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# Modeling Cascading Failure Propagation through Dynamic Bayesian Networks

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**Abstract:** Capturing dependencies in dependability studies is one of the most challenging tasks and requires suitable modeling techniques. In recent years the growing interest toward complex critical infrastructures and their interdependencies has solicited new efforts in the area of modeling and analysis of large interdependent systems. Cascading failures are a typical phenomenon of dependencies of components inside a system or among systems. New research in the area of the analysis of cascading phenomena has been also dictated by the recent occurrence of large scale electrical blackouts both in USA and in Europe that have caused the shortage of electrical power to millions of citizens. The present paper proposes to model cascading failures by means of Dynamic Bayesian Networks (DBN). In contrast with available techniques, DBN offer a good trade-off between the analytical tractability and the representation of the propagation of the cascading event. A failure scenario, taken from the literature, is considered as a final example.

*Keywords:* Cascading failure, Dynamic Bayesian Networks, Electrical Grid.

## 1. INTRODUCTION

Dependencies increase the risk of failure and the vulnerability in complex critical infrastructure and among infrastructures. As an example, most major power grid blackouts that have occurred in the past were initiated by a single event (or multiple related events) that gradually led to cascading failures and eventual collapse of the entire system [Pourbeik et al. (2006)]. Rinaldi et al. (2001) classify dependency-related disruptions or outages as cascading, escalating, or common cause. A cascading phenomenon occurs when the failure in one component of a system induces an overload in adjacent components increasing their failure probability. If the overload can be compensated by the strength of the adjacent components the cascade may be arrested, otherwise the cascade may become an avalanche causing a progressive and rapid disruption of all the system. Recent electrical blackouts that occurred both in USA and Europe are typical cascading phenomena, and since their effect has been catastrophic for millions of citizens, they have stimulated further research as also witnessed by the launch of public research programs in the EU (CRUTIAL: http://crutial.cesiricerca.it, IRRIIS: http://www.irriis.org) and USA (The Complex Interactive Networks/Systems Initiative (CIN/SI), funded equally by EPRI and U.S. Department of Defense (DOD)).

Developing modeling frameworks for understanding interdependencies among critical infrastructures and analyzing their impact is a necessary step for building interconnected infrastructures on which a justified level of confidence can be placed with respect to their robustness to potential vulnerabilities and disruptions. Modeling can provide useful insights of how component failures might propagate and lead to disturbances on the service delivered to the users.

The definition and implementation of a modeling framework for the propagation of cascading failures is an open problem in the study of the dependability analysis of critical infrastructures. Two approaches can be accounted in the literature: *i)* - a pure statistical approach; *ii)* - a phenomenological approach.

The aim of the *statistical approach* is to model how the appearance of a failure and the consequent overload can be redistributed, and to study the propagation of the cascade. A series of papers [Dobson et al. (2002, 2004, 2005)] propose a statistical model called *CASCADE* which is based on an abstract view of the cascading phenomenon. The system is assumed to be composed of many identical components randomly loaded. An initial disturbance causes some components to fail by exceeding the loading limit. The failure of a component causes a fixed load increase for other components. Models of the dynamic redistribution of the load have been explored also in [Motter and Lai (2002); Crucitti et al. (2004); Kinney et al. (2005)] where the load redistribution is based on the definition of node efficiency which is a centrality measure based on shortest paths computation. The idea of computing centrality measures using only the shortest paths has been criticized in [Newman (2005)] with the motivation that the intrinsic redundancy of the network is neglected.

In the *phenomenological approach*, an attempt is made to build a physical scenario that leads to the comprehension of the propagation of the cascading phenomenon. Typical examples of this approach in the literature can be found in [DeMarco (2001); Chowdhury and Baravc (2006); Lininger et al. (2007); Faza et al. (2007)]. In particular, Chowdhury and Baravc (2006) discuss the possible evolution of various failure scenarios related to the IEEE 118 bus test system. An effort to propose a formal approach to a failure scenario is discussed in [Faza et al. (2007)] resorting to a chain of conditional probabilities. The proposal is, however, incomplete: it does not take into account the dynamic of the cascade and it is not suitable for quantitative evaluations.

