Towards Energy-Balanced Data Transmission for Lifetime Optimization in Wireless Sensor Networks

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***Abstract*—Energy balance is a critical issue in wireless sensor networks. Several mixed data transmission (MDT) schemes have been proposed to achieve energy balance. However, most existing works are lack of theoretical study, especially understanding the relationship between network-wide energy balancing and lifetime optimization. In this paper, we conduct comprehensive theoretical analysis to the two-level based MDT scheme when applying to network-wide energy balancing, and eventually to maximize the network lifetime. We propose a novel network model, named energy balance area (EBA), and formally analyze its characteristics under the two-level based MDT scheme. To maximize the network lifetime, we convert the transmission probability allocation problem in the MDT scheme into an EBA partitioning (EBA-PT) problem, which is shown to be NP-hard. We then propose a heuristic approximation algorithm to determine the optimal configuration of EBAs, which is proven in this paper to be the key for maximizing the network lifetime. In this way, we obtain a near-optimal result. Our experimental studies show that network lifetime can be further improved as compared the hop-by-hop and the two-level based MDT schemes.**

I. Introduction

Wireless Sensor Networks (WSNs) have been widely used in a variety of applications. Since sensor nodes are usually powered by battery, how to effectively utilize the batter power of each sensor node so that the network lifetime can be maximized is a critical issue in WSNs. It has been revealed that communication is a dominant factor of energy consumption in WSNs. Therefore, the design of an energy-efficient data transmission scheme is crucial to prolong the network lifetime.

A sensor node typically has a limited transmission range, and data packets are usually transmitted to the sink node via multihops [8]. The convergent traffic pattern of WSNs leads to the early death of the nodes lying closest to the sink, which would disable the entire network. To alleviate the energy overuse of the nodes near the sink, recent research works suggest a joint power control and routing technique [4] [5]. The basic idea is that if the transmission range of a node can be increased properly, we may skip those nodes which previously have heavier loads. The joint power control and routing technique has been used in several mixed data transmission (MDT) schemes [7]-[9]. In these schemes, a node is assumed to have adjustable transmission power levels. To achieve energy balance, a probability (i.e., *transmission probability*) is assigned to each transmission power level. When transmitting data packets, sensor nodes randomly select their transmission power with corresponding transmission probabilities. However, it is a non-trivial task to determine the transmission probability for each sensor node, which is critical to the system performance. If the corresponding transmission probability is set too small, it may not effectively alleviate the loads of bottleneck nodes. On the other hand, if the transmission probability is set too large, those further-away nodes may become the new bottlenecks, which may result in a shorter network lifetime. Therefore, it is really a challenge to allocate a proper transmission probability for each node to optimize the network lifetime, which is formulated as the *transmission probability allocation (TPA)* problem.

There have been several studies on the MDT schemes in WSNs [7] [8], which provide some insights into the TPA problem. However, all of these studies have an explicit assumption that all sensor nodes can directly reach the sink in one hop, which is impossible, if not impractical, for real applications since the range of a network will be limited by the transmission range of sensor nodes. Having a more realistic assumption, Jarry et al. [10] proposes a two-level based MDT scheme, which enables each node to probabilistically switch its transmission range between the one-hop and two-hop distance. An algorithm is proposed to compute the transmission probability for each node. The energy consumption of sensor nodes achieves balance in some way, however no evidence in their paper shows that how the network lifetime is maximized. Moreover, a more recent study [11] shows that the two-level based MDT scheme achieves energy balance only if the network range is less than a certain threshold, which significantly limits the scheme when applying to network-wide energy balance. The understanding of network-wide energy balance is lack of theoretical study; it is still an open issue.

In this paper, we conduct comprehensive theoretical analysis to understand how the two-level based MDT scheme achieves network-wide energy balance in wireless sensor networks, so as to maximize the network lifetime. While most of the MDT schemes adopt a slice network model which divides the entire network area into n slices, we show mathematically that the slice model under the two-level based MDT scheme is not able to achieve network-wide energy balance. Leveraging the slice

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model as a basis, we propose a novel network model named energy balance area (EBA). Each EBA consists of one or more slices, and our goal is to achieve energy balance within EBAs such that the network lifetime can be maximized. We then formally model and analyze the characteristics of the two-level based MDT scheme within a single EBA. Through our analysis, we reveal that there exists an upper bound for the size of an EBA (termed as *energy balance bound*) such that by determining a proper size for each EBA, the network lifetime can be maximized. We conduct theoretical analysis on the energy balance bound, and eventually we convert the original TPA problem into an EBA partitioning (EBA-PT) problem, which is shown to be NP-hard. We propose a heuristic approximation algorithm to determine the optimal configuration of EBAs. Through our experimental evaluation, we demonstrate that our approach improves the network lifetime by 29.5% and 4.2%, respectively, as compared with the hop-by-hop approach and the two-level based MDT scheme proposed in [10].

II. Related Work

The joint power control and routing technique has been extensively studied in the literature. The existing methods can be roughly categorized into: hop-by-hop based and mixed data transmission (MDT) schemes. To attain energy balance, the hop-by-hop based schemes enable nodes to select the one-hop transmission range based on their traffic loads [6]. Instead of using a fixed one-hop transmission range, each node is allowed to switch between certain ranges by MDT schemes. Prior efforts [7] [8] [9] mainly focus on the TPA problem of a specific MDT scheme, with which each node can transmit with the one-hop distance or to the sink directly. In [7], the authors propose an analytical model for the TPA problem and formally derive the transmission probabilities. In [8] and [9], the TPA problem is formulated as linear program models, which can be solved by classical methods in a centralized manner. However, these works fail to take the transmission range constraints of sensor nodes into account, and thus cannot be applied in large WSNs.

