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Generating effective defined-contribution pension plan using simulation optimization approach

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ABSTRACT

This paper presents an optimization approach to analyze the problems of portfolio selection for long-term investments, taking into consideration the specific target replacement ratio for defined-contribution (DC) pension scheme; the purpose is to generate an effective multi-period asset allocation that reaches an amount matching the target liability at retirement date and reduce the downside risk of the investment. A multi-period asset liability simulation model was used to generate 4000 asset return predictions, and an evolutionary algorithm, *evolution strategies*, was incorporated into the model to generate multi-period asset allocations under four conditions, considering different weights for measuring the importance of matching the target liability and different periods of downside risk measurement. Computational results showed that the evolutionary algorithm, *evolution strategies*, is a very robust and effective approach to generate promising asset allocations under all the four cases. In addition, computational results showed that the promising asset allocations revealed valuable information, which is able to help fund managers or investors achieve a higher average investment return or a lower level of volatility under different conditions.

1. Introduction

The core purpose of pension funds is to serve as an attractive form of savings for employees with the ultimately goal of providing them with benefit payments when they have ended their active income earning careers. There are two types of pension plans: defined contribution (DC) and defined benefit (DB). There has recently been a rapid trend of employees around the world shifting from the DB scheme to the DC scheme with an increasing number of the new workforce joining defined contribution schemes. A DC pension plan is relatively simple; each participant accumulates his contributions and investment returns in a distinct personal pension account. Typically, a longer tenure is associated with a greater probability of being better rewarded in a DB plan. Under the DC scheme, employers transfer the pension fund investment risk to the employees. Such a scheme usually performs very badly in periods of high inflation because wages and salaries rise as fast as or faster than prices, whereas the value of funds often does not. No one knows if the DC plan will be able to provide a good pension benefit when the day of retirement arrives. Therefore, it is essential for the employees to choose optimal investment strategies during the accumulation phase so that they will have sufficient funds accumulated on retirement.

The traditional single-period mean–variance (MV) approach (Markowitz, 1959) has dominated the portfolio selection process in the investment management profession for over a decade (Sharpe & Tint, 1990; Wilkie, 1985; Wise, 1984a, 1984b, 1987a, 1987b; Sherris, 1992). The MV approach is applied to single period investments and solves the problem of single-period asset allocation under a restrictive set of assumptions; however, this method is not suitable for a long-term investments, where multi-period asset allocation is more appropriate, since holding the same proportions of assets for thirty years may have a lower average investment return or a higher volatility than a so called "life cycle" or "top–down" investment strategy. In addition, the MV approach has the disadvantage of being a single-point forecast. A different mean and variance of the forecast may result in very different asset allocations (Chopra & Ziemba, 1993; Koskosidis & Duarte, 1997).

Merton introduced a multi-period context of portfolio strategy (Merton, 1971, 1990) and his dynamic programming (DP) technique is widely applied to the financial optimization in a continuous-time model (Basak & Shapiro, 2001; Battocchio & Menoncin, 2004; Cuoco & Cvitanic, 1998; Devolder, Princep, & Fabian, 2003; Gerrard, Haberman, & Vigna, 2004; Haberman & Sung, 2005; Haberman & Vigna, 2002; Hipp & Taksar, 2000; Josa-Fombellida & Rincon-Zapatero, 2004; Lioui & Poncet, 2001; Yiu, 2004). However, it is sometimes difficult to apply this technique to realistic problems because it generally needs very strong assumptions to obtain closed-form solutions in a continuous-time model. For example, if,

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according to some regulations, the weight of a specific asset must be lower than a specified proportion of the portfolio, say 50%, then the DP technique will not be able to attain a closed-form solution. It is even more difficult to consider more complicated constraints such as the monitoring of downside risk. In addition, a multi-period context of portfolio strategy in a discrete-time model normally leads to sets of recursive equations (Huang & Cairns, 2004). The difficulty of applying constraints further increases in discrete-time models and prevents the DP technique from being applicable in realistic problems.