The present paper is aimed at showing that Bayesian Networks (BN) [Langseth and Portinale (2007)] - and more properly Dynamic BN [Montani et al. (2008)] - can provide a very valuable framework for modeling and quantitatively analyzing dynamically dependent failure phenomena as those arising in the propagation of cascading failures, while at the same time representing the evolution of the cascading phenomenon in a more physical way with respect to the abstract statistical models. After a brief illustration of the main features of BN in reliability, we introduce Dynamic BN (DBN) (Section 2), and we show how this formalism may capture the propagation of failure phenomena (Section 3). For the sake of illustration, we then apply the DBN model to one of the scenarios on the IEEE 118 bus test system, taken from [Chowdhury and Baravc (2006)], and we show how the model can be solved to provide quantitative results (Section 4).

## 2. BAYESIAN NETWORKS IN RELIABILITY

Traditional modeling approaches in system dependability may be classified as combinatorial or state-space. Since combinatorial models (like Fault trees [Sahner et al. (1996)]) assume that components are statistically independent, they cannot be invoked in the present case. State-space models (like Markov chains or Petri nets [Sahner et al. (1996)]) rely on the specification of the whole set of the possible system states so that the stochastic behavior of each component may depend on the state of all the other components. This flexibility is very seldom exploited in practice and the state space description appears over-specified with respect to the real modeling needs. Furthermore these models incur rapidly in the state space explosion.

An interesting trade off between combinatorial and state space models are Bayesian Networks (BN) [Langseth and Portinale (2007); Montani et al. (2008)]. We argue that BN are a possible suitable approach to model qualitatively and quantitatively cascading failures.

BN are a widely used formalism for representing uncertain knowledge in probabilistic systems and have been applied to a variety of real-world problems. BN are defined by a directed acyclic graph in which discrete random variables are assigned to nodes, together with the conditional dependence on the parent nodes. Root nodes are nodes with no parents, and marginal prior probabilities are assigned to them. Since each node can conditionally depend only on the parent nodes, BN provide an interesting framework to include localized dependencies like probabilistic gates, multi-state variables, dependent failures, uncertainty in model parameters. Furthermore, BN allow not only a forward (or predictive) analysis but also a backward (diagnostic) analysis, where the posterior probability of any set of variables can be computed.

Dynamic Bayesian Networks (DBN) extend the BN formalism by providing an explicit discrete temporal dimension [Weber and Jouffe (2003); Montani et al. (2008)]. They represent a probability distribution over the possible histories of a time-invariant process, thus allow to represent more complex kinds of dependencies with respect to BN. The advantage with respect to a classical probabilistic temporal model like Markov Chains, on the other hand, is that a DBN is a stochastic transition model factored over a number of random variables, over which a set of conditional dependency assumptions is defined. The assumption of time invariance ensures that the dependency model of the variables is the same at any point in time. Typically in a DBN, 2 time slices are considered in order to model the system temporal evolution, and a DBN represents a discretized Markov chain process.

Given a set of time-dependent state variables $X_1 \dots X_n$ and given a BN $N$ defined on such variables, a DBN is essentially a replication of $N$ over two time slices $t - \Delta$ and $t$ (being $\Delta$ the so called discretization step), with the addition of a set of arcs representing the transition model. Letting $X_i^t$ denote the copy of variable $X_i$ at time slice $t$, the transition model is defined through a distribution $P[X_i^t|X_i^{t-\Delta}, Y^{t-\Delta}, Y^t]$ where $Y^{t-\Delta}$ is any set of variables at slice $t - \Delta$ different than $X_i$ (possibly the empty set), and $Y^t$ is any set of variables at slice $t$ different than $X_i$ (possibly the empty set). Arcs interconnecting nodes at different slices compose the so-called inter-slice dependency model, while arcs interconnecting nodes at the same slice compose the intra-slice dependency model. The two slices of a DBN are often called the *anterior* and the *ulterior* layer.

Finally, it is useful to define the set of *canonical variables* as $\{Y : Y^{t-\Delta} \in \bigcup_k Parents[X_k^t]\}$; they are the variables having a direct edge from the anterior layer to another variable in the ulterior layer. A DBN is in *canonical form* if the canonical variables are represented only at slice $t-\Delta$ (i.e. the anterior layer contains only variables having influence on the same variable or on another variable at the ulterior layer). In the following, we will consider DBN in canonical form.

