# Digital Image Processing

Lecture #7

Ming-Sui (Amy) Lee

# Announcement

- **The following schedule**

| | | | |
|---|---|---|---|
| 02/17 | Lecture 1 | 04/14 | Lecture 8 |
| 02/24 | Lecture 2 | 04/21 | Proposal |
| 03/03 | Lecture 3 | 04/28 | Lecture 9 |
| 03/10 | Lecture 4 | 05/05 | Lecture 10 |
| 03/17 | Lecture 5 | 05/12 | Lecture 11 |
| 03/24 | Lecture 6 | 05/19 | Demo |
| 03/31 | Lecture 7 | 05/26 | Demo |
| 04/07 | Midterm | 06/02 | Final Package Due |

- **Midterm Exam**
  - **Apr. 07, 2022**

# Announcement

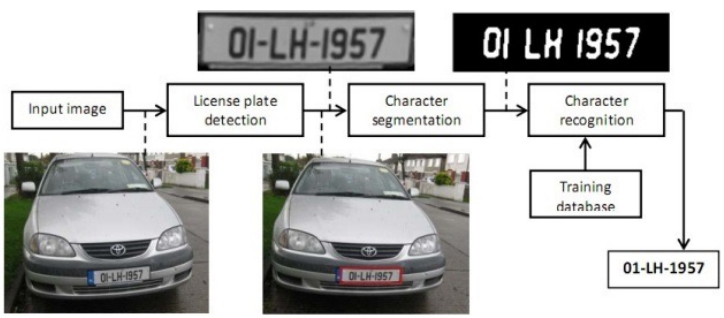
- Midterm
  - Apr. 07, 2022 @ 102
  - Closed-book exam
  - Photo/Student ID
  - One-line calculator
- Proposal
  - Apr. 21, 2022
  - Please form a team with 2 students
  - Email TA the member list by Apr. 08

    Update the member list on NTU COOL by Apr. 09

# Shape Analysis & Document Processing

# Shape Analysis & Document Processing

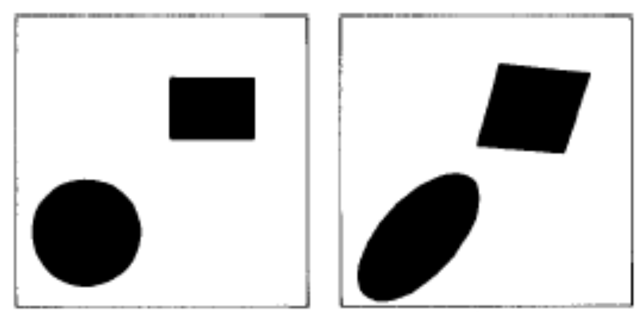
- OCR – Optical Character Recognition
  - Extract features from
    - characters (A~Z)
    - numerals (0~9)
    - other special symbols (*&^%$#)




# Shape Analysis & Document Processing

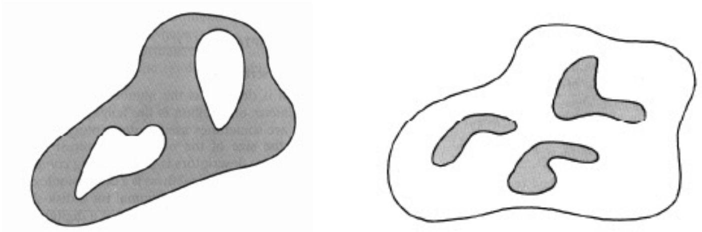
- Topological Attributes
  - May design your own attributes
  - Properties are invariant under the rubber-sheet transformation

A A T T



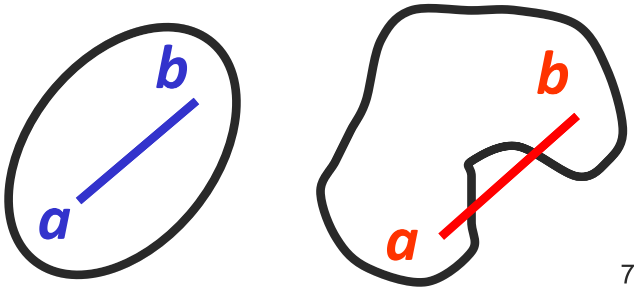
# Shape Analysis & Document Processing

- Topological Attributes
  - C: number of connected object components
  - H: number of holes
  - E: Euler number E=C-H



  - Convex Set
    - An object C is convex if

$$\forall a, b \in C$$
$$\Rightarrow ta + (1-t)b \in C, \quad 0 \le t \le 1$$



# Shape Analysis & Document Processing

- **Convex Hull & Convex Deficiency**
  - Convex Hull
    - The convex hull of a set is the smallest convex set that contains the set
  - Convex Deficiency
    - The set of points within the convex hull but not in the object form the convex deficiency
    - Divide subsets of convex deficiency into two types
      - Lake and Bay
      - L: number of lakes
      - B: number of bays


(a) Object (b) Convex hull (c) Bays and lake

# Shape Analysis & Document Processing

- **Examples**

C: number of connected object components
H: number of holes
E: Euler number E=C-H
Convex Hull? Convex Deficiency: Lake? Bay?

