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Optimal CEO Compensation with Search: Theory and Empirical Evidence

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ABSTRACT

We integrate an agency problem into search theory to study executive compensation in a market equilibrium. A CEO can choose to stay or quit and search after privately observing an idiosyncratic shock to the firm. The market equilibrium endogenizes CEOs' and firms' outside options and captures contracting externalities. We show that the optimal pay-to-performance ratio is less than one even when the CEO is risk neutral. Moreover, the equilibrium pay-to-performance sensitivity depends positively on a firm's idiosyncratic risk and negatively on the systematic risk. Our empirical tests using executive compensation data confirm these results.

TWO QUESTIONS CONCERNING EXECUTIVE compensation deserve particular attention. The first is: how does a firm's risk affect CEO's pay-to-performance sensitivity (hereafter PPS), that is, the ratio of incentive pay to firm performance? Standard agency models predict that PPS does not change with firm risk if the agent is risk neutral and decreases with firm risk if the agent is risk averse. Notable examples are Bolton and Dewatripont (2005, pp. 160–162), Holmstrom (1982), and Murphy (1999, pp. 27–28). In contrast to this theoretical prediction, however, the empirical evidence on the effect of firm risk on PPS is mixed. For example, Core and Guay (1999) and Oyer and Shaefer (2005) find a

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positive relationship while Aggarwal and Samwick (1999) document a negative relationship.[1]

The second question pertains to the large increase in CEO compensation that has accompanied the increase in firm size over the past three decades. This large increase has generated intense debate among both the public and academics as to whether CEOs are overcompensated. Although the increase in firm value contributed in part to the increase in CEO pay, a closer look at the data (see Section IV for a detailed description of the data) reveals that incentive pay, which is the predominant component of CEO pay, increased more rapidly than the increase in firm value. From 1994 to 2009, median incentive pay increased by 244% in real terms, compared with a 40% increase in median firm value, and its share of total pay increased from 41% to 78.8%. Further, total CEO pay outpaced firm value. The ratio between CEO pay and firm value increased from \$1.59 in 1994 to \$1.73 in 2009 per \$1,000. These observations suggest that the key to understanding the increase in CEO compensation is understanding what factors determine PPS.

We argue that two factors, both arising from the notion that executive contracts should be designed to maximize firm value in a market economy, are important for PPS. One is CEO job mobility. When different firms compete for CEOs, each firm has an incentive to design contracts to increase the probability of retention. Thus, changes in market conditions can affect PPS by affecting the severity of competition for CEOs. The second factor is the composition of risks faced by a firm. By switching from one firm to another, a CEO can change the amount of idiosyncratic risk to which he is exposed, but not aggregate systematic risk since all firms face the same systematic risk. PPS should therefore depend on these two types of risks differently.

To incorporate these factors, we integrate an agency model into search theory to determine incentive contracts in a market equilibrium, and then empirically evaluate the model. Search theory endogenizes CEOs' and firms' outside options and enables us to distinguish idiosyncratic risks from systematic risks. The integrated model captures the intuitive mechanism that competition among firms for CEOs affects incentive contracts in the equilibrium by affecting a CEO's incentive to participate in a firm. To distinguish the effect of competition on the incentive contract from the effect of risk aversion, we focus on risk-neutral and effort-averse CEOs.

In our model, there are many firms and CEOs. In each period, a firm's output depends on an aggregate shock, an idiosyncratic shock, and the CEO's effort. The aggregate shock is publicly observed while the idiosyncratic shock, which captures the match quality between the firm and the CEO at a particular point in time, is the CEO's private information. The firm offers an incentive contract that can be contingent on its output and the aggregate shock, but not directly on the idiosyncratic shock and the CEO's effort. The CEO decides whether to accept the offer after observing the idiosyncratic shock. If he quits, he can search for a new job. Due to the competition among firms, a CEO's outside option

[1] Prendergast (2002) summarizes additional conflicting empirical evidence on this relationship.

depends on the probability of getting a new job and the compensation at the new job. This link between a CEO's outside option and other firms' contracts implies that a market equilibrium must determine all firms' contracts and agents' outside options simultaneously. We focus on a stationary and symmetric equilibrium in which all firms offer the same type of contracts.

To determine the equilibrium, we first analyze an individual firm's optimal contract under arbitrarily fixed outside options for CEOs and firms. The value of the outside option to a contract participant is defined as the difference between the value of quitting and searching on the one hand and the future value of staying in the contractual relationship on the other. We prove that the optimal PPS is less than one, in spite of a risk-neutral CEO. This result arises because a CEO can choose whether to quit after privately observing the idiosyncratic shock. If the idiosyncratic shock is contractible or the CEO is forbidden to quit, the optimal contract would set PPS to one, as is well known in agency models with a risk-neutral agent. Such a contract would align the CEO's effort perfectly with the objective of maximizing the joint surplus of the match, and the firm would vary the base wage with the idiosyncratic shock to obtain the maximum share of the joint surplus. However, because the idiosyncratic shock is the CEO's private information, it is not feasible to make the base wage contingent on such a shock. The CEO will choose to stay to obtain the high payoff when the idiosyncratic shock is high, and will quit to insulate himself from the low payoff when the shock is low. In this setting, it is optimal for the firm to set PPS below one to get part of the high surplus when the idiosyncratic shock is high and compensate for the low payoff when the CEO quits. In fact, the firm chooses PPS and the base wage to obtain the optimal trade-off between the retention probability and the expected profit conditional on retention.

When the optimal PPS is below one, it can be affected by aggregate and idiosyncratic risks. When the outside options are arbitrarily fixed, the two risks have the same qualitative effect on PPS. Specifically, they affect PPS negatively if and only if the joint value of the CEO's and the firm's outside options for the current period is positive.[2] This effect of the risks on PPS arises from a new mechanism in our model whereby a firm trades off retention and profit conditional on retention, not from risk aversion as in the standard agency models cited above. To see this, consider the case in which the risk (either aggregate risk or idiosyncratic risk) increases. An increase in the risk increases a firm's expected profit conditional on retention, which increases a firm's incentive to retain the CEO. However, when the value of the CEO's outside option is high, the probability of retaining the CEO is low. Therefore, it is optimal for the firm to increase retention probability by increasing the base wage and reducing PPS. The opposite holds when the risk decreases. Thus, overall, PPS is negatively related to aggregate and idiosyncratic risks under positive outside options.

[2] A negative joint value of outside options means that the joint value of breaking the relationship now falls below the joint *future* value of continuing the relationship. In an intertemporal setting, positive and negative joint values of outside options are both possible.

Next, we endogenize the outside options, determine the market equilibrium, and explore new predictions of the equilibrium. In contrast to the partial equilibrium with fixed outside options, the two risks now have opposite effects on PPS in the equilibrium. This difference stems from two externalities due to endogenizing outside options: the interactions between firm contracts and the dependence of the matching probability on the contracts through competitive entry of vacancies. Under these externalities, the joint value of a firm's and a CEO's outside options increases with idiosyncratic risk and decreases with aggregate risk. The intuition is as follows. An increase in idiosyncratic risk increases the dispersion in match value, which induces both the CEO and the firm to search for a new match with a higher expected profit. In contrast, an increase in aggregate risk increases the profit of all firms uniformly, and thus reduces the motivation for the CEO and the firm to search. This link between risks and outside options serves as a bridge between risks and PPS. Specifically, an increase in idiosyncratic risk increases the CEO's outside option, which intensifies the competition among firms for CEOs. As a result, firms must increase the equilibrium PPS so that the CEO can capture more of the surplus and is less likely to quit.[3] In contrast, an increase in aggregate risk reduces the CEO's incentive to search, which weakens the competition and triggers firms to lower the equilibrium PPS. It should be noted that the opposite effect of the two risks on the optimal PPS is unique to the market equilibrium with search. When the outside options are exogenous as commonly assumed in the agency literature, PPS responds to the two risks in the same direction, as discussed.

Finally, we empirically test two new predictions of our model. First, the equilibrium PPS depends negatively on systematic risk and positively on idiosyncratic risk. Second, because PPS responds to the two risks differently, so does the ratio of a CEO's total compensation to firm value in the equilibrium. This ratio depends negatively on systematic risk and positively on idiosyncratic risks. The empirical tests find robust support for these predictions.

We contribute to the labor search literature (e.g., Mortensen and Pissarides (1994)) by integrating incentive contracts into a search model to examine CEO compensation and empirically testing the model's implications.[4] To the principal-agent literature (e.g., Bolton and Dewatripont (2005) and references therein), our paper contributes along three dimensions. First, we explicitly model CEOs' quitting decisions and study incentive contracts that induce both optimal effort and optimal retention. Second, we endogenously determine the effects of market conditions on a CEO's outside option. Third, we analyze the optimal contract in a dynamic equilibrium in which firms interact in the CEO job market. This dynamic equilibrium structure contrasts with typical agency

[3] An alternative way to retain a CEO is to increase his salary. But in equilibrium increasing salary is not an effective way to retain the CEO for two reasons. First, while a firm only wants to retain a CEO with a high matching quality, increasing salary would increase the retention probability of all types of CEOs. Second, increasing PPS can also increase the optimal effort level, which is particularly valuable when the realized matching quality is high.

[4] In a summary of the new perspectives of search theory, Shi (2008) points out that integration of contract theory with search theory is a promising research agenda.

models, such as Jin (2002) and Garvey and Milbourn (2003), that analyze the optimal contract in a static setting with a single agent–firm pair where the joint outside option value is assumed to be positive since the CEO has a positive reservation utility while the principal has zero reservation value. The common conclusion of these studies is that PPS decreases with idiosyncratic risk. The negative effect of idiosyncratic risk on PPS is consistent with our partial equilibrium analysis when the joint outside option value is positive. However, in a dynamic equilibrium setting such as that in our model, the joint outside option value for the current period can be either positive or negative. Specifically, when the future value of continuing the match is higher than the value of breaking up the match, the joint outside option value for the current period is negative, in which case PPS increases with idiosyncratic risk. More importantly, our market equilibrium analysis shows that the equilibrium PPS increases with idiosyncratic risk due to contracting externalities. This new result offers a possible explanation for the mixed evidence on the empirical relationship between a firm's total risk and PPS.[5]

Our paper is also related to Oyer (2004) and Edmans, Gabaix, and Landier (2009). Similar to our model, Oyer (2004) recognizes that an agent may choose not to participate in a contract in certain states of the world. However, he assumes that the outside option is exogenous and he does not study a market equilibrium. Moreover, he studies broad-based stock option plans for lower-ranked workers and abstracts from the effort-inducing mechanism on the ground that such plans have limited incentive effects on workers. Edmans, Gabaix, and Landier (2009) use the same assumption as we do, that the shocks and the agent's effort are multiplicative in a firm's profit function, but they study a different mechanism (i.e., positive assortative matching) and their objective is to explain the negative relationship between the CEO's effective equity stake and firm size; they do not analyze the effects of risks on PPS.

The rest of the paper is organized as follows. Section I describes the model, formulates individuals' decision problems, and defines the market equilibrium. Section II examines the optimal contract under fixed outside options. Section III discusses the contracting externalities and determines the market equilibrium. Section IV presents the empirical analyses, and Section V concludes the paper. Proofs and tables are provided in the Appendices.

I. A Search Market with Incentive Contracts

In this section we describe the environment of the model economy, set up individual CEOs' and firms' decision problems, and define the market equilibrium.

[5] A few other studies focus on the relationship between firm risk and PPS from different perspectives. For example, Shi (2011) differentiates respondable and nonrespondable risk. Guo and Ou-Yang (2006) focus on the wealth effects while Bhattacharyya and Lafontaine (1995) focus on the role of risk in franchising.

A. The Environment of the Model Economy

Consider an infinite-horizon economy in discrete time. There are many infinitely lived CEOs whose measure is normalized to one. In each period, a CEO is either employed or unemployed. If a CEO is unemployed, he receives utility B in the period, which includes the utility of extra leisure as well as monetary benefit during unemployment. In addition, the CEO can search for a job. If a CEO is employed, he chooses the level of effort to exert on the job, e, and earns income w. Utility in the period is $u(w, e) = w - \frac{c}{2}e^2$, where the constant $c > 0$ reflects a CEO's effort aversion. Note that a CEO is risk neutral in income. This assumption ensures that risk aversion is not a determinant of PPS, as emphasized in the agency literature. Instead, we focus on a new mechanism that centers on the interactions between firms in the market equilibrium.

There are also many firms whose measure is endogenously determined by job creation. In each period, a firm either has or does not have a CEO. A firm without a CEO can incur a recruiting cost H to search for a CEO. If a firm has a CEO, profit in the period before paying the CEO is

$$\pi \equiv \pi(e, x, y) = ey^{\alpha} x^{1/2}, \quad \alpha > 1/2, \tag{1}$$

where x is a shock specific to the firm–CEO pair in the period and y is an aggregate or systematic shock in the period. The idiosyncratic shock x is identically and independently distributed on $[\underline{x}, \bar{x}]$ across matches and over time, where $\bar{x} > \underline{x} > 0$. We assume that the cumulative distribution function, $F_1(x)$, is uniform so that the mean of x is $\mu_x = (\underline{x} + \bar{x})/2$ and the standard deviation is $\sigma_x = (\bar{x} - \underline{x})/(2\sqrt{3})$. The aggregate shock y is identically and independently distributed over time according to the cumulative distribution function $F_2(y)$, with mean μ_y and standard deviation σ_y^2. Note that the profit function is multiplicative between effort e and the shocks (x, y), which captures the intuitive notion that the marginal productivity of effort is higher when a firm experiences higher shocks.[6] When CEOs are risk neutral, this multiplicative specification is necessary for a nontrivial analysis. In particular, if the profit function is additive between effort and the shocks, then the shocks do not affect the optimal choice of effort and, with risk neutrality, this implies that the risks generated by the shocks have no effect on PPS.

Idiosyncratic risk can be understood as the match quality between the CEO and the firm in the current period, rather than a permanent characteristic of the firm, the CEO, or the match. For example, a high match quality means that a CEO's talent, experience, education, and personal objective match well in this particular period with the firm's size, nature of business, strategic direction, organizational culture, and so on. A CEO who is well matched with a firm at one point in time may not be well matched with the firm at another point in

[6] For example, in the literature on CEO compensation, Edmans, Gabaix, and Landier (2009) promote the multiplicative specification by arguing that a majority of CEO actions are "rolled out" across the entire company and hence have a greater effect in a larger firm. It is useful to note that a generalization of the profit function is $\pi = ey^{\alpha}x^{\beta}$. The analytical results are the same for all $\beta \geq 1/2$, but the algebra is simpler with $\beta = 1/2$.

time if one of the above-mentioned CEO or firm characteristics has changed.[7] To capture the realistic feature that a CEO might have a better idea than a firm about the match quality, we assume that the realization of the idiosyncratic shock x is a CEO's private information. This assumption on x is central to the results in this paper because a CEO's quitting decision is nontrivial only when x is noncontractible, as we demonstrate later.

A firm offers a sequence of one-period contracts to the CEO.[8] As in the literature, effort is a CEO's private information and not verifiable. In contrast, the aggregate shock y and profit π are publicly observed. However, knowing π and y is not sufficient for an outsider to disentangle effort e and the idiosyncratic shock x. To simplify the analysis and to facilitate comparison with well-known models, we assume that the contract in each period has the following linear form:

$$w = w(\psi, \pi) = a + b\pi, \quad \text{where } \psi \equiv (a, b).$$

That is, total CEO compensation consists of a base wage, a, and a profit-sharing payment, $b\pi$. The profit-sharing ratio, b, is referred to as PPS. Note that a and b can be functions of y but not of the unobservable e and x. For brevity, we refer to ψ as a contract.

To clarify the elements of the economy, we depict the timing of events in each period in Figure 1.

A period consists of four stages. The first stage is exogenous separation in which a CEO separates exogenously from the firm into unemployment with probability $\delta \in [0, 1)$. This exogenous separation represents the turnover of CEOs caused by reasons other than those modeled explicitly here, such as job separation caused by family relocation. The second stage involves contract offers and quitting decisions. In this stage, a firm with a CEO offers a contract to the CEO, after which the idiosyncratic shock x is realized and the CEO then chooses whether to accept the contract or to quit and become unemployed. The third stage is effort choice and production, whereby the aggregate shock y is realized, a CEO who stays with the firm chooses effort, profit is generated, and the CEO is paid according to the contract.[9] The fourth stage is search and

[7] We focus on idiosyncratic heterogeneity occurring ex post rather than ex ante. It is well known that a large fraction of the wage differential among workers cannot be explained by observable heterogeneity (see Mortensen (2005)). This is also likely to be the case for managers. An excellent CEO in a mining firm may or may not be a good CEO in a software firm. Specifically, Graham, Harvey, and Puri (2010) document evidence that CEOs' personal or behavioral traits such as optimism and managerial risk-aversion are related to corporate financial policies. They also show that certain types of firms appear to attract executives with particular psychological profiles and that CEOs' behavioral traits help explain compensation structure.

[8] In Appendix C, we add a long-term retention reward to the contract, which resembles the extend of option grants in practice. We show that the qualitative features of the incentive contract remain the same. We do not consider fully dynamic (recursive) contracts because such contracts would be extremely difficult to deal with.

[9] Whether y is realized before or after a CEO chooses effort matters only slightly for the analysis. If y is realized before the effort choice, y affects the decisions through $\mathbb{E}(y^{2\alpha})$, as shown in Sections II and III. If y is realized after the effort choice, y affects the decisions through $[\mathbb{E}(y^{\alpha})]^2$. If $\alpha \neq 1$ and if α is not too small, then the two terms have the common property that they increase in the variance of y. This is the property we need in the analysis.

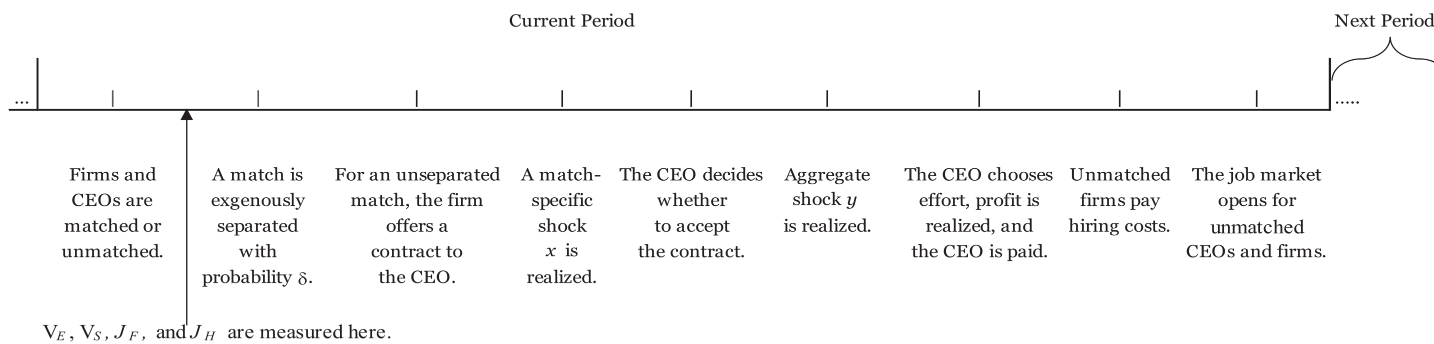

Figure 1. Timing of events in each period. In the above figure, V_E is the value function of an employed CEO, V_S is the value function of a CEO searching for a job, J_F is the value function of a firm with a matched CEO, and J_H is the value function of a firm with a vacant CEO position.

matching. Here, an unmatched CEO receives the benefit B and searches for a match, while a firm without a CEO pays the recruiting cost H to seek a CEO. Entry of vacancies is competitive. After search and matching, the period ends and another period starts.

The matching process is modeled as in Mortensen and Pissarides (1994). Denote by v the number of vacancies and by s the number of searching CEOs at the search/matching stage of a period. The total number of new matches is given by the matching function $m(v, s) = vs/(v + s)$.[10] Denote job market tightness by $\theta = s/v$, the matching probability of a searching CEO by $\lambda \equiv m(v, s)/s$, and the matching probability of a vacancy by $q \equiv m(v, s)/v$. We have

$$\lambda = 1/(1 + \theta) \quad \text{and} \quad q = \theta/(1 + \theta) = 1 - \lambda.$$

These expressions reflect the intuitive property that, when there are more searching CEOs per vacancy, the matching probability is lower for a searching CEO and higher for a vacancy. Each CEO or firm takes the tightness and matching probabilities as given, because these characteristics depend only on the aggregate numbers of vacancies and searching CEOs.