Given a DBN, inter-slice edges connecting a variable in the anterior layer to the same variable in the ulterior layer are called *temporal arcs*; in other words, a temporal arc connects variable $X_i^{t-\Delta}$ to variable $X_i^t$. Their role is in defining the nodes they are connecting as copies of the same variable at different slices.

Concerning the analysis of a DBN, different kinds of inference algorithms are available. In particular, let $X^t$ be a set of variables at time $t$ and $y_{a:b}$ any stream of observations from time point $a$ to time point $b$ (i.e. a set of instantiated variables $Y_i^j$ with $a \leq j \leq b$). The following tasks can be performed over a DBN:
- **Prediction**: computing $P(X^{t+h}|y_{1:t})$ for some horizon $h > 0$, i.e. predicting a future state taking into consideration the observation up to now (if $h = 0$ the task is more properly called **Filtering** or **Monitoring**);
- **Smoothing**: computing $P(X^{t-l}|y_{1:t})$ for some $l < t$, i.e. estimating what happened $l$ steps in the past, given all the evidence (observations) up to now.

Different algorithms, either exact (i.e. computing the exact probability value that is required by the task) or approximate can be exploited in order to implement the above tasks.

## 3. MODELING CASCADING FAILURES BY DYNAMIC BAYESIAN NETWORKS

In the following, we consider a portion of the IEEE 118 Bus Test Case described in [Chowdhury and Baravc (2006)] and depicted in Figure 1, which discusses various cascading failure scenarios. We will concentrate on one of them, namely the one that describes the sequence of events originated from the failure of line 5-4, until they propagate up to node 12, and we show how it can be tackled by using a DBN.

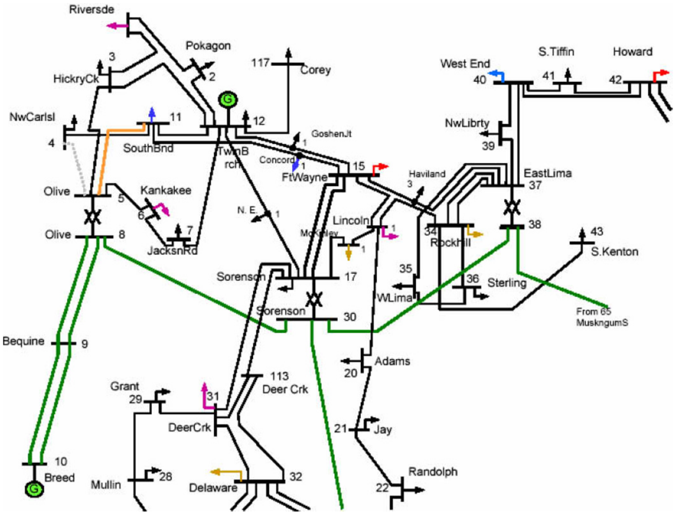


Fig. 1. The Electrical Grid of the IEEE 118 Bus Test Case [Chowdhury and Baravc (2006)]

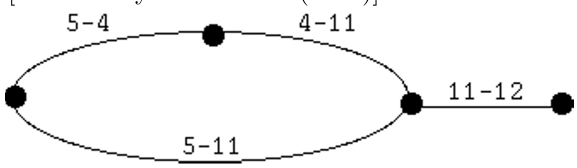


Fig. 2. Series/parallel diagram for the cascading failure example

For the sake of simplicity, we will concentrate on a subpart of the whole network described in [Chowdhury and Baravc (2006)], which is represented in the series/parallel diagram of Figure 2. As illustrated in the diagram, the subnet is composed by a parallel module in series with line 11-12. The parallel module is made by two branches, the first of which, in turn, is composed by a series of two lines (5-4 and 4-11), while the second coincides with line 5-11.