H i ? Q @ *a*

g B *m* a D H

# Shape Analysis & Document Processing

- Geometrical Properties
  - Distance
    - Euclidean distance $d_E = \left[(j_1 - j_2)^2 + (k_1 - k_2)^2\right]^{1/2}$
    - Magnitude distance $d_M = |j_1 - j_2| + |k_1 - k_2|$
    - Maximum value distance $d_X = MAX\{|j_1 - j_2|, |k_1 - k_2|\}$
  - Perimeter
    - The number of “sides” which separate pixels with different values
  - Area
    - Total number of pixels with F(j,k)=1
    - The “enclosed area” is the total number of pixels with F(j,k)=0 or 1 within the outer perimeter of an object

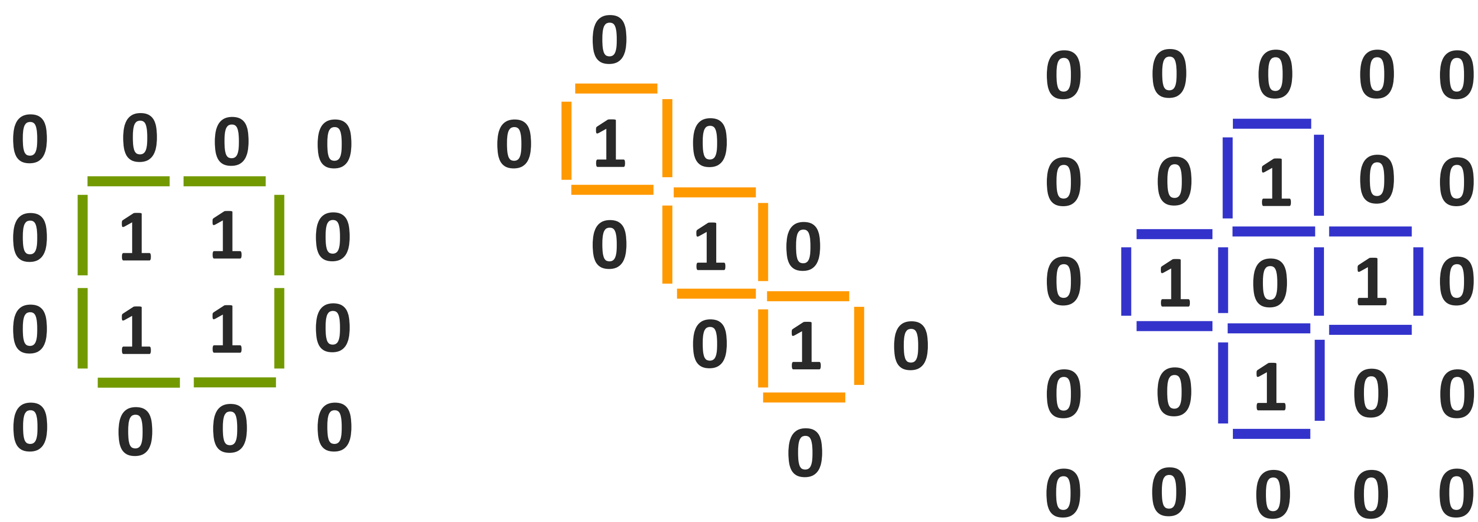
# Shape Analysis & Document Processing

- **Examples**

- Area? [Total number of pixels with F(j,k)=1]

- Perimeter? [The number of "sides" which separate pixels with different values]

- Enclosed Area? [the total number of pixels with F(j,k)=0 or 1 within the outer perimeter of an object]



A=4; P=8; EA=4

A=3; P=12; EA=3

A=4; P=16; EA=5

# Shape Analysis & Document Processing

- **Geometrical Properties**
  - **Relative measure**
    - **Scaling-invariant**
    - **Normalized area/perimeter**
    - **Normalized w.r.t. the bounding box**
  - **Computation of several attributes with local patterns**
    - **Bit Quads**
      - **Let n{Q} represent the count of the number of matches between image pixels and pattern Q**

$$Q = 1 \Rightarrow n\{Q\} = Area$$

$$Q = [0 \quad 1] or [1 \quad 0] or \begin{bmatrix} 1 \\ 0 \end{bmatrix} or \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow n\{0 \quad 1\} + n\{1 \quad 0\} + n\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} + n\begin{Bmatrix} 0 \\ 1 \end{Bmatrix} = Perimeter$$

# Shape Analysis & Document Processing

- Geometrical Properties
  - Bit Quads (Gray's algorithm)
    - A systematic way to compute geometric attributes based on local pattern matching