To overcome the transmission range constraints, Jarry et al. [10] propose the two-level based MDT scheme. An algorithm is proposed to compute the transmission probabilities. However, no evidence shows that the network lifetime is maximized. In [11], the authors investigate the energy balance bounds of the two-level based MDT schemes in 1D networks. Meanwhile, the lifetime performance is neglected. In this work, we extend the analysis of [11], and further investigate how to exploit the energy balance feature of the two-level based MDT scheme to optimize the network lifetime.

III. System Model and Assumptions

We consider an event detection WSN where the sensor nodes are uniformly deployed. The network area is represented as a concentric slice model, with the slice width of r (r equals to the one-hop transmission range of sensor nodes). Figure 1 shows a network being divided into five slices, i.e., S_1, S_2, $\cdots$, S_5. We denote the number of nodes in S_i by N_i. Once an event is detected by a node, a message will be created and reported to the sink. Events are uniformly generated in the network. We denote the data generating rate in S_i by λ_i.

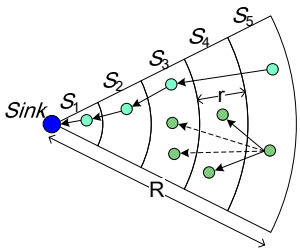


Fig. 1. The network model

To complete the task of event reporting, we adopt the two-level based MDT scheme [10]. As shown in Fig.1, once a message arrives at a node, it will be transferred one slice or two slices away. The probability of a message from S_i being transmitted to S_{i-1} is defined as the *transmission probability* (p_i) for the nodes in S_i. Figure 1 shows a possible delivering path of messages generated in S_5. To enable the MDT scheme, we assume that each node has two kinds of transmission range, i.e. r_1 and r_2, with $r_2 = 2r_1$.

Since our focus is on the energy consumption of communications, we assume that the network employs a TDMA based MAC protocol, with which the communication channel is free of interference. Let c_0 be the per-bit receiving energy, c_1 and c_2 be the per-bit transmitting energy corresponding to r_1 and r_2, respectively. The sink node has continuous power supply. We assume that the energy consumption of nodes in the same slice are well balanced (i.e., intra-slice balance), and denote the whole energy budgets of all nodes in S_i by E_i. Essentially, the intra-slice energy balance [9] is orthogonal to our analysis and beyond the scope of this paper.

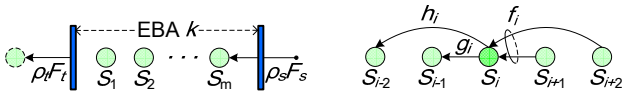
IV. Balancing the Energy among Slices

This section addresses the energy balance issues of a single EBA (i.e., Energy Balance Area). We first propose an analytical model to formally derive the transmission probabilities. Then, by utilizing the unique structure of EBAs, we investigate the energy balance bound and reveal the relations between EBA parameters and network lifetime, which provides essential guidelines for lifetime optimization.

A. Deriving the transmission probability

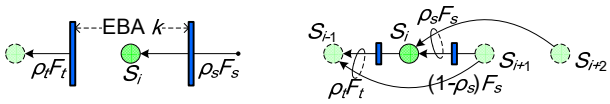
An Energy Balance Area (EBA) is composed by m ($m \geq 2$) adjacent slices, i.e., $S_1, S_2, \cdots, S_m$, as shown in Fig.2a. Let F_s and F_t denote the incoming and outgoing traffic of the EBA. Clearly, $F_t = F_s + \sum_{i=1}^{m} \lambda_i$. Note that only a fraction of F_s flows into S_m, we denote such fraction by ρ_s. Accordingly, the fraction of F_t flows into the next slice of S_1 is denoted by ρ_t ($0 \leq \rho_s, \rho_t \leq 1$). We assume that the parameters m, ρ_t and ρ_s of an EBA are available. Let g_i and h_i be the average rates of traffic from S_i to S_{i-1} and S_{i-2}, respectively (see Fig.2b). With f_i standing for the receiving traffic rate of S_i, we have

$$f_i = g_{i+1} + h_{i+2}, \quad i = 1, 2, \cdots, m-2 \tag{1}$$



(a) The EBA model (b) The flow model

Fig. 2. Illustration of an energy balance area (EBA)



(a) A single-slice EBA (b) The flows of a single-slice EBA

Fig. 3. Illustration of a single-slice EBA

Specifically, $f_m = \rho_s F_s$ and $f_{m-1} = (1-\rho_s)F_s$. For uniformity of notation, we write $f_0 = \rho_t F_t$.