According to [Chowdhury and Baravc (2006)], we assume that each line can be in one of the following states: *working*, *outaged* (failed) or *overloaded*. Initially a line is working, and it can turn to the overloaded state as a consequence of the overload or the outage of another line. During the working state or the overloaded state, a line may fail. We suppose that the outage probability of all the lines is ruled by the negative exponential distribution, and that the failure rate has the same value $\lambda$ for all the working lines. Moreover, we assume that the value of the failure rate of a certain line is increased during the overloaded state. In particular, if the overload of the line $l_1$ is caused by the overload of the line $l_0$ on which $l_1$ depends, the failure rate of $l_1$ becomes $\alpha\lambda$. If instead the overload of $l_1$ is caused by the outage of $l_0$, the failure rate of $l_1$ becomes $\beta\lambda$. In other words, the failure probability of an overloaded line is greater than the failure probability of a working line. Such increase depends on the cause of the overload: $\beta > \alpha > 1$.

**Building the DBN model.** We propose a methodology for automatically converting a series-parallel diagram, like the one in Figure 2, into a DBN. The methodology proceeds in a modular fashion, by properly composing the translation of the simplest modules that can be found in the diagram, i.e. the *series* module and the *parallel* module, according to the rules explained in the following.

Figure 3.a shows the DBN translating a *series* module composed by two lines A and B (where the number of lines can be trivially extended to three or more). In the DBN, the inter-slice dependency model captures the following semantics: if line A becomes overloaded, it overloads line B at the following time slice. On the other hand, the node $S$, which summarizes the behavior of the whole module, is set as follows: if A or B fail, $S$ immediately fails. If A is overloaded, $S$ gets immediately overloaded; it can never happen that A works properly and B is overloaded. Finally, if A and B work properly, $S$ works properly.

Figure 3.b shows the DBN translating a *parallel* module composed by two lines A and B (where the number of lines can be trivially extended to three or more). The inter-slice dependency model captures the following semantics: if line A fails or becomes overloaded, it overloads line B at the following time slice, and vice-versa. On the other hand, the node $P$, which summarizes the behavior of the whole module, is set as follows: if A and B fail, $P$ immediately fails. If only A fails, $P$ works properly; the same holds if only B fails. If A gets overloaded, $P$ gets immediately overloaded; the same holds if B gets overloaded. Finally, if A and B work properly, $P$ works properly. Therefore, all variables in Figures 3.a and 3.b are three-state variables.

All non-trivial compositions of a parallel and a series modules are managed by three rules:
1) to connect a parallel module in series with a series module: the $P$ node summarizing the behavior of the first module affects the first line in the series module, in the inter-slice dependency model;
2) to connect any module (series or parallel) in series with a parallel module: the $S$ or $P$ node summarizing the behavior of the first module affects the $P$ node summarizing the behavior of the second module, in the inter-slice dependency model;
3) to connect a series module in parallel with any (series or parallel) module: the $S$ node summarizing the behavior of the first module affects the first line in the second module (if it is a series one), or all the first lines in the parallel branches (if the second module is in turn a parallel one), in the inter-slice dependency model.

Following such composition rules, we have been able to translate the diagram in Figure 2 into the DBN in Figure 4. By iterating the rules application, more complex models can be automatically translated as well.

It is worth noting that the work in [Torres-Toledano and Sucar (1998)] introduced an algorithm for automatically converting a Reliability Block Diagram (RBD) [Sahner et al. (1996)], which is also based on series and parallel connection types, into a BN. In particular, according to [Torres-Toledano and Sucar (1998)], lines in parallel have to be connected through an AND node (since the overall parallel circuit fails iff all the lines are outaged), while lines in series have to be connected through an OR node (since the overall series circuit fails if at least one of the lines is outaged). With respect to our methodology, such algorithm is limited to binary variables. On the other hand, we generalize the OR and AND nodes into the $S$ and $P$ nodes, which are multi-state (namely three-state) variables, able to account also for the overload condition.

Moreover, if a line gets outaged/overloaded, the lines affected by it get overloaded, and do not directly fail: they might fail subsequently. A BN, which could manage multi-state variables, cannot deal with this kind of temporal dependency, and this justifies our choice of relying on a DBN.