// Bit quad patterns //

$$Q_0 \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$Q_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$Q_2 \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$Q_3 \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$Q_4 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$Q_D \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A = \frac{1}{4}\left[n\{Q_1\} + 2n\{Q_2\} + 3n\{Q_3\} + 4n\{Q_4\} + 2n\{Q_D\}\right]$$

$$P = n\{Q_1\} + n\{Q_2\} + n\{Q_3\} + 2n\{Q_D\}$$

# Shape Analysis & Document Processing

- **Example**

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 |

$$A = \frac{1}{4}\left[n\{Q_1\} + 2n\{Q_2\} + 3n\{Q_3\} + 4n\{Q_4\} + 2n\{Q_D\}\right]$$

$$P = n\{Q_1\} + n\{Q_2\} + n\{Q_3\} + 2n\{Q_D\}$$

$$Q_0 \quad \begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$$Q_1 \quad \begin{bmatrix}1&0\\0&0\end{bmatrix} \begin{bmatrix}0&1\\0&0\end{bmatrix} \begin{bmatrix}0&0\\0&1\end{bmatrix} \begin{bmatrix}0&0\\1&0\end{bmatrix}$$

$$Q_2 \quad \begin{bmatrix}1&1\\0&0\end{bmatrix} \begin{bmatrix}0&1\\0&1\end{bmatrix} \begin{bmatrix}0&0\\1&1\end{bmatrix} \begin{bmatrix}1&0\\1&0\end{bmatrix}$$

$$Q_3 \quad \begin{bmatrix}1&1\\0&1\end{bmatrix} \begin{bmatrix}0&1\\1&1\end{bmatrix} \begin{bmatrix}1&0\\1&1\end{bmatrix} \begin{bmatrix}1&1\\1&0\end{bmatrix}$$

$$Q_4 \quad \begin{bmatrix}1&1\\1&1\end{bmatrix}$$

$$Q_D \quad \begin{bmatrix}1&0\\0&1\end{bmatrix} \begin{bmatrix}0&1\\1&0\end{bmatrix}$$

# Shape Analysis & Document Processing

- Geometrical Properties
  - Bit Quads (Duda's algorithm)
    - More accurate to represent the area of a continuous object that has been coarsely discretized than Gray's
    - 2x2 patterns

$$A = \frac{1}{4} n\{Q_1\} + \frac{1}{2} n\{Q_2\} + \frac{7}{8} n\{Q_3\} + n\{Q_4\} + \frac{3}{4} n\{Q_D\}$$

$$P = n\{Q_2\} + \frac{1}{\sqrt{2}} \left[ n\{Q_1\} + n\{Q_3\} + 2n\{Q_D\} \right]$$

# Shape Analysis & Document Processing

- **Geometrical Properties**
  - **Bit Quads**

    $Q_1$ $\begin{bmatrix}1&0\\0&0\end{bmatrix}$ $\begin{bmatrix}0&1\\0&0\end{bmatrix}$ $\begin{bmatrix}0&0\\0&1\end{bmatrix}$ $\begin{bmatrix}0&0\\1&0\end{bmatrix}$

    $Q_3$ $\begin{bmatrix}1&1\\0&1\end{bmatrix}$ $\begin{bmatrix}0&1\\1&1\end{bmatrix}$ $\begin{bmatrix}1&0\\1&1\end{bmatrix}$ $\begin{bmatrix}1&1\\1&0\end{bmatrix}$

    $Q_D$ $\begin{bmatrix}1&0\\0&1\end{bmatrix}$ $\begin{bmatrix}0&1\\1&0\end{bmatrix}$

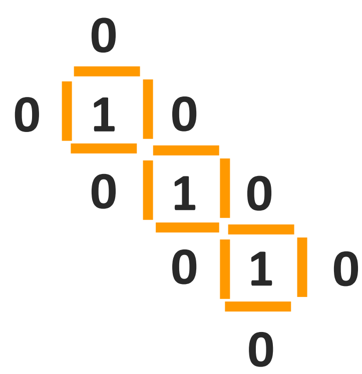
    - Easy to determine the “Euler number” of an image
  - **Euler Number (Gray’s)**
    - Four-connectivity

$$E = \frac{1}{4}\left[n\{Q_1\} - n\{Q_3\} + 2n\{Q_D\}\right]$$

    - Eight-connectivity

$$E = \frac{1}{4}\left[n\{Q_1\} - n\{Q_3\} - 2n\{Q_D\}\right]$$

e.g.



