第一周习题.

习题8.1.

4.(2)不成立.(只要 $b-c$ 与 a 平行即可).

(4).不成立.

(6).不成立.无交换律.

6. $(a,b,c)\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\vec{x}$.

8. $0=3+2s \Rightarrow s=-\frac{3}{2}$

9. $\frac{\pi}{3}$.

11. $|\vec{a}\times\vec{b}|=\sqrt{3}$. (1).3 (2).300.

13. $(a+b+c)\times a$ 或 b 或 $c=0$.

$\therefore a+b+c=0$.

16. $|a-b|=14$. $(\frac{2}{7},-\frac{3}{7},\frac{6}{7})$.

17. $(\frac{3}{13},\frac{4}{13},-\frac{12}{13})$.

19. $\overrightarrow{AB}=(1,3,4)$. $\overrightarrow{AC}=(2,6,8)$.

$\therefore$ 共线.

24.(1).(5,1,7).

(2).(20,4,28).

27.(1). $\begin{vmatrix}2&3&-1\\1&-1&3\\1&9&-11\end{vmatrix}=0$ $\therefore$ 共面.

(2). $\begin{vmatrix}3&-2&1\\2&1&2\\3&-1&-2\end{vmatrix}=-25$ $\therefore$ 不共面.

28. $\begin{vmatrix}-1&-1&6\\-2&0&2\\1&-1&4\end{vmatrix}=0$. $\therefore$ 共面.

习题8.2.

4.(1).平行 (3).重合.

5. ~~(2,0,-1)~~

π 的方向为 $(2,0,-1)\times(0,1,0)=(1,0,$

$\therefore x-3+2(z-1)=0$.

7.(1). $\frac{\pi}{3}$.

9.(1) $\frac{21}{7}=3$.

11. $x+y-2z+1=0$.

13. (x,x,x) 到 $x+y+z-1=0$ 距离为

$\frac{|(x-1,x,x)\cdot(1,1,1)|}{\sqrt{3}}=\frac{|3x-1|}{\sqrt{3}}=x$.

$\therefore x=\frac{3+\sqrt{3}}{6}$ 或 $\frac{3-\sqrt{3}}{6}$.

(舍).

19. l_2: $\frac{x-4}{3}=\frac{y+4}{-2}=\frac{z}{1}$.

$\therefore l_1 \parallel l_2$.

$d=\frac{|\overrightarrow{M_1M_2}\times\vec{v}|}{|\vec{v}|}=\sqrt{5}$

23.(1). $M_1=(9,-2,0)$ $M_2=(0,-7,2)$.

由 $\det(\overrightarrow{M_1M_2},\vec{v_1},\vec{v_2})\neq 0$ 知异面.

$\vec{v}=\vec{v_1}\times\vec{v_2}=(-15,-10,30)$.

取 $\vec{v}=(-3,-2,6)$.

则 $d=\frac{|\overrightarrow{M_1M_2}\cdot\vec{v}|}{|\vec{v}|}=7$

第二周习题.

习题8.3.

1.(1).是 (2).否 (5).否 (7).是

2.(1).线 面.
(3).圆 圆柱.
(5).抛物线 柱.

3.(1).$\exists M_0\in$曲线. 且 $M_0=(x_0,y_0,z_0)$.

$$\begin{cases} y_0^2-\frac{z_0^2}{4}=1 \\ x_0=0 \\ (x-x_0,y-y_0,z-z_0)\cdot(0,0,1)=0 \\ x^2+y^2+z^2=x_0^2+y_0^2+z_0^2 \end{cases}$$

$\Rightarrow x^2+y^2-\frac{z^2}{4}=1$.

单叶双曲面.

(3).
$$\begin{cases} 4x_0^2+9y_0^2=36 \\ z_0=0 \\ (x-x_0,y-y_0,z-z_0)\cdot(0,1,0)=0 \\ x^2+y^2+z^2=x_0^2+y_0^2+z_0^2 \end{cases}$$

$\Rightarrow x^2+\frac{9}{4}y^2+z^2=9$. 椭球.