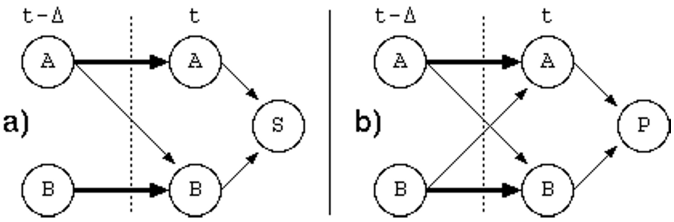


Fig. 3. a) DBN for the series module. b) DBN for the parallel module

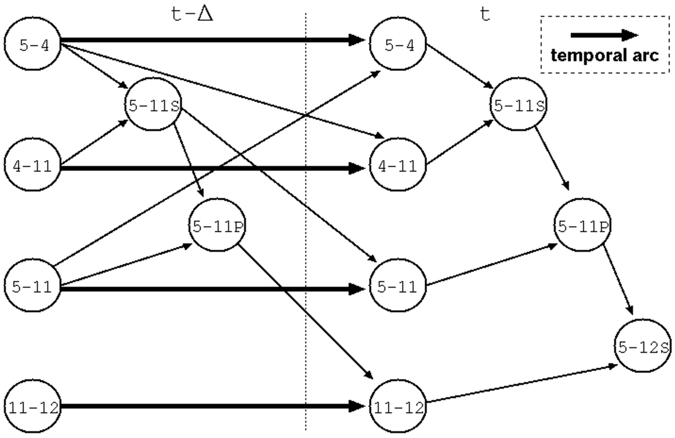


Fig. 4. DBN for the cascading failure example. An $S$ identifies a *series* node, and a $P$ identifies a *parallel* one (e.g. *5-11S* summarizes the module obtained by taking the series of 5-4 and 4-11, while *5-11P* summarizes the parallel between such a series and line 5-11 - see Figure 2)

The dependencies of a node are quantified in a DBN by means of Conditional Probability Tables (CPT). The probability in every table entry has to be set according to the state of the parent nodes (including the historical copy of the node at hand) at time $t - \Delta$. For instance, in the DBN model in Figure 4, the CPT expressing the probability that node *5-4* fails at time $t$, depending on its state at time $t-\Delta$, and on the state of its parent node *5-11*

| ***5-4* failed at $t$** | ***5-4* failed at $t-\Delta$** | ***5-4* over. at $t-\Delta$** | ***5-4* work. at $t-\Delta$** |
|---|---|---|---|
| ***5-11* failed at $t-\Delta$** | 1 | 1-$e^{-\beta\lambda\Delta}$ | 0 |
| ***5-11* over. at $t-\Delta$** | 1 | 1-$e^{-\alpha\lambda\Delta}$ | 0 |
| ***5-11* work. at $t-\Delta$** | 1 | * | 1-$e^{-\lambda\Delta}$ |

Table 1. CPT for node *5-4* failed (i.e. outaged), depending on its historical copy at time $t - \Delta$ and on its parent node *5-11* at time $t-\Delta$ (the symbol * stands for any value). $\lambda$ is the failure rate; $\alpha$ and $\beta$ are constants increasing $\lambda$ in case of overload

| ***5-4* over. at $t$** | ***5-4* failed at $t-\Delta$** | ***5-4* over. at $t-\Delta$** | ***5-4* work. at $t-\Delta$** |
|---|---|---|---|
| ***5-11* failed at $t-\Delta$** | 0 | $e^{-\beta\lambda\Delta}$ | 1 |
| ***5-11* over. at $t-\Delta$** | 0 | $e^{-\alpha\lambda\Delta}$ | 1 |
| ***5-11* work. at $t-\Delta$** | 0 | * | 0 |

Table 2. CPT for node *5-4* overloaded

<table>
<tr><th>5-4 working at $t$</th><th colspan="2">5-4 failed at $t-\Delta$</th><th colspan="2">5-4 over. at $t-\Delta$</th><th>5-4 work. at $t-\Delta$</th></tr>
<tr><td>5-11 failed at $t-\Delta$</td><td colspan="2">0</td><td colspan="2">0</td><td>0</td></tr>
<tr><td>5-11 over. at $t-\Delta$</td><td colspan="2">0</td><td colspan="2">0</td><td>0</td></tr>
<tr><td rowspan="2">5-11 work. at $t-\Delta$</td><td>$\mu = 0$</td><td>$\mu > 0$</td><td>$\mu = 0$</td><td>$\mu > 0$</td><td rowspan="2">$e^{-\lambda\Delta}$</td></tr>
<tr><td>0</td><td>$1 - e^{-\mu\Delta}$</td><td>*</td><td>1</td></tr>
</table>

Table 3. CPT for node *5-4* working, assuming the absence of repair ($\mu = 0$) or its presence ($\mu > 0$). $\mu$ is the repair rate

at time $t - \Delta$, is the one provided in Table 1. The CPT expressing the probability that node *5-4* is overloaded at time $t$ is provided in Table 2.

