Answer sheet. —— Week 9.

习题 11.2

1. (1) $\sqrt{2}\pi$. (3) $8a^2$ (5) $\frac{2\sqrt{2}-1}{3}\pi$

2. (1) 9 (3) $\frac{\sqrt{2}+1}{2}\pi$ (别漏了上表面) (5) $\sqrt{2}\pi$ (7) $\frac{125\sqrt{5}-1}{420}$

3. (2) $\left(\iint_S x\,ds=\iint_S y\,ds=0\right)$ πa^3

习题 11.3

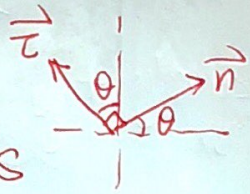
1. (1) $\frac{4}{3}$ (3) 0 (5) $e^{\frac{3}{2}}-e$

2. $\frac{k}{2}(a^2-b^2)$

4. (1) $+2$ (3) 0 (5) $\frac{2}{3}$

5. (1) $\frac{3\pi a^2}{8}$ ~~7. (1)~~

7 (1) $\oint \frac{\partial f}{\partial n}ds=\oint \nabla f\cdot\vec{n}\,ds$



$=\oint (f_x'\cos\theta+f_y'\sin\theta)ds$

设 $\vec{\tau}=(\cos\varphi,\sin\varphi)$ 则 $\begin{cases}\cos\varphi=-\sin\theta\\ \sin\varphi=\cos\theta\end{cases}$ 及 $\begin{cases}\sin\varphi\,ds=dy\\ \cos\varphi\,ds=dx\end{cases}$

$\therefore \oint \frac{\partial f}{\partial n}ds=\oint (f_x'\sin\varphi-f_y'\cos\varphi)ds$

$=\oint f_x'\,dy-f_y'\,dx \overset{\text{Green}}{=\!=} \iint_D \Delta f\,dx\,dy$

Answer Sheet — Week 10.

习题 11.4

1. (1) $\frac{4}{3}\pi abc$ (3) $\frac{2}{105}R^7$ (原式 $=\iint_D y^2z^2\sqrt{R^2-y^2-z^2}\,dydz$ 无负号.)

(5) $\frac{1}{4}\pi$ (7) $\frac{2}{5}\pi a^5$

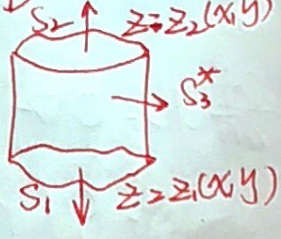
习题 11.5

1. (1) $\frac{1}{2}$ (3) $\frac{8}{3}\pi(a+b+c)R^3$ (5) $\frac{\pi}{2}$

3. 4π

I. 大家基本用 Gauss 公式证, 下面给出更"基础"的证明:

$$V=\iiint_V 1\,dV=\iint_D dxdy\int_{z_1(x,y)}^{z_2(x,y)}dz=\iint_D z_2(x,y)dxdy-\iint_D z_1(x,y)dxdy$$



$$=\iint_{S_2(上侧)} z\,dxdy+\iint_{S_1(下侧)} z\,dxdy+\underbrace{\iint_{S_3^*(外侧)} z\,dxdy}_{=0}$$

$$=\oiint_{S^*(外侧)} z\,dxdy \quad \text{同理, } V=\oiint_{S^*(外)} x\,dydz=\oiint_{S^*(外)} y\,dzdx$$

$$\therefore V=\frac{1}{3}\oiint_{S^*(外侧)} x\,dydz+y\,dzdx+z\,dxdy$$

8. (1) $-\frac{3}{2}$ (3) $-\frac{9}{2}a^3$ (5) $\sqrt{3}\pi a^2$

11. 0.