5. $(x,y,z)\in L_0$. 有 $\exists (x_0,y_0,z_0)\in L$ s.t.

$$\begin{cases} (x-x_0,y-y_0,z-z_0)\parallel(1,-1,2) \\ \frac{x_0-1}{1}=\frac{y_0}{1}=\frac{z_0-1}{-1} \\ x-y+2z-1=0 \end{cases}$$

$\therefore \begin{cases} x-3y-2z+1=0 \\ x-y+2z-1=0 \end{cases}$

$\therefore L_0: \frac{x-2}{4}=\frac{y-1}{2}=\frac{z}{-1}$.

绕y轴转:

$$\cancel{\begin{cases} \frac{x_0-2}{2}=\frac{y_0-1}{0}=\frac{z_0}{1} \\ (x-x_0,y-y_0,z-z_0)\cdot(0,1,0)=0 \\ x^2+y^2+z^2=x_0^2+y_0^2+z_0^2 \end{cases}}$$

$$\begin{cases} \frac{x_0-2}{4}=\frac{y_0-1}{2}=\frac{z_0}{-1} \\ y=y_0 \\ x^2+y^2+z^2=x_0^2+y_0^2+z_0^2 \end{cases}$$

$\Rightarrow x^2+z^2-\frac{17}{4}y^2+\frac{y}{2}=\frac{1}{4}$.

8. $x^2+y^2+z^2=\frac{|z-4|^2}{1}$

$\therefore \cancel{x^2+y^2+(z+\frac{1}{2})^2=\frac{17}{4}}$

$x^2+y^2=-8z+16$.

Recording start from here

习题8.4.

1.(1) $r^2=9$.

(3). $r^2+4z^2=10$.

(5). $r^2\sin\theta-2r^2\cos^2\theta=0$.

(7). $x^2+y^2+3z^2=4$

(9). $r\cdot\cos\theta+r\sin\theta=4$.

3. $\begin{cases}2x_0^2-z_0^2=2.\\ y_0=0.\\ z=z_0.\\ x^2+y^2+z^2=x_0^2+y_0^2+z_0^2.\end{cases}$

$\therefore\ x^2+y^2-\frac{z^2}{2}=1$.

$\therefore\ r^2-\frac{z^2}{2}=1$.

5. 二点.$(a\cdot\sin\theta_1\cos\varphi_1,\ a\sin\theta_1\sin\varphi_1,\ a\cos\theta_1)$.

$\therefore \cos r=\frac{\vec{OA}\cdot\vec{OB}}{|\vec{OA}|\cdot|\vec{OB}|}$.

$=\frac{\sin\theta_1\sin\theta_2\cos\varphi_1\cos\varphi_2+\sin\theta_1\sin\theta_2\sin\varphi_1\sin\varphi_2+\cos\theta_1\cos\theta_2}{}$

$=\sin\theta_1\sin\theta_2\cos(\varphi_1-\varphi_2)+\cos\theta_1\cos\theta_2$ 四

习题9.1.

1.(1) $\forall x\in(A\cap B)^c\Rightarrow x\notin A\cap B$.

$\therefore x\in A\backslash B\cup B\backslash A\cup(A\cup B)^c=A^c\cup B^c$.

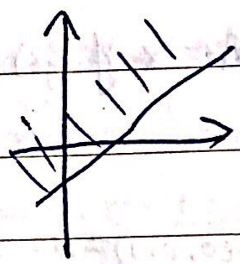
$\forall x\in A^c\cup B^c\Rightarrow x\in A^c$ 或 $x\in B^c$.

$\therefore x\in(A\cap B)^c$.

$\therefore (A\cap B)^c=A^c\cup B^c$.

类似可证(2).

3. $a>0$.



边界点满足 $y=ax+b$.

5. 设$(x_n,y_n)\longrightarrow(x_0,y_0)$.

则对1. $\exists N$ s.t. $\forall n>N$有.

$\|(x_n-x_0,\ y_n-y_0)\|<1$.

$\therefore$取. $R=\max\{\|(x_i-x_0,\ y_i-y_0)\|\}$ $i=1,\cdots$

则有界. 四

7.(1). $x+y\geq0$. 闭区域

(3). $(x-1)^2+y^2<1$. 开区域

(5). $\frac{x^2}{a^2}+\frac{y^2}{b^2}<1$. 开区域.

