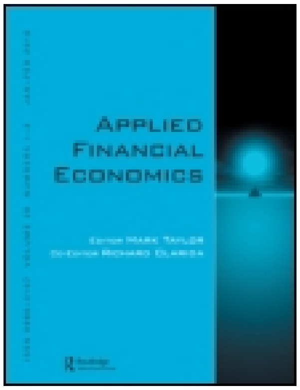

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Applying simulation optimization to dynamic financial analysis for the asset-liability management of a property-casualty insurer

Tzu-Yi Yu [a], Chenghsien Tsai [b], Hsiao-Tzu Huang [c] & Chuen-Lung Chen [c]

[a] Department of Information Management, National Chi Nan University, Puli, Nantou, Taiwan, ROC

[b] Department of Risk Management and Insurance, National Chengchi University, Taipei, Taiwan, ROC

[c] Department of Management Information System, National Chengchi University, Taipei, Taiwan, ROC

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Applying simulation optimization to dynamic financial analysis for the asset–liability management of a property–casualty insurer

Tzu-Yi Yu[a], Chenghsien Tsai[b,*], Hsiao-Tzu Huang[c] and Chuen-Lung Chen[c]

[a]*Department of Information Management, National Chi Nan University, Puli, Nantou, Taiwan, ROC*
[b]*Department of Risk Management and Insurance, National Chengchi University, Taipei, Taiwan, ROC*
[c]*Department of Management Information System, National Chengchi University, Taipei, Taiwan, ROC*

The Dynamic Financial Analysis (DFA) system is a useful decision-support system for the insurer, but it lacks optimization capability. This article applies a simulation optimization technique to a DFA system and use the enhanced system to search an Asset–Liability Management (ALM) solution for a Property–Casualty (P&C) insurance company. The simulation optimization technique used herein is a Genetic Algorithm (GA), and the optimization problem is a constrained, multi-period asset allocation problem that takes account of insurance liability dynamics. We find that coupling a DFA system with simulation optimization results in significant improvements over the search method currently available to the DFA system. The results were robust across random number sets. Furthermore, the resulting asset allocations changes with the asset–liability setting in a way that is consistent with the differences in the settings. Applying simulation optimization to a DFA system is therefore promising.

I. Introduction

A tool capable of assessing both the liability and asset risks is essential for sound management of a Property–Casualty (P&C) insurer. The Dynamic Financial Analysis (DFA) system can be such a tool. A company-wide DFA system can simulate the distribution of an insurer's surplus/equity at some point of time in the future under various assumptions about the insurer's underwriting and investment strategies, underwriting outcome and evolution of financial markets. Alternative underwriting and/or investment strategies can then be evaluated based on how they affect the insurer's future surplus distribution. For instance, an insurer may employ a DFA system to assess investment strategies by examining the impacts of alternative strategies on the surplus distribution over a target time horizon. The DFA system can therefore help managers make investment and business decisions in a comprehensive

*Corresponding author. E-mail: ctsai@nccu.edu.tw

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way and implement genuine Asset–Liability Management (ALM).

The construction of a DFA system for the P&C insurance company dates back to about two decades ago. Insurance and actuarial scholars started conceptual discussions in the late 1980s (Pentikainen, 1988; Taylor and Buchanan, 1988; Coutts and Devitt, 1989; Paulson and Dixit, 1989; Taylor, 1991). The British Institute of Actuaries Working Party on Insurance Solvency and actuaries soon developed P&C insurance company simulation models that could be used to evaluate the solvency of a company (Daykin *et al.*, 1989, 1994; Daykin and Hey, 1991). The Casualty Actuarial Society (CAS) of the US embarked on a long-term, multi-stage project entitled 'Dynamic Financial Analysis' in the mid-1990s. The potential of DFA was demonstrated by Cummins *et al.* (1999) in which the scenario analysis conducted using a simple cash flow model outperformed regulatory Financial Analysis Solvency Tools (FAST) and risk-based capital requirements in predicting solvencies of US P&C insurers. D'Arcy and Gorvett (2004) utilized a DFA system to investigate whether an optimal growth rate exists for a P&C insurer.

The DFA system, albeit powerful, tells us only which proposed strategy is better. It cannot tell us what the optimal strategy is. A DFA system generates surplus distributions given users' input about initial positions and strategies. It does not have the mechanism/algorithm to search for the optimum. Managers therefore have to educate guesses on what the optimal strategy may look like and employ the trial-and-error method to shoot for a good strategy. Trying all possible strategies to seek for the optimum is infeasible due to the large number of decision variables. A DFA system without an optimization mechanism is therefore incapable of helping managers maximize the shareholders' value. The goal of this article is to illustrate how simulation optimization may enhance the power of the DFA system to be a better decision-making tool.

Simulation optimization can be regarded as the process of determining the values of controllable input variables that optimize the values of stochastic output variables generated by a simulation model. The controllable input variables, also called decision variables, in the case of a DFA system may include asset allocation, capital structure, business growth, business allocation, reinsurance arrangement, etc. The output variables, also called the response variables, are usually a function of simulated surplus, insolvency probability and other concerns of the board (e.g. meeting the capital requirements). The simulation model itself (a DFA system in this article) acts as a complex function mapping controllable input values to response values. The simulation optimization problem can therefore be characterized as a stochastic search over a feasible exploration region (Keys and Rees, 2004).[1]

We perform simulation optimization by integrating a DFA system with one optimization technique and use the integrated system to conduct ALM analysis for a P&C insurance company. Our DFA system contains four asset classes (cash, bonds, stocks and short-term investments) and two types of insurance businesses (long- and short-tail businesses) to capture the essence of the insurer's operations. Although the DFA system is simple when compared with commercial packages, it is complex enough to preclude one from finding optimal decision variables analytically.[2] We therefore resort to the techniques used in simulation optimization. We choose one of the most popular Evolutionary Algorithms (EAs), the Genetic Algorithms (GAs), as the optimization algorithm for our DFA system. The optimal ALM problem is formulated as searching multi-period asset allocations with short-sale constraints while taking account of insurance business dynamics to maximize an objective function incorporating expected discounted surplus and insolvency probability.

Our optimization problem is not merely a short-sale-constrained, multi-period asset allocation problem but rather than an asset–liability 'matching' problem. The loss ratios of insurance businesses are correlated with asset returns, and the claim payments and premium incomes involve selling and buying assets. The explicit consideration of insurance business cash flows between times of asset allocations through simulations differentiates this article from the vast applications of GA to the portfolio construction problem. The majority of these applications (e.g. Lin and Yang, 2003; Venugopal *et al.*, 2004; Oh *et al.*, 2005; Lai and Li, 2008; Lin and Liu, 2008) were within the single-period framework as the

[1] The feasible region is defined by the practical limits on the ranges of the controllable inputs. Examples of practical limits include short-sale constraints and upper bounds on portfolio weights faced by most financial institutions.

