# Accuracy and Precision of Earthquake Locators: Insights from a Synthetic 2019 Ridgecrest Sequence Experiment

Yifan Yu, William Ellsworth, Gregory Beroza

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Stanford University

# Introduction – Earthquake Location

Fundamental but Challenging Topic in Seismology

**Earthquake**
- Seismogenic Zone
- Induced Seismicity

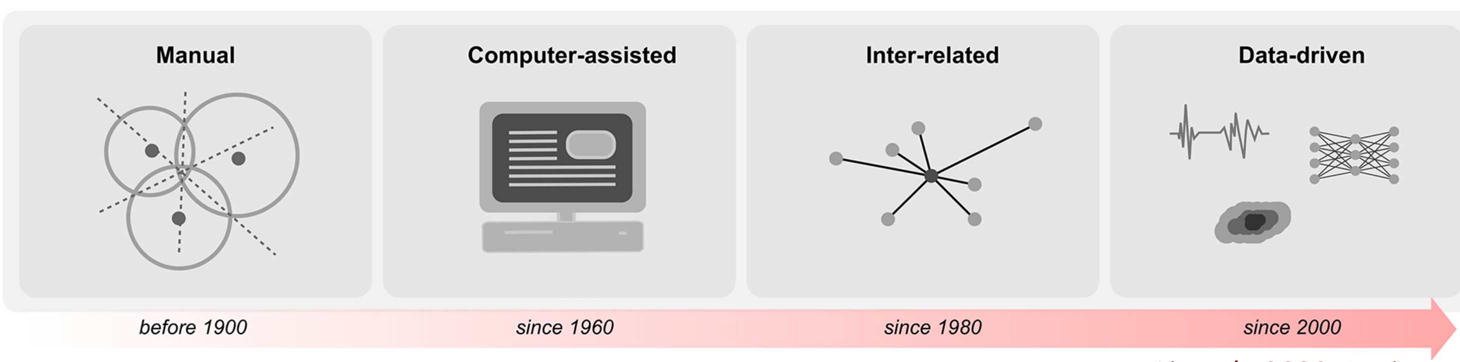
**Volcano**
- Eruption Activity
- Magma Chamber

**Precise Location**

**Explosion**
- Nuclear Monitoring
- Wartime Activity

**Icesheet Fracture**

……

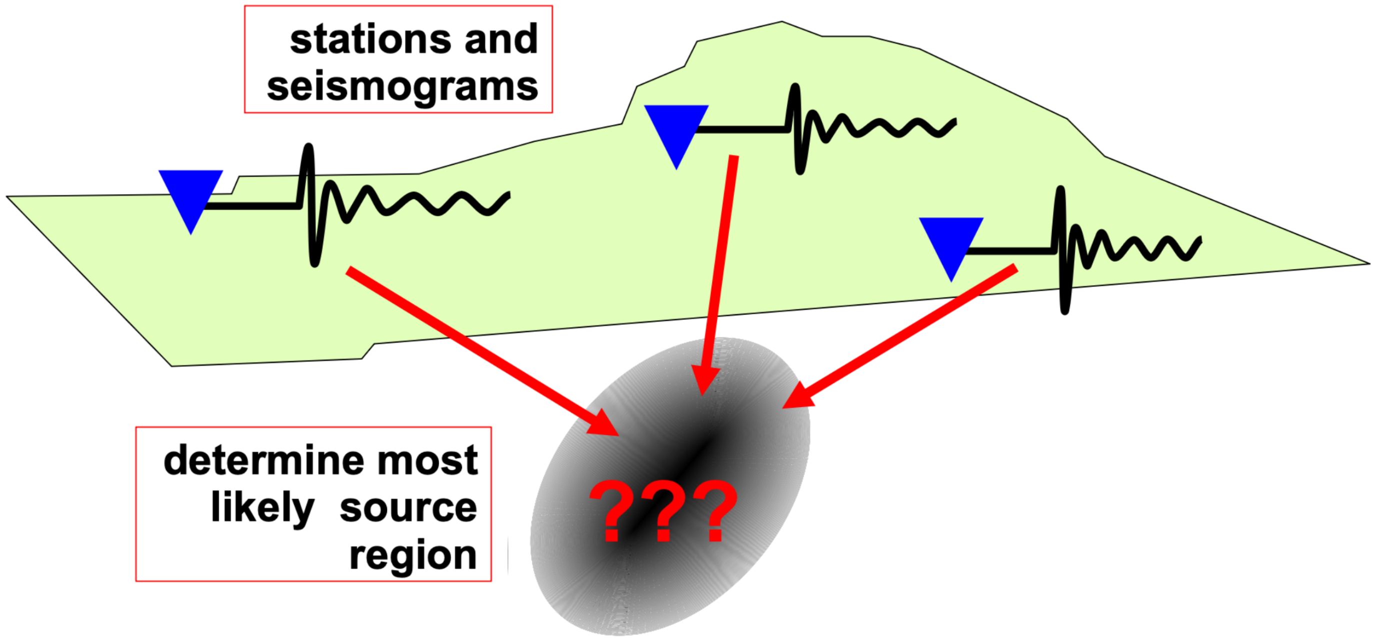



Li et al., 2020, Reviews of Geophysics

# Introduction – Earthquake Location

A Well Defined Nonlinear Inverse Problem

$$t_{arr} = t_0 + \int_r \frac{1}{v(s)} ds \qquad d_{obs} = (t_{arr}^1, \ldots, t_{arr}^n) = G(m) = G([x, y, z, t_0])$$




From: NonLinLoc

# Introduction – Hypocenter Locators

$$R = G * dM$$
$$dM = (G^T G)^{-1} G^T R$$

**Hypoinverse** An iterative linearized inversion scheme

**VELEST**

Constant Error

$$t_{res} = H * h + M * m + e = Gm + e$$

Hypocenter Velocity Model

**Non_Lin_Loc** Nonlinear inversion: Grid Search

**EDT (Equal Differential Time)**

$$pdf(x) \propto \boxed{\sum_{obs_a, obs_b}} e^{-\frac{\left[\boxed{[Tobs_a(x) - Tobs_b(x)]} - \boxed{[TTcalc_a(x) - TTcalc_b(x)]}\right]^N}{\sigma^2}}$$

"satisfy ***the most pairs*** of observations"

- no origin time estimate required

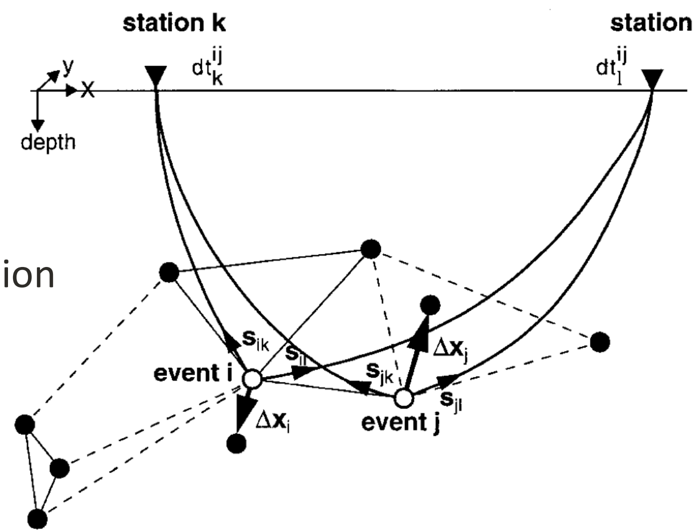
**HypoSVI** Machine Learning model!
Grid search with ML traveltime calculator