B. Decisions of Individual CEOs and Firms

In each period, decisions are made in the following order: firms choose contracts, CEOs choose whether to quit or stay, and in the case CEOs stay, they choose the level of effort to exert. We analyze these decisions recursively in this subsection. In making their decisions, an individual firm or CEO takes other firms' and CEOs' choices as given. Also, because a contract is assumed to be a sequence of one-period contracts, individuals take as given future contracts that affect the current period's choices only through the future value functions.

We first examine the optimal choice of effort by a CEO who has chosen to stay with the firm in the current period. Given the contract $\psi = (a, b)$ and the realizations of (x, y), the CEO chooses effort e to maximize utility $u(w, e)$, where $w = a + b\pi(e, x, y)$. Under the specified forms of u and π, the optimal choice of effort is given by the following first-order condition:

$$e^* = e^*(\psi, x, y) \equiv by^{\alpha}x^{1/2}/c. \tag{2}$$

As expected, optimal effort decreases in the effort-aversion parameter c, increases in PPS, and is independent of the base wage. In addition, because effort and the shocks are multiplicative in the profit function, higher shocks induce higher effort. Given any contract ψ and the induced effort e^*, we denote profit, the CEO's income, and the CEO's utility, respectively, as follows:

[10] The specific matching function has constant returns to scale and is strictly concave in the two arguments, v and s. The intuition for the main results of our paper should hold for more general matching functions, but the algebra becomes more complicated.

$$\pi^* = \pi^*(\psi, x, y) \equiv \pi(e^*(\psi, x, y), x, y) = by^{2\alpha}x/c,$$
$$w^* = w^*(\psi, x, y) \equiv a + b\pi^* = a + b^2y^{2\alpha}x/c,$$
$$u^* = u^*(\psi, x, y) \equiv a + \frac{1}{2}b^2y^{2\alpha}x/c. \tag{3}$$

Next, we examine a CEO's quitting decision after observing x. If the CEO chooses to quit, he becomes unmatched. The value of this unmatched CEO is the same as that of a CEO who is unemployed at the beginning of the period, which is denoted by V_S. If the CEO chooses to stay with the firm, his utility in the current period is $\mathbb{E}_y(u^*)$, where $\mathbb{E}_y$ denotes the expectation over y. In addition, the CEO will start the next period as matched, the value of which is denoted by $V_{E,+1}$, where the subscript $+1$ indicates the next period. The CEO accepts the contract if and only if $\mathbb{E}_y(u^*) + \beta V_{E,+1} \geq V_S$, where $\beta \in (0, 1)$ is the discount factor. We write this acceptance condition as

$$\mathbb{E}_y[u^*(\psi, x, y)] \geq \underline{u} \equiv V_S - \beta V_{E,+1}. \tag{4}$$

Let us call $\underline{u}$ the *CEO's (effective) outside option for the current period*. We substitute u^* from (3) and express (4) as a cutoff rule on the idiosyncratic shock x. That is, the CEO accepts the contract if and only if the realization of x satisfies $x \geq \rho\bar{x}$, where the cutoff ratio ρ is

$$\rho(\psi, \underline{u}) \equiv \frac{2c[\underline{u} - \mathbb{E}_y(a)]}{\bar{x}\,\mathbb{E}_y(b^2y^{2\alpha})}. \tag{5}$$

We keep a and b^2 inside the expectation operator $\mathbb{E}_y$ because in principle these terms can be contingent on y. As expected, the cutoff is higher and quitting is more likely if the CEO's effective outside option for the period is higher. Also, a more generous base wage and a higher PPS both reduce the cutoff and make the CEO less likely to quit, provided $\rho > 0$.

Let us compute the value function of a CEO. If a CEO enters a period as matched, the value function is V_E. If the CEO separates from the firm, either exogenously or endogenously, the CEO obtains the value V_S. If the CEO is not separated from the firm in the current period, the additional value above V_S that the CEO obtains is $u^* + \beta V_{E,+1} - V_s = u^* - \underline{u}$. Because the CEO works for the firm if and only if he is not separated from the firm exogenously and if the realization of x is no less than $\rho\bar{x}$, a matched CEO's value obeys

$$V_E = V_S + (1-\delta)\int_{\bar{x}\rho(\psi,\underline{u})}^{\bar{x}} \mathbb{E}_y[u^*(\psi, x, y) - \underline{u}]\, dF_1(x). \tag{6}$$

The integral over x reflects the fact that x is not realized when V_E is measured. If a CEO enters a period as unmatched, he receives utility B and starts searching. With probability λ, the CEO finds a match at the end of the period, in which case the CEO enters the next period as a matched CEO whose discounted value is $\beta V_{E,+1}$. With probability $1-\lambda$, the CEO fails to find a match, in which case the CEO's discounted value is $\beta V_{S,+1}$. Thus, V_S obeys

$$V_S = B + \lambda\beta V_{E,+1} + (1-\lambda)\beta V_{S,+1}. \tag{7}$$

Now we examine a firm's contract offer and value function. Let J_F and J_H denote the value of a firm that enters the period with and without a CEO, respectively. Denote by $\underline{J} = J_H - \beta J_{F,+1}$ a *firm's (effective) outside option for the period*. We derive J_F similarly to V_E. For a firm that starts the period with a CEO, the firm may lose the CEO through exogenous separation or endogenous quits in the period, in which case the firm's value is J_H. If the CEO stays, the firm obtains net profit in the current period, $\pi^* - w^*$, and the discounted value in the future, $\beta J_{F,+1}$. The additional value above J_H is $\pi^* - w^* + \beta J_{F,+1} - J_H = \pi^* - w^* - \underline{J}$. Thus, J_F obeys

$$J_F = J_H + (1-\delta)\max_{\psi} \int_{\bar{x}\rho(\psi,\underline{u})}^{\bar{x}} \mathbb{E}_y[\pi^*(\psi,x,y) - w^*(\psi,x,y) - \underline{J}]\,dF_1(x). \tag{8}$$

We denote the optimal contract for the maximization problem in (8) as $\psi^*(\underline{u},\underline{J},y)$ to emphasize its potential dependence on the two sides' outside options $(\underline{u},\underline{J})$ and the aggregate shock. Notice that (8) incorporates the CEO's participation decision through the cutoff rule $\rho(\psi,\underline{u})$ and incentive compatibility of the effort choice through $e^*(\psi,x,y)$, which is embedded in (π^*, w^*).

The value of a firm with a vacant CEO position, J_H, can be computed similarly to V_S. The firm incurs a cost H to recruit in the period. With probability q, the firm will get a match in the period, in which case the firm's value will be $\beta J_{F,+1}$. With probability $1-q$, the firm will fail to get a match in the period, in which case the firm's value will be $\beta J_{H,+1}$. Thus,

$$J_H = -H + \beta[qJ_{F,+1} + (1-q)J_{H,+1}]. \tag{9}$$

C. Definition of a Market Equilibrium

Because the outside options depend on the matching probabilities, which are functions of market tightness, we need to determine the number of searching CEOs, s, and the number of firms with vacant CEO positions, v. These numbers are measured immediately before the search process starts (see Figure 1). Free entry of vacancies determines v. To determine s, we compute the change in the number of searching CEOs between the beginning of the search stages in the current and next periods, $s_{+1} - s$. In the current period, the number of new matches created and hence the flow out of the group of searching CEOs is λs, where λ is the matching probability for a searching CEO. Exogenous separation and endogenous quits in the next period generate the flow into the group of searching CEOs. This inflow is $(1-s+\lambda s)[\delta + (1-\delta)F_1(\bar{x}\rho_{+1})]$, where $1-s+\lambda s$ measures the number of CEOs in matches at the beginning of the next period and $\delta + (1-\delta)F_1(\bar{x}\rho_{+1})$ measures the probability with which a matched CEO will become unemployed through exogenous separation and endogenous quits in the next period. Thus,

$$s_{+1} - s = (1 - s + \lambda s)[\delta + (1 - \delta)F_1(\bar{x}\rho_{+1})] - \lambda s. \tag{10}$$

Notice that the number of firms is endogenously determined as $v + 1 - s$, where v is the number of firms that are searching for CEOs and $(1 - s)$ is the number of CEO-staffed firms.

We define an equilibrium as follows. A stationary and symmetric *market equilibrium* consists of an individual firm's contract $\psi^*(\underline{u}, \underline{J}, y)$, an individual CEO's quitting rule $\rho(\psi, \underline{u})$ and effort rule $e^*(\psi, x, y)$, other firms' contracts $\tilde{\psi} = (\tilde{a}, \tilde{b})$, other CEOs' choices $(\tilde{\rho}, \tilde{e}^*)$, value functions (V_E, V_S, J_F, J_H), the implied effective outside options $(\underline{u}, \underline{J})$, and the numbers of searching CEOs and searching firms, (s, v), such that the following requirements are satisfied:

(i) given any $(\psi, \underline{u})$, a CEO's decision rules $e^*(\psi, x, y)$ and $\rho(\psi, \underline{u})$ satisfy (2) and (5);
(ii) given any $(\underline{u}, \underline{J})$, a firm's contract $\psi^*(\underline{u}, \underline{J}, y)$ solves the maximization problem in (8);
(iii) the value functions satisfy (6), (7), (8), and (9), while the effective outside options are given as $\underline{u} = V_s - \beta V_{E,+1}$ and $\underline{J} = J_H - \beta J_{F,+1}$;
(iv) symmetry: $\psi = \tilde{\psi}$ and $(\rho, e^*) = (\tilde{\rho}, \tilde{e}^*)$;
(v) the number of searching CEOs, s, obeys the dynamics in (10);
(vi) competitive entry of vacancies: v is such that the net value of creating a vacancy is $J_H = 0$; and
(vii) stationarity: $s_{+1} = s$, $J_{F,+1} = J_F$, $J_{H,+1} = J_H$, $V_{E,+1} = V_E$, and $V_{S,+1} = V_S$.

We focus on a stationary equilibrium because the economy described in Section I.A is stationary. We focus on a symmetric equilibrium because all CEOs and firms conditional on their status (employed or unemployed, with or without a CEO, respectively) are ex ante homogeneous. Note that stationarity and symmetry imply only that the decision rules are time-invariant and symmetric between individuals/firms in the same status. Since these rules are functions of the realizations of the shocks, the values of a firm's or a CEO's choices can still vary from period to period as different realizations of the shocks make firms or CEOs heterogeneous ex post.

The model presented here differs from a standard static agency model with a single agent–firm pair along three dimensions: (i) the model distinguishes a firm's idiosyncratic risk from systematic risk, as opposed to lumping them together as the firm's total risk; (ii) a CEO can choose to quit after privately observing idiosyncratic risk; and (iii) there are contracting interactions/externalities among firms in the market equilibrium that work through endogenous outside options. In the next section we explore the importance of elements (i) and (ii) by analyzing the optimal contract under any arbitrarily fixed outside options, $(\underline{u}, \underline{J})$. In Section III, we examine the role of element (iii) and determine the equilibrium.

II. Optimal Contract under Fixed Outside Options

In this section we determine the optimal contract when the effective outside options for the current period, $(\underline{u}, \underline{J})$, are fixed. We also analyze how the two risks and the quitting decision affect the optimal contract. Even with fixed $(\underline{u}, \underline{J})$, our model differs from a static agency model not only in the presence of the two risks and the allowance for quitting, but also in the admissible region of the outside options. A static model assumes that $(\underline{u}, \underline{J})$ are nonnegative. This assumption is unlikely to be valid in an intertemporal setting because the future value of a match can exceed the value of breaking up the match, in which case the outside option for the current period is negative. Specifically, a firm's outside option for a period, $\underline{J} = J_H - \beta J_{F,+1}$, is negative because $J_H = 0$ by the free-entry condition of vacancies. A CEO's outside option for a period, $\underline{u} = V_S - \beta V_{E,+1}$, can also be negative when the discount factor is sufficiently close to one. Thus, the case with $\underline{u} + \underline{J} < 0$ is the normal case in the equilibrium. For generality in this section and for comparison with the literature, we do not restrict the sign of $\underline{u} + \underline{J}$.

A. The Optimal Contract

As is well known from standard static agency models in which a firm's profit is contractible and the manager's effort is not, the optimal contract is to set $b^* = 1$ for a risk-neutral manager who does not have limited liability (e.g., Bolton and Dewatripont (2005, pp. 160–162), Holmstrom (1982), and Murphy (1999, pp. 27–28)). However, the optimal contract in our setting is very different from this well-known result because the risk-neutral CEO can quit and the idiosyncratic shock x is not publicly observable, despite the fact that the firm's profit and the aggregate shock are contractible.

We now examine the contracting problem in (8) where a firm chooses $\psi = (a, b)$ by anticipating that the CEO's cutoff rule for quitting responds to the contract according to $\rho(\psi, \underline{u})$. To emphasize the importance of the quitting choice, we reformulate the contracting problem by using (b, ρ) as the firm's choices. That is, the firm chooses b and ρ, leaving the base wage a to ensure that the cutoff ρ is consistent with the CEO's optimal quitting rule $\rho(\psi, \underline{u})$. This reformulation, presented in Appendix A, simplifies the analysis of the optimal contract. To ease exposition, denote $\Omega \equiv \frac{\bar{x}}{2c}\mathbb{E}_y(y^{2\alpha})$ and rewrite the firm's objective function as $[1 - F_1(\rho\bar{x})]p(b, \rho)$, where $p(b, \rho)$ is the firm's expected surplus over y conditional on retaining the CEO and is defined as

$$p(b, \rho) \equiv [b^2\rho + b(1 - b)(1 + \rho)]\Omega - (\underline{u} + \underline{J}). \tag{11}$$

PROPOSITION 1: *Assume that $(\underline{u}, \underline{J})$ are fixed and satisfy $\frac{\mu_x}{2\bar{x}}(\frac{3x}{\bar{x}} - 1)\Omega < \underline{u} + \underline{J} < \Omega$.*

(i) The optimal choices (b^*, ρ^*) are unique and independent of the realization of the aggregate shock y.

(ii) *b^* and ρ^* are interior and satisfy the first-order conditions*:

$$\rho^* = 2b^* - 1, \quad \rho^* = \frac{1}{3}\left[1 + \frac{2(\underline{u} + \underline{J})}{b^*\Omega}\right]. \tag{12}$$

The expected base wage is $E_y(a^*) = \underline{u} - b^{*^2}\rho^*\Omega$. The unique (admissible) solution to (12) is

$$b^* = \frac{1}{3} + \frac{1}{3}\left[1 + \frac{3}{\Omega}\left(\underline{u} + \underline{J}\right)\right]^{1/2}, \quad \rho^* = \frac{2}{3}\left[1 + \frac{3}{\Omega}\left(\underline{u} + \underline{J}\right)\right]^{1/2} - \frac{1}{3}. \tag{13}$$

(iii) *An increase in either $\underline{u}$ or $\underline{J}$ increases (b^*, ρ^*) and the CEO's incentive pay $b^*\pi^*$. A higher $\underline{u}$ increases the expected base wage $E_y(a^*)$ but a higher $\underline{J}$ reduces the expected base wage.*

(iv) *An increase in either aggregate or idiosyncratic risk (σ_y or σ_x) reduces (b^*, ρ^*) when $\underline{u} + \underline{J} > 0$ and increases (b^*, ρ^*) when $\underline{u} + \underline{J} < 0$. Also, an increase in either risk increases the CEO's incentive pay $b^*\pi^*$ at $x = \rho^*\bar{x}$ and reduces the expected base wage $E_y(a^*)$.*

The optimal cutoff ratio ρ^* is independent of y because a CEO's quitting decision is made before observing y. The driving force for $b^* < 1$, which is explained later, is a CEO's option to quit after privately observing x. Because this driving force is independent of the realization of y, so is the optimal b^*. For b^* and ρ^* to lie in the interior of $(0, 1)$, it is necessary and sufficient that $(\underline{u} + \underline{J})$ satisfies the bounds in the proposition. These bounds are satisfied by the endogenous outside options in the equilibrium under certain restrictions on the parameters specified later in Proposition 2. In fact, these bounds on $(\underline{u} + \underline{J})$ yield $\frac{1}{2}(1 + \frac{\underline{x}}{\bar{x}}) < b^* < 1$ and $\rho^* \in (\underline{x}/\bar{x}, 1)$.

Our result $b^* < 1$ contrasts sharply with the well-known result $b^* = 1$ in standard static agency models for a risk-neutral agent. The difference is caused by the assumption that x is only observed by a CEO who can quit. It is easy to see the role of quitting: if a CEO is forbidden to quit, then it is always optimal for a firm to extract all surplus by setting $b = 1$ to induce effort.

As for the role of private information about x, we first consider the alternative assumption that x is publicly observable and contractible while a CEO can still quit. In this case, a contract takes the form $\psi(\underline{u}, \underline{J}, x, y)$ instead of $\psi(\underline{u}, \underline{J}, y)$. Moreover, it is optimal for the firm to induce the CEO to quit if the realization of x is so low that the joint surplus of the match is negative. When x is high enough to generate a positive joint surplus, the firm will use the contract to squeeze the CEO's expected surplus over y to zero. That is, (4) holds with equality for such x, which yields $\mathbb{E}_y(a) = \underline{u} - \frac{x}{2c}\mathbb{E}_y(b^2 y^{2\alpha})$. Then, for such x, after substituting (π^*, w^*) from (3) and $\mathbb{E}_y(a)$, we obtain the firm's expected surplus as

$$\mathbb{E}_y(\pi^* - w^* - \underline{J}) = \frac{x}{c}\mathbb{E}_y\left\{\left[\frac{b^2}{2} + b(1-b)\right]y^{2\alpha}\right\} - (\underline{u} + \underline{J}).$$

For each pair (x, y), the derivative of the firm's expected surplus with respect to $b(\underline{u}, \underline{J}, x, y)$ is $\frac{x}{c}[1 - b(\underline{u}, \underline{J}, x, y)]y^{2\alpha}$, which is strictly positive for all $b < 1$.

Thus, when x is contractible, the optimal PPS is $b(\underline{u}, \underline{J}, x, y) = 1$ for all x high enough to make it worthwhile for the firm to keep the CEO. With $b^* = 1$, the firm makes the CEO's incentive in the effort choice perfectly aligned with the goal of maximizing the total expected surplus of the match. Notice that this contract with $b^* = 1$ requires the expected base wage over y to vary with the idiosyncratic shock in the form that $\mathbb{E}_y(a) = \underline{u} - \frac{x}{\bar{x}}\Omega$. That is, when x is high, the firm rewards the CEO through incentive pay by giving $b = 1$, but at the same time reduces the base wage. In fact, the firm adjusts the CEO's base wage conditional on x to the extent that the CEO's value in the period's match is equal to the effective outside option for the period.

Now we return to the model where x is the CEO's private information. In this case, the incentive contract cannot be contingent on the realization of x. Thus, the firm cannot squeeze the CEO's expected surplus to zero by adjusting the base wage. Rather, the CEO is shielded from the negative surplus in the case $x < \rho^*\bar{x}$ by quitting. For all $x > \rho^*\bar{x}$, the CEO's expected surplus, $\mathbb{E}_y(u^* - \underline{u})$, is strictly positive and increases with the size of the "pie" generated by a higher x. Since the firm cannot condition the base wage on x, the only way to get a share of this larger pie is to set PPS below one.[11]

When $b^* < 1$, the optimal PPS interacts with the quitting decision and is affected by the two risks, as stated in parts (iii) and (iv) of Proposition 1 and explained in the next two subsections.

B. The Interactions between the Optimal PPS and the Quitting Decision

As explained immediately before Proposition 1, the optimal choices (b^*, ρ^*) maximize a firm's expected surplus, $[1 - F_1(\rho\bar{x})]p(b, \rho)$. The first-order conditions are stated in (12) and depicted in Figure 2.

In each panel, the straight line depicts the first-order condition of b^*, the curve depicts the first-order condition of ρ^*, and the intersection of the two is the optimal pair (b^*, ρ^*). The dotted curve represents the first-order condition of ρ^* with a higher $\underline{u}$ or $\underline{J}$, relative to the solid curve.