[2] The impossibility is due to three reasons. First, the system contains several types of stochastic processes. The function represented by the simulation model is thus unknown. Second, the variations of the outcomes generated by the system are significant, heterogeneous over the feasible region, and not normally distributed. Third, the system has 12 controllable variables over real intervals. The feasible region is rather large.

famous mean–variance analysis was. Some papers such as Chan *et al*. (2002) and Yang (2006) extended the framework from single period to multiple periods using scenario trees. Few papers (e.g. Zhang and Zhang, 2009) analysed the multi-period asset allocation problem using simulations which are often more flexible and general than scenario trees.[3] We found only one paper (Consiglio *et al*., 2006) that employs simulations to consider (insurance) liabilities when making asset allocation decisions. This ALM problem is however analysed in the context of one specific type of life insurance products without disclosing the optimization algorithm. This article is the first one that applies GA to a company-wide simulation model for the ALM of the P&C insurer. Since very few companies use scenario trees in their decision-making while more and more companies and regulations (e.g. the Basel II of the Basel Committee on Banking Supervision, the Solvency II of the Committee of European Insurance and Occupational Pensions Supervisors, the CAS of the US, etc.) adopt and/or promote the uses of simulations, our application not only makes a contribution to the literature and but also has implications to the industry.

Our results show that applying simulation optimization to the DFA system is promising. Our GA introduces a significant improvement over the search method currently available to a DFA system. The resulting 'optimal' asset allocations look reasonable without extreme positions.[4] The advantages of our proposed application can be further illustrated by the robustness across different random numbers and asset–liability settings. The value of the objective function is insensitive to the generated random numbers. Different asset–liability settings result in different optimal asset allocations, as expected, and the changes in the optimal solutions are comprehensible with respect to the differences in the settings. Simulation optimization seems to be capable of enhancing the usefulness of DFA for P&C insurers.

The rest of this article is organized as follows. Section II describes our DFA system and the ALM problem of the insurer. Section III starts with an introduction to simulation optimization, followed by a brief review on GAs and detailed descriptions of our proposed algorithm. The application results are discussed in Section IV. We compare the results from the only search method currently available to the DFA system with those from our simulation optimization. Section V concludes this article.

II. The DFA System

Financial and insurance markets

In this section, we set up five types of markets and specify their stochastic processes, respectively. We assume that the process for the 1-year spot rate at time t, $r(t)$, is

$$\mathrm{d}r(t) = q(m - r(t))\mathrm{d}t + \bar{\sigma}_r\sqrt{r(t)}\mathrm{d}\bar{W} \tag{1}$$

where t is zero or a positive integer, m stands for the long-term average of spot rates, q reflects the speed of mean reverting $(0<q<1)$, $\bar{\sigma}_r = [v \quad 0 \quad 0 \quad 0 \quad 0]$ and $\mathrm{d}\bar{W} = [\mathrm{d}W_r, \mathrm{d}W_S, \mathrm{d}W_{STI}, \mathrm{d}W_{LR(L)}, \mathrm{d}W_{LR(S)}]'$. $\mathrm{d}\bar{W}$ represent the differentials of five-dimension Wiener processes including the processes of the 1-year spot rate (r), the equity index (S), the short-term investments (STI), the loss ratio of long-tail insurance liabilities ($LR(L)$) and the loss ratio of short-tail lines ($LR(S)$). It has a correlation matrix $\Re$ specifying the correlations among the Wiener processes. The mapping from short rates to Treasury bond prices has been derived in Cox *et al*. (1985): the price at time t of a default-free zero-coupon bond that pays \$1 at time T equals

$$P_T(t, r) = A_0(T - t)e^{-B(T-t)r} \tag{2}$$

where T is a positive integer, $T \geq t$, $B(x) = \frac{2(e^{rx}-1)}{(r+q)(e^{rx}-1)+2\gamma}$,

$$A_0(x) = \left[\frac{2\gamma e^{\frac{x}{2}(q+\gamma)}}{(q+\gamma)(e^{rx}-1)+2\gamma}\right]^{\frac{2qm}{\sigma^2}} \quad \text{and}$$
$$\gamma = \sqrt{q^2 + 2v^2}$$

The equity index is assumed to evolve according to the following interest-rate-adjusted geometric Brownian motion process:

$$\frac{\mathrm{d}S(t)}{S(t)} = (r(t) + \pi_S)\mathrm{d}t + \overline{\sigma}_s\,\mathrm{d}\overline{W} \tag{3}$$

where the constant parameter π_S denotes the risk premium on the stock index investment, and $\overline{\sigma}_S = \begin{bmatrix}0 & \sigma_S & 0 & 0 & 0\end{bmatrix}$. We assume that the short-term investments follow a geometric Brownian

[3] The articles using scenario trees usually considered several periods with dozens of scenarios only, while those utilizing simulations allow for more periods with thousands of scenarios.

[4] We are aware that simulation optimization is a heuristic search method. The existence of the optimal solution is not proven, and there is no verification theorem to show that the resulted solution from simulation optimization is at least as good as all other solutions. The word 'optimal' is used loosely in this article to mean 'the best known solution'.

motion

$$\frac{\mathrm{d}STI(t)}{STI(t)} = \mu_{STI}\,\mathrm{d}t + \overline{\sigma}_{STI}\,\mathrm{d}\overline{W} \qquad (4)$$

where the constant parameter μ denotes the expected return of the short-term investment per period with continuous compounding, and $\overline{\sigma}_{STI} = \begin{bmatrix} 0 & \sigma_{STI} & 0 & 0 & 0 \end{bmatrix}$.

In the insurance markets, insurers underwrite both long-tail and short-tail noncatastrophe businesses. We follow Harrington and Niehaus (1999) in assuming that the loss ratios are normally distributed. More specifically, we assume that the loss ratio of the long-tail businesses follows:

$$LR(L)(t) = \mu_{LR(L)} + \overline{\sigma}_{LR(L)} \times \mathrm{d}\overline{W} \qquad (5)$$

where $\overline{\sigma}_{LR(L)} = \begin{bmatrix} 0 & 0 & 0 & \sigma_{LR(L)} & 0 \end{bmatrix}$.[5] The loss ratio of the short-tail lines has a similar process to Equation 5 but with a different volatility $\overline{\sigma}_{LR(S)} = [0 \quad 0 \quad 0 \quad 0 \quad \sigma_{LR(S)}]$.

The parameters of the above five models are specified in Table 1.

