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An iterated local search heuristic for multi-capacity bin packing and machine reassignment problems

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ABSTRACT

This paper proposes an efficient Multi-Start Iterated Local Search for Packing Problems (MS-ILS-PPs) metaheuristic for Multi-Capacity Bin Packing Problems (MCBPP) and Machine Reassignment Problems (MRP). The MCBPP is a generalization of the classical bin-packing problem in which the machine (bin) capacity and task (item) sizes are given by multiple (resource) dimensions. The MRP is a challenging and novel optimization problem, aimed at maximizing the usage of available machines by reallocating tasks/processes among those machines in a cost-efficient manner, while fulfilling several capacity, conflict, and dependency-related constraints. The proposed MS-ILS-PP approach relies on simple neighborhoods as well as problem-tailored shaking procedures. We perform computational experiments on MRP benchmark instances containing between 100 and 50,000 processes. Near-optimum multi-resource allocation and scheduling solutions are obtained while meeting specified processing-time requirements (on the order of minutes). In particular, for 9/28 instances with more than 1000 processes, the gap between the solution value and a lower bound measure is smaller than 0.1%. Our optimization method is also applied to solve classical benchmark instances for the MCBPP, yielding the best known solutions and optimum ones in most cases. In addition, several upper bounds for non-solved problems were improved.

1. Introduction

Emerging cloud computing platforms, such as Google Apps and Amazon EC2, are an effective way to support many kinds of Internet-based services (Armbrust et al., 2010). These platforms are deployed in large-scale server clusters (data centers) composed of several hundreds or thousands of inter-connected machines and allow hosting multiple individual services characterized by a set of interacting processes/tasks. The machines in these platforms provide different types of resources, such as computing (CPU), memory (RAM) and storage (DISK). The processes of a given service are allowed to run on those machines and consume their resources.

The problem of allocating a set of processes across a pool of server machines is basically a bin-packing problem where each of the machines is a bin. The goal is to allocate as many processes as possible into each bin, while respecting multiple capacity constraints. This problem is referred in the literature as the Multi-Capacity Bin Packing Problem (MCBPP), also known as the Vector Bin Packing Problem. In a more complex scenario, we consider the Machine Reassignment Problem (MRP) which consists of optimizing the usage of available machine resources by reallocating processes to different machines in a cost-efficient manner. An optimal reassignment decision must maximize load balancing/spreading objectives while satisfying resource capacity constraints with minimal migration costs of processes among the machines. This difficult optimization problem was originally proposed by Google for the 2012 ROADEF/EURO challenge (Roadef, 2012).

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Despite its relevance in today's data center applications, the specific characteristics of the MRP have not been adequately and extensively addressed in the literature. In fact, few resolution approaches are capable of handling very large size instances, such as those proposed by Google, that consist of several constraints related to service dependencies, spread and conflicts in order to meet load balancing and fault tolerance needs. Typically, the workload of multiple services in a cluster can change over time. This requires to periodically solve an optimization problem and use the solution to reconfigure the cluster. We refer to Petrucci et al. (2011) for an optimization control loop design and implementation in a real server cluster, which can benefit from the underlying optimization problem investigated and solved in this work.

In this paper, we propose a Multi-Start Iterated Local Search for Packing Problems (MS-ILS-PP) tailored for large-scale machine reassignment and packing problems with multiple resources. The method is based on the Iterated Local Search framework (Lourenço et al., 2010), which is known to be simple and successful on a wide range of combinatorial optimization problems (Derbel et al., 2012; Elbenani et al., 2012; Ferreira et al., 2012; Silva et al., 2012; Subramanian et al., in press). Our method iteratively applies a local-search procedure to improve the solutions, along with shaking moves to escape from local optima. To reduce the computational time, shaking moves and restarts are dynamically triggered whenever the improvement is estimated to be too small with respect to the total solution cost. Extensive experimental evaluations demonstrate the remarkable performance of the proposed method on MRP instances with up to 50,000 processes. The MCBPP is also addressed by iteratively decrementing the number of machines and solving a Lagrangian relaxation of the packing problem. The method performs very well on classical instances for this problem, provided by Caprara and Toth (2001). All the upper bounds and optimal solutions found by previous algorithms are retrieved, and several of them are improved.

The remainder of this paper is organized as follows. Section 2 formally defines the MCBPP and presents related works. The MRP definition and a mathematical model are given in Section 3, along with a simple approach for computing lower bounds used to assess the quality of the solutions produced by our method. Section 4 describes the proposed MS-ILS-PP metaheuristic. Section 5 is dedicated to the computational experiments, describing the benchmark instances used, parameters calibration and sensitivity analysis, and finally the results of MS-ILS-PP on the MRP and MCBPP. Conclusions and future directions are discussed in Section 6. A summary of the abbreviations and the notations used in the article are reported in Tables 6 and 7.

2. The multi-capacity bin packing problem

The MCBPP can be formally defined as follows. Let P be a set of objects and R a set of resources. Any object $p \in P$ is characterized by a vector of $|R|$ resource consumptions. A set of homogeneous bins is available to pack the objects. Let C_r be the bin capacity for a resource $r \in R$. The objective of the MCBPP is to pack the objects of P in as few bins as possible, without exceeding the capacity of the bins for any resource.

The literature on bin packing is wide, as well as the number of related applications. Two main families of multi-dimensional bin packing problems shall be discerned: those concerned with "geometrical packing" of objects in space, which are outside the scope of the present paper, and those concerned with "vector packing", where each object is simply characterized by some independent resource consumptions without any immediate notion of geometrical space.

A number of related papers specific to vector packing in the context of multiprocessor systems were proposed over the past 40 years. Some early heuristics were first introduced by Graham (1972) and Kou and Markowsky (1977). Concerning the MCBPP with two dimensions, Spieksma (1994) developed a branch-and-bound algorithm, while Caprara and Toth (2001) put forward exact and heuristic approaches as well as worst-case performance analysis. Kellerer and Kotov (2003) developed a $\mathcal{O}(n \log n)$ approximation algorithm with absolute worst-case performance ratio 2. Chang et al. (2005) modeled the real-life problem of packing steel products into special containers as a two dimensional MCBPP and proposed a heuristic algorithm.

For problems with more resources, the multicomputer task allocation as MCBPP was modeled by Beck (1996), and a comparison between 256 packing algorithms was conducted. Leinberger and Karypis (1999) addressed the MCBPP for application in parallel computing system. Finally, Epstein and Tassa (2003) proposed a general framework for MCBPP and studied the monotonicity of the objective function through problems that occurs in video transmission. A heuristic approach using set-covering formulation was proposed by Monaci and Toth (2006). Wilcox et al. (2011) developed a genetic algorithm to solve the MCBPP as a task placement problem for packing processes onto a cluster of servers. Panigrahy et al. (2011) studied heuristics for task placement problems inspired by FFD (first-fit decreasing) variants. Shachnai and Tamir (2012) derived approximation algorithms for data placement problem in the context of multimedia storage systems.

Surveying the literature, it is notable that very few advanced metaheuristics are currently available for the MCBPP. Instances of moderate size (less than 200 objects) have been most often addressed. We contribute to fill this methodological gap by proposing a simple and efficient metaheuristic which can efficiently deal with large-scale MCBPP and MRP instances with up to 50,000 objects.

3. The machine reassignment problem

The MRP is a novel optimization problem that integrates several optimization goals and constraints from other specific packing problems addressed in the literature, such as the MCBPP presented in Section 2. Several related heuristics have been proposed for the cost-efficient workload consolidation in server clusters. Doddavula et al. (2011) and Feller et al. (2011) studied the specific case of multidimensional resource aspect of the machine/task reassignment problem, showing improved resource utilization over simple greedy algorithms, such as FFD, commonly used in workload consolidation. Stillwell et al. (2012) proposed an optimization solution for the machine resource allocation considering both the multi-resource dimension and machine heterogeneity cases, which are combined to provide a single optimization algorithm. Their work basically extends existing machine/task reassignment packing approaches, traditionally used for the homogeneous and single resource cases. A heuristic based on column generation technique was developed by Kramer et al. (2012) in which the authors dealt with the heterogeneous machine reassignment case. Nevertheless, their optimization approach is limited to a single resource dimension (CPU capacity) variant of the machine reassignment/task packing problem.