由 扫描全能王 扫描创建

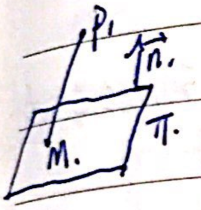
解析几何重点.

1. $\vec{a}\times\vec{b}=\left(\begin{vmatrix} a_2 & a_3\\ b_2 & b_3\end{vmatrix}, \begin{vmatrix} a_3 & a_1\\ b_3 & b_1\end{vmatrix}, \begin{vmatrix} a_1 & a_2\\ b_1 & b_2\end{vmatrix}\right)$.

$=\left(\begin{vmatrix} a_2 & a_3\\ b_2 & b_3\end{vmatrix}, -\begin{vmatrix} a_1 & a_3\\ b_1 & b_3\end{vmatrix}, \begin{vmatrix} a_1 & a_2\\ b_1 & b_2\end{vmatrix}\right)$.

2. 点到平面间的距离.

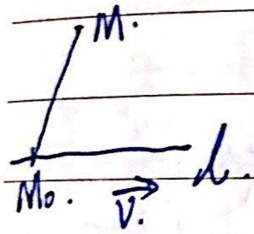
点P_1到平面π的距离.



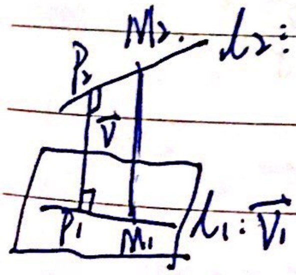
$$d=\frac{|\overrightarrow{P_1M}\cdot\vec{n}|}{|\vec{n}|}=\frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$$

3. 点到直线间的距离

$$d=\frac{|\overrightarrow{MM_0}\times\vec{v}|}{|\vec{v}|}.$$



4. 两直线之间的距离.



$$d=|\overrightarrow{P_2P_1}|=\frac{|\overrightarrow{M_1M_2}\cdot\vec{v}|}{|\vec{v}|}=\frac{|\overrightarrow{M_1M_2}\cdot(\vec{v}_1\times\vec{v}_2)|}{|\vec{v}_1\times\vec{v}_2|}.$$

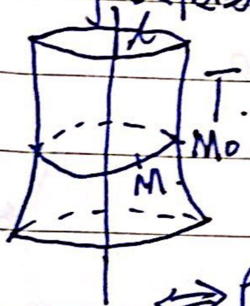
5. (旋转曲面). 一条曲线Γ绕直线l旋转所得曲面称为旋转曲面. l为轴. Γ为母线.

l: 过点 $M_1(x_1,y_1,z_1)^T$.

方向为 $(l,m,n)^T$.

$\Gamma:\begin{cases}F(x,y,z)=0.\\ G(x,y,z)=0.\end{cases}$

问旋转曲面?



M在曲面上$\Leftrightarrow$ $\exists$点 $M_0\in\Gamma$. s.t. $\overrightarrow{MM_0}\perp l$. 且$M$与$M_0$到$l$距离相同

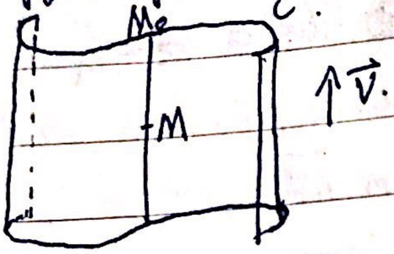
$\Leftrightarrow\begin{cases}F(x_0,y_0,z_0)=0.\\ G(x_0,y_0,z_0)=0.\\ l(x-x_0)+m(y-y_0)+n(z-z_0)=0.\\ |\overrightarrow{MM_1}|=|\overrightarrow{M_0M_1}|.\end{cases}$

eg. $\frac{x}{2}=\frac{y}{1}=\frac{z-1}{-1}$ 绕 $x=\frac{y}{-1}=\frac{z-1}{2}$ 转.

