Two-Way ANOVA

Now we consider two factors, Factor A and Factor B, called **main effects**.

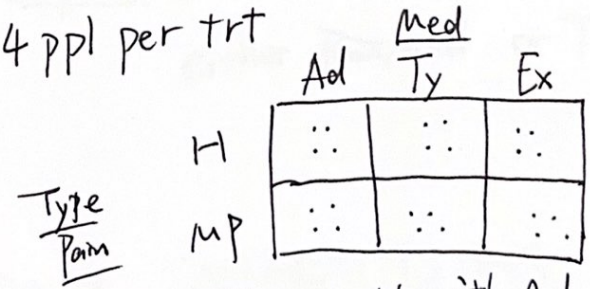
- Factor A has a levels (groups).
- Factor B has b levels (groups).

What's different?

- Treatments are combinations of factor levels # trt = $a \times b$
- Interactions between 2 factors are possible
  - interaction plots
  - test for interaction using ANOVA table.
- Replications = # obs per trt.

Example: Which pain medicine gives fastest relief?
Suppose we want to study 3 pain meds (Advil, Tylenol, Excedrin) on 2 types of pain: headaches and muscle pain. We are interested in studying the mean time to pain relief. Suppose we assign 4 people to each treatment.

- **Response:** time to pain relief
- **Factors:** 2 Factors. Pain Meds ($a=3$ levels) + Type Pain ($b=2$ levels)
- **# of Treatments:** $3 \times 2 = 6$ trts.
- **Experimental Units:** $N = 4 \times 6 = 24$ ppl.
- **Replications:** $n_i = n_j = 4$ ppl per trt



Trts: H with Ad
H .. Ty
H .. Ex
MP .. Ad
MP .. Ty
MP .. Ex

= Plots of trt means.

Interaction Plots

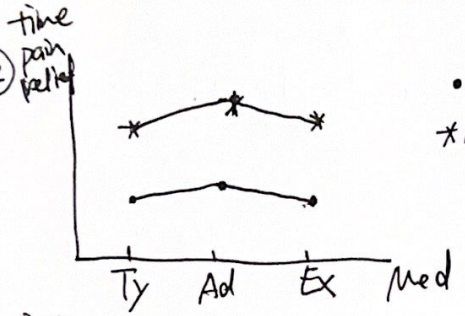
What are some possibilities for effects of the factors on the response variable? We can use plots of Treatment MEANS to determine whether there are interaction effects and main effects.

① 

•H
*MP

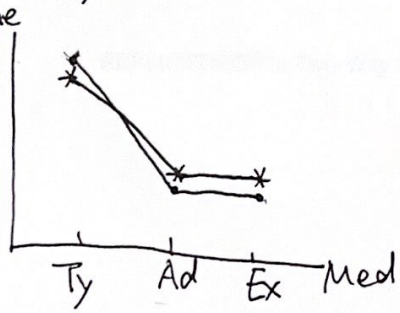
Expect to see: SIG DIFF due to both Type of pain & Meds.

- Time pain relief for H shorter than MP (expect SIG DIFF due to type of pain)
- Time pain relief w/Advil larger than Ty & Ex (expect SIG DIFF due to Med)

② 

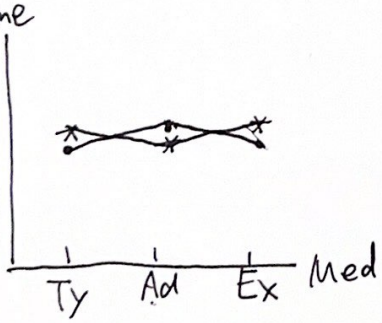
•H
*MP

Expect: SIG DIFF due to Type of Pain but NOT SIG DIFF in Med.

③ 

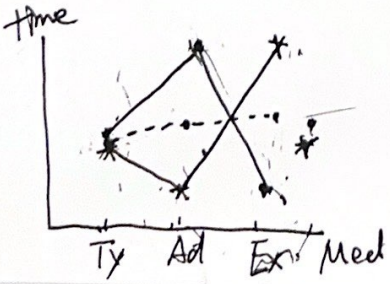
•H
*MP

Expect: SIG DIFF due to Med, but NOT SIG DIFF due to Type of Pain.

④ 

•H
*MP

Expect: NO SIG DIFF due to either ~~Pain~~ Med or Type of Pain.