In this work, we address the more general case of the machine reassignment problem which is based on the problem formulation proposed by Google (Roadef, 2012). More precisely, the MRP seeks to optimize the assignment of a set of *processes*, P, intended to run on a set M of heterogeneous *machines*. A set R of *resources* is considered, the *capacity* of a machine $m \in M$ for any resource $r \in R$ is noted $C_{m,r}$. Any process $p \in P$ *requires* $R_{p,r}$ units of each resource $r \in R$. The MRP assumes a feasible initial solution S_{init}, in which each process $p \in P$ is assigned to a machine $M_0(p)$. This initial

assignment, however, may be improved by moving some processes among machines while respecting several constraints. No time dimension is considered in the model and all processes are moved simultaneously without any time-processing delay. The MRP includes several constraints (Section 3.1), and objectives combined in a weighted sum function (Section 3.2). Given those constraints and objectives, we propose a mathematical formulation for the MRP (Section 3.3) and lower bound (LB) measure (Section 3.4).

3.1. Constraints

- A *capacity constraint* limits, for each resource r, the total resource consumption of the processes running on a machine m to $C_{m,r}$. Note that a subset $TR \subseteq R$ of resources is considered as *transient*. During the migration process from one machine to the other, such transient resources are consumed on both current and newly allocated machines.
- Let S be a set of *services*. Any process p belongs to a unique service. A *conflict constraint* then forbids two processes of the same service from being run on the same machine.
- Machines are grouped into *locations* and *neighborhoods* denoted by L and N, respectively. Each machine is associated to only one location and neighborhood. Each service $s \in S$ must respect a *spread constraint* that imposes the minimum number of distinct locations, given by SM_s, on which the processes of the service should run.
- Additionally, one service can be dependent upon other services. The set of services on which service $s \in S$ depends is denoted by D^s. The *dependency constraint* requires that if a given process of service s runs on a machine of neighborhood $n \in N$, at least one process of each service of D^s has to run on a machine that belongs to neighborhood n.

3.2. Objectives

- For each machine m and each resource r a safety capacity, $SC_{m,r}$, is defined. Then, the *load cost* is defined as the sum of excess resource consumptions, over $SC_{m,r}$, on all machines and resources.
- Another cost, called *balance cost*, penalizes unbalanced free resources within machines. Indeed, having a large quantity of a free resource $r_1 \in R$ on a machine, but no more free resource of another type, often results in wasted capacity. Let $FR(m,r)$ be the amount of free resource $r \in R$ on machine $m \in M$ w.r.t. to its capacity $C_{m,r}$. The balance cost is decomposed into a set B of cost components. Each such component $b \in B$ is associated with a cost $C_{BAL}(b) = \sum_{m \in M} \max\left(0, T^b FR(m, r_1^b) - FR(m, r_2^b)\right)$, where r_1^b and r_2^b are two resources, and T^b is a ratio.
- Any movement of a process p leads to a *process move cost* Φ_p. This move cost depends upon the process. Any process moved between a machine pair (m_1, m_2) leads to an additional *machine move cost* $\Phi_{m_1 m_2}$. Finally, *service move cost* proportional to the maximum number of moved processes from the same service is incurred.

3.3. Integer programming formulation

We introduce a mathematical formulation for the MRP in expressions (1)–(12), which may be solved to optimality for small/medium instances using standard solvers. This formulation relies on the binary decision variables $x_{m,p}$, taking value one if and only if the process $p \in P$ is operated on machine $m \in M$. Also, the binary variables $y_{l,s}$ take value one if and only if at least one process of service $s \in S$ is operated in location $l \in L$, and the binary variables $z_{n,p}$ indicate if process $p \in P$ is in neighborhood $n \in N$.

$$\begin{aligned}\text{Minimize } & \sum_{r \in R} \Phi_r \sum_{m \in M} \max\left\{0, \left(\sum_{p \in P} x_{p,m} R_{p,r}\right) - SC_{m,r}\right\} \\ & + \sum_{b \in B} \Phi_b \sum_{m \in M} \max\left\{0, T^b \left(C_{m,r_1^b} - \sum_{p \in P} x_{p,m} R_{p,r_1^b}\right) \right. \\ & \left. - \left(C_{m,r_2^b} - \sum_{p \in P} x_{p,m} R_{p,r_2^b}\right)\right\} + \sum_{p \in P} \sum_{m \in M \setminus M_0(p)} (\Phi_p + \Phi_{M_0(p),m}) x_{m,p} \\ & + \Phi_s \max_{s \in S}\left(\sum_{p \in s} \sum_{m \in M \setminus M_0(p)} x_{m,p}\right) \end{aligned} \tag{1}$$

$$\text{s.t. } \sum_{m \in M} x_{m,p} = 1 \qquad \forall p \in P \tag{2}$$

$$\sum_{p \in P} x_{m,p} R_{p,r} \leqslant C_{m,r} \qquad \forall m \in M,\ r \in R \setminus TR \tag{3}$$

$$\sum_{p \in P, M_0(p) \neq m} x_{m,p} R_{p,r} + \sum_{p \in P, M_0(p) = m} R_{p,r} \leqslant C_{m,r} \qquad \forall r \in TR,\ m \in M \tag{4}$$

$$\sum_{p \in s} x_{m,p} \leqslant 1 \qquad \forall m \in M,\ s \in S \tag{5}$$

$$\sum_{p \in s} \sum_{m \in l} x_{m,p} \geqslant y_{l,s} \qquad \forall l \in L,\ s \in S \tag{6}$$

$$\sum_{l \in L} y_{l,s} \geqslant SM_s \qquad \forall s \in S \tag{7}$$

$$z_{n,p} = \sum_{m \in n} x_{m,p} \qquad \forall n \in N,\ p \in P \tag{8}$$

$$z_{n,p} \leqslant \sum_{k \in s_b} z_{n,k} \qquad \forall n \in N,\ p \in P,\ s_b \in D^{s(p)} \tag{9}$$

$$x_{m,p} \in \{0,1\} \qquad \forall m \in M,\ p \in P \tag{10}$$

$$y_{l,s} \in \{0,1\} \qquad \forall l \in L,\ s \in S \tag{11}$$

$$z_{n,p} \in \{0,1\} \qquad \forall n \in N,\ p \in P \tag{12}$$

The objective function (1) aims at minimizing the weighted sum of the five MRP costs, presented in the same order as previously (Section 3.2). Following the description from Section 3.1, constraints (2) guarantee that each process is assigned to one machine. Constraints (3) state that the capacity of each machine for each resource must be respected. Constraints (4) enforce transient resources capacity constraints. Constraints (5) prevent two processes of the same service to be assigned to the same machine. Constraints (6) and (7) enforce the spread constraints. Constraints (8) and (9) enforce the dependency constraints.