For any given ρ, the first-order condition of PPS, b^*, gives a positive relationship between the optimal PPS and the cutoff. To explain this relationship, we start with the fact that, for any given ρ, the retention probability does not depend on b. Thus, the optimal PPS maximizes the firm's surplus conditional on retention, $p(b, \rho)$, given by (11). At the optimal PPS, the marginal benefit of b to the conditional surplus is zero and diminishing. Moreover, the marginal benefit of b is increasing in ρ, that is, b and ρ are complementary with each other in the conditional surplus. As a result, when the cutoff is higher, the marginal benefit of PPS to the conditional surplus is amplified, in which case

[11] We do not model the possibility of contract renegotiation when the CEO chooses to quit. Such renegotiation complicates the analysis significantly because a CEO might pretend to quit just to renegotiate the contract even when x is high. Also, we assume that the firm has some inalienable knowledge asset necessary for the operation, which makes it not optimal for the firm to sell the operation to the CEO. This assumption is implicit in the contract that all payments between the firm and the CEO must occur after production is carried out.

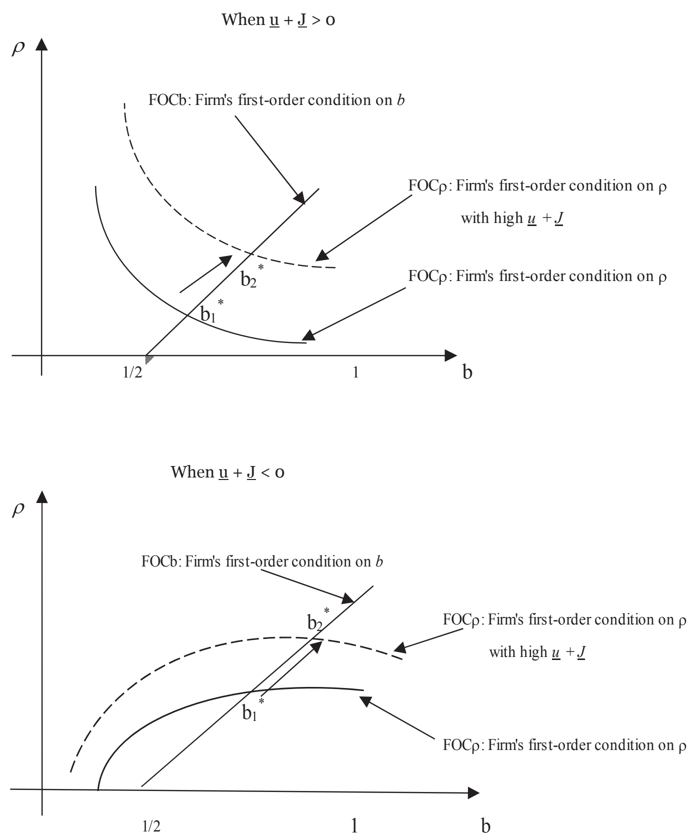

Figure 2. Determination of optimal b and ρ.

a higher PPS is optimal in order to restore the marginal benefit of PPS to zero. This gives the positive relationship between b and ρ. The reason for the complementarity between b and ρ is that a higher cutoff means that the idiosyncratic shock conditional on retention of the CEO is higher in the first-order stochastic dominance. In this case, an increase in the CEO's effort induced by a higher PPS increases the firm's profit by a larger amount, because effort and the idiosyncratic shock are multiplicative in the profit function.

In contrast to PPS, the cutoff affects both the retention probability and the firm's conditional surplus. A higher cutoff reduces the retention probability. At the same time, it improves the distribution of the idiosyncratic shock conditional on retention and hence increases the firm's conditional surplus. For any given b, the optimal cutoff for retention, ρ^*, achieves the optimal trade-off between the retention probability and the conditional surplus by maximizing the firm's expected surplus, $[1 - F(\rho\bar{x})]p(b, \rho)$. The resulting relationship between PPS and the cutoff is positive when $\underline{u} + \underline{J} < 0$ and negative when $\underline{u} + \underline{J} > 0$. That is, when b increases, the net marginal effect of ρ on the firm's expected surplus increases if and only if $\underline{u} + \underline{J} < 0$, in which case a higher cutoff is needed to maintain the net marginal effect of ρ at zero. To explain this result, note that PPS affects the trade-off between the marginal effects of ρ on the conditional surplus and the retention probability. A higher PPS increases the marginal benefit of ρ to the conditional surplus, because b and ρ are complementary with each other in the conditional surplus. This effect of b is weighted by the retention probability because the effect is operative only when the firm can retain the CEO. On the cost side, a higher b affects the level of the conditional surplus that will be lost when the retention probability falls with ρ. This cost effect is ambiguous. Overall, when b increases, the effect on the marginal benefit of ρ dominates the effect on the marginal cost of ρ if and only if the retention probability is sufficiently large to give the first effect a sufficiently high weight. In turn, for the retention probability to be sufficiently high, the sum of the two sides' effective outside options must be lower than a critical level. This critical level of $(\underline{u} + \underline{J})$ turns out to be zero.

C. The Effects of the Outside Options and the Two Risks

We now explain the last two parts of Proposition 1. Part (iii) describes the effect of the outside options on the optimal contract. Higher outside options increase the optimal contract (b^*, ρ^*) through the sum $(\underline{u} + \underline{J})$, as shown in (13). This is also apparent in both panels of Figure 2, where an increase in $(\underline{u} + \underline{J})$ shifts up the solid curve FOCρ to the dotted curve while leaving FOCb intact, resulting in a higher optimal PPS and a higher cutoff. Although outside options exert their impacts as a sum, intuition can be obtained by examining the effects of the two components separately. Let us start with the cutoff. A higher outside option for a CEO reduces the CEO's likelihood to stay, which induces an increase in the cutoff. Also, a higher outside option for a firm raises the profit expectation for an existing match, and a higher profit necessitates a higher idiosyncratic shock conditional on the shock being above the cutoff, which in turns calls for a higher cutoff. Given the above insights, the positive relation between PPS and the outside options can be easily understood since, as shown in the last section, PPS and the cutoff are complementary.

Part (iv) of Proposition 1 states how risks impact the optimal contract (b^*, ρ^*), where risk is defined as the variance of the associated shock with the mean fixed. Aside from the outside options $(\underline{u} + \underline{J})$, Ω is the only term that enters the optimal contract as shown in (13). Thus, with fixed outside options, the two risks exert their impact on optimal contracts exclusively through Ω. An increase in idiosyncratic risk σ_x increases Ω directly, and an increase in systematic risk σ_y increases Ω by increasing the term $\mathbb{E}(y^{2\alpha})$. When Ω increases, the line FOCb in Figure 2 remains unchanged while the solid curve FOCρ shifts to the dotted curve. Therefore, an increase in either risk reduces (b^*, ρ^*) when $\underline{u} + \underline{J} > 0$ and increases (b^*, ρ^*) when $\underline{u} + \underline{J} < 0$.

To facilitate deeper understanding of the above effects, let us trace a cause-effect link from risks to profits and then from profits to optimal contracts, notwithstanding that the equilibrium is solved simultaneously. To begin, for any given contract, an increase in either risk increases the firm's conditional surplus. Consider first an increase in aggregate risk. Insofar as the conditional surplus function in (11) depends positively on $\mathbb{E}(y^{2\alpha})$ via Ω and higher aggregate risk leads to a higher $\mathbb{E}(y^{2\alpha})$, an increase in aggregate risk leads to higher profit. Since the overall size of the "pie" has increased, the firm's conditional surplus would increase under any given contract. Consider next an increase in idiosyncratic risk. For any given contract, higher idiosyncratic risk increases the firm's conditional surplus by increasing the average value of x conditional on the CEO's acceptance of the match. Specifically, because the CEO stays only if $x \geq \rho^*\bar{x}$, the firm's profit behaves like a call option written on idiosyncratic shock, with a strike price set to the reservation value $\rho^*\bar{x}$. Higher idiosyncratic risk widens the interval of possible realizations of x on both sides of the mean. However, the widening of the interval of x on the left side of the mean has no impact on the firm's profit since the CEO quits if the realization is below $\rho^*\bar{x}$. In contrast, the widening of the interval of x on the right side of the mean increases the firm's profit and conditional surplus.

Having established the positive link from the risks to a firm's conditional surplus, we now link the latter to the optimal contract. When a firm's conditional surplus increases with either risk, the firm must re-optimize to restore the optimal trade-off between the retention probability and the conditional surplus. The required adjustments in (b^*, ρ^*) depend on $(\underline{u} + \underline{J})$, the sign of which dictates whether the firm is more concerned with the retention probability or the conditional surplus, as alluded to above. When $\underline{u} + \underline{J} > 0$, the cutoff is relatively high and the firm is more concerned about the retention probability. In this case, it is optimal for the firm to increase the retention probability by inducing the cutoff ρ^* to fall. The opposite is true when $\underline{u} + \underline{J} < 0$. In both cases, PPS adjusts in the same direction as ρ^* due to their complementarity in the firm's conditional surplus. Hence, a higher conditional surplus leads to higher b^* and ρ^*. Juxtaposing the two links, we obtain that an increase in either idiosyncratic risk or systematic risk will increase (b^*, ρ^*) when $\underline{u} + \underline{J} < 0$ and decrease (b^*, ρ^*) when $\underline{u} + \underline{J} > 0$.

Finally, to understand the second claim in part (iv) of Proposition 1, we need to realize that, to induce effort, the firm uses part of the increase in profit to increase the CEO's incentive pay. Thus, even when b^* and ρ^* fall in the case $\underline{u} + \underline{J} > 0$, the CEO's incentive pay at $x = \rho\overline{x}$ still increases with Ω. Because the total expected pay to the CEO is fixed at $\underline{u}$, the increase in the incentive pay must be accompanied by a fall in the expected base wage.

At this point, we would like to point out two key differences between our model and the standard static agency models with a risk-averse agent (Murphy (1999, pp. 27–28) and Bolton and Dewatripont (2005, pp. 160–162)). First, the optimal contract in our model responds to changes in risks because of the optimal trade-off between the retention probability and conditional surplus, not because of risk aversion. In contrast, in the static agency models, PPS falls with increased risks because the agent is risk averse.[12] Second, standard models presume that the outside options are always nonnegative, a stipulation unlikely to be valid in an intertemporal setting with sufficiently patient players. When the joint outside option is negative, our analysis in this section shows that the risks can increase PPS.

III. Contracting Externalities and Properties of the Market Equilibrium

This section explains contracting externalities and describes the market equilibrium. We first show how individual firms' choices generate contracting externalities via effective outside options and matching probabilities. We then state Proposition 2 and show that the two risks affect the optimal contract differently in equilibrium, in contrast to the result with fixed outside options. These equilibrium properties form the main hypotheses for our empirical tests in Section IV.

Contracting externalities arise because the outside options $(\underline{u}, \underline{J})$ and the matching rates (λ, q) are endogenous in equilibrium but are taken as given by individuals. To understand these externalities, we show how the outside options $(\underline{u}, \underline{J})$ respond to b. To this end, suppose that all firms choose a contract $\psi = (a, b)$. Given the contract ψ and the induced cutoff ρ, we can derive the following equations (see Appendix B):

$$\underline{u} = B - qL\Omega b^2(1-b)^2\ , \quad \underline{J} = -\beta J_F = -2L\Omega b(1-b)^2, \tag{14}$$

$$q = H/[2L\Omega b(1-b)^2], \quad \text{where } L \equiv \beta(1-\delta)\bar{x}/(\sigma_x\sqrt{3}). \tag{15}$$

The above equations reveal two externalities: (i) individual firms and CEOs ignore the equilibrium relationship between the matching probability and the contract; (ii) individual firms and CEOs ignore the effect of their own contract on the outside options of other firm–CEO pairs.

The first externality arises because of the dependence of the matching probability on the contracts through competitive entry of vacancies. To see how the matching probability and the contract component b interact, we note that free entry of vacancies requires $q = H/(\beta J_F)$, which increases in b since J_F is a decreasing function of b (see (14)). When the value of a CEO-staffed firm falls with PPS, fewer vacancies are created in order for each vacancy to break even. This equilibrium response of q to b reduces a searching CEO's matching probability $\lambda = 1 - q$ and hence reduces a CEO's outside option $\underline{u}$ (see (14)).

The second externality stems from the fact that one firm's contract directly affects other managers' outside options. This externality appears in (14): for

[12] Standard models typically assume that the profit function is additive in the agent's effort and the shocks, rather than multiplicative as in our model. However, even if standard models are modified to have a multiplicative profit function, risk aversion will continue to be the reason why PPS is below one, and so the optimal PPS will likely be decreasing with the two risks.

a given matching probability q, the outside options $\underline{u}$ and $\underline{J}$ increase in b, provided $\rho = 2b - 1 > 0$. To see this, recall that, when a firm chooses a higher PPS, the firm also increases the cutoff ρ and reduces the expected base wage. For any $b > 1/2$, the loss to a CEO from the reduction in the expected base wage outweighs the gain from the increased PPS, causing an employed CEO's expected surplus in the current period to fall. The falling expected surplus increases the CEO's desire to quit and search. Thus, for a given q, competition among firms intensifies in a bid to retain their CEOs, which reduces the value of a CEO-staffed firm. Thus, the value of a searching firm relative to the value of a matched firm increases.

Simple algebra reveals that, as far as a CEO's outside option is concerned, the consequence of the first externality dominates the second one, resulting in an overall negative relation between PPS and a CEO's outside option.

Because of these externalities, the properties of PPS in the equilibrium differ from those under fixed outside options. Before stating these properties, let us denote the ratio of a CEO's total expected pay to firm value as $R_{pay/size}$, where firm value is J_F and total expected pay is $(1-\delta)\int_{\rho^*\bar{x}}^{\bar{x}} \mathbb{E}_y(a^* + b^*\pi^*)dF_1(x)$. The following proposition is proven in Appendix B:

PROPOSITION 2: *A unique market equilibrium exists in a nonempty region of the parameters* $(H, \mu_x/\sigma_x, B)$ *specified in Appendix B. Moreover, the equilibrium PPS, the cutoff* ρ^**, and the matching probability* q *are all interior. Assuming* $\beta > 1/[2(1-\delta)]$*, the equilibrium has the following properties:*

- *(i) equilibrium PPS decreases with* μ_y *and* σ_y*;*
- *(ii) equilibrium PPS increases with idiosyncratic risk* σ_x *if and only if* $b^* < b_2 \in (2/3, 1)$*, where* b_2 *is defined by (B4); and*
- *(iii) the pay–size ratio* $R_{pay/size}$ *is* $\frac{2b^*-1}{1-b^*} + (1-b^*)L$*, decreases with* μ_y *and* σ_y*, and increases with idiosyncratic risk* σ_x *if and only if* $b^* < b_3 \in (\mu_x/\bar{x}, 2/3)$*, where* b_3 *is defined by (B5).*

Let us elaborate on the restrictions on the parameters $(H, \mu_x/\sigma_x, B)$ for an interior equilibrium to exist (see (B3) in Appendix B for the precise restrictions).[13] First, the hiring cost H should not be very high. If H is very high, the vacancy-filling probability must be one for a firm to create a vacancy. In this case, a CEO's job-finding probability is zero. Second, the ratio μ_x/σ_x should not be too high. If μ_x/σ_x is very high, idiosyncratic risk is low relative to the mean, in which case a CEO never quits. Third, the benefit to a searching CEO should be bounded below and above. If this benefit is very low, there is no gain for a CEO to quit one job and search for another. If this benefit is very high, quitting happens often. In this case, a firm needs to set PPS to be very high, which is not profitable. In addition, we impose the assumption $\beta > 1/[2(1-\delta)]$ to simplify the algebra. This assumption is easily satisfied when the exogenous separation

[13] Because these restrictions guarantee that b^* and ρ^* are interior, they ensure that the sum $(\underline{u} + \underline{J})$ generated by the equilibrium satisfies the bounds imposed in Proposition 1.

rate δ is small. For example, when $\delta = 0$, the assumption requires only that $\beta > 1/2$.

We now turn to parts (i) and (ii) of Proposition 2, which describe how the two risks affect the equilibrium PPS. First, an increase in μ_y or σ_y always reduces the equilibrium PPS, while it does so under the partial equilibrium only when the joint outside option value is positive. Second, the two risks can affect the equilibrium PPS in opposite directions. In particular, an increase in σ_x increases PPS when PPS is low, while σ_y always reduces the equilibrium PPS. In contrast, in the partial equilibrium with fixed outside options, the two risks always affect PPS in the same way. These differences demonstrate in a concrete way the importance of the market equilibrium. Also, the finding of opposite effects of the two risks on PPS contrasts with the agency literature with a single agent–firm pair, where the two risks both reduce PPS.

To shed light on why the two risks can affect the equilibrium PPS in opposite directions, we explain how these risks affect the outside options. To this end, we combine the two externalities discussed earlier and derive the following expression (see Appendix B):

$$\underline{u} + \underline{J} = B - \frac{H}{2}b - 2L\Omega b(1-b)^2, \tag{16}$$

where L is stated in (15). For any given b, the joint outside option value, $\underline{u} + \underline{J}$, depends on σ_y entirely through $\Omega = \frac{\bar{x}}{2c}\mathbb{E}_y(y^{2\alpha})$ and on σ_x entirely through $\bar{x}^2/\sigma_x$, which enters in $L\Omega$. The higher μ_y or σ_y, the higher Ω. An increase in σ_x reduces $\bar{x}^2/\sigma_x$, even after considering the relationship $\bar{x} = \mu_x + \sigma_x\sqrt{3}$. Thus, for a given b, $\underline{u} + \underline{J}$ decreases in σ_y and increases in σ_x.

It is intuitive that the two risks affect the outside options in opposite directions. As is well known in search theory, the return to search is a convex function of the underlying match value because the return is truncated at zero. That is, a match with a high value is accepted, but a match with a low value is rejected, in which case a searcher retains the value of search. Convexity implies that the return to search increases when match value is more dispersed. An increase in σ_x increases dispersion in match value and hence increases the return to search relative to staying in a match for both the CEO and the firm. In contrast, an increase in σ_y affects all matches in the same way and reduces the return to search relative to staying in the existing match.

Because of their different impacts on outside options, the two risks can affect the equilibrium PPS in opposite directions. To see this, recall from Proposition 1 that an increase in the joint outside option value ($\underline{u} + \underline{J}$) boosts PPS. When aggregate risk increases, the ratio $(\underline{u} + \underline{J})/\Omega$ falls and, by (13), the equilibrium PPS falls. When idiosyncratic risk increases, the joint outside option value goes up, which increases PPS. In addition, for any given joint outside option value, an increase in idiosyncratic risk has the direct effect of increasing PPS if and only if the joint outside option value is negative. The overall effect of a higher σ_x on the equilibrium PPS is positive if $\underline{u} + \underline{J} < 0$ or if PPS itself is not too high. Specifically, since $(\underline{u} + \underline{J}) > 0$ if and only if $b^* > 2/3$ (see (13)), an increase in σ_x

increases PPS if and only if $b^* < b_2 \in (2/3, 1)$ (see Appendix B for details). The above property reveals that the response of the equilibrium PPS to the risks is often dictated by outside options' response to the risks. As such, a partial equilibrium model is prone to erroneous conclusions about risks' impact on PPS simply due to its fixing outside options.

At this point, we compare our results to those in Jin (2002) and Garvey and Milbourn (2003), who study incentive contracts in a static partial equilibrium setting in which idiosyncratic and systematic risks are explicitly modeled. In their static setting, the joint outside option value is assumed to be positive since the CEO has a positive reservation utility while the principal has zero reservation value. Jin (2002) shows that the optimal b^* decreases with firm-specific risk but is not affected by market risk when the CEO can trade the market. When the CEO cannot trade the market, the effect of systematic risk is indeterministic. Garvey and Milbourn (2003) show that the optimal b^* decreases with firm-specific risk, while it decreases with market risk when hedging is costly or is not affected by costless hedging. Since we do not explicitly model the CEO's optimal portfolio or hedging decision, it is impossible to compare the effect of systematic risk on PPS in our model to that in Jin (2002) and Garvey and Milbourn (2003). We thus focus comparison on the effect of idiosyncratic risk on PPS.

