

# Poster: Energy Balance Bounds of Mixed Data Transmission in Wireless Sensor Networks

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**Abstract**—Mixed data transmission strategies have been proposed to address the energy imbalance problem in wireless sensor networks. However, whether the energy of all nodes can be balanced by this approach, especially under the communication range constraints of sensor nodes, is lack of study. In this paper, we present a novel analytical model for the two-hop based mixed data transmission and derive its energy balance bounds. It is found that the limits of the mixed data transmission scheme on energy balance are only determined by the power profiles of the communication system. This power profile is indicated by a newly discovered parameter which is defined as the premium power ratio of the system.

## I. INTRODUCTION

Energy imbalance is an inherent problem in wireless sensor networks, leading to the energy holes and impairing the network lifetime. The mixed data transmission scheme [1] is proposed to deal with this issue, and the energy balance may be achieved through the combination of hop-by-hop and direct data transmission schemes. Several approaches [2] [3] have adopted the mixed data transmission scheme, where the sink cannot be reached by all the nodes directly. In [3], Jarry etc. proposed a two-hop based mixed data transmission scheme which stochastically transmits data one hop or two hops away; however, the efficiency of the scheme had not been theoretically studied. Moreover, the problem of whether the perfect energy balance can be achieved by the two-hop based mixed data transmission is lack of study. In this paper, we aim to address the energy balance bounds of the scheme. This work would contribute to understanding the fundamental limits of the mixed data transmission scheme on energy balance.

## II. PROBLEM STATEMENTS

We consider a data collection sensor network where the transmission range of each node is usually much shorter than the network radius. Let  $r_1$  and  $r_2$  represent the one-hop and two-hop transmission range of each node, where  $r_2 = 2r_1$ . We raise the following questions:

- could the energy of all nodes be perfectly balanced by the two-hop based mixed data transmission?
- what are the fundamental limits of the two-hop based mixed data transmission on energy balance?

By answering these questions, we are able to understand to what extents of energy balance that a specific network can achieve; moreover, a designer could acquire if the performance meets the application requirements at early stages.

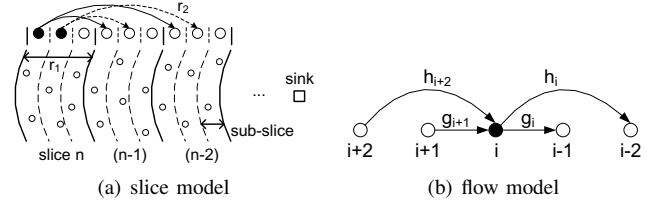


Fig. 1. Models for the two-hop based mixed data transmission.

## III. ANALYTICAL MODEL

We first study these problems in 1-D networks as shown in Fig. 1. We assume that sensors are uniformly distributed and the sink is located at one end of the chain. In case of a middle-placed sink, the network can be splitted into two chains with the common sink. We consider a randomly generated data flow with the rate of  $k$ .

The network area is equally partitioned into  $n$  slices with the width of  $r_1$  (Fig. 1(a)). Data from the slice  $i$  is randomly transmitted to the slice  $(i-1)$  or  $(i-2)$ , to achieve the inter-slice energy balance. Furthermore, for the sake of intra-slice energy balance, each slice is divided into  $m$  sub-slices. Data is only allowed to be transmitted between the corresponding sub-slices of two slices as shown in Fig. 1(a).

The flow model of the network is presented in Fig. 1(b). Let  $g_i$  and  $h_i$  represent the rate of data transmitted from the slice  $i$  to  $(i-1)$  and  $(i-2)$ , respectively. For the sake of simplicity, we denote  $E[g_i]$  ( $E[h_i]$ ) as  $G_i$  ( $H_i$ ) in the rest of the paper. Define  $F_i$  as the average data receiving rate of the slice  $i$ , apparently,  $F_i = G_{i+1} + H_{i+2}$ .

Let  $\varepsilon_i$  denote the average energy consumption rate of all the nodes in the slice  $i$ . With the prerequisite of perfect energy balance,  $\varepsilon_i = \varepsilon_j$  holds for any two slices, i.e.,  $i$  and  $j$ . Then, according to energy balance and flow conservation, we have

$$F_i - (\alpha + 1)F_{i-1} + \alpha F_{i-2} = \alpha k, \quad i = 2, 3, \dots, n,$$

with  $F_0 = nk$  and  $F_n = 0$ , where  $\alpha = \frac{c_2 - c_1}{c_0 + c_1}$ ,  $c_0$  is the receiving power of the radio and  $c_1$  ( $c_2$ ) is the transmitting power corresponding to  $r_1$  ( $r_2$ ). This is a fundamental recurrence relation for the flows in an energy-balanced network.

By solving the above recurrence relation, we could obtain the expressions of  $F_i$  and  $G_i$  with the following constraints:

- 1)  $G_i \leq F_i + k, \quad i = 1, 2, \dots, n,$
- 2)  $G_i \geq 0, \quad i = 1, 2, \dots, n,$
- 3)  $F_i \geq 0, \quad i = 1, 2, \dots, n,$

TABLE I  
TYPICAL VALUES OF  $\alpha$ .