3.4. A simple lower bound for the MRP

For the largest instances considered in this work (50,000 processes and 5000 machines), even computing the optimal value of the linear relaxation of Problem (1)–(12) is not practical. Thus, we introduce a simpler lower bound (LB) for the MRP which can be evaluated in linear time relatively to the problem size. This LB is an essential tool to assess the effectiveness of the proposed method. Let us first introduce Problem (P′):

$$\begin{aligned}(\mathrm{P}') \quad \text{Minimize } & \sum_{r \in R} \Phi_r \sum_{m \in M} \max\left\{0, \left(\sum_{p \in P} x_{p,m} R_{p,r}\right) - SC_{m,r}\right\} \\ & + \sum_{b \in B} \Phi_b \sum_{m \in M} \max\left\{0, T^b \left(C_{m,r_1^b} - \sum_{p \in P} x_{p,m} R_{p,r_1^b}\right) - \left(C_{m,r_2^b} - \sum_{p \in P} x_{p,m} R_{p,r_2^b}\right)\right\} \end{aligned} \tag{13}$$

$$\text{s.t. } \sum_{m \in M} x_{m,p} = 1 \qquad \forall p \in P \tag{14}$$

$$x_{m,p} \in \{0,1\} \quad \forall m \in M, p \in P \tag{15}$$

It can be observed that (P′) is a relaxation of the MRP in which the objectives considered are only the load cost and the balance cost. The conflict, dependency, capacity and spread constraints are relaxed. We also put forward the following relations:

$$\max\left\{0, \sum_{p\in P} R_{p,r} - \sum_{m\in M} SC_{m,r}\right\} = \max\left\{0, \sum_{m\in M}\left(\sum_{p\in P} x_{p,m}R_{p,r}\right) - SC_{m,r}\right\} \leqslant \sum_{m\in M} \max\left\{0, \left(\sum_{p\in P} x_{p,m}R_{p,r}\right) - SC_{m,r}\right\} \quad \forall r \in R \tag{16}$$

$$\max\left\{0, T^b\left(\sum_{m\in M} C_{m,r_{b_1}} - \sum_{p\in P} R_{p,r_{b_1}}\right) - \left(\sum_{m\in M} C_{m,r_{b_2}} - \sum_{p\in P} R_{p,r_{b_2}}\right)\right\} \leqslant \sum_{m\in M} \max\left\{0, T^b\left(C_{m,r_1^b} - \sum_{p\in P} x_{p,m}R_{p,r_1^b}\right) - \left(C_{m,r_2^b} - \sum_{p\in P} x_{p,m}R_{p,r_2^b}\right)\right\} \quad \forall b \in B \tag{17}$$

Relations (16) and (17) lead to the following LB for (P′). As (P′) is a relaxation of the MRP, the expression above is also a LB of the MRP. The LB measure is given as follows:

$$LB = \max\left\{0, \sum_{p\in P} R_{p,r} - \sum_{m\in M} SC_{m,r}\right\} + \max\left\{0, T^b\left(\sum_{m\in M} C_{m,r_{b_1}} - \sum_{p\in P} R_{p,r_{b_1}}\right) - \left(\sum_{m\in M} C_{m,r_{b_2}} - \sum_{p\in P} R_{p,r_{b_2}}\right)\right\} \tag{18}$$

4. Proposed MS-ILS-PP metaheuristic

This section describes the proposed MS-ILS-PP for machine reassignment and multi-capacity bin packing problems. The general description of the method is given in Sections 4.1–4.3, and some specificities related to its application to the MCBPP are discussed in Section 4.4.

4.1. General algorithm structure

The MS-ILS-PP metaheuristic combines a Local-Search (LS) improvement procedure with problem-tailored shaking moves and specialized restart procedures. The MRP assumes an initial feasible solution S_{INIT}, which can be directly used as starting point by the ILS. The stochastic nature of the method enables to explore different parts of the search space when applying restarts.

The pseudo-code of MS-ILS-PP is presented in Algorithm 1. In this algorithm, as well as in the remainder of this paper, we assume the availability of a pseudo-random number generator Alea() which produces uniformly distributed random real numbers in [0,1]. As described in lines 6–7, the local search is applied with equal probability on either the best solution, with a view of intensifying the search around good solution characteristics, or on the current solution to give more search opportunities. The resulting solution replaces the best known in case of improvement, and a shaking move is applied. Two types of alternating shaking moves are used. LS and shaking are applied as long as the improvement of the current best solution is significant, a *Restart* being triggered whenever the best solution improvement has been smaller than a threshold $\gamma_{RESTART}$ during the last five LS and shaking iterations. The method terminates once a maximum time limit T_{max} is reached, and returns the best overall solution.

Algorithm 1. Multi-start iterated local search for the MRP

1: $S_{BESTALL} \leftarrow S_{INIT}$
2: **while** time() $< T_{max}$ **do**
3: $S_{CURR} \leftarrow S_{INIT}$; $S_{BEST} \leftarrow S_{INIT}$ *//New start from the initial solution*
4: $It_{SHAK} = 0$ *//Counter for the number of shaking phases*
5: **while** the relative improvement of the best solution during the last 5 shaking phases is greater than $\gamma_{RESTART}\%$ **do**
6: **if** (*Alea*() < 0.5) *//50% chance to work on the current best solution*
7: **then** $S_{CURR} \leftarrow$ LocalSearch(S_{BEST})
8: **else** $S_{CURR} \leftarrow$ LocalSearch(S_{CURR})
9: **if** Cost(S_{CURR}) < Cost(S_{BEST}) **then** $S_{BEST} \leftarrow S_{CURR}$
10: **if** ($It_{SHAK}\%2 == 0$) *//Two alternating shaking moves*
11: **then** $S_{CURR} \leftarrow$ Shaking1(S_{CURR})
12: **else** $S_{CURR} \leftarrow$ Shaking2(S_{CURR})
13: **end while**
14: **if** Cost(S_{BEST}) < Cost($S_{BESTALL}$) **then** $S_{BESTALL} \leftarrow S_{BEST}$
15: **end while**
16: **return** $S_{BESTALL}$ *//Returning best overall solution*

4.2. Local-search procedure

The LS procedure consists in iteratively exhausting a set of moves and applying only improving ones. During each iteration of the LS, a subset of machines $\overline{M}$ is selected and, for every machine $m \in \overline{M}$, a subset of moves involving m is evaluated. The best move found on m is then applied. At each loop, the selection of machines in $\overline{M}$ is driven by their associated cost, e.g. the sum of the load and balance cost of processes on this machine. This selection is done by ranking the machines in decreasing order of cost, and then choosing the first $|M| \times [0.1 + 0.9 \times Alea()^2]$ machines in this order.

Algorithm 2. LS improvement procedure

1: **while** the relative solution improvement during the last 5 loops is greater than $\gamma_{SHAKING}\%$ **do**
2: Sort(Machine List)
3: fractionMachines $\leftarrow |M| \times [0.1 + 0.9 \times Alea()^2]$
4: **for all** $i \in \{1,\ldots,$fractionMachines$\}$ **do**
5: m $\leftarrow$ Machine List[i]
6: exploreRelocateMoves(m)
7: exploreSwapMoves(m)
8: $S_{CURR} \leftarrow$ performBestMove(S_{CURR})
9: **end for**
10: **end while**
11: **return** S_{CURR}

Two kinds of moves are considered in the LS. The first one is a relocate move that transfers a single process from machine m to another one. The second kind of move is a swap move that consists of interchanging one or two processes from machine m with one or two process from a distinct machine. After exploring the relocate and swap moves of a machine, the best improving move (if any) is performed. The general rationale of this LS is described in Algorithm 2. The LS continues as long as the total solution improvement during the last five loops is significant, i.e. larger than a threshold $\gamma_{SHAKING}\%$. A detailed description of each subset of moves is given below (illustrated in Fig. 1).

Relocate neighborhood. The relocate neighborhood includes all moves that consist of relocating a single process from $m \in M$ to a different machine. This neighborhood contains $|P_m^{sol}|(|M|-1)$ moves, where P_m^{sol} stands for the set of processes currently assigned to machine m in solution *sol*. As the number of processes and machines considered in the instances can be large (up to 50,000 processes and 5000 machines), exploring complete neighborhoods may be prohibitively slow. Therefore, only a random subset of this neighborhood is explored, in such a way that on average $\varphi/3$ moves are evaluated for a given machine, where φ is a parameter of the method.