Simulation techniques, such as the dynamic financial analysis system, have been a commonly used tool for financial analysis, and it certainly is an appropriate tool to deal with the DC pension plan problem. It allows users to take into consideration all kinds of constraints to simulate real world problems and helps users make appropriate decisions under different conditions. Although simulation techniques are powerful, it usually generates decisions by employing users' professional knowledge and the trial-and-error method and cannot guarantee promising solutions. Therefore, the approach of integrating simulation techniques with optimization methods is valuable for researchers and practitioners to conduct financial analysis. Furthermore, since simulation models for financial analysis can be very complicated, optimization methods should be carefully chosen and properly applied so that promising decisions can be obtained. Lately, evolutionary algorithms have become the most important techniques for optimization problems. The SCI and SSCI database contains more than ten thousand technical papers developed in the past decade that have reported successful applications of evolutionary algorithms in many different research fields. Several of the papers applied genetic algorithms to solve portfolio optimization problems: Abiyev and Menekay (2007), Baglioni, Pereira, Sorbello, and Tettamanzi (2000), Chan, Wong, Cheung, and Tang (2002), Chang, Meade, Beasley, and Sharaiha (2000), Chang, Yang, and Chang (2009), Oh, Lim, and Min (2005), Lin and Ko (2009) and Yang (2006). To our knowledge, the papers of Baglioni et al. (2000), Chan et al. (2002) and Yang (2006) were the only research papers considering simulation models for multi-period asset allocation and applied basic genetic algorithms to determine effective asset allocations. However, all the applications were in some ways preliminary; therefore, in this research an evolutionary algorithm, evolution strategies, is chosen to be integrated with simulation models for the thorough investigation of the DC pension plan problem.

We developed a multi-period discrete-time asset liability simulation model and integrated an evolution strategies algorithm with the model to generate a DC pension plan that can match a target liability and decrease the downside risk. For the purpose of illustration, Wilkie's investment model is adopted (Wilkie, 1995) to simulate a representative set of equal-probability plausible scenarios of future returns. Each scenario represents one possible uncertain return over the planning horizon. A large set of scenarios is generated to adequately represent highly unlikely market swings, and the proposed simulation optimized approach is applied to obtain a promising investment strategies. In this research, 4000 equal-probability scenarios of returns in 40 years were generated for the simulation model, and the proposed approach was applied to generate investment strategies under conditions considering different weights for measuring the importance of matching the target liability and the period to reduce downside risk. Computational results showed that the proposed approach is effective for finding promising investment strategies to match the target liability and decrease the downside risk during the accumulation phase in a DC plan. In addition, the investment strategies generated under different conditions provide valuable information for fund managers or investors to make proper decisions under different conditions.

The rest of this paper is organized as follows. Section 2 describes our asset liability management models for pension funds. Section 3 introduces the evolution strategies algorithm and explains how the algorithm is applied to generate promising multi-period asset allocations to achieve the objectives of the models. The computational results are discussed in Sections 4 and 5 concludes some findings in this paper.

2. Asset liability management for pension funds

Generally speaking, the main goal of asset liability management for pension funds is to find acceptable investment returns and contributions that ensure that the fund is sufficient during the planning horizon. In this paper, we investigate the investment allocation and the downside risk faced by retiring members of DC plans. We assume that each individual has a pre-specified target liability (a specific income replacement ratio). Hence, achieving the asset–liability matching is the major objective of the participants of DC plans. A trade-off between investment returns and insolvency is an important factor that must be considered. Usually, the solvency is evaluated by the amounts of unfunded liabilities, which is the difference between liabilities and assets. Generally speaking, when compared with a single-period investment strategy, a multi-period asset allocation strategy provides a larger allowance for temporary underfunding, which allows investors to hold more equity at the beginning of the term and possibly enhances investment returns during the entire period. Therefore, a multi-period investment strategy is more likely to provide a higher average return, subject to a certain allowance of risk, than a single period investment strategy. It is then important to construct a model of a multi-period investment strategy for a long-term liability.

