UQ 6310 Homework1

1B. (a) Repeat Example 1 yourself. The example is showing Fourier-based spectral method for simulating the random process. Compare your result with the results given in the examples. (b) Repeat Example 1 using K-L expansion. Compare the K-L results for the autocorrelation of the generated random process with the Fourier results in part (a).

Solution:

(a) Given the autocorrelation function $R(\tau) = e^{-\tau}$, the spectral densit is calculated based on Eq. (3) in the tutorial materials, which is as follows:

$$S_{gg}(\omega) = \frac{1}{\pi}\int_0^\infty e^{-\tau}\cos(\omega\tau)d\tau = \frac{1}{\pi(1+\omega^2)} \tag{1}$$

Also, the time goes from 0 to 1, with the step 0.01. At each time, 100 samples are simulated. ω is from 0 to 10π with step 0.01π. The following figure show the targeted and simulated autocorrelation function.

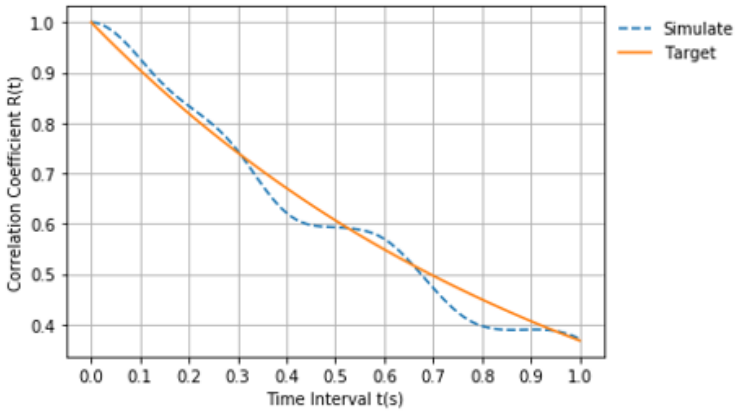

Figure 1: Simulation Gaussian process using spectral method

(b) Given the autocorrelation function $R(\tau) = e^{-\tau}$, the correlation matrix is generated by calculating the value of $R(\tau)$ at different time difference, from 0 to 1 with time step 0.01. The eigenvalue and eigenvector are further calculated. Choosing the first few eigenvector and eigenvalue, by equation (9) (To make the simulation more accurate, we use all of the eigenvectors and eigenvalues), the simulation result is as follows:

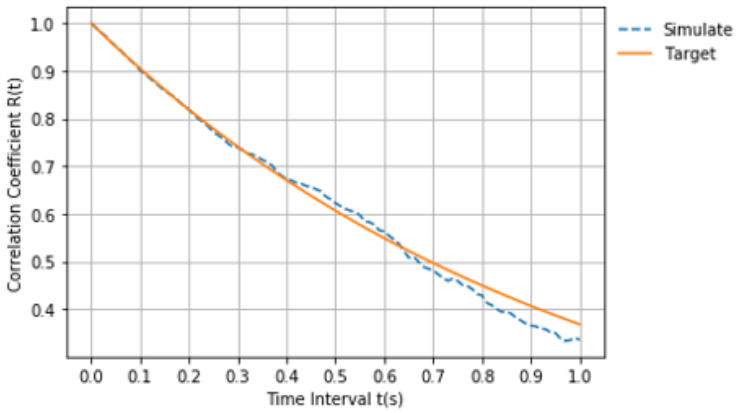

Figure 2: Simulation Gaussian process using K-L method

Comparing the simulation results using above two methods, the K-L expansion simulation is better than spectral method.