Assignment 3
MA/FIM 548 Monte Carlo Methods for FM
Spring 2021

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Deadline: 3 pm, April 8, 2021

Instructions:

- Please submit one work per group.
- It counts for 10% of your final grade.
- Submit your work as single pdf file on Moodle before 3 pm on April 8.
- Late assignments will not be accepted.
- NO interactions between groups.
- Each group member must make a substantial contribution to each part of the assignment.
- It is not acceptable, e.g., to divide the assignments amongst the team members.
- You can use any software (Matlab, Python, C/C++, R etc.)

1. Let us consider d asset prices driven by geometric Brownian motions under the risk-neutral measure Q

$$dS_t^i = (r - \delta_i)S^i dt + \sigma_i S_t^i dW_t^i$$

where r is the interest rate, δ_i is the dividend yield of asset i, σ_i is the volatility of asset i, and W^i are SBMs under Q with cross-correlations $\rho_{i,j}$. The parameters are: $r = 0.05$, $\delta_i = 0.02$, $\sigma_i = 0.3$, $\rho_{i,j} = 0.2$.

Use Monte-Carlo method to price the following multi-asset options

(a) European basket option with payoff

$$(S_T^1 + S_T^2 - S_T^3 - K)^+$$

where $S_0^1 = S_0^2 = S_0^3 = K = 100$, $T = 1$.

(b) Bermudan basket option with payoff (using least-squares Monte-Carlo method)

$$(S_\tau^1 + S_\tau^2 - S_\tau^3 - K)^+$$

where $S_0^1 = S_0^2 = S_0^3 = K = 100$, $T = 1$. This contract can be exercised at $t_i = i/12$, $i = 1, \ldots, 12$.

Report option price and variance of estimators. Choose yourself the set of basis functions, number of simulations to get reasonable accuracy.

2. A bank has sold a call option on one stock and a put option on another stock. For the first option the stock price is 50, the strike price is 51, the volatility is 28% per annum, and the time to maturity is nine months. For the second option the stock price is 20, the strike price is 19, the volatility is 25% per annum, and the time to maturity is one year. Neither stock pays a dividend, the risk-free rate is 6% per annum, and the correlation between stock price returns is 0.4. Assume Black-Scholes model for stock prices, and hence you can use Black-Scholes formulas to obtain option prices, deltas, and gammas. Calculate a 10-day 99% VaR

(a) Using only deltas.

(b) Using the delta-gamma approximation simulation approach.

(c) Using the full simulation approach

Formulas that you can use (option prices, deltas, gammas for call and put)

$$C = S_0 N\left(\frac{\log(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}\right) - Ke^{-rT}N\left(\frac{\log(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}\right)$$

$$P = Ke^{-rT}N\left(\frac{\log(K/S_0) - (r - \sigma^2/2)T}{\sigma\sqrt{T}}\right) - S_0 N\left(\frac{\log(K/S_0) - (r + \sigma^2/2)T}{\sigma\sqrt{T}}\right)$$

$$\delta_C = N\left(\frac{\log(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}\right)$$

$$\delta_P = N\left(\frac{\log(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}\right) - 1$$

$$\Gamma_C = \Gamma_P = \frac{n\left(\frac{\log(S_0/K)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}\right)}{S_0\sigma\sqrt{T}}$$

where N is cdf and n is pdf of $N(0,1)$.

3. Assume that firm's asset value follows geometric Brownian motion

$$dV(t) = \mu V(t)dt + \sigma V(t)dW_t$$

where μ, σ are constant parameters, and W is a SBM. Now let us assume the default time is given by

$$\tau = \inf\{t > 0 : V(t) \leq B\}$$

where B is constant default boundary. Apply Monte-Carlo method to estimate the probability of default before time T

$$\mathbb{P}(\tau < T).$$

The parameters set: $V_0 = 100, \mu = 0.03, \sigma = 0.4, B = 70, T = 5$.

For this, first you can choose time step $h = T/m$ and simulate $(V(t_1), V(t_2), \ldots, V(t_m))$ with $t_{i+1} - t_i = h$. Secondly, let us simulate minimum values of M at each interval (t_i, t_{i+1}) for $i = 1, 2, \ldots, m-1$, given values of $V(t_i)$ and $V(t_{i+1})$ as follows

$$M_i = \exp\left\{\frac{1}{2}\left(\log(V(t_{i+1})V(t_i)) - \sqrt{(\log(V(t_{i+1})/V(t_i)))^2 - 2\sigma^2 h \log(U_i)}\right)\right\}$$

where $U_0, U_1, \ldots, U_{m-1}$ i.i.d $\sim U(0,1)$. If you are interested in details of this result see Chapter 6.4 in Glasserman's textbook. The value of μ is irrelevant.

Then the minimum value of V on $[0,T]$ can be estimated as follows

$$\min_{t\in[0,T]} V(t) \approx \min(M_0, M_1, \ldots, M_{m-1}).$$

Finally, recall that the two events below are equivalent

$$\{\tau < T\} \quad \text{and} \quad \left\{\min_{t\in[0,T]} V(t) \leq B\right\}.$$

Choose the number of simulations and time step h yourself but make sure you obtain a reasonable accuracy.