The purpose of this paper is to establish an effective asset allocation to a long-term liability such as pension benefits. With the asset allocation, fund managers or investors are able to follow their schedules to meet their target obligations as long as the market is consistent all the time. If the market changes after a period, fund managers or investors need to simulate the new market information and apply the model of the multi-period investment strategy to obtain another effective asset allocation for the rest of the period of the target liability. A pension fund has long-term obligations; and therefore, a long-term view of investment strategy is required because of its long planning horizon. Thus, we use the standard asset classes used by pension funds such as short-term bonds, consols, index-linked gilts and equities.

In order to evaluate the asset value of the portfolio, we define.

P_{kj} : proportion held in asset type j at the kth year,

where $j = 1$ is for short-term bonds; $j = 2$ is for consols; $j = 3$ is for ILGs; $j = 4$ is for equities.

$A(0)$: total initial asset holding.
$A(k)$: total asset value at the kth year.
$A(n)$: total asset value at the end of the term.
c%: contribution rate, a percentage of an individual's salary contributed to his (or her) pension account each year in order to reach a target liability.
S_1: initial salary.
S_k: salary at the kth year.
S_n: salary at the last year of the term.
$r_i(k)$ is the investment return of the ith asset at the kth year.

The value of the total asset at the time of the kth year is:

$$A(k) = (A(k-1) + c\% \times S_k) \times \left[\sum_{j=1}^{4} P_{kj} \times (1 + r_j(k))\right], \ j = 1, \ldots, 4 \quad (1)$$

where $k = 1, \ldots, n$.

Thus, the value of the total asset at the maturity date is:

$$A(n) = (A(n-1) + c\% \times S_n)\left[\sum_{j=1}^{4} P_{nj} \times (1 + r_j(n))\right]$$
$$= \sum_{t=1}^{n} c\% \times S_t \prod_{i=t}^{n}\left[\sum_{j=1}^{4} P_{ij} \times (1 + r_j(i))\right]. \quad (2)$$

Maximizing investment surplus (positive tracking errors) is a one of the main purposes for most investors. However, for most pension plan sponsors, asset–liability matching is widely recognized as a sensible goal because of its conservative property. The income replacement rate is a popular index to judge the required amount at retirement. Thus, we define the target liability at the retirement date as follows:

$$L(n) = 80\% \times S_x \times \ddot{a}_x, \quad (3)$$

where S_x is the salary at age x and x is the retirement age. $\ddot{a}_x$ is the value of an annuity of 1 per annum payable annually in advance for a life attaining age x. For the simplicity, $\ddot{a}_x$ is calculated with a fixed interest rate.

Although the final wealth at retirement date is important in pension schemes, the investment performance during the accumulation period should not be ignored. Therefore, we define the annual minimum increase of the target liability as the larger value between 5% and the inflation rate. In other words, we aim to have a target annual minimum return of 5% or the inflation rate. The target liability at time T_k is as follows:

$$L(T_k) = [L(T_k - 1) + c\% \times S_{T_k - 1}] \times \max\{1.05, rpi_{T_k}\},$$
$$L(0) = 0, \quad T_k = 1, \ldots, n-1, \quad (4)$$

where rpi_{T_k} is the inflation rate at time T_k.

As mentioned previously, in this research, we not only consider the minimization of the tracking error at the terminal date, but also make an effort to reduce the downside risk by checking the downside risk regularly. If we check the value of the pension fund against the target liability to determine its sufficiency every five years, then we can define the objective function as follows:

$$\text{Min } \theta \times E[(A(n) - L(n))^2] - \sum_{k=1}^{k=\frac{n}{5}-1} E[A(T_{5\times k}) - L(T_{5\times k})], \quad (5)$$

where θ is a weight for adjusting the importance of asset–liability matching at the retirement date (Haberman & Vigna (2002)). If we check the sufficiency of the pension fund against the target liability every year, then we define the objective function as follows:

$$\text{Min } \theta \times E[(A(n) - L(n))^2] - \sum_{k=1}^{k=\frac{n}{5}-1} E[A(T_k) - L(T_k)]. \quad (6)$$

Note that also, in this research, although the sufficiency of the pension fund against the target liability can be checked every year, the proportions of asset allocation are changed every five years. This is because if the proportions are changed every year, the number of variables considered in the model increases significantly and optimally solving the simulation model will become computationally intractable.

