2. Tukey's Honest Significant Difference.

- Family confidence is 95%
- Individual comparisons have higher confidence level.
- pairwise confidence interval for $\mu_i - \mu_j$
- Interpretation: Look for 0. If 0 is NOT inside the pairwise c.I. for $\mu_i - \mu_j$, we say there is significant difference between μ_i and μ_j.

3. Fisher's Least Significance Difference.

- Individual confidence is 95%
- Family confidence is smaller, but NOT quite as small as if we constructed individual confidence intervals.
- ~~pairwise~~ pairwise confidence interval for $\mu_i - \mu_j$.
- Interpretation: Look for 0.

4. Bonferroni's method.

- Family confidence is set to be what we want (say, 95%)
- Individual confidence is higher.
- pairwise confidence interval for $\mu_i - \mu_j$.

CI for $\mu_i - \mu_j$: $\quad \text{est} \pm \overbrace{(t \text{ or } z) * \text{stderr}}^{\text{margin of error}}$

$$\bar{y}_{i\cdot} - \bar{y}_{j\cdot} \pm t \cdot S_p \sqrt{\frac{1}{n_i} + \frac{1}{n_j}}$$

$S_p = \sqrt{MSE}$

$t_{N-g, \alpha/2k}$ for k comparisons

If $k = \binom{g}{2} = \frac{g(g-1)}{2}$, then $t_{N-g, \frac{\alpha}{g(g-1)}}$

n_i, n_j: sample sizes for 2 groups being compared.

t: From t-table w/ Individual CONF LEVEL and df from ERROR.

$t_{N-g,\alpha/2K}$: If a r.v. $X \sim t$ distribution with $df = N-g$

then $t_{N-g,\alpha/2K}$ is the $(1-\alpha/2K)$-th percentile.

i.e., $t_{N-g,\alpha/2K}$ is the cutoff such that

$P(X > t_{N-g,\alpha/2K}) = \alpha/2K$.

α: 1 − family confidence level.

(e.g. If family confidence level = 95%, then $\alpha = 5\%$) $t_{N-g}(\alpha/2K)$ $t_{\alpha/2K}(N-g)$

· Example :

① df for t-table : from ERROR → $N-g = 12-3 = 9$

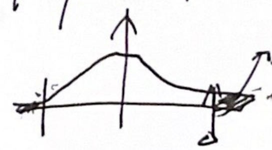
② Figure out Indiv. Conf. :

$g=3$ diets # comparisons $= \frac{g(g-1)}{2} = \frac{3\times 2}{2} = 3$. $\begin{pmatrix} 1-2 \\ 1-3 \\ 2-3 \end{pmatrix}$

- Want 94% Family CONF.
- willing to give up 6% total.
- Give up for each interval: $\frac{6\%}{3} = 2\%$

* Individual Conf. level : 98%

③ Find value from t-table with 98% Conf. level. & 9 df. $\boxed{2.821}$



④ $S_p = \sqrt{MSE} = \sqrt{6.75} = 2.598$

⑤ $n_i = n_j = 4$

⑥ Compute margin of error : $t \cdot S_p \sqrt{\frac{1}{n_i} + \frac{1}{n_j}} = 2.821 \times 2.598 \times \sqrt{\frac{1}{4} + \frac{1}{4}}$

$= 5.18$

⑦ Construct three pairwise C.I.s

Groups	$\bar{y}_{i.} - \bar{y}_{j.}$	± m.e.	C.I.		SIG DIFF?
1−2	21.5 − 24.5 = −3	5.18	(−3−5.18, −3+5.18) = (−8.18, 2.18)	(−, +)	NO
1−3	21.5 − 29.25 = −7.75	5.18	(,)	(−, −)	YES
2−3	24.5 − 29.25 = −4.75	5.18	(,)	(−, +)	NO

⑧ Conclusions: Bonferroni's method shows that diet 1 and diet 3 have SIG DIFF in the effect of wt loss, with 94% Family CONF.

→ Easier way − Skip the intervals and instead:

m.e. 5.18

Diet 1 21.5

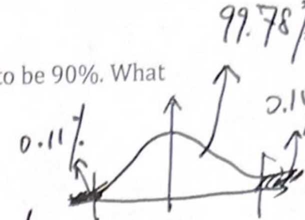
Diet 2 24.5

Diet 3 29.25

→ SIG DIFF.

Groups that are NOT connected have SIG DIFF.

Diet 1	21.5	A	
Diet 2	24.5	A	B
Diet 3	29.25		B



Example 1: For $g = 10$ groups, suppose we want the family (simultaneous) confidence to be 90%. What individual confidence level should we use?

- Give up total 10%
- # comp $= \frac{10 \times 9}{2} = 45$
- Each comp we give up $\frac{10\%}{45} = 0.222\cdots\% \approx 0.22\%$

$\Rightarrow$ Individual CONF LEVEL: $100\% - 0.22\% = 99.78\%$ $\quad t_{N-g,\,0.0011}$ $\quad \frac{\alpha}{g(g-1)}$ $\quad g=10$

Example 2: For $g = 4$ groups, suppose we want the family (simultaneous) confidence to be 94%. What individual confidence level should we use?

- Give up total 6%
- # comp $= \frac{4 \times (4-1)}{2} = 6$
- Each comp we give up $6\% / 6 = 1\%$

$\Rightarrow$ Individual CONF LEVEL: 99% $(100\% - 1\% = 99\%)$ $\quad t_{N-g,\,0.005}$

$\frac{6\%}{4 \times 3} = \frac{6\%}{12} = 0.005$ $\qquad \frac{10\%}{10 \times 9} = \frac{10\%}{90} = 0.0011$

Example 3: For $g = 6$ groups, we want each comparison to have **individual** 99% confidence. What is the family confidence?

- # comp $= \frac{6 \times 5}{2} = 15$
- Each comp we give up 1% $(100\% - 99\% = 1\%)$
- Give up total $15 \times 1\% = 15\%$

$\Rightarrow$ Family conf $= 85\%$ $(100\% - 15\% = 85\%)$

$\frac{\alpha}{6(6-1)} = \frac{1\%}{2}$ $\quad t_{N-g,\,0.005}$ $\quad \Downarrow$ $\alpha = 15\%$

Example 4: For $g = 5$ groups, we want family confidence to be 95%. What individual confidence level should we use?

- Give up total 5%
- # comp $= \frac{5 \times 4}{2} = 10$
- Each comp we give up $\frac{5\%}{10} = 0.5\%$

$\Rightarrow$ Individual conf $= 99.5\%$

$\frac{5\%}{5 \times 4} = \frac{1\%}{4} = 0.0025$ $\quad t_{N-g,\,0.0025}$

Example 5: Suppose the group means are A: 105 B: 110 C: 112 D: 99 E: 103 F: 122

Determine if there are significant differences among the means if the margin of error is:

me = 3.5	me = 11.3	me = 1.5	me = 29
D 99	D 99	D 99	D 99
E 103	E 103	E 103	E 103
A 105	A 105	A 105	A 105
B 110	B 110	B 110	B 110
C 112	C 112	C 112	C 112
F 122	F 122	F 122	F 122
		No connecting groups!	No SIG DIFF