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A Genetic Algorithm for a 2D Industrial Packing Problem

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Cutting and packing problems are encountered in many industries, with different industries incorporating different constraints and objectives. The wood-, glass- and paper industry are mainly concerned with the cutting of regular figures, whereas in the ship building, textile and leather industry irregular, arbitrary shaped items are to be packed. In this paper two genetic algorithms are described for a rectangular packing problem. Both GAs are hybridised with a heuristic placement algorithm, one of which is the well-known Bottom-Left routine. A second placement method has been developed which overcomes some of the disadvantages of the Bottom-Left rule. The two hybrid genetic algorithms are compared with heuristic placement algorithms. In order to show the effectiveness of the design of the two genetic algorithms, their performance is compared to random search.

Keywords: two-dimensional orthogonal packing problem, nesting, combinatorial optimisation, genetic algorithms, random search, heuristics, simulation

INTRODUCTION

Packing problems are optimisation problems that are concerned with finding a *good* arrangement of multiple *items* in larger containing regions (*objects*). The usual objective of the allocation process is to maximise the material utilisation and hence to minimise the "wasted" area. This is of particular interest to industries involved with mass-production as small improvements in the layout can result in savings of material and a considerable reduction in production costs. The development of an algorithm to solve an industrial packing problem clearly must consider the complexity of the problem, determined by the geometry of the objects and the constraints imposed. In addition the algorithm must be easy to adapt to the present competitive market with frequent product introductions, changing product designs and "shorter time to market" strategy. The flexibility achieved by manual packing is no longer a competitive solution due to high labour and liability costs. Conventional automated packing methods do not offer this flexibility, since they are mostly tailored to a particular packing task (Hinxman, 1980; Sarin, 1983; Hässler and Sweeney, 1991). This calls for a new approach to packing problems, which implements automation, but also maintains the flexibility which is offered by manual composition of packing layouts. One of the main aspects in the development of flexible packing systems is the integration of intelligent search processes in order to find good packing patterns. Intelligent search processes such as genetic algorithms are highly flexible since they describe the packing problem in the form of general search principles rather than a set of special placement rules.

Our work is concerned with a two-dimensional packing problem frequently encountered in the wood-, glass- and paper industry. The problem consists of packing rectangular items onto a rectangular object while minimising the used object space. The packing process has to ensure that there is no overlap between the items, which are allowed to rotate by 90°. So far only a few researchers have applied genetic algorithms to this problem type. Genetic algorithms for packing problems mainly concentrate on guillotineable packing problems (Kröger, 1995; András, 1996) and bin-packing (Hwang, 1992; Falkenauer, 1994). Smith (1985) developed an order-based genetic for a rectangle packing problem, where the orientation of the items is fixed. The genetic algorithm by Kröger et al. (1991) includes rotation and is based on a tree structure to encode the problem. Since its performance is compared to well-known packing heuristics, a relative comparison with our technique is possible.

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In this paper two hybrid genetic algorithms are introduced that are combined with two different heuristic routines to decode the genotype. The first technique has been developed by Jakobs (1996) and is used for comparison purposes, since it has been demonstrated on similar problems to ours and also uses order-based chromosome structure. For the second genetic algorithm the placement routine has been modified to overcome certain drawbacks. Results indicate performance of the hybrid genetic algorithm is strongly dependent on the nature of the placement routine.

GENETIC ALGORITHMS

Genetic algorithms are a relatively new technique with respect to the packing industry (Hopper and Turton, 1998), however they have already been used successfully in a vast number of industrial applications such as structural design, planning of telecommunication networks, electronic circuit design and pattern recognition. They utilise search and optimisation procedures that operate in a similar way to the evolutionary processes observed in nature. The search is guided towards improvement using the "survival of the fittest principle". This is achieved by extracting the most desirable features from a generation of solutions and combining them to form the next generation. The quality of each solution is evaluated and the "fitter" individuals are selected for the reproduction process. Continuation of this process through a number of generations will result in optimal or near-optimal solutions. Further theoretical and practical details can be found in (Davies, 1991; Goldberg, 1989).

HYBRID GENETIC ALGORITHMS FOR 2D RECTANGULAR PACKING PROBLEM

A common feature found in most genetic algorithms developed for packing problems is their two-stage approach, where a genetic algorithm is used to explore and manipulate the solution space. Since the solutions generated by the genetic algorithm do not show the admissibility and the quality of the packing scheme they represent, a second ("non-genetic") procedure is needed to decode the solutions generated by the genetic algorithm into the corresponding packing patterns. A solution to the packing problem in this case consists of a sequence of integer numbers indicating the order, in which the rectangles are placed onto the object. The exact location in the layout is then determined by a placement routine. The two placement algorithms that have been combined with the order-based genetic algorithm are described below. They are both designed such that any partial layout solution satisfies the non-overlap constraint.