# Introduction – Hypocenter Locators

**HypoDD**

$$dr_k^{ij} = (t_k^i - t_k^j)^{obs} - (t_k^i - t_k^j)^{cal}$$

$$\frac{\partial t_k^i}{\partial \boldsymbol{m}} \Delta \boldsymbol{m}^i - \frac{\partial t_k^i}{\partial \boldsymbol{m}} \Delta \boldsymbol{m}^j = dr_k^{ij}$$




**Growclust**

L1 Norm, more resilient to outliers

Traveltime table search instead of matrix inversion

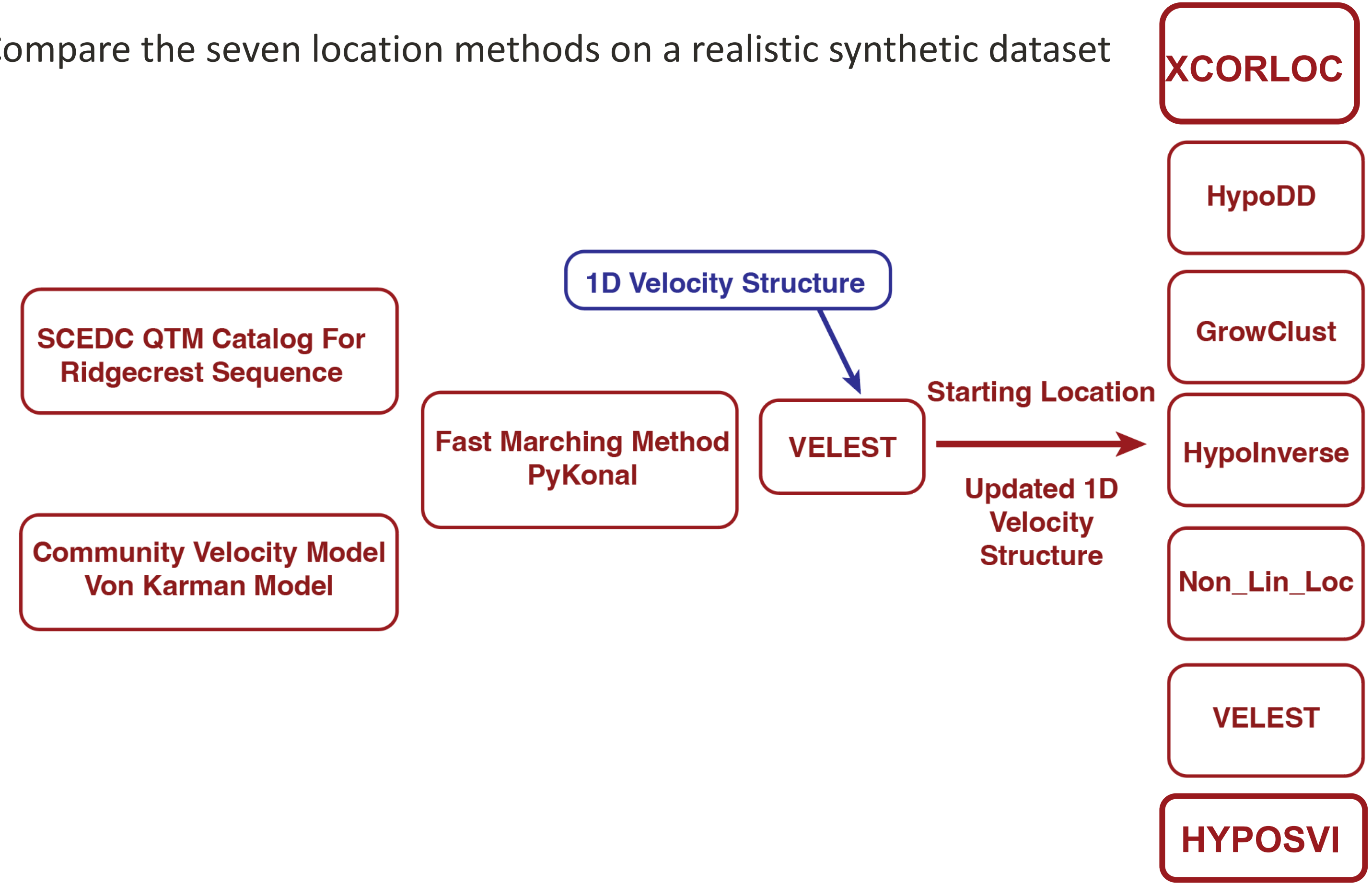
**XCORLOC**

Similar to Growclust

But with Source Station Specific Term

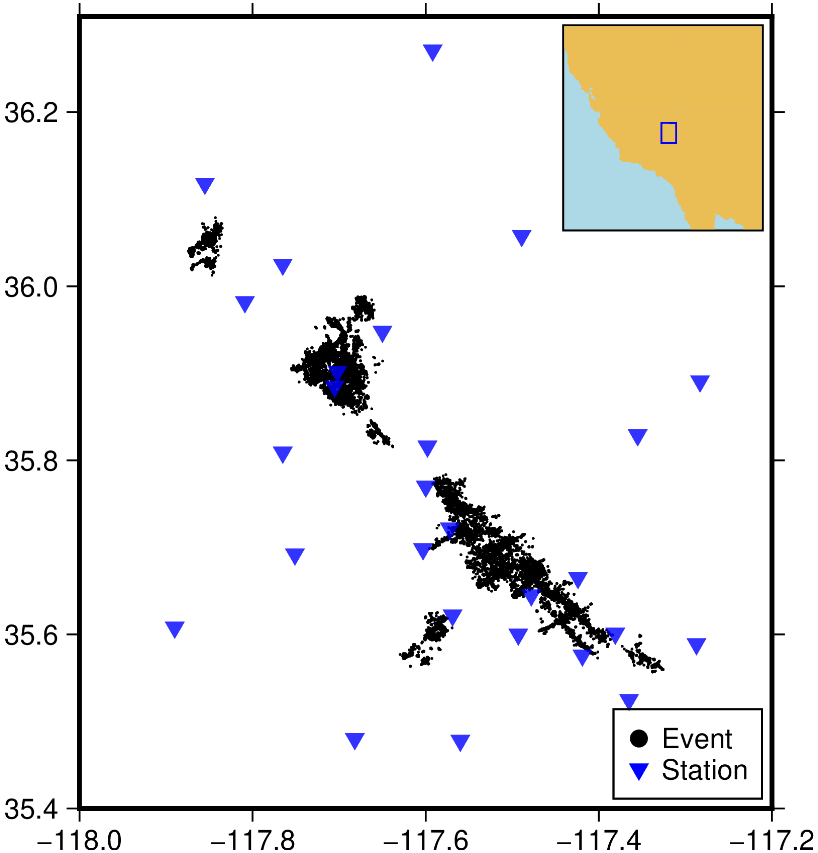
# How to compare their performance?

