* Sidenote: In principle, more info leads to more powerful tests. Parametric methods rely on stronger ~~st~~ assumptions, so they utilize more info regarding population of interest ⇒ Parametric methods are typically more powerful than non-parametric methods.

Sign Test for the Median

This test is similar to Wilcoxon Signed Rank Test, but uses less information, so it is less powerful. It only counts how many observations were below the hypothesized value of the population median, and how many are above. Like Wilcoxon Signed Rank, it ignores any data points exactly equal to that hypothesized value.

Hypotheses for Sign Test: $H_0: \eta = \eta_0$ → some pre-specified constant

$H_a: \eta \neq \eta_0$ ($>$, $<$) OR $H_0: p = \frac{1}{2}$, $H_a: p \neq \frac{1}{2}$ ($>$, $<$)

• p = proportion of successes → scores that are above certain value η_0.

Test Statistic for Sign Test:

X = # successes

p-value and Conclusions

p-value is calculated based on binomial distribution.

p-val = $2P_1$ (two-sided)
P_1 (one-sided ">")
$1-P_1$ (one-sided "<")

e.g. 10, 7, 6, 9, 8, 11, 17.
Test for $H_0: \eta = 8$
$H_a: \eta \neq 8$
⇔ Test for $H_0: p = \frac{1}{2}$, $H_a: p \neq \frac{1}{2}$ where p = prop of values that are larger than 8 in population

Example: Revisit the Turkish and Colombian coffee example and conduct the Sign Test for the Median

$H_0: \eta_d = 0$
$H_a: \eta_d \neq 0$
η_d = median diff in population

OR

$H_0: p = 1/2$
$H_a: p \neq 1/2$
p = prop of succ in population
succ = positive DIFF

TS: $X = 7$

sketchy.

p-val: $Bin(10, \frac{1}{2})$

$[P(7) + P(8) + P(9) + P(10)] \times 2$

$= \left[\binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10}\right] \times \left(\frac{1}{2}\right)^{10} \times 2$

$\binom{10}{7} = \binom{10}{3} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$

$= 0.3438$ → Fail to rej H_0

Signed Test is weaker (less powerful) than Wilcoxon Signed Rank Test.

Conclusions: We do not have enough evidence to say there is a SIG (median) DIFF in strength of T + C coffees

More about p-value: Binomial Distribution

The binomial distribution gives probabilities for the number of successes observed in n Bernoulli trials, each with success probability p.

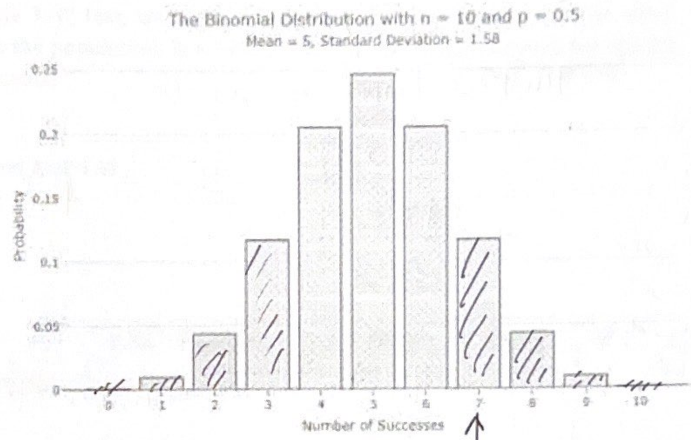
Change the values of n and p to see how the shape of the distribution changes. Hover over a bar in the graph to see the probability for the corresponding number of successes, or consult the table below.

Number of Bernoulli Trials (n):

Probability of Success (p):

Probability Table:

x	P(X = x)
0	0.0010
1	0.0098
2	0.0439
3	0.1172
4	0.2051
5	0.2461
6	0.2051
7	0.1172
8	0.0439
9	0.0098
10	0.0010



X

$$p\text{-val} = [P(7) + P(8) + P(9) + P(10)] \times 2$$

Kruskal-Wallis H-Test

The Kruskal-Wallis H-test is a nonparametric alternative to ANOVA for comparing means of three or more **independent** groups. With the K-W test, we want to see if all the treatments have the same **distribution** of response variable in the population. Since the distributions may be skewed, we should again use median as the measure of center. not normal

How do the assumptions differ from ANOVA?

