理论作业三

一、多层感知机

h接受的输入是:

$$\begin{cases} h_1 = w_{11}x_1 + w_{13}x_2 + b_1 = 0.3232 \\ h_2 = w_{12}x_1 + w_{14}x_2 + b_1 = 0.3232 \end{cases}$$

经过激活函数后:

$$egin{cases} a_1 = sigmoid(h_1) = 0.58010392 \ a_2 = sigmoid(h_2) = 0.58010392 \end{cases}$$

故u得到的值是:

$$\begin{cases} y_1 = w_{21}a_1 + w_{23}a_2 + b_2 = 1.209109116 \\ y_2 = w_{22}a_1 + w_{24}a_2 + b_2 = 1.093088332 \end{cases}$$

此时损失函数:

$$L = loss = rac{1}{2}((y_1 - label_1)^2 + (y_2 - label_2)^2) = 0.682004107$$

$$\begin{cases} rac{\partial L}{\partial y_1} = y_1 - label_1 = 1.159109116 \\ rac{\partial L}{\partial y_2} = y_2 - label_2 = 0.143088332 \end{cases}$$

首先更新第二层的权重参数:

$$egin{aligned} rac{\partial L}{\partial w_{2j}} &= rac{\partial L}{\partial y_i} \cdot rac{\partial y_i}{\partial w_{2j}} \ &= (y_i - label_i) \cdot a_k \ i &= egin{cases} 1 & if \ (j = 1, 3) \ 2 & if \ (j = 2, 4) \end{cases}, \quad k &= egin{cases} 1 & if \ (j = 1, 2) \ 2 & if \ (j = 3, 4) \end{cases} \end{aligned}$$

$$egin{aligned} rac{\partial L}{\partial b_2} &= rac{\partial L}{\partial y_1} \cdot rac{\partial y_1}{\partial b_2} + rac{\partial L}{\partial y_2} \cdot rac{\partial y_2}{\partial b_2} \ &= rac{\partial L}{\partial y_1} + rac{\partial L}{\partial y_2} \end{aligned}$$

故:

$$\begin{cases} \frac{\partial L}{\partial w_{21}} = (y_1 - label_1) \cdot a_1 = 0.672403742 \\ \frac{\partial L}{\partial w_{22}} = (y_2 - label_2) \cdot a_1 = 0.083006102 \\ \frac{\partial L}{\partial w_{23}} = (y_1 - label_1) \cdot a_2 = 0.672403742 \\ \frac{\partial L}{\partial w_{24}} = (y_2 - label_2) \cdot a_2 = 0.083006102 \\ \frac{\partial L}{\partial b_2} = (y_1 - label_1) + (y_2 - label_2) = 1.302197448 \end{cases}$$

所以第二层权重应更新为:

$$\left\{egin{aligned} &w_{21}^* = w_{21} - lpha rac{\partial L}{\partial w_{21}} = 0.1638 \ &w_{22}^* = w_{22} - lpha rac{\partial L}{\partial w_{22}} = 0.4085 \ &w_{23}^* = w_{23} - lpha rac{\partial L}{\partial w_{23}} = 0.2138 \ &w_{24}^* = w_{24} - lpha rac{\partial L}{\partial w_{24}} = 0.3585 \ &b_2^* = b_2 - lpha rac{\partial L}{\partial b_2} = -0.0511 \end{aligned}
ight.$$

现在更新第一层权重。已知sigmoid函数的导数是:

$$sigmoid'(h) = rac{d}{dh}rac{1}{1+e^{-h}} = rac{e^{-h}}{(1+e^{-h})^2} = sigmoid(h) \cdot (1-sigmoid(h))$$

以 w_{11} 为例,其通过 $h_1 o a_1$ 传播,之后同时通过 y_1,y_2 传播。因此:

$$\begin{split} \frac{\partial L}{\partial w_{11}} &= \frac{\partial L}{\partial y_1} \frac{\partial y_1}{\partial w_{11}} + \frac{\partial L}{\partial y_2} \frac{\partial y_2}{\partial w_{11}} \\ &= \frac{\partial L}{\partial y_1} \frac{\partial y_1}{\partial a_1} \frac{\partial a_1}{\partial h_1} \frac{\partial h_1}{\partial w_{11}} + \frac{\partial L}{\partial y_2} \frac{\partial y_2}{\partial a_1} \frac{\partial a_1}{\partial h_1} \frac{\partial h_1}{\partial w_{11}} \\ &= \left(\frac{\partial L}{\partial y_1} \frac{\partial y_1}{\partial a_1} + \frac{\partial L}{\partial y_2} \frac{\partial y_2}{\partial a_1} \right) \frac{\partial a_1}{\partial h_1} \frac{\partial h_1}{\partial w_{11}} \end{split}$$

