2 indep groups

Example: Compare the reaction times (in milliseconds) of subjects under 2 drugs (different subjects for each drug).

Drug A	Drug B
1.96 (4)	2.43 (9)
2.24 (7)	2.07 (5)
1.71 (2)	2.71 (11)
2.41 (8)	2.50 (10)
1.62 (1)	6.84 (13)
1.93 (3)	2.88 (12)
	2.11 (6)

- Which statistical inference procedures could we use on this data? Which one should we use?

→ Wilcoxon Rank Sum Test for 2 independent groups

• Better b/c small samples w/ one outlier.

→ CI/Sig Test for $\mu_1 - \mu_2$

- Using the output below, conduct the test and interpret the results.

Assumptions: SRS of ppl/~~subjects~~ subjects randomly assigned to Drug A/B? representative?

Hypotheses: $H_0: \eta_A = \eta_B$ $H_a: \eta_A \neq \eta_B$

Test Stat: $W_A = 25$

P-value: ~~p~~ 0.0184   Rej H_0 at $\alpha = 0.10, 0.05$, not 0.01

Conclusion: Pretty strong evidence to say there is a SIG DIFF in reaction times for drugs A and B.
(median reaction times
OR distribution of reaction times)
assuming subjects ~~or~~ were random + rep.

Mann-Whitney Test and CI: DrugA, DrugB

```
       N  Median
DrugA  6   1.945
DrugB  7   2.500

Point estimate for ETA1-ETA2 is -0.520
96.2 Percent CI for ETA1-ETA2 is (-1.260,-0.110)
W = 25.0
Test of ETA1 = ETA2 vs ETA1 not = ETA2 is significant at 0.0184
```

Wilcoxon Signed Rank Test

The Wilcoxon Signed Rank Test is a nonparametric alternative to a **matched pairs** t-test (i.e. we have **dependent samples**). Again, when the distributions might not be normal, it is better to use the median as the measure of center.

Matched pairs / Dependent samples:

2 treatments given to SAME experimental units or VERY SIMILAR ones that have been matched by all confounding variables we can think of.

Hypotheses for Wilcoxon Signed Rank Test: ~~**Test**~~

$H_0: \eta_d = 0$ (median difference is zero)

$H_a: \eta_d \neq 0$, $>$, $<$

Statistic for Wilcoxon Signed Rank Test:

1. Find DIFF in response variable for trt 1 − trt 2 for each pair
2. Take absolute values of the differences
3. Rank those absolute values
4. Compute
   - W_+ = sum of ranks corresponding to positive DIFF
   - W_- = sum of ranks corresponding to negative DIFF

p-value and Conclusions

Look at output produced by computer

small p-val → Rej H_0 → Sig diff in medians

Example: Compare Turkish coffee and Colombian coffee to determine which one is stronger. Eleven professional testers try both coffees, in random order, blind test, and assign a score to each one on a scale of 1 (weak) to 10 (strong). The results are below:

Judge	Turkish	Colombian	DIFF	|DIFF|	RANK
1	6	4	2	2	5
2	8	5	3	3	7.5
3	4	5	-1	1	2
4	9	8	1	1	2
5	4	1	3	3	7.5
6	7	9	-2	2	5
7	6	2	4	4	9
8	5	3	2	2	5
9	6	7	-1	1	2
10	8	2	6	6	10
~~11~~	~~7~~	~~7~~	~~0~~		

→ eliminate diff = 0.

$n = 10$ DIFF instead of 11.

Diff = 1, 3 of them $\frac{1+2+3}{3} = 2$

Diff = 2, 3 of them $\frac{4+5+6}{3} = 5$

Diff = 3, 2 of them $\frac{7+8}{2} = 7.5$

- Which statistical inference procedures could we use on this data? Which one should we use?

could use,
t-test (matched pairs)
Wilcoxon Signed Rank Test

should use. Check assumptions.
- Wilcoxon: SRS
- t-test: SRS
  $n \geq 30$ ($n = 11$ X) OR The original distribution of DIFF is normal.
  → can check No major outliers
  → not too bad

BUT Data is subjective
non-parametric methods better. ⇒ should use Wilcoxon Signed Rank Test.

- Using the output below, conduct the test and interpret the results.

$H_0: \eta_d = 0 \quad H_a: \eta_d \neq 0$

TS; $W_+ = 5 + 7.5 + 2 + 7.5 + 9 + 5 + 10 = 46$

$W_- = 2 + 5 + 2 = 9$

p-val: 0.067 $(> 0.05 \quad < 0.1)$

conclusions. Some evidence to say there is a DIFF in strength of T and C coffees (median diff in strength of T & C coffees is NOT zero)

Data suggests Turkish coffee is stronger than Colombian coffee.

```
Wilcoxon Signed Rank Test: Difference

Test of median = 0.000000 versus median not = 0.000000

                 N for  Wilcoxon           Estimated
             N   Test   Statistic      P      Median
Difference  10     10        46.0  0.067       2.000
```

$X \sim Bin(n, p)$

Suppose we have n independent trials. Each trial has a probability of success p. Let X be the number of trials that are successes, then $X \sim Bin(n, p)$.

X takes on values in the set $\{0, 1, 2, 3, \cdots, n\}$

$$P(X=i) = \binom{n}{i} p^{i} (1-p)^{n-i}$$

p^i → i successes

$(1-p)^{n-i}$ → $(n-i)$ failures

$\binom{n}{i}$ = $\frac{n!}{i!(n-i)!}$ All possible combinations of i successes and $(n-i)$ failures on n trials

1	2	3	4	$\cdots$	n
S	F	S	S		F
S	S	F	F		S
F	F	S	F		S
S	S	S	S		S

$\vdots$

$Bin(\overset{n}{6}, \overset{p}{0.5})$