Compare the seven location methods on a realistic synthetic dataset



# Setup – Earthquake Location

2019 Ridgecrest Earthquake Sequence – Data Mine



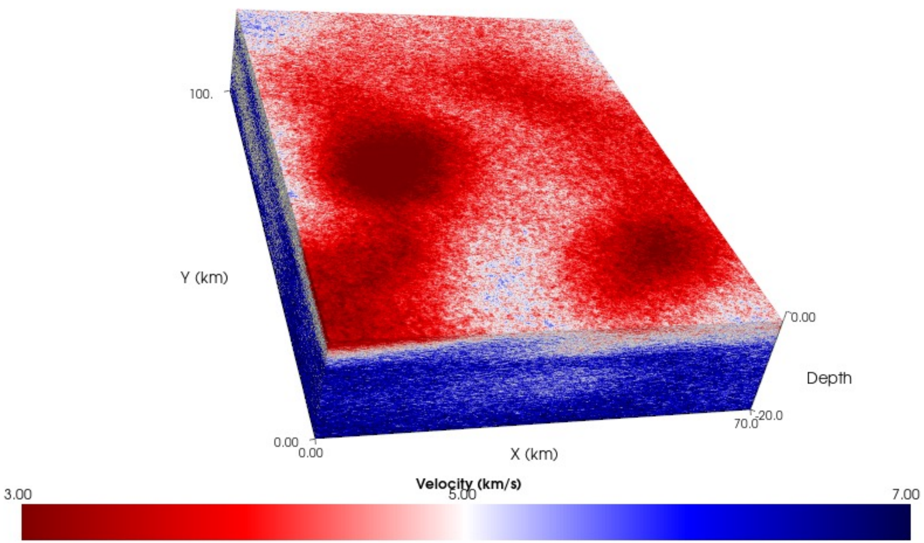

SCEDC QTM Catalog For Ridgecrest

We randomly draw 1000 sources out of the catalog (cluster > 400) as true locations

Real stations distribution from SCEDC

# Setup – Velocity Structure

3D, Heterogeneous




- SCEC Community Velocity
- Von Karman Perturbation
  (a_z=100m, a_h=500m, $\epsilon = 0.05$)
- $70km \times 100km \times 20km\ depth + (max.\, 2km\ elevation)$
  (dx=dy=100m, dz=50m)

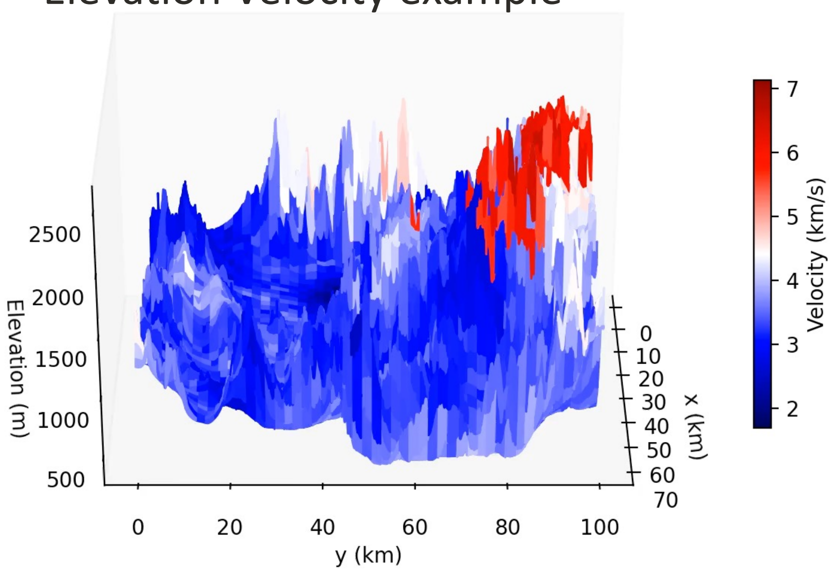
# Setup – Elevation Effect

Mountains and basins could impact the location!

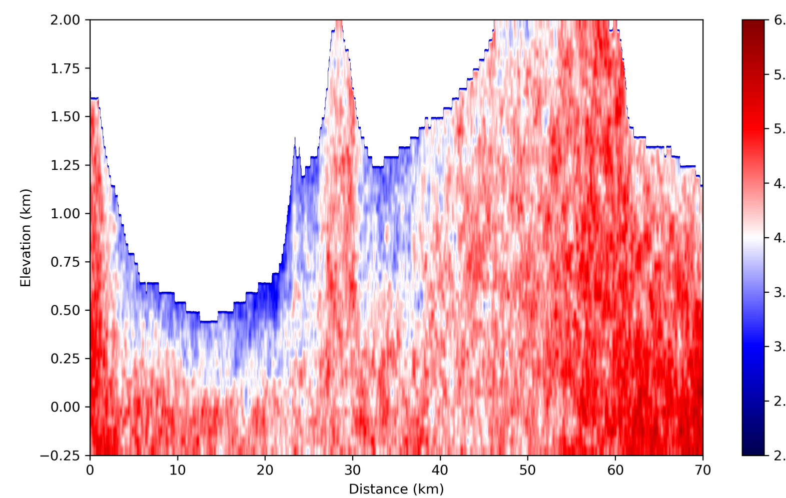
1. Classify as Granite or Sediments

2. Velocity decrease with elevation in different slope based on empirical relations from experiment (Brocher, 2008)

Elevation Velocity example




Velocity profile example



# Setup – Traveltime Calculator

Eikonal Function Based
Fast Marching Method: **PyKonal**

RESEARCH ARTICLE | JUNE 03, 2020

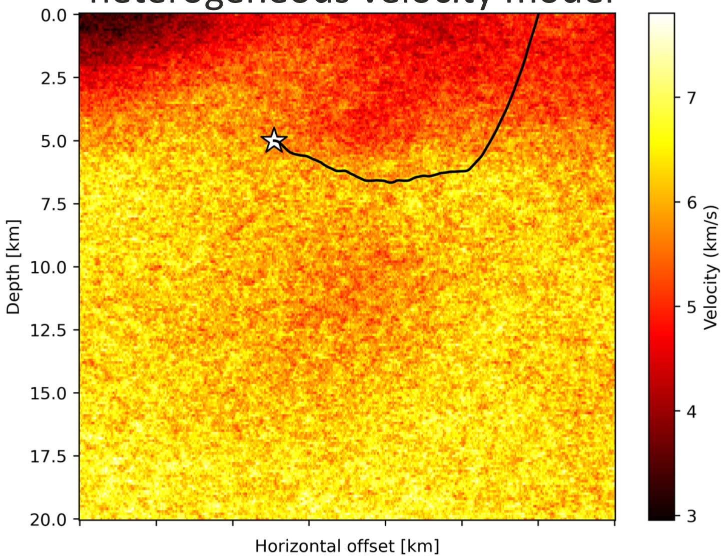
PyKonal: A Python Package for Solving the Eikonal Equation in Spherical and Cartesian Coordinates Using the Fast Marching Method

