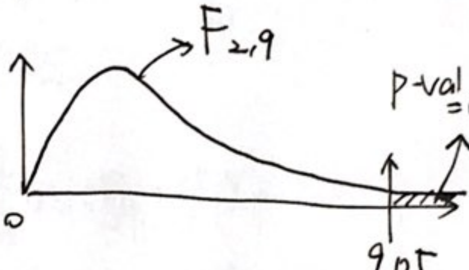
$H_0: \mu_1 = \mu_2 = \mu_3$ $\quad$ H_a: ~~~~ At least one of μ_is different from one another.

ANOVA table

Source	df	SS	MS	F	p-val
Group	2 (3−1)	122.17 (182.92−122.17)	61.085 (122.17/2)	9.05 $\left(\frac{61.085}{6.75}\right)$	0.007
Error	9 (12−3)	60.75 (Given)	6.75 (60.75/9)		
Total	11 (12−1)	182.92 (Given)			

$df_{num} = 2$

$df_{den} = 9$



p-val $= 0.7\% = 0.007 < 0.01$

Decision: Rej H_0

Conclusion: (Since p-val is tiny) we have very strong evidence to say average wt loss with these 3 diets are NOT all the same in population.

Which ones are different? Multiple comparisons.

Sums of Squares

Total SS: $SST = \sum_{i=1}^{g} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{..})^2$ DF: $N-1$

Group SS: $SSG = \sum_{i=1}^{g} n_i(\bar{y}_{i.} - \bar{y}_{..})^2$ DF: $g-1$

Error SS: $SSE = \sum_{i=1}^{g} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i.})^2$ DF: $N-g$

$$SSE + SSG = SST$$

$$\sum_{i=1}^{g} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{..})^2 = \sum_{i=1}^{g} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i.} + \bar{y}_{i.} - \bar{y}_{..})^2$$

$$= \sum_{i=1}^{g} \sum_{j=1}^{n_i} \Big[\underbrace{(y_{ij} - \bar{y}_{i.})^2}_{I_1} + \underbrace{(\bar{y}_{i.} - \bar{y}_{..})^2}_{I_2} + \underbrace{2(y_{ij} - \bar{y}_{i.})(\bar{y}_{i.} - \bar{y}_{..})}_{I_3} \Big]$$

$$I_1 = \sum_{i=1}^{g} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i.})^2 = SSE$$

$$I_2 = \sum_{i=1}^{g} \sum_{j=1}^{n_i} (\bar{y}_{i.} - \bar{y}_{..})^2 = \sum_{i=1}^{g} n_i (\bar{y}_{i.} - \bar{y}_{..})^2 = SSG$$

$$I_3 = 2\sum_{i=1}^{g} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i.})(\bar{y}_{i.} - \bar{y}_{..}) = 2\sum_{i=1}^{g} \Big[(\bar{y}_{i.} - \bar{y}_{..}) \underbrace{\sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i.})}_{=0} \Big]$$

$$\sum_{j=1}^{n_i} y_{ij} - \sum_{j=1}^{n_i} \bar{y}_{i.} = \sum_{j=1}^{n_i} y_{ij} - n_i \bar{y}_{i.} = 0$$

$$\bar{y}_{i.} = \frac{1}{n_i} \sum_{j=1}^{n_i} y_{ij}$$

$$n_i \bar{y}_{i.} = \sum_{j=1}^{n_i} y_{ij}$$

MS = Mean Squares

$$SSE = \sum_{i=1}^{g} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i.})^2 = \sum_{i=1}^{g} (n_i - 1) S_i^2$$

$$S_i^2 = \frac{1}{(n_i - 1)} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i.})^2$$

$$~~SSG~~ \; MSG = \frac{SSG}{g-1}$$

$$MSE = \frac{SSE}{N-g}$$

$\downarrow$ estimator

variance

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

· Multiple comparison of means

Before: CI for μ: $\bar{x} \pm t \cdot \frac{s}{\sqrt{n}}$

$$\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

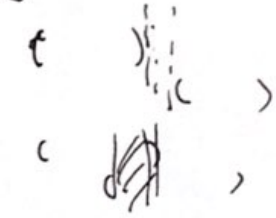
1. Individual C.I. for each ~~g~~ mean μ_i:

$$\bar{y}_{i\cdot} \pm t \cdot \frac{S_p}{\sqrt{n_i}}$$

S_p = pooled stdev
= average within variability
b/c assumption equal variance.

$S_p = \sqrt{MSE}$ $\quad t: df = df_{err} = N - g$

If C.I. for 2 trts DO NOT overlap
→ SIG DIFF btwn those 2 groups.



95% C.I. 5% μ_1 μ_2 μ_3

$\mu_1 - \mu_2$ $\quad \mu_1 - \mu_3$ $\quad \mu_2 - \mu_3$

Conf loses each time: 5% 5% 5%

Conf loss Total: 15%

$\boxed{\text{Family confidence}}$: 100% − 15% = 85%

What if we had 4 groups? How many comparisons?

If g = # groups, # comparisons = $\binom{g}{2} = \frac{g(g-1)}{2}$

1−2 2−3 3−4
1−3 2−4
1−4

30%