Swap neighborhood. The swap neighborhood includes all moves that swap one or two processes from m with one or two processes from a different machine. Again, only a randomly-selected subset of φ moves from this neighborhood is explored. Note that the randomly-selected moves may be different from one LS iteration to another.

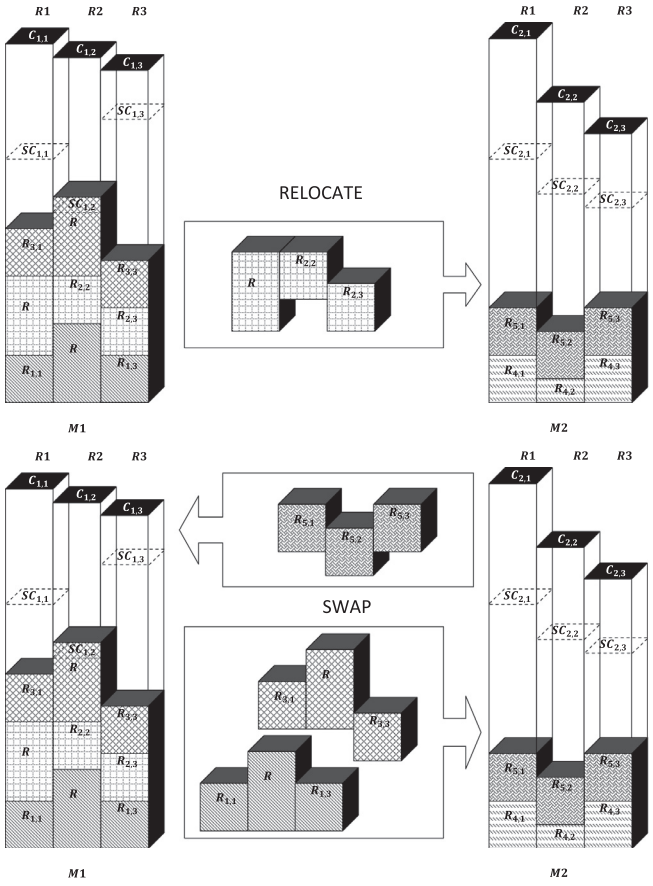


Fig. 1. Illustration of the relocate and exchange/swap move explored by our algorithm.

4.3. Shaking operators

Shaking operators are potentially-deteriorating moves which provide the means to escape from locally optimal solutions. Two shaking operators are considered in the proposed method, and called in turn. Note that only feasible shaking moves are applied.

The *Home Relocate* shaking operator selects randomly ξ processes that are currently not hosted on their initial machine, and relocates them to their initial machine. This move tends to be particularly efficient for reducing the process move costs in MRP context.

The *K-Swap* shaking operator randomly selects ξ times a pair of machines, and performs a swap of random groups of either 3, 4, or 5 processes, with equal probability, among these machines. These two operators modify the solution in such a way that it cannot be directly reversed by simply applying a single local-search move.

4.4. Addressing the MCBPP

This concise description highlights the simplicity of the method, contrasting with the apparent complexity of the MRP. In addition, since the MRP is a fairly general problem, it encompasses many packing settings as special cases, with multiple resources, potential heterogeneous bins, spread/conflict constraints. We adapted our methodology to address the MCBPP, which is highly relevant in the context of multiprocessors systems, to demonstrate the method capability in producing high-quality solutions on other classic problem variants.

In the MCBPP context, it is necessary to tackle the objective of bin minimization rather than process re-arrangement. We thus apply a *relax-and-decrement* strategy, in which the algorithm iteratively imposes the number of machines (i.e. bins) and works on a relaxed version of the MCBPP with linearly-penalized capacity violations. The resulting relaxed packing problem, for a fixed number of bins, is modeled as a MRP with safety capacities and load costs. A feasible packing is found with the current number of bins whenever the penalty reaches zero. This number is iteratively decremented until no zero-penalty packing can be found, and the minimum number of bins yielding a feasible solution is returned. This additional step is highlighted in gray on Fig. 2.

5. Computational experiments

Extensive computational experiments have been conducted to calibrate the search parameters, evaluate their impact, and analyze the performance and scalability of the approach on MRP and MCBPP benchmark instances of various sizes. This section describes the benchmark instances used for the tests (Section 5.1), presents the parameter calibration and sensitivity analyses (Section 5.3), and finally assesses the performance of the method on the MRP and MCBPP benchmark instances (Sections 5.4 and 5.5) with different run-time limits. The MS-ILS-PP algorithm has been coded in C++ and compiled using "g++-O3". All tests were conducted on an Opteron 2.4 GHz with 4 GB of RAM memory running Linux OpenSuse 11.1.

5.1. Benchmark instances

The benchmark dataset of the MRP considered in this work is composed of 3 sets (Sets A, B and X) of 10 instances that were used in the 2012 ROADEF Challenge (Roadef, 2012). The size of these instances ranges from 4 machines and 100 processes to 5000 machines and 50,000 processes. The instances of Set A tends to be of smaller size than the others. Sets B and X are larger, and very similar in nature.

The MCBPP instances used to evaluate the performance of our heuristic are taken from Spieksma (1994), Caprara and Toth (2001), and also addressed in Monaci and Toth (2006). A total of 10 different classes of instances are presented. Each class is composed of 40 instances, containing 10 instances of four different sizes. Classes 2, 3, 4, 5 and 8 are known to be easily solvable by simple greedy heuristics (Monaci and Toth, 2006), and thus we focus our experiments on the remaining classes, leading to a total of 200 instances. We refer to Caprara and Toth (2001) for a thorough description of the specificities of each class. In particular, the instances of Class 10 have been generated by *cutting* the bin resources into triplets of objects, in such a way that there is not a single unit of capacity slack in these solutions. Thus, for this class, the optimal solutions are in most cases known as a consequence of the instance generation process, but not a result of any bin packing algorithm.

5.2. Solution-quality measure for the MRP

Some MRP instances considered in this work may have arbitrarily low optimal costs. Thus, assessing the solution quality as a Gap(%) from the best solution cost would give too much importance to problems for which the expected final solution value is low, and, in some cases even, dividing per zero. We thus rely on another alternative, which involves measuring and comparing the *cost gains* relative to the initial solution S_{INIT}.

Define the *cost gain* $G(X)$ of a method X yielding a final solution $S_{FINAL}(X)$ as $G(X) = S_{INIT} - S_{FINAL}(X)$. Also, define the upper bound on cost gains G_{MAX} as the difference between the initial solution value and the lower bound (LB) on the solution cost: $G_{MAX} = S_{INIT} - LB$.