In Tables 1 and 2 it can never happen that *5-4* at time $t - \Delta$ is overloaded and *5-11* at time $t - \Delta$ is working - therefore any value could be inserted in the corresponding table entry. Moreover, in Table 1 the probability that *5-4* is failed at time $t$, given that *5-4* at time $t-\Delta$ is working and *5-11* at time $t-\Delta$ is failed, is 0. In fact, *5-4* needs to become overloaded before it fails.

**Modeling the repair.** The possibility of repairing outaged lines is a realistic option which can improve the overall availability of the system. The modeling approach presented in this paper can be extended in such a way that the DBN model can represent the repair of lines as well, according to a specific repair policy. For instance, if we suppose that a line undergoes repair as soon as it becomes outaged, then the repair can be modeled directly in the CPT of the corresponding node. Assuming that $\mu$ is the repair rate, the CPT expressing the probability that the node *5-4* is working (Table 3) shows how to model both the absence of repair ($\mu = 0$) and its presence ($\mu > 0$). However, in this paper, we suppose that there is no repair in the case study under exam.

**The cascading failure scenario.** In the net in Figures 2 and 4, several cascading failure scenarios may occur as a consequence of the outage of lines. We will concentrate on one of these scenarios, namely the one that describes the sequence of events originated from the failure of line 5-4, until they propagate up to the line 11-12: initially all the lines are working, so their failure rate is $\lambda$. If the outage of line 5-4 occurs, it influences the state of the series composed by 5-4 and 4-11, which becomes outaged in turn. This series is in parallel with the line 5-11, so the outage of the series determines the overload of the line 5-11: the failure rate of 5-11 is multiplied by the constant $\beta$. The overload state of line 5-11 determines in turn the overload of the whole parallel module whose branches are the series composed by 4-5 and 5-11, and the line 5-11. Since such module composes a series together with the line 11-12, the overload state of the parallel module causes the overload state of the line 11-12 in turn. As a consequence of the overload, the failure rate of the line 11-12 is multiplied by the constant $\alpha$.

In this situation, the series may fail if at least one among the parallel module and the line 11-12 fails. In particular, in the parallel module, the line 5-4 is already outaged, and the state of the line 4-11 has no effect on the rest of the subnet. Therefore the failure of the parallel module occurs if the line 5-11, currently in overload state, fails.

## 4. QUANTITATIVE RESULTS

In this section, we compute several prediction and smoothing measures (see Section 2) by inferencing the DBN model depicted in Figure 4 and representing the behavior of the subnet in Figure 2, as described in Section 3. The DBN model has been designed and analyzed by means of the Radyban tool [Montani et al. (2008)] assuming that the parameters $\lambda$, $\alpha$ and $\beta$ determining the failure rate of the lines in the working or overload state, are set to 0.0001 $h^{-1}$, 1.2 and 1.5 respectively. Moreover, we assume that all the lines are in the working state at the initial time. In particular, we consider the scenario deriving from the outage of the line 5-4 and described at the end of Section 3. We suppose that we observe the outage of 5-4 at time $t_1$=300 $h$ in all the experiments. Such observation is associated with the node *5-4*.

First, we predict the probability of the states of the lines 5-11 and 11-12[1], together with the general state of the subnet, for a mission time varying from 0 to 1000 $h$. To this aim, we make inference on the DBN model and we query the nodes *5-11*, *11-12* and *5-12S*.