Placement Algorithms

In Bottom-Left packing (BL) each item is moved as far as possible to the bottom of the object and then as far a possible to the left (Jakobs, 1996). A valid position is found when the rectangle collides with the partial layout at its lower and left side. Figure 1 shows the placement of a sequence of rectangles which is described by the permutation (2, 6, 4, 3, 0, 1, 5). The major disadvantage of the BL-routine consists of the creation of empty areas in the layout, when larger items block the movement of successive ones. In order to overcome this drawback a placement algorithm (Bottom-Left-Fill algorithm, BLF) has been developed that allows to place each item at the lowest available position of the object. Since the generation of the layout with the BLF algorithm is based on the allocation of the lowest sufficiently large region in the partial layout rather than on a series of bottom-left moves, it is capable of filling existing 'gaps'.

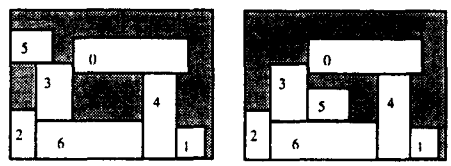

Fig.:1: BL-rule (left) and BLF-rule (right)

Genetic Algorithm

The quality of the layout which is constructed using the above placement algorithms depends on the sequence in which the rectangles are presented to the routine. Since the number of combinations is too large to be explored exhaustively in reasonable amount of time, a genetic algorithm has been used as a more efficient search strategy. Hence the packing task is split been the genetic algorithm, which searches for a good ordering and the placement routine, which interprets the permutation and evaluates its quality. Since an order-based encoding is used for this problem, care has to be taken that valid chromosomes are generated during the cross-over and mutation operations. Partially matched cross-over (PMX) (Goldberg, 1989) and order-based mutation (Syswerda, 1991) are suitable for this type of encoding and have been used in this case. Rotation of the rectangles is permitted in this packing problem. In order to allow the genetic algorithm to explore the orientation of the items a second mutation operator is used which changes the orientation of each rectangle in the sequence with a certain probability. The quality of a packing pattern is first of all determined by its height, since the unused rectangular area can be re-used. However, the variable height is not sufficient to express how tight the items are packed. For the fitness function a weighted sum has been used considering height with 70% and packing density with 30%. Further techniques that have been implemented in the genetic algorithm include elitism and seeding (Goldberg, 1989). Since heuristic placements with pre-ordered input sequences can yield to packing patterns, whose quality can lies above average (Coffman et al., 1984), the initial population has been seeded with the permutation, which describes the rectangles sorted according to decreasing height.

SIMULATION

Performance of the two genetic algorithms has been tested with five problems, which belong to the class of non-guillotineable packing problems (Dyckhoff, 1990). The problem consists a fixed number of items, which are packed onto a single object of fixed width and unlimited height. The problems have been constructed such that (at least) one optimal solution is known. The genetic algorithm has been combined with each of the placement routines in turn and simulated over 1000 generations, each consisting of 50 individuals. In order to establish the efficiency of the search process carried out by the GA, Random Search has been applied over the same amount of iterations (i.e. 50000). The results presented below are the average of 10 simulations, showing the current best average solution. The heuristic methods have been run 50 times with and without pre-ordering. DH denotes an input sequence in which the items are sorted by decreasing height respectively width (DW).

RESULTS AND DISCUSSION

The comparison of the two genetic algorithms shows that the combination with the BLF placement rule achieves layouts with lower packing height on average. Both algorithms achieve the highest performance gain within the first 2000 generations (Fig. 2). With increasing problem size the difference between the best solution and the optimum height increases for the GA with the BL rule (up to 25%). The GA using the BLF rule, however, achieves packing heights which are less than 10% from the optimum regardless of the problem size (Table 1). Since the BLF routine attempts to first fill the gaps in the layout, the majority of the small items will be "absorbed" within the existing partial layout and not contribute further to the packing height, which is mainly determined by the larger items (Fig 4). With the BL-rule, however, unused regions in layout cannot be accessed and smaller rectangles also contribute to the height. Fig. 3 and 4 show initial and best layouts for both GAs.

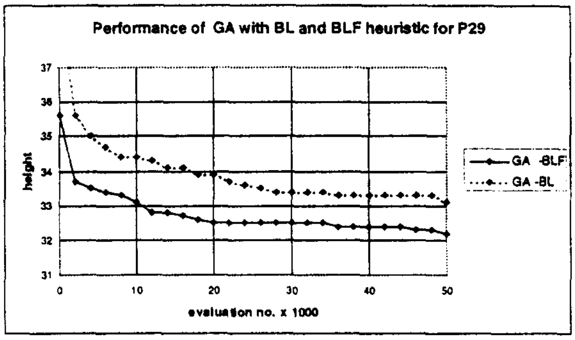

Fig. 2: Performance of both GAs for problem P29

Table 1: Packing heights achieved by the genetic and heuristic algorithms with random and ordered input