The flow-conservation constraint in S_i can be described as

$$f_i + \lambda_i = g_i + h_i, \quad i = 1, 2, \cdots, m \tag{2}$$

By combining (1) and (2) to eliminate h_i, along with elementary manipulations, we obtain

$$g_i = f_i + f_{i-1} - F_t + \sum_{k=1}^{i} \lambda_k, \quad i = 1, 2, \cdots, m \tag{3}$$

Hence, once f_i's are obtained, the transmission probability can be calculated with $p_i = \frac{g_i}{f_i + \lambda_i}$. Next, we derive the expression of f_i by utilizing the constraint of energy balance:

$$\frac{E_1}{e_1} = \frac{E_2}{e_2} = \cdots = \frac{E_m}{e_m} = T \tag{4}$$

where, T signifies the common lifetime of slices in the EBA, e_i is the energy consumption rate of S_i, which is written as

$$e_i = c_0 \cdot f_i + c_1 \cdot g_i + c_2 \cdot (f_i + \lambda_i - g_i)$$

Then, substituting (3) into this expression, we have

$$e_i = a_i + \phi_i \tag{5}$$

where, $a_i = (c_0 + c_1) f_i - (c_2 - c_1) f_{i-1}$ and $\phi_i = (c_2 - c_1)(F_t - \sum_{k=1}^{i} \lambda_k) + c_2 \lambda_i$. We note that ϕ_i is a constant depends on i, and the variables, f_i and f_{i-1}, are only contained in a_i. Then, once a_i is determined, f_i can be obtained with

$$f_i = \frac{1}{c_0 + c_1} \sum_{k=1}^{i} \left(a_k \cdot \alpha^{i-k} \right) + \alpha^i \cdot \rho_t F_t \tag{6}$$

where, $\alpha = \frac{c_2 - c_1}{c_1 + c_0}$. Since $a_i = (c_0 + c_1) f_i - (c_2 - c_1) f_{i-1}$, By adding the m a_i's together in a delicate way, we can get

$$\frac{1}{c_0 + c_1} \sum_{k=1}^{m} \left(a_k \cdot \alpha^{m-k} \right) = \rho_s F_s - \alpha^m \rho_t F_t \tag{7}$$

Recall the mathematical relation that $\frac{a}{b} = \frac{c}{d} = \frac{m \cdot a + n \cdot c}{m \cdot b + n \cdot d}$. By applying it to Eq. (4), we have

$$T = \frac{\frac{1}{c_0 + c_1} \sum_{k=1}^{m} \left(E_k \cdot \alpha^{m-k} \right)}{\frac{1}{c_0 + c_1} \sum_{k=1}^{m} \left(e_k \cdot \alpha^{m-k} \right)}$$

Since $e_k = a_k + \phi_k$, the denominator can be written as

$$\frac{1}{c_0 + c_1} \sum_{k=1}^{m} \left(a_k \cdot \alpha^{m-k} \right) + \frac{1}{c_0 + c_1} \sum_{k=1}^{m} \left(\phi_k \cdot \alpha^{m-k} \right)$$

Substituting (7) into the above expression, we can finally get

$$T = \frac{\sum_{k=1}^{m} \left(E_k \cdot \alpha^{m-k} \right)}{(c_0 + c_1)(\rho_s F_s - \alpha^m \rho_t F_t) + \sum_{k=1}^{m} \left(\phi_k \cdot \alpha^{m-k} \right)} \tag{8}$$

As all the parameters contained in (8) are available, T is essentially a constant. Once T is derived, e_i, as well as a_i, can be determined accordingly. Finally, we can get f_i, g_i (with Eqs. (6) and (3)), and obtain the transmission probability (p_i).

B. Determining the energy balance bound

This section studies the performance limit of MDT schemes on energy balance. We name such limit as energy balance bound, which is expressed as the maximum size of an EBA.

As an EBA is composed by adjacent slices that are expected to have the identical lifetime. A single slice naturally forms an EBA. To derive the energy balance bound, the basic idea is as follows. We initially consider each slice as an individual EBA. If we can tune the marginal parameters, i.e., ρ_s and ρ_t, of two neighbor EBAs in such a way that their lifetimes finally reach the same level, then, as a result, the size of the original EBA is increased by one. We call this operation an *EBA expanding*. If we proceed in such a way, the EBA can finally expand from a single slice to its maximal size, and thus the energy balance bound is obtained.

To this end, it is important to begin by analyzing the single-slice EBA (see Fig.3). Noting that the traffics received by S_{i-1} are from either S_i, the single-slice EBA, or S_{i+1}, we have

$$\rho_t F_t = (\rho_s F_s + \lambda_i) \cdot p_i + (1 - \rho_s) F_s$$

As $0 \le p_i \le 1$, substituting in the above equation, we get

$$\max(0,\ 1 - \frac{F_t}{F_s} \rho_t) \le \rho_s \le 1 \tag{9}$$

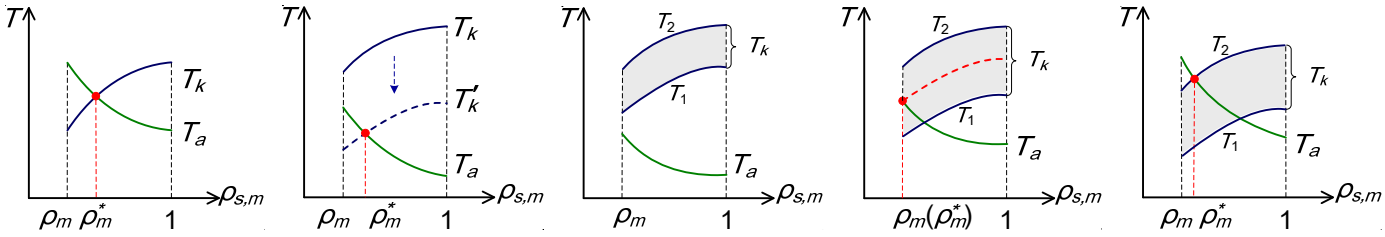
It essentially gives the minimum value of ρ_s. Using the same methods in Section IV-A, we can derive the lifetime of a single-slice EBA. Due to page limit, we omit the process. Our results reveal that the expression of the lifetime of a single-slice EBA is a special case of (8) when $m = 1$.