3. Evolution strategies

Evolutionary algorithms (EAs) are randomized search methods that incorporate the nature of evolution into its processes (Michalewicz, 1999; Michalewicz & Fogel, 2002). Evolutionary algorithms, unlike traditional optimization techniques, use “populations” instead of single points to search and solve complex optimization problems. The population for the initial generation is usually generated randomly. From the members (parents) in the population, genetic operators are then used to produce offsprings, and the favorable offsprings, based on the “survival of the fittest theory” in the biological world, are chosen to constitute the population for the next generation. The process continues for generations until a termination criterion is satisfied and a superior solution is acquired.

Genetic algorithms (GA), genetic programming (GP), and evolution strategies (ES) are some of the commonly used evolutionary algorithms. GA is the most popular one and is good for general optimization problems; GP is appropriate for rule-based optimization problems, ES is developed exclusively for continuous variable problems (Rechenberg, 1973; Schwefel, 1981). Since the problem at hand involves continuous variables, we choose to apply ES in solving the candidate problem in this research. The steps of a (μ, λ) ES algorithm are presented below (Back, 1996; Nissen & Biethahn, 1995), where μ is the number of parents in the current population and λ is the number of offspring produced by the parents, and λ is about seven times μ. Back (1996) suggests a default set of (μ, λ) to be (15, 100), and it is used in this application.

Step 1: Generate a population for the initial generation. A population of μ solutions (members) is generated. Each solution is usually represented by a row vector consisting of two parts. The elements in the first part are the values of the decision variables (x_j) considered in a given application, and the elements in the second part are the mutation step sizes (σ_j) corresponding to the decision variables in the first part. The decision variables in our application are the contribution rate and the proportions of the four asset allocations in every period. Note that in our simulation model, the investment strategy is to change the proportions of the asset allocations every five years and then keep the same proportions over the said period. Therefore, with a forty year simulation horizon we have to determine the contribution rate and proportions of the four asset allocations in each of the eight periods, and there are a total of 33 decision variables included in a solution. If the investment strategy is to change the proportions of the asset allocations every year instead of every five years, the number of decision variables increases largely from 33 to 161. This will cause optimally solving the simulation model computationally intractable. We thus choose to change the proportions of the asset allocations every five years in our simulation model. The contribution rate and the proportions of the asset allocations are real values in the range of [0, 1], and the sum of the proportions of the four asset allocations in a period is equal to one. Therefore, for each solution we first randomly generate the contribution rate and the proportions of the first three asset allocations of the eight periods from the uniform distribution with a range of [0, 1]. A simple method, denoted as the feasibility-keeping method, is then applied to the generated proportions of the three asset allocations for each period as follows. If the sum of the three proportions is greater than one, then all the proportions will be multiplied by 0.9 consecutively until their sum is less than or equal to one; the proportion of the fourth asset allocation is then obtained by subtracting the sum from one. This feasibility-keeping method is able to reduce the number of the decision variables, which are needed to be determined by the ES, from forty to thirty-two. In addition, in each solution, all the mutation step sizes are set to 3.0 (Back, 1996).

Step 2: Apply recombination and mutation to the parents in the current population to produce λ offspring. A pair of parents, A and B, is randomly chosen from the population, and recombination and mutation are applied to A and B

to produce a child C. Discrete recombination is used to determine the first part, the decision variable values of child C. The value of each decision variable in C is randomly and equally chosen from the value of the same variable in A and B. Intermediate recombination is used to determine the second part, the mutation step sizes of C. The jth mutation step size in C is simply determined by the average of the jth mutation step size in A and B ($\sigma_j(C) = 0.5(\sigma_j(A + \sigma_j(B))$.

The generated child C is then mutated by first modifying its mutation step sizes and then adding these step sizes to mutate the corresponding decision variables. Each mutation step size $\sigma_j(C)$ is modified by the following equation:

$$\sigma'_j(C) = \sigma_j(C)\exp(\tau' N(0,1) + \tau N_j(0,1)), \quad (7)$$

where $N(0,1)$ is a standard-normally distributed random variable, and the values of τ and τ' are set to 1.0 with each decision variable x_j is mutated by the following equation:

$$x'_j(C) = x_j(C) + N_j\left(0, \sigma'_j(C)\right). \quad (8)$$

When child C is generated, the feasibility-keeping method is applied to the asset allocations of each period to maintain its feasibility. The reproduction procedure is repeated until λ offspring are produced.