Malcolm C. A. White; Hongjian Fang; Nori Nakata; Yehuda Ben-Zion

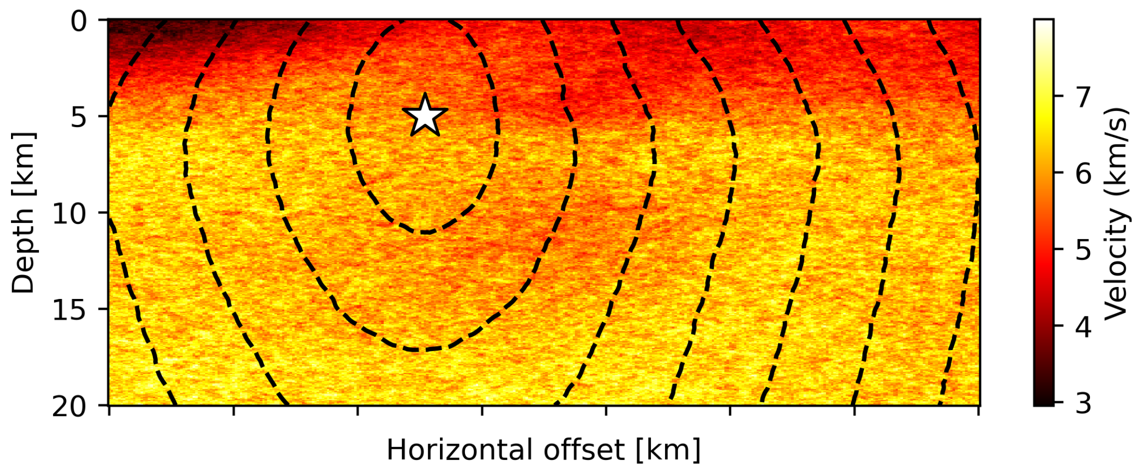
+ Author and Article Information

Seismological Research Letters (2020) 91 (4): 2378–2389.
https://doi.org/10.1785/0220190318 Article history

Ray tracing verification on heterogeneous velocity model

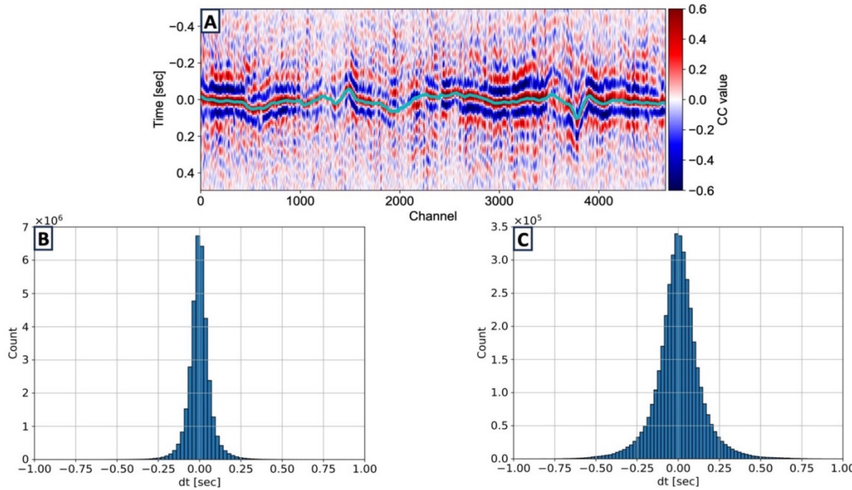



Traveltime contour verification



# Setup – Realistic Error

The observation is not perfect.