  - Note// We are not able to compute the number of connected components *C* and the number of holes *H* (*E=C-H*) separately by local neighborhood computation

# Shape Analysis & Document Processing

- **Examples**

$$Q_0 \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$Q_1 \quad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$Q_2 \quad \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$Q_3 \quad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$Q_4 \quad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$Q_D \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 |

**Four-connectivity**

$$E = \frac{1}{4}\left[n\{Q_1\} - n\{Q_3\} + 2n\{Q_D\}\right]$$

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

**Eight-connectivity**

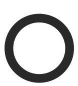
$$E = \frac{1}{4}\left[n\{Q_1\} - n\{Q_3\} - 2n\{Q_D\}\right]$$

# Shape Analysis & Document Processing

- **Other Attributes and Properties**
  - **Symmetry property**
    - Horizontally symmetric / vertically symmetric
  - **Circularity (thinness ratio)**

$$C_0 = \frac{4\pi A_0}{(P_0)^2}$$

- E.g.



$$A_0 = \pi r^2; \quad P_0 = 2\pi r \Rightarrow C_0 = \frac{4\pi \pi r^2}{(2\pi r)^2} = \frac{4\pi^2 r^2}{4\pi^2 r^2} = 1$$

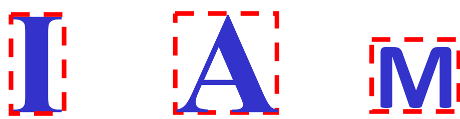
$$A_0 = a^2; \quad P_0 = 4a \Rightarrow C_0 = \frac{4\pi a^2}{(4a)^2} = \frac{\pi}{4} \approx 0.8$$

$b >> a$

$$A_0 = ab; \quad P_0 = 2(a+b) \Rightarrow C_0 = \frac{4\pi ab}{(2(a+b))^2} = \frac{\pi ab}{a^2 + b^2 + 2ab} \approx \frac{\pi a}{b}$$

# Shape Analysis & Document Processing

- Other Attributes and Properties
  - Width and height
    - Bounding box



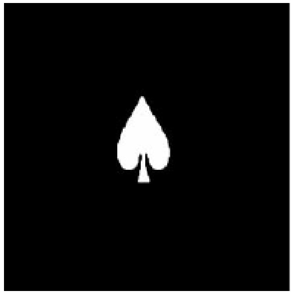
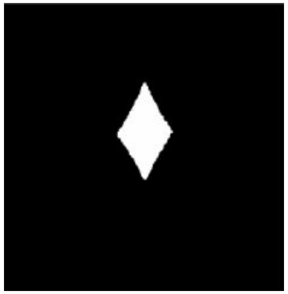
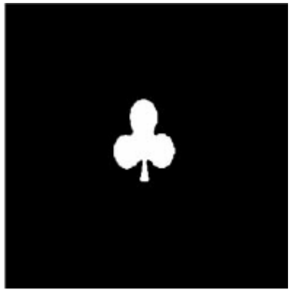

- Width ratio: $b/(a+b)$

- Height ratio: $a/(a+b)$

    - An image with many components but fewer holes
      - Euler number may be an approximation of # of components
      - Average area: $A_A = \frac{A_0}{E}$
      - Average perimeter: $P_A = \frac{P_0}{E}$
    - Thin objects (typewritten or script  characters)
      - Average length $\approx L_A = \frac{P_A}{2}$
      - Average width $\approx W_A = \frac{2A_A}{P_A}$

# Shape Analysis & Document Processing

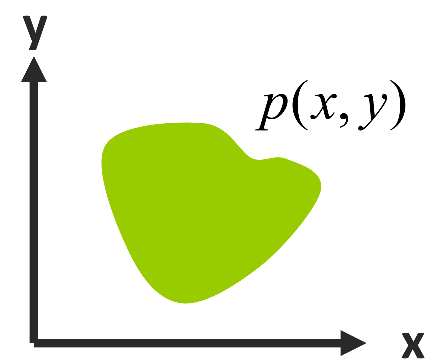
- Examples

| Attribute | spade | heart | diamond | club |
|---|---|---|---|---|
| Outer perimeter | 652 | 512 | 548 | 668 |
| Enclosed area | 8421 | 8681 | 8562 | 8820 |
| Average area | 8421 | 8681 | 8562 | 8820 |
| Average perimeter | 652 | 512 | 548 | 668 |
| Average length | 326 | 256 | 274 | 334 |
| Average width | 25.8 | 33.9 | 31.3 | 26.4 |
| Circularity | 0.25 | 0.42 | 0.36 | 0.25 |

# Shape Analysis & Document Processing

- **Other attributes and properties**
  - **Spatial moments**
    - **Treat the object shape as a pdf,** $p(x,y)$
    - **For a joint pdf,** $p(x,y)$ **, its** $(m,n)^{th}$ **moment is defined as**



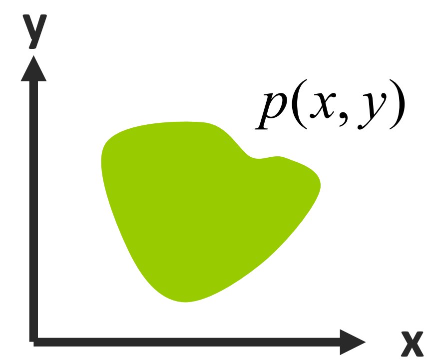

$$M(m,n) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^m y^n p(x,y)\,dxdy$$

$$M(0,0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x,y)\,dxdy = A;$$

$$M(1,0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xp(x,y)\,dxdy = \eta_x;\; M(0,1) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} yp(x,y)\,dxdy = \eta_y$$