这样不妨记
$$\frac{\partial L}{\partial a_i} = \frac{\partial L}{\partial y_1} \frac{\partial y_1}{\partial a_i} + \frac{\partial L}{\partial y_2} \frac{\partial y_2}{\partial a_i}, i = 1, 2$$
。对这个整体计算如下:

$$\begin{cases} \frac{\partial L}{\partial a_1} = \frac{\partial L}{\partial y_1} \frac{\partial y_1}{\partial a_1} + \frac{\partial L}{\partial y_2} \frac{\partial y_2}{\partial a_1} \\ = w_{21}(y_1 - label_1) + w_{22}(y_2 - label_2) \\ = 0.643944307 \\ \frac{\partial L}{\partial a_2} = \frac{\partial L}{\partial y_1} \frac{\partial y_1}{\partial a_2} + \frac{\partial L}{\partial y_2} \frac{\partial y_2}{\partial a_2} \\ = w_{23}(y_1 - label_1) + w_{24}(y_2 - label_2) \\ = 0.694745347 \end{cases}$$

另外计算 $\frac{\partial a_i}{\partial h_i}$:

$$egin{cases} rac{\partial a_1}{\partial h_1} = a_1(1-a_1) = 0.243583362 \ rac{\partial a_2}{\partial h_2} = a_2(1-a_2) = 0.243583362 \end{cases}$$

据此计算 $\frac{\partial L}{\partial h_i} = \frac{\partial L}{\partial a_i} \frac{\partial a_i}{\partial h_i}$:

$$\begin{cases} \frac{\partial L}{\partial h_1} = \frac{\partial L}{\partial a_1} \frac{\partial a_1}{\partial h_1} = 0.156854119\\ \frac{\partial L}{\partial h_2} = \frac{\partial L}{\partial a_2} \frac{\partial a_2}{\partial h_2} = 0.169228407 \end{cases}$$

所以:

$$\begin{cases} \frac{\partial L}{\partial w_{11}} = \frac{\partial L}{\partial h_1} \frac{\partial h_1}{\partial w_{11}} = 0.012548329 \\ \frac{\partial L}{\partial w_{12}} = \frac{\partial L}{\partial h_2} \frac{\partial h_2}{\partial w_{12}} = 0.013538272 \\ \frac{\partial L}{\partial w_{13}} = \frac{\partial L}{\partial h_1} \frac{\partial h_1}{\partial w_{13}} = 0.018822494 \\ \frac{\partial L}{\partial w_{14}} = \frac{\partial L}{\partial h_2} \frac{\partial h_2}{\partial w_{14}} = 0.020307409 \end{cases}$$

所以:

$$egin{cases} w_{11}^* = w_{11} - lpha rac{\partial L}{\partial w_{11}} = 0.2337 \ w_{12}^* = w_{12} - lpha rac{\partial L}{\partial w_{12}} = 0.2932 \ w_{13}^* = w_{13} - lpha rac{\partial L}{\partial w_{13}} = 0.1906 \ w_{14}^* = w_{14} - lpha rac{\partial L}{\partial w_{14}} = 0.1498 \end{cases}$$

对于偏置项 b_1 , 其传播路径如下:

$$egin{aligned} b_1
ightarrow egin{cases} h_1
ightarrow a_1 \ h_2
ightarrow a_2 \end{pmatrix}
ightarrow egin{cases} y_1 \ y_2 \end{pmatrix}
ightarrow L \end{aligned}$$