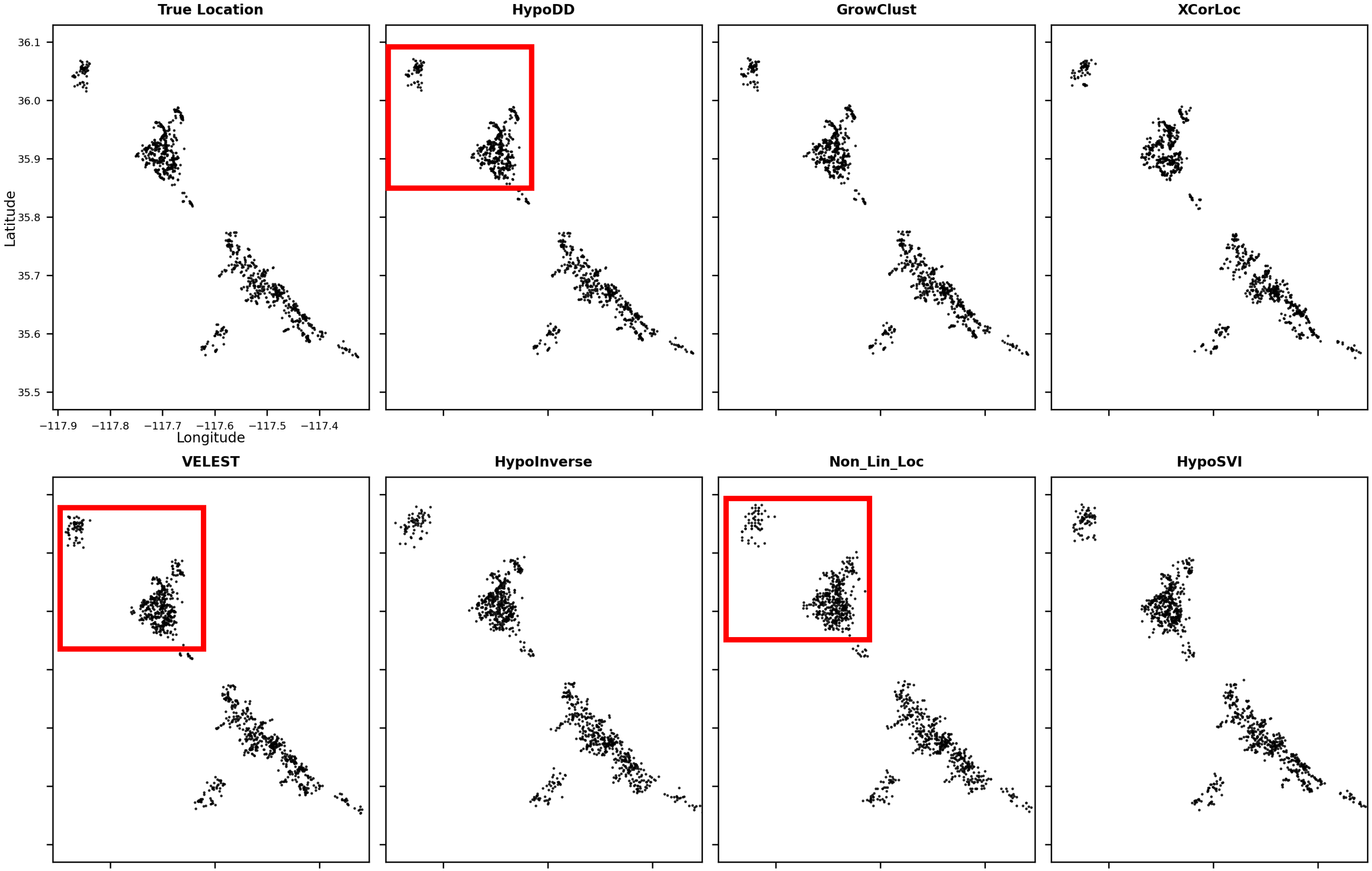



Biondi et al., 2023

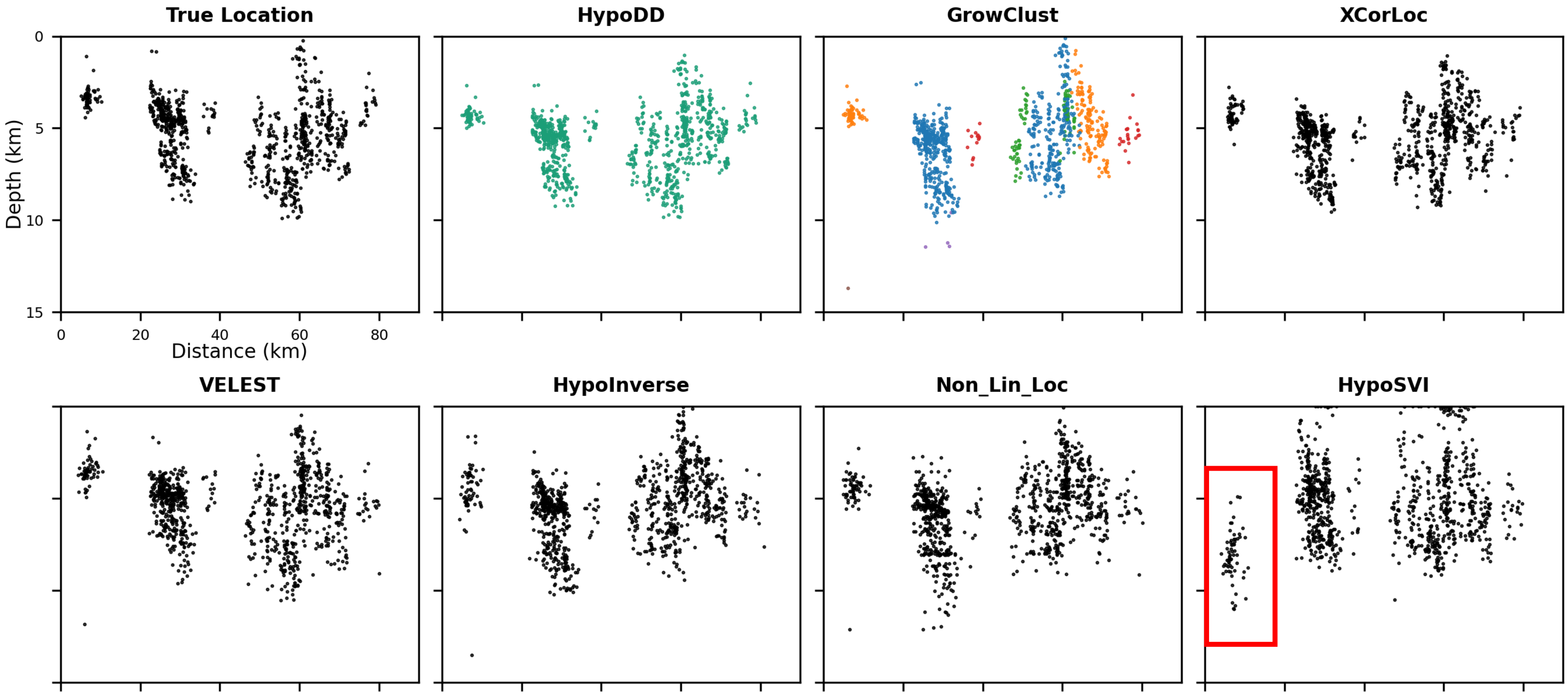
While both cross correlation and direct arrival phases error can be considered in Laplacian distributions, CC is more accurate than direct. And P phase is more accurate than S.

- Realistic traveltime reading error in Laplacian distribution
- Phase availability for each stations in probability: 0.67 for P, 0.5 for S
- 1% P phase and 4 % S phase outliers exsit

# Results – Mapview

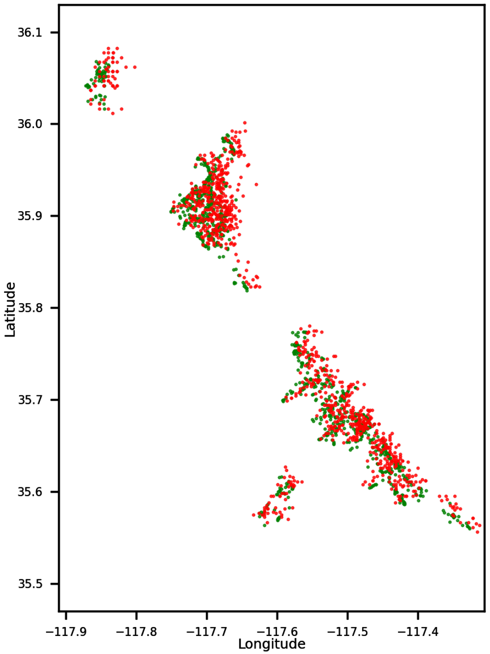


# Results - Profile

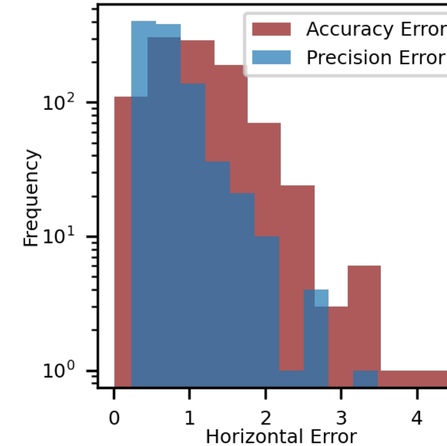


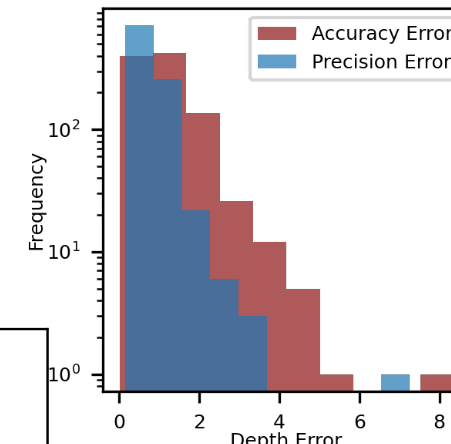
# Results - NonLinLoc

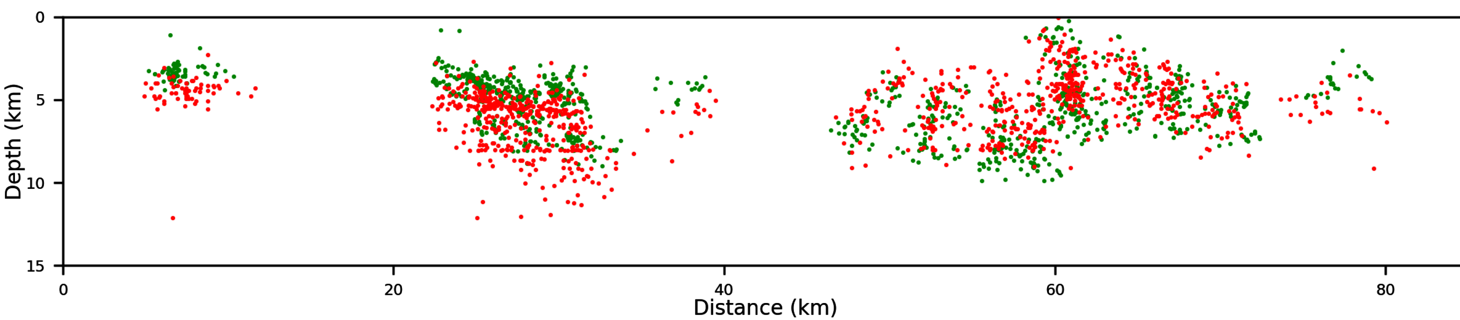
Green: true location
Red: recovered location