	P25	P29	P49	P97	P197
Optimum	15	30	60	120	240
GA + BL	17	33	66	137	298
GA + BLF	16	32	63	127	252
BL	23	50	107	193	411
BLF	20	43	87	148	293
BL - DH	18	44	74	144	290
BLF - DH	18	33	65	125	248
BL - DW	20	39	79	155	297
BLF - DW	18	35	65	125	250

The results obtained from the hybrid genetic algorithms are between 15 and 60% better compared to the performance of the heuristic methods on their own (BL and BLF in Table 1). The performance gains for the GA + BL combination are higher than for the hybrid GA that has been combined with the BLF routine. Pre-ordering the input to the heuristics routines improves the outcome. In particular sorting the sequence of rectangles in terms of height yields to better packs. In comparison the parallel GA developed by Kröger et al. (1991), whose data structure is based on a graph, achieved layouts that where approximately 10% better than those obtained by the BL heuristic.

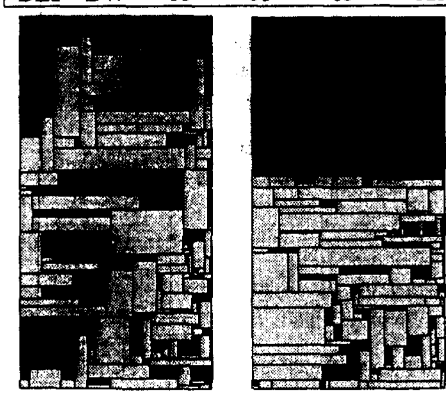

Fig. 3: Initial and best layout for GA + BL (P97)

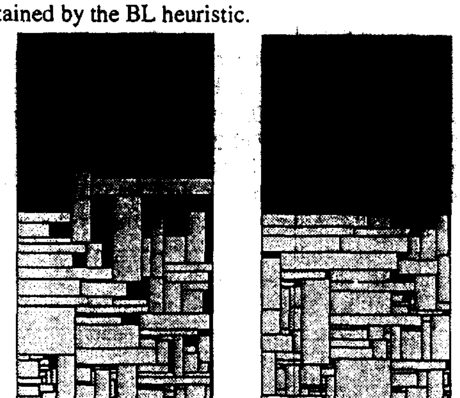

Fig. 4: Initial and best layout for GA + BLF (P97)

In order to establish how well the genetic algorithm explores the search space a Random Search process has been applied to the packing problems. The heuristic methods have been evaluated using 50000 random input sequences (i.e. number of iterations in one GA run). As it can be seen in Fig. 5 the genetic algorithm performs better in both cases. Whereas the random search only 'explores' the solution space, the in-built search mechanisms allow the genetic algorithm to 'exploit' good regions. The difference between the GA and Random Search for the BLF case is smaller, which indicates that the 'exploitation' of the solution space is limited. Since the packing heights achieved with the BLF on its own are already very close to the optimum height, the GA cannot improve the performance of the heuristic as much as in the BL case. In other words by using the 'poorer' BL-decoder, the GA is needed to find a good input sequence, whereas applying the better BLF-heuristic layouts are achieved only in a fifth of the number of iterations. This questions the use of a GA combined with a 'poor' decoder for this type of packing problem. In order to achieve high quality layouts, which is the industrial objective, the improved BLF heuristic is sufficient applied over a small number of iterations.

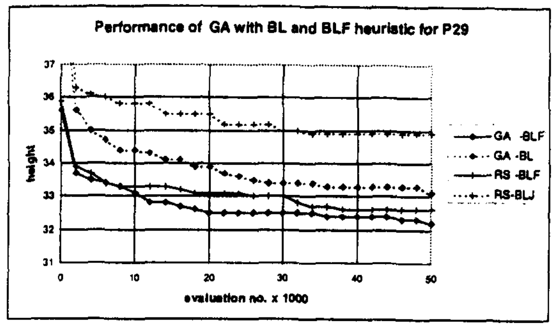

Fig. 5: Performance of GAs and Random Search (P29)

CONCLUSIONS

Two hybrid genetic algorithms for the rectangle packing problem have been introduced. Whereas the first technique uses a well-known heuristic from literature (BL), for the second one an improved version of this heuristic has been developed (BLF). The hybrid GAs as well as their heuristic algorithms have been tested on a number of packing problems. The genetic algorithm combined with the improved BLF heuristic outperforms the GA using the BL method as well as the heuristic methods. However, the difference between the hybrid GA (with the BLF routine) and the BLF heuristic is smaller than expected. Given time the genetic algorithm is able to find better layouts, although the improvements are only marginal. For industrial applications the improved heuristic is sufficient to achieve high quality layouts. Since the performance difference between the two hybrid GAs is only due to the improved heuristic, the decoder has a larger effect on the outcome of the hybrid technique than the GA. In order to improve the GA encoding schemes need to be studied that incorporate more layout specific knowledge.

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