A Flow-Based Optimization Framework to Simulate Interdependent Infrastructure Networks

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***Abstract*—Critical infrastructure networks are becoming increasingly interdependent, which adversely impacts their performance following disruptions due to the cascading effect of initial failures. There is a need to model the interdependent infrastructure networks, estimate their performance after disruptions and prioritize resources during restoration. However, empirical data of individual networks and their interdependency is typically hard to obtain. Furthermore, the existing network simulation methods are either not applicable to infrastructure networks or fail to fully characterize their features. This paper aims to build a framework to simulate interdependent infrastructure networks. Determining the location of the infrastructure facilities are mathematically formalized as an optimal distribution problem. An improved heuristic search method, annealing simulation with KD-tree search is proposed to solve the facility-location problem. A random graph algorithm tailored to infrastructure networks is developed to generate adjacency based on the prescribed degree distribution. Network flows are simulated by solving a mix-integer nonlinear optimization problem to minimize the overall cost considering multiple types of physical constraints. Several topology metrics are statistically analyzed for networks simulated with our framework and other existing methods. The results are compared with three real infrastructure networks to demonstrate the effectiveness of the proposed framework.**

***Index Terms*—Interdependent infrastructure networks, annealing simulation, mix-integer nonlinear optimization, statistical network analysis.**

I. Introduction

CRITICAL Critical infrastructure networks such as power grids, water and gas distribution networks, telecommunication networks, and multi-modal transportation networks provide essential services to a modern society [?], [?], [?]. As a result of economic and demographic development, these networks have grown in scale and complexity [?]. The interactions across the networks have become more critical [?], [?], [?]. For instance, power networks require water for cooling effect when generating electricity; pumping stations in water networks depend on electricity from power networks to extract water from nearby rivers [?]. As such, interdependencies between individual infrastructure networks guarantee their regular operations and connect them to form a system. Assessing performance of this system helps decision-makers to strategically allocate resources for mitigation and recovery of infrastructure networks post-disruption [?], [?], [?], which ensures the continuity of the economy development. Therefore, it is important to evaluate the system-level performance of interdependent infrastructure networks.

Assessing system-level performance of interdependent infrastructure networks relies on two essential factors: the analysis method and the system model to implement the method. Current literature is more focused on developing unique methods to capture system features [?]. These methods differ both in their research object, e.g., analyzing system vulnerability or system resilience [?], and in assumptions they make, e.g., treating physical interdependency as a conditional failure probability without any consideration of operational constraints [?]. Despite the great efforts devoted in devising analyzing methods, few works research on developing system models, and this is because scholars usually take real systems as testbeds to examine the effectiveness of their methods. In order to establish these system models, complete information of network topology and flow, as well as their interdependencies is required. Typically, this information is obtained from real data, e.g., topology of transportation networks is extracted from maps [?] and traffic flow is obtained by consulting state agencies [?], data of water and power networks can be gained from utilities. However, due to proprietary reasons and lack of documents, data for most infrastructure networks are not publicly available [?], [?], [?]. Some efforts have been dedicated to approach this issue and the most successful way is to simulate interdependent infrastructure networks []. The following summarizes five general types of simulation methods in the current literature and their similarity and difference are compared as well.

The first general type of methods [?], [?], [?], [?] considers interdependent power and water networks. They assume there are only two types of nodes: source nodes which generate and supply resources to demand nodes, demand nodes which receive and distribute resources to users. The source nodes are isolated and randomly distributed within the interval [0, 1], and the demand nodes are added one by one with random coordinates. Newly added demand nodes are connected via undirected links to geographically nearest existing nodes and the initial flow is simulated based on link betweenness. The physical interdependency between water and power networks is considered in all these methods. However, [] treats the interdependency as a conditional failure probability $Prob(W_i|P_j)$ which denodes a possibility that two nodes in different networks, water node i and power node j, enables failure to propagate from one node to the other []. When $Prob(W_i|P_j) = Prob(W_i)$, the interdependency strength between these two nodes is weak enough to be ignored while when $Prob(W_i|P_j) = 1$, the interdependency strength is

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extremely strong and the failure of power node j necessarily leads to the malfunction of water node i. The other three [] account the interdependency as an absolute dependency, i.e., water node i may not be operational unless its interdependent node, power node j, is also operational. The second method [] is the same as the first one [] in defining types of nodes and links, and determining node locations but in determining network connection, the second method add extra links to guarantee the connection of the simulated network and ensure that the degree distribution deviates little from the real infrastructure networks. The third method [] considers the interdependency between power and gas networks based on topological constraints, i.e., if there is none of paths from any gas pumps to the corresponding gas-based generator, then the generator deems to be failed. The network flow is simulated by considering its physical features , i.e., the gas pipeline model [] and the DC power flow model []. In the fourth method [], a virtual city 'Centerville' was designed to allow system researchers maximum flexibility in investigating performance of urban cities. Centerville is envisioned as being a typical middle-class city situated in a Midwestern State which has 7 types of water nodes and 5 types of power nodes. The network links are set up along the right-of-way for transportation networks. The last method [] assumes all nodes fall into one type, i.e., all facilities share the same purpose in serving the systems. The nodes are distributed with probability proportional to the clustering coefficient. The network connection is simulated based on both geographical proximity and preferential attachment.