The common conclusion of the two previous studies is that PPS decreases with idiosyncratic risk. Such a result is consistent with the partial equilibrium analysis when the joint outside option value is positive (see part (iv) of Proposition 1). However, in a dynamic equilibrium setting such as our model, the joint outside option value can be positive or negative. When the joint outside option value is negative, PPS increases with idiosyncratic risk. More importantly, our market equilibrium analysis shows that the equilibrium PPS increases with idiosyncratic risk due to contracting externalities. The comparison between our model and these two studies further indicates that the equilibrium analysis imposes stronger discipline on the relationship between idiosyncratic risk and PPS, which serves as proper guidance for empirical tests.

The properties of the pay–size ratio described in part (iii) of Proposition 2 follow from the equilibrium effects of the two risks on PPS. When σ_y increases, the equilibrium PPS decreases and the retention probability increases, as apparent in (13). Both responses increase a firm's expected value. A CEO's total expected pay may also increase, but it increases by a smaller proportion because the CEO's profit-sharing ratio is reduced. Overall, the pay–size ratio falls with σ_y. In contrast, an increase in σ_x can increase the pay–size ratio by increasing PPS and incentive pay. Because an increase in σ_x increases PPS only when PPS is relatively small, not surprisingly it increases the pay–size ratio only when PPS is relatively small.

IV. Empirical Analysis

To facilitate the empirical analyses, let us summarize. We construct a theoretical model to integrate an agency problem in CEO compensation with search theory and analyze the market equilibrium with many firms and CEOs. Each firm offers an incentive contract to the CEO that achieves the optimal trade-off between the probability of retaining the CEO and expected profit conditional on retention. This trade-off implies that the optimal PPS is below one, despite CEOs being risk neutral. More importantly, the search process endogenously determines CEOs' and firms' outside options, which reflect the externalities in the market equilibrium. The externalities lead to novel effects of the risks on incentive contracts. While an increase in idiosyncratic risk boosts search by widening dispersion in match value, an increase in aggregate risk dampens search by compressing dispersion in match value. As a result, idiosyncratic risk increases PPS when PPS is not too large, but aggregate risk always reduces PPS. In addition, the two risks affect the pay–size ratio differently. The different effects of the two risks on PPS and the pay–size ratio arise from the externalities, which are unique to the market equilibrium with search. When the outside options are fixed at positive levels as in the agency literature, the two risks have the same qualitative impact on PPS, contrary to our equilibrium predictions.

The objective of our empirical analyses is threefold: (i) to evaluate our model's predictions on PPS, (ii) to provide an explanation for the mixed evidence on the relationship between firm risks and PPS, and (iii) to provide new evidence on the relative growth between CEO pay and firm size. Because the empirical PPS value is rather small, the condition $b^* < b_3$ in Proposition 2 is easily satisfied. In this case, our model yields the following testable predictions:

Prediction 1: PPS, b, decreases with a firm's systematic risk and increases with the firm's idiosyncratic risk.

Prediction 2: The relative growth of total pay to firm size, $R_{pay/size}$, decreases with a firm's systematic risk and increases with the firm's idiosyncratic risk.

A. Data and Variable Definitions

CEO compensation data are retrieved from ExecuComp for the period 1992 to 2009. Firm characteristics and returns are obtained from COMPUSTAT and CRSP. To follow the tradition of this literature (e.g., Coles, Daniel, and Naveen (2006)), we exclude financial and utility firms (SIC codes 6000 to 6999 and 4900 to 4999, respectively). We exclude observations for which there are fewer than 48 months of stock return data, as in Aggarwal and Samwick (1999). We also exclude firm-years that experienced CEO turnover because the reported compensation for those years covers only a portion of the year, making comparison across years difficult. Our final sample consists of 12,890 firm-years for 1,890 firms and 3,181 CEOs.

We identify empirical measures for PPS and $R_{pay/size}$. A typical compensation package includes salary, bonus, and restricted stock and option grants (see Murphy (1999)).[14] Since most incentive payments are related to equities, we

[14] "Salary" in the empirical section corresponds to the "base wage" in the theoretical model.

focus on PPS related to stock and option grants. Following Jensen and Murphy (1990), we define PPS as the change in the value of CEO pay with respect to a \$1,000 change in shareholder wealth. We use the "ex ante" PPS measure, which is the implied change in CEO wealth for a \$1,000 change in firm value. The ex ante PPS measure is computed based on the number of stock and option grants, and is consistent with the definition of b in our model and more convenient for empirical tests. This approach is used by Core and Guay (1999), Coles, Daniel, and Naveen (2006), and Jin (2002).

For completeness, we calculate two versions of PPS. The first, called "new equity incentives," is based on the stock and option grants for the current year, which are straightforward to obtain from the ExecuComp data set. The second, called "total equity incentives," is computed from the accumulated stock and option grants up to the current year. The calculation of total equity incentives is complicated by the fact that details on past option grants only became available after 2006. For years prior to 2006, researchers have used the 1-year approximation method developed by Core and Guay (2002a) to estimate the incentive level of past option grants. These authors show that the 1-year approximation method generates very similar estimates as using the actual details of option grants. For robustness, we calculate total equity incentives in two ways. First, to take advantage of the more detailed compensation data post-2006, we use the full details of option grants to calculate the exact total equity incentives after 2006 but use the 1-year approximation method prior to 2006. Second, we calculate total equity incentives using the 1-year approximation method for the whole sample period. The latter removes the potential impact of the data change on our empirical results. Consistent with Core and Guay (2002a), we find that the two versions of total equity incentives for the period 2006 to 2009 are very close to each other, with a high correlation of 0.983.

Total compensation is taken as flow compensation (TDC1), which consists of salary, bonus, other annual short-term compensation, total value of restricted stock granted, total value of stock options granted (using Black–Scholes), long-term incentive payouts, and all other long-term compensation. We calculate $R_{pay/size}$ as the ratio between annual total compensation and firm size, where firm size is proxied by sales, market capitalization, or book assets.

Next, we formulate the two major explanatory variables: a firm's systematic risk and idiosyncratic risk. A firm's risk is proxied by the volatility of its stock returns, an approach adopted by many authors such as Core and Guay (1999, 2002a), Coles, Daniel, and Naveen (2006), and Demsetz and Lehn (1985). Under this approach, a firm's total risk is the volatility of stock returns over the 60 months prior to the fiscal year. The market model is used to obtain the firm's beta using the same set of monthly return data. A firm's systematic risk is equal to the firm's beta multiplied by stock market risk, while the firm's idiosyncratic risk is the square-root of total return variance minus systematic return variance.

It is worth noting that, in contrast to Aggarwal and Samwick (1999), Jin (2002), and Garvey and Milbourn (2003), we do not use the volatility of dollar returns as our risk measure. These authors use dollar risk measures because a firm's profit in their static models is the sum of CEO effort and the noise term, which forces profits and the volatility of the noise term to be measured in the same units. One drawback of these dollar risk measures is the high correlation between systematic risks and idiosyncratic risks. This correlation is as high as 0.906 for our sample. Such a high correlation creates multicollinearity problems in the regressions that make it difficult to disentangle the effects of the two risks. Furthermore, the dollar risk measures are highly correlated with firm size, an important control variable used in the regression. For example, the correlation between dollar idiosyncratic (systematic) risk and market capitalization is 0.847 (0.759) for our sample. These high correlations also create multicollinearity problems in the regressions that make it difficult to separate the effects of the risks from the effect of firm size. Therefore, as argued by Core and Guay (2002b), the dollar risk measures can be viewed as noisy proxies for firm size and hence are not appropriate for capturing firm risks. Our model enables us to avoid these problems. In our model, a firm's profit is the product (instead of the sum) of the aggregate shock, the firm-specific shock, and CEO effort. Even if the CEO's effort is measured in dollars, the aggregate shock and the match-specific shock can be measured in other units without violating consistency. Consistent with our model, we use return volatilities as risk measures that have relatively low correlations with other explanatory variables.

The aggregate shock in our model is an important factor in determining the compensation contract. It includes all changes common to the industry that affect the marginal contribution of a CEO's effort to firm profit. Aggregate shocks to total factor productivity, such as the one measured by the Solow residual, are part but not the only part of this aggregate shock. For example, industry-wide changes in inputs of physical capital, energy, and regular (non-CEO) labor can all change the marginal contribution of a CEO's effort to firm profit. For this reason, we use four broad measures as proxies for the aggregate state: industry sales, GDP, the commercial paper spread, and the credit spread. Intuitively, high growth in industry sales represents a healthy business environment for that industry while high GDP growth indicates a booming economy. The use of the commercial paper spread is based on Bernanke and Blinder (1992), who suggest that a high commercial paper spread at the beginning of the year signals a bad economy since this measure tends to rise sharply during credit crunches. For ease of interpretation of the regression results, we use the negative lagged commercial paper spread as a proxy for aggregate shocks. Use of the negative credit spread is based on Gilchrist, Yankov, and Zakrajsek (2009) and Gomes and Schmid (2010).[15] Annual growth in industry sales is computed from COMPUSTAT while annual GDP growth is retrieved from the website of the Bureau of Economic Analysis. The commercial paper spread is the difference between the 3-month commercial paper rate and the T-bill rate (see Friedman and Kuttner (1993) and Korajzyk and Levy (2003)), and the credit spread is

[15] We thank the Editor for suggesting the use of the credit spread as a proxy.

the difference between the average yields of Baa bonds and Aaa bonds. Both rates are obtained from the Federal Reserve Board.

To control for additional heterogeneity in the data but not in our model, we include other variables. Following Milbourn (2003) and Garvey and Milbourn (2003), we control for the CEO's age and tenure, and a firm's size and growth. Tenure is defined as the number of years a person has been CEO of a firm. Firm growth is proxied by the firm's sales growth or Tobin's q, while its size is proxied by its sales, book assets, or market capitalization. Jensen (1986) argues that equity incentives can mitigate the agency problem caused by high free cash flow. Thus, following Core and Guay (1999), we measure a firm's free cash flow as operating cash flow minus dividends over assets. Also, we follow Jin (2002) to control for capital over sales, research and development (R&D) expense over capital, advertising expense over capital, investment over capital, and dummy variables for missing observations on R&D and advertising expenses. These variables are intended to capture cross-sectional differences in how managerial effort affects firm value.[16]

We use two measures to combat the potential undue effect of outliers existing in the highly skewed compensation data. First, we winsorize the compensation data and firm characteristics at the 1% and 99% levels, a standard procedure used in the compensation literature (e.g., Garvey and Milbourn (2006), and Coles, Daniel, and Naveen (2006)). Second, following Aggarwal and Samwick (1999), in addition to OLS regressions, we run median regressions to further reduce the potential impact of outliers.

Table I provides summary statistics for the winsorized compensation data as well as CEO and firm characteristics, and unwinsorized macroeconomic proxies. All monetary variables are inflation adjusted using 2005 dollars. As a result, industry sales growth and GDP growth are in real terms.

The average annual salary for a CEO is about $727,000, slightly higher than the average annual bonus of $635,000. However, the median annual salary of $672,000 is double the median bonus of $324,000, indicating that bonuses are more skewed toward the high end. Similar patterns of skewness exist in equity-related pay and total pay. In particular, the average equity-related pay and total compensation are $3,362,000 and $4,787,000, respectively, which are roughly twice the corresponding median values. It is worth noting that the average equity-related pay is more than four times the average annual salary, indicating that a CEO derives his pay mainly from equity-related compensation. The average new equity incentive granted in a fiscal year is $1.96 per $1,000 change in shareholder wealth, compared to the average accumulated total equity incentive of $28.37. The average CEO is about 55 years old and stays with a firm for approximately 8 years. The youngest CEO is 39 years old while the oldest is 75. The longest tenure is 37 years while the shortest job duration is 6 months.

Panel B of Table I indicates that firms in the sample are skewed toward large size. In particular, the average market capitalization is $6,845 million,

[16] We thank an anonymous referee for suggesting these additional control variables.

Table I
Summary Statistics

This table reports summary statistics on CEO compensation and characteristics, firm characteristics, and macroeconomic variables for 12,890 firm-years over the period 1992 to 2009. CEO compensation and characteristics data are retrieved from ExecuComp, and firm characteristics data are from COMPUSTAT and CRSP. Firm total risk is stock return volatility over the 60 months prior to the fiscal year. New equity incentives are PPS for a CEO based on the stock and option grant for the fiscal year with respect to a $1,000 change in shareholder wealth. Total equity incentives are PPS for a CEO based on the cumulative stock and option grants with respect to a $1,000 change in shareholder wealth. Two versions of total equity incentives are obtained: Version 1 is based on all available information while Version 2 is computed using the 1-year approximation method proposed by Core and Guay (2002a). CEO tenure is the number of years a person has been the CEO of a firm. Firm systematic risk is equal to a firm's beta multiplied by the stock market risk while firm idiosyncratic risk is the square root of the difference between total return variance and systematic return variance. The commercial paper spread is defined as the difference between the annualized rate on 3-month commercial paper and the 3-month T-bill rate, while the credit spread is the difference between the yield of Baa and Aaa bonds. All monetary variables are deflated using 2005 dollars.

Variables	Mean	Std. Dev.	Min.	25% Percentile	Median	75% Percentile	Max.	Skewness	Kurtosis
Panel A: CEO Compensation and Characteristics									
Salary (thousand)	$727	$338	$38	$473	$672	$923	$1,911	0.92	4.02
Bonus (thousand)	$635	$931	$0	$0	$324	$818	$5,322	2.73	11.95
Equity incentive pay (thousand)	$3,362	$5,003	$0	$491	$1,513	$3,985	$28,863	2.91	12.72
Total compensation (thousand)	$4,787	$5,798	$251	$1,396	$2,757	$5,641	$34,271	2.83	12.41
New equity incentives									
(Per $1,000 change in shareholder wealth)	$1.96	$2.92	$0.00	$0.27	$0.98	$2.36	$18.07	3.14	14.94
Total equity incentives: Version 1									
(Per $1,000 change in shareholder wealth)	$28.34	$49.28	$0.09	$3.27	$11.15	$29.12	$331.26	3.52	17.31
Total equity incentives: Version 2									
(Per $1,000 change in shareholder wealth)	$28.37	$49.59	$0.11	$2.94	$10.82	$29.64	$335.14	3.50	17.22
CEO tenure	7.88	7.15	0.50	2.75	5.67	10.59	37.02	1.72	6.22
CEO age	55.37	7.01	39.00	51.00	56.00	60.00	75.00	0.06	2.87

(*Continued*)

Table I—*Continued*

Variables	Mean	Std. Dev.	Min.	25% Percentile	Median	75% Percentile	Max.	Skewness	Kurtosis
Panel B: Firm Characteristics									
Firm total risk	43%	20%	16%	29%	38%	53%	115%	1.29	4.59
Firm idiosyncratic risk	40%	19%	14%	26%	35%	49%	107%	1.23	4.47
Firm systematic risk	15%	10%	1%	9%	13%	19%	54%	1.44	5.65
Market capitalization (million)	$6,845	$17,003	$40	$543	$1,486	$4,771	$119,311	4.72	27.69
Sales (million)	$4,993	$10,244	$27	$520	$1,398	$4,287	$66,097	3.93	20.32
Sales growth	11%	24%	−49%	0%	8%	19%	110%	1.28	7.31
Panel C: Macroeconomic Variables									
Industry sales growth	4%	11%	−43%	−2%	4%	9%	61%	0.28	6.34
Commercial paper spread (basis points)	27.72	17	1	17	27	38	73	0.78	4.12
GDP growth	3%	2%	−3%	2%	3%	4%	5%	−1.47	5.11
Credit spread (basis points)	94	39	60	69	83	92	198	1.74	4.90

almost five times the median value of \$1,486 million; the average sales revenue is \$4,993 million, almost four times the median sales of \$1,398 million. The average firm's total risk represented by return volatility is 43%, which is slightly higher than the median 38%. The average systematic risk is 15%, about one-third of average total risk.

During the period 1992 to 2009, the growth rate of industry sales ranges from −43% to 61%, with a mean and median of 4%. The average GDP growth rate is 3%, with a standard deviation of 2%, indicating significant fluctuation. The commercial paper spread and credit spread are even more volatile, with a standard deviation of 17 and 39 basis points, respectively, in comparison with the corresponding means of 27.72 and 94 basis points.

Table II presents the correlations among the explanatory variables.

As expected, there is a positive correlation between industry sales growth and GDP growth, and between the commercial paper spread and the credit spread. Also, the two spreads are negatively correlated with the two growth series. For example, the credit spread has a correlation of −0.356 with industry sales growth, and −0.862 with GDP growth. Lastly, the low correlations among most of the explanatory variables reassure the absence of potential multicollinearity problems.

B. Test of Prediction 1: Effects of Idiosyncratic and Systematic Risks on PPS

To test the opposite effects of the idiosyncratic and systematic risks on PPS, we run the following regression:

$$\begin{aligned} b = {} & a_1 + a_2 \textit{Firm idiosyncratic risk} + a_3 \textit{ Firm systematic risk} \\ & + a_4 \textit{Macro proxy} + a_5 \textit{CEO Age} + a_6 \textit{CEO Tenure} + a_7 \log(\textit{Firm size}) \\ & + a_8 \textit{Firm growth} + a_9 \textit{Free cash flow} + a_{10} \textit{Capital / sales} \\ & + a_{11} \textit{R\&D / capital} + a_{12} \textit{R\&D missing dummy} + a_{13} \textit{Advertising / capital} \\ & + a_{14} \textit{Advertising missing dummy} + a_{15} \textit{ Investment / capital} + \varepsilon, \end{aligned} \tag{17}$$

where *Macro proxy* represents industry sales growth, GDP growth, the negative lagged commercial paper spread, and the negative credit spread, respectively.[17] The two risk measures, *Firm idiosyncratic risk* and *Firm systematic risk*, are computed from the market model as described in the previous subsection. Regression (17) is run separately for two versions of b. The first version is PPS from new equity grants. The second version is PPS from total equity grants. We refer to each version as the base case. Furthermore, for each base case, we run four alternative regressions as robustness checks.

The first robustness test runs (17) with a nonlinear transformation of the two risk measures. The transformation is performed using the cumulative

[17] For ease of exposition, hereafter "commercial paper spread" or "NCP spread" will be used to stand for "negative lagged commercial paper spread." Likewise, "credit spread" or "NCredit Spread" will stand for "negative credit spread."

Table II

Correlations

This table reports the correlations among explanatory variables and control variables for 12,890 firm-years over the period 1992 to 2009. CEO compensation and characteristics data are retrieved from ExecuComp, and firm characteristics data are from COMPUSTAT and CRSP. CEO tenure is the number of years a person has been the CEO of a firm. Firm total risk is stock return volatility over the 60 months prior to the fiscal year. Firm systematic risk is equal to a firm's beta multiplied by the stock market risk while firm idiosyncratic risk is the square root of the difference between total return variance and systematic return variance. The commercial paper spread (CP spread) is defined as the difference between the annualized rate on 3-month commercial paper and the 3-month T-bill rate, while the credit spread is the difference between the yield of Baa and Aaa bonds. All monetary variables are deflated using 2005 dollars.

	Industry Sales Growth	Lagged CP Spread	GDP Growth	Credit Spread	Market Capitalization	Sales	Sales Growth	CEO Tenure	CEO Age	Firm Total Risk	Firm Idiosyncratic Risk	Firm Systematic Risk
Industry sales growth	1.000											
Lagged CP spread	0.421	1.000										
GDP growth	−0.204	−0.430	1.000									
Credit spread	−0.356	−0.862	0.459	1.000								
Market capitalization	0.015	0.009	0.008	−0.005	1.000							
Sales	−0.008	−0.033	0.019	0.034	0.736	1.000						
Sales growth	0.277	0.204	−0.110	−0.191	0.023	−0.017	1.000					
CEO tenure	0.028	0.045	0.000	−0.045	−0.054	−0.080	0.061	1.000				
CEO age	0.009	0.047	−0.012	−0.050	0.073	0.082	−0.038	0.415	1.000			
Firm total risk	0.008	0.001	−0.270	−0.004	−0.214	−0.270	0.115	0.027	−0.185	1.000		
Firm idiosyncratic risk	0.012	0.018	−0.264	−0.015	−0.235	−0.288	0.124	0.030	−0.187	0.987	1.000	
Firm systematic risk	−0.009	−0.076	−0.179	0.055	−0.044	−0.093	0.025	0.008	−0.109	0.654	0.532	1.000

distribution function approach proposed by Aggarwal and Samwick (1999). In the second robustness check, we run (17) using risk measures computed from the Fama–French three-factor model. The last two robustness tests are run to account for potential endogeneity of firm risks. As pointed out by Tufano (1996) and Coles, Daniel, and Naveen (2006), endogeneity may exist because executive compensation may affect a firm's risk-taking behavior. Therefore, in the third robustness check, we replace an individual firm's risk measures with average industry risk measures. This approach is advocated by Jin (2002), who argues that CEOs are less likely to control for industry risk and thus average industry risk measures should be a better representation of environmental risk. The last robustness check involves running a typical instrumental variable regression to account for potential endogeneity effects, whereby the average industry risk measures are taken as the instrumental variables. Since there is no standard procedure to run median regressions with instrumental variables, we only perform the last robustness check for OLS regressions.