The inference results for the general state of the subnet are reported in Table 4 where we can notice that the probability of the system to be working is zero for times higher than 300 $h$. This is due to the observation of the outage of the line 5-4 at time $t_1$=300 $h$: this event has several cascading effects causing the overload of the whole subnet, as described in Section 3. Before the outage of the line 5-4, the probability of the subnet to be overloaded is very small, but it becomes close to 1 just after the outage of the line 5-4, and decreases as time elapses because of the increase of the probability of outage of the subnet. A similar effect concerns the state probabilities of the lines 5-11 (Table 5) and 11-12 (Table 6).

The prediction results have been validated by modeling and analyzing the system also in form of *Generalized Stochastic Petri Nets* (GSPN) [Sahner et al. (1996)] where the observations have been represented by properly setting the initial marking of places. The fifth column in Tables 4, 5 and 6 reports the outaged state probability computed on the GSPN model.

[1] the state of the line 4-11 is not relevant because it composes a series with the outaged line 5-4.

| time | working | overloaded | outaged | outaged (GSPN) |
|---|---|---|---|---|
| 100 $h$ | 0.961076 | 0.028678 | 0.010246 | 0.010257 |
| 200 $h$ | 0.923389 | 0.055636 | 0.020974 | 0.010257 |
| 300 $h$ | 0.002972 | 0.922583 | 0.074445 | 0.077806 |
| 400 $h$ | 0 | 0.900898 | 0.099101 | 0.102372 |
| 500 $h$ | 0 | 0.876898 | 0.123102 | 0.126284 |
| 600 $h$ | 0 | 0.853537 | 0.146463 | 0.149559 |
| 700 $h$ | 0 | 0.830798 | 0.169202 | 0.172213 |
| 800 $h$ | 0 | 0.808665 | 0.191335 | 0.194265 |
| 900 $h$ | 0 | 0.787121 | 0.212878 | 0.215728 |
| 1000 $h$ | 0 | 0.766152 | 0.233848 | 0.236620 |

Table 4. Prediction of the subnet state prob.

| time | working | overloaded | outaged | outaged (GSPN) |
|---|---|---|---|---|
| 100 $h$ | 0.970638 | 0.019365 | 0.009996 | 0.009999 |
| 200 $h$ | 0.941950 | 0.038061 | 0.019989 | 0.019994 |
| 300 $h$ | 0.003063 | 0.953441 | 0.043497 | 0.044002 |
| 400 $h$ | 0 | 0.942262 | 0.057738 | 0.058235 |
| 500 $h$ | 0 | 0.928232 | 0.071767 | 0.072256 |
| 600 $h$ | 0 | 0.914412 | 0.085588 | 0.086069 |
| 700 $h$ | 0 | 0.900796 | 0.099203 | 0.099675 |
| 800 $h$ | 0 | 0.887384 | 0.112616 | 0.113080 |
| 900 $h$ | 0 | 0.874171 | 0.125828 | 0.126284 |
| 1000 $h$ | 0 | 0.861155 | 0.138844 | 0.139292 |

Table 5. Prediction of line 5-11 state prob.

| time | working | overloaded | outaged | outaged (GSPN) |
|---|---|---|---|---|
| 100 $h$ | 0.961367 | 0.028657 | 0.009976 | 0.009979 |
| 200 $h$ | 0.923672 | 0.056419 | 0.019909 | 0.009979 |
| 300 $h$ | 0.006043 | 0.961559 | 0.032397 | 0.035360 |
| 400 $h$ | 0.000096 | 0.955964 | 0.043939 | 0.046866 |
| 500 $h$ | 0.000095 | 0.944561 | 0.055344 | 0.058235 |
| 600 $h$ | 0.000094 | 0.933293 | 0.066612 | 0.069469 |
| 700 $h$ | 0.000094 | 0.922160 | 0.077746 | 0.088057 |
| 800 $h$ | 0.000093 | 0.911160 | 0.088747 | 0.091536 |
| 900 $h$ | 0.000092 | 0.900291 | 0.099617 | 0.102372 |
| 1000 $h$ | 0.000091 | 0.889551 | 0.110358 | 0.113080 |

Table 6. Prediction of line 11-12 state prob.