	$r_1$	$r_2$	$c_1$	$c_2$	$c_0$	$\alpha$
CC2420	5	10	8.5	9.9	18.8	0.0513
	10	20	9.9	15.2	18.8	0.1847
	5	20	8.5	15.2	18.8	0.2454
$d^2$ model	5	10	2.55	10.05	0.05	2.8846
	10	20	10.05	40.05	0.05	2.9703
	5	20	2.55	40.05	0.05	14.4231
$d^4$ model	5	10	50.0008	50.0130	50	0.000122
	10	20	50.0130	50.2080	50	0.001950
	5	20	50.0008	50.2080	50	0.002072

where constraint 1 is guaranteed by the flow conservation at each slice. Thus, according to the other two constraints, we have the following lemma.

*Lemma 1:*  $G_i$ 's and  $F_i$ 's are valid energy-balanced flows if and only if  $G_n \geq 0$ .

#### IV. PRELIMINARY RESULTS

In this section, we present the sufficient and necessary condition of energy balance, and address the fundamental limits of the mixed data transmission scheme.

*Theorem 1:* In a uniform 1-D network, the perfect energy balance can only be achieved iff  $n \leq n_0$ , where  $n$  is the slice number,  $n_0 = \max \{n | n \in \mathbb{N} \wedge G_n \geq 0\}$ .

The proof of the theorem is omitted here due to the limited space. We demonstrate the correctness of the theorem by the following discussion. First, according to the mathematical properties of  $G_n$ , we have  $G_n > 0$  while  $n = 1$ ;  $G_n$  first grows with  $n$  and then decreases to a negative number. Therefore, there must exist  $n_0$  satisfying the conditions:  $G_n \geq 0$  while  $n \leq n_0$ , and  $G_n < 0$  while  $n > n_0$ . Here,  $n_0$  indicates the largest area within which the energy of all nodes can be balanced. We call  $n_0$  as the energy balance bound of the two-hop based mixed data transmission.

On the other hand, the optimal probability of the one-hop ( $r_1$ ) transmission at each slice can be derived from the expressions of  $G_i$  and  $F_i$ . We prove that under the condition of  $n \leq n_0$ , the energy of all nodes can be perfectly balanced with the optimal transmission probabilities. Therefore, the condition in theorem 1 is sufficient and necessary.

It can be proved that the perfect energy balance can always be achieved if  $n = 2$ . This provides a baseline for  $n_0$ . From the definition of  $n_0$ , we are able to obtain its expression:  $n_0 = 2$  while  $0 < \alpha < 1$ ;  $n_0 = 3$  while  $1 \leq \alpha < 2$ ; and  $n_0 = \left\lfloor \frac{1}{\alpha+1} \left( \alpha^2 - \frac{1}{\alpha-1} \right) \right\rfloor$  while  $\alpha \geq 2$ . Then, as a result,  $n_0$  is upper bounded by  $\lfloor \alpha + 2 \rfloor$ .

We find that  $n_0$  only depends on  $\alpha$ , which is the ratio of  $(c_2 - c_1)$ , the extra power consumption has been introduced, and  $(c_0 + c_1)$ , the benchmark power profile which provides a basic communication cost of the network. This implies that the limits of the two-hop based mixed data transmission on energy balance are only determined by the power profiles of the communication system. We define  $\alpha$  as the premium power

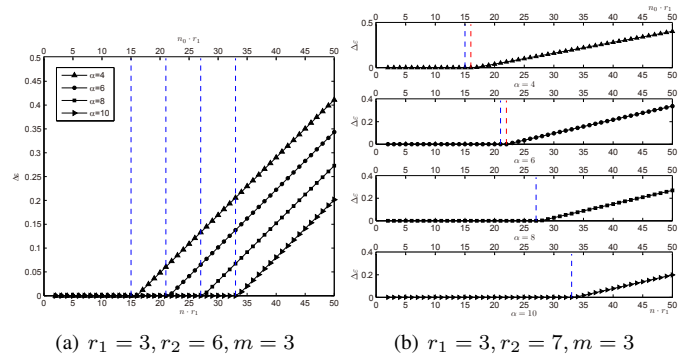


Fig. 2. Energy balance status of 1-D networks with different size.

ratio of the system. In particular,  $\alpha$  is independent of any radio platforms and communication models. Therefore, the above analytical results can be easily extended to any network models. The typical values of  $\alpha$  are listed in TABLE I.

#### V. EVALUATION

Our analytical results are validated through extensive simulations. We construct a linear programming model for the two-hop based mixed data transmission and solve it in LINGO11. The largest size ( $\hat{n}_0$ ) of a perfectly energy-balanced 1-D network is deduced from the optimal solution of the model and then compared with our theoretical prediction results ( $n_0$ ).

The preliminary evaluation results are demonstrated in Fig. 2, where  $n_0$ 's and  $\hat{n}_0$ 's are marked differently with blue and red dash lines.  $\Delta\varepsilon = \max\{\varepsilon_i - \varepsilon_j\}$ , and  $\Delta\varepsilon = 0$  indicates that the perfect energy balance is achieved. As shown in Fig. 2,  $n_0$  is consistent with  $\hat{n}_0$  while  $r_2 = 2r_1$ . Even  $r_2 \neq 2r_1$ , our analytical model can accurately calculate the limits of the two-hop based mixed data transmission on energy balance.

#### VI. CONCLUSIONS AND FUTURE WORK

In this paper, we theoretically investigate the fundamental limits of the two-hop based mixed data transmission on energy balance. We have proposed an analytical model and derived the energy balance bound of the scheme, which is verified by the simulation results. In our future work, we will extend the analysis to a 2-D random network and study the suboptimal energy balance problem in networks with  $n$  greater than  $n_0$ .

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