- Capture the general positions
- The depth profile is diffusive
- Overall error is in big range

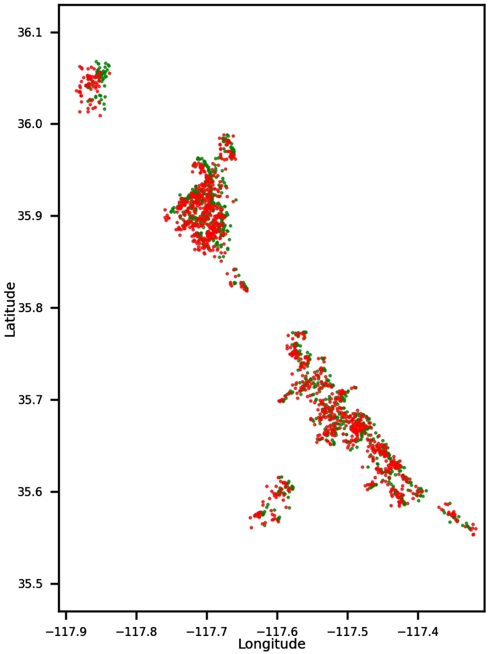


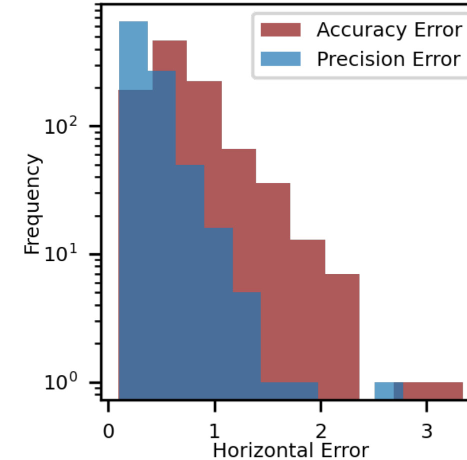



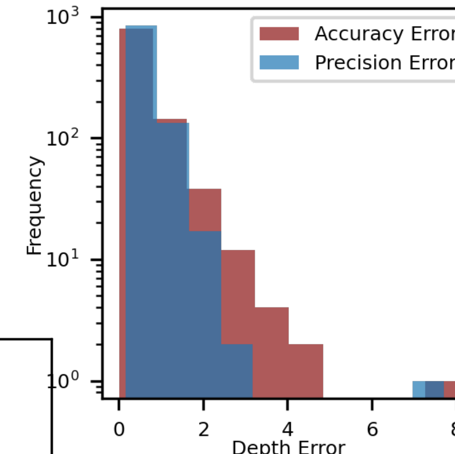
# Results - VELEST

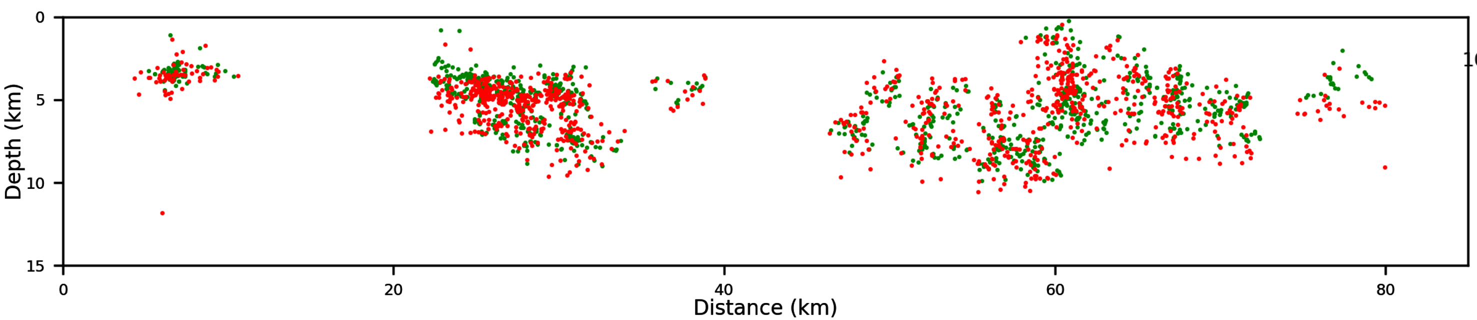
Green: true location
Red: recovered location



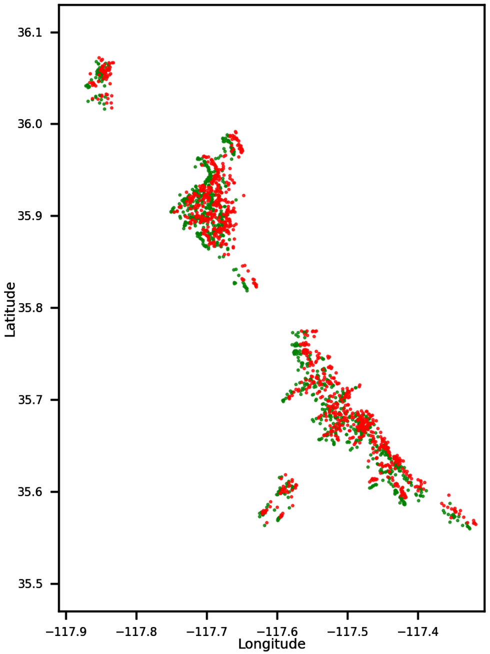

- Well recovered positions with small systematic shifts
- Well recovered depth
- Overall error is in small range