$\begin{cases}\frac{x_0}{2}=\frac{y_0}{1}=\frac{z_0-1}{-1}.\\ (x-x_0)-(y-y_0)+2(z-z_0)=0.\\ x^2+y^2+(z-1)^2=x_0^2+y_0^2+(z_0-1)^2.\end{cases}$

$\Rightarrow 5x^2+5y^2+23z^2-12xy+24xz-24yz-24x+24y-46z+23=0.$

6. 柱面：一条直线 l 沿着空间曲线 C 平行移动所产生的曲面. l 称为母线. C 为准线



柱面母线方向 $\vec{v}(l,m,n)^T$.

准线 C 方程: $\begin{cases} F(x,y,z)=0 \\ G(x,y,z)=0 \end{cases}$.

则 $M\in$ 柱面 $\Leftrightarrow \exists M_0$ s.t. $\overrightarrow{M_0M}\parallel\vec{v}$, $M_0\in C$.

$$\Leftrightarrow \begin{cases} F(x_0,y_0,z_0)=0 \\ G(x_0,y_0,z_0)=0 \\ x=x_0+lt \\ y=y_0+mt \\ z=z_0+nt \end{cases}$$

消去 (x_0,y_0,z_0) 得

$$\begin{cases} F(x-lt,\ y-mt,\ z-nt)=0 \\ G(x-lt,\ y-mt,\ z-nt)=0 \end{cases}$$

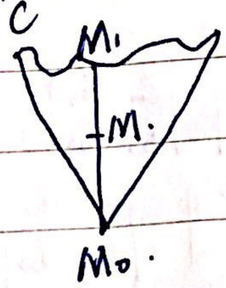
再消去 t 得柱面方程.

eg. 准线为 $\begin{cases} xy=4 \\ z=0 \end{cases}$. 母线方向为 $(1,-1,1)^T$.

$$\begin{cases} x_0y_0=4 \\ z_0=0 \\ \frac{x-x_0}{1}=\frac{y-y_0}{-1}=\frac{z-z_0}{1} \end{cases}$$

$\Rightarrow (x-z)(y+z)=4$.

7. 锥面. 由曲线 C 和不在 C 上的点 M_0 的连线构成.



设顶点 $M_0(x_0,y_0,z_0)$.

曲线 C: $\begin{cases} F(x,y,z)=0 \\ G(x,y,z)=0 \end{cases}$.

则锥面方程. $M\in$ 锥面 $\Leftrightarrow \exists M_1\in C$ s.t. M 在 $\overline{M_1M_0}$ 上.

$$\Leftrightarrow \begin{cases} F(x_1,y_1,z_1)=0 \\ G(x_1,y_1,z_1)=0 \\ x_1=x_0+(x-x_0)t \\ y_1=y_0+(y-y_0)t \\ z_1=z_0+(z-z_0)t \end{cases}$$

消去 x_1,y_1,z_1,t 即可.

eg. 顶点为 $(4,0,-3)^T$. 准线为 $\begin{cases} \frac{x^2}{25}+\frac{y^2}{9}=1 \\ z=0 \end{cases}$.

$$\begin{cases} \frac{x_1^2}{25}+\frac{y_1^2}{9}=1 \\ z_1=0 \\ \frac{x_1-4}{x-4}=\frac{y_1}{y}=\frac{z_1+3}{z+3} \end{cases}$$

$\Rightarrow \cancel{9x^2+y^2+3z^2+24xz-6z=9}$

$9x^2+25y^2-9z^2+24xz-150z=225$

Recording start from here

由 扫描全能王 扫描创建

§. 二次曲面的分类

一. 椭球面.

(1). $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

(2). $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=-1$.

(3). $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=0$.

二. 双曲面.

(4) $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$.

(5). $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$.

三. 抛物面

(6) $\frac{x^2}{p}+\frac{y^2}{q}=2z$.

(7) $\frac{x^2}{p}-\frac{y^2}{q}=2z$.

四. 二次锥面.

(8). $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$.

五. 二次柱面.

(9). $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(10) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=-1$.

(11). $\frac{x^2}{a^2}+\frac{y^2}{b^2}=0$.