Besides predictions, we can compute smoothing measures on the DBN model. This can be useful in order to diagnose the possible causes of the system outage. Given the outage of the line 5-4, the outage of the whole subnet can be caused by the outage of the line 11-12, or by the outage of the line 5-11 (see Section 3). In order to evaluate the probability of such causes, we can execute the inference on the DBN model in Figure 4, but in this case we perform smoothing instead of prediction. We take into account further observations: the subnet is overloaded at time $t_1$=300 $h$ and is outaged at time $t_2$=800 $h$.

In the DBN model (see Figure 4), such observations are associated with the node *5-12S* in the representing the state of the whole subnet. According to this, we compute the probability to be working, overloaded or outaged, of the line 5-11 (Table 7), and of the line 11-12 (Table 8).

By the comparison of the values in such tables, we can notice that the probability of both lines to be working becomes zero after 300 $h$ because they become overloaded and eventually outaged as a consequence of the outage of the line 5-4 at time $t_1$=300 $h$. Moreover, we can notice that the probability of the line 5-11 to be outaged is a bit higher than the same probability concerning the line 11-12; this depends on the fact that the failure rate of the line 5-11 ($\beta\lambda$) is higher than the failure rate of 11-12 ($\alpha\lambda$), as described in Section 3 ($\beta > \alpha$).

| time | working | overloaded | outaged |
|---|---|---|---|
| 100 $h$ | 0.660737 | 0.339263 | 0 |
| 200 $h$ | 0.326283 | 0.673717 | 0 |
| 300 $h$ | 0 | 1.000000 | 0 |
| 400 $h$ | 0 | 0.882104 | 0.117896 |
| 500 $h$ | 0 | 0.765963 | 0.234037 |
| 600 $h$ | 0 | 0.651552 | 0.348448 |
| 700 $h$ | 0 | 0.538844 | 0.461155 |
| 800 $h$ | 0 | 0.427815 | 0.572185 |
| 900 $h$ | 0 | 0.421445 | 0.578554 |
| 1000 $h$ | 0 | 0.415170 | 0.584830 |

Table 7. Smoothing of line 5-11 state prob.

| time | working | overloaded | outaged |
|---|---|---|---|
| 100 $h$ | 0.664123 | 0.335877 | 0 |
| 200 $h$ | 0.329587 | 0.670413 | 0 |
| 300 $h$ | 0.003220 | 0.996780 | 0 |
| 400 $h$ | 0 | 0.905548 | 0.094452 |
| 500 $h$ | 0 | 0.812220 | 0.187780 |
| 600 $h$ | 0 | 0.720005 | 0.279995 |
| 700 $h$ | 0 | 0.628890 | 0.371110 |
| 800 $h$ | 0 | 0.538862 | 0.461138 |
| 900 $h$ | 0 | 0.532434 | 0.467566 |
| 1000 $h$ | 0 | 0.526083 | 0.473917 |

Table 8. Smoothing of line 11-12 state prob.

If we compare the probability values of the lines 5-11 and 11-12 in the prediction experiment (Tables 5 and 6) and in the smoothing experiment (Tables 7 and 8), we can notice that the outage probability for both lines is much higher in the smoothing experiment than in the prediction one. This is due to the observation in the smoothing experiment of the outage of the subnet at time $t_2$=800 $h$. Such event can only be caused by the occurrence of outage of the line 5-11 or of the outage of 11-12. The increase of the outage probability determines the decrease of the overload probability.

## 5. CONCLUSIONS

In this paper, we have shown how cascading failures can be modeled by means of the DBN framework. DBN are a powerful instrument for modeling and analyzing dynamically dependent failure phenomena. Moreover, they enable a representation of the evolution of the cascading phenomenon in a more physical way with respect to the abstract statistical models. We have also proposed a modular algorithm for automatically converting a series/parallel diagram into a DBN, which allows a quick DBN creation. The overall approach has been exemplified by means of an example, taken from [Chowdhury and Baravc (2006)]. In the future, we plan to further investigate the potentialities of DBN in this application domain, by taking into consideration more complex cascading failure scenarios. In such an analysis, we will also consider computational complexity issues. As a general comment, in DBN inference complexity depends on: (1) how much the net can be factorized; (2) how fine is the required discretization. We will evaluate how to deal with complexity in situations in which items (1) and (2) assume different values, also studying possible trade-offs between approximations and computation time (as regards e.g. item (2)).

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