所以:

$$\begin{split} \frac{\partial L}{\partial b_1} &= \frac{\partial L}{\partial y_1} \frac{\partial y_1}{\partial b_1} + \frac{\partial L}{\partial y_2} \frac{\partial y_2}{\partial b_1} \\ &= \frac{\partial L}{\partial y_1} \left(\frac{\partial y_1}{\partial a_1} \frac{\partial a_1}{\partial b_1} + \frac{\partial y_1}{\partial a_2} \frac{\partial a_2}{\partial b_1} \right) \\ &\quad + \frac{\partial L}{\partial y_2} \left(\frac{\partial y_2}{\partial a_1} \frac{\partial a_1}{\partial b_1} + \frac{\partial y_2}{\partial a_2} \frac{\partial a_2}{\partial b_1} \right) \\ &= \frac{\partial L}{\partial y_1} \left(\frac{\partial y_1}{\partial a_1} \frac{\partial a_1}{\partial b_1} \frac{\partial h_1}{\partial b_1} + \frac{\partial y_1}{\partial a_2} \frac{\partial a_2}{\partial b_2} \frac{\partial h_2}{\partial b_1} \right) \\ &\quad + \frac{\partial L}{\partial y_2} \left(\frac{\partial y_2}{\partial a_1} \frac{\partial a_1}{\partial h_1} \frac{\partial h_1}{\partial b_1} + \frac{\partial y_2}{\partial a_2} \frac{\partial a_2}{\partial h_2} \frac{\partial h_2}{\partial b_1} \right) \\ &= \left(\frac{\partial L}{\partial y_1} \frac{\partial y_1}{\partial a_1} + \frac{\partial L}{\partial y_2} \frac{\partial y_2}{\partial a_1} \right) \frac{\partial a_1}{\partial b_1} \frac{\partial h_1}{\partial b_1} \\ &\quad + \left(\frac{\partial L}{\partial y_1} \frac{\partial y_1}{\partial a_2} + \frac{\partial L}{\partial y_2} \frac{\partial y_2}{\partial a_2} \right) \frac{\partial a_2}{\partial h_2} \frac{\partial h_2}{\partial b_1} \\ &= \frac{\partial L}{\partial a_1} \frac{\partial a_1}{\partial h_1} \frac{\partial h_1}{\partial b_1} + \frac{\partial L}{\partial a_2} \frac{\partial a_2}{\partial h_2} \frac{\partial h_2}{\partial b_1} \\ &= \frac{\partial L}{\partial a_1} \frac{\partial a_1}{\partial h_1} + \frac{\partial L}{\partial a_2} \frac{\partial a_2}{\partial h_2} \\ &= \frac{\partial L}{\partial h_1} + \frac{\partial L}{\partial h_2} \\ &= 0.156854119 + 0.169228407 \\ &= 0.326082526 \end{split}$$

所以 b_1 应更新为:

$$b_1^* = b_1 - \alpha \frac{\partial L}{\partial b_1} = 0.1170$$

综上所述,一次反向传播后,权重和偏置更新为:

$$\left\{ egin{aligned} &w_{11}^*=0.2337 \ &w_{12}^*=0.2932 \ &w_{13}^*=0.1906 \ &w_{14}^*=0.1498 \ &b_1^*=0.1170 \end{aligned}
ight. \,, \left\{ egin{aligned} &w_{21}^*=0.1638 \ &w_{22}^*=0.4085 \ &w_{23}^*=0.2138 \ &w_{24}^*=0.3585 \ &b_2^*=-0.0511 \end{aligned}
ight.$$

\blacksquare , MDP

1.

轨迹一:

$$\mathcal{R}eward = R(科目一) + \gamma R(科目二) + \gamma^2 R(科目三) + \gamma^3 R(通过) + \gamma^4 R(睡觉)$$

= $-2 + 0.5 \times (-2) + 0.5^2 \times (-2) + 0.5^3 \times (10) + 0.5^4 \times (0)$
= -2.25

轨迹二:

$$\mathcal{R}eward = R(科目一) + \gamma R(玩手机) + \gamma^2 R(玩手机)$$

+ $\gamma^3 R(科目一) + \gamma^4 R(科目二) + \gamma^5 R(睡觉)$
= $-2 + 0.5 \times (-1) + 0.5^2 \times (-1) + 0.5^3 \times (-2) + 0.5^4 \times (-2) + 0.5^5 \times (0)$
= -3.125

2.

$$egin{aligned} V &= E(E(U_t)) \ &= \sum p_i (R_{t,i} + \gamma V_{t_1,i}) \ &= 0.6 imes (-2 + 1 imes 10) + 0.4 imes (-2 + 1 imes (-8.5)) \ &= 0.6 \end{aligned}$$