(12). $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

(13) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$.

(14). $x^2=2py$

(15). $x^2=a^2$

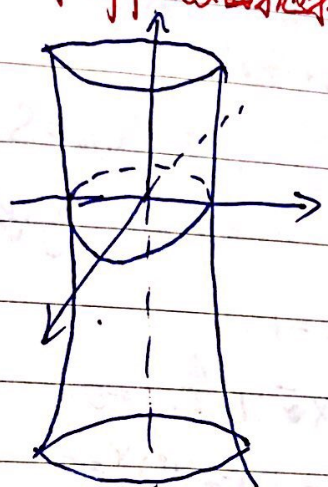
(16) $x^2=-a^2$

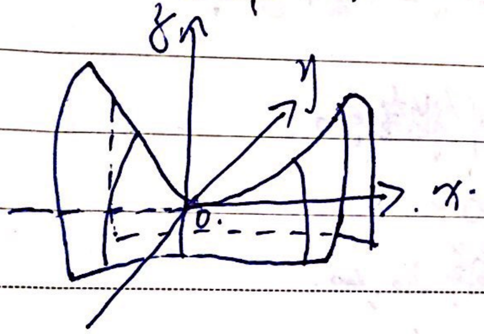
(17). $x^2=0$.

共17种. 原因在于三元的二次型最简型式共17种.

9. 直纹面: ∃一族直线. 这一族直线均在S上. 且S的每个点均在这一族某个直线上.

有2种: 双曲抛物面+单叶双曲面.





Thm. 单叶双曲面为直纹面.

$S: \frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$.

$\therefore (\frac{x}{a}-\frac{z}{c})(\frac{x}{a}+\frac{z}{c})=(1-\frac{y}{b})(1+\frac{y}{b})$.

$\therefore \frac{\frac{x}{a}-\frac{z}{c}}{1-\frac{y}{b}}=\frac{1+\frac{y}{b}}{\frac{x}{a}+\frac{z}{c}} \triangleq \frac{\lambda}{\mu}$.

则 $\begin{cases}\mu(\frac{x}{a}-\frac{z}{c})-\lambda(1-\frac{y}{b})=0. \\ \mu(1+\frac{y}{b})-\lambda(\frac{x}{a}+\frac{z}{c})=0.\end{cases} \triangleq l_{\lambda,\mu}$

其中 λ,μ 不全为0.

下证 $l_{\lambda,\mu}$ 为一族直线, 构成单叶双曲面.

(I). $l_{\lambda,\mu}\in S$.

$\because$ 方程 $\begin{cases} \mu(\frac{x}{a}-\frac{z}{c})-\lambda(1-\frac{y}{b})=0 \\ \mu(1+\frac{y}{b})-\lambda(\frac{x}{a}+\frac{z}{c})=0 \end{cases}$ 有非零解.

$\therefore \begin{vmatrix} \frac{x}{a}-\frac{z}{c} & -(1-\frac{y}{b}) \\ 1+\frac{y}{b} & -(\frac{x}{a}+\frac{z}{c}) \end{vmatrix}=0.$

$\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1 \Rightarrow l_{\lambda,\mu}\in S.$

(II) $\forall (x_0,y_0,z_0)\in S$. $\exists\ l_{\lambda,\mu}$ s.t. $(x_0,y_0,z_0)\in l_{\lambda_0,\mu_0}$.

$\because \frac{x_0^2}{a^2}+\frac{y_0^2}{b^2}-\frac{z_0^2}{c^2}=1.$

而方程 $\begin{cases} (\frac{x_0}{a}-\frac{z_0}{c})X-(1-\frac{y_0}{b})Y=0. \\ (1+\frac{y_0}{b})X-(\frac{x_0}{a}+\frac{z_0}{c})Y=0. \end{cases}$

中. $1\pm\frac{y_0}{b}$ 不可能全为0.

且系数行列式为0.

$\therefore$ 方程有非零解 λ_0,μ_0.

$\therefore (x_0,y_0,z_0)\in l_{\lambda_0,\mu_0}$

$\therefore$ 单叶双曲面为直纹面.