# Shape Analysis & Document Processing

- **Other attributes and properties**
  - **Spatial moments**
    - **Usually, the central moments are more interesting since they are invariant under translation (shift-invariant)**

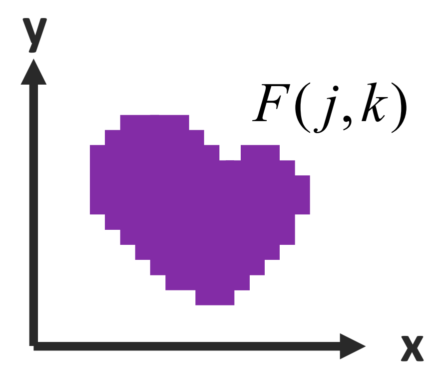



$$U(m,n) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\eta_x)^m (y-\eta_y)^n \, p(x,y)\,dx\,dy$$

where $\eta_x$ and $\eta_y$ are marginal means of $p(x,y)$

# Shape Analysis & Document Processing

- **Other attributes and properties**
  - **Discrete Image Spatial Moments**
    - **The $(m,n)^{th}$ spatial geometric moment is defined as**




$$M(m,n) = \frac{1}{J^m K^n} \sum_{j=1}^{J} \sum_{k=1}^{K} (x_j)^m (y_k)^n F(j,k)$$

$$M(0,0) = \sum_{j=1}^{J} \sum_{k=1}^{K} F(j,k)$$ **< Image surface >**

$$M(1,0) = \frac{1}{J} \sum_{j=1}^{J} \sum_{k=1}^{K} x_j F(j,k); \quad M(0,1) = \frac{1}{K} \sum_{j=1}^{J} \sum_{k=1}^{K} y_k F(j,k)$$

# Shape Analysis & Document Processing

- Other attributes and properties
  - Spatial moments
    - Examples
    - Table 18.3-1 ( p. 635 )

| Image | $M(0,0)$ | $M(1,0)$ | $M(0,1)$ | $M(2,0)$ | $M(1,1)$ | $M(0,2)$ | $M(3,0)$ | $M(2,1)$ | $M(1,2)$ | $M(0,3)$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Spade | 8,219.98 | 4,013.75 | 4,281.28 | 1,976.12 | 2,089.86 | 2,263.11 | 980.81 | 1,028.31 | 1,104.36 | 1,213.73 |
| Rotated spade | 8,215.99 | 4,186.39 | 3,968.30 | 2,149.35 | 2,021.65 | 1,949.89 | 1,111.69 | 1,038.04 | 993.20 | 973.53 |
| Heart | 8,616.79 | 4,283.65 | 4,341.36 | 2,145.90 | 2,158.40 | 2,223.79 | 1,083.06 | 1,081.72 | 1,105.73 | 1,156.35 |
| Rotated heart | 8,613.79 | 4,276.28 | 4,337.90 | 2,149.18 | 2,143.52 | 2,211.15 | 1,092.92 | 1,071.95 | 1,008.05 | 1,140.43 |
| Magnified heart | 34,523.13 | 17,130.64 | 17,442.91 | 8,762.68 | 8,658.34 | 9,402.25 | 4,608.05 | 4,442.37 | 4,669.42 | 5,318.58 |
| Minified heart | 2,104.97 | 1,047.38 | 1,059.44 | 522.14 | 527.16 | 535.38 | 260.78 | 262.82 | 266.41 | 271.61 |
| Diamond | 8,561.82 | 4,349.00 | 4,704.71 | 2,222.43 | 2,390.10 | 2,627.42 | 1,142.44 | 1,221.53 | 1,334.97 | 1,490.26 |
| Rotated diamond | 8,562.82 | 4,294.89 | 4,324.09 | 2,196.40 | 2,168.00 | 2,196.97 | 1,143.83 | 1,108.30 | 1,101.11 | 1,122.93 |
| Club | 8,781.71 | 4,323.54 | 4,500.10 | 2,150.47 | 2,215.32 | 2,344.02 | 1,080.29 | 1,101.21 | 1,153.76 | 1,241.04 |
| Rotated club | 8,787.71 | 4,363.23 | 4,220.96 | 2,196.08 | 2,103.88 | 2,057.66 | 1,120.12 | 1,062.39 | 1,028.90 | 1,017.60 |
| Ellipse | 8,721.74 | 4,326.93 | 4,377.78 | 2,175.86 | 2,189.76 | 2,226.61 | 1,108.47 | 1,109.92 | 1,122.62 | 1,146.97 |