To derive the energy balance bound, we need to first analyze the prerequisites for EBA expanding.

1) Prerequisites: The energy balance bound is supposed to be obtained by incrementally expanding an EBA. However, such a method works only if a) the lifetime of two neighbor EBAs could eventually reach the same level by adjusting ρ_s or ρ_t, and b) the tuning of ρ_s, or ρ_t, would not break the energy-balanceness of the original EBAs. Prerequisite a) will be analyzed afterward. Here, to testify the certainty of b), we provide the following theorem.

Theorem 1: In an arbitrary EBA, let ρ_0 be the minimum ρ_s that ensures $0 \le p_i \le 1$ for all slices, then the energy balance would always be attained while ρ_s increases from ρ_0 to 1.

The proof is omitted due to the limited space. We briefly demonstrate the correctness of Theorem 1 as follows. By Eqs.



(a) $T_a > T_k$ (b) $T_a < T_k$ (c) Case A (d) Case B (e) Case C

Fig. 4. Illustration of the relations between T_a and T_k

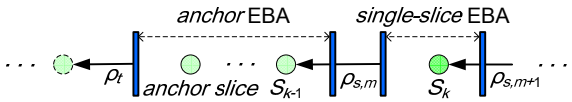


Fig. 5. A snapshot of the EBA expanding

(6) and (8), we can get that both f_i and g_i are the increasing functions of ρ_s. As $p_i = \frac{g_i}{f_i+\lambda_i}$, and ρ_0 is the minimum ρ_s that satisfies $p_i \geq 0$, we have $p_i \geq 0$ while $\rho_0 \leq \rho_s \leq 1$. In addition, from Eq. (3), we can get that $p_i \leq 1$ always holds. Therefore, increasing ρ_s keeps the energy-balanceness.

Now, let us turn to prerequisite a). Consider one step of the EBA expanding process (see Fig.5). The starting slice and the EBA started from this slice are referred to as the *anchor slice* and *anchor EBA*, respectively. Let $\rho_{s,m}$, $\rho_{s,m+1}$ and $F_{s,m}$, $F_{s,m+1}$ denote the ρ_s and F_s parameters of the current anchor EBA and its following single-slice EBA (S_k). ρ_m and ρ_{m+1} correspond to the minimum value of $\rho_{s,m}$ and $\rho_{s,m+1}$. Denote the lifetime of the anchor EBA and S_k by T_a and T_k. We consider how T_a and T_k would vary, had $\rho_{s,m}$ been increased from ρ_m to 1. Note that when $\rho_{s,m} = 1$, $T_a < T_k$ (using (8), we focus on the lifetime of S_{k-1} and S_k and obtain $T_{k-1} < T_k$ when $\rho_{s,m} = 1$; we note $T_{k-1} = T_a$). Based on the relations between T_a and T_k when $\rho_{s,m} = \rho_m$, there are two cases:

- if $T_a > T_k$ when $\rho_{s,m} = \rho_m$, as shown in Fig.4a, T_a and T_k would reach the same level if we tune $\rho_{s,m}$ to ρ_m^*.
- if $T_a < T_k$ when $\rho_{s,m} = \rho_m$ (see Fig.4b), the situation becomes complex and we discuss it in detail below.

If $T_a < T_k$ when $\rho_{s,m} = \rho_m$, simply tuning $\rho_{s,m}$ cannot make T_a and T_k equal. In addition, we need to reduce T_k by increasing $\rho_{s,m+1}$ (see Fig.4b). If the minimum T_k is smaller than T_a, then the anchor EBA can be expanded as above. Otherwise, the EBA can not be expanded any more. To summarize, we have proved Lemma 1.

Lemma 1: The anchor EBA can be expanded to the next slice only if $T_a \geq T_k$ when $\rho_{s,m} = \rho_m$ and $\rho_{s,m+1} = 1$.

Therefore, in order to decide whether an EBA expanding operation can be conducted, we need to first obtain ρ_m, the minimum value of $\rho_{s,m}$. In what follows, we show that such a task can be accomplished while we incrementally expand the anchor EBA to approach the energy balance bound.

2) EBA expanding: The EBA expanding is conducted by increasing $\rho_{s,m}$ from ρ_m to a certain value, say ρ_m^*, so that S_k can be just combined into the anchor EBA and $\rho_{s,m+1} = \rho_{m+1}$. It is worth to note that, the minimum ρ_s of the new anchor EBA, i.e., ρ_{m+1}, will be generated by such an operation. Specifically, ρ_m^* and ρ_{m+1} are derived as below.