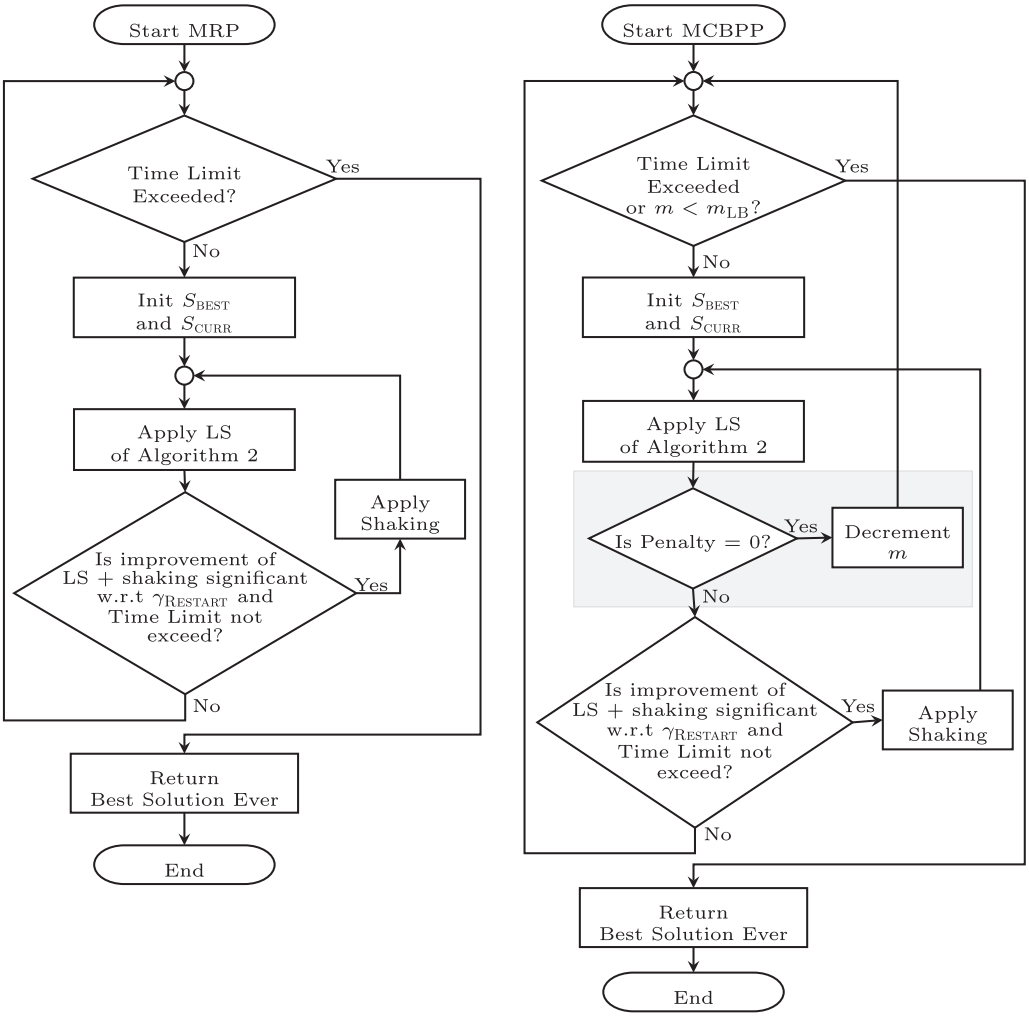


Fig. 2. MS-ILS-PP applied to MRP and MCBPP.

Finally define the *score* $C(X)$ of a method X, in Eq. (19), as its relative percentage of gap to G_{MAX} in terms of cost gain (percentage taken relatively to the initial solution cost).

$$C(X) = 100 \times \frac{G_{MAX} - G(X)}{S_{INIT}} = 100 \times \frac{S_{FINAL}(X) - LB}{S_{INIT}} \quad (19)$$

A score of 1 means that it is not possible, in the better case, to achieve an additional system improvement by more than 1% of the initial cost. This performance indicator matches most industrial application realities.

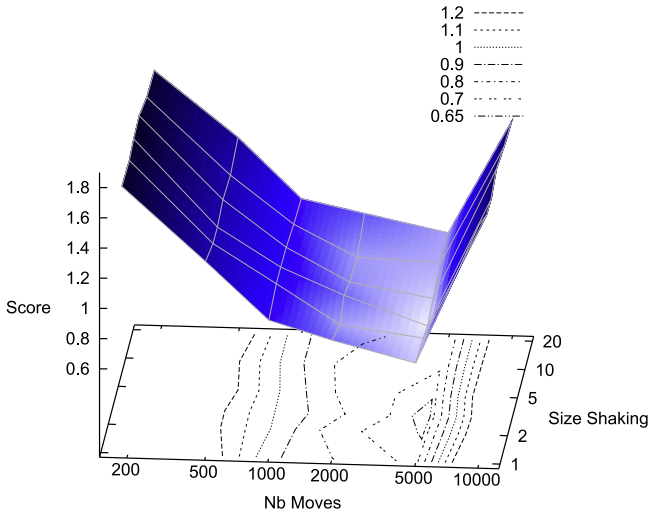
5.3. Parameters calibration

The MS-ILS-PP algorithm relies on four main parameters: the thresholds $\gamma_{RESTART}$ and $\gamma_{SHAKING}$ for triggering a restart or a shaking phase. The size of the shaking operator ξ and the pruning criteria φ.

The first two parameters were independently calibrated in order to trigger the restart or the shaking when visibly no significant solution improvement occurs. Our experiments revealed that the best values for these parameters were $\gamma_{RESTART} = 0.5\%$ and $\gamma_{SHAKING} = 0.05\%$. These parameters have an influence on the run time by avoiding unnecessary search phases, but not a large impact on solution quality.

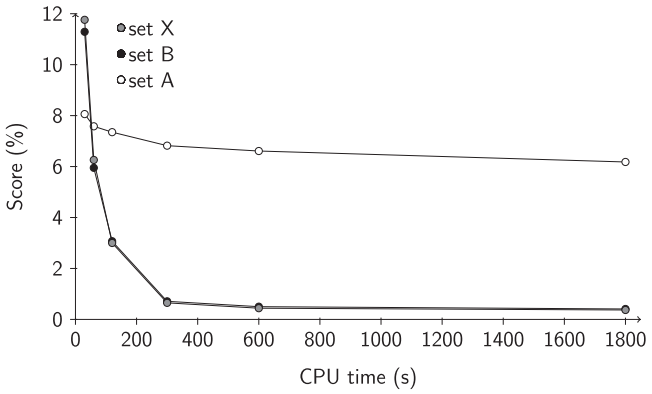
In contrast, the solution quality is very sensitive to the two last parameters, ξ and φ. We thus present the calibration results on 4 training instances: A2-2, B-1, B-5, and B-10 with different parameters values, selected in a range that appeared to be relevant during prior analyses: $\xi \in \{1,2,3,5,10,20\}$ and $\varphi \in \{200,500,1000,2000,5000,10{,}000\}$. The instances of Set X were not included in the calibration, since experiments on this set aims to reproduce a real setting where no prior instance-specific calibration has been conducted. Average results on 30 runs are considered. Also, to avoid giving higher importance to instances for which scores are larger (such as A2-2 for example), we applied an affine scaling of the score values in such a way that for each instance the best scaled score becomes 0 and the worst 1. Fig. 3 reports the sum of the four scaled scores for each parameter setting.

It can be observed that the good parameter values are located around $\xi = 3$ and $\varphi = 5000$. The method seems very sensitive to the value of the pruning parameter φ. More precisely, high values for this parameter means evaluating more moves on a machine before applying the best, leading to more elitist but time-consuming searches. This kind of elitism appeared to be critical on some instances, especially B-1, where good decisions in the beginning of the search have a high impact on the overall performance. However, if the elitism is too high, e.g. with $\varphi = 10{,}000$, the method does not have enough time to converge to a local optimum of the LS, and thus the solution quality rapidly decreases.



$\varphi \setminus \xi$	1	2	**3**	5	10	20
200	1.780	1.763	**1.796**	1.811	1.745	1.732
500	1.300	1.230	**1.229**	1.272	1.245	1.307
1000	0.923	0.977	**0.976**	0.969	0.933	0.914
2000	0.803	0.720	**0.803**	0.752	0.727	0.828
5000	**0.669**	**0.645**	**0.616**	**0.658**	**0.707**	**0.716**
10000	1.594	1.470	**1.462**	1.528	1.484	1.463

Fig. 3. Average scaled score on four training instances with different parameter settings.



CPU Time	30 s	60 s	2 min	5 min	10 min	30 min
Score Set A	8.06%	7.58%	7.35%	6.82%	6.61%	6.18%
Score Set B	11.29%	5.95%	3.07%	0.71%	0.50%	0.41%
Score Set X	11.76%	6.26%	3.00%	0.65%	0.44%	0.37%

Fig. 4. Solution quality versus run time.