⑤ 

•H
*MP

Expect: INTERACTIONS between Med + Type of Pain

"Best" pain med DEPENDS on Type of Pain

TWO-WAY ANOVA table to determine significant effects.

Source	df	SS	MS	F	p-val
Factor A (main effect)	a−1	SSA	$MSA = \frac{SSA}{a-1}$	$\frac{MSA}{MSE}$	*
Factor B (main effect)	b−1	SSB	$MSB = \frac{SSB}{b-1}$	$\frac{MSB}{MSE}$	*
Interaction AB	(a−1)×(b−1)	SSAB	$MSAB = \frac{SSAB}{(a-1)(b-1)}$	$\frac{MSAB}{MSE}$	*
Error (Random Error variability)	leftovers	SSE	$MSE = \frac{SSE}{leftovers}$		
Total	N−1	SST.			

test for main effects

test for interaction

★ **Always test for interaction first**

H_0: NO interaction present H_a: There IS interaction.

IF p-val for interaction is:

Small (say ≤ 0.05) [SIG]

- Interaction between Factor A+B
- Relationship between A+B complicated
- look at levels of Factor A separately for each level of Factor B.

Ex: order Meds for each Type of pain.

Headache	MP	When ignored FactorB
~~Ty~~ Ex	Ad	Ty
~~Ad~~ Ty	Ty	Ad (useless)
Ex Ad	Ex	Ex

- Don't even look at p-val for Main Effects (A+B)

Large [NOT SIG]

- NO interaction
- Look at p-vals for main effects (A+B) + interpret
- Look at factors separately

Ex:

Med	Type Pain
Ty	MP
Ad	H
Ex	

Two-Way ANOVA Examples

SCANDAL

Can "Low-Fat" Nutrition Labels Lead to Obesity?

BRIAN WANSINK and PIERRE CHANDON
JOURNAL OF MARKETING RESEARCH, NOVEMBER 2006

Study examines whether low-fat nutrition labels increase the actual consumption of hedonic chocolate candies by overweight and normal-weight consumers. To achieve this, we asked adult family members (53% males, 31 years old, 25.3 body mass index [BMI]) participating in a university open house to serve themselves unusual colors of M&M's (gold, teal, purple, and white), which were clearly labeled either as "New Colors of Regular M&M's" (regular-label condition) or as "New 'Low-Fat' M&M's" (low-fat-label condition). We then measured how many calories of M&M's they ate.

Two-way ANOVA: Consumption versus BodyMass, Label

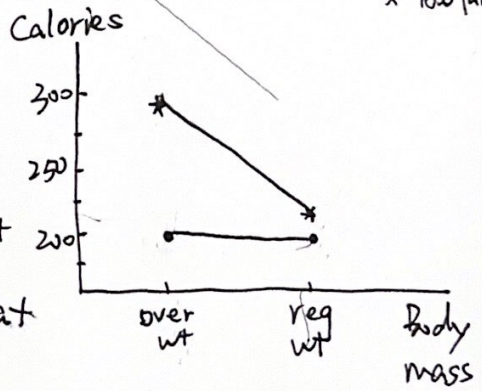
Source	DF	SS	MS	F	P
BodyMass	1	11458.2	11458.2	12.34	0.001
Label	1	35581.2	35581.2	38.32	0.000
Interaction	1	9030.0	9030.0	9.72	0.004
Error	36	33430.5	928.6		
Total	39	89500.0			

S = 30.47 R-Sq = 62.65% R-Sq(adj) = 59.53%

	BodyMass	
Label	overweight	regular weight
"regular" m&m's	Mean=191.6 Stdev=45.0 N= 10	Mean=187.80 Stdev=17.37 N=10
"low fat" m&m's	Mean=281.3 Stdev=34.6 N=10	Mean=217.4 Stdev=13.56 N=10

- Response: How many calories they ate.
- Factors: Label (reg, low-fat) Body mass (overwt, reg wt.)
- levels: reg, low-fat; overwt, reg wt.
- Trts: 2 × 2 = 4 → { overwt with reg; overwt ... low-fat; reg wt -- reg; reg wt -- low-fat }
- Replications: 10 ppl per group

• reg
* low-fat



Expect:
p-val for interaction – SIGNIFICANT.
- Reg label works similarly for both types of body mass (eat about same # of calories on average)
- Low-fat label both groups eat more calories But overwt ppl eat much more than reg wt ppl.