Algorithm 1 The energy balance bound determining algorithm

Input: F_t, ρ_t at the anchor slice, and the network size n.
Output: The energy balance bound (m_0), and $\{\rho_m\}$, the minimum of $\rho_{s,m}$ for all $m = 1, \cdots, m_0$.

```
Let m = 1, and calculate ρ_1 by ρ_1 = f(ρ_t).
while m ≤ n do
    Let ρ_{s,m} = ρ_m, and calculate T_a by T_a = T_a(ρ_m).
    Let T_1 = T_k(ρ_m, 1), T_2 = T_k(ρ_m, f(ρ_m)).
    if T_a < T_1 then
        Break out of the loop. {Obtain the EB bound.}
    else if T_a ≤ T_2 then
        Obtain ρ̄_{m+1} by solving T_k(ρ_m, ρ̄_{m+1}) = T_a(ρ_m).
        Set ρ_{m+1} to be ρ̄_{m+1}.
    else
        Obtain ρ*_m by solving T_a(ρ*_m) = T_k(ρ*_m, f(ρ*_m)).
        Set ρ_{m+1} to be f(ρ*_m).
    end if
    Let m = m + 1. {Expand the size of the EBA by 1.}
end while
Set m_0 to be m.
Output m_0 and {ρ_m} as the results.
```

As ρ_1 is given by (9), we attempt to find an approach to calculate ρ_{m+1} from ρ_m. Thus, we focus on the process of expanding the anchor EBA from the size of m to $m+1$. Let $\rho_{s,m} = \rho_m$. From (9), we have $f(\rho_m) \leq \rho_{s,m+1} \leq 1$, where $f(\rho_m) = \max(0,\ 1 - \frac{F_{s,m}}{F_{s,m+1}}\rho_m)$. Denote the minimum and maximum value of T_k by T_1 and T_2, respectively. According to the relations between T_a and T_k when $\rho_{s,m} = \rho_m$, there are three cases. (For clarity, we introduce $T_a(\rho_1)$, $T_k(\rho_1, \rho_2)$ to denote the value of T_a, T_k when $\rho_{s,m} = \rho_1$ and $\rho_{s,m+1} = \rho_2$.)

Case A: If $T_a < T_1$ (see Fig.4c), then, according to Lemma 1, we cannot expand the anchor EBA any more, which means that m is the maximum size of the anchor EBA. Hence, the obtained m is the energy balance bound.

Case B: If $T_1 \leq T_a \leq T_2$ (Fig.4d), then $\rho_m^* = \rho_m$, $\rho_{m+1} = \bar{\rho}_{m+1}$, where $\bar{\rho}_{m+1}$ is obtained by solving $T_k(\rho_m, \bar{\rho}_{m+1}) = T_a(\rho_m)$.

We prove $\rho_{m+1} = \bar{\rho}_{m+1}$ as below. If there exists another

$\rho_{s,m}$, say ρ'_m, satisfying $T'_1 \leq T'_a \leq T'_2$. Note that $\rho'_m > \rho_m$. By (8), we have $T'_a < T_a$. After the anchor EBA being expanded, the lifetime of the EBA satisfies $T_{m+1} = T_a$ and $T'_{m+1} = T'_a$. Note that $T'_{m+1} < T_{m+1}$, which, in turn, deduces $\bar{\rho}'_{m+1} > \bar{\rho}_{m+1}$. Therefore, $\rho_{m+1} = \bar{\rho}_{m+1}$.

Case C: If $T_a > T_2$, then $\rho_{m+1} = f(\rho^*_m)$, and ρ^*_m can be obtained by solving $T_a(\rho^*_m) = T_k(\rho^*_m, f(\rho^*_m))$.

The rationale behind can be described as follows. According to (9), the spectrum of T_k keeps shifting upward while $\rho_{s,m}$ is increasing (see Fig.4e). Hence, by increasing the $\rho_{s,m}$, we can gradually reach *Case B* and find the ρ^*_m that makes the lifetime of the anchor EBA equal to T_2, i.e., $T_a(\rho^*_m) = T_k(\rho^*_m, f(\rho^*_m))$. Therefore, $\rho_{m+1} = f(\rho^*_m)$.

Based upon this, we are able to calculate the ρ_m's and the energy balance bound in a single algorithm as below.

3) The algorithm: The energy balance bound determining algorithm is presented as Algorithm 1, which produces m_0, as well as the minimum value of $\rho_{s,m}$ for all $m = 1, \cdots, m_0$. The algorithm starts from a single-slice EBA (line 1), then gradually expands the EBA size. Lines 2-15 correspond to one single EBA expanding. It obtains the energy balance bound in line 16 and outputs the results in line 17. Clearly, the computational complexity of the algorithm is $O(n)$.

V. Optimizing the Network Lifetime

The lifetime optimization problem is handled in this section. We illustrate the NP-hardness of the problem, and propose a heuristic approximation algorithm that can obtain near-optimal solutions in polynomial time.

A. The EBA partitioning problem

Recall that, for each EBA, parameter ρ_s varies continuously and the maximum lifetime is obtained when ρ_s is minimized. Therefore, to optimize the network lifetime, we let $\rho_s = \rho_m$ and focus on how to determine the size of each EBA, which is defined as the *EBA partitioning (EBA-PT) problem*.

Given the fact that the size of an EBA varies from 1 to $m_{i,0}$ (i.e., the energy balance bound) and the EBA lifetime also varies accordingly. We can consider each EBA as an object of the knapsack problem. Thus, the lifetime and size of an EBA can be taken as the value and the weight of the corresponding object, respectively. Let $x_{i,j}$ represent the EBA that starts from S_i with the size of j. Then, the EBA-PT problem can be formulated as

$$\max \min_{\substack{i=1,\cdots,n \\ j=1,\cdots,m_{i,0}}} \{p_{i,j} | x_{i,j} = 1\} \tag{10}$$

$$s.t. \sum_{i=1}^{n} \sum_{j=1}^{m_{i,0}} j \cdot x_{i,j} = n \tag{11}$$

$$\sum_{j=1}^{m_{i,0}} x_{i,j} \leq 1, \ i = 1, \cdots, n \tag{12}$$

$$x_{i,j} = 0 \text{ or } 1, \ i = 1, \cdots, n; \ j = 1, \cdots, m_{i,0} \tag{13}$$

where, $p_{i,j}$ represents the value of the object, i.e., the lifetime of EBA $x_{i,j}$, which can be calculated by Eq. (8). The constraint (11) states that the weight of all the selected objects should exactly equal to the capacity of the knapsack. Constraints (12) and (13) ensure that at most one object would be selected from the group of EBAs with the same anchor slice. The objective function (10) maximize the network lifetime which is determined by the EBA with the shortest lifetime.