ANOVA	K-W
• SRS	• SRS
• Normal distribution of response variable in population for each group	• min 5 obs per group
• Equal variances	

Hypotheses for the Kruskal-Wallis H-test:

Ho: Response variable in population has the same median/distribution for all groups

Ha: Some DIFF / at least one DIFF / Not all same

Kruskall-Wallis Test Statistic:

Notations: y_{ij} = Response for observation j in group i

R_{ij} = Rank of observation j in group i

g = # groups

n_i = # obs group i

N = total # obs

Group 1	Group 2	. . .	Group g
$y_{11}(R_{11})$	$y_{21}(R_{21})$	...	$y_{g1}(R_{g1})$
$y_{12}(R_{12})$	$y_{22}(R_{22})$	...	$y_{g2}(R_{g2})$
⋮	⋮		⋮
$y_{1n_1}(R_{1n_1})$	$y_{2n_2}(R_{2n_2})$	...	$y_{gn_g}(R_{gn_g})$

p-value and Conclusions:

$\bar{R}_i$ = avg rank group i

$\bar{R}$ = avg of all ranks

Compute Kruskal Wallis TS:

- Rank all obs from the smallest to the largest
- Compute $\bar{R}_i$ and $\bar{R}$

  where $\bar{R}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} R_{ij}$ and $\bar{R} = \frac{1}{N}\sum_{i=1}^{g}\sum_{j=1}^{n_i} R_{ij}$
- IF having ties, compute the adjustment factor D:

$$D = 1 - \frac{\sum(t^3 - t)}{(N-1)N(N+1)}$$

  where t is the number of ties for each rank value.
- TS: $H = \frac{12}{N(N+1)}\sum_{i=1}^{g} n_i(\bar{R}_i - \bar{R})^2$

  If no ties, H is the TS; otherwise, the TS is $\frac{H}{D} \triangleq H_{adj}$

Remarks:

$(a_n - a_{n-1} = a_{n-1} - a_{n-2} = \cdots = a_2 - a_1)$

- Recall for an arithmetic sequence $\{a_1, a_2, \cdots, a_n\}$, the sum $\sum_{i=1}^{n} a_i = \frac{(a_1 + a_n)n}{2}$. So, it follows that

$$\bar{R} = \frac{1}{N}\#\text{Total raks} = \frac{1}{N}(1 + 2 + \cdots + N) = \frac{1}{N}\cdot\frac{(N+1)N}{2} = \frac{N+1}{2}.$$

- $\sum(t^3 - t)$: e.g. Say we have ranks 2, 2, 2, 4, 5.5, 5.5.

  For 2, three values are in a tie, so $t = 3$ and $t^3 - t = 3^3 - 3 = 24$

  For 4, no tie

  For 5.5, two values are in a tie, so $t = 2$ and $t^3 - t = 2^3 - 2 = 6$

  So, $\sum(t^3 - t) = 24 + 6 = 30$.
- Since $\bar{R} = \frac{N+1}{2}$, we can rewrite H as $H = \frac{12}{N(N+1)}\sum_{i=1}^{g} n_i\left(\bar{R}_i - \frac{N+1}{2}\right)^2$

Simplified procedure.

- Rank all obs from the smallest to the largest

- $H = \frac{12}{N(N+1)} \sum_{i=1}^{g} n_i (\bar{R}_i - \frac{N+1}{2})^2$, where $\bar{R}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} R_{ij}$

  If no ties, this is the T.S.; otherwise, proceed.

- $D = 1 - \frac{\sum (t^3 - t)}{(N-1)N(N+1)}$, where t is the number of ties for each rank value.

  $\Rightarrow H_{adj} = \frac{H}{D}$