Based on the above literature review, we have identified 5 gaps of the current network simulation methods. 1) Most of the methods [] only consider one or two types of infrastructure facilities but real networks have three or even more types of facilities, which have different functions and therefore require different modeling characterization. 2) Almost all methods except the last one assign facility locations randomly but in reality, physical space plays a significant role on constraining facility locations. For example, spatial distribution of community can influence where a particular water storage tank is constructed. 3) The first two methods [] cannot guarantee generated networks are connected: it is highly possible that there is an isolated supply node whose resources have nowhere to go, or there is a single component including only demand nodes where there are no resource input. Both of these two situations are inconsistent with reality. 4) Network flow is not fully characterized by existing methods: quantifying flow by the betweenness [**?**], [**?**], [**?**], [**?**] utilizes only the network topology information without physical constraints, e.g., whether the flow is conserved or satisfies community demands. 5) All methods model the interdependency ambiguously. Modeling interdependency as a conditional failure probability [**?**] fails to explain the statistical property of interdependency and the probability value is taken casually without any justification, Modeling interdependency as an absolute relationship or via topological constraints [**?**], [**?**], [**?**], [**?**] totally ignores the operational dependency, e.g., the malfunction of a gas-based generator may arise even when it is connected to a gas pump, as long as it cannot receive enough resource input.

This paper aims to fill the previous gaps by making the following contributions:

1) Within each infrastructure network, we classify facilities into three geenral types and model them separately using three different nodes.

2) We formalize the determination of node locations as an optimal distribution problem and solve it via annealing simulation with KD tree.

3) We abstract the structure of the infrastructure network as a two level bipartite graph and devising an algorithm to simulate the network topology.

4) We bring up a new type of interdependency: operational interdependency, differentiate it from the traditional physical interdependency and formalize it as physical constraints based on their own features.

5) We initialize the system flow by considering physical constraints within networks and interdependent constraints between networks, and solve a mixed integer nonlinear programming problem.

The remainder of this article is organized as follows: Section II interprets three basic infrastructure networks and the interdependency between them. Section III is devoted to the presentation of the methodological framework, including the simulation of network topology, interdependency and flow. A case study is presented in Section IV to demonstrate the framework and several graph metrics are introduced to compare the simulated networks against real networks. Concluding remarks are provided in Section V.

II. Interpretation of the interdependent infrastructure networks

In order to model infrastructure networks and analyze their performance such as vulnerability and resilience, information of network topology, system interdependency and flow moving intro or inter networks are required. The features of infrastructure networks cannot be fully characterized without clearly understanding the operational mechanism of each individual networks and their coupling effects or interdependency in the system. This section provides a detailed interpretation of the system operation, which lays the foundation for the declaration of modeling network topology, different interdependency and the initialization of flow in Section III. Among multiple types of infrastructure networks, gas, power and water networks are the most critical ones to be considered. As a result, the focus of this paper has primarily been on these three infrastructure networks. Their operations and interdependency are visualized as follows:

A. Subsection Heading Here

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III. Conclusion

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Appendix A
Proof of the First Zonklar Equation

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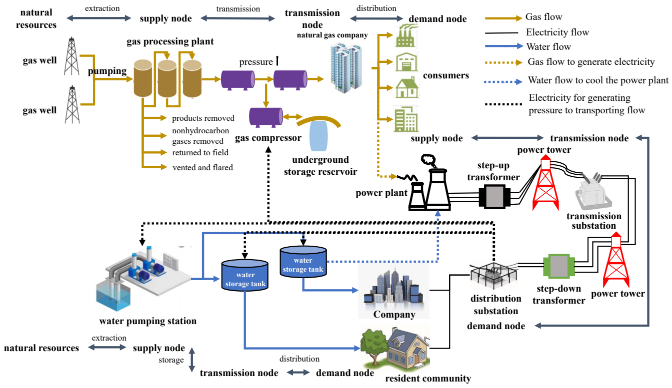

Fig. 1. The network topology and the connection pattern of the simulated Water-Power-Gas system

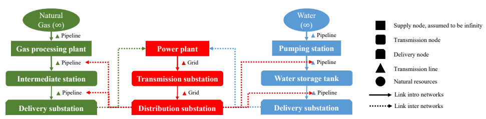

Fig. 2. Simulation results for the network.

APPENDIX B

Appendix two text goes here.

ACKNOWLEDGMENT

The authors would like to thank...

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Michael Shell Biography text here.

John Doe Biography text here.

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