# Shape Analysis & Document Processing

- **Other attributes and properties**
  - Row moment of inertia

$$\mu'_{20} = \mu_{20} / \mu_{00} = M_{20} / M_{00} - \bar{x}^2$$

  - Column moment of inertia

$$\mu'_{02} = \mu_{02} / \mu_{00} = M_{02} / M_{00} - \bar{y}^2$$

  - Row-column cross moment of inertia

$$\mu'_{11} = \mu_{11} / \mu_{00} = M_{20} / M_{00} - \bar{x}\bar{y}$$

# Shape Analysis & Document Processing

- **Other attributes and properties**
  - **Covariance Matrix**

$$U = \mathrm{cov}[I(x,y)] = \begin{bmatrix} \mu'_{20} & \mu'_{11} \\ \mu'_{11} & \mu'_{02} \end{bmatrix}$$

- **Perform SVD of the covariance matrix** $E^T U E = \Lambda$

**The columns of** $E = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix}$ **are the eigenvectors**

**of U and** $\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

# Shape Analysis & Document Processing

- Other attributes and properties
  - Eigenvalues can be derived explicitly

$$\lambda_1 = \frac{1}{2}[\mu'_{20} + \mu'_{02}] + \frac{1}{2}\left[(\mu'_{20})^2 + (\mu'_{02})^2 - 2\mu'_{20}\mu'_{02} + 4(\mu'_{11})^2\right]^{1/2}$$

$$\lambda_2 = \frac{1}{2}[\mu'_{20} + \mu'_{02}] - \frac{1}{2}\left[(\mu'_{20})^2 + (\mu'_{02})^2 - 2\mu'_{20}\mu'_{02} + 4(\mu'_{11})^2\right]^{1/2}$$

  - Let $\lambda_M = MAX\{\lambda_1, \lambda_2\}$ and $\lambda_N = MIN\{\lambda_1, \lambda_2\}$

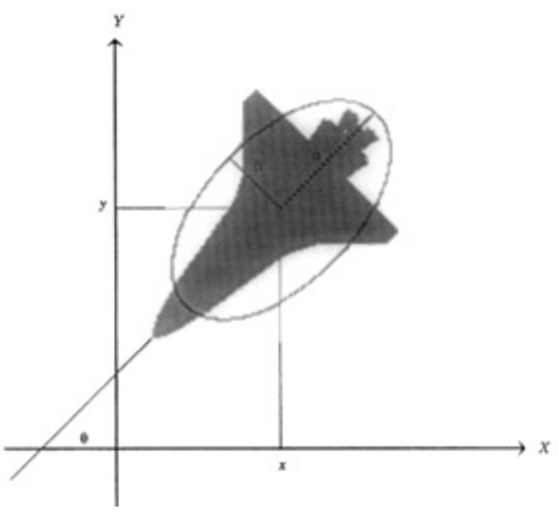
    The eigenvalue ratio is $\lambda_N / \lambda_M$
  - The orientation is $\theta = \frac{1}{2}\arctan\left\{\frac{2\mu_{11}}{\mu_{20} - \mu_{02}}\right\}$

# Shape Analysis & Document Processing

- **Other attributes and properties**
  - **The orientation is**

$$\theta = \frac{1}{2}\arctan\left\{\frac{2\mu_{11}}{\mu_{20}-\mu_{02}}\right\}$$

  - **Eclipse defined by 2 eigenvectors and orientation angle $\vartheta$**



# Shape Analysis & Document Processing

- **Other attributes and properties**
  - **Table 18.3-3**

| Image | Largest Eigenvalue | Smallest Eigenvalue | Orientation (radians) | Eigenvalue Ratio $= \lambda_N / \lambda_M$ |
|---|---|---|---|---|
| Spade | 33.286 | 16.215 | –0.153 | 0.487 |
| Rotated spade | 33.223 | 16.200 | –1.549 | 0.488 |
| Heart | 36.508 | 16.376 | 1.561 | 0.449 |
| Rotated heart | 36.421 | 16.400 | –0.794 | 0.450 |
| Magnified heart | 589.190 | 262.290 | 1.562 | 0.445 |
| Minified heart | 2.165 | 0.984 | 1.560 | 0.454 |
| Diamond | 42.189 | 13.334 | 1.560 | 0.316 |
| Rotated diamond | 42.223 | 13.341 | –0.030 | 0.316 |
| Club | 37.982 | 21.831 | –1.556 | 0.575 |
| Rotated club | 38.073 | 21.831 | 0.802 | 0.573 |
| Ellipse | 47.149 | 11.324 | 0.785 | 0.240 |

# Shape Analysis & Document Processing

- **Seven invariant moments [Hu's 1962]**
  - A Comparative Study of Three Moment-Based Shape Descriptors
  - invariant to rotation, scaling and translation