Step 3: Evaluate the λ offspring and choose the best μ offspring to constitute the population for the next generation. The decision variables of a child are submitted to the simulation model, and the result, the tracking error generated by the model, is the fitness value of the child.

Step 4: Check the termination criterion. If the termination criterion is satisfied, stop; otherwise go to Step 2. For all the examples solved in this research, we plotted evolutionary curves and found that the ES converged within 500 generations in all of the examples. Therefore, the termination criterion used in this application is a maximum of 500 generations.

In order to verify the performance of the ES with the given parameters, we coded the ES in C language and applied it in solving the following complicated benchmark examples in Michalewicz (1999), Michalewicz & Fogel (2002). For purposes of convenience, the ES with the given parameters is denoted as BES (Basic ES):

Case 1:

$$\begin{aligned} \text{Minimize} \quad f(x) &= 100\left(x_2 - x_1^2\right)^2 + (1 - x_1)^2 + 90\left(x_4 - x_3^2\right)^2 \\ &\quad + (1 - x_3)^2 + 10.1\left((x_2 - 1)^2 + (x_4 - 1)^2\right) \\ &\quad + 19.8(x_2 - 1)(x_4 - 1) \\ \text{Subject to} : & -10.0 \leqslant x_i \leqslant 10.0, \quad i = 1, 2, 3, 4. \end{aligned}$$

Case 2:

$$\begin{aligned} \text{Minimize} \quad f(x, y) &= 0.5 + \left(\sin(sqrt(x^2 + y^2))^2 - 0.5\right) \\ &\quad /\left(1.0 + 0.001(x^2 + y^2)\right)^2 \\ \text{Subject to} : & -100.0 \leqslant x, \quad y \leqslant 100.0. \end{aligned}$$

For Case 1, Colville (1968) presented this problem in his study. The global optimal solution for this problem is $\boldsymbol{x}^* = (1.0, 1.0, 1.0, 1.0)$ and $f(\boldsymbol{x}^*) = 0$. Michalewicz (1999) applied an effective genetic algorithm, GENOCOP (Genetic algorithm for Numerical Optimization for Constrained Problems), to the problem and found a solution, $\boldsymbol{x}$, close $\boldsymbol{x}^*$, such that $f(\boldsymbol{x}) = 10^{-8}$ in about 10,000 iterations. We applied BES to solve this problem 20 times using different initial solutions. All the applications produced solutions very close to $\boldsymbol{x}^*$, and their objective values were all less than 10^{-8} in about 6000 iterations. These solutions were improved to make the objective values approach 10^{-15} in about 10,000 iterations. As to Case 2, Dozier presented this question in his study (Dozier, Homaifar, Tunstel, & Battle, 2001), and he noted that the algorithm should find the optimum solution $f(x,y) > 0.99754$ within 4000 iterations. We also applied BES to solve this problem 20 times using different initial solutions. All the applications found $f(x,y) > 0.99754$ within 100 iterations. These results demonstrate effectiveness, efficiency, and robustness of BES. Robustness is especially important for simulation models because the conditions generated by simulation models, with the same parameters, may be different. Therefore, it is believed that BES is an appropriate tool for the candidate problem.

4. Analysis of numerical results

In this section, Wilkie's investment model (1995) is adopted to simulate a representative set with 4000 equal-probability plausible scenarios of asset return predictions. We assume that a new employee is twenty-five years old and he will contribute a percentage of his salary, the contribution rate, into his pension account each year in order to reach a target liability of 80% income replacement when he retires at the age of sixty-five. Also, he will face 4000 equal-probability scenarios of asset return predictions every year in the following 40 years. A pension manager must find an effective investment strategy for the new employee. The investment strategy is to determine the contribution rate and change the proportions of asset allocation every five years. We integrated BES with the simulation model to find effective investment strategies for the employee under different cases and observed the sharpe ratio and distributions of surplus and deficit generated by the investment strategies. Four cases, considering different weight θ and different frequency of downside risk measurement, were developed, and for each case, BES was executed five times with randomly generated initial populations. The BES was executed using a Linux platform, a Gentoo Linux 2.6.11 operation system, and the CPU is Intel Xeon CPU 3.00 GHz with 1024 KB cache size memory. The computation in the simulation model was quite time consuming; it took BES about 5,400 s to finish an execution for each case.