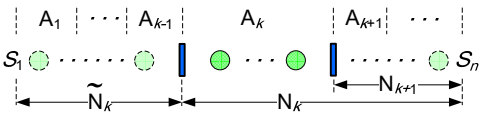


Fig. 6. Illustration of an EBA partitioning scheme

As we can see that the EBA-PT problem is actually modeled as a multidimensional 0-1 knapsack problem (MKP). It is well known that the MKP is NP-hard [12], which indicates that the EBA-PT problem is also NP-hard. Therefore, it is difficult to compute an optimal solution with polynomial time complexity; we try to find an approximate solution instead.

B. Properties of the solution

In this section, we analyze the impact of EBA configurations on network lifetime, which can give us some insights into the algorithm design.

Given an EBA partitioning scheme, we denote the i'th EBA by A_i. The parameters of A_i are denoted by $\rho_{s,i}$, $\rho_{t,i}$ and m_i. ρ_{m_i} corresponds to the minimum value of $\rho_{s,i}$. In particular, let N_k denote the sub-network from A_k to the last EBA. As shown in Fig.6, A_k has divided the whole network area into three sub-areas, namely, $\widetilde{N}_k$, A_k and N_{k+1}. For clarity, let T_k be the lifetime of A_k, and L_k be the lifetime of N_k.

Consider two neighbor EBAs, A_i and A_{i+1}. While $T_i > T_{i+1}$, recall Fig.4a, by increasing $\rho_{s,i}$, we can combine A_i and A_{i+1} into a new EBA. We call this operation an *EBA merging*. Different from the EBA expanding in Section IV-B2, EBA merging keeps the values of $\rho_{t,i}$, $\rho_{s,i+1}$, and does not require A_{i+1} to be a single-slice EBA. Specifically, if A_{i+1} is the critical EBA, i.e., $T_{i+1} = \min\{T_j\}$, then the network lifetime can be prolonged by the *EBA merging*. Furthermore, we have Lemma 2.

Lemma 2: In the optimal solution of the EBA-PT problem, the first EBA is, in the meanwhile, the critical EBA.

Proof: We prove this lemma by the reduction to absurdity. If A_i, instead of A_1, is the critical EBA, then $T_{i-1} > T_i$ must hold. Thus, by merging A_i with A_{i-1}, we can further improve the network lifetime, which means that the original EBA partitioning scheme is actually not the optimal solution. This is a contradiction. ■

The network lifetime is mainly determined by A_1, and thus the size of A_1 need to be delicately selected. To derive the optimal m_1, we first assume that the optimal lifetime of N_2, denoted by $\widehat{L}_2$ can be achieved. Noting that both the value of $\widehat{L}_2$ and T_1 would be affected by m_1, we explicitly denote them by $\widehat{L}_2(m_1)$ and $T_1(m_1)$, respectively. Since $1 \leq m_1 \leq m_{1,0}$, by Lemma 2, combined with the fact that T_1 is a non-increasing function of m_1, we can obtain the optimal m^*_1,

where $m_1^* = \min\{m_1 | T_1(m_1) < \widehat{L}_2(m_1)\}$.

Specifically, if $m_{1,0} \leq 2$, then $m_1^* = m_{1,0}$. We explain the situation of $m_{1,0} = 2$ as follows. If the size of A_1 is set to 1, there are two cases: if $T_1 > T_2$, then, according to Lemma 2, the network lifetime can be improved by merging A_1 and A_2. Hence, we have $m_1^* = 2$. On the other hand, if $T_1 < T_2$, since $\rho_{s,1} = \rho_1$, according to Section IV-B2, we have $T_1(2) = T_1(1)$. Therefore, $m_1^* = 2$ still holds. It also indicates that if more than one slice are allowed in A_1, then setting m_1 to 2, instead of 1, will yield a better lifetime.

To summarize, we can get the solution of the EBA-PT problem by setting $m_1 = m_1^*$. Further, we have Theorem 2 that captures the relation between the solution of EBA-PT problem and $\widehat{L}_1$, the optimal network lifetime.

Theorem 2: $\widehat{L}_1$ is bounded by $T_1(m_1^*) \leq \widehat{L}_1 < T_1(m_1^* - 1)$, where $m_1^* = \min\{m_1 | T_1(m_1) < \widehat{L}_2(m_1)\}$.