5.4. MS-ILS-PP performance on MRP benchmark instances

The MS-ILS-PP metaheuristic was run 40 times for each instance. A time limit of five minutes is imposed in order to match practical application contexts. Table 1 presents the results found on the MRP benchmark dataset. In the columns are reported, in turn, the instance name, number of machines and processes, initial solution cost, our best LB, average solution and score of MS-ILS-PP on the 40 runs, and best solution and score of MS-ILS-PP. This best LB has been computed, for the Set A, by running the CPLEX optimization solver for solving the mathematical model of Section 3.3, and for Sets B and X, using the expressions presented in Section 3.4. The last lines of the Table present the average score per group of instances.

Table 1
Results of MS-ILS-PP on MRP benchmark instances with T_{max} = 300 s.

Inst.	$\|M\|$	$\|P\|$	Init. cost	LB	MS-ILS-PP	avg (%)	MS-ILS-PP	best (%)
A1-1	4	100	49,528,750	44,306,501	44,306,501	0.00	44,306,501	0.00
A1-2	100	1000	1,061,649,570	777,531,000	787,593,254	0.95	780,499,081	0.28
A1-3	100	1000	583,662,270	583,005,918	583,006,343	0.00	583,006,015	0.00
A1-4	50	100	632,499,600	242,406,000	263,449,642	3.33	258,024,574	2.47
A1-5	12	1000	782,189,690	727,578,000	727,578,639	0.00	727,578,412	0.00
A2-1	100	1000	391,189,190	126	515,038	0.13	167	0.00
A2-2	100	1000	1,876,768,120	537,253,000	1,006,392,553	25.00	970,536,821	23.09
A2-3	100	1000	2,272,487,840	1,031,400,000	1,488,682,473	20.12	1,452,810,819	18.54
A2-4	50	1000	3,223,516,130	1,680,230,000	1,721,276,657	1.27	1,695,897,404	0.49
A2-5	50	1000	787,355,300	307,403,000	444,758,671	17.45	412,613,505	13.36
Score Set A						6.82		5.82
B1-1	100	5000	7,644,173,180	3,290,754,940	3,643,207,680	4.61	3,516,215,073	2.95
B1-2	100	5000	5,181,493,830	1,015,153,860	1,034,641,974	0.38	1,027,393,159	0.24
B1-3	100	20,000	6,336,834,660	156,631,070	165,037,128	0.13	158,027,548	0.02
B1-4	1000	10,000	9,209,576,380	4,677,767,120	4,677,962,023	0.00	4,677,940,074	0.00
B1-5	100	40,000	12,426,813,010	922,858,550	926,221,613	0.03	923,857,499	0.01
B1-6	200	40,000	12,749,861,240	9,525,841,820	9,525,923,099	0.00	9,525,913,044	0.00
B1-7	4000	40,000	37,946,901,700	14,833,996,360	15,372,961,904	1.42	15,244,960,848	1.08
B1-8	100	50,000	14,068,207,250	1,214,153,440	1,220,521,241	0.05	1,214,930,327	0.01
B1-9	1000	50,000	23,234,641,520	15,885,369,400	15,885,635,887	0.00	15,885,617,841	0.00
B1-10	5000	50,000	42,220,868,760	18,048,006,980	18,155,878,491	0.26	18,093,202,104	0.11
Score Set B						0.69		0.44
X1-1	100	5000	7,422,426,760	3,023,565,050	3,348,966,927	4.38	3,209,874,890	2.51
X1-2	100	5000	5,103,634,830	1,001,403,470	1,026,504,981	0.49	1,018,646,825	0.34
X1-3	100	20,000	6,119,933,380	0	3,151,834	0.05	1,965,401	0.03
X1-4	1000	10,000	9,207,188,610	4,721,558,880	4,721,814,589	0.00	4,721,786,173	0.00
X1-5	100	40,000	12,369,526,590	0	696,174	0.01	615,277	0.00
X1-6	200	40,000	12,753,566,360	9,546,930,520	9,547,005,000	0.00	9,546,992,887	0.00
X1-7	4000	40,000	37,763,791,230	14,251,967,330	14,893,088,510	1.70	14,701,830,252	1.19
X1-8	100	50,000	11,611,565,600	0	369,803	0.00	309,080	0.00
X1-9	1000	50,000	23,146,106,380	16,125,494,300	16,125,775,950	0.00	16,125,753,242	0.00
X1-10	5000	50,000	42,201,640,770	17,815,790,830	17,928,266,694	0.27	17,867,789,754	0.12
Score Set X						0.69		0.42

Table 2
Performance of MS-ILS on the MCBPP instances.

Size	Class	BKLB	BKUB	ILS avg	ILS best	nb BKUB	Avg T (s)
25	1	69	69	69.0	69	10/10	12.7
25	6	101	101	101.0	101	10/10	21.3
25	7	96	96	96.0	96	10/10	18.6
25	9	73	73	73.0	73	10/10	20.3
24	10	80	80	80.0	80	10/10	11.1
50	1	135	135	135.0	135	10/10	72.3
50	6	214	215	215.0	215	10/10	68.6
50	7	196	197	197.0	197	10/10	88.0
50	9	144	145	145.0	145	10/10	199.2
51	10	170	170	170.0	170	10/10	68.9
100	1	255	260	257.0	257	10/10	294.5
100	6	405	410	410.0	410	10/10	300.0
100	7	398	405	402.1	402	10/10	285.9
100	9	257	267	267.0	267	10/10	300.0
99	10	330	330	330.0	330	10/10	232.4
200	1	503	510	503.0	503	10/10	300.0
200	6	803	811	812.4	811	10/10	300.0
200	7	799	802	802.0	802	10/10	300.0
200	9	503	513	513.0	513	10/10	300.0
201	10	670	670	679.8	678	2/10	300.0

In general, the quality of the solutions produced by the method is very high, and for 21/30 instances the score is smaller than 1%, meaning that more than 99% of the possible solution improvements have been achieved. On Set A, some results with larger gaps are reported. We believe that for these instances, the LB that we use may be of insufficient quality, thus complicating the task to assess the method performance on these few instances. The method seems to produce high-quality solutions in a consistent manner, with only a small gap between the average results and the best results. Also, very small scores are achieved on the largest instances with up to 50,000 processes. As such, the proposed method provides a valuable tool to optimize large scale multiprocessors systems.

To further analyze the performance of the method as the allowed CPU time grows, we conducted a set of 10 runs with different time limits, ranging from 30 s to 30 min. The average scores on each problem set are reported in Fig. 4. From these results, it appears that the method is particularly well calibrated for achieving high-quality results in five minutes, with a score of 0.71% and 0.65% on the large-scale Sets B and X, respectively. On the Set A again, the lack of good lower bound tends to complicate the

Table 3
Detailed best solutions for the MCBPP instances.