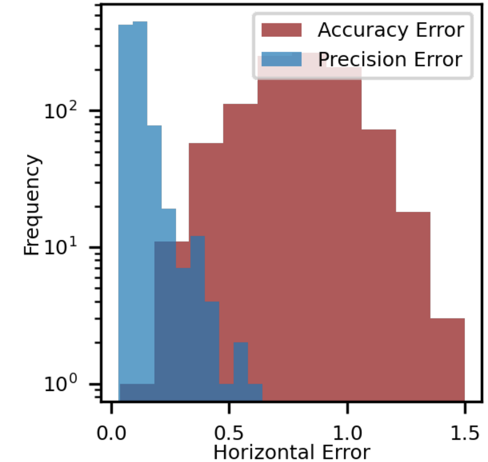



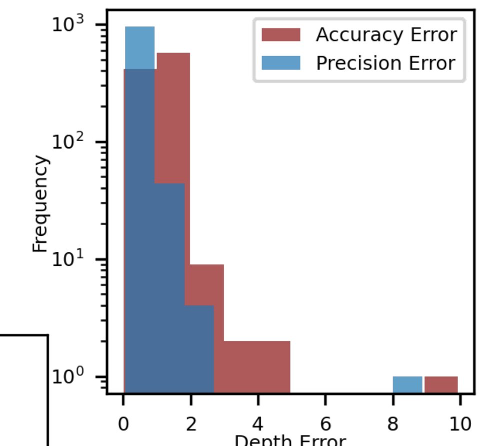
# Results - Growclust

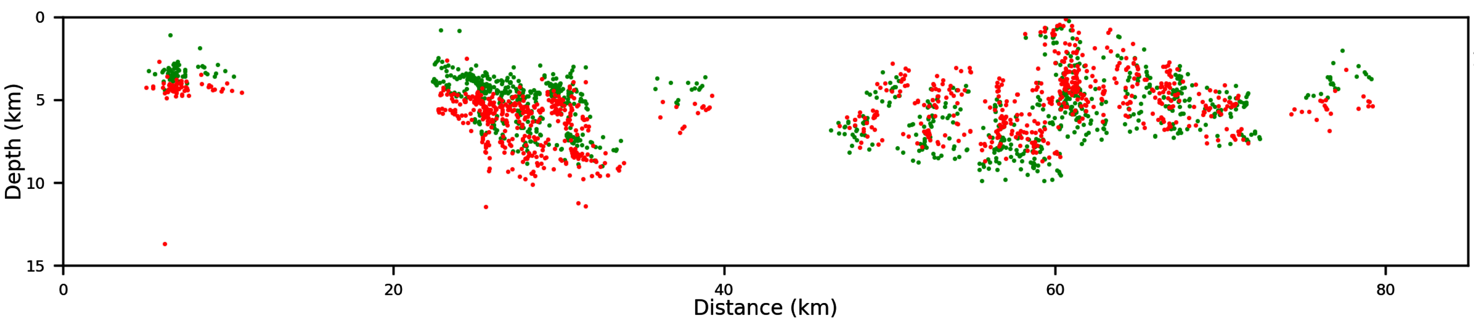



Green: true location
Red: recovered location

- The relative locations are well recovered
- Good relative structure in systematic shift
- Much smaller error in horizontal

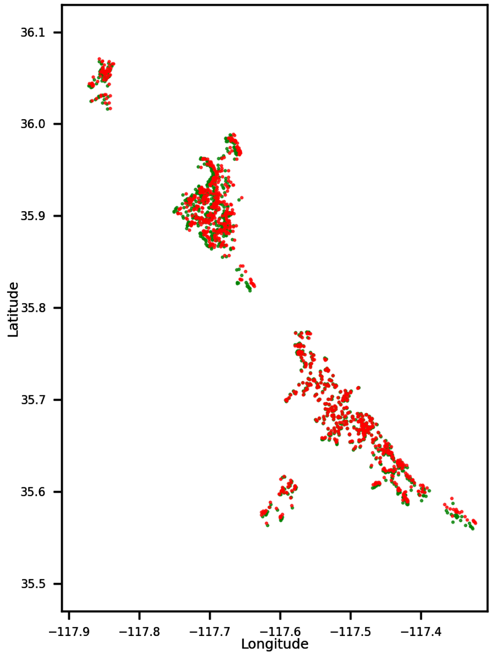


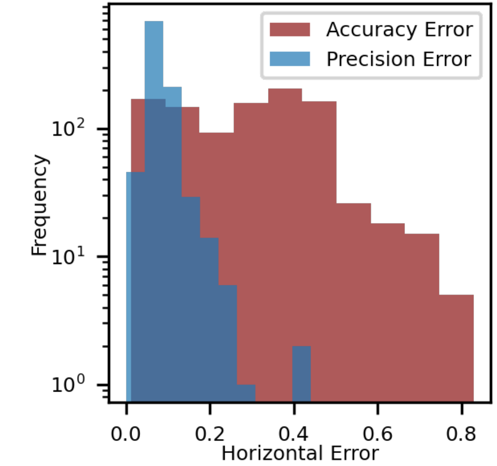
# Results - HypoDD

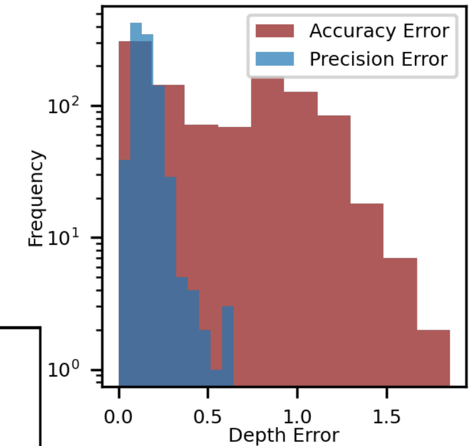
Green: true location
Red: recovered location

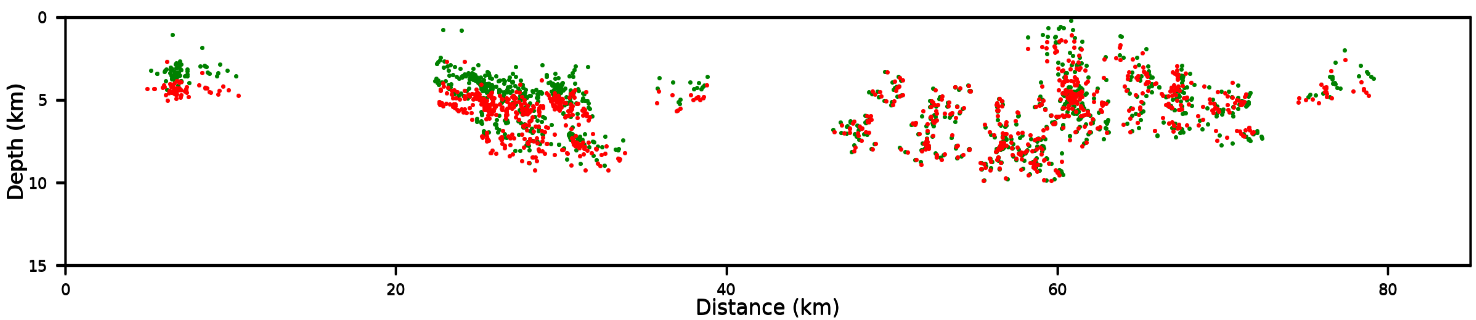



- The relative locations both in mapview and depth profile are well recovered and in small shift

