Assignment 2
MA/FIM 548 Monte Carlo Methods for FM
Spring 2022

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Deadline: 11:45 am, March 2, 2022

Instructions:

- Please submit one work per group.
- It counts for 10% of your final grade.
- Submit your work as single pdf file on Moodle before 11:45 am on March 2.
- Late assignments will not be accepted.
- NO interactions between groups.
- Each group member must make a substantial contribution to each part of the assignment.
- It is not acceptable, e.g., to divide the assignments amongst the team members.
- You can use any software (Matlab, Python, C/C++, R etc.)

$X_1 = Z_1$, $X_2 = \rho_{12} Z_1 + \sqrt{1-\rho_{12}^2}\, Z_2$

$X_3 = \rho_{13} Z_1 + a Z_2 + b Z_3$

1. Assume that you are able to simulate independent standard normal variables. Provide formulas that simulate three standard normal variables X_1, X_2, X_3 with correlations $\rho_{1,2}, \rho_{1,3}, \rho_{2,3}$.

2. Assume that the stock price follows Black-Scholes model under a risk-neutral measure

$$dS_t = rS_t dt + \sigma S_t dW_t$$

with $S_0 = 100, r = 0.04, \sigma = 0.3$.

Employ MC method to price the autocallable reverse convertible contract with the following no-arbitrage price

$$V(c) = \mathbb{E}\left[\sum_{i=1}^{n} e^{-rt_i} c_i \times \underline{I_{\tau \geq t_i}} + e^{-r\tau} S_0 \times \underline{I_{\tau < T}} + e^{-rT}(S_T \times \underline{I_{S_T \leq K}} + S_0 \times \underline{I_{S_T > K}}) \times \underline{I_{\tau \geq T}}\right]$$

where c is the annual coupon, $c_i = cT/m$, $0 = t_0 < t_1 < \ldots < t_m = T$ are such that $h = t_i - t_{i-1} = T/m$ for $i = 1, \ldots, m$, and τ is a random time that represents the date the contract is autocalled

突破点

$$\tau = \min\{t_i : i_0 \leq i \leq m, S_{t_i} \geq S_0\}$$

for some $1 \leq i_0 \leq m$, and $\tau = T$ if $S_{t_i} < S_0$ for $i_0 \leq i \leq m$. Assume $K = 55$, $T = 1$ year, $m = 4$, $i_0 = 2$. Your goal is to search for $c > 0$ such that

$$V(c) = S_0 = 100.$$

Choose yourself the number of simulations N but make sure that the **answer is accurate to two decimals when c is written in %.** Reverse convertible contracts are structured products that are very popular nowadays as they can help enhance yields in times of low interest rates. See the example of an actual contract below

https://structuredproducts.raiffeisen.ch/isin/CH0588781218

3. Assume that the stock price follows the local volatility model

$$\underline{dS_t} = rS_t \underline{dt} + \sigma(S_t, t) S_t \underline{dW_t}$$

under Q and the goal is to price European call option. The parameters are $S_0 = 100$, $K = 110$, $r = 0.05$, $\sigma(S,t) = 0.5e^{-t}(100/S)^{0.3}$, and $T = 1$. Apply Monte-Carlo method to price this option. Make sure that the **answer is accurate to one decimal.**

$C = (S_T - K)^+$

- Euler scheme

$$X_{t_{j+1}} = X_{t_j} + \mu(X_{t_j})\Delta t + \sigma(X_{t_j})\sqrt{\Delta t} Z_j$$

where Z_j are i.i.d. $N(0,1)$.

S_t S_0

4. Assume that the stock price follows Heston stochastic volatility model under a risk-neutral measure

$$dS_t = rS_t dt + \sqrt{v_t} S_t dW_t^s$$
$$dv_t = \kappa(\theta - v_t)dt + \sigma_v \sqrt{v_t} dW_t^v$$

1.

where (W^s, W^v) are correlated Brownian motions with correlation parameter ρ. The parameters are $S_0 = 100, v_0 = 0.04, r = 0.03, \kappa = 2, \theta = 0.04, \sigma_v = 0.5, \rho = -0.7$.

Use MC method to price lookback call option with fixed strike

$$\mathbb{E}\left[e^{-rT}\left(\max_{1\le i\le m} S_{t_i} - K\right)^+\right]$$

with $T = 0.5, K = 120$. The observation dates are $0 < t_1 < \ldots < t_m = T$ with $t_i - t_{i-1} = T/m$ for all i. Determine option prices for $m = 3, m = 6, m = 12$. Choose yourself the discretization step Δt (e.g. for Euler scheme) and the number of simulations N but make sure that the **answer is accurate to one decimal.**

5. Let us consider the variance gamma process as the stock price model with parameters $\theta = -0.1, \sigma = 0.2, \nu = 0.2$. The current stock price is $S_0 = 100$, the interest rate is $r = 0.03$ and the dividend yield is $\delta = 0$.

For this model, let us price a reset strike option of call type. It has maturity is $T_2 > 0$ and the strike price is $K > 0$. The holder of the reset strike option can reset the strike at a pre-determined time $T_1 < T_2$ and make it at-the-money if it is favorable. Hence, the payoff is

$$(S_{T_2} - \min(K, S_{T_1}))^+$$

Use MC method to price this contract with $K = 100$, $T_1 = 0.25$, $T_2 = 0.5$. Make sure that the **answer is accurate to two decimals.**

1.

$$d = 3 \ , \quad \Sigma = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{21} & 1 & \rho_{23} \\ \rho_{31} & \rho_{32} & 1 \end{bmatrix} \qquad Z_1, Z_2, Z_3 \overset{iid}{\sim} N(0,1)$$

$$\therefore \ q_{11} = \sqrt{\Sigma_{11} - 0} = 1 \ , \quad q_{22} = \sqrt{\Sigma_{22} - q_{21}^2} = \sqrt{1 - \rho_{21}^2}$$

$$q_{21} = \frac{1}{q_{11}} (\Sigma_{21} - 0) = \rho_{21}$$

$$q_{31} = \frac{1}{q_{11}} (\Sigma_{31} - 0) = \rho_{31}$$

$$q_{32} = \frac{1}{q_{22}} (\Sigma_{32} - q_{31} \cdot a_{21}) = \frac{1}{\sqrt{1-\rho_{21}^2}} \cdot (\rho_{32} - \rho_{21}\rho_{31})$$

$$q_{33} = \sqrt{\Sigma_{33} - q_{31}^2 - q_{32}^2} = \sqrt{1 - \rho_{31}^2 - \frac{\rho_{32} - \rho_{21}\rho_{31}}{1-\rho_{21}^2}}$$

$$(X_1, X_2, X_3) = (q_{11} Z_1 \ , \ q_{21} Z_1 + q_{22} Z_2 \ , \ q_{31} Z_1 + q_{32} Z_2 + q_{33} Z_3)$$

$$\therefore \ X_1 = Z_1 \ , \quad X_2 = \rho_{21} Z_1 + \sqrt{1 - \rho_{21}^2} \cdot Z_2$$

$$X_3 = \rho_{31} \cdot Z_1 + \frac{\rho_{32} - \rho_{21} \cdot \rho_{31}}{\sqrt{1-\rho_{21}^2}} \cdot Z_2 + \sqrt{1 - \rho_{31}^2 - \frac{\rho_{32} - \rho_{31}\rho_{31}}{1-\rho_{21}^2}} \cdot Z_3$$