#	Size	Class 1			Class 6			Class 7			Class 9			Class 10		
		LB	UB	ILS	LB	UB	ILS	LB	UB	ILS	LB	UB	ILS	LB	UB	ILS
1	S1	6	6	6	10	10	10	9	9	9	7	7	7	8	8	8
2		7	7	7	10	10	10	9	9	9	7	7	7	8	8	8
3		7	7	7	10	10	10	10	10	10	7	7	7	8	8	8
4		7	7	7	10	10	10	10	10	10	7	7	7	8	8	8
5		7	7	7	10	10	10	10	10	10	7	7	7	8	8	8
6		7	7	7	10	10	10	10	10	10	7	7	7	8	8	8
7		7	7	7	10	10	10	10	10	10	7	7	7	8	8	8
8		7	7	7	10	10	10	9	9	9	8	8	8	8	8	8
9		7	7	7	10	10	10	10	10	10	8	8	8	8	8	8
10		7	7	7	11	11	11	9	9	9	8	8	8	8	8	8
1	S2	13	13	13	21	21	21	21	21	21	14	14	14	17	17	17
2		13	13	13	21	21	21	18	18	18	14	14	14	17	17	17
3		13	13	13	21	21	21	21	21	21	14	14	14	17	17	17
4		13	13	13	21	21	21	18	18	18	14	14	14	17	17	17
5		13	13	13	21	21	21	21	21	21	14	14	14	17	17	17
6		14	14	14	22	22	22	18	18	18	14	15	15	17	17	17
7		14	14	14	21	22	22	21	22	22	15	15	15	17	17	17
8		14	14	14	22	22	22	18	18	18	15	15	15	17	17	17
9		14	14	14	22	22	22	22	22	22	15	15	15	17	17	17
10		14	14	14	22	22	22	18	18	18	15	15	15	17	17	17
1	S3	25	26	**25**	40	41	41	40	41	41	25	26	26	33	33	33
2		26	26	26	41	41	41	39	40	**39**	26	27	27	33	33	33
3		26	26	26	41	41	41	41	41	41	26	27	27	33	33	33
4		25	26	**25**	40	41	41	39	40	**39**	25	26	26	33	33	33
5		25	26	**25**	40	41	41	40	41	41	25	26	26	33	33	33
6		25	26	26	40	41	41	39	40	**39**	26	27	27	33	33	33
7		25	26	26	40	41	41	40	41	41	26	27	27	33	33	33
8		26	26	26	41	41	41	39	40	40	26	27	27	33	33	33
9		26	26	26	41	41	41	41	41	41	26	27	27	33	33	33
10		26	26	26	41	41	41	40	40	40	26	27	27	33	33	33
1	S4	50	51	**50**	80	81	81	80	80	80	50	51	51	67	67	68
2		50	51	**50**	80	81	81	80	80	80	50	51	51	67	67	68
3		50	51	**50**	80	81	81	80	80	80	50	51	51	67	67	68
4		50	51	**50**	80	81	81	80	80	80	50	51	51	67	67	68
5		50	51	**50**	80	81	81	80	80	80	50	51	51	67	67	68
6		50	51	**50**	80	81	81	79	80	80	50	51	51	67	67	68
7		50	51	**50**	80	81	81	80	81	81	50	51	51	67	67	67
8		51	51	51	81	81	81	79	80	80	51	52	52	67	67	68
9		51	51	51	81	81	81	81	81	81	51	52	52	67	67	68
10		51	51	51	81	82	82	80	80	80	51	52	52	67	67	67

For the instances of Class 1 to 9, (S1,S2,S3,S4) = (25,50,100,200).
For the instances of Class 10, (S1,S2,S3,S4) = (24,51,99,201).

Table 4
Impact of the multi-start on MRP instances.

	Average		Best	
	MS-ILS-PP ()	ILS-PP (%)	MS-ILS-PP (%)	ILS-PP (%)
Set A	6.82	7.05	5.82	6.32
Set B	0.69	1.03	0.44	0.79
Set X	0.69	1.00	0.42	0.78

Table 5
Impact of the multi-start on MCBPP instances.

	Average		Best	
	MS-ILS-PP	ILS-PP	MS-ILS-PP	ILS-PP
Class 1	24.10	24.10	24.10	24.10
Class 6	38.46	38.46	38.43	38.43
Class 7	37.43	37.43	37.43	37.43
Class 9	24.95	24.95	24.95	24.95
Class 10	31.50	31.49	31.45	31.38

analysis. Longer run times may lead to additional solution increases, but most of the possible improvement (more than 99% for Sets B and X) is already achieved after five minutes.

5.5. MS-ILS-PP performance on MCBPP benchmark instances

The performance of the method is also assessed on the classical MCBPP benchmark instances of Spieksma (1994) and Caprara and Toth (2001), using the problem transformation described in Section 4.4. Since these instances are smaller, the shaking phase is triggered whenever the LS has reached a local optimum, and 5000 shaking iterations without improvement of the current best solution are allowed before triggering a restart.

Table 2 presents the results of MS-ILS on 10 runs, and compare them with the current state-of-the-art solutions – Best Known Lower Bounds (BKLBs) and Upper Bounds (BKUBs) – reported by Monaci and Toth (2006) and, in the case of Class 10, the optimal number of bins known by instance construction. These results are aggregated by problem class, i.e. for each class the cumulated number of bins of the 10 instances is reported. Each column presents, in turn, the problem size, the problem class, the best known upper and lower bound, the average and the best number of bins generated with the MS-ILS metaheuristic, the number of BKUB found by MS-ILS, and the average CPU time per instance.

These experiments demonstrate the remarkable performance of the proposed MS-ILS approach on the classic MCBPP instances. All known best known upper bounds and optimal solutions obtained by previous algorithms have been found. For some of the largest instances of Class 10, some optimal solutions–known only from instance construction but not from any past algorithm–are not found. Nevertheless, finding these solutions is very difficult due to the way the instances were generated, without any unit of free resource in the optimal assignment (see discussion in Section 5.1).

In addition, 13 new best upper bounds have been found. Ten of these solutions are for Class 1 with sizes 100 and 200, and three of these solutions are for Class 7 with size 100. These new upper bounds match the known lower bounds from Monaci and Toth (2006) and thus are now proven optimal. The best solutions of MS-ILS are reported in Table 3 in details in the column (ILS) along with the previous lower (LB) and upper bounds (UB). New optimal solutions are underlined.

5.6. Evaluation of the multi-start component

In order to evaluate the impact of the use of restarts in the methods, an experimental analysis was performed on the MRP and the MCBPP with and without the restart mechanism. The results are presented in Tables 4 and 5. Table 4 reports the average and best score for each class of MRP instances, for the multi-start ILS (MS-ILS-PP) and without restart (ILS-PP). The results of MS-ILS-PP are systematically better than those of ILS-PP. Table 5 reports the average and best (minimum) number of bins for the five classes of MCBPP benchmark instances. In this latter case the difference between both versions, in terms of solution quality, is not significant. Therefore, the restarts do not enhance the exploration capabilities of the method, at least for the considered MCBPP benchmark instances, possibly because the shaking moves already contribute to efficiently diversify the search while maintaining a low objective value.

6. Conclusions

This paper introduced an Iterated Local Search algorithm for Machine Reassignment and Multi-Capacity Bin Packing Problems. The proposed solution approach is very simple and relies on few parameters. It produces high-quality solutions in a reasonable computational time, even for large-scale instances of both problems. For the MRP, we can prove that in most cases more than 99% of the possible improvement from the initial solution have been achieved by reorganizing the processes with our method. For the MCBPP, all the current best known upper bounds and optimal solutions available from previous algorithms on classic benchmark instances have been retrieved, and 13 new best upper bounds have been produced. These new solutions match the lower bounds from the literature, and thus are optimal.

The MRP is a relatively new problem which opens many promising avenues of research. Future works involve developing new procedures for better managing the dependency relationships, better compound neighborhoods, exact components within the resolution, i.e. the so-called math-heuristics, and the use of lower bound information on sub-problems to guide the search. Further evaluation of the effectiveness of our optimization solution can be achieved by conducting cluster management simulations using traces of workloads provided by Google as input data (see Hellerstein, 2010). We may also consider implementing and evaluating our optimization solution in a real cluster management infrastructure, leveraging the work described by Petrucci et al. (2011).

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Table 6
Summary of the abbreviations.

Abbreviation	Meaning
BKLB	Best Known Lower Bounds
BKUB	Best Known Upper Bounds
BPP	Bin Packing Problem
CPU	Computing
DISK	Storage
FFD	First-Fit Decreasing
ILS	Iterated Local Search
LB	Lower Bound
LS	Local-Search
MCBPP	Multi-Capacity Bin Packing Problem
MRP	Machine Reassignment Problem
MS-ILS-PP	Multi-Start Iterated Local Search for Packing Problems
RAM	Memory
UB	Upper Bound

Table 7
Notations.