The first case investigates the asset allocation and contribution rate on the condition where $\theta = 1.0$ and the downside risk is checked every five years. The proportions of the asset allocations and the contribution rate produced by BES for each period are presented in Table 1. The results show that we need to contribute close to 16% of the annual salary to reach the 80% income replacement rate. Also, the results illustrate that we should hold more assets in consols, P_{k2}, at the start of the term and then gradually shift them to index-linked gilts, P_{k3}. We should hold more short-bonds, P_{k1}, in the last period in order to reduce liquidity risk. Furthermore, it is found that the index-linked gilts is the most important asset because it is most correlated to the concept of the target liability, salary and inflation rate; conversely, equities, P_{k4}, is the least important asset because it is least correlated to the target liability.

Case 2 investigates the condition where $\theta = 1.0$ and the downside risk is checked every year. The proportions of the asset allocations in Table 2 shows that index-linked gilts, P_{k3}, is still the most important asset; however, we should hold more equities, P_{k4}, at the start of the term for the purpose of increasing the fund value quickly to avoid the downside risk every year. This, as expected, reveals a fact that when investors change their measurement of the downside risk from every five years to every year, the portfolio composition will change towards more risky investments (more equities held in the portfolio) and the contribution rate will be decreased.

Table 1
Optimal asset allocation and contribution rate for Case 1.

Time k	1	6	11	16	21	26	31	36
P_{k1}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0092	0.0303	0.2499
P_{k2}	1.0000	0.4227	0.3021	0.1783	0.1534	0.1208	0.0900	0.0000
P_{k3}	0.0000	0.5097	0.6848	0.8046	0.8466	0.8642	0.8797	0.7295
P_{k4}	0.0000	0.0675	0.0131	0.0171	0.0000	0.0057	0.0000	0.0255
Contribution rate = 0.1559 = 15.59 %								

Table 2
Optimal asset allocation and contribution rate for Case 2.

Time k	1	6	11	16	21	26	31	36
P_{k1}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2411
P_{k2}	0.8182	0.3331	0.2333	0.1350	0.1294	0.0996	0.0858	0.0000
P_{k3}	0.0000	0.4466	0.6507	0.7811	0.8311	0.8649	0.9045	0.7361
P_{k4}	0.1818	0.2203	0.1160	0.0839	0.0395	0.0355	0.0097	0.0228
Contribution rate = 0.1494 = 14.94 %								

Table 3
Optimal asset allocation and contribution rate for Case 3.

Time k	1	6	11	16	21	26	31	36
P_{k1}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2645
P_{k2}	0.6026	0.2705	0.1807	0.1008	0.1106	0.0786	0.0695	0.0000
P_{k3}	0.0000	0.3974	0.6310	0.7688	0.8239	0.8709	0.9245	0.7304
P_{k4}	0.3974	0.3321	0.1883	0.1304	0.0654	0.0506	0.0060	0.0051
Contribution rate = 0.1458 = 14.58 %								

Table 4
Optimal asset allocation and contribution rate for Case 4.

Time k	1	6	11	16	21	26	31	36
P_{k1}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3417
P_{k2}	0.0000	0.0797	0.0000	0.0000	0.0260	0.0000	0.0000	0.0000
P_{k3}	0.0000	0.0000	0.2899	0.4956	0.6281	0.7907	0.9449	0.6583
P_{k4}	1.0000	0.9203	0.7101	0.5044	0.3458	0.2093	0.0551	0
Contribution rate = 0.1212 = 12.12 %								

Case 3 investigates the condition where θ is reducing from 1.0 to 0.1 and the downside risk is checked every five years. Since the importance of the asset–liability matching at the terminal date is low, an appropriate investment strategy should hold more risky assets in order to increase investment returns quickly to reduce the downside risk during the accumulation period. The generated proportions of the asset allocations in Table 3 confirm this expectation that we should hold more equities, P_{k4}, in the first few periods of the term.