B.1. The Case for New Equity Incentives

Since the incentive contract in our model is offered to a CEO period by period, it is natural to test the effects of the two risks on new equity incentives, which is the focus of the current section. For brevity, we only present results using sales and sales growth to measure firm size and growth. The results based on our other proxies for firm size and growth are qualitatively similar.[18] Table III presents OLS and median regression results.

The main findings are as follows. First, consistent with our predictions, b depends positively on idiosyncratic risk and negatively on systematic risk in all regressions. Coefficients for idiosyncratic risk are all significant at the 1% level while most of the coefficients for systematic risk are significant at the 1% level. We use results from median regressions to discuss the impact of a firm's risks on PPS for new equity incentives. Suppose GDP growth is used as the macro proxy. An increase of one standard deviation in firms' idiosyncratic risk (i.e., 19%) increases new equity incentive pay by \$459,237 ($= 2.029 \times 19\% \times$ \$6,845 million $\times 17.10\%/1{,}000$), while an increase of one standard deviation in firms' systematic risk (i.e., 10%) decreases new equity incentive pay by \$98,673 ($= 0.843 \times 10\% \times$ \$6,845 million $\times 17.10\%/1{,}000$). Thus, the impacts of firms' idiosyncratic risk and systematic risk on PPS are economically significant.[19]

Second, to compare our predictions with those of a standard agency model, we rerun (17) by replacing *idiosyncratic risk* and *systematic risk* with the firm *total*

[18] We also perform regressions with industry fixed effects. Results are qualitatively similar. As argued by Jin (2002), the intrinsic problem with fixed effect regressions is that if idiosyncratic risk and systematic risk do not change much over time, then fixed effects will absorb much of the risk-induced cross-sectional variation in PPS. This concern is especially relevant for firm systematic risk since all firms in an industry should have similar exposure to market risk.

[19] For our sample period, \$6,845 million is the average market value of equity, and 17.10% is the average stock return. Therefore, \$6,845 million$\times$17.10% is the average change in shareholder wealth during a year.

Table III

Base Test of Prediction 1: Effects of Firm Risks on Pay-to-Performance Sensitivity of New Equity Incentive Grants

This table reports the base case results of the regression (17) for 12,890 firm-years over the period 1992 to 2009. CEO compensation and characteristics data are retrieved from ExecuComp, and firm characteristics data are from COMPUSTAT and CRSP. New equity incentives are PPS for a CEO based on the stock and option grant for the fiscal year with respect to a $1,000 change in shareholder wealth. Firm total risk is stock return volatility over the 60 months prior to the fiscal year. Firm systematic risk is equal to a firm's beta multiplied by stock market risk while firm idiosyncratic risk is the square root of the difference between total return variance and systematic return variance. CEO tenure is the number of years a person has been the CEO of a firm. Firm size is measured by sales. Firm growth is sales growth. Free cash flow is operating cash flow minus dividends over assets. NCP spread is the negative lagged commercial paper spread and NCredit spread is the negative credit spread. The commercial paper spread is defined as the difference between the annualized rate on 3-month commercial paper and the 3-month T-bill rate, while the credit spread is the difference between the yield of Baa and Aaa bonds. All monetary variables are deflated using 2005 dollars. We also run regressions by replacing "idiosyncratic" and "systematic" risks with "total risk." The coefficient and t-value for total risk are reported at the bottom of the table. For OLS regressions, standard errors are clustered at the firm level. For median regressions, standard errors are calculated by bootstrapping with 500 replications; t-statistics are in parentheses. *, **, and *** indicate significance at 10%, 5%, and 1% levels, respectively.

		New Equity Incentive PPS							
Predictions: This Model		OLS				Median			
Firm idiosyncratic risk	+	2.716***	3.369***	2.754***	2.731***	1.960***	2.505***	2.029***	2.000***
		(10.574)	(12.284)	(10.719)	(10.650)	(14.341)	(17.313)	(14.878)	(14.521)
Firm systematic risk	−	−1.105***	−0.924**	−1.259***	−1.192***	−0.735***	−0.574***	−0.843***	−0.778***
		(−2.823)	(−2.365)	(−3.214)	(−3.044)	(−4.090)	(−3.025)	(−4.671)	(−4.272)
Industry sales growth		−0.780***				−0.977***			
		(−3.028)				(−8.986)			
NCP spread (basis points)			−0.020***				−0.011***		
			(−12.497)				(−15.718)		
GDP growth				−11.787***				−10.179***	
				(−7.906)				(−15.534)	
NCredit spread (basis points)					−0.007***				−0.006***
					(−9.553)				(−18.150)
CEO age		−0.020***	−0.017***	−0.019***	−0.018***	−0.012***	−0.010***	−0.008***	−0.008***
		(−3.406)	(−2.872)	(−3.176)	(−3.108)	(−5.845)	(−5.635)	(−4.024)	(−4.039)
CEO tenure		−0.012**	−0.014**	−0.012**	−0.012**	−0.007***	−0.008***	−0.007***	−0.006***
		(−2.258)	(−2.577)	(−2.256)	(−2.273)	(−3.780)	(−4.274)	(−3.507)	(−3.230)
log(Firm Size)		−0.383***	−0.361***	−0.391***	−0.395***	−0.206***	−0.188***	−0.208***	−0.208***
		(−15.142)	(−14.230)	(−15.456)	(−15.621)	(−24.534)	(−20.656)	(−22.991)	(−23.582)

(*Continued*)

Table III—*Continued*

		New Equity Incentive PPS							
Predictions: This Model		OLS				Median			
Firm growth		−0.017	0.038	0.069	0.089	−0.003	−0.028	0.013	0.013
		(−0.119)	(0.269)	(0.483)	(0.623)	(−0.045)	(−0.475)	(0.212)	(0.223)
Free cash flow		−1.614***	−1.541***	−1.695***	−1.709***	−1.242***	−1.167***	−1.228***	−1.321***
		(−2.749)	(−2.623)	(−2.892)	(−2.916)	(−5.153)	(−4.968)	(−4.843)	(−5.563)
Capital/Sales		−0.419***	−0.435***	−0.421***	−0.420***	−0.197***	−0.207***	−0.197***	−0.191***
		(−6.595)	(−6.812)	(−6.612)	(−6.610)	(−8.787)	(−11.852)	(−10.013)	(−10.425)
RD/Capital		0.035	0.005	0.011	−0.001	0.165***	0.128***	0.138***	0.114***
		(0.526)	(0.069)	(0.165)	(−0.010)	(4.754)	(3.654)	(3.802)	(3.133)
RD missing dummy		0.138*	0.159**	0.135*	0.134*	0.027	0.041	0.024	0.014
		(1.746)	(2.042)	(1.713)	(1.712)	(1.082)	(1.622)	(1.025)	(0.586)
Advertising/Capital		0.011	0.023	0.028	0.033	0.023	0.069	0.034	0.032
		(0.065)	(0.141)	(0.170)	(0.202)	(0.261)	(0.748)	(0.394)	(0.379)
Advertising missing dummy		−0.040	−0.057	−0.011	0.001	0.017	0.007	0.036	0.029
		(−0.541)	(−0.779)	(−0.157)	(0.018)	(0.658)	(0.270)	(1.440)	(1.220)
Investment/Capital		0.326	0.079	0.388	0.422	0.114	−0.126	0.134	0.199
		(1.206)	(0.292)	(1.435)	(1.558)	(0.892)	(−1.039)	(1.061)	(1.641)
Adjusted or pseudo-R^2		0.132	0.144	0.136	0.138	0.088	0.093	0.094	0.096
Predictions: Traditional Model		OLS				Median			
Firm total risk	–	2.057***	2.718***	2.038***	2.043***	1.466***	1.976***	1.432***	1.446***
		(9.931)	(12.042)	(9.874)	(9.915)	(13.043)	(16.945)	(12.054)	(11.663)
Adjusted or pseudo-R^2		0.129	0.140	0.133	0.135	0.085	0.090	0.091	0.093

risk. For brevity, we only report the coefficient and t-value for the firm *total risk* as well as the corresponding R^2. The relationship between PPS and a firm's total risk is positive and significant at the 1% level for all regressions. This finding is consistent with the results in Core and Guay (1999) but inconsistent with the prediction of a standard agency model.

So far, we have shown that our model's predictions are generally borne out by the empirical evidence for new equity incentives. We also run four alternative regressions described earlier as robustness checks. Since regression results are qualitatively similar to those in Table III, for brevity we only report coefficients of firm idiosyncratic risk and firm systematic risk in Table IV.

Under all four alternative specifications, the results show that firm idiosyncratic risk continues to exert a positive and significant (at the 1% level) impact on PPS. The results for firm systematic risk are slightly weaker but the regression coefficients are still significantly negative under most of the specifications.

B.2. The Case for Total Equity Incentives

While our model is more applicable to new equity incentives, the empirical compensation literature mostly focuses on cumulative equity grants. Assuming that a firm's risk level is relatively stable or highly autocorrelated over time, our predictions of the two risks would also apply to total equity grants. In this section, we test the effects of the two risks on cumulative equity grants by running regression (17) for the two versions of total equity incentives described in Section IV.A. We also perform the four robustness checks for the two versions of total equity incentives. By and large, the results are qualitatively similar to those for new equity incentives. For brevity, we only report the coefficients for the two risk measures in Table V.

Consistent with the results for new equity incentives, firm idiosyncratic risk has a significantly positive effect while firm systematic risk has a significantly negative effect on both versions of total equity incentives. Most of the coefficients are significant at the 1% level, and the magnitudes are larger compared to those in Tables III and IV. Intuitively, risks should have a larger impact on total equity incentives than on new equity incentives due to the cumulative effect.

To summarize, Prediction 1 of our model is strongly supported by various empirical specifications for both new and total equity incentives. In particular, PPS of new or total equity grants is affected positively by a firm's idiosyncratic risk and negatively by its systematic risk.

It is worth noting that our empirical finding of a positive impact of idiosyncratic risk on total equity incentives can neither be taken as evidence against, nor be compared to, the negative impact documented by Jin (2002) and Garvey and Milbourn (2003) because our risk measures are different from theirs. In particular, our risk measures are volatilities of stock returns while theirs are return volatilities multiplied by the firm's market capitalization. As discussed earlier, our risk measures do not lead to the multicollinearity problems that exist in these papers. Moreover, our finding of a positive effect of firm

Table IV

Robustness Test of Prediction 1: Effects of Firm Risks on Pay-to-Performance Sensitivity of New Equity Incentive Grants

This table reports results of four robustness tests for the PPS of new equity incentives based on the regression (17). The sample period is 1992 to 2009 and the sample size is 12,890 firm-years. CEO compensation and characteristics data are retrieved from ExecuComp, and firm characteristics data are from COMPUSTAT and CRSP. The first test uses the cumulative distribution functions (CDFs) of the risk measures from the market model (base case), while the second test uses the risk measures obtained from the Fama–French three-factor model. The third test uses the industry average risk measures to proxy for the individual risk measures. The last robustness test runs an instrumental variable regression where the industry average risk measures are taken as the instrumental variables. All risk measures are annualized and calculated over the 60 months prior to the fiscal year. New equity incentives are PPS for a CEO based on the stock and option grant for the fiscal year with respect to a $1,000 change in shareholder wealth. CEO tenure is the number of years a person has been the CEO of a firm. Firm size is measured by sales. Firm growth is sales growth. Free cash flow is operating cash flow minus dividends over assets. Four macro proxies are used in the regressions: industry sales growth, negative lagged commercial paper spread (NCP spread), GDP growth, and negative credit spread (Ncredit spread). The commercial paper spread is defined as the difference between the annualized rate on 3-month commercial paper and the 3-month T-bill rate, while the credit spread is the difference between the yield of Baa and Aaa bonds. All monetary variables are deflated using 2005 dollars. For brevity, only coefficients on risk measures are reported. For OLS regressions, standard errors are clustered at the firm level. For median regressions, standard errors are calculated by bootstrapping with 500 replications; t-statistics are in parentheses. *, **, and *** indicate significance at 10%, 5%, and 1% levels, respectively.

			New Equity Incentive PPS							
			OLS				Median			
Robustness Tests	Aggregate State Proxy	Prediction: This Model	Industry Sales Growth	NCP Spread	GDP Growth	Ncredit Spread	Industry Sales Growth	NCP Spread	GDP Growth	Ncredit Spread
CDF of risks	Firm idiosyncratic risk	+	1.848***	2.306***	1.866***	1.845***	0.996***	1.281***	0.998***	0.966***
			(13.270)	(15.364)	(13.407)	(13.287)	(19.456)	(24.116)	(17.158)	(16.443)
	Firm systematic risk	−	−0.207*	−0.207*	−0.280**	−0.251**	−0.107**	−0.076*	−0.143***	−0.118***
			(−1.756)	(−1.764)	(−2.357)	(−2.122)	(−2.568)	(−1.785)	(−3.202)	(−2.612)
Risks from Fama–French 3-factor model	Firm idiosyncratic risk	+	3.059***	3.550***	3.078***	3.017***	2.114***	2.514***	2.131	2.014***
			(9.403)	(10.675)	(9.465)	(9.304)	(12.434)	(16.323)	(12.057)	(11.643)
	Firm systematic risk	−	−0.811*	−0.362	−0.875**	−0.773*	−0.477**	−0.135	−0.437**	−0.342*
			(−1.935)	(−0.863)	(−2.086)	(−1.848)	(−2.352)	(−0.719)	(−2.258)	(−1.760)
Average risks	Firm idiosyncratic risk	+	1.846***	2.521***	2.000***	1.992***	0.762***	1.085***	0.840***	0.835***
			(7.072)	(9.016)	(7.603)	(7.607)	(7.660)	(9.476)	(7.609)	(7.502)
	Firm systematic risk	−	−1.162**	−0.840*	−1.551***	−1.486***	−0.090	0.054	−0.371*	−0.286
			(−2.420)	(−1.758)	(−3.208)	(−3.082)	(−0.445)	(0.253)	(−1.855)	(−1.441)
Instrument variable analysis	Firm idiosyncratic risk	+	2.691***	3.893***	2.875***	2.869***				
			(7.561)	(9.915)	(8.063)	(8.074)				
	Firm systematic risk	−	−1.518***	−1.222**	−1.941***	−1.873***				
			(−2.820)	(−2.290)	(−3.593)	(−3.474)				

Table V

Tests of Prediction 1: Effects of Firm Risks on Pay-to-Performance Sensitivity of Total Equity Incentive Grants

This table reports results for PPS of total equity incentives based on the regression (17). The sample period is 1992 to 2009 and the sample size is 12,890 firm-years. CEO compensation and characteristics data are retrieved from ExecuComp, and firm characteristics data are from COMPUSTAT and CRSP. Total equity incentives are calculated with cumulative stock and option grants up to the fiscal year with respect to a $1,000 change in shareholder wealth. Two versions of total equity incentives are obtained: Version 1 in Panel A is based on all available information while Version 2 in Panel B is computed using the 1-year approximation method proposed by Core and Guay (2002a). In the base case, the risk measures are estimated using the market model. There are four robustness tests. The first test uses the cumulative distribution functions (CDFs) of the risk measures from the market model (base case), while the second test uses the risk measures obtained from the Fama–French three-factor model. The third test uses the industry average risk measures to proxy for the individual risk measures. The last robustness test runs an instrumental variable regression where the industry average risk measures are taken as the instrumental variables. All risk measures are annualized and calculated over the 60 months prior to the fiscal year. CEO tenure is the number of years a person has been the CEO of a firm. Firm size is measured by sales. Firm growth is sales growth. Free cash flow is operating cash flow minus dividends over assets. Four macro proxies are used in the regressions: industry sales growth, negative lagged commercial paper spread (NCP spread), GDP growth, and negative credit spread (Ncredit spread). The commercial paper spread is defined as the difference between the annualized rate on 3-month commercial paper and the 3-month T-bill rate, while the credit spread is the difference between the yield of Baa and Aaa bonds. All monetary variables are deflated using 2005 dollars. For brevity, only coefficients on risk measures are reported. For OLS regressions, standard errors are clustered at the firm level. For median regressions, standard errors are calculated by bootstrapping with 500 replications. t-statistics are in parentheses. *, **, and *** indicate significance at 10%, 5%, and 1% levels, respectively.

Panel A: Total Equity Incentives Measured Using Available Information

			OLS				Median			
	Aggregate State Proxy	Prediction: This Model	Industry Sales Growth	NCP Spread	GDP Growth	Ncredit Spread	Industry Sales Growth	NCP Spread	GDP Growth	Ncredit Spread
Base Case										
Risks measured as return volatility	Firm idiosyncratic risk	+	35.391***	37.418***	35.250***	35.335***	28.269***	30.540***	28.511***	28.410***
			(6.642)	(6.636)	(6.627)	(6.642)	(24.557)	(25.792)	(24.140)	(24.808)
	Firm systematic risk	–	−22.466***	−21.785***	−21.835***	−22.312***	−11.412***	−10.866***	−11.401***	−10.906***
			(−3.053)	(−2.961)	(−2.967)	(−3.035)	(−7.188)	(−7.174)	(−6.974)	(−6.745)
Robustness tests										
CDF of risks	Firm idiosyncratic risk	+	22.684***	24.221***	22.618***	22.679***	14.153***	15.458***	14.279***	14.296***
			(6.798)	(6.786)	(6.789)	(6.807)	(25.451)	(24.731)	(24.642)	(25.250)
	Firm systematic risk	–	−6.348***	−6.334**	−6.036**	−6.269**	−1.880***	−1.892***	−2.135***	−2.122***
			(−2.581)	(−2.578)	(−2.442)	(−2.546)	(−3.746)	(−3.815)	(−4.280)	(−4.420)

(*Continued*)

Table V—*Continued*

			OLS				Median			
	Aggregate State Proxy	Prediction: This Model	Industry Sales Growth	NCP Spread	GDP Growth	Ncredit Spread	Industry Sales Growth	NCP Spread	GDP Growth	Ncredit Spread
Panel A: Total Equity Incentives Measured Using Available Information										
Risks from Fama–French 3-factor model	Firm idiosyncratic risk	+	40.673***	41.984***	40.619***	40.752***	31.297***	32.752***	31.376***	31.025***
			(5.904)	(5.951)	(5.895)	(5.917)	(20.572)	(21.829)	(21.156)	(20.701)
	Firm systematic risk	–	−17.455**	−16.144**	−17.151**	−17.536**	−7.994***	−6.458***	−7.772***	−7.586***
			(−2.242)	(−2.055)	(−2.199)	(−2.253)	(−4.678)	(−3.880)	(−4.665)	(−4.532)
Average risks	Firm idiosyncratic risk	+	27.405***	28.746***	26.983***	27.325***	17.305***	18.427***	17.652***	17.845***
			(4.241)	(4.113)	(4.147)	(4.199)	(13.611)	(13.805)	(13.607)	(14.292)
	Firm systematic risk	–	−29.910***	−29.286***	−28.813***	−29.736***	−7.193***	−5.519***	−6.686***	−7.138***
			(−3.281)	(−3.211)	(−3.151)	(−3.258)	(−3.578)	(−2.831)	(−3.250)	(−3.531)
Instrument variable analysis	Firm idiosyncratic risk	+	38.711***	42.308***	38.024***	38.451***				
			(4.128)	(3.985)	(4.045)	(4.089)				
	Firm systematic risk	–	−36.410***	−35.556***	−34.824***	−35.918***				
			(−3.493)	(−3.426)	(−3.344)	(−3.454)				

(*Continued*)

Table V—*Continued*

Panel B: Total Equity Incentives Measured Using 1-Year Approximation Method										
			OLS				Median			
	Aggregate State Proxy	Prediction: This Model	Industry Sales Growth	NCP Spread	GDP Growth	Ncredit Spread	Industry Sales Growth	NCP Spread	GDP Growth	Ncredit Spread
Base Case										
Risks measured as return volatility	Firm idiosyncratic risk	+	35.868***	38.263***	35.807***	35.850***	30.234***	32.112***	30.275***	30.262***
			(6.640)	(6.688)	(6.639)	(6.646)	(25.817)	(27.897)	(26.712)	(27.039)
	Firm systematic risk	−	−21.916***	−21.055***	−21.458***	−21.849***	−11.277***	−10.444***	−11.317***	−10.945***
			(−2.938)	(−2.823)	(−2.874)	(−2.931)	(−6.548)	(−6.698)	(−6.860)	(−6.633)
Robustness tests										
CDF of risks	Firm idiosyncratic risk	+	23.365***	25.203***	23.339***	23.373***	15.377***	16.321***	15.378***	15.369***
			(6.891)	(6.934)	(6.893)	(6.904)	(27.828)	(27.846)	(27.632)	(27.054)
	Firm systematic risk	−	−6.132**	−6.084**	−5.894**	−6.093**	−1.941***	−1.664***	−1.981***	−1.921***
			(−2.456)	(−2.441)	(−2.347)	(−2.438)	(−4.023)	(−3.383)	(−4.125)	(−3.969)
Risks from Fama–French 3-factor model	Firm idiosyncratic risk	+	42.620***	44.196***	42.647***	42.703***	33.364***	34.887***	33.426***	33.359***
			(6.125)	(6.199)	(6.132)	(6.142)	(23.167)	(23.560)	(23.392)	(22.746)
	Firm systematic risk	−	−19.291**	−17.653**	−19.090**	−19.362**	−7.519***	−6.748***	−7.942***	−7.661***
			(−2.462)	(−2.234)	(−2.432)	(−2.472)	(−4.253)	(−4.016)	(−4.630)	(−4.357)
Average risks	Firm idiosyncratic risk	+	28.175***	30.016***	27.893***	28.207***	19.064***	20.392***	19.087***	18.940***
			(4.329)	(4.260)	(4.264)	(4.309)	(15.130)	(15.351)	(16.076)	(16.378)
	Firm systematic risk	−	−28.608***	−27.660***	−27.831***	−28.672***	−5.719***	−5.016***	−5.911***	−5.818***
			(−3.101)	(−2.996)	(−3.005)	(−3.102)	(−2.869)	(−2.705)	(−3.205)	(−3.103)
Instrument variable analysis	Firm idiosyncratic risk	+	40.072***	44.513***	39.548***	39.949***				
			(4.238)	(4.157)	(4.181)	(4.219)				
	Firm systematic risk	−	−35.100***	−33.880***	−33.839***	−34.862***				
			(−3.331)	(−3.231)	(−3.214)	(−3.315)				

idiosyncratic risk on PPS remains the same after controlling for the effects of dollar risk measures (please see the Internet Appendix).[20] Our finding is also consistent with findings in Demsetz and Lehn (1985), Core and Guay (1999, 2002a, and 2002b), and Coles, Daniel, and Naveen (2006), who use the same risk measures as ours.