$$\phi_1 = \eta_{20} + \eta_{02}$$

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^{\ 2}$$

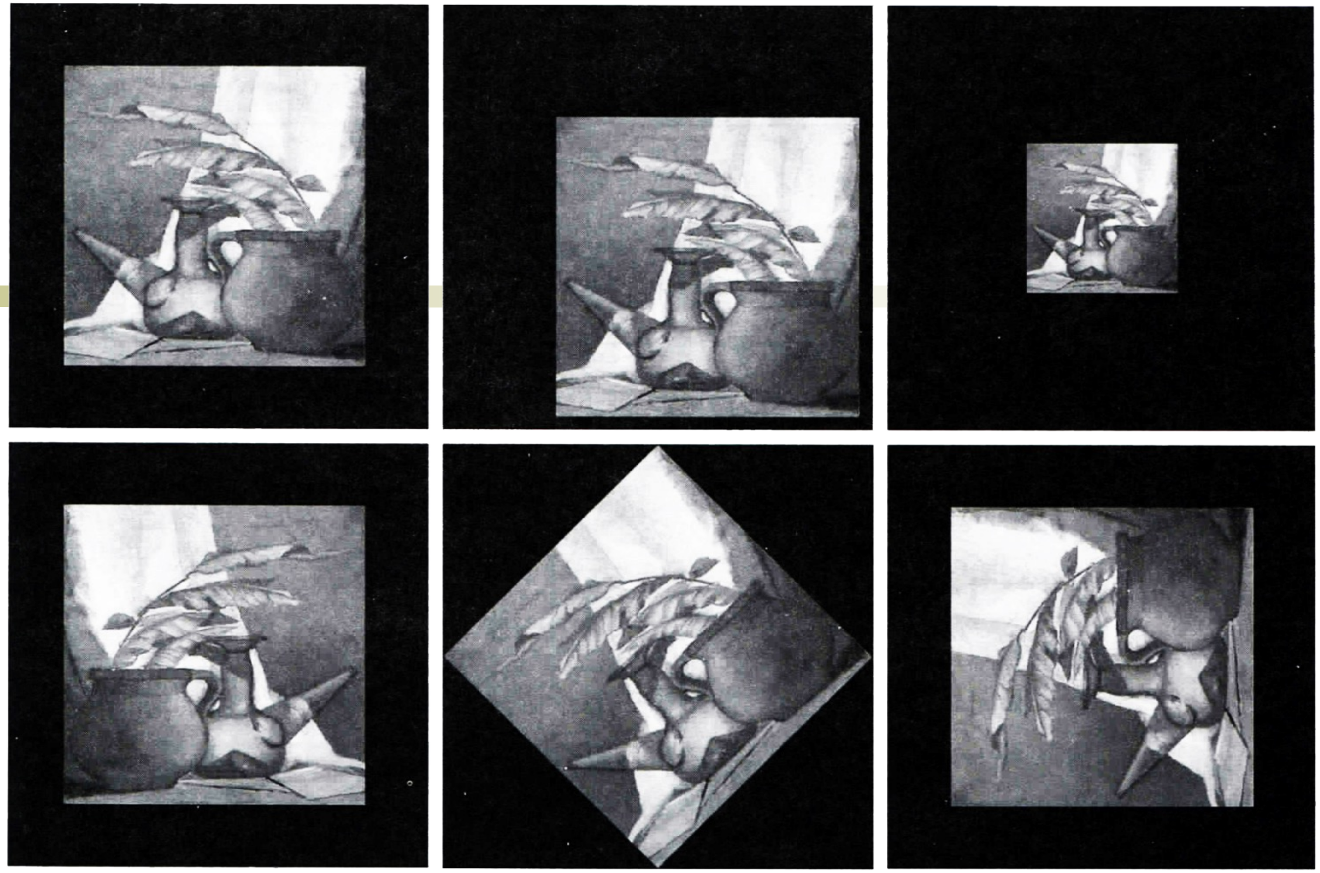
$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$$

$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$

$$\begin{aligned}\phi_5 = {} & (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})\left[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2\right] \\ & + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})\left[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right]\end{aligned}$$

$$\begin{aligned}\phi_6 = {} & (\eta_{20} - \eta_{02})\left[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right] \\ & + 4\eta_{11}^{\ 2}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03})\end{aligned}$$

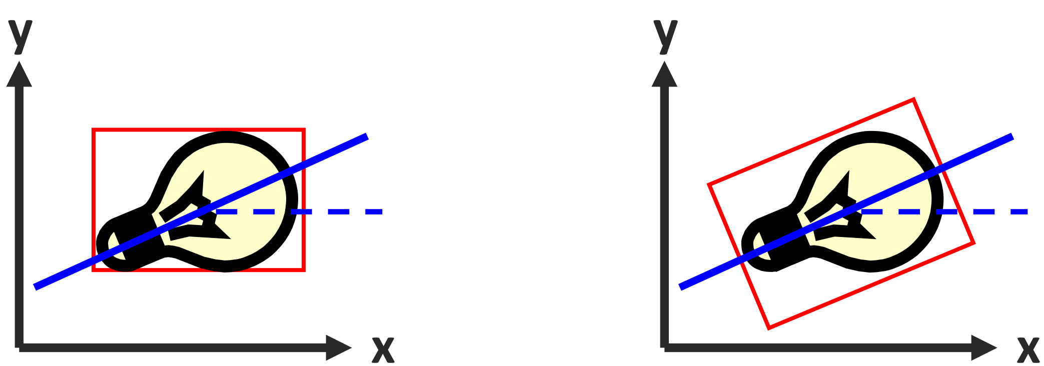
$$\begin{aligned}\phi_7 = {} & (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})\left[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2\right] \\ & + (3\eta_{12} - \eta_{30})(\eta_{21} + \eta_{03})\left[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right]\end{aligned}$$