Case 4 investigates the condition where the frequency of measurement for the downside risk is reduced from every five years to every year in the case of $\theta = 0.1$. This means that we focus almost entirely on reducing the downside risk during the accumulation period. Therefore, we must speed up the accumulation of the pension fund in order to reduce the downside risk. Table 4 presents the generated proportions of the asset allocations and the contribution rate. It shows that the majority of the assets held during the early periods is in equities, P_{k4}, and they are switched to index-linked gilts, P_{k3}, and short-bonds, P_{k1}, to reduce the volatility in the later periods of the term.

In order to investigate the performance of the investment strategies generated in the previous four cases, we calculate the annual investment return, the Sharpe ratio, and the terminal residuals of deficit and surplus for each strategy. The deficit residual includes mean, standard deviation and conditional tail expectation at 5% ($CTE_{5\%}$) of the shortfall between the pension fund value and the targeted pension liability, and the surplus residual includes mean, standard deviation and conditional tail expectation at 5% ($CTE_{5\%}$) of the positive surplus between the pension fund value and the targeted pension liability. Table 5 presents the calculated results for the investment strategies generated under the four cases. As discussed above, we will engage in riskier investments if we measure the level of the downside risk more frequently during the accumulation period and lower the importance of the asset–liability matching at the terminal date. This means that we will hold riskier assets as we progress from the objective stated in Case 1 to that in Case 4; Table 5 shows that riskier investments will increase the annual average return (from 8.6% to 9.92%) and decrease the requirement of the contribution rate (from 15.59% to 12.12%). However, Table 5 also shows that we will have larger deficits with risky investment. For example, the value of deficit as represented by $CTE_{5\%}$ in Case 4 is much larger than those in the other cases. This means that we may have a terminal fund that is much lower than the target liability at retirement date. On the other hand, the value

Table 5
Performance of different investment strategies.

Case	θ	Measurement of downside risk	Contribution rate	Annual average return	Sharpe ratio	Deficit residual			Surplus residual		
						μ	σ	$CTE_{5\%}$	μ	σ	$CTE_{5\%}$
1	1	Every 5 years	15.59%	8.60%	1.309	12.9	12.8	52.44	12.0	13.1	53.35
2	1	Every year	14.94%	8.90%	1.401	13.7	12.5	51.18	14.2	14.5	55.04
3	0.1	Every 5 years	14.58%	9.02%	1.454	14.0	12.6	51.54	15.3	16.0	59.19
4	0.1	Every year	12.12%	9.92%	1.787	18.7	16.7	68.96	32.0	37.9	118.21

of surplus $CTE_{5\%}$ (118.21) in Case 4 is much larger than those of the other cases. This explains the better annual average return (9.92%) and the lower required contribution rate (12.12%).

5. Summary

This paper proposed a simulation optimization approach to analyze and solve the problems of portfolio selection by applying multi-period asset allocation for a practical objective function, considering both asset–liability matching at the retirement date and the frequency of checking downside risk. Computational results showed that with the plausible simulation of future predictive returns, this simulation optimization approach, using a powerful optimization algorithm (evolution strategies), is able to find promising multi-period asset investment strategies that avoid the disadvantage of being highly sensitive to the single-point forecast. In addition, computational results showed that the promising multi-period investment strategies revealed useful information, which is able to help fund managers or investors achieve a higher average investment return or a lower level of volatility under different conditions. Furthermore, with regular monitoring of downside risk, risk-seeking fund managers or investors are able to further enhance their investment performance.

Since simulation techniques have been a commonly used tool for financial analysis, the proposed simulation optimization approach can be easily applied to solve other financial problems. Computational burden may be a barrier for researchers and practitioners to apply this simulation optimization approach, so finding a way to improve efficiency of the approach is worthwhile.

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