Proof: Note that in the EBA-PT problem, $\rho_{s,1}$ is constrained by $\rho_{s,1} = \rho_{m_1}$. Recalling the process of expanding A_1 from the size of $m_1^* - 1$ to m_1^*, $\rho_{s,1}$ increases from $\rho_{m_1^*-1}$ to $\rho^*_{m_1^*-1}$. If there exists a $\rho'_{s,1}$, $\rho_{m_1} < \rho'_{s,1} < \rho^*_{m_1}$, that satisfies $T_1 < \widehat{L}_2$ when $m_1 = m_1^* - 1$ and $\rho_{s,1} = \rho'_{s,1}$, then it can yield a better result than $T_1(m_1^*)$, according to the analysis in Section IV-B. It happens when $\rho_{s,2}$ or m_2 can be further decreased. Thus, we have $T_1(m_1^*) \leq \widehat{L}_1 < T_1(m_1^* - 1)$. ∎

Theorem 2 indicates that the solution of the EBA-PT problem is a good approximation to the optimal network lifetime. Considering that the calculation of m_1^* depends on the optimal lifetime of N_2, i.e., $\widehat{L}_2$, which is non-trivial to obtain, we attempt to seek a heuristic-based approach to approximate the optimal lifetime of N_k, without knowing $\widehat{L}_{k+1}$.

Note that the merging of A_i and A_{i+1} would increase the size of A_i. However, as m_i is upper-bounded by $m_{i,0}$ (the energy balance bound of A_i), the merging of A_i and A_{i+1} will be *blocked* if we set $m_i = m_{i,0}$. Then we call A_i an ***EBA blockage***. In this case, according to Lemma 2, we can conclude that A_i is the critical EBA of N_i. Thus, $L_i = T_i$ holds. Since $1 \leq m_i \leq m_{i,0}$ and T_i is a non-increasing function of m_i, we have $\widehat{L}_i \geq T_i(m_{i,0})$. Essentially, this gives the lower bound for $\widehat{L}_i$. Therefore, the optimal lifetime of N_i can be estimated by the lifetime of A_i, if we set A_i as the EBA blockage.

By virtue of Theorem 2 and the notion of EBA blockage, we can approximately solve the EBA-PT problem. Our algorithm will be presented in the next section.

C. The approximation algorithm

As analyzed above, to get the approximate solution of the EBA-PT problem, we can set A_2 as the EBA blockage and apply Theorem 2 to derive m_1^*. As $\widehat{L}_2$ is approximated by $T_2(m_{2,0})$, the accuracy of $\widehat{L}_2$ significantly affects the computation of m_1^*. To improve the solution, we recursively apply the above process to the sub-network N_2, by setting A_3 as the EBA blockage. Intuitively, if we proceed as above, the solution can be improved gradually. Therefore, in our approximation algorithm, we generally set A_k as the EBA blockage and solve the EBA-PT problem in a recursive manner.

Algorithm 2 The approximation algorithm (AOPT-k)

Input: The anchor slice S_i and an integer k.
Output: The approximate optimal lifetime T_1^*, and $\{m_j\}$, the size of $A_{(k-k'+1)-j}$, $j = 1, \cdots, k-k'$.

1: Calculate $m_{1,0}$ by Algorithm 1.
2: **if** ($m_{1,0} \leq 2$ or $k == 0$) **then**
3: Let $k' = k$. {Setting the EBA blockage.}
4: **return** $T_1^* = T_1(m_{1,0})$.
5: **end if**
6: **for** each $m_1 = 2, \cdots, m_{1,0}$ **do**
7: Recursively calculate $\widehat{L}_2$ by Algorithm 2 with the input parameters of S_{i+m_1} and $k-1$.
8: **if** ($T_1(m_1) \leq \widehat{L}_2$) **then**
9: Set $m(k-k') = m_1$. {The size of the current EBA.}
10: **return** $T_1^* = T_1(m_1)$.
11: **end if**
12: **end for**

The proposed algorithm, AOPT-k, is presented as Algorithm 2. It starts by calculating the energy balance bound (line 1), and proceeds with recursions. Lines 2-5 correspond to the endpoints of the recursion. In each step of the recursion (lines 6-12), the size of the corresponding EBA is determined.

After the algorithm is terminated, $A_{k-k'+1}$ should be set as the EBA blockage. Since the EBA configuration after $A_{k-k'+1}$ does not affect the network lifetime, we simply set the EBA size to the energy balance bound. The approximation ratio of the algorithm is demonstrated by Proposition 1.

Proposition 1: Denote the optimal network lifetime by $\widehat{L}_1$. Let $d_i = T_i(m_i^* - 1) - T_i(m_i^*)$ and $b_i = T_i(1) - T_i(m_i^*)$. Then, we have $\widehat{L}_1 < T_1^* + \sum_{i=1}^{k-k'} d_i + b_{k-k'+1}$, where, $T_i^* = T_i(m_i^*)$, and m_i^* is the size of A_i produced by Algorithm 2.