C. Test of Prediction 2: Effects of Idiosyncratic and Systematic Risks on the Ratio of Total Compensation to Firm Size

Before conducting the regression analysis, it is useful to examine the time-series behavior of the main variables. In Table VI, we report the year-by-year medians of the key variables.

We also present the corresponding plots in Figures 3 through 6.

Let us first examine compensation. Since the samples for 1992 and 1993 are relatively small and biased toward large firms, we use the 1994 sample as the base case for the following discussions. There is an upward trend in annual compensation, as apparent in Table VI and Figure 3. The median equity-related pay and total compensation increased from \$805,000 and \$1,962,000 in 1994 to \$2,771,000 and \$3,516,000 in 2009, respectively. The corresponding percentage increases are 244% and 79%. The numbers suggest that the increase in total pay is mostly due to the increase in equity-related pay.

To examine the trend in firm size, in Table VI we report the annual medians for firm sales and market capitalization (the trend for book assets is similar). Clearly, there is positive growth in median firm size during the sample period. Market capitalization increased 40% from 1994 to 2009. This growth rate is much lower than those in equity-related pay and total pay. The differential growth rates are manifested by the trend in the median ratio of total pay to firm size. The median ratio exhibits a positive trend (see Figure 4), increasing from \$1.59 per \$1,000 in 1994 to \$1.94 per \$1,000 in 2009 (see Table VI). As for the risk measures, the median firm idiosyncratic risk is much higher than the median firm systematic risk, as apparent in Figure 5. Moreover, the median idiosyncratic risk increased from 1994 to 2004 and declined afterwards. This nonmonotonic pattern is consistent with the evidence documented in the asset pricing literature.[21]

The most salient and relevant observations from Table VI and Figures 3 to 6 are: (1) equity-related compensation and total compensation have increased, and (2) the increase in total compensation has outpaced the increase in firm size.

[20] The Internet Appendix may be found in the online version of this article.

[21] In the asset pricing literature, Campell et al. (2001) first document a noticeable increasing trend in firm-specific risk for the period 1962 to 1997. However, Brandt et al. (2010) show that, during recent years, idiosyncratic volatility has fallen substantially, reversing any time trend documented by Campell et al. (2001). They also find that the late 1990s surge and 2000s reversal in idiosyncratic volatility is most evident in firms with low stock prices and limited institutional ownership. Another recent paper by Bekaert, Hodrick, and Zhang (2010) examines aggregate idiosyncratic volatility in 23 developed equity markets and finds no evidence of upward trends.

Table VI

Median Statistics for Annual Pay, Firm Size, Ratio between Annual Pay and Firm Size, and Firm Risks during 1992–2009

This table reports median statistics for annual compensation, firm size, the ratio between annual pay and firm size, and firm risks for 12,890 firm-years over the period 1992 to 2009. CEO compensation and characteristics data are retrieved from *ExecuComp*, and firm characteristics data are from *COMPUSTAT* and *CRSP*. Firm size is proxied by firm sales or market capitalization. Total firm return volatility is stock return volatility over the 60 months prior to the fiscal year. Firm systematic risk is equal to a firm's beta multiplied by stock market risk while firm idiosyncratic risk is the square root of the difference between total return variance and systematic return variance. All risk measures are annualized. The commercial paper spread is defined as the difference between the annualized rate on 3-month commercial paper and the 3-month T-bill rate, while the credit spread is the difference between the yield of Baa and Aaa bonds. All monetary variables are deflated using 2005 dollars.

Year	Total	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Sample size	12,890	122	412	572	637	694	745	783	796	809	861	939	977	1,009	986	567	640	672	669
Annual pay (millions)																			
Equity incentive pay		$1.020	$0.765	$0.805	$0.720	$0.862	$1.143	$1.174	$1.370	$1.415	$1.624	$1.582	$1.380	$1.659	$1.800	$2.691	$3.197	$3.301	$2.771
Total compensation		$2.850	$1.995	$1.962	$1.902	$1.999	$2.422	$2.520	$2.566	$2.827	$2.854	$2.913	$2.652	$3.130	$3.123	$3.939	$4.006	$4.159	$3.516
Firm size (billions)																			
Sales		$4.988	$1.951	$1.343	$1.262	$1.310	$1.377	$1.378	$1.171	$1.238	$1.205	$1.188	$1.169	$1.257	$1.284	$1.578	$1.821	$1.915	$1.624
Market capitalization		$4.832	$1.779	$1.231	$1.224	$1.176	$1.386	$1.427	$1.242	$1.179	$1.227	$1.132	$1.356	$1.650	$1.618	$2.173	$2.423	$1.530	$1.726
$R_{pay/size}$ = Total pay/size																			
Total Pay/Sales ($/a thousand $)		$0.476	$0.947	$1.308	$1.395	$1.424	$1.579	$1.745	$1.910	$2.076	$2.058	$2.148	$2.090	$2.159	$2.219	$2.129	$2.114	$2.003	$2.185
Total pay/market cap ($/a thousand $)		$0.519	$1.025	$1.587	$1.500	$1.542	$1.501	$1.772	$2.010	$2.150	$2.185	$2.354	$1.870	$1.814	$1.831	$1.615	$1.613	$2.467	$1.944
Firm risks																			
Firm idiosyncratic risk		22%	26%	29%	30%	30%	30%	31%	33%	37%	42%	44%	46%	46%	42%	35%	30%	28%	29%
Firm systematic risk		21%	16%	14%	14%	11%	8%	8%	12%	13%	14%	15%	16%	16%	16%	15%	13%	11%	15%
Macroeconomic variables																			
Industry sales growth		3.8%	1.8%	6.6%	12.0%	5.5%	3.7%	7.1%	6.9%	4.6%	−3.3%	−2.6%	5.5%	7.6%	1.4%	5.7%	3.1%	3.6%	−8.5%
Commercial paper spread (basis points)		33	24	15	29	27	26	38	43	40	31	17	5	8	1	20	25	44	73
GDP growth		3.4%	2.9%	4.1%	2.5%	3.7%	4.5%	4.4%	4.8%	4.1%	1.1%	1.8%	2.5%	3.6%	3.1%	2.7%	1.9%	0.0%	−2.6%
Credit spread (basis points)		84	71	66	61	68	60	69	83	75	87	131	110	76	83	89	92	181	198

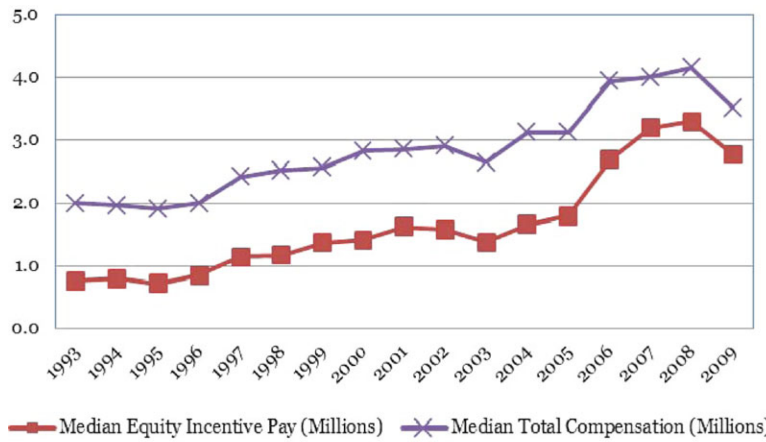

Figure 3. Time trend for median annual pay.

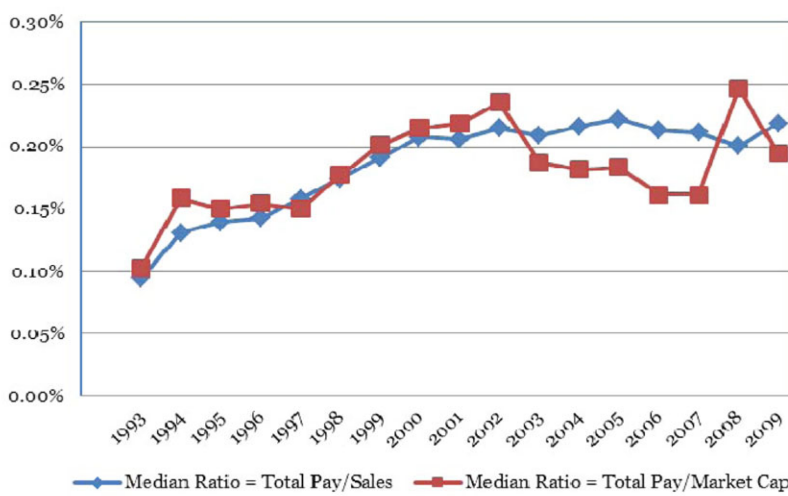

Figure 4. Time trend for median ratio between pay and size.

Next, we formally investigate how firms' risks affect the ratio of CEO pay to firm size. Specifically, we run the following regression to test Prediction 2:

$$
\begin{aligned}
R_{pay/size} = {} & a_1 + a_2 \mathit{Firm\ idiosyncratic\ risk} + a_3 \mathit{Firm\ systematic\ risk} \\
& + a_4 \mathit{Macro\ proxy} + a_5 \mathit{CEO\ Age} + a_6 \mathit{CEO\ Tenure} + a_7 \log(\mathit{Firm\ size}) \\
& + a_8 \mathit{Firm\ growth} + a_9 \mathit{Free\ cash\ flow} + a_{10} \mathit{Capital/sales} \\
& + a_{11} \mathit{R\&D/capital} + a_{12} \mathit{R\&D\ missing\ dummy}
\end{aligned}
$$

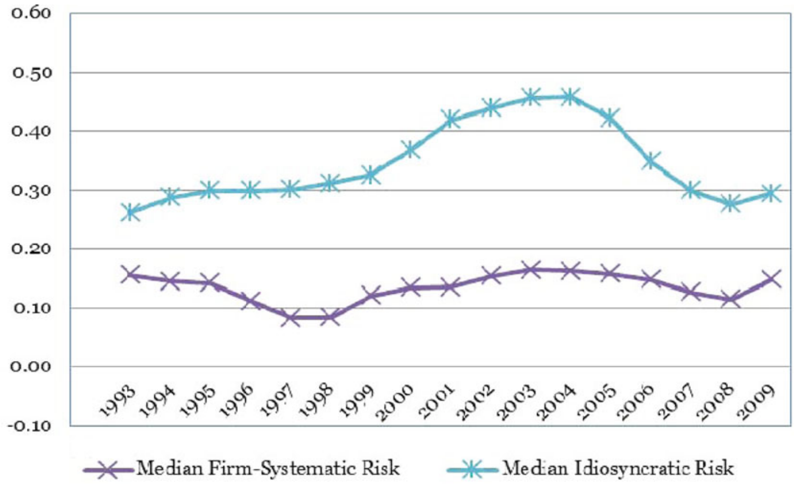

Figure 5. Time trend for median firm risks.

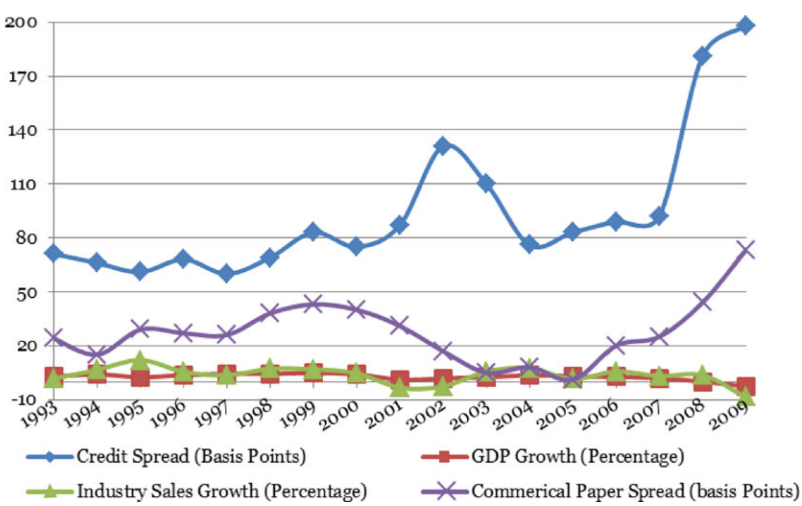

Figure 6. Time trend for macro proxies.

$$+ a_{13} Advertising/capital + a_{14} Advertising\ missing\ dummy$$
$$+ a_{15}\ Investment/capital + \varepsilon. \quad (18)$$

The pay–size ratio is computed based on three measures of firm size: (1) market capitalization, (2) sales, and (3) book assets. For brevity, we only report the results based on sales in Table VII. The results based on book assets and market capitalization are qualitatively similar and are presented in the Internet Appendix. Panel A of Table VII presents the complete results for the base

Table VII

Tests of Prediction 2: Effects of Firm Risks on Pay–Size Ratio

This table reports results for the pay–size ratio based on the regression (18). The sample period is 1992 to 2009 and the sample size is 12,890 firm-years. CEO compensation and characteristics data are retrieved from ExecuComp, and firm characteristics data are from COMPUSTAT and CRSP. The pay–size ratio is the ratio between CEO total compensation and the firm's sales. Panel A presents the base case results where the risk measures are estimated using the market model. We also run regressions by replacing "idiosyncratic" and "systematic" risks with "total risk." The coefficient and t-value for total risk are reported at the bottom of the table. Panel B presents results of four robustness tests. The first test uses the cumulative distribution functions (CDFs) of the risk measures from the market model (base case), while the second test uses the risk measures obtained from the Fama–French three-factor model. The third test uses the industry average risk measures to proxy for the individual risk measures. The last robustness test runs an instrumental variable regression where the industry average risk measures are taken as the instrumental variables. All risk measures are annualized and calculated over the 60 months prior to the fiscal year. CEO tenure is the number of years a person has been the CEO of a firm. Firm size is measured by sales. Firm growth is sales growth. Free cash flow is operating cash flow minus dividends over assets. Four macro proxies are used in the regressions: industry sales growth, negative lagged commercial paper spread (NCP spread), GDP growth, and negative credit spread (Ncredit spread). The commercial paper spread is defined as the difference between the annualized rate on 3-month commercial paper and the 3-month T-bill rate, while the credit spread is the difference between the yield of Baa and Aaa bonds. All monetary variables are deflated using 2005 dollars. For brevity, only coefficients on risk measures are reported. For OLS regressions, standard errors are clustered at the firm level. For median regressions, standard errors are calculated by bootstrapping with 500 replications; t-statistics are in parentheses. *, **, and *** indicate significance at 10%, 5%, and 1% levels, respectively.