- Much smaller error



# Results – Overall Performance

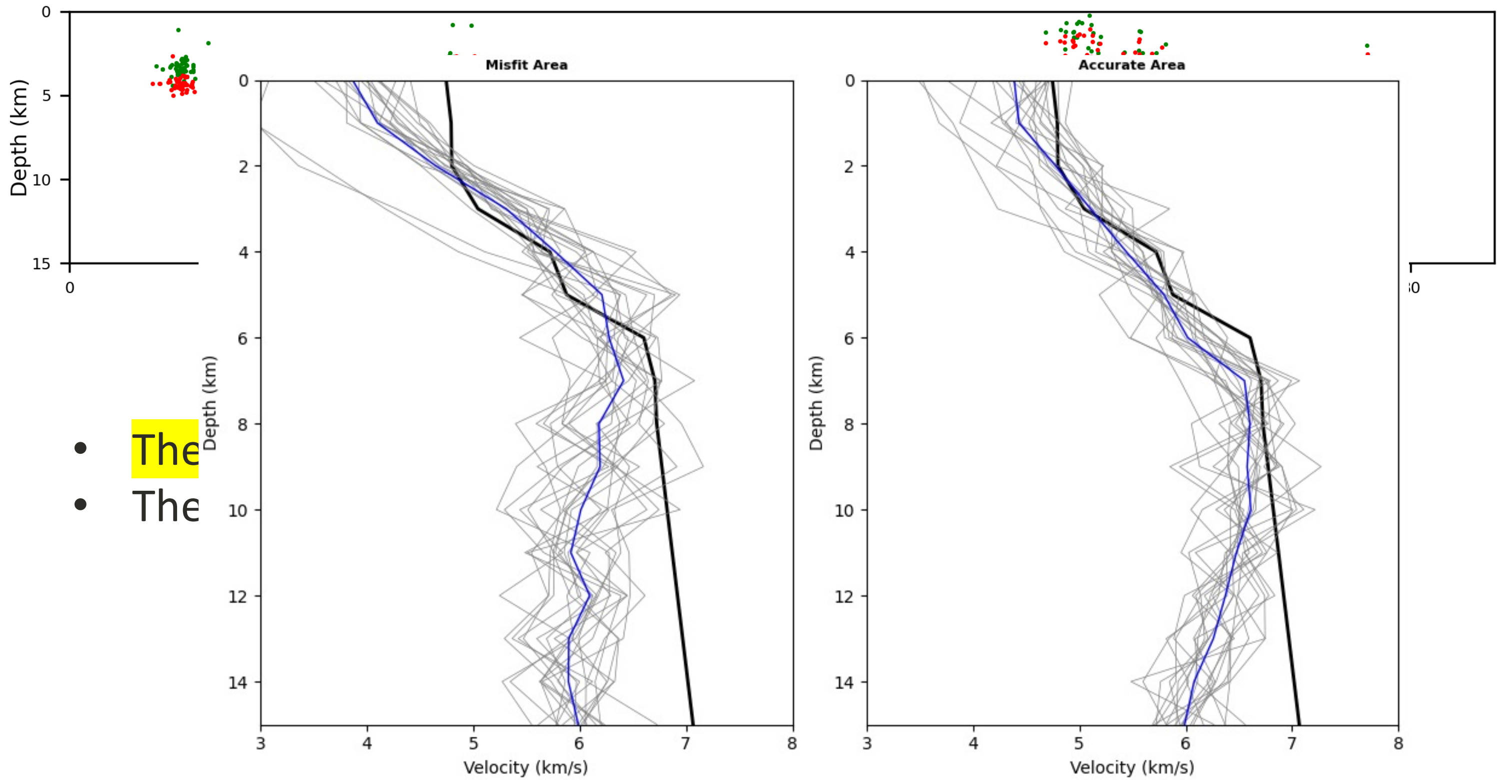
Chamfer distance:

$$d_{CD}(S_1, S_2) = \sum_{x \in S_1} \min_{y \in S_2} \|x - y\|_2^2 + \sum_{y \in S_2} \min_{x \in S_1} \|x - y\|_2^2$$

| Method | Mean Accuracy Error (km) | | Mean Precision Error (km) | | Chamfer | Execution |
|---|---|---|---|---|---|---|
| | Horizontal | Depth | Horizontal | Depth | Distance | Time (s) |
| VELEST | 0.696 | 0.559 | 0.372 | 0.644 | 1.170 | 60 |
| HypoDD | 0.289 | 1.124 | 0.079 | 0.147 | 0.946 | 17 |
| Growclust | 0.802 | 1.071 | 0.110 | 0.297 | 1.702 | 140 |
| HypoInverse | 0.824 | 1.118 | 0.560 | 0.671 | 1.617 | 2 |
| Non_Lin_Loc | 1.070 | 1.124 | 0.705 | 0.799 | 1.750 | 2000 |
| XCorLoc | 2.376 | 1.486 | 1.233 | 0.669 | 1.629 | 120 |
| HypoSVI | 0.911 | 1.152 | 0.454 | 0.951 | 1.854 | 2000 |

# Discussion – Limits of 1D Velocity Structure

What causes the depth misfit variation across the region?

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# Discussion – Uncertainties Analysis

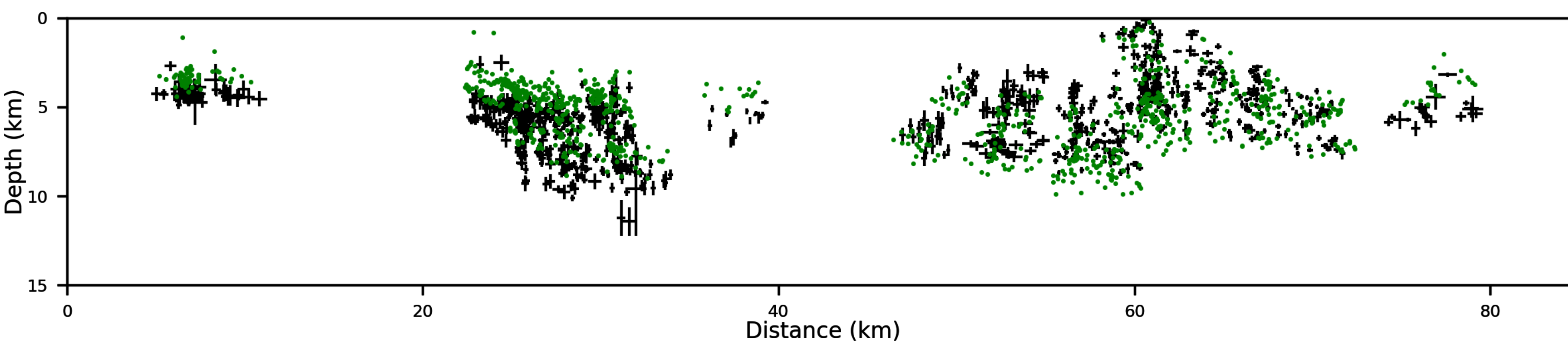
1. Bootstrapping: Growclust, XCORLOC

2. Estimating from likelihood function: NonLinLoc, HypoSVI

3. Covariance Matrix: Hypoinverse

They all underestimate the error range

Example from Growclust's bootstrapping error analysis



# Discussion – Near Source S Phase

Gomberg et al. (1990) argues that the S phase within distance of 1.4 * source depth is important for constraining the depth

Randomly take down one near source S phase V.S. take down one far S phase

| VELEST | Mean Accuracy Error (km) | | Mean Precision Error (km) | |
|---|---|---|---|---|
| | Horizontal | Depth | Horizontal | Depth |
| Without near S | 0.64 | 0.61 | 0.39 | 0.70 |
| Without far S | 0.58 | 0.51 | 0.35 | 0.62 |

# Conclusion

- This controlled experiment proves the effectiveness of the relative location methods over absolute one, while VELEST and HypoDD stands out.
- 1D velocity structure is limited to represent the realistic situation.
- Our findings should motivate revisiting earthquake location uncertainty assessment.

# Thank You