The starting value of the short-term interest rate is 6%. Furthermore, the correlation matrix $\Re$ is specified as follows.[6]

	$\mathrm{d}W_S$	$\mathrm{d}W_r$	$\mathrm{d}W_{LR(L)}$	$\mathrm{d}W_{STI}$
$\mathrm{d}W_S$	1	−0.31	−0.19	0.36
$\mathrm{d}W_r$	−0.31	1	−0.004	−0.03
$\mathrm{d}W_{LR(L)}$	−0.19	−0.004	1	−0.47
$\mathrm{d}W_{STI}$	0.36	−0.03	−0.47	1

Table 1. Asset–liability setting

Model parameters			
Short rate	$m = 6\%$	$q = 0.3$	$v = 2\%$
Equity index	$\pi_S = 6\%$	$\sigma_S = 20\%$	
Short-term investments	$\mu_{STI} = 15\%$	$\sigma_{STI} = 35\%$	
Loss ratio (long)	$\mu_{LR(L)} = 75\%$	$\sigma_{LR(L)} = 30\%$	
Loss ratio (short)	$\mu_{LR(S)} = 80\%$	$\sigma_{LR(S)} = 25\%$	

The dynamics of the insurer's financial positions

Suppose that a newly established P&C insurer starts to underwrite insurance businesses with the surplus (i.e. shareholders' equities) of $IS(0)$ million dollars. It receives premiums of $IP(0)$ million dollars in cash at the beginning of year 1 (i.e. at time 0) with $B(0)$ $(0 \le B(0) \le 1)$ being the proportion of the businesses in long-tail lines. To underwrite these businesses, the insurer incurs and pays underwriting expenses in cash and upfront. The underwriting expense ratios of the long- and short-tail businesses are assumed to be $\mathrm{Exp}(L)$ and $\mathrm{Exp}(S)$ respectively, where both ratios are positive but smaller than one.

The remaining cash and the initial surplus are then invested in cash, Treasury bonds, equity index and short-term investment with the proportion vector $\overline{\theta}(0)$, where $\overline{\theta}(0) = [\theta_1(0) \quad \theta_2(0) \quad \theta_3(0) \quad \theta_4(0)]'$, $\theta_1(0)$, $\theta_2(0)$, $\theta_3(0)$ and $\theta_4(0)$ are the proportion of the wealth invested in cash, equity index, Treasury bonds and short-term investment, respectively. We assume that $\sum_{i-1}^{4} \theta_i(t) = 1$. We further assume that $\theta_i(t) \ge 0$ since insurers are almost always subject to short-sale constraints from regulation. The maturity of invested bonds ranges from 1 year to 15 years, and the invested proportions are assumed to be even across the maturities for the sake of simplicity. Assuming that the fair value of the reserves equal to the premiums written net of expenses, we get the balance sheet of the insurer at the beginning of year 1 shown in Table 2.

At the end of the year, investment returns and loss ratios are realized according to the stochastic models in the Section 'Financial and insurance markets'.[7] To account for loss development and business growth, we assume that the insurer's long-tail businesses grow $G(L)$ annually and have a 10-year development period with a loss development function $D_L(\mathrm{d}y)$, where $\mathrm{d}y = 1, 2, 3, \ldots,$ or 10, $0 \le D_L(\mathrm{d}y) \le 1$ and $\sum_{\mathrm{d}y-1}^{10} D_L(\mathrm{d}y) = 1$. The short-tail businesses have an annual growth rate of $G(S)$ and a 3-year development period with a loss development function $D_S(\mathrm{d}y)$, where $\mathrm{d}y = 1, 2$ or 3, $0 \le D_S(\mathrm{d}y) \le 1$ and $\sum_{\mathrm{d}y-1}^{3} D_S(\mathrm{d}y) = 1$. Simulated loss ratios represent a multiple of the ultimate loss divided by the premiums written, where the ultimate losses for the businesses written in any given year are defined as the total payments across all development years paid for the

[5] The normal distribution might be reasonable for aggregate losses arising from a population with a known mean and independent individual losses (Bustic, 1994). Eling and Holzmüller (2008) also modelled the sum of small claims using a normal distribution.

[6] The correlation coefficients are estimated using the historical data on S&P 500 index, indexes of All Publicly Traded Real Estate Investment Trusts (REITs), Treasury-Bill rates and the loss ratios published in *Best's Aggregates and Averages*. The sampling period is from 1972 to 1999. $\mathrm{d}W_{LR(S)}$ is not in the matrix because we assume that the loss ratio of short-tail businesses is independent of other processes. We use REITs, a securitization investment tool, as a subjective proxy to the insurer's short-term investments.

[7] We assume that the return on cash is $r(t)$.

Table 2. Balance sheet of the insurer at the beginning of year 1

Assets		Liabilities and surplus	
Cash	$\$(\theta_1(0) * \text{Total assets})$	Liabilities of long-tail businesses	$\$B(0) * IP(0) * (1 - \text{Exp}(L))$
Stocks	$\$(\theta_2(0) * \text{Total assets})$	Liabilities of short-tail businesses	$\$(1 - B(0)) * IP(0) * (1 - \text{Exp}(S))$
Treasury bonds	$\$(\theta_3(0) * \text{Total assets})$		
Short-term investments	$\$(\theta_4\ (0) * \text{Total assets})$	Surplus	$\$IS(0)$
Total assets	$\$(IS(0) + B(0) * IP(0) * (1 - \text{Exp}(L)) + (1 - B(0)) * IP(0) * (1 - \text{Exp}(S)))$	Total liabilities and surplus	$\$(IS(0) + B(0) * IP(0) * (1 - \text{Exp}(L)) + (1 - B(0)) * IP(0) * (1 - \text{Exp}(S)))$

written businesses.[8] More specifically, the ultimate losses for the businesses written in year t equal

$$\frac{\text{Simulated loss ratio} \times \text{Premiums written in year } t}{\text{An adjustment factor}}$$

The loss payment in development year dy for the businesses written in year t is then equal to the t-th year's ultimate loss times $D_L(\mathrm{d}y)$ or $D_S(\mathrm{d}y)$.

To pay losses, the insurer sells assets proportionally. Specifically, we assume that the insurer sells each type of invested assets, including cash, Treasury bonds, stocks and short-term investment by the proportion of the asset's market value to the total asset's value. The asset allocation of the insurer will thus be unaffected by the sale of assets. We then deduct the amount of losses paid from reserves and attain the year-end balance sheet for year 1.[9]

At the beginning of year 2, the insurer underwrites $IP(1)$ million dollars of businesses, pays underwriting expenses and invests the net amount in the four asset classes with the proportion vector $\overline{\theta}(1)$.[10] Procedures similar to the previous paragraphs are then repeated for 25 years, or, repeated until the insurer becomes insolvent. The insurer is deemed insolvent whenever its surplus ($IS(t)$), the difference between the market value of assets and the fair value of reserves, is smaller than zero.

The parameters of the representative insurer are set as follows: $IS(0) = 120$, $IP(0) = 200$, $B(0) = 50\%$, $\text{Exp}(L) = 25\%$, $\text{Exp}(S) = 20\%$, $G(L) = 5\%$, $G(S) = 4\%$ and the following.[11]

dy	1	2	3	4	5	6	7	8	9	10
$D_L(\mathrm{d}y)$ (%)	50	30	10	5	3	1	0.5	0.3	0.1	0.1
$D_S(\mathrm{d}y)$ (%)	80	15	5							

The above asset–liability settings including the parameters associated with the financial and insurance markets as well as the parameters of the representative insurer are chosen so that the insurer has an 'adequate' insolvency probability to facilitate subsequent analyses. The different asset–liability settings used to illustrate the robustness of our proposed application is shown in the Appendix.