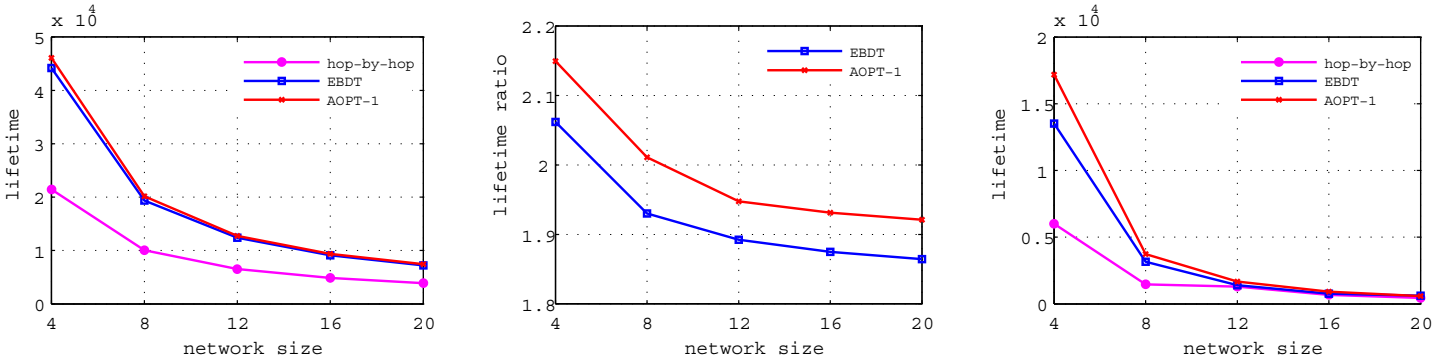
Proof: According to Algorithm 2, T_1^* is produced if the condition in line 8 holds. In fact, $\widehat{L}_2$ is approximated by T_2^*. Hence, the above condition is essentially $T_1(m_1) \leq T_2^*$. We have $T_1(m_1^*) \leq T_2^*$ and $T_1(m_1^* - 1) > T_2^*$. Let $\epsilon_i = \widehat{L}_i - T_i^*$. Then, $T_1(m_1^*) + \epsilon_2 \leq \widehat{L}_2$ and $T_1(m_1^* - 1) + \epsilon_2 > \widehat{L}_2$.

Therefore, there must exists a $j \in \{1, \cdots, m_1^* - 1\}$ that satisfies $T_1(j) \leq \widehat{L}_2$. Consequently, $\widehat{L}_1 = T_1(\widehat{m}_1)$, where $\widehat{m}_1 = \min\{j | T_1(j) \leq \widehat{L}_2\}$. Thus, we have $\epsilon_1 = \widehat{L}_1 - T_1^* \leq \widehat{L}_2 - T_1(m_1^*) < T_1(m_1^* - 1) + \epsilon_2 - T_1(m_1^*)$. That is, $\epsilon_1 < d_1 + \epsilon_2$. Similarly, we can get $\epsilon_2 < d_2 + \epsilon_3$. In general, we have $\epsilon_i < d_i + \epsilon_{i+1}$. Thus, $\epsilon_1 < \sum_{i=1}^{k-k'} d_i + \epsilon_{k-k'+1} < \sum_{i=1}^{k-k'} d_i + b_{k-k'+1}$. This completes the proof. ∎

We now analyze the computational complexity of the algorithm. Since it executes at most k levels of the recursion, let $T(k)$ denote the worst-case time complexity, we have $T(k) = O(n)$ while $k = 0$; otherwise, $T(k) = O(n) + nT(k-1)$. With some elementary manipulations, we can get that the complexity of the entire algorithm is $O(n^{k+1})$, where n is the number of slices in the network.

VI. Simulation Results

The performance of our algorithm is evaluated via OMNeT++ simulations. We conduct experiments in both 1D and



(a) 1D networks (b) Zoom-in view of AOPT-1 vs. EBDT in Fig.7a (c) 2D networks

Fig. 7. Comparing the lifetime performance (network size is indicated by the number of slices, i.e., the maximum number of data transfer hops)

2D uniform networks. The communication parameters are set according to [13]. We repeat each experiment for 50 times and compute the average as the final results.

To the best of our knowledge, the only two-level based MDT scheme is proposed in [10], which is referred to as EBDT. We evaluate the lifetime performance of our algorithm in comparison with the hop-by-hop scheme and the best result of EBDT. The hop-by-hop scheme is used as a baseline. We set $k = 1$ for our algorithm. The network lifetime is represented by the number of reported events until the first node dies.

The results are shown in Fig.7. It can be seen that our scheme substantially improves the network lifetime in both 1D and 2D networks. Compared with the hop-by-hop scheme, the average lifetime gain of AOPT-1 is at least 1.9 times in 1D networks, and 1.3 times in 2D networks. AOPT-1 also outperforms the EBDT. For a clear presentation, we specifically compare AOPT-1 with EBDT in Fig.7b, where the lifetime ratio between AOPT-1 (or EBDT) and the hop-by-hop scheme is illustrated. Here, we only use the results of 1D networks as an example (similar results are observed in 2D networks). From Fig.7b, we can see that AOPT-1 makes a further 4.2% improvement upon the best result of EBDT. Such an improvement is produced by optimally setting the transmission probabilities for the two-level based MDT scheme. This validates the effectiveness of our algorithm.

As expected, we can also see from Fig.7 that the lifetime gain of both AOPT-1 and EBDT decreases while the network size is increasing. The reason is as follows: Since the transmission ranges are limited, while the network size, i.e., the slice number, is large enough (for example, when $n \geq 12$ in the 2D networks), data must be transmitted in a multi-hop manner. In this case, the network lifetime is mainly determined by the overwhelming traffic loads, and thus the contribution of energy balance on prolonging network lifetime is limited. Nevertheless, AOPT-1 still improves the lifetime by 29.5%, as compared with the hop-by-hop scheme.

VII. Conclusions

In this paper, we investigate how to exploit the energy balance feature of two-level based MDT schemes to optimize network lifetime. We analyze the performance limits of the scheme and reveal that energy balance can only be achieved within local areas, i.e., Energy Balance Areas (EBAs). By utilizing the unique structure of EBAs, we transform the lifetime optimization problem into the EBA-PT problem, which is NP-hard. We propose a heuristic based approximation algorithm to solve the problem, which can produce near-optimal solution in polynomial time. Simulation results indicate that the network lifetime can be further improved by our algorithm.

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