Panel A: $R_{pay/size}$ = Annual Total Pay/Sales ($/a thousand $)

		OLS				Median			
	Prediction: This Model	Industry Sales Growth	NCP Spread	GDP Growth	Ncredit Spread	Industry Sales Growth	NCP Spread	GDP Growth	Ncredit Spread
Firm idiosyncratic risk	+	12.660***	13.493***	12.668***	12.660***	6.388***	6.837***	6.428***	6.398***
		(12.446)	(12.642)	(12.425)	(12.439)	(27.959)	(31.204)	(31.183)	(28.483)
Firm systematic risk	−	−8.487***	−8.094***	−8.522***	−8.495***	−3.402***	−3.404***	−3.776***	−3.585***
		(−5.507)	(−5.317)	(−5.519)	(−5.518)	(−10.277)	(−10.049)	(−11.799)	(−11.097)
Industry sales growth		0.536				−0.574***			
		(0.842)				(−3.018)			
NCP spread (basis points)			−0.031***				−0.012***		
			(−6.979)				(−10.824)		
GDP growth				−2.620				−7.969***	
				(−0.689)				(−7.826)	
NCredit spread (basis points)					−0.001				−0.004***
					(−0.399)				(−8.396)

(*Continued*)

Table VII—*Continued*

Panel A: $R_{pay/size}$ = Annual Total Pay/Sales ($/a thousand $)

	Prediction: This Model	OLS: Industry Sales Growth	OLS: NCP Spread	OLS: GDP Growth	OLS: Ncredit Spread	Median: Industry Sales Growth	Median: NCP Spread	Median: GDP Growth	Median: Ncredit Spread
CEO age		−0.058***	−0.052***	−0.057***	−0.057***	−0.011***	−0.009***	−0.010***	−0.010***
		(−2.959)	(−2.678)	(−2.932)	(−2.937)	(−3.229)	(−3.007)	(−3.101)	(−2.789)
CEO tenure		0.046***	0.043***	0.046***	0.046***	0.014***	0.015***	0.016***	0.016***
		(2.900)	(2.702)	(2.908)	(2.904)	(4.491)	(4.443)	(5.034)	(4.822)
Firm growth		3.207***	3.488***	3.304***	3.285***	0.360**	0.439***	0.455***	0.426***
		(5.959)	(6.515)	(6.160)	(6.178)	(2.494)	(2.986)	(3.027)	(2.979)
Free cash flow		−24.659***	−24.445***	−24.666***	−24.659***	−0.969**	−0.829*	−0.953**	−1.024**
		(−7.470)	(−7.458)	(−7.484)	(−7.486)	(−2.094)	(−1.827)	(−2.067)	(−2.146)
Capital/Sales		2.860***	2.835***	2.864***	2.864***	0.901***	0.878***	0.923***	0.923***
		(8.018)	(7.939)	(8.009)	(8.011)	(14.726)	(15.302)	(14.617)	(15.036)
RD/Capital		3.401***	3.337***	3.395***	3.397***	2.613***	2.578***	2.635***	2.597***
		(9.485)	(9.329)	(9.448)	(9.440)	(16.761)	(15.617)	(16.504)	(16.066)
RD missing dummy		−0.680***	−0.653***	−0.681***	−0.681***	0.055	0.082**	0.045	0.048
		(−2.874)	(−2.780)	(−2.881)	(−2.879)	(1.336)	(2.007)	(1.249)	(1.258)
Advertising/Capital		0.185	0.215	0.195	0.192	0.685***	0.725***	0.747***	0.739***
		(0.304)	(0.355)	(0.321)	(0.316)	(4.098)	(4.726)	(4.871)	(4.952)
Advertising missing dummy		0.512**	0.494**	0.528**	0.525**	0.241***	0.247***	0.286***	0.298***
		(2.475)	(2.390)	(2.545)	(2.524)	(5.328)	(5.510)	(6.704)	(6.853)
Investment/Capital		4.623***	4.251***	4.656***	4.649***	1.373***	1.239***	1.445***	1.525***
		(4.985)	(4.600)	(5.025)	(5.011)	(6.206)	(5.706)	(6.571)	(6.814)
Adjusted or pseudo-R^2		0.372	0.375	0.372	0.372	0.148	0.151	0.150	0.150
Alternative Model									
Firm total risk		9.144***	10.072***	9.139***	9.139***	5.061***	5.542***	4.990***	4.966***
		(11.132)	(11.499)	(11.121)	(11.118)	(25.847)	(25.741)	(25.282)	(25.190)
Adjusted or pseudo-R^2		0.360	0.363	0.360	0.360	0.139	0.141	0.140	0.140

(*Continued*)

Table VII—*Continued*

Panel B: $R_{pay/size}$ = Annual Total Pay/Sales ($/a thousand $)

	Aggregate State Proxy	Prediction This Model	OLS Industry Sales Growth	OLS NCP Spread	OLS GDP Growth	OLS Ncredit Spread	Median Industry Sales Growth	Median NCP Spread	Median GDP Growth	Median Ncredit Spread
Robustness tests										
CDF of risks	Firm idiosyncratic risk	+	6.25***	6.73***	6.26***	6.25***	2.92***	3.13***	2.89***	2.89***
			(15.407)	(15.568)	(15.382)	(15.403)	(37.908)	(34.884)	(36.939)	(35.368)
	Firm systematic risk	−	−1.51***	−1.48***	−1.52***	−1.51***	−0.71***	−0.67***	−0.76***	−0.72***
			(−4.231)	(−4.162)	(−4.207)	(−4.219)	(−9.370)	(−8.886)	(−10.241)	(−9.174)
Risks from Fama–French 3-factor Model	Firm idiosyncratic risk	+	12.5***	13.09***	12.49***	12.49***	6.59***	6.86***	6.64***	6.59***
			(9.751)	(10.047)	(9.741)	(9.752)	(24.320)	(26.985)	(25.298)	(25.406)
	Firm systematic risk	−	−2.65	−1.92	−2.64	−2.65	−1.37***	−1.05***	−1.492***	−1.4***
			(−1.537)	(−1.119)	(−1.536)	(−1.538)	(−3.445)	(−2.730)	(−3.888)	(−3.562)
Average risks	Firm idiosyncratic risk	+	9.53***	10.22***	9.59***	9.57***	3.68***	3.87***	3.76***	3.68***
			(10.963)	(11.038)	(10.945)	(10.960)	(20.997)	(21.041)	(21.411)	(20.932)
	Firm systematic risk	−	−10.27***	−9.93***	−10.41***	−10.34***	−2.87***	−2.76***	−3.25***	−3.02***
			(−5.736)	(−5.565)	(−5.792)	(−5.783)	(−7.764)	(−7.073)	(−8.456)	(−8.236)
Instrument variable analysis	Firm idiosyncratic risk	+	12.62***	13.44***	12.628***	12.62***				
			(12.411)	(12.603)	(12.390)	(12.404)				
	Firm systematic risk	−	−8.49***	−8.1***	−8.52***	−8.49***				
			(−5.506)	(−5.319)	(−5.517)	(−5.516)				

case. For the same reason as for Table IV, Panel B of Table VII reports only the coefficients of the two risk measures for the robustness tests.

Our main finding is that the ratio $R_{pay/size}$ is affected positively by firm idiosyncratic risk and negatively by firm systematic risk, confirming Prediction 2. Based on the magnitude of the regression coefficients, $R_{pay/size}$ is much more sensitive to firm idiosyncratic risk than to firm systematic risk. To illustrate, let us focus on the median regression in Panel A when the macro variable is proxied by the negative commercial paper spread. An increase of one standard deviation in idiosyncratic risk (approximately 19%) results in an increase of \$1.299 per \$1,000 ($= 6.837 \times 19\%$) in the ratio while an increase of one standard deviation in systematic risk (approximately 10%) results in a reduction of \$0.340 per \$1,000 ($= 3.404 \times 10\%$). Given that the median pay–size ratio of the sample is \$1.81 per \$1,000, the impact of firm risk measures on the pay–size ratio is not only statistically significant, but also economically important.

Turning to the robustness tests, even though the magnitude of the coefficients varies among the different specifications, firm idiosyncratic risk consistently has a significantly positive impact on the pay–size ratio while firm systematic risk has a significantly negative impact. In sum, the empirical results support Prediction 2 on the relationship between risks and the pay–size ratio.

V. Conclusion

This paper addresses two questions regarding executive compensation: (1) how does PPS depend on systematic and idiosyncratic risks, and (2) how does the pay–size ratio depend on these risks? To address these questions, we integrate an agency problem into search theory and analyze the market equilibrium with many firms and CEOs. Our model differs from a standard static agency model with a single agent–firm pair along three dimensions. First, instead of focusing on total risk as in the extant literature, our model distinguishes a firm's idiosyncratic risk from systematic risk. Second, a CEO can choose to quit after privately observing the idiosyncratic shock. Third, there are contracting interactions/externalities among firms in the market equilibrium that work through endogenous outside options and matching probabilities. In our setup, each firm offers an incentive contract to the CEO that achieves the optimal trade-off between the probability of retaining the CEO and the expected profit conditional on retention. This trade-off generates an optimal PPS that is less than one for a risk-neutral CEO. More importantly, the search process endogenously determines CEOs' and firms' outside options, which reflect the externalities in the market equilibrium. The externalities induce novel effects of the risks on incentive contracts, which are confirmed by our empirical tests using CEO compensation data from 1992 to 2009. First, the equilibrium PPS depends positively on a firm's idiosyncratic risk, and negatively on a firm's systematic risk. This is in contrast to agency models with exogenous outside options, where the two risks always affect PPS in the same way. This result offers a plausible explanation for the ambiguous empirical relationship between PPS and a firm's total risk. Second, the ratio of a CEO's total compensation to firm value depends positively on firm idiosyncratic risks and negatively on firm systematic risks.

A natural extension of the current study is to investigate whether the model predictions hold for executive compensation practices worldwide. For example, Oxelheim, Wihlborg, and Zhang (2010) show that macroeconomic influences on Swedish CEOs' compensation are substantial. It would be relevant to investigate how firms' systematic and idiosyncratic risks affect European executive compensation. Moreover, although European executives receive less compensation than their American counterparts, their compensation has also risen in recent years (see Oxelheim and Wihlborg (2008)). It would be interesting to see how European compensation has evolved relative to firm size and whether firm risk factors significantly affect the relative growth of compensation to firm size.

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Appendix A: Proof of Proposition 1

We start by reformulating the contracting problem with (b,ρ) as a firm's choices, instead of (b,a). To do so, we invert (5) to obtain $\mathbb{E}_y(a) = \underline{u} - \frac{\rho\bar{x}}{2c}\mathbb{E}_y(b^2y^{2\alpha})$. Substituting this expression and using (3) to compute $\mathbb{E}_y(\pi^* - w^*)$, we rewrite (8) as

$$J_F = J_H + (1-\delta)\max_{(b,\rho)}\int_{\rho\bar{x}}^{\bar{x}}\left[\frac{\rho\bar{x}}{2c}\mathbb{E}_y(b^2y^{2\alpha}) + \frac{x}{c}\mathbb{E}_y[b(1-b)y^{2\alpha}] - (\underline{u}+\underline{J})\right]dF_1(x). \tag{A1}$$

The maximization problem above is the reformulated contracting problem.

Parts (i) and (ii) of Proposition 1: Consider the maximization problem in (A1). For any given $(\underline{u}, \underline{J})$, the objective function in (A1) is continuous in the choices (b,ρ). Because the set of feasible choices is $(b,\rho)\in[0,1]\times[\underline{x}/\bar{x}, 1]$, which is compact, the Theorem of the Maximum (see page 62 in Stokey and Lucas with Prescott (1989)) implies that the maximum is attained by a feasible choice. In this proof, let us denote the optimal choices as $(b^*(y), \rho^*)$ by suppressing their dependence on $(\underline{u}+\underline{J})$. Note that ρ is independent of y. Also note that the objective function is twice continuously differentiable in (b,ρ). With differentiability, we can verify that the objective function is strictly concave in b and ρ separately. However, the objective function is not necessarily concave in (b,ρ) jointly. To circumvent this problem, we use a two-step procedure to prove that the optimal choices are unique. First, for any fixed ρ, we prove that the optimal choice of b is unique. Second, taking into account the dependence of the optimal choice of b on ρ obtained in the first step, we prove that the resulting objective function is strictly concave in ρ and hence the optimal choice ρ^* is unique. In this procedure we also prove that $b^*(y)$ is independent of y and that the interior optimal choices satisfy (12).

Take the first step. For any given $\rho \in [\underline{x}/\bar{x}, 1]$, denote the optimal choice of b as $\hat{b}(\rho, y)$. Clearly, $\hat{b}(\rho^*, y) = b^*(y)$. Because the objective function in (A1) is strictly concave in $b(y)$ for any given ρ, the optimal choice, $\hat{b}(\rho, y)$, is unique. If $\hat{b}(\rho, y)$ is at either corner of $[0, 1]$, then clearly it is independent of y. So, consider interior $\hat{b}(\rho, y)$. Because the objective function is continuously differentiable in $b(y)$, the interior $\hat{b}(\rho, y)$ is characterized by the first-order condition, which is $\hat{b}(\rho, y) = (\rho + 1)/2$. Because ρ is independent of y, $\hat{b}(\rho, y) = \hat{b}(\rho)$ is independent of y. It follows that $b^*(y) = \hat{b}(\rho^*)$ is independent of y. This procedure also establishes that the first equation in (12) holds whenever b^* is interior.

Take the second step. Substituting $b(y) = \hat{b}(\rho)$, we write the objective function in (A1) as $f(\rho) = [1 - F_1(\rho\bar{x})]p(\hat{b}(\rho), \rho)$, where $p(b, \rho)$ is defined by (11). Using the uniform distribution F_1, we can directly verify $f''(\rho) < 0$. Thus, the optimal choice ρ^* is unique and independent of y. This implies that b^* is also unique, because $b^* = \hat{b}(\rho^*)$ and because $\hat{b}(\rho)$ is unique for any given ρ . If ρ^* is interior, then $f'(\rho^*) = 0$, which can be written as $b^* - 2\rho^* + \frac{\underline{u}+\underline{J}}{b^*\Omega} = 0$. Substituting $b^* = (\rho^* + 1)/2$ into the first term of this condition, we get the second equation in (12).

Now we prove that, under the restrictions on $(\underline{u} + \underline{J})$ specified in Proposition 1, the equations in (12) have a unique solution that is interior. Substituting ρ^* from the second equation into the first equation in (12), we get

$$3b^2 - 2b - (\underline{u} + \underline{J})/\Omega = 0.$$

This quadratic equation has a real solution if and only if $\underline{u} + \underline{J} \geq -\Omega/3$. Maintain this condition. Then the above quadratic equation has two real solutions generically. Because the quadratic expression is minimized at $b = 1/3$, the smaller solution is less than or equal to 1/3 and hence less than 1/2, in which case the first equation in (12) implies $\rho^* < 0 < \underline{x}/\bar{x}$. Thus, the smaller solution is not admissible. The larger solution for b^* to the quadratic equation above is given by the first equation in (13), and the implied solution for ρ^* is given by the second equation. This solution satisfies $b^* < 1$ if and only if $\underline{u} + \underline{J} < \Omega$. This condition also guarantees $\rho^* < 1$, because $\rho^* = 2b^* - 1$ by the first equation in (12). Moreover, $\rho^* > \underline{x}/\bar{x}$ if and only if $b^* > \frac{1}{2}(\frac{\underline{x}}{\bar{x}} + 1) = \frac{\mu_x}{\bar{x}}$, which is equivalent to $\underline{u} + \underline{J} > \frac{\Omega\mu_x}{2\bar{x}}(\frac{3\underline{x}}{\bar{x}} - 1)$. Note that this lower bound on $(\underline{u} + \underline{J})$ is greater than $-\Omega/4$ and hence greater than $-\Omega/3$, which was imposed earlier for b^* to be a real number. Thus, the unique solution for the pair (b^*, ρ^*) to (12) is interior if and only if $\frac{\Omega\mu_x}{2\bar{x}}(\frac{3\underline{x}}{\bar{x}} - 1) < \underline{u} + \underline{J} < \Omega$. Finally, (5) yields $\mathbb{E}_y(a^*) = \underline{u} - (b^*)^2\rho^*\Omega$.

Parts (iii) and (iv) of Proposition 1: Consider the solution for (b^*, ρ^*) given by (13). Clearly, $\underline{u}$ and $\underline{J}$ affect (b^*, ρ^*) only through the sum $(\underline{u} + \underline{J})$. Also, (13) shows that an increase in $(\underline{u} + \underline{J})$ increases b^* and ρ^*. Because the CEO's incentive pay is equal to $(b^*)^2\frac{\underline{x}}{\bar{x}}\Omega$, it increases with $\underline{u}$ and $\underline{J}$. Expected salary is equal to $\mathbb{E}_y(a^*) = \underline{u} - (b^*)^2\rho^*\Omega$. By increasing (b^*, ρ^*), an increase in $\underline{J}$ reduces $\mathbb{E}_y(a^*)$. Using (13) we can calculate

$$\frac{d(\mathbb{E}_y(a^*))}{d\underline{u}} = \frac{1}{3}\left\{2 - \left[1 + \frac{3}{\Omega}(\underline{u} + \underline{J})\right]^{1/2}\right\}.$$

This is positive if and only if $\underline{u} + \underline{J} < \Omega$, which is maintained in Proposition 1.

A higher σ_y is reflected by a higher value of $\mathbb{E}_y(y^{2\alpha})$ and a higher σ_x by a higher value of $\bar{x}$. Both risks increase Ω. It is clear from (13) that the two risks affect (b^*, ρ^*) only through Ω. Also, (13) shows that an increase in Ω increases (b^*, ρ^*) if and only if $\underline{u} + \underline{J} < 0$. The incentive pay at $x = \rho^* \bar{x}$ is equal to $(b^*)^2 \rho^* \Omega$. We can compute

$$\frac{d}{d\Omega}[(b^*)^2\rho^*\Omega] = \frac{b^*}{9}\left\{1 - \frac{3}{\Omega}(\underline{u} + \underline{J}) + \left[1 + \frac{3}{\Omega}(\underline{u} + \underline{J})\right]^{1/2}\right\}.$$

This is clearly positive if $\underline{u} + \underline{J} \le \Omega/3$. Consider the case in which $\underline{u} + \underline{J} > \Omega/3$. In this case, the above derivative is positive if and only if $1 + \frac{3}{\Omega}(\underline{u} + \underline{J}) > [\frac{3}{\Omega}(\underline{u} + \underline{J}) - 1]^2$. This condition is equivalent to $\underline{u} + \underline{J} < \Omega$, which is maintained. Thus, an increase in Ω increases $(b^*)^2\rho^*\Omega$. Because $\mathbb{E}_y(a^*) = \underline{u} - (b^*)^2\rho^*\Omega$, the expected salary decreases in Ω. Q.E.D.

Appendix B: Proof of Proposition 2

According to the definition in Section I.C, we determine a market equilibrium by solving for the contract $\psi = (a, b)$, the induced choices by the CEO (ρ, e^*), the value functions (V_E, V_S, J_F, J_H), and the measures (v, s). The effort level e^* is given by (2) as a function of ψ. In Proposition 1, we have already solved (b, ρ) and $\mathbb{E}_y(a)$ as functions of $(\underline{u}, \underline{J})$. We derive (14) and (15) below, which give $(\underline{u}, \underline{J})$ and q as functions of b. We also solve $(\underline{u}, \underline{J})$ and b jointly from (13)–(15). Once this is done, we can determine other equilibrium objects easily. Specifically, we can recover J_F and q from (14) and (15), compute a searching CEO's matching probability as $\lambda = 1 - q$, compute the value function J_H as $J_H = 0$, and compute the value functions (V_E, V_S) from (6) and (7). Moreover, we can compute s from (10) by setting $s_{+1} = s$, solve $\theta = \lambda^{-1} - 1$, and recover $v = s/\theta$. In the procedure below, we suppress the superscript $*$ on optimal choices and the tilde on other firms' choices.

To derive (14) and (15), we impose symmetry between firms' choices and stationarity, as required by the equilibrium. Substituting the properties of the optimal contract, $\rho = 2b - 1 > 0$ and $\mathbb{E}_y(a) = \underline{u} - b^2\rho\Omega$, we can solve V_E from (6), V_S from (7), and J_F from (A1). With stationarity, the outside options are $\underline{u} = V_S - \beta V_E$ and $\underline{J} = -\beta J_F$, where we use the free-entry condition $J_H = 0$. Substituting the solutions of (V_E, V_S) into the expression for $\underline{u}$ and using $\lambda = 1 - q$, we get $\underline{u}$ as in (14). Inverting (13) to get $\underline{u} + \underline{J} = b(3b - 2)\Omega$ and substituting it into the solution just obtained for J_F, we get J_F and $\underline{J}$ as in (14). Setting $J_H = 0$ in (9) yields $q = H/(\beta J_F)$, which produces (15) after substituting J_F from (14).

To solve $(\underline{u}, \underline{J})$ and b jointly from (13)–(15), we substitute q from (15) into (14) to get $\underline{u} = B - \frac{H}{2}b$. Adding this result to the expression for $\underline{J}$ in (14), we get (16). Substituting (16) into the expression for b in (13), we find that the equilibrium

PPS solves $G(b) = 0$, where

$$G(b) \equiv 4Lb(1-b)^2 + 2b(3b-2) + \frac{Hb - 2B}{\Omega}. \tag{B1}$$

The constant L is defined in (15) as $L = \beta(1-\delta)\bar{x}/(\sigma_x\sqrt{3})$. Let us determine the admissible interval for b in the equilibrium. From Proposition 1 we know that b must satisfy $1 > b > \frac{1}{2}(1 + \frac{\underline{x}}{\bar{x}}) = \frac{\mu_x}{\bar{x}}$. Note that $\frac{\mu_x}{\bar{x}} > \frac{1}{2}$ because $\underline{x} > 0$. In addition, the equilibrium must satisfy $q \in (0, 1)$. From (15), we know that q lies in $(0, 1)$ if and only if $b(1-b)^2 > H/(2L\Omega)$. For all $b \in (\mu_x/\bar{x}, 1)$, the function $b(1-b)^2$ is strictly decreasing and hence achieves the maximum at the left corner $\mu_x/\bar{x}$. Also, the function is equal to zero at $b = 1$. Thus, a necessary condition for $q \in (0, 1)$ is $H < 2L\Omega\frac{\mu_x}{\bar{x}}(1 - \frac{\mu_x}{\bar{x}})^2$. Under this condition, $q \in (0, 1)$ if and only if $b \in (\mu_x/\bar{x}, b_1)$, where $b_1 \in (\mu_x/\bar{x}, 1)$ is defined by

$$b_1(1-b_1)^2 = H/(2L\Omega). \tag{B2}$$

Note that b_1 is independent of the parameter B. The admissible interval for b is $(\mu_x/\bar{x}, b_1)$.