The ALM problem

We assume that the insurer's objective is to maximize a utility function over the time horizon $[0, H]$. The utility function consists of two components, expected discounted surplus and ruin probability, to reflect the return-risk tradeoff faced by

[8] The multiple, also called the adjustment factor in this article, is to account for the effect of growth and time value of money. For an insurer that does not have growth in premiums written, calendar-year loss ratios are equal to the ratio of the ultimate losses to the premiums written. For a growing insurer, however, calendar-year loss ratios will be less than the ratio of the ultimate losses to the premiums written because the denominators of loss ratios grow with time. Furthermore, time value of money should be considered.

[9] Reserves might be smaller than loss payments in extreme cases. We set reserves as zero in these cases and deduct the deficit from surplus.

[10] Notice that $IP(t+1) = IP(t) * B(t) * (1 + G(L)) + IP(t) * (1 - B(t)) * (1 + G(S))$.

[11] After a simple spreadsheet work, we obtain an adjustment factor of 0.9449 for the long-tail businesses given a growth rate of 5%, a discount rate of 7% and the specified $D_L(\mathrm{d}y)$. The adjustment factor for the short-tail businesses is 0.9905 given a growth rate of 4%, a discount rate of 7% and the specified $D_S(\mathrm{d}y)$.

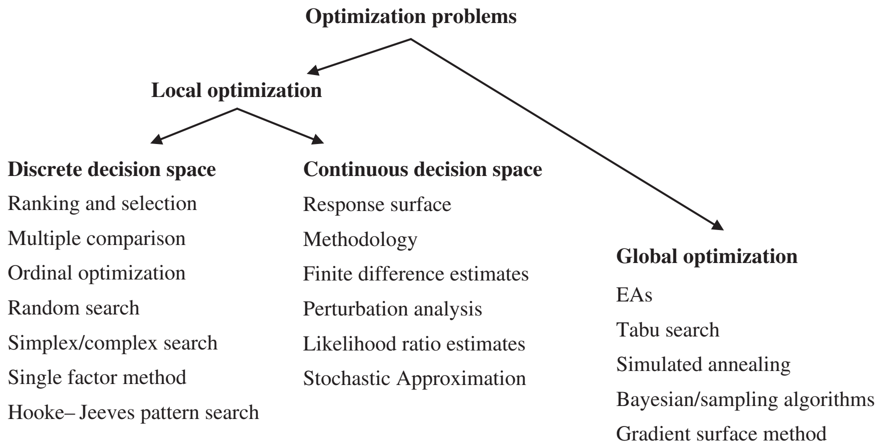

Fig. 1. Classification of optimization methodologies

the insurer. More specifically, the optimization problem of the insurer is

$$\max_{\overline{\theta}(t)} \frac{\sum_{i=1}^{I}\left[\frac{1}{H}\left(\sum_{t-1}^{H}\frac{IS(t)}{(1+s)^t}\right)\right]}{I} - k(ruin\_probability - x) \qquad (6)$$

where I is the number of the simulated paths in which no insolvency occurs, s is a constant chosen subjectively by the insurer to reflect time preference on future surplus $IS(t)$, k is also a constant chosen by the insurer to indicate the relative importance of excessive insolvency probability to expected discounted surplus and x is the tolerable insolvency probability of the insurer. The decision variables used to maximize the objective function are contained in the asset allocation vector, $\overline{\theta}(t)$.

Since both terms of the objective function involve insurance liabilities, the optimization problem is not merely an asset allocation problem but an asset–liability 'matching' problem.

In the following simulation, we set H at 25 years and the number of simulated paths at 5000. The discount rate for future surplus per period s is assumed to be 3%, k is chosen to be 4×10^{10} and $x = 2\%$. Without losing the generality of multi-period asset allocations, we simplify the optimization problem from 25 years to four periods for the sake of computation time. More specifically, the insurer makes asset allocation decisions at $t = 0$, 6, 12 and 18, and keeps the allocation the same as the previous year's at all other times.[12] The number of controllable variables is thus reduced to 12. A single-period asset allocation problem will have only three controllable variables, which may be solved using simpler techniques.

III. The Simulation Optimization Techniques

An introduction to optimization via simulation

To solve the optimization problem set up in Section II, we make use of simulation optimization techniques. Various simulation optimization techniques have been proposed. Several survey papers such as Fu (1994), Andradottir (1998) and Tekin and Sabuncuoglu (2004) provided comprehensive coverage on the foundations, theoretical developments and applications of these techniques. Existing techniques can be classified into two types: local optimization and global optimization. Local optimization techniques are further classified in terms of discrete and continuous decision spaces.[13] Figure 1, made from Tekin and Sabuncuoglu (2004), demonstrates the aforementioned classification.

The major difference between local and global optimization techniques lies in the assumption about the shape of the response value surface. Local optimization techniques are not suitable for those cases in which the function represented by the simulation model is complex and multimodal. Without an effective method to find good initial solutions, these algorithms are usually trapped in a local optimum and generate poor solutions. Global optimization techniques, such as simulated annealing

[12] The last period starting from the fourth asset allocation is therefore 7 years.
[13] In a discrete space, decision variables take a discrete set of values such as the number of machines in a system. The feasible region in a continuous space, on the other hand, consists of real-valued decision variables such as the release time of factory orders.

and EAs, are developed to help a search escape from the local optimum and produce better solutions.

Among the global optimization techniques, we chose EAs for the DFA system. Using EAs in simulation optimization is on the increase lately because they require no restrictive assumptions or prior knowledge about the shape of the response surface (Back and Schwefel, 1993). EAs work on a population of solutions in such a way that poor solutions become extinct while good solutions evolve to reach for the optimum. The most popular EAs are GAs, Evolutionary Programming (EP) and Evolution Strategies (ES). EP and ES have not yet been widely used in simulation optimization, but GAs have been successfully applied to the optimization problems arising in complex manufacturing systems (Tekin and Sabuncuoglu, 2004). We therefore chose to employ a GA to be the optimization technique for the DFA system. The following section gives a brief introduction on GAs and then describes our GA in detail.

Genetic algorithms

The search procedure of GAs combines reproduction and recombination to mimic the process of natural evolution. The solution space of the problem is viewed as the environment of evolution. A solution of the problem is a member of a species in the environment. A generation of the species is presented as a population of solutions. The objective value of a solution is a measure of its fitness to the environment. The better the fitness of the solution, the higher the probability that the solution can be chosen as a parent to produce new solutions (offspring) for the next population (generation). Genetic operators (usually crossover and mutation) are applied to the chosen parents to produce offspring. As this process continues for generations, the fitness of the members improves.