| Invariant Moment | $\Phi_1$ | $\Phi_2$ | $\Phi_3$ | $\Phi_4$ | $\Phi_5$ | $\Phi_6$ | $\Phi_7$ |
|---|---|---|---|---|---|---|---|
| Original image | 2.8662 | 7.1265 | 10.4109 | 10.3742 | 21.3674 | 13.9417 | -20.7809 |
| Shift | 2.8662 | 7.1265 | 10.4109 | 10.3742 | 21.3674 | 13.9417 | -20.7809 |
| Half size | 2.8664 | 7.1267 | 10.4107 | 10.3719 | 21.3924 | 13.9383 | -20.7724 |
| Mirrow | 2.8662 | 7.1265 | 10.4109 | 10.3742 | 21.3674 | 13.9417 | 20.7809 |
| Rotate $45^\circ$ | 2.8661 | 7.1266 | 10.4115 | 10.3742 | 21.3663 | 13.9417 | -20.7813 |
| Rotate $90^\circ$ | 2.8662 | 7.1265 | 10.4109 | 10.3742 | 21.3674 | 13.9417 | -20.7809 |

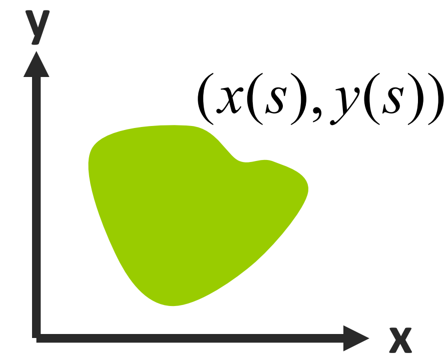
# Shape Analysis & Document Processing

- Other attributes and properties
  - Shape Orientation Descriptors
    - Trace the edge points along the contour
    - The direction of connected neighbors (clock-wise or counter-clockwise)
    - Image-oriented bounding box
    - Object-oriented bounding box
      - Height/width/area/ratio/min v.s. max radius/radius angle/radius ratio



# Shape Analysis & Document Processing

- Other attributes and properties
  - Fourier Descriptors
    - Polar coordinates: z(s)=x(s)+iy(s)
    - Total length = L → x(s+L)=x(s); y(s+L)=y(s)
    - Time, s → parameter of a parameterized curve
    - Fourier series expansion
    - Apply Fourier analysis to x(s) and y(s)




  - Wavelet Descriptors
    - Total length = L → x(s+L)=x(s); y(s+L)=y(s)
    - Time, s → parameter of a parameterized curve
    - Apply wavelet transform to x(s) and y(s)

# Shape Analysis & Document Processing

- **Attributes/Features**

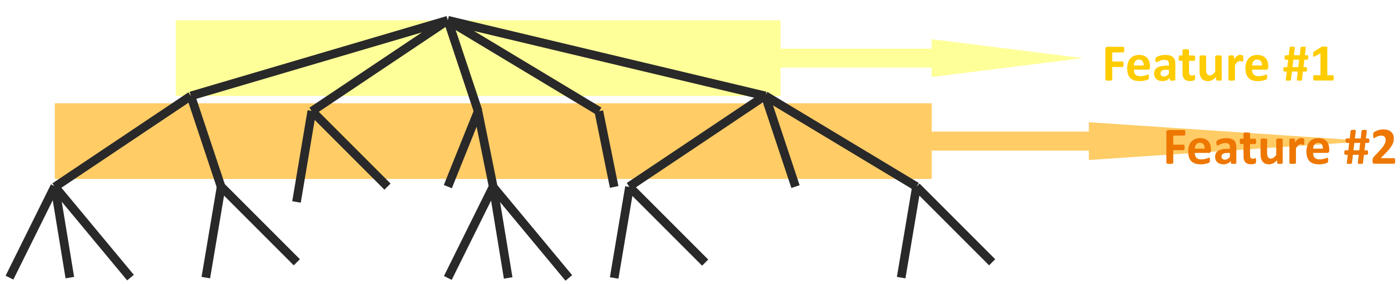
// Training data //

| symbol | index | E | C | L | … | … | … |
|---|---|---|---|---|---|---|---|
| A | 1 | | | | | | |
| B | 2 | | | | | | |
| … | … | | | | | | |

// Test data //

| input | E | C | L | … | … | … |
|---|---|---|---|---|---|---|
| | | | | | | |
| | | | | | | |
| … | | | | | | |

- Identify a set of features
  - Parallel classification
    - Form a feature vector
    - Consider them simultaneously
  - Sequential classification
    - Apply one feature at a time




    - A leaf represents one object