Now we establish that a unique solution for equilibrium b exists. Compute

$$G'(b) = 4(3b-1)[1 - L(1-b)] + \frac{H}{\Omega}, \quad G''(b) = 8L(3b-2) + 12.$$

Assume that $G''(1/2) \geq 0$, $G(\mu_x/\bar{x}) < 0$, and $G(b_1) > 0$, which we support with explicit restrictions on the parameters. Because $G''(b)$ is strictly increasing in b, the assumption $G''(1/2) \geq 0$ implies that for all $b > 1/2$, $G''(b) > 0$ and hence $G'(b)$ is strictly increasing. It is evident that $G'(1) > 0$. If $G'(1/2) \geq 0$, then $G'(b) > 0$ for all $b > 1/2$. If $G'(1/2) < 0$, then there exists $b_0 \in (1/2, 1)$ such that $G'(b) < 0$ for $b \in (1/2, b_0)$ and $G'(b) > 0$ for $b \in (b_0, 1)$. In both cases, the assumptions $G(\mu_x/\bar{x}) < 0$ and $G(b_1) > 0$ ensure that a unique solution exists in the admissible interval $(\mu_x/\bar{x}, b_1)$. Furthermore, the solution has the property $G'(b) > 0$.

To summarize the above proof, we find that there is a unique and admissible solution for equilibrium b if the following conditions hold: $H < 2L\Omega\frac{\mu_x}{\bar{x}}(1 - \frac{\mu_x}{\bar{x}})^2$, $G''(1/2) \geq 0$, $G(\mu_x/\bar{x}) < 0$, and $G(b_1) > 0$. Let us express these conditions more explicitly as follows:

$$\begin{aligned}
&H < H_1 \equiv 2\beta(1-\delta)\Omega\frac{\mu_x\sigma_x\sqrt{3}}{\bar{x}^2},\\
&\frac{\mu_x}{\sigma_x\sqrt{3}} \leq \frac{3}{\beta(1-\delta)} - 1,\\
&B > B_1 \equiv \frac{\mu_x}{\bar{x}}\left\{\frac{\Omega}{\bar{x}}\left[\mu_x - 2\sigma_x\sqrt{3}(1 - \beta(1-\delta))\right] + \frac{H}{2}\right\},\\
&B < B_2 \equiv H\left(1 + \frac{b_1}{2}\right) + \Omega b_1(3b_1 - 2).
\end{aligned} \tag{B3}$$

There is a nonempty region of $(H, \mu_x/\sigma_x)$ that satisfies the first two conditions, and this region is independent of the parameter B. Given $(H, \mu_x/\sigma_x)$ in this region, the interval (B_1, B_2) that satisfies the last two conditions is nonempty by construction. Thus, there is a nonempty region of the parameters $(H, \mu_x/\sigma_x, B)$ that satisfies all conditions above.

For comparative statics, let us establish the results $L > \frac{\bar{x}}{2\sigma_x\sqrt{3}} > 1$, $B_1 > H/4$, and $b^* < 2B/H$ under the restriction $\beta \geq \frac{1}{2(1-\delta)}$. Because $\underline{x} > 0$, we have $\mu_x > \sigma_x\sqrt{3}$, $\bar{x} > 2\sigma_x\sqrt{3}$, and $2\mu_x > \bar{x}$. The restriction on β implies $L > \frac{\bar{x}}{2\sigma_x\sqrt{3}} > 1$ and $B_1 > \mu_x H/(2\bar{x}) > H/4$. To prove $b^* < 2B/H$, note that it is equivalent to $G(2B/H) > 0$, which in turn is equivalent to

$$L > \left(1 - \frac{3B}{H}\right) \Big/ \left(1 - \frac{2B}{H}\right)^2.$$

Because $B > B_1 > H/4$, $1 - 6B/H < 0$ and so the right-hand side of the above inequality is an increasing function of B/H. A sufficient condition for the inequality to hold is that it holds at $B = H/4$, which is equivalent to $L > 1$, which we just established.

We now establish results (i) and (ii) stated in Proposition 2. The aggregate shock affects b^* exclusively through $\mathbb{E}_y(y^{2\alpha})$, which appears in Ω in the function $G(b)$. A higher μ_y or σ_y leads to a higher $\mathbb{E}_y(y^{2\alpha})$ and hence a higher Ω. Compute

$$\frac{db^*}{d\Omega} = \frac{-\partial G/\partial\Omega}{G'(b^*)} = -\frac{2B - Hb^*}{\Omega^2 G'(b^*)} < 0.$$

The inequality follows from the result $b^* < 2B/H$, which we just established, and the fact that $G'(b^*) > 0$. To examine the effect of σ_x on b^*, note that $\bar{x} = \mu_x + \sigma_x\sqrt{3}$ and $\frac{d}{d\sigma_x}(\frac{\bar{x}}{\sigma_x\sqrt{3}}) = -\frac{\mu_x}{\sigma_x^2\sqrt{3}}$. Using $G(b^*) = 0$, we can derive

$$\frac{db^*}{d\sigma_x} = \frac{2\sqrt{3}b^*}{\bar{x}G'(b^*)}\left[2L(1-b^*)^2\left(\frac{\mu_x}{\sigma_x\sqrt{3}} - 1\right) - 3b^* + 2\right].$$

Thus, $db^*/d\sigma_x > 0$ if and only if $2L(1-b^*)^2(\frac{\mu_x}{\sigma_x\sqrt{3}} - 1) - 3b^* + 2 > 0$. The left-hand side of this inequality is a strictly decreasing function of b^*. It is positive at $b^* = 2/3$ and negative at $b^* = 1$. Thus, there exists $b_2 \in (2/3, 1)$ such that

$$2L(1-b_2)^2\left(\frac{\mu_x}{\sigma_x\sqrt{3}} - 1\right) - 3b_2 + 2 = 0. \tag{B4}$$

Moreover, $db^*/d\sigma_x > 0$ if $b^* < b_2$ and $db^*/d\sigma_x < 0$ if $b^* > b_2$.

Finally, we turn to part (iii) of Proposition 2. Recall $J_F^* = \frac{2}{\beta}L\Omega b^*(1-b^*)^2$ from (14) and $\mathbb{E}_y(a^*) = \underline{u} - b^{*^2}\rho^*\Omega$ from Proposition 1. Also, since $b^*\pi^* = b^{*^2}y^{2\alpha}x/c$, we have $\mathbb{E}_y(b^*\pi^*) = \frac{2x}{\bar{x}}b^{*^2}\Omega$. Substituting $\mathbb{E}_y(a^*)$ and $\mathbb{E}_y(b^*\pi^*)$, and integrating over x, we get

$$
\begin{aligned}
&(1-\delta)\int_{\rho^*\bar{x}}^{\bar{x}} \mathbb{E}_y(a^* + b^*\pi^*)dF_1(x) \\
&\quad = (1-\delta)\int_{\rho^*\bar{x}}^{\bar{x}} \left[\underline{u} + b^{*^2}\left(\frac{2x}{\bar{x}} - \rho^*\right)\Omega\right] dF_1(x) \\
&\quad = \frac{\bar{x}(1-\delta)(1-b^*)}{\sigma_x\sqrt{3}}[\underline{u} + b^{*^2}\Omega] = \frac{2}{\beta}L\Omega b^*(1-b^*)(2b^*-1) + (1-b^*)LJ_F^* \\
&\quad = J_F^*\left[\frac{2b^*-1}{1-b^*} + (1-b^*)L\right].
\end{aligned}
$$

The ratio between expected total pay and firm value, denoted by $R_{pay/size}$, is computed as

$$
R_{pay/size} = \frac{\mathbb{E}(a^* + b^*\pi^*)}{J_F^*} = \frac{2b^*-1}{1-b^*} + (1-b^*)L.
$$

We obtain $\frac{\partial R_{pay/size}}{\partial b} = \frac{1}{(1-b^*)^2} - L$. Since $(1-b^*)^{-2} > 4$ for $b^* > \frac{1}{2}$ and $L < 3$ due to $\frac{\mu_x}{\sigma_x\sqrt{3}} \leq \frac{3}{\beta(1-\delta)} - 1$, we have $\frac{\partial R_{pay/size}}{\partial b} > 1$. It is easy to establish the following results:

$$
\frac{\partial R_{pay/size}}{\partial \mu_y} = \frac{\partial R_{pay/size}}{\partial b}\frac{\partial b^*}{\partial \mu_y} < 0 \quad \text{and} \quad \frac{\partial R_{pay/size}}{\partial \sigma_y} = \frac{\partial R_{pay/size}}{\partial b}\frac{\partial b^*}{\partial \sigma_y} < 0.
$$

To examine the effect of σ_x on $R_{pay/size}$, we obtain

$$
\begin{aligned}
\frac{\partial R_{pay/size}}{\partial \sigma_x} &= \frac{\partial R_{pay/size}}{\partial b}\frac{\partial b}{\partial \sigma_x} - (1-b^*)L\frac{\mu_x}{\bar{x}\sigma_x} \\
&= \frac{1}{\bar{x}\sigma_x G'(b^*)}\left[2\sqrt{3}b^*(2-3b^*)\left(\frac{1}{(1-b^*)^2} - L\right) + 4L\underline{x}b^*[1 - L(1-b^*)^2]\right. \\
&\qquad \left. - (1-b^*)L\mu_x G'(b^*)\right].
\end{aligned}
$$

Tedious algebra shows that $\frac{\partial R_{pay/size}}{\partial \sigma_x} > 0$ for $b^* \in (\frac{\mu_x}{\bar{x}\sigma_x}, b_3)$, where $b_3 \in (\frac{\mu_x}{\bar{x}\sigma_x}, \frac{2}{3})$ is determined by

$$
2\sqrt{3}b_3(2-3b_3)\left(\frac{1}{(1-b_3)^2} - L\right) + 4L\underline{x}b_3\big[1 - L(1-b_3)^2\big] - (1-b_3)L\mu_x G'(b_3) = 0. \tag{B5}
$$

This completes the proof of part (iii) in Proposition 2. Q.E.D.

Appendix C: Equilibrium Long-Term Incentive Contract

Now we extend the contract studied in Section III to include a retention reward payment. This extension is motivated by existing option grant practice, which exhibits the following features. First, a firm normally grants options to its CEO on a yearly basis. These grants are intended to increase retention and incentives. Second, an option grant has a vesting period that normally lasts 5 years and a CEO can only exercise a fraction of his accumulated options that have vested. If the CEO is fired, he may get part of the retention account as settlement but, if the CEO voluntarily quits, he has to forgo the remaining unvested options. Based on these features, we introduce a retention reward mechanism as follows. If the CEO is newly matched with a firm, he starts with a zero balance in the retention account. The firm will put amount i into the account in the current period. The CEO can receive a fraction ϕ of the retention account balance if he works for the firm next period. This mechanism is carried out as long as the CEO continues to work for the firm. Denote a contract with this reward mechanism as $\psi = (a, b, i)$, where a and b have the same meanings as in the baseline model. Denote the balance of the retention account at the beginning of a period as k. The balance in the next period is

$$k_{+1} = (1 + r)[i + (1 - \phi)k],$$

where $(1 - \phi)\,k$ is the amount in the retention account immediately after paying the CEO in the current period. This amount is augmented by the firm's injection into the account, i. The retention account earns risk-free interest at the rate r.

At the beginning of the current period, if a matched CEO is separated from the firm exogenously, he can negotiate with the firm to settle the retention account. This settlement can be understood as the CEO's severance pay if he is fired by the firm. Negotiation is costly for both sides. Let us model the negotiation cost as a reduction in the retention account of $n(k)$. We use the Nash bargaining rule to determine the split of the remaining amount, $k - n(k)$, between the firm and the CEO, where the CEO's bargaining power is $\eta \in (0, 1)$. That is, the CEO receives $\eta[k - n(k)]$ and the firm receives $(1 - \eta)[k - n(k)]$. However, if a CEO who has survived the exogenous job separation shock decides to quit, he has to forgo the money in the retention account. This treatment is similar to the forgone unvested options in practice. In this case, the firm will claim the entire balance of the retention account.

The value function of a searching CEO is still V_S. Note that, in the event that a CEO just separated from a job exogenously and settled an account k with the firm, V_S is measured after the settlement is already paid. If a searching CEO gets a match, he will start the next period with zero balance in the retention account, and so the future value function in this case will be $V_E(k_{+1} = 0)$. Modifying (7), we have

$$V_S = B + \beta[\lambda V_E(k_{+1} = 0) + (1 - \lambda)V_{S,+1}].$$

The value function of a CEO who enters a period as matched is now denoted as $V_E(k)$. Similar to the baseline model, we can compute an employed

CEO's expected utility over y in the current period as $\mathbb{E}_y(u) = a + \frac{b^2}{2c}x\mathbb{E}_y(y^{2\alpha})$. A CEO accepts a contract if and only if $a + \frac{b^2}{2c}x\mathbb{E}_y(y^{2\alpha}) + \beta V_E(k_{+1}) > V_S$. This acceptance condition can be written as $x \geq \rho(k)\bar{x}$, where the cutoff ratio $\rho(k)$ is

$$\rho(k) = \frac{1}{b^2\Omega}[V_S - \beta V_E(k_{+1}) - a]. \tag{C1}$$

If a CEO separates from the job exogenously, he obtains the settlement income $\eta[k - n(k)]$. Incorporating this income, we modify (6) as

$$\begin{aligned} V_E(k) &= \delta\{\eta[k - n(k)] + V_S\} \\ &\quad + (1-\delta)\left[\int_{\rho\bar{x}}^{\bar{x}} \left(a + b^2\frac{x}{\bar{x}}\Omega + \beta V_E(k_{+1})\right) dF_1(x) + F_1(\rho\bar{x})V_S\right]. \end{aligned}$$

Substituting salary a in terms of ρ from (C1) and integrating over x, we obtain:

$$V_E(k) = V_S + \delta\eta[k - n(k)] + (1-\delta)[1 - F_1(\rho\bar{x})]\frac{b^2}{2}(1-\rho)\Omega.$$

Denote the value function of a firm with a CEO as $J_F(k)$ and the value function of a firm without a CEO as J_H. If a searching firm just settled a retention account with a separating CEO, J_H is measured after the settlement receipts are counted. If a searching firm gets a match, the firm will start the next period with zero balance in the retention account, and so the future value function in this case will be $J_F(k_{+1} = 0)$. Thus,

$$J_H = -H + \beta[qJ_F(k_{+1} = 0) + (1-q)J_{H,+1}].$$

For a firm with a matched CEO, the value function is

$$J_F(k) = \max_{(a,b,i)} \left\{ \begin{array}{l} \delta[(1-\eta)(k - n(k)) + J_H] + (1-\delta)(k + J_H)F_1(\rho\bar{x}) \\ + (1-\delta)\displaystyle\int_{\rho(\psi)\bar{x}}^{\bar{x}} \left[b(1-b)\frac{x}{\bar{x}}2\Omega - a - i + \beta J_F(k_{+1})\right] dF_1(x) \end{array} \right\} \tag{C2}$$

s.t.

$$a = V_S - \beta V_E(k_{+1}) - b^2\rho\Omega \quad \text{and} \quad i = \frac{k_{+1}}{1+r} - k(1-\phi).$$

As in the baseline model, $J_F(k)$ is computed before x and y are realized and hence is independent of x and y. The Bellman equation (C2) modifies the one in the baseline model in three ways. First, the firm receives $(1-\eta)(k - n(k))$ from the retention account when the firm's CEO separates exogenously and receives k when the CEO quits. Second, the firm injects i in the retention account when the CEO stays in the current period, in addition to paying salary and the incentive amount. As a result, the firm's profit in the current period is $\pi - w = 2b(1-b)\Omega\frac{x}{\bar{x}} - a - i$, which appears inside the integral in (C2). Third, because the future balance in the retention account k_{+1} will depend on the current injection i, the firm incorporates this dependence as a constraint.

Since there is a one-to-one mapping between i and k_{+1}, we can restate the contract as $\psi = (a, b, k_{+1})$. By doing so, we can easily formulate the dynamic contracting problem by taking the retention account balance in the current period k as the state variable. In other words, the firm chooses the contract for the current period, $\psi = (a,\ b, k_{+1})$, while taking J_H, $J_{H,+1}$, and $J_F(k_{+1})$ as given. Also, the firm anticipates that the CEO's effort e^* and acceptance rule $\rho(k)$ will depend on the contract. Solving the dynamic maximization problem in (C2) leads to the following optimal contract:

$$k_{+1} \text{ is solved from } n'(k_{+1}) = \frac{1}{\delta}\left(1 - \frac{1}{\beta(1+r)}\right),$$

$$b = \frac{1}{3} + \frac{1}{3}\left[1 + \frac{3}{\Omega}\left(\underline{u}(k_{+1}) + \underline{J}(k_{+1}) + \frac{k_{+1}}{1+r}\right)\right]^{1/2},$$

$$a = \underline{u}(k_{+1}) - \phi k - \Omega b^2(2b-1),$$

$$\rho(k) = 2b - 1,$$

with $\underline{u}(k_{+1}) = V_S - \beta V_E(k_{+1})$ and $\underline{J}(k_{+1}) = J_H - \beta J_F(k_{+1})$ interpreted as the effective outside options for a CEO and a firm, respectively. The above optimal contract indicates that, when $\underline{u}(k_{+1}) + \underline{J}(k_{+1}) + \frac{k_{+1}}{1+r} > 0$, PPS decreases with σ_y and σ_x. The effects of σ_y and σ_x on PPS are opposite when $\underline{u}(k_{+1}) + \underline{J}(k_{+1}) + \frac{k_{+1}}{1+r} < 0$. However, we will show that σ_y and σ_x have different effects on PPS in a market equilibrium when V_S, J_H, $V_E(k_{+1})$, and $J_F(k_{+1})$ are endogenously determined.

For the market equilibrium, we can modify the definition in Section I.C to incorporate the retention account. The equilibrium values of (a, b, k_{+1}, ρ), (V_E, V_S, J_F, J_H), and (q, λ) can be solved from the following set of equations:

$$\underline{u}(k_{+1}) = V_S - \beta V_E(k_{+1}), \tag{C3}$$

$$\underline{J}(k_{+1}) = J_H - \beta J_F(k_{+1}), \tag{C4}$$

$$n'(k_{+1}) = \frac{1}{\delta}\left(1 - \frac{1}{\beta(1+r)}\right), \tag{C5}$$

$$a = \underline{u}(k_{+1}) - \phi k - \Omega b^2(2b-1), \tag{C6}$$

$$b = \frac{1}{3} + \frac{1}{3}\left[1 + \frac{3}{\Omega}\left(\underline{u}(k_{+1}) + \underline{J}(k_{+1}) + \frac{k_{+1}}{1+r}\right)\right]^{1/2}, \tag{C7}$$

$$\rho(k) = 2b - 1, \tag{C8}$$

$$J_F(k) = k\,(1-\eta\delta) - \delta\,(1-\eta)\,n(k) + (1-\delta)\,b(1-b)^2 \frac{2\bar{x}\Omega}{\sigma_x\sqrt{3}}, \tag{C9}$$

$$J_H = -H + \beta[q\,J_F(0) + (1-q)J_H], \tag{C10}$$

$$V_S = B + \beta[\lambda V_E(0) + (1-\lambda)V_S], \tag{C11}$$

$$V_E(k) = V_S + \eta\delta[k - n(k)] + (1-\delta)b^2(1-b)^2 \frac{\bar{x}\Omega}{\sigma_x\sqrt{3}}, \tag{C12}$$

$$s_{+1} = s + (1 - s + \lambda s)[\delta + (1-\delta)F_1(\bar{x}\rho_{+1})] - \lambda s, \tag{C13}$$

$$\beta q\,J_F(k) = H, \tag{C14}$$

$$q = 1 - \lambda. \tag{C15}$$

First, we find the expressions for V_S based on V_E. Working with (C11) and (C12) yields

$$V_S = \frac{1}{1-\beta}\left(B - \frac{b}{2}H + A\beta\,(1-\delta)\,b^2(1-b)^2\frac{\bar{x}\Omega}{\sqrt{3}\sigma_x}\right),$$

which is the same as in the baseline case. We then substitute the expressions for V_S, $V_E(k_{+1})$, and $J_F(k_{+1})$ in (C7). Using (C14) and (C15), we can rewrite (C7) as follows:

$$g(b) = G(b) + 2\left(\beta k - \frac{k}{1+r} - \beta\delta n(k)\right) = 0,$$

where $G(b)$ is defined in (B1) in Appendix B. Since $g(b)$ depends on b in the same way $G(b)$ does, we conclude that the dependence of the equilibrium PPS on μ_y, σ_y, and σ_x remains the same as in Proposition 2. In other words, the introduction of the long-term incentive reward changes the level of b but not the dependence of b on the risks.

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