UQ 6310 Homework2

2. Solution: Given the autocorrelation function $R(\Delta l) = e^{-\Delta l}$, the correlation matrix is generated by calculating the value of $R(\Delta l)$ at different location difference, which are $0, 1L, 2L, 3L$ with time step L. The eigenvalue and eigenvector are further calculated. Choosing the first few eigenvector and eigenvalue, by equation (9) (To make the simulation more accurate, we use all of the eigenvectors and eigenvalues), the simulation result is as follows:

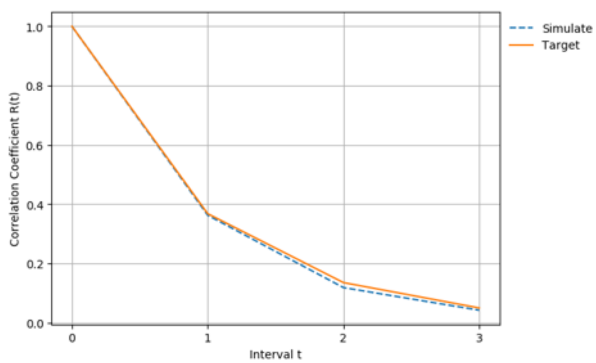


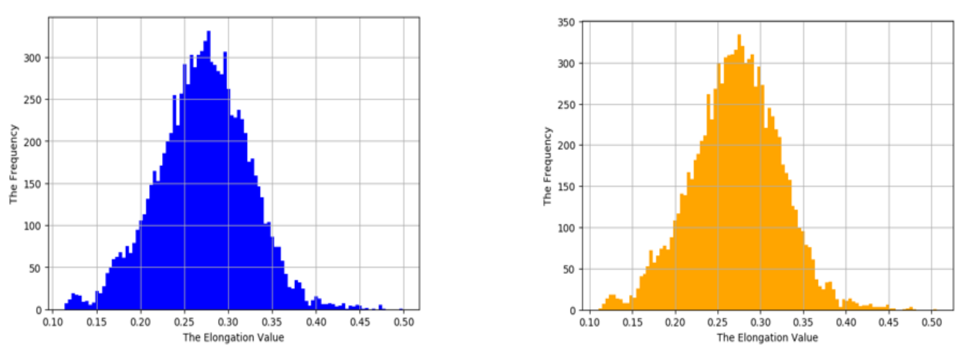
Figure 1: Simulation Gaussian process using K-L method

Comparing the simulated Correlation Coefficient curve with the Given Curve, the random process of E is simulated very well.

For each $E_i, i \in 1, 2, 3, 4$ and P, we sample 100 data points. And since the random process of E we generated is the normalized one, we need to unnormalize their corresponding value. And by our physical model as follows:

$$Y = PL/(E_1 A) + PL/(E_2 A) + PL/(E_3 A) + PL/(E_4 A) \tag{1}$$

we can calculate the elongation value, the distribution of it is shown in Fig(2a)



(a) Calculated by physical model (b) Calculated by surrogate model

Figure 2: Distribution of the elongation calculated using two ways

Here we have 4 variables for different E and 1 variable for P, so the Chaos Expansion here is 5 dimensional and 2 order, which gives us 21 items. The regression form is as follows:

$$\begin{aligned}\{\Phi\} = &\{1, \xi_1, \xi_2, \xi_3, \xi_4, \xi_5\} + \{\xi_1^2 - 1, \xi_2^2 - 1, \xi_3^2 - 1, \xi_4^2 - 1, \xi_5^2 - 1\} \\ &+ \{\xi_1\xi_2, \xi_1\xi_3, \xi_1\xi_4, \xi_1\xi_5\} + \{\xi_2\xi_3, \xi_2\xi_4, \xi_2\xi_5\} + \{\xi_3\xi_4, \xi_3\xi_5\} + \{\xi_4\xi_5\}\end{aligned} \tag{2}$$

$$\{\Phi\} = \left(X^{\mathrm{T}}X\right)^{-1} X^{\mathrm{T}}Y \tag{3}$$

Where X is the 10000×21 training points and Y is the 10000×1 elongation vector. By equation (3), the coefficient is as follows:

$$\begin{aligned}\Phi = [&2.69 \times 10^{-1}, 5.39 \times 10^{-2}, -6.86 \times 10^{-3}, -6.73 \times 10^{-3}, -7.03 \times 10^{-3}, -6.87 \times 10^{-3}, \\ &- 2.60 \times 10^{-17}, 6.88 \times 10^{-4}, 7.61 \times 10^{-4}, 7.45 \times 10^{-14}, 6.90 \times 10^{-4}, -1.37 \times 10^{-3}, \\ &- 1.33 \times 10^{-3}, -1.48 \times 10^{-3}, -1.39 \times 10^{-3}, -6.70 \times 10^{-3}, 1.05 \times 10^{-4}, -2.04 \times 10^{-5}, \\ &- 7.80 \times 10^{-5}, 3.74 \times 10^{-6}, 7.40 \times 10^{-5}]^{\mathrm{T}}\end{aligned} \tag{4}$$

Using the equation (5), we can calculate the elongation value based on our trained surrogate model, the distribution of which is shown as Fig (2b). Comparing the two distributions, it can be found that the surrogate model performs well on its trained data. But it doesn't mean the surrogate model performs well on other data, which requires further validation.

$$Y = \Phi X \tag{5}$$