Notation	Meaning
B	Set of cost components of a balance cost
$C_{BAL}(b)$	Balance cost of component $b \in B$
$C_{m,r}$	Capacity of machine $m \in M$ for resource $r \in R$
C_r	Bin capacity for resource $r \in R$
$C(X)$	Score of method X
D^s	Set of services on which $s \in S$ is dependent
Φ_b	Cost of exceeding the balance between r_1^b and r_2^b by one unit
Φ_{m_1,m_2}	Cost of moving a process from a machine m_1 to a machine m_2
Φ_p	Cost of moving process $p \in P$
Φ_r	Cost of exceeding the safety load of $r \in R$ by one unit
Φ_s	Cost per process moved in the service in which the maximum number of processes is moved
φ	Pruning criteria
$FR(m,r)$	Amount of free ressource $r \in R$ on machine $m \in M$ w.r.t capacity $C_{m,r}$
$\gamma'_{RESTART}$	Threshold for triggering a restart
$\gamma'_{SHAKING}$	Threshold for triggering a shaking phase (MRP)
G_{MAX}	Upper bound on cost gains
$G(X)$	Cost gain of a method X
L	Set of locations
$\bar{M}$	Subset of machine selected to be eventually modified during the current iteration of the LS
M	Set of heterogeneous machines
$M_0(p)$	Machine to which $p \in P$ is assigned in the initial solution
N	Set of neighborhoods
P	Set of objects/processes
P_m^{sol}	Set of processes assigned to $m \in M$ in solution sol
R	Set of resources
$R_{p,r}$	Requirement of $p \in P$ regarding resource $r \in R$
S	Set of services
$S_{FINAL}(X)$	Final solution found by method X
S_{init}	Feasible initial solution
$SC_{m,r}$	Safety capacity of ressource $r \in R$ on machine $m \in M$
SM_s	Minimum number of locations on which processes of s should run
T^b	Ratio between ressource r_1^b and r_2^b of component $b \in B$
TR	Set of resources considered as transient
ξ	Size of the shaking operator
$x_{m,p}$	Binary variable indicating if $p \in P$ is assigned to $m \in M$
$y_{l,s}$	Binary variable indicating if at least one process of $s \in S$ is operated in $l \in L$
$z_{n,p}$	Binary variable indicating if $p \in P$ is in $n \in N$

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Appendix A. Abbreviations and notations

Tables 6 and 7.

References

Armbrust, M., Fox, A., Griffith, R., Joseph, A., Katz, R., Konwinski, A., et al. (2010). A view of cloud computing. *Communications of the ACM, 53*(4), 50–58.

Beck, J. (1996). Modeling multicomputer task allocation as a vector packing problem. In *Proceedings of 9th international symposium on systems synthesis* (pp. 115–120).

Caprara, A., & Toth, P. (2001). Lower bounds and algorithms for the 2-dimensional vector packing problem. *Discrete Applied Mathematics, 111*(3), 231–262.

Chang, S., Hwang, H.-C., & Park, S. (2005). A two-dimensional vector packing model for the efficient use of coil cassettes. *Computers & Operations Research, 32*(8), 2051–2058.

Derbel, H., Jarboui, B., Hanafi, S., & Chabchoub, H. (2012). Genetic algorithm with iterated local search for solving a location-routing problem. *Expert Systems with Applications, 39*(3), 2865–2871.

Doddavula, S. K., Kaushik, M., & Jain, A. (2011). Implementation of a fast vector packing algorithm and its application for server consolidation. In *Proceedings of the 2011 IEEE third international conference on cloud computing technology and science, CLOUDCOM '11* (pp. 332–339). Washington, DC, USA: IEEE Computer Society.

Elbenani, B., Ferland, J., & Bellemare, J. (2012). Genetic algorithm and large neighbourhood search to solve the cell formation problem. *Expert Systems with Applications, 39*(3), 2408–2414.

Epstein, L., & Tassa, T. (2003). Vector assignment problems: a general framework. *Journal of Algorithms, 48*(2), 360–384.

Feller, E., Rilling, L., & Morin, C. (2011). Energy-aware ant colony based workload placement in clouds. In *Proceedings of the 2011 IEEE/ACM 12th international conference on grid computing, GRID '11* (pp. 26–33). Washington, DC, USA: IEEE Computer Society.

Ferreira, C., Ochi, L., Parada, V., & Uchoa, E. (2012). A GRASP-based approach to the generalized minimum spanning tree problem. *Expert Systems with Applications, 39*(3), 3526–3536.

Graham, R. (1972). Bounds on multiprocessing anomalies and related packing algorithms. In *AFIPS '72 (Spring): proceedings of the spring joint computer conference*.

Hellerstein, J. L. (2010). Google cluster data, Google research blog, posted at <http://googleresearch.blogspot.com/2010/01/google-cluster-data.html>.

Kellerer, H., & Kotov, V. (2003). An approximation algorithm with absolute worst-case performance ratio 2 for two-dimensional vector packing. *Operations Research Letters, 31*(1), 35–41.

Kou, L. T., & Markowsky, G. (1977). Multidimensional bin packing algorithms. *IBM Journal of Research and Development, 21*(5), 443–448.

Kramer, H. H., Petrucci, V., Subramanian, A., & Uchoa, E. (2012). A column generation approach for power-aware optimization of virtualized heterogeneous server clusters. *Computers & Industrial Engineering, 63*(3), 652–662.

Leinberger, W., & Karypis, G. (1999). Multi-capacity bin packing algorithms with applications to job scheduling under multiple constraints. In *ICPP '99 proceedings of the 1999 international conference on parallel processing* (pp. 404–412).

Lourenço, H., Martin, O., & Stützle, T. (2010). Iterated local search: framework and applications. In M. Gendreau & J.-Y. Potvin (Eds.), *Handbook of metaheuristics* (pp. 363–397). Boston, MA: Springer.

Monaci, M., & Toth, P. (2006). A set-covering-based heuristic approach for bin-packing problems. *INFORMS Journal on Computing, 18*(1), 71–85.

Panigrahy, R., Talwar, K., & Uyeda, L. (2011). *Heuristics for vector bin packing*. Technical Report, Microsoft.

Petrucci, V., Carrera, E. V., Loques, O., Leite, J., & Mossé, D. (2011). Optimized management of power and performance for virtualized heterogeneous server clusters. In *11th IEEE/ACM international symposium on cluster, cloud and grid (CCGrid'11)*.

Roadef (2012). ROADEF/EURO Challenge 2012: machine reassignmenthttp://challenge.roadef.org/2012/en/index.php. Accessed 08.11.12.

Shachnai, H., & Tamir, T. (2012). Approximation schemes for generalized two-dimensional vector packing with application to data placement. *Journal of Discrete Algorithms, 10*, 35–48.

Silva, M., Subramanian, A., Vidal, T., & Ochi, L. (2012). A simple and effective metaheuristic for the minimum latency problem. *European Journal of Operational Research, 221*(3), 513–520.

Spieksma, F. (1994). A branch-and-bound algorithm for the two-dimensional vector packing problem. *Computers & Operations Research, 21*(1), 19–25.

Stillwell, M., Vivien, F., & Casanova, H. (2012). Virtual machine resource allocation for service hosting on heterogeneous distributed platforms. In *Proceedings of the 2012 IEEE 26th international parallel and distributed processing symposium, IPDPS '12* (pp. 786–797). Washington, DC, USA: IEEE Computer Society.

Subramanian, A., Uchoa, E., & Ochi, L. (in press). A hybrid algorithm for a class of vehicle routing problems. *Computers & Operations Research*, ISSN 03050548. http://dx.doi.org/10.1016/j.cor.2013.01.013.

Wilcox, D., McNabb, A. W., & Seppi, K. D. (2011). Solving virtual machine packing with a reordering grouping genetic algorithm. In: *IEEE congress on evolutionary computation* (pp. 362–369).