1 第16教学周12.19

定义1.1 (order). *We say that $u \in \mathcal{D}'(\Omega)$ is of order $N \in \mathbb{N}_0$, if $\forall$ compact $K \subset \Omega$, $\exists C_K > 0$ such that*

$$|\langle u, \varphi \rangle| \leq C_K \sup_{x \in K, |\alpha| \leq N} |\partial^\alpha \varphi(x)|. \tag{1.1}$$

u is said to be of infinite order if it is not of order N for any N; otherwise it is said to be of finite order. The order of u is the smallest N that can be used, resp. ∞.

1.4 The support of a distribution

定义1.2 (support). *Let $u \in \mathscr{D}'(\Omega)$.*

1. *We say that u is 0 on the open subset $\omega \subset \Omega$ when*

$$\langle u, \varphi \rangle = 0 \quad \textit{for all } \varphi \in C_c^\infty(\omega).$$

2. *The support of u is defined as the set*

$$\operatorname{supp} u = \Omega \setminus \left(\bigcup \{ \omega \mid \omega \textit{ open } \subset \Omega, u \textit{ is } 0 \textit{ on } \omega \} \right).$$

引理1.3. *Let $(\omega_\lambda)_{\lambda \in \Lambda}$ be a family of open subsets of Ω. If $u \in \mathcal{D}'(\Omega)$ is 0 on ω_λ for each $\lambda \in \Lambda$, then u is 0 on the union $\bigcup_{\lambda \in \Lambda} \omega_\lambda$.*

Distribution with compact support.

$$\mathcal{D}_c'(\Omega) := \{ u \in \mathcal{D}'(\Omega) : \operatorname{supp} u \text{ is a compact subset of } \Omega \}. \tag{1.2}$$

定理1.4. *The spaces $\mathcal{D}_c'(\Omega)$ and $\mathcal{E}'(\Omega)$ are algebraically isomorphic.*

命题1.5. *If $u \in \mathcal{D}'(\mathbb{R}^d)$ and $\operatorname{supp} u \subset \{0\}$, then u has a unique representation of the form:*

$$u = \sum_{|\alpha| \leq m} C_\alpha \partial^\alpha \delta_0 \tag{1.3}$$

for some $m \geq 0$ and $C_\alpha \in \mathbb{C}$.

The structure theorem.

定理1.6 (structure theorem). *Let Ω be open $\subset \mathbb{R}^n$ and let $u \in \mathcal{E}'(\Omega)$. Let V be an open neighborhood of* $\operatorname{supp} u$ *with $\overline{V}$ compact $\subset \Omega$, and let M be an integer $> (N+d)/2$, where N is the order of u. There exists a system of continuous functions f_α with support in V for $|\alpha| \leq 2M$ such that*

$$u = \sum_{|\alpha| \leq 2M} D^\alpha f_\alpha$$

Moreover, there exists a continuous function g on $\mathbb{R}^d$ such that $u = (1-\Delta)^M g$ (and one can obtain that $g \in H^{d/2+1-\varepsilon}\left(\mathbb{R}^d\right)$ for any $\varepsilon > 0$).

作业

P_{291} **13, 15;** P_{299} **19;**