Based on this explanation, the procedure of a basic GA can be described as follows.[14] Let $S(t)$ denote the population in the t-th generation, $s_i(t)$ the i-th member in $S(t)$, $f(s_i(t))$ the fitness of $s_i(t)$, $Totfit$ the sum of $f(s_i(t))$ in $S(t)$, *popsize* the population size and *maxgen* the maximum number of generations for convergence. Then a GA usually has the following steps.

Step 1: Generate an initial population, $S(t)$, where $t = 0$.

Step 2: Calculate the fitness value for each member, $f(s_i(t))$, in population $S(t)$.

Step 3: Calculate the selection probability for each member, which is defined as $f(s_i(t))/Totfit$.

Step 4: Select a pair of members (parents) randomly according to the selection probability.

Step 5: Apply genetic operators to the parents to produce the offspring for the next population, $S(t+1)$. If the size of the new population is equal to *popsize*, then go to Step 6; otherwise, go to Step 4.

Step 6: If the current generation, $t+1$, is equal to *maxgen*, then stop; else go to Step 2.

According to the above basic procedure, a GA must consider the following factors: (A) representation of a solution, (B) initial population, (C) selection probability, (D) genetic operators, (E) termination criterion and (F) three parameters: population size, crossover rate and mutation rate. In the following, each of these factors and parameters will be discussed in detail for a GA to search the optimal ALM solutions through our DFA system.

Representation of a solution. A solution for GA application is usually represented by a row vector. The value of an element in the vector refers to the allocation to an asset in a period. The number of elements in the vector depends upon the number of investable assets considered in a period and the number of periods considered in a problem. Since four assets are considered in four periods with the constraint that the sum of the allocations equals one, 12 elements are included in the vector in the current application.

Initial population

The initial population of our GA is randomly generated as most applications of GAs are. Since the value of an element in the vector is in the range of [0, 1], we first generate three random numbers from the uniform [0, 1] distribution. If the sum of the three generated elements is greater than one, then the elements will be multiplied by 0.9 consecutively until their sum is less than or equal to one. We call this procedure a feasibility-keeping procedure. The purpose of the procedure is to ensure that the fourth element will not be negative and violate the short-sale constraint. Note that the feasibility-keeping procedure has to be implemented in all four periods in each vector.

[14] The following descriptions are drawn from Chen *et al.* (1995, 1996).

Selection probability

The selection probability of a member in a population should generally project the performance measure of the member. A member with a better fitness in a population would usually have a higher probability of being selected. Since the candidate problem is to find asset allocations that maximize a specified utility function, the value of the utility function is used to measure the fitness of asset allocations. The larger the value of the utility function, the higher the probability the set of allocations will be selected. The following procedure is an application of the well-known roulette-wheel selection scheme for calculating the selection probability of a member in a population.

Step 1: Calculate the fitness $f(i)$ for each member in the population.

Step 2: Calculate the total fitness, $Totfit$, of all the members in the population.

Step 3: Calculate the selection probability for each member that is equal to $f(i)/Totfit$.

A complementary selection strategy (elitist strategy) is also considered in the current application. More specifically, the member with the best fitness value in each population will always survive and automatically become a member in the next generation. This is to preserve the best solution so that the search always covers certain good solution regions.

Genetic operators

Genetic operators are performed on the parents to generate offspring. Crossover and mutation are two common genetic operators of GAs.

Crossover. An effective crossover operator, BLX-0.5 (Eshelman and Schaffer, 1993), is used in this research. Two selected parents, vectors A and B, are given and denote the values of an element in A and B as x and y, respectively. The BLX-0.5 is implemented to x and y to produce a value z for the element in the offspring generated by A and B as follows.

Step 1: Let $\Delta = 0.5 * |y - x|$.

Step 2: Randomly generate z from the range of $(x - \Delta,\ y + \Delta)$ if $x < y$; let $x - \Delta = 0$ if $x - \Delta < 0$, and let $y + \Delta = 1$, if $y + \Delta > 1$. Otherwise, randomly generate z from the range of $(y - \Delta,\ x + \Delta)$; let $y - \Delta = 0$ if $y - \Delta < 0$, and let $x + \Delta = 1$ if $x + \Delta > 1$.

For example, let $(x_1, x_2) = (0.5, 0.2)$ be the values of the first two elements in A and let $(y_1, y_2) = (0.1, 0.8)$ be the values of the first two elements in B. The values of the first two elements, (z_1, z_2) of the offspring of A and B can then be generated as follows. Let $\Delta_1 = 0.5 * |y_1 - x_1| = 0.5 * |0.1 - 0.5| = 0.2$ and $\Delta_2 = 0.5 * |y_2 - x_2| = 0.5 * |0.8 - 0.2| = 0.3$. Then randomly generate z_1 from the range of $(y_1 - \Delta_1,\ x_1 + \Delta_1) = (0.1 - 0.2,\ 0.5 + 0.2) = (0,\ 0.7)$ and randomly generate z_2 from the range of $(x_2 - \Delta_2, y_2 + \Delta_2) = (0.2 - 0.3,\ 0.8 + 0.3) = (0,\ 1.0)$.

Mutation. When a solution is produced by crossover, a mutation operator is applied to the solution. Michalewicz (1996) developed a nonuniform mutation operator and showed that the operator outperformed other mutation operators. A non uniform mutation operator is applied in this GA application by the following procedure.

Step 1: Randomly select k elements out of the 12 elements in the solution, where $k = 1 + \mathrm{Int}[rnd * 12]$ and Int is an integer function.

Step 2: Apply the nonuniform mutation operator to each of the k elements. Let element i be one of the k elements and z_i^t the value of element i in the current generation t. The value of element i in generation $t+1$, z_i^{t+1}, is generated as follows: $z_i^{t+1} = z_i^t - \Delta(t, 1.0 - z_i^t)$ if $rnd < 0.5$; otherwise $z_i^{t+1} = z_i^t - \Delta(t, z_i^t)$, where $\Delta(t, v) = v * (1.0 - rnd^b)$, $b = (1.0 - t/T)^5$ and T is equal to $maxgen$ (the maximum number of generations for convergence). Note that $b = (1.0 - t/T)^5$ is approaching 0 when t is close to T; when b is approaching 0, (t, v) also approaches 0. Michalewicz (1996) pointed out that this property causes the nonuniform mutation operator to search uniformly the solution space initially (when t is small) and search the solution space locally at later stages. Also note that after applying the mutation operator to the solution, the feasibility-keeping procedure has to be implemented in the solution.

Population size, crossover rate and mutation rate. Population size ($popsize$) is the number of members generated in each generation. Crossover rate is the probability that a crossover operator applies to the chosen parents, and the mutation rate is the probability that a mutation operator applies to the offspring. The population size of 60, as used in all the examples in Michalewicz (1996), is also used in the current application. Both crossover rate and mutation rate are set to be one after several trial runs for the candidate problem.

Termination criteria

The maximum number of generations (*maxgen*) is the most widely used termination criterion for GAs. It is usually determined by trial and error. We found that our GA converged within 2000 iterations in all the trial runs in the current application. Therefore, *maxgen* is set to be 2000.

We are now in the position to present the procedure of applying our GA to the DFA.

Step 0: Let $popsize = 60$, $maxgen = 2000$, and $t = 0$.

Step 1: Generate initial population with twelve-element solutions (vectors) $\boldsymbol{V_i}$ ($i = 1, 2, \ldots, popsize$). For each solution, call *rnd* to generate an asset allocation to each of its elements and apply the feasibility-keeping procedure to satisfy the constraint.

Step 2: Calculate the fitness, $f(\boldsymbol{V_i})$, for each solution by conducting the DFA simulation with the asset allocation given in $\boldsymbol{V_i}$.

Step 3: Select two parent solutions based on their fitness. The selection probability of a parent solution $\boldsymbol{V_i}$ is $p(i) = f(\boldsymbol{V_i})/Totfit$.

Step 4: Apply crossover operator, BLX-0.5, to the selected parent solutions.

Step 5: Apply the nonuniform mutation operator to the solution generated in Step 4, and apply the feasibility-keeping procedure to maintain the feasibility of the solution.

Step 6: If the total number of offspring solutions generated is equal to *popsize*, go to Step 7; otherwise, go to Step 3.

Step 7: Let $t = t + 1$. If t is equal to *maxgen*, stop; otherwise go back to Step 2.

The optimization program is coded in C language and is executed using a Linux platform (the OS is the RedHat AS 3.0). The CPU is Intel Itanium 21.5 GHz. One of the features of this optimization program is the usage of the dynamic memory allocation. The required memory used is around 70 MB when running the program. The program is compiled using the Intel C/C++ compiler in the Linux System, yet it can also be compiled and executed using the Microsoft Window XP platform with Visual Studio C/C++ compiler. For 2000 iterations, this program took about 18 200 seconds to finish.

IV. Results

Results of a primitive search method

Without using the simulation optimization technique, the insurer has only a primitive search method appropriate in a single-period framework in hand and it works as follows. We first list all possible asset allocations using the grid size of 20%. Inserting these allocations into the DFA system and assuming that these allocations are kept to the end of the simulation, we then obtain the values of the objective function. This method is obviously incapable and ineffective, but it is the only method available to the DFA system before simulation optimization is introduced. The objective values along with corresponding average discounted surpluses and ruin probabilities are shown in Table 3.

Table 3 shows that the best asset allocation is $\vec{\theta}^*(t) = \begin{bmatrix} 0 & 0.4 & 0.4 & 0.2 \end{bmatrix}'$. It results in an objective function value of 1 137 275 044 with an average discounted surplus of \$881 275 044 and a ruin probability of 1.36%. Although the zero-cash allocation looks odd, it is reasonable because we did not consider liquidity in the model setting. Furthermore, new premiums can cover the loss payments in most cases. The runner-up is $\vec{\theta}(t) = \begin{bmatrix} 0.2 & 0.4 & 0.2 & 0.2 \end{bmatrix}'$ with an objective function value of 1 132 889 250, an average discounted surplus of \$876 889 250 and a ruin probability of 1.36%. This second best allocation generates a little bit less expected surplus, which is reasonable because the return on cash on average is smaller.[15] Number three is $\vec{\theta}(t) = \begin{bmatrix} 0.4 & 0.4 & 0 & 0.2 \end{bmatrix}'$ which results in an objective function value of 1 097 049 222, an average discounted surplus of \$873 049 222, and a ruin probability of 1.44%. The higher insolvency probability is probably because cash does not generate adequate returns. Number four is $\vec{\theta}(t) = \begin{bmatrix} 0 & 0.2 & 0.6 & 0.2 \end{bmatrix}'$. This asset allocation produces an objective function value of 1 022 434 859, an average discounted surplus of \$710 434 859 and a ruin probability of 1.22%. Although it generates the smallest ruin probability among all the asset allocations, it produces an inferior average discounted surplus compared to the top three allocations. The fifth winner is $\vec{\theta}(t) = \begin{bmatrix} 0 & 0.4 & 0.2 & 0.4 \end{bmatrix}'$ which results in an objective function value of 1 017 769 551, an average discounted surplus of \$1 225 769 551 and a ruin probability of 2.52%. Allocating more assets to higher-risk

[15] Remember that the return from cash is the 1-year short rate. Since the simulated yield curve is usually upward-sloping, the 1-year bond is smaller than longer-maturity bonds. The insolvency probability of the runner-up is the same as the number-one choice implies that the risk of the bond portfolio is small.

Table 3. The results of the primitive research method

Cash	Stock	Bond	Short-term investment	Value of the objective function	Average discounted surplus	Ruin probability
1	0	0	0	−72 581 897	351 418 103	0.0306
0.8	0.2	0	0	551 978 871	463 978 871	0.0178
0.8	0	0.2	0	−46 112 487	353 887 513	0.0300
0.8	0	0	0.2	552 303 659	552 303 659	0.0200
0.6	0.4	0	0	637 421 327	605 421 327	0.0192
0.6	0.2	0.2	0	570 938 385	466 938 385	0.0174
0.6	0.2	0	0.2	971 157 610	699 157 610	0.0132
0.6	0	0.4	0	12 156 152	356 156 152	0.0286
0.6	0	0.2	0.2	587 409 029	555 409 029	0.0192
0.6	0	0	0.4	336 229 618	840 229 618	0.0326
0.4	0.6	0	0	280 468 735	776 468 735	0.0324
0.4	0.4	0.2	0	664 738 139	608 738 139	0.0186
0.4	0.4	0	0.2	1 097 049 222	873 049 222	0.0144
0.4	0.2	0.4	0	589 926 635	469 926 635	0.0170
0.4	0.2	0.2	0.2	975 076 921	703 076 921	0.0132
0.4	0.2	0	0.4	865 202 808	1 017 202 808	0.0238
0.4	0	0.6	0	70 408 056	358 408 056	0.0272
0.4	0	0.4	0.2	614 605 036	558 605 036	0.0186
0.4	0	0.2	0.4	379 816 127	843 816 127	0.0316
0.4	0	0	0.6	−1 437 343 176	1 234 656 824	0.0868
0.2	0.8	0	0	−979 818 603	988 181 397	0.0692
0.2	0.6	0.2	0	331 768 042	779 768 042	0.0312
0.2	0.6	0	0.2	920 627 902	1 072 627 902	0.0238
0.2	0.4	0.4	0	707 867 321	611 867 321	0.0176
0.2	0.4	0.2	0.2	1 132 889 250	876 889 250	0.0136
0.2	0.4	0	0.4	996 985 281	1 220 985 281	0.0256
0.2	0.2	0.6	0	585 128 292	473 128 292	0.0172
0.2	0.2	0.4	0.2	1 002 693 284	706 693 284	0.0126
0.2	0.2	0.2	0.4	916 984 155	1 020 984 155	0.0226
0.2	0.2	0	0.6	−262 968 275	1 425 031 725	0.0622
0.2	0	0.8	0	120 775 989	360 775 989	0.0260
0.2	0	0.6	0.2	641 808 556	561 808 556	0.0180
0.2	0	0.4	0.4	423 394 768	847 394 768	0.0306
0.2	0	0.2	0.6	−1 377 259 696	1 238 740 304	0.0854
0.2	0	0	0.8	−4 699 438 942	1 764 561 058	0.1816
0	1	0	0	−3 230 097 737	1 249 902 263	0.1320
0	0.8	0.2	0	−881 219 802	990 780 198	0.0668
0	0.8	0	0.2	192 104 867	1 304 104 867	0.0478
0	0.6	0.4	0	351 750 776	783 750 776	0.0308
0	0.6	0.2	0.2	948 987 876	1 076 987 876	0.0232
0	0.6	0	0.4	608 774 850	1 456 774 850	0.0412
0	0.4	0.6	0	727 242 728	615 242 728	0.0172
0	0.4	0.4	0.2	1 137 275 044	881 275 044	0.0136
0	0.4	0.2	0.4	1 017 769 551	1 225 769 551	0.0252
0	0.4	0	0.6	−153 875 069	1 662 124 931	0.0654
0	0.2	0.8	0	604 121 085	476 121 085	0.0168
0	0.2	0.6	0.2	1 022 434 859	710 434 859	0.0122
0	0.2	0.4	0.4	953 078 642	1 025 078 642	0.0218
0	0.2	0.2	0.6	−187 400 811	1 428 599 189	0.0604
0	0.2	0	0.8	−3 183 750 687	1 968 249 313	0.1488
0	0	1	0	131 373 027	363 373 027	0.0258
0	0	0.8	0.2	661 117 051	565 117 051	0.0176
0	0	0.6	0.4	451 314 002	851 314 002	0.0300
0	0	0.4	0.6	−1 309 777 975	1 242 222 025	0.0838
0	0	0.2	0.8	−4 639 208 243	1 768 791 757	0.1802
0	0	0	1	−8 421 281 758	2 482 718 242	0.2926

Table 4. The optimal asset allocations of GA

t	0	6	12	18
Cash (%)	16.1906	2.7821	14.4054	3.3340
Stock (%)	32.4451	45.2235	47.1426	40.3112
Bond (%)	29.6427	12.7891	5.7029	7.0734
Short-term investment (%)	21.7216	39.2053	32.7491	49.2814

equities produces a significantly higher average surplus, but results in a higher ruin probability.

The ranking of the top five asset allocations looks reasonable. However, we do not spot any pattern revealing between which two asset allocations the optimal allocation might be. Reducing the grid size of 20% is therefore the only way to go if we have no optimization algorithm. However, this reduction will increase possible asset allocations dramatically. The total number of asset allocations increases to 1771, for instance, when the grid size is 5%. Furthermore, the total number of possible asset allocations will explode when the problem is expanded to multi-period. Therefore, this primitive search method is not feasible in finding the optimal asset allocations.

Results of the GA

Our GA produces significantly better asset allocations than the primitive search method. The value of the objective function is 1 343 396 299 implying an 18% improvement over the above primitive search method. The GA results in a significantly higher average discounted surplus ($1 055 396 299 that is 20% higher than that of the winner in Section A) and a lower ruin probability (1.28%). The optimal asset allocations are shown in Table 4:

We plotted the resulting 1st percentile, 25th percentile, mean, 75th percentile, and 99th percentile surplus over time from the optimal asset allocations in Fig. 2.

Advantages of the simulation optimization

The advantages of applying simulation optimization to a DFA system can be further illustrated by the robustness across different random numbers and asset–liability settings. We tried two other sets of random numbers and two other sets of parameters used in specifying the risks and initial positions of the insurer's assets and liabilities.[16] Using different sets of random number results in small changes in the objective function value. The resulting values are 1 338 413 716 and 1 358 066 783. Since these values represent a −0.37% and 1.09% difference, respectively, from the benchmark case described in the previous paragraph, our application is robust across random numbers.

The other two sets of parameters representing alternative asset–liability settings generate significantly different results. Alternative asset–liability setting 1 (Appendix) resulted in an objective function value of 1 445 571 649, an average discounted surplus of $829 571 650, and a ruin probability of 0.46%. The optimal asset allocations are given in Table 5.

The features of this setting are high-risk, high-return financial and insurance markets with a moderately positive correlation between the long-tail insurance and financial markets. The position correlation possibly contributes to the lower ruin probability than that in the benchmark case and thus generates a higher objective function value. The lower average discounted surplus might be a result from the relatively conservative investments during the first 12 years.

Alternative asset–liability setting 2 (Appendix) results in an objective function value of 998 583 316, an average discounted surplus of $198 583 316 and zero ruin probability. The optimal asset allocations are shown in Table 6:

The features of alternative asset–liability setting 2 are large, positive correlations between the long-tail insurance and the financial markets, relatively safe insurance markets, a fairly profitable but highly volatile stock market and a low-return but high-risk bond market. The allocations to bonds and cash are thus minimal. Most of the funds are allocated to short-term investment with some funds to stocks for higher returns. The average discounted surplus under asset–liability setting 2 is the smallest among the three asset–liability settings because the discount rate for surplus is the highest (45% versus 3% and 15%). The lower returns in the financial markets might also contribute to the smallest average discounted surplus. The zero insolvency probability is probably due to the low risk in the insurance markets and the high correlation between the financial and long-tail insurance markets. Finally, the significantly lower average discounted surplus under asset–liability setting 2 than those in other two settings results in the smallest objective among the three settings.

[16] The alternative parameter sets are described in the Appendix. They are specified rather arbitrarily. We intentionally make them 'unreasonable' to see whether our GA can still find solutions under odd settings. The random number set used for these two alternative parameter sets is the same as the one used in the benchmark case.

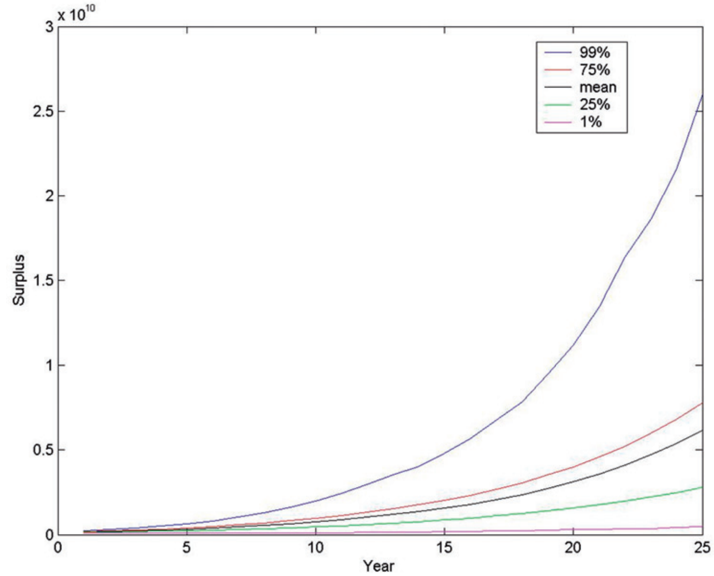

Fig. 2. The simulated surplus statistics under the optimal asset allocations

Table 5. The optimal asset allocations of GA with alternative asset–liability setting 1

t	0	6	12	18
Cash (%)	34.4349	18.2585	0.1492	2.5802
Stock (%)	19.6516	15.9641	36.8198	49.8485
Bond (%)	17.8915	30.4849	2.1546	10.3742
Short-term investment (%)	28.0221	35.2925	60.8764	37.1971

Table 6. The optimal asset allocations of GA with alternative asset–liability setting 2

t	0	6	12	18
Cash (%)	0.0010	0.0010	0.0010	0.0183
Stock (%)	14.3156	17.6433	25.7770	21.0308
Bond (%)	0.0010	0.0013	0.0053	0.1233
Short-term investment (%)	85.6824	82.3544	74.2167	78.8276

V. Summaries and Conclusions

A P&C insurer faces not only asset risks but also liability risks, and the DFA system is useful in implementing ALM. A company-wide DAF system can take full account of the static and dynamic relations among asset variables and liability variables. The major output of a DFA system is the distribution of an insurer's future surplus that can be further used to compare alternative investment strategies, business strategies and reinsurance arrangements, among others. Insurance regulators can use a DFA system to perform an early warning analysis as well as set up minimal capital requirements. The Swiss Solvency Tests is one real-world example.

The main drawback of the DFA system is the lack of an optimization mechanism. Users can perform only comparative analysis with no way of knowing what the optimal strategy is. Simulation optimization is receiving considerable interest in the field of operations research and may find a nice application to the DFA system. By integrating a DFA system with optimization features, the system turns from a descriptive model into an operational tool to solve various decision-making problems. The contribution of this article is to illustrate the potential of the integration by applying an integrated DFA system to the ALM problem of a P&C insurer.

We first built up a simplified DFA system in which an insurer underwrites both short- and long-tail businesses and invest in four types of assets. Then we formulated the ALM problem as a constrained, multi-period one. It is an ALM problem because insurance losses are correlated with asset values and cash flows associated with insurance result in selling and purchasing assets. We set up the problem in the multi-period framework because the accumulation of a sequence of single-period optimal decisions across periods may not be optimal for these periods taken as a whole and most stakeholders of the P&C insurer care about the long-term profitability and solvency. We also considered the short-sale constraints faced by insurers when making investments. The capability of solving a constrained, multi-period ALM problem illustrates the advantage of simulation optimization, although we must keep in mind that the found solution as a result of simulation optimization cannot be proved to be the optimum. The simulation optimization technique used in this article is a GA.

We performed a search for the optimal ALM solution of a P&C insurer using our integrated DFA system. The resulting asset allocations produce a significantly higher value of the objective function compared to the allocation found from the only search method currently available to the DFA system. The optimal allocations produced a higher average discounted surplus and a lower ruin probability. Using different sets of random number generated similar values of objective function and demonstrated the robustness of our integrated DFA system across random numbers. The optimal asset allocations are sensitive to the asset–liability setting reflected by the parameters of financial market models and insurance business specifications, with reasonable changes. These advantages demonstrate that the P&C insurers that are using or are interested in DFA shall consider applying simulation optimization to their DFA systems.

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Appendix: Alternative Asset–Liability Settings

Alternative asset–liability setting 1

Model parameters			
Short rate	$m=20\%$	$q=0.1$	$v=10\%$
Equity index	$\pi_S=10\%$	$\sigma_S=35\%$	
Short-term investment	$\mu_{STI}=30\%$	$\sigma_{STI}=60\%$	
Loss ratio (long)	$\mu_{LR(L)}=30\%$	$\sigma_{LR(L)}=100\%$	
Loss ratio (short)	$\mu_{LR(S)}=20\%$	$\sigma_{LR(S)}=80\%$	

The starting value of the short-term interest rate is 1%. The correlation matrix $\Re$ is given as follows.

	dW_S	dW_r	$dW_{LR(L)}$	dW_{STI}
dW_S	1	0.60	0.60	0.60
dW_r	0.60	1	0.60	0.60
$dW_{LR(L)}$	0.60	0.60	1	0.59
dW_{STI}	0.60	0.60	0.59	1

The parameters of the representative insurer are set as follows: $IS(0)=100$, $IP(0)=110$ $B(0)=\frac{10}{11}$, $\text{Exp}(L)=50\%$, $\text{Exp}(S)=50\%$, $G(L)=2\%$, $G(S)=1\%$ and the following.

dy	1	2	3	4	5	6	7	8	9	10
$D_L(dy)$ (%)	10	10	10	10	10	10	10	10	10	10
$D_S(dy)$ (%)	30	40	30							

The discount rate for future surplus is assumed to be 15% while the discount rate for the reserves is set at 1%. The new investments to bonds are allocated one-third to 1-year bonds, one-third to 7-year bonds and one-third to 15-year bonds.

Alternative asset–liability setting 2

Model parameters			
Short rate	$m=2\%$	$q=0.7$	$v=6\%$
Equity index	$\pi_S=20\%$	$\sigma_S=50\%$	
Short-term investment	$\mu_{STI}=10\%$	$\sigma_{STI}=13\%$	
Loss ratio (long)	$\mu_{LR(L)}=90\%$	$\sigma_{LR(L)}=20\%$	
Loss ratio (short)	$\mu_{LR(S)}=95\%$	$\sigma_{LR(S)}=10\%$	

The starting value of the short-term interest rate is 12%. The correlation matrix $\Re$ is given as follows.

	dW_S	dW_r	$dW_{LR(L)}$	dW_{STI}
dW_S	1	0.99	0.99	0.99
dW_r	0.99	1	0.99	0.99
$dW_{LR(L)}$	0.99	0.99	1	0.99
dW_{STI}	0.99	0.99	0.99	1

The parameters of the representative insurer are set as follows: $IS(0)=250$, $IP(0)=1{,}110$ $B(0)$ $\frac{1}{11}$, $\text{Exp}(L)=10\%$, $\text{Exp}(S)=5\%$, $G(L)=12\%$, $G(S)=20\%$ and the following.

dy	1	2	3	4	5	6	7	8	9	10
$D_L(dy)$ (%)	0.1	0.1	0.3	0.5	1	3	5	10	30	50
$D_S(dy)$ (%)	5	15	80							

The discount rate for future surplus is assumed to be 45% while the discount rate for the reserves is set at 10%. The new investments to bonds are all allocated 7-year bonds.