\hat{e}'\hat{e}(Z'Z)^{-1}Z'\hat{e}}{\hat{\sigma}^2} \xrightarrow{d} Z'[I_k - Q(Q'Q)^{-1}Q']Z$   $M_Q$  is idempotent.

"  $Z'M_Q Z \longrightarrow M_Q Z \sim N(0, M_Q I_k M_Q')$

$Z'M_Q Z = \sum_{i=1}^k \lambda_i X_i^2(1)$   $\lambda_i$  means eigenvalue of  $M_Q$  and  $\lambda_i = 1$  or  $0$  since idempotent.

$\text{rank}(M_Q) = \text{rank}(I_k - P_Q) = k - k$ , hence  $\lambda_i = 0 \quad \forall i > k$

$(M_Q Z)^2 = \sum_{i=1}^{k-k} \lambda_i X_i^2(1)$

$\